Adaptive curvelet-domain primary-multiple separation

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Motivation

Seismic processing flows often require data-adaptive matching

- matched-filtering in primary-multiple separation
- estimation of scaling in migration-amplitude recovery

Inadequate amplitude matching leads to

- remnant multiples & deterioration primary energy

Introduce two-stage approach

- adaptive curvelet-domain matching
- Bayes separation

Combination exploits *smoothness* and *sparsity* in phase space ...

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Motivation cont'd

Kinematics are generally well predicted.

Non-adaptive curvelet-domain separation adds **robustness**.

Large errors in the location, dip and amplitude of the predicted multiples remain a problem.

Present an **adaptive** method assuming

- successful removal of the **global** "seismic wavelet"
- slowly varying amplitude errors in **phase space**
- roughly correct **kinematics**

Design a technique that exploits the invariance of curvelets under a certain class of operators.

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Multiple prediction SRME

SRME-multiple prediction

$$\mathbf{\Delta p} \mapsto \breve{\mathbf{m}}^{(1)}(s,r,t) = \big(\mathbf{\Delta P}\boldsymbol{\mathcal{A}} *_{t,x} \mathbf{\Delta p}\big)(s,r,t)$$

with

$$\begin{aligned} \mathbf{\Delta p} &= \text{vector with the primaries} \\ \breve{\mathbf{m}}^{(1)} &= \text{vector with predicted first-order multiples} \\ \mathbf{\Delta P} &= \mathbf{F}^H \text{block diag}\{\Delta \mathbf{p}\}\mathbf{F} \quad \text{(Kronecker products)} \\ \mathbf{F} &= \text{temporal Fourier transform} \\ \boldsymbol{\mathcal{A}} &= \text{inverse wavelet.} \end{aligned}$$

In practice, $\mathbf{p} \mapsto \mathbf{\Delta p}$, $\mathbf{P} \mapsto \mathbf{\Delta P}$, with $\mathbf{p}$ the total data, so

$$\breve{\mathbf{m}}^{(1)} \approx \mathbf{P}\boldsymbol{\mathcal{A}}\mathbf{p}$$

Conventional SRME

Issues:

- multiples are predicted by a multidimensional convolution
- predicted multiples contain the “source-receiver directivity & surface reflectivity” twice
- limitation in acquisition and other unknown factors lead to unknown mismatch between actual and predicted multiples

Remedies:

- windowed matched filtering
- iterative SRME

Problems:

- overfitting **=>** loss of primary energy
- incomplete data **=>** inaccurate high-order predictions

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Multiple prediction SRME

Matched filter

$$\tilde{\mathbf{a}} = \arg\min_{\mathbf{a}} \|\mathbf{p} - \mathbf{a} *_t \breve{\mathbf{m}}^{(1)}\|_2$$

yielding

$$\begin{aligned} \tilde{\Delta \mathbf{p}} &= \mathbf{p} - \tilde{\mathbf{a}} *_t \breve{\mathbf{m}}^{(1)} \\ \boldsymbol{\mathcal{A}} &= \text{block diag}\{\tilde{\mathbf{a}}\} \end{aligned}$$

for each offset.

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Problems

Assumes the filter to be ***stationary*** (diagonal in Fourier space)

Wavelet changes as a function of (s,r,t).

Windowed matched-filtering techniques have been proposed

- window sizes arbitrary
- under fit (remnant primary energy)
- over fit (removal of primary energy)
- no control over the variations of the estimated filters amongst different windows

Propose a curvelet-domain matched filtering approach.

Alternative approach

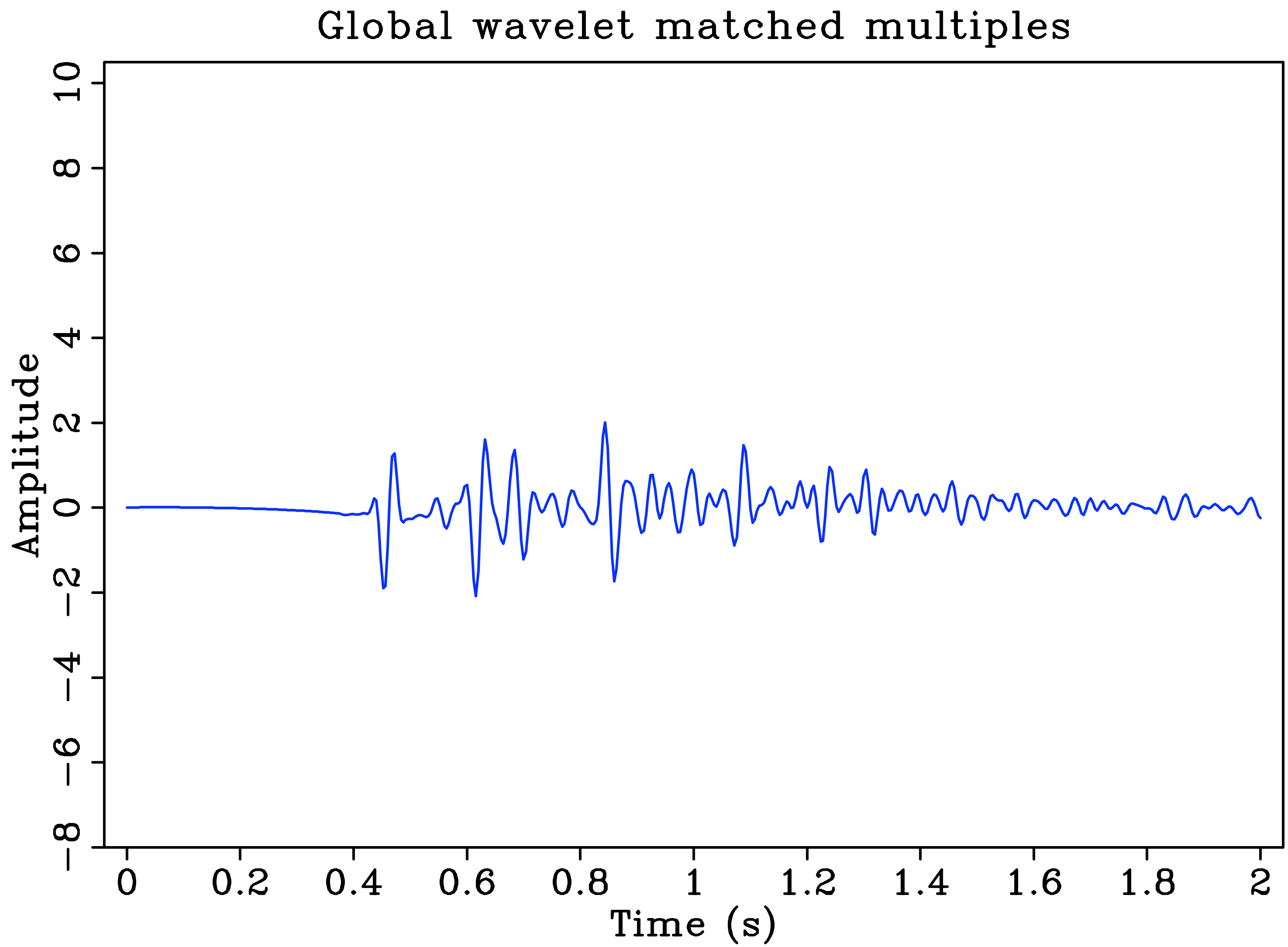
Replace

- aggressive windowed least-squares matching by global matching.
- iterative SRME by single-iteration SRME.

Different steps:

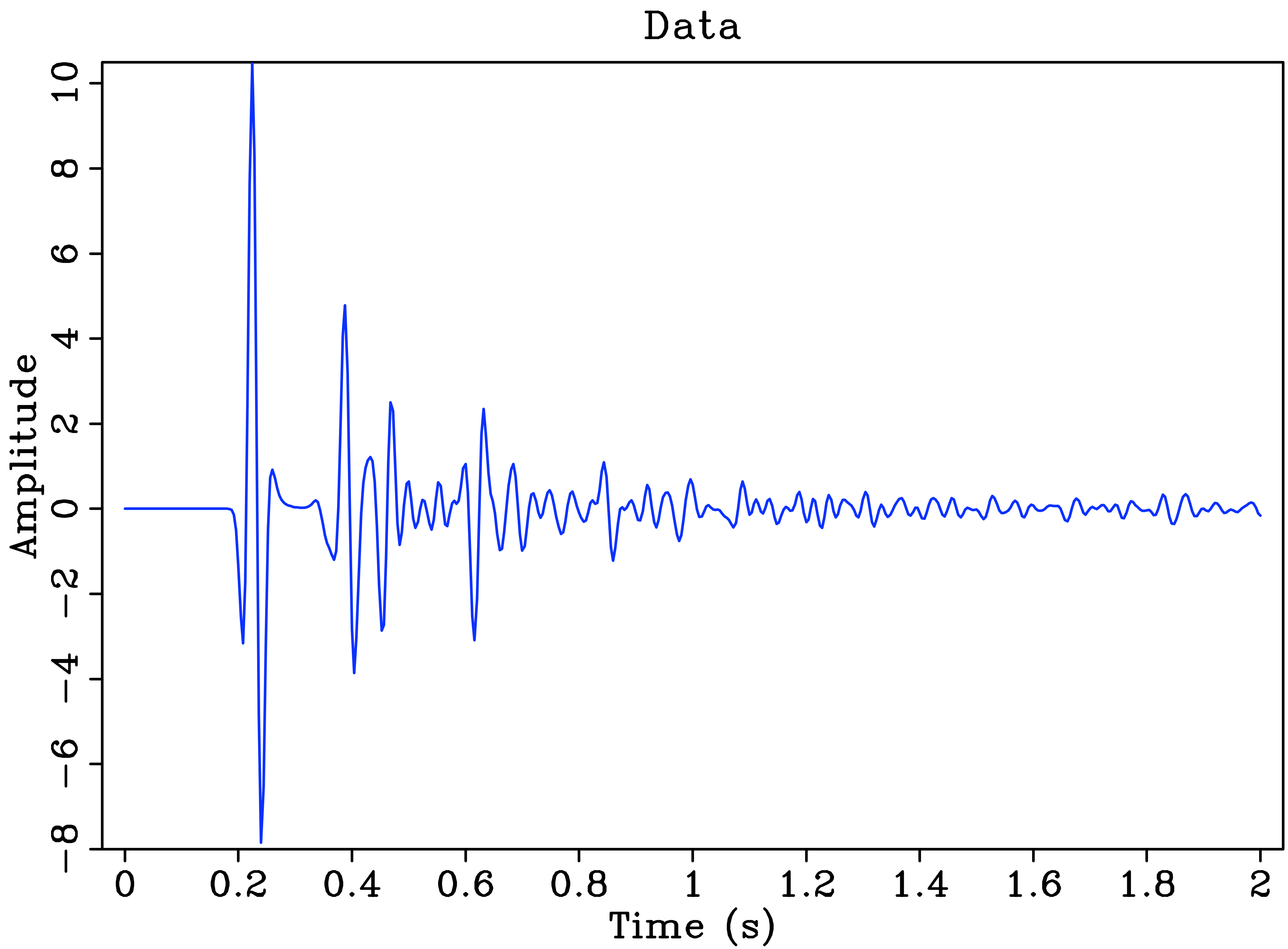
- Single-term prediction with global matching
- Amplitude correction by our adaptive curvelet-domain matched filter with *phase-space smoothness* control
- Curvelet-domain separation with sparsity promotion and separation control

Traditional SRME



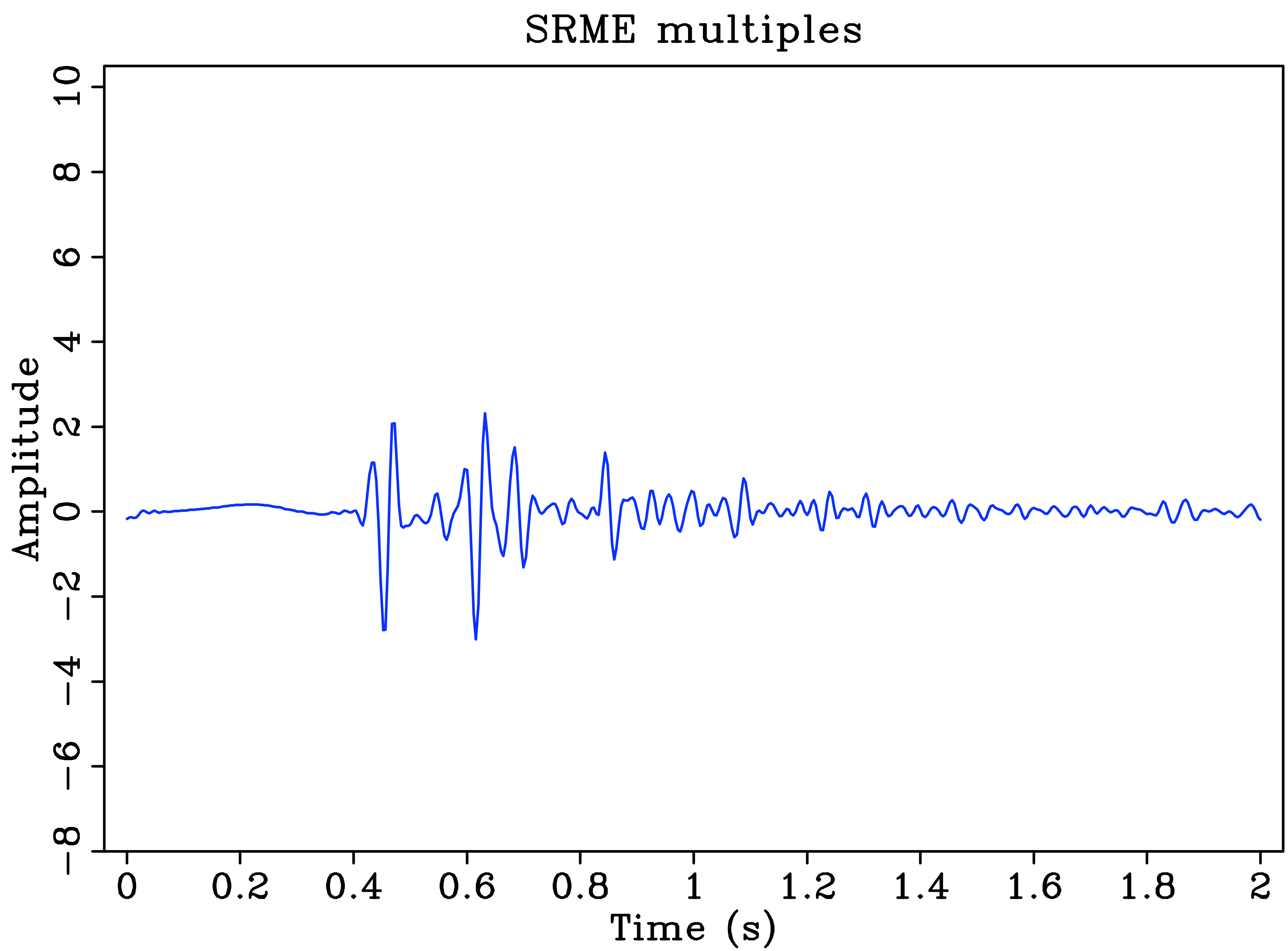
Only global wavelet matching no curvelet matching

Traditional SRME



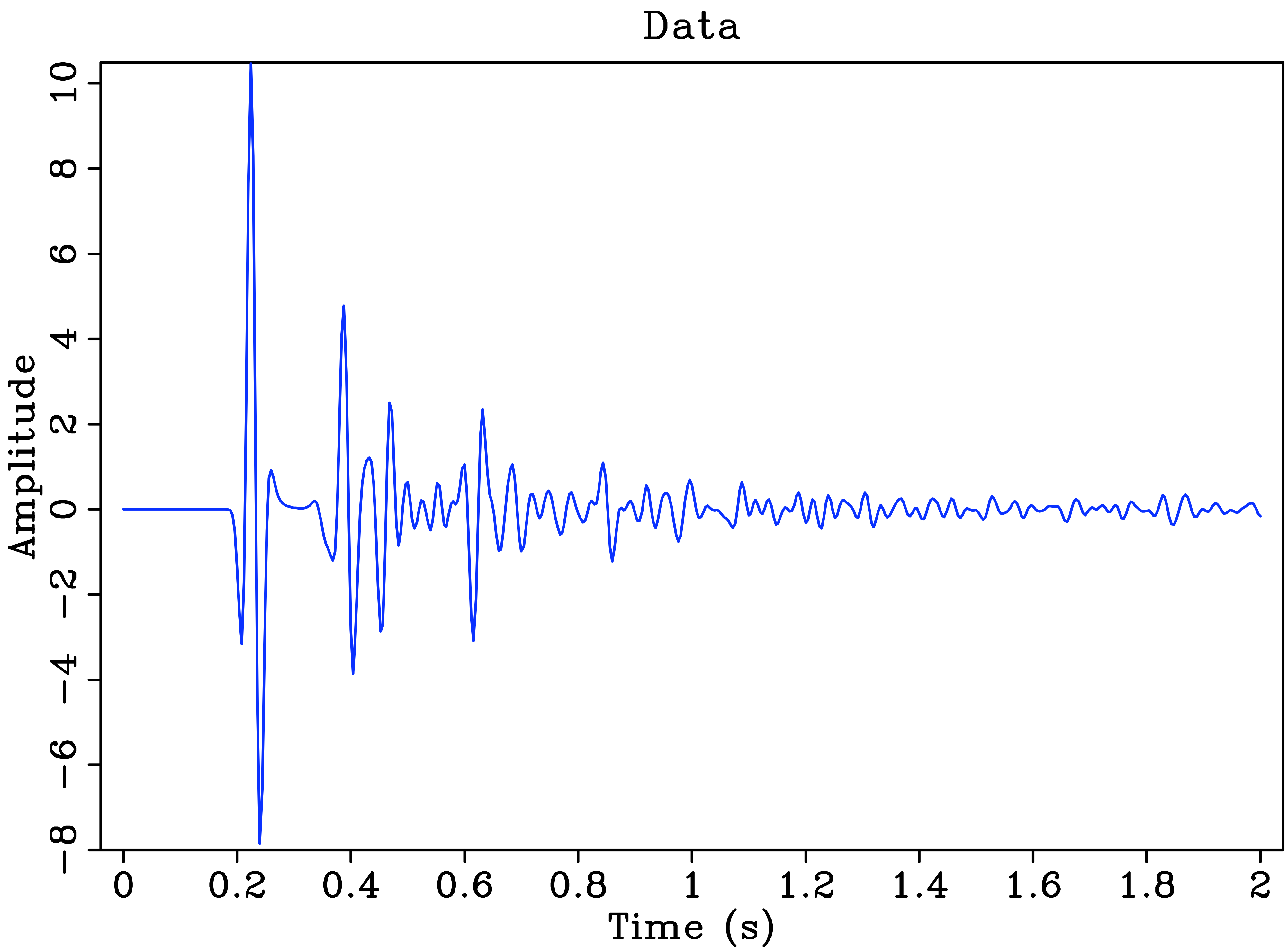
Total data

Traditional SRME



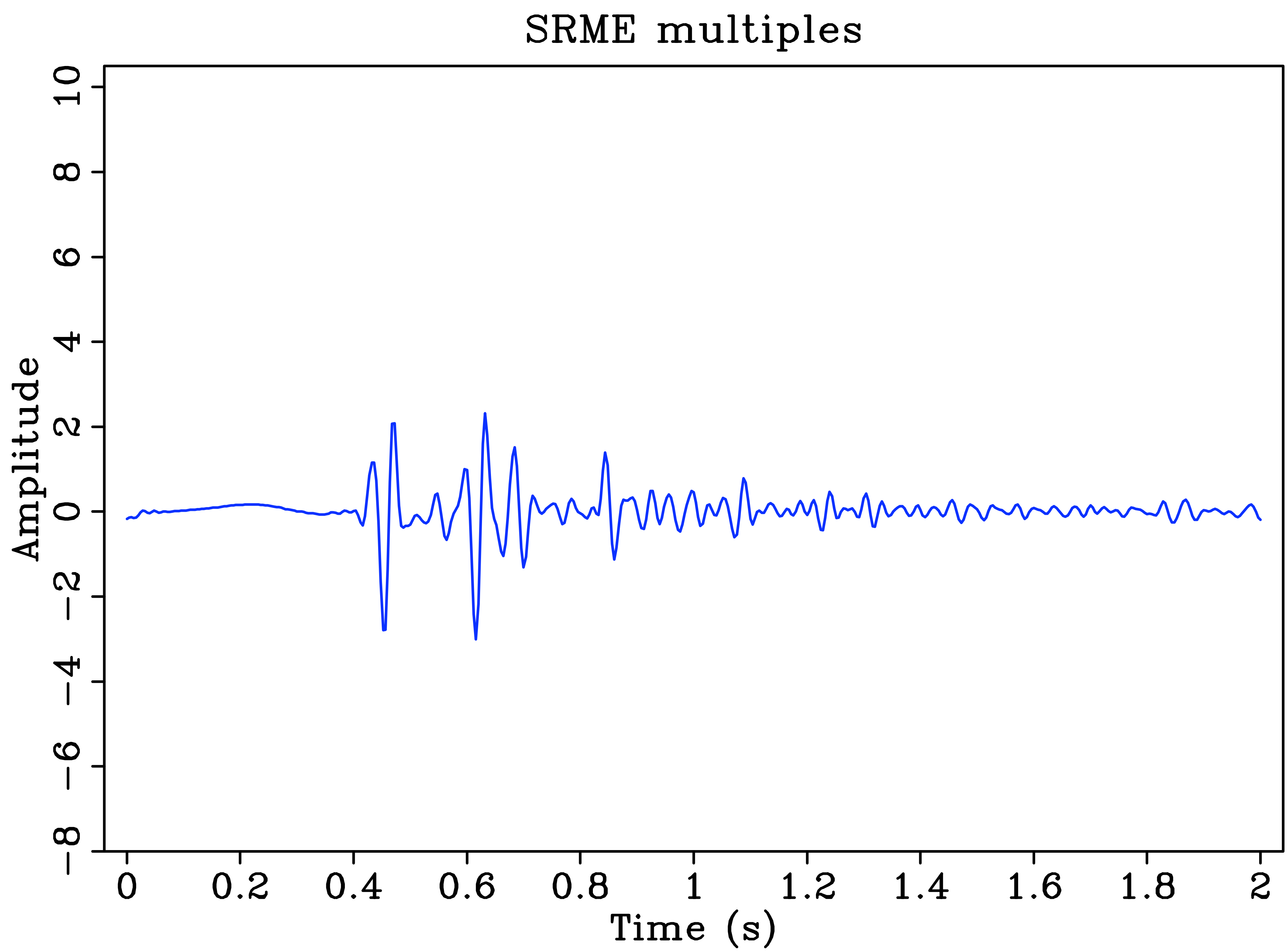
SRME windowed amplitude matched multiples

Traditional SRME



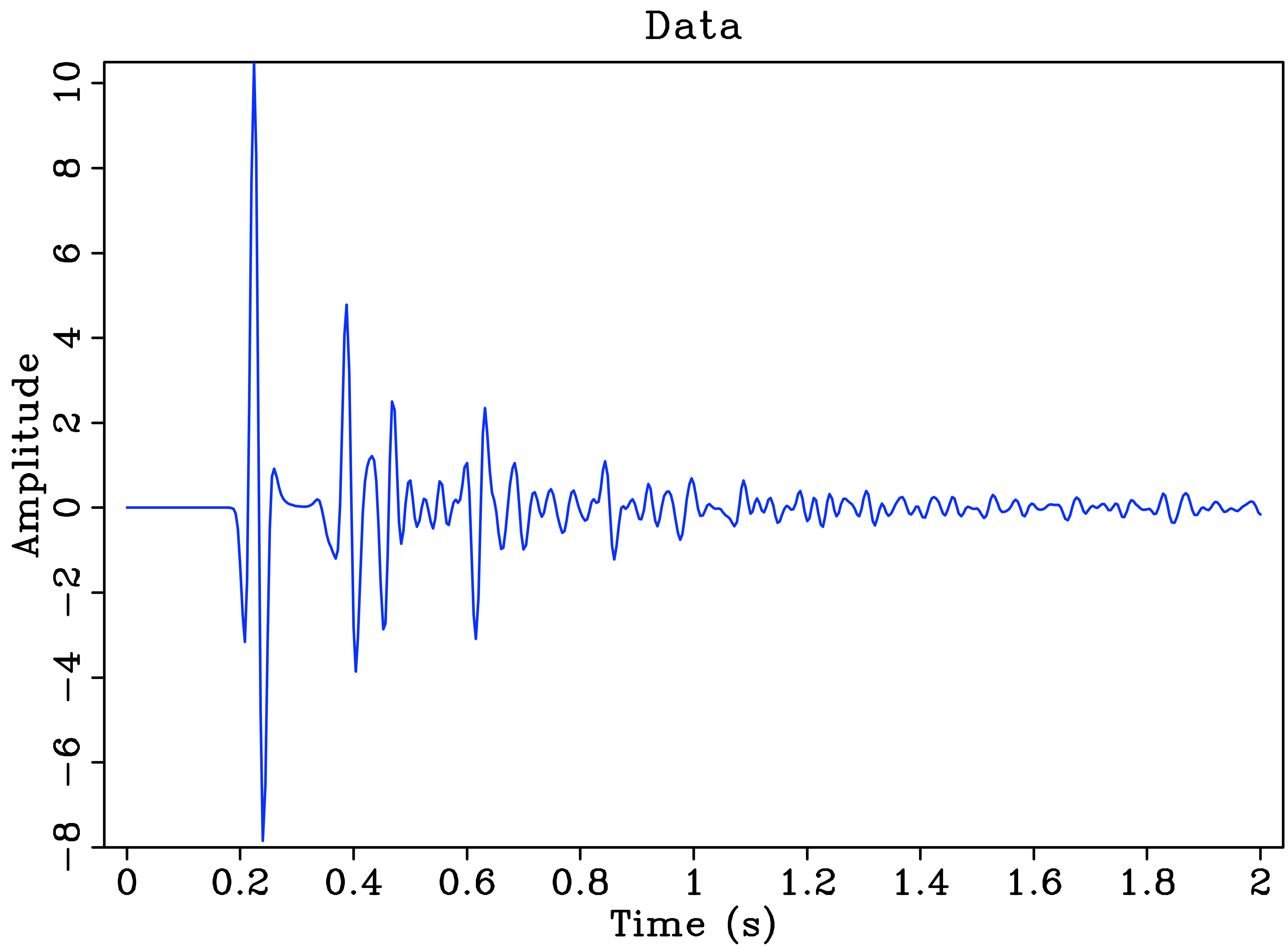
Total data

Traditional SRME



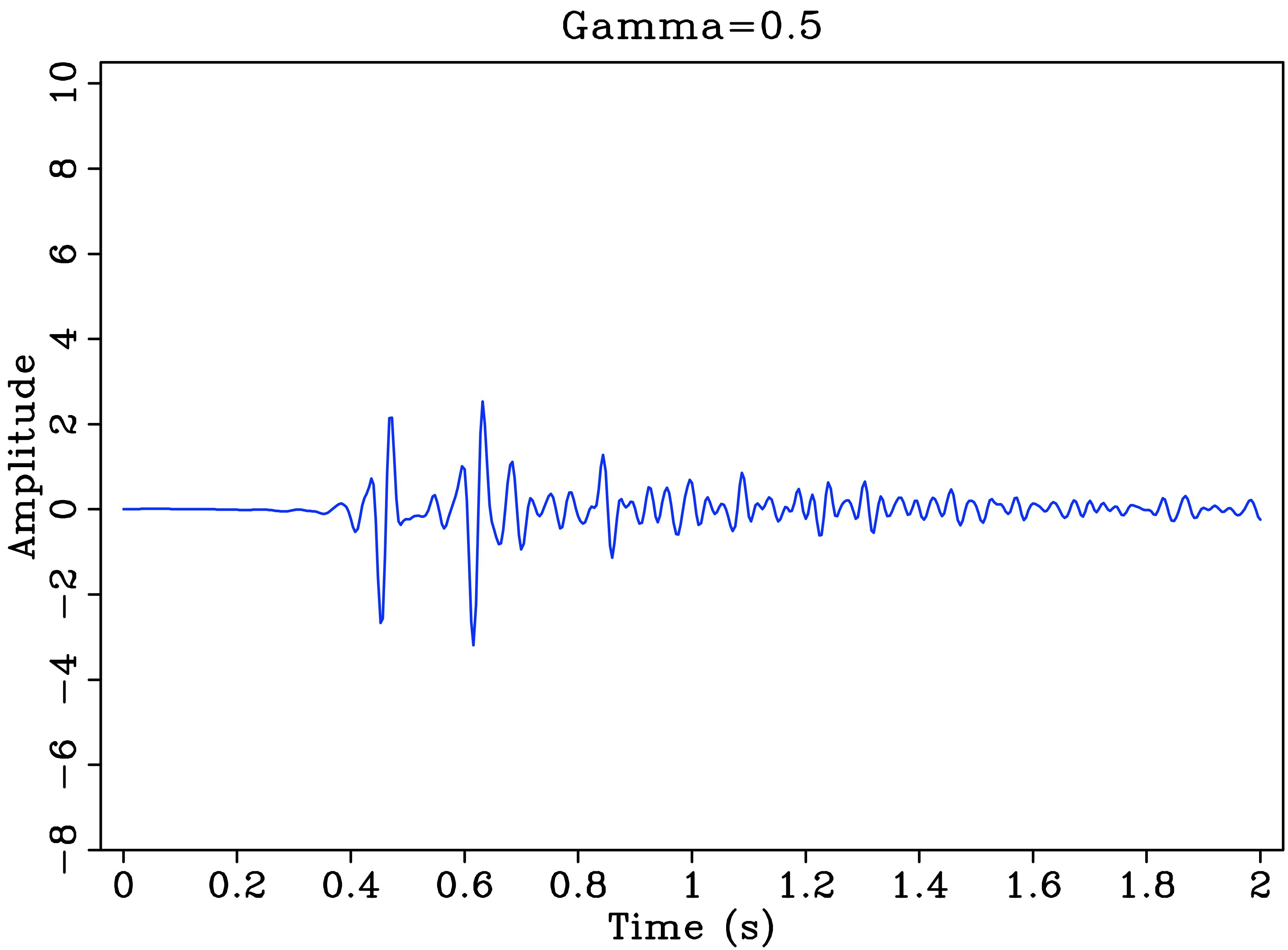
SRME windowed amplitude matched multiples

Traditional SRME



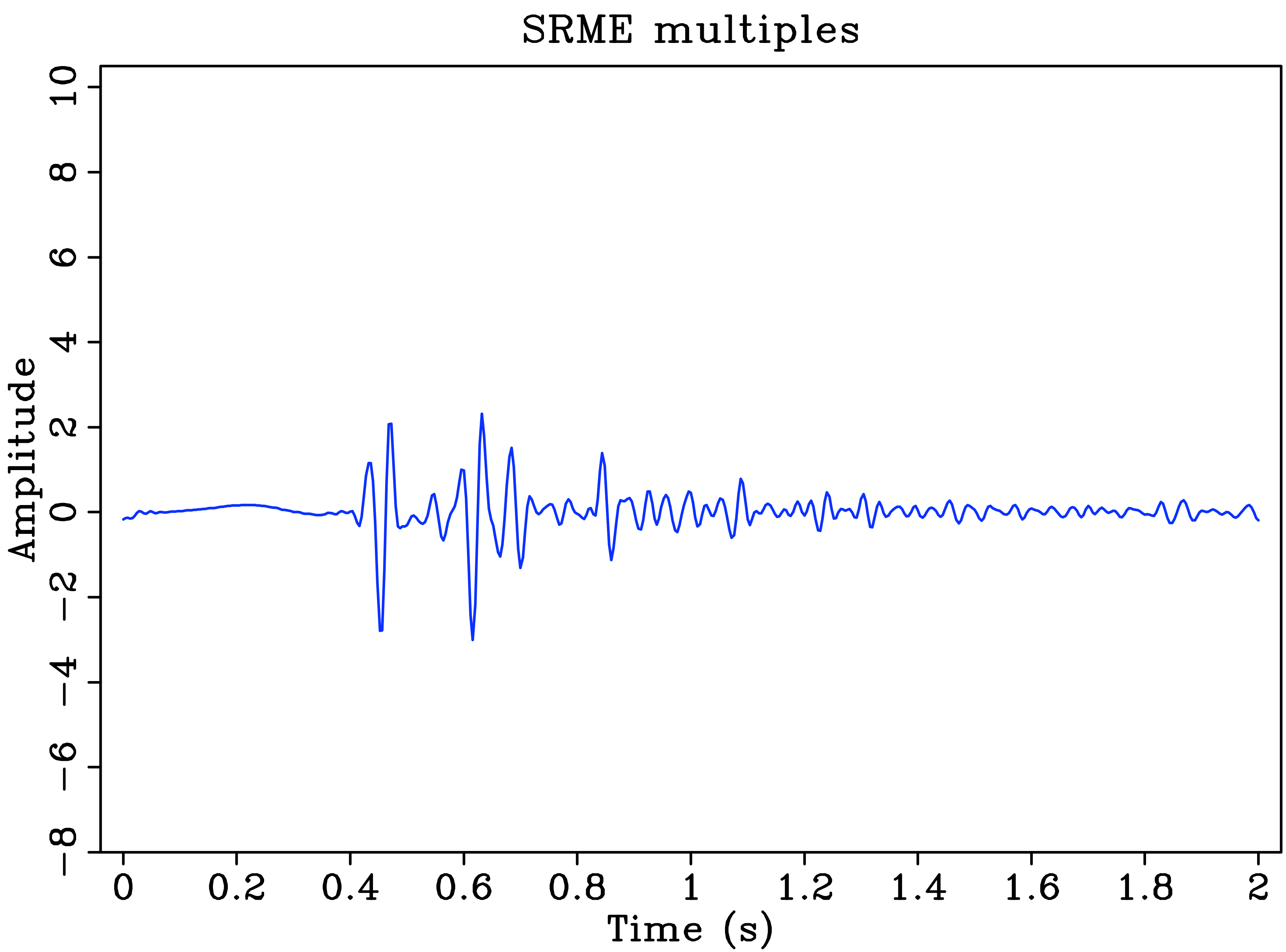
Total data

Smoothness penalty



$\gamma = 0.5$

Traditional SRME



SRME windowed amplitude matched multiples

Curvelet-domain matched filtering

"Sparsity- and continuity-promoting seismic imaging with curvelet frames" by Felix J. Herrmann, P. P. Moghaddam and C. C. Stolk to appear in the Journal of Applied and Computational Harmonic Analysis

http://dx.doi.org/10.1016/j.acha.2007.06.007

Forward model

Linear model for amplitude mismatch:

$$
\begin{aligned}
(Bf)(x) &= \int_{x\in\mathbb{R}^d} e^{jk\cdot x} b(x,k)\hat{f}(k)\mathrm{d}k \\
B &= \text{Pseudodifferential operator} \\
b(x,k) &= \text{the symbol}
\end{aligned}
$$

- spatially-varying dip filter
- zero-order Pseudo

After discretization:

$$
\mathbf{f} = \mathbf{B}\mathbf{g}
$$

- linear operator
- **f** and **g** known
- matrix **B** is full and not known ....

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Forward model

Diagonal approximation in the curvelet domain:

$$\begin{aligned}\mathbf{f} &= \mathbf{B}\mathbf{g} \\ &\approx \mathbf{C}^T \mathrm{diag}\{\mathbf{w}\}\mathbf{C}\mathbf{g}\end{aligned}$$

- curvelet domain scaling
- opens the way to an estimation of **w**

Examples:

	B	**f**	**g**
migration	$\mathbf{K}^T\mathbf{K}$	**migrated "image"**	**"reflectivity"**
multiple removal	**obliquity factor**	**total data**	**predicted multiples**

Key idea

Problems with estimating **w**

- inversion of an *underdetermined* system
- *over* fitting
- *positivity* and reasonable *scaling* by **w**

Solution:

- use *smoothness* of the symbol
- formulate *nonlinear* estimation problem that minimizes

$$J_\gamma(\mathbf{z}) = \frac{1}{2}\|\mathbf{d} - \mathbf{F}_\gamma e^{\mathbf{z}}\|_2^2,$$

with

$$\mathrm{grad} J(\mathbf{z}) = \mathrm{diag}\{e^{\mathbf{z}}\}\left[\mathbf{F}^T\left(\mathbf{F}e^{\mathbf{z}} - \mathbf{d}\right)\right]$$

Solve with l-BFGS-B

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Key idea

Impose *smoothness* via following system of equations

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\breve{\mathbf{s}}_2\} \\ \gamma \mathbf{L} \end{bmatrix} \mathbf{w}$$

where

$$\mathbf{d} = \mathbf{F}_\gamma \mathbf{w}$$

and with

$$\mathbf{L} = \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \mathbf{D}_\theta^T \end{bmatrix}^T$$

first-order differences in *space* and *angle* directions for each *scale*.

Assure positivity with nonlinear least-squares ...

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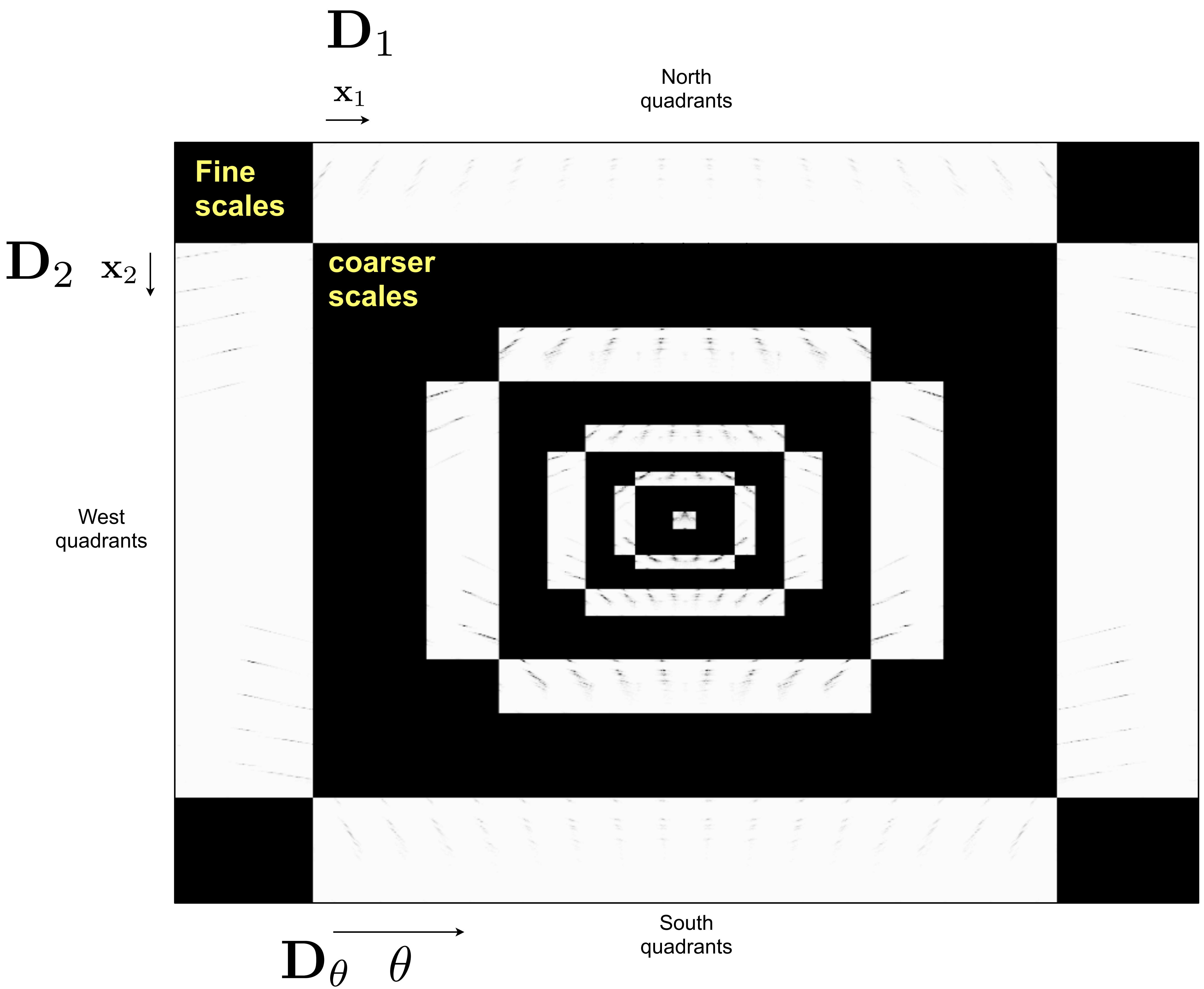
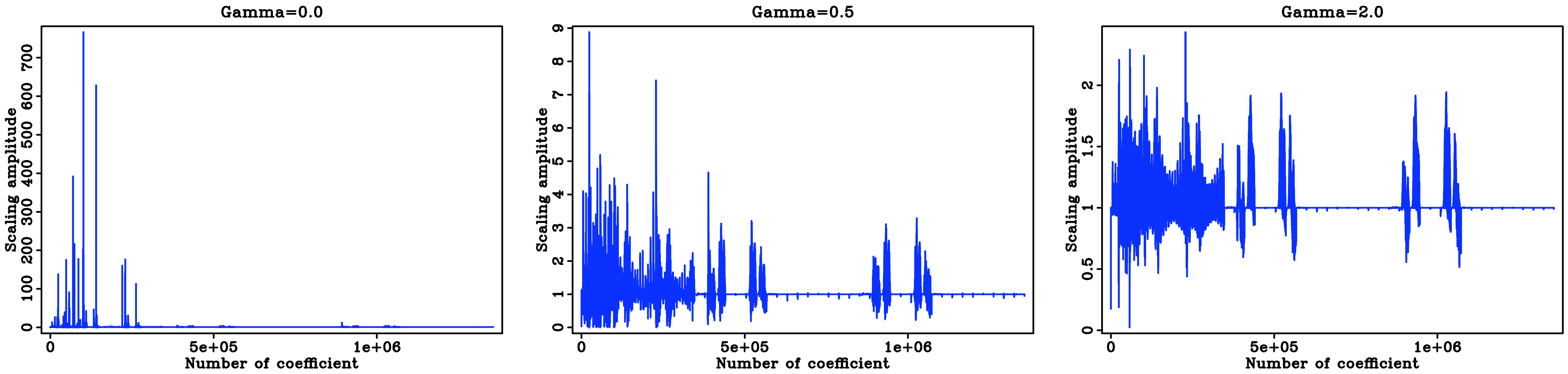
Key idea



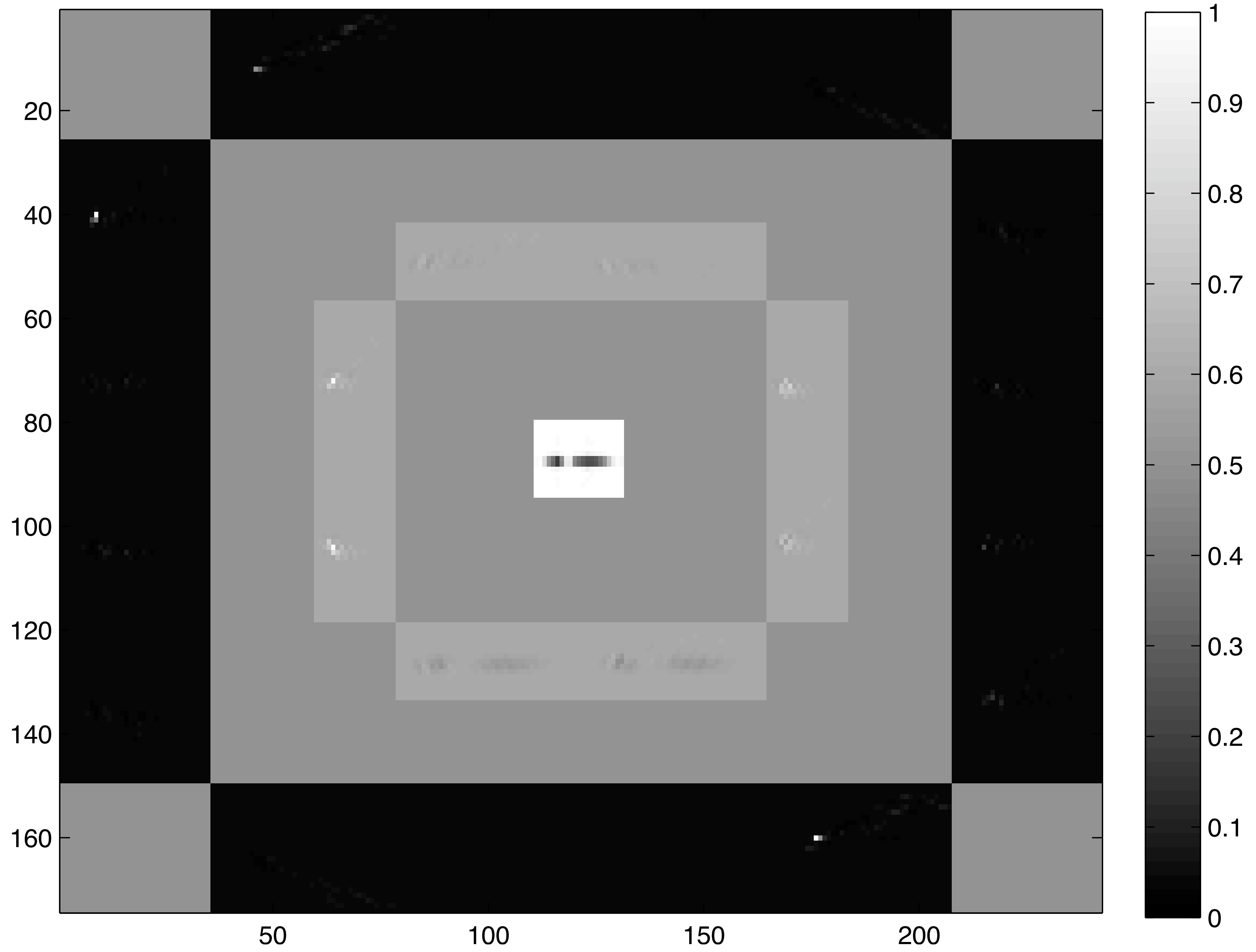
$\mathbf{D}_1$
$\mathbf{x}_1$
North
quadrants
Fine
scales
$\mathbf{D}_2$
$\mathbf{x}_2$
coarser
scales
West
quadrants
East
quadrants
South
quadrants
$\mathbf{D}_\theta$
θ

Smoothness penalty



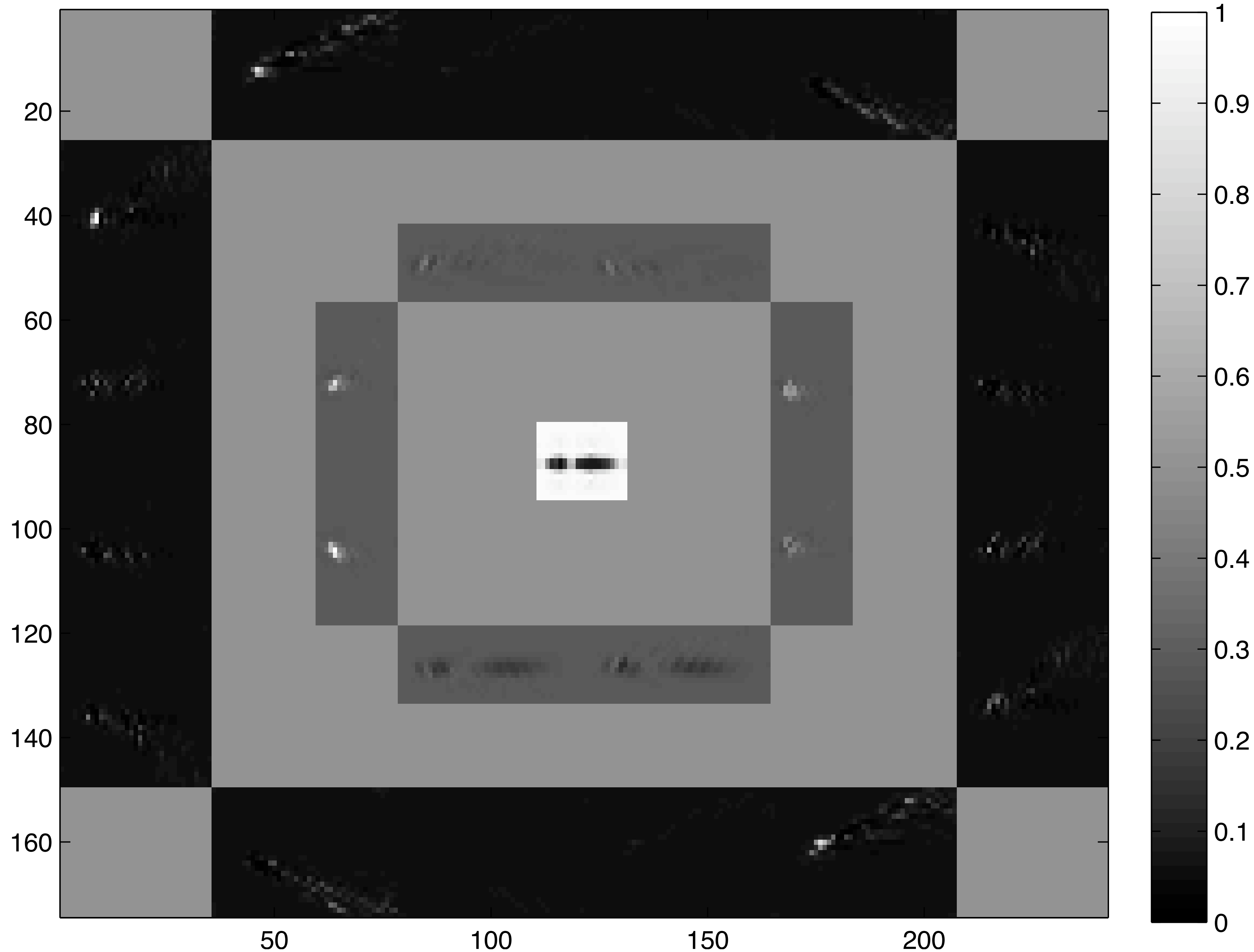
- reduces overfitting
- scaling is positive and reasonable

Smoothness penalty



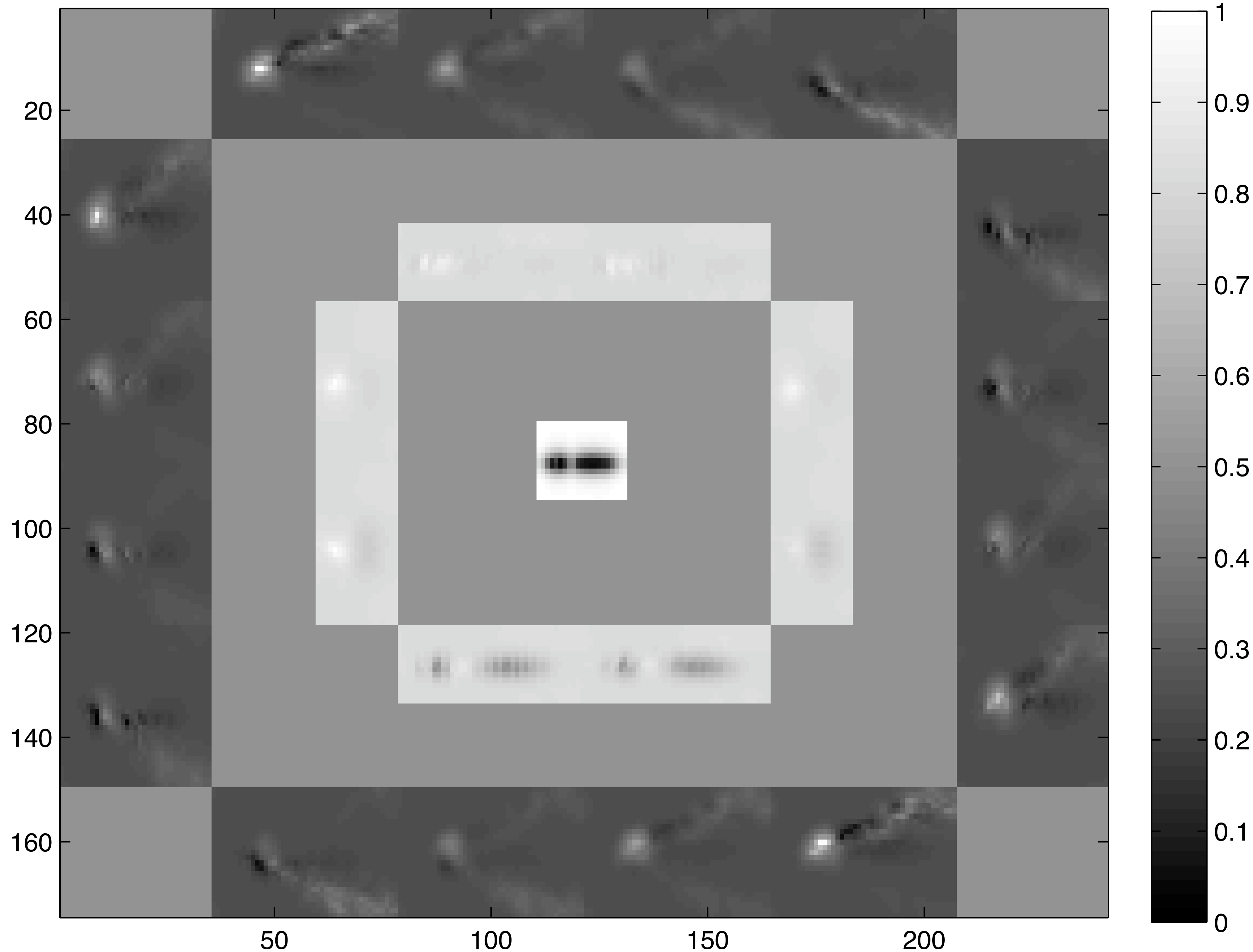
$\gamma = 0.0$

Smoothness penalty



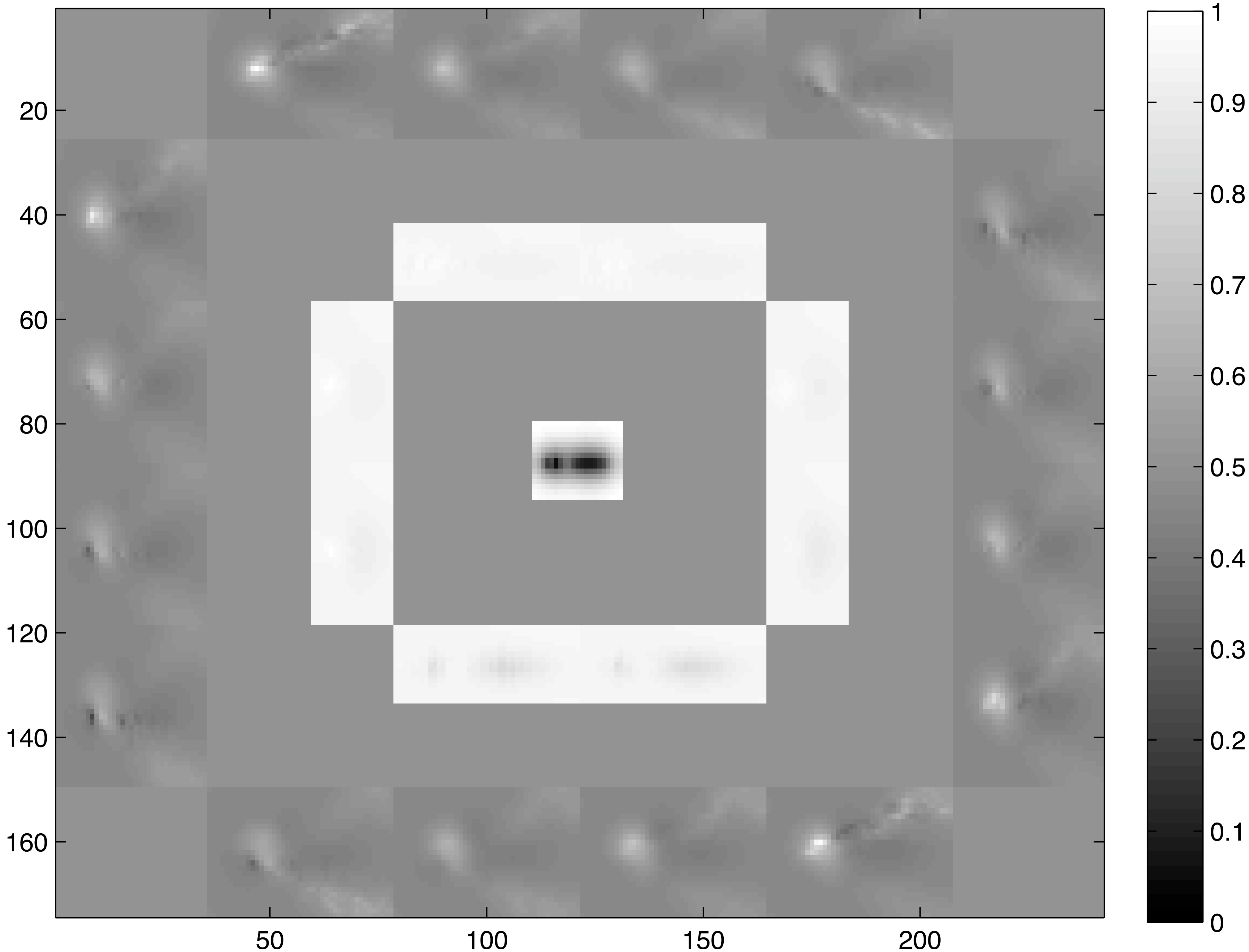
$\gamma = 0.1$

Smoothness penalty



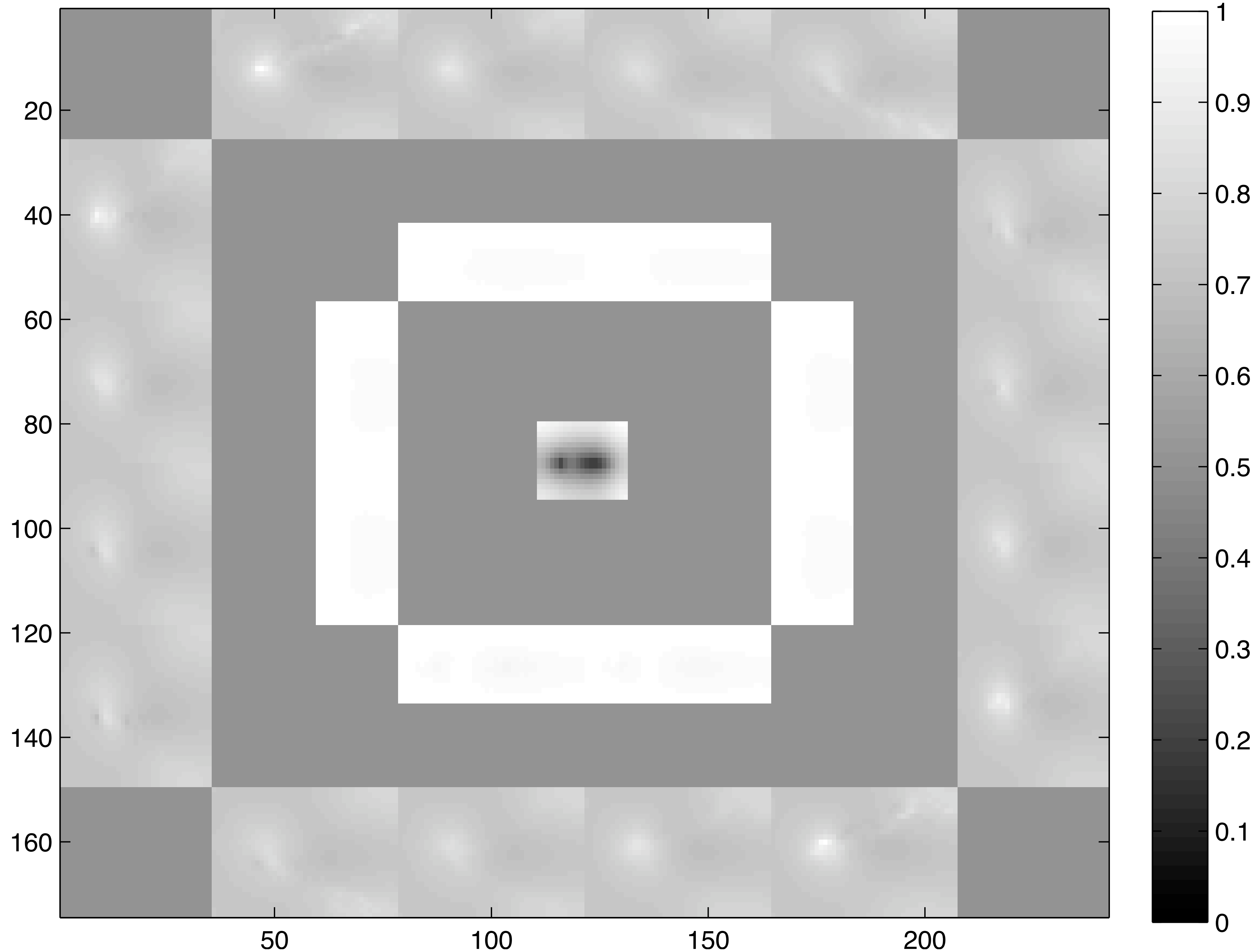
$\gamma = 0.5$

Smoothness penalty



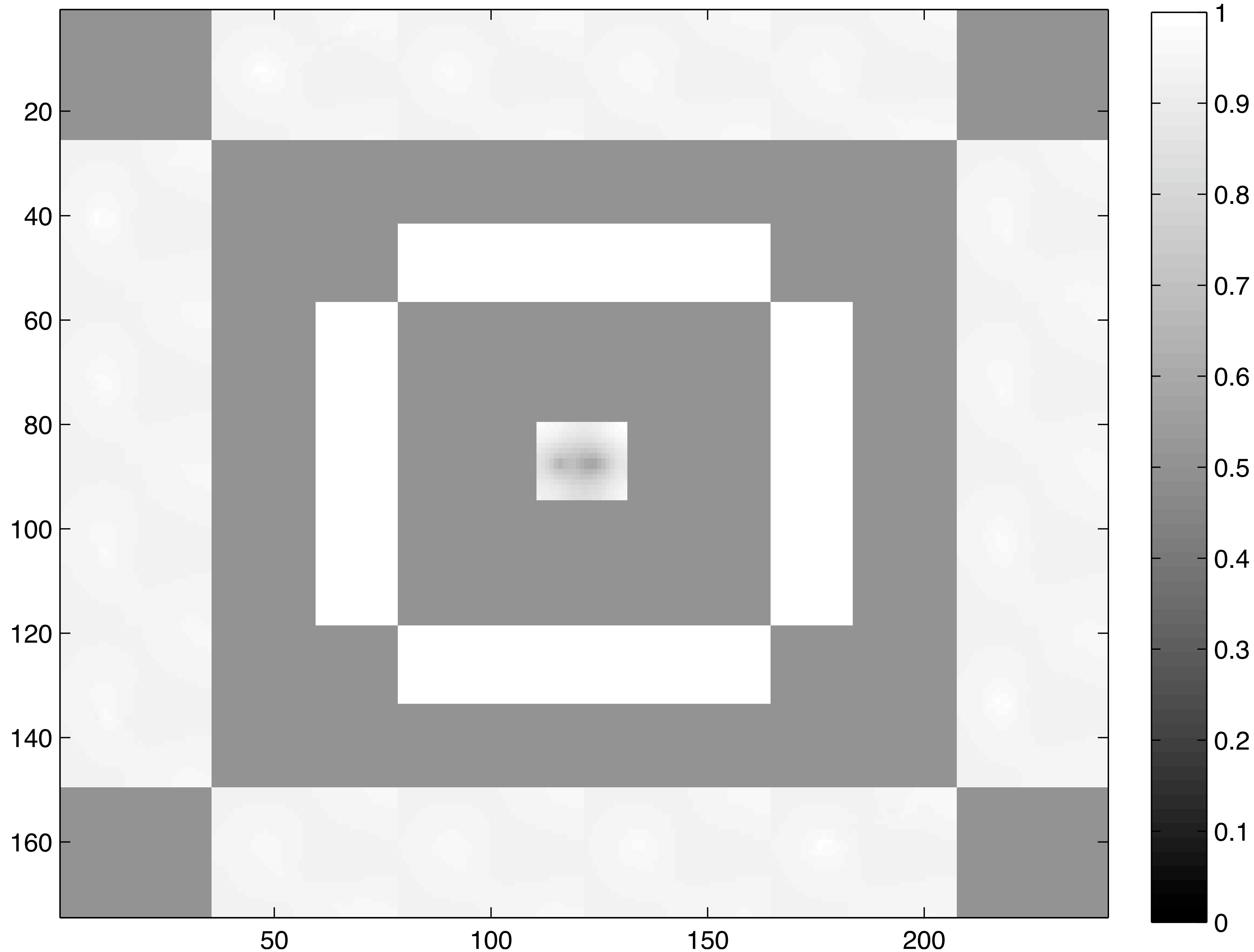
$\gamma = 1.0$

Smoothness penalty



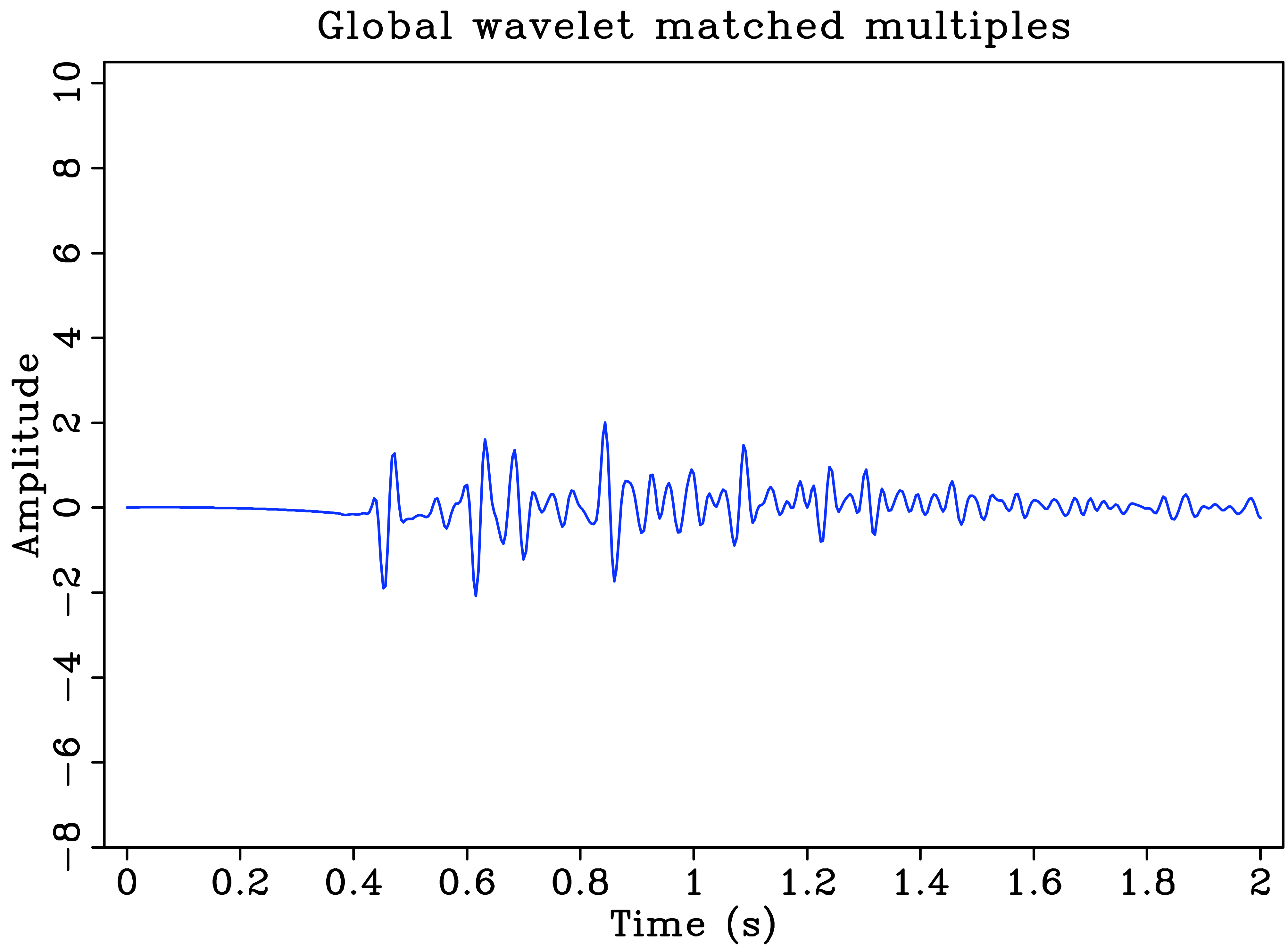
$\gamma = 2.0$

Smoothness penalty



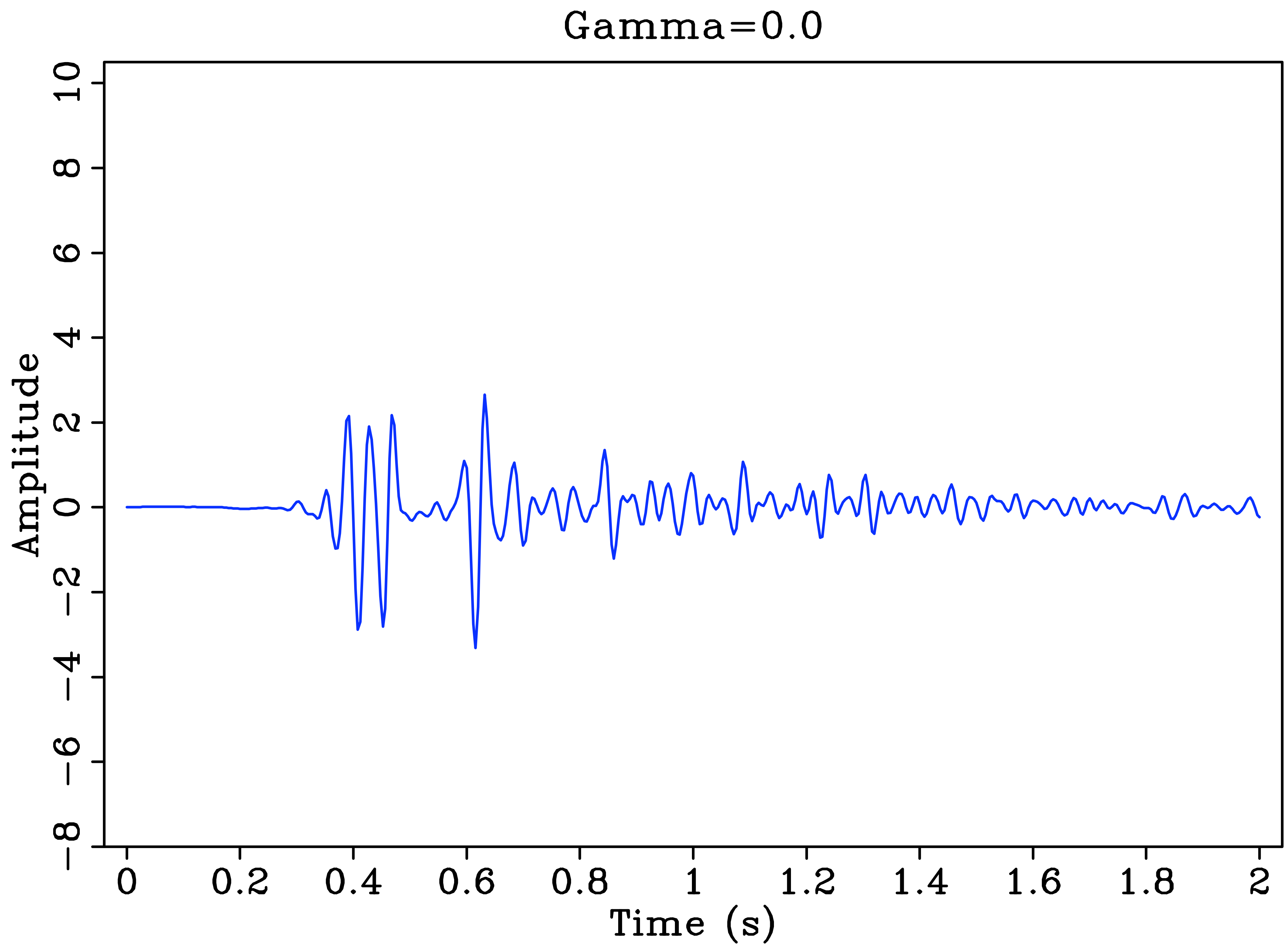
$$\gamma = 5.0$$

Smoothness penalty



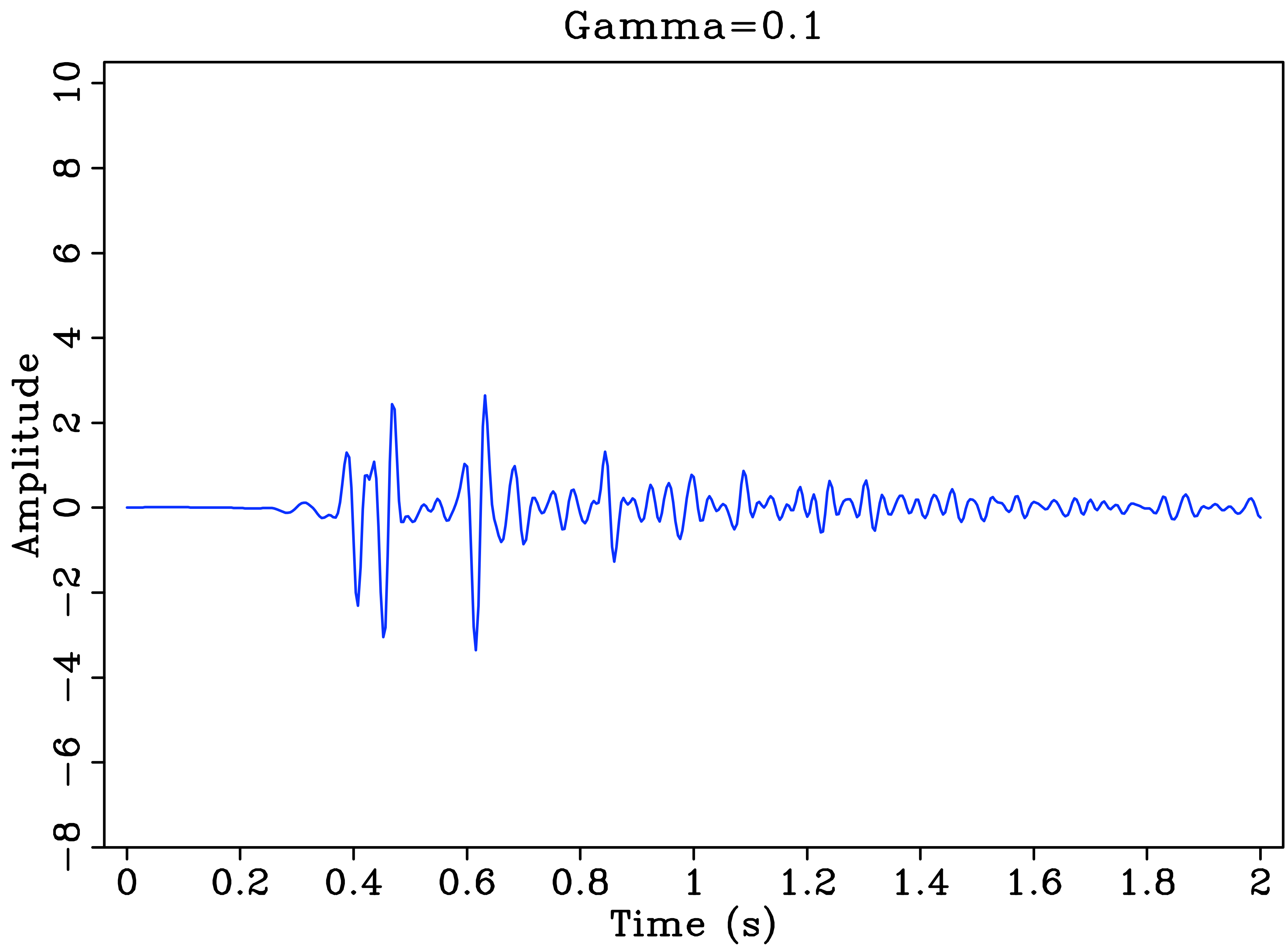
Only global wavelet matching no curvelet matching

Smoothness penalty



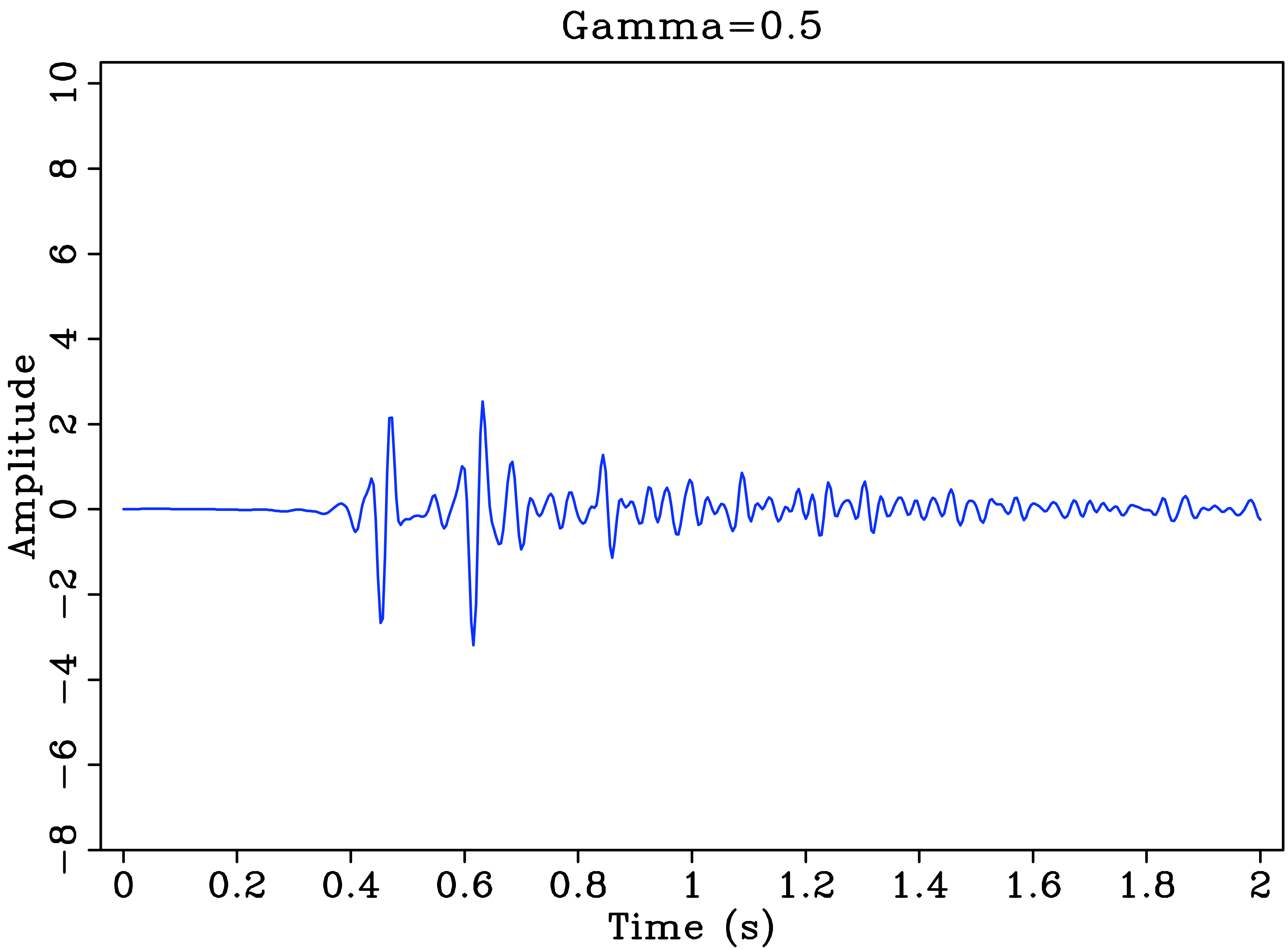
$\gamma = 0.0$

Smoothness penalty



$\gamma = 0.1$

Smoothness penalty



$\gamma = 0.5$

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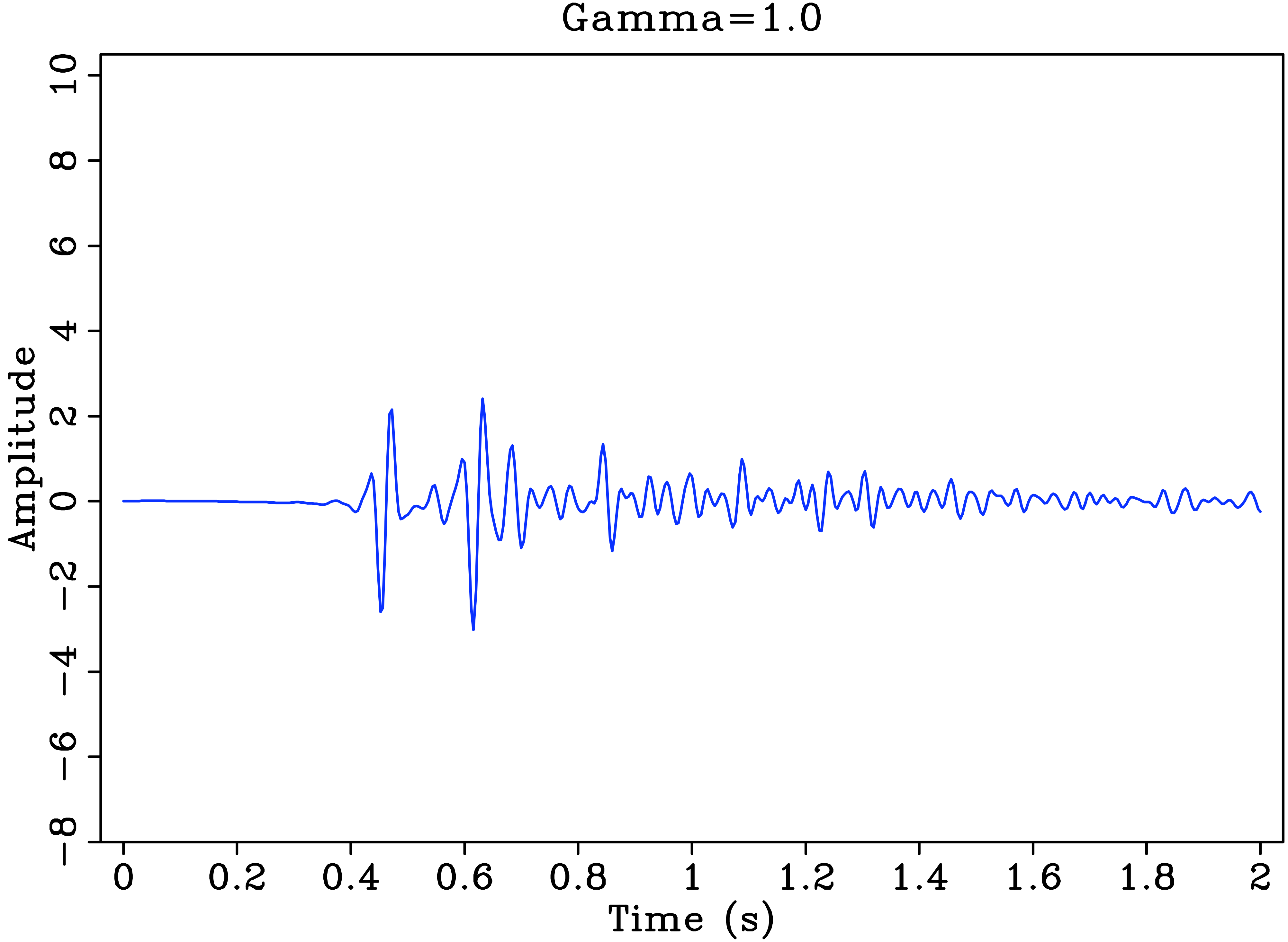
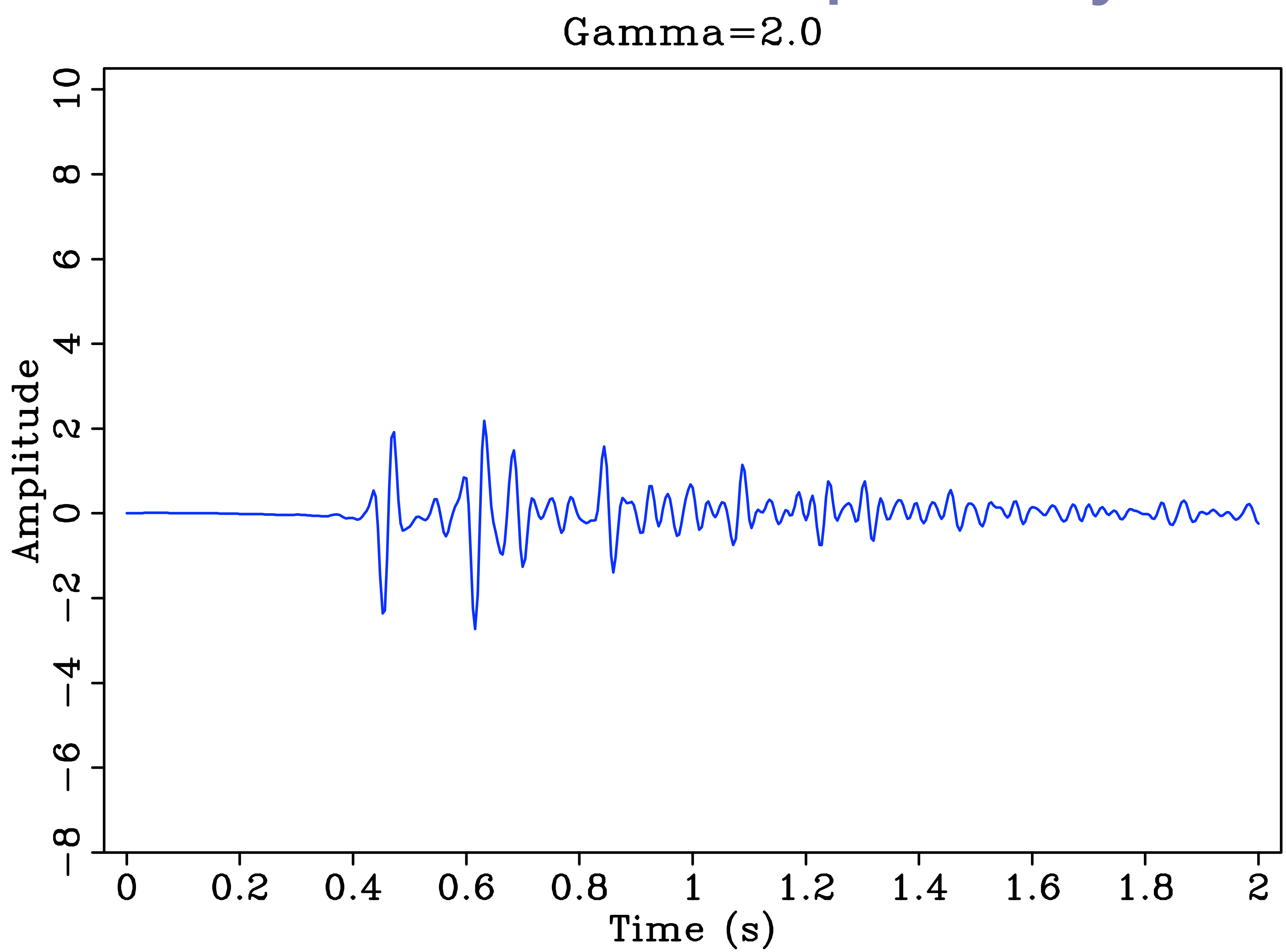
Smoothness penalty



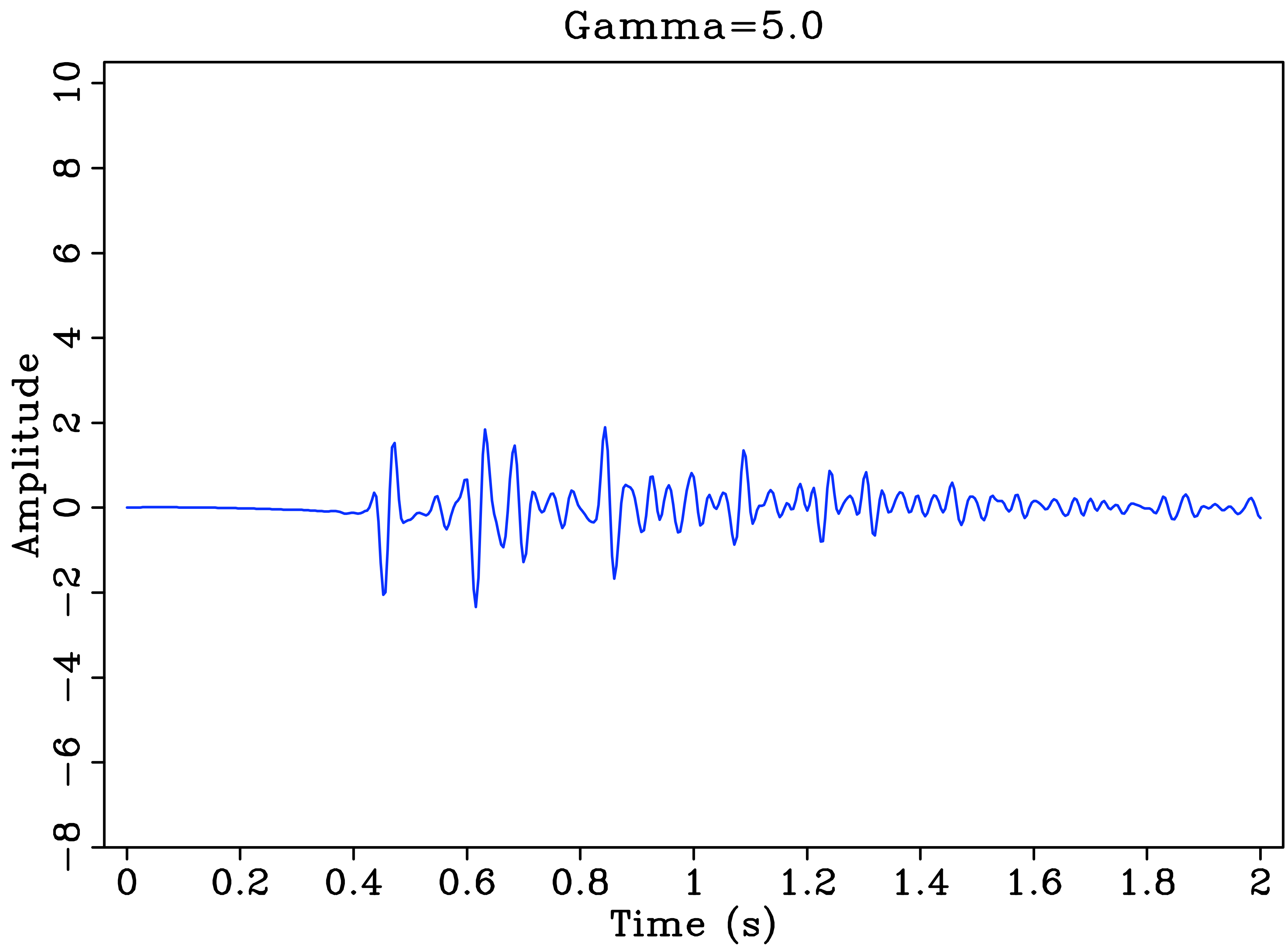
$\gamma = 1.0$

Smoothness penalty



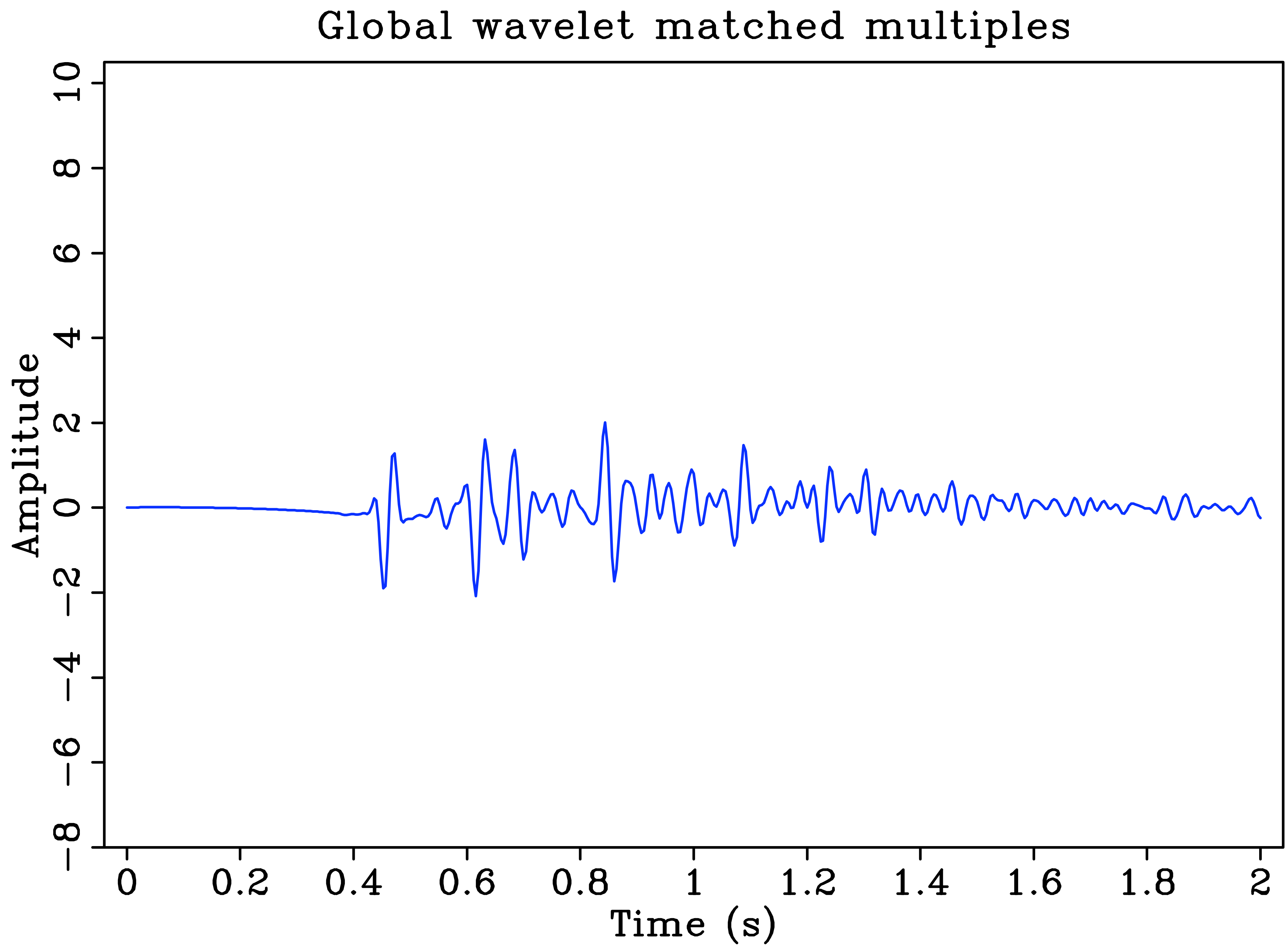
$\gamma = 2.0$

Smoothness penalty



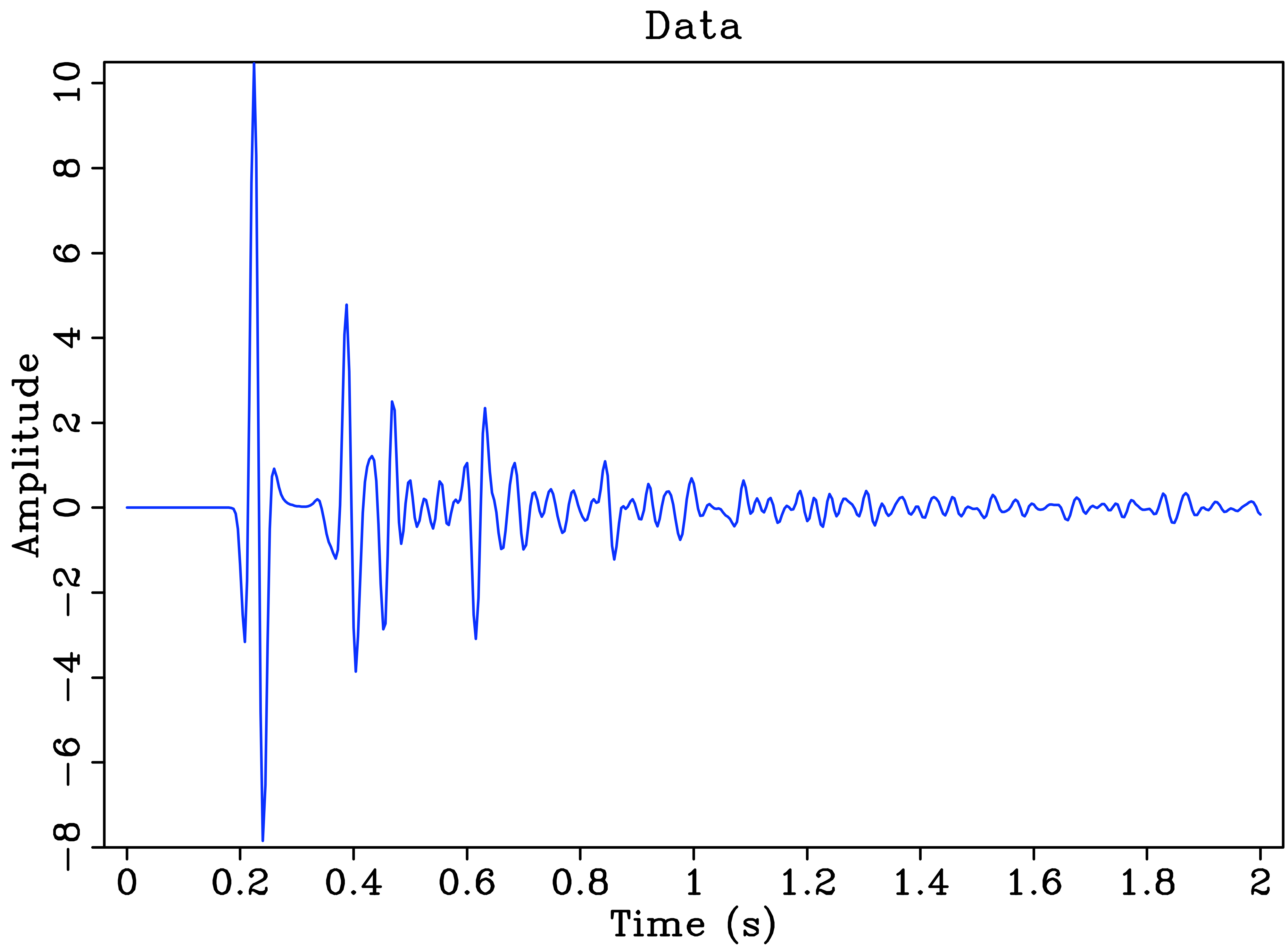
$\gamma = 5.0$

Smoothness penalty



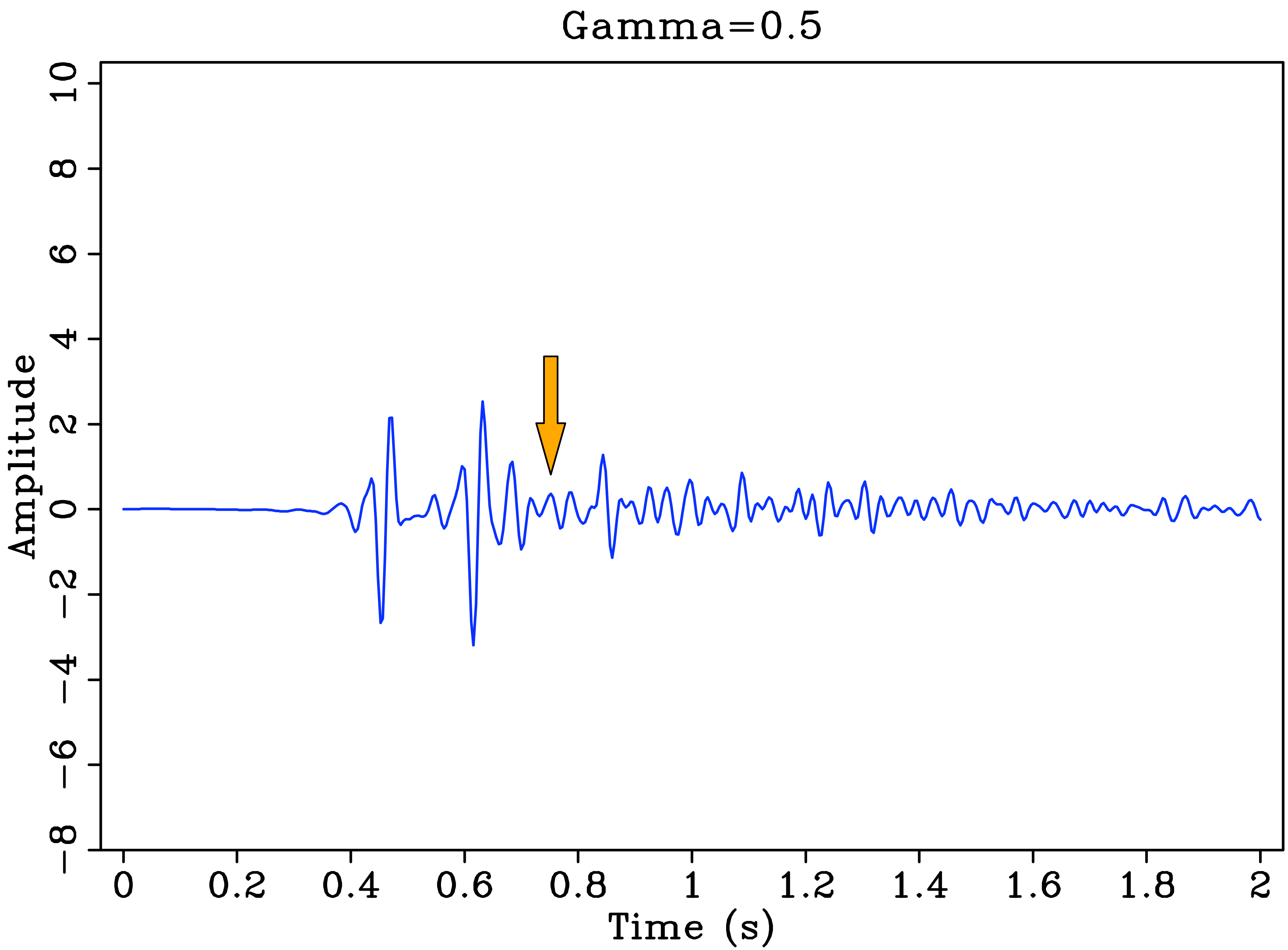
Only global wavelet matching no curvelet matching

Smoothness penalty



Total data

Smoothness penalty



Correctly curvelet matched

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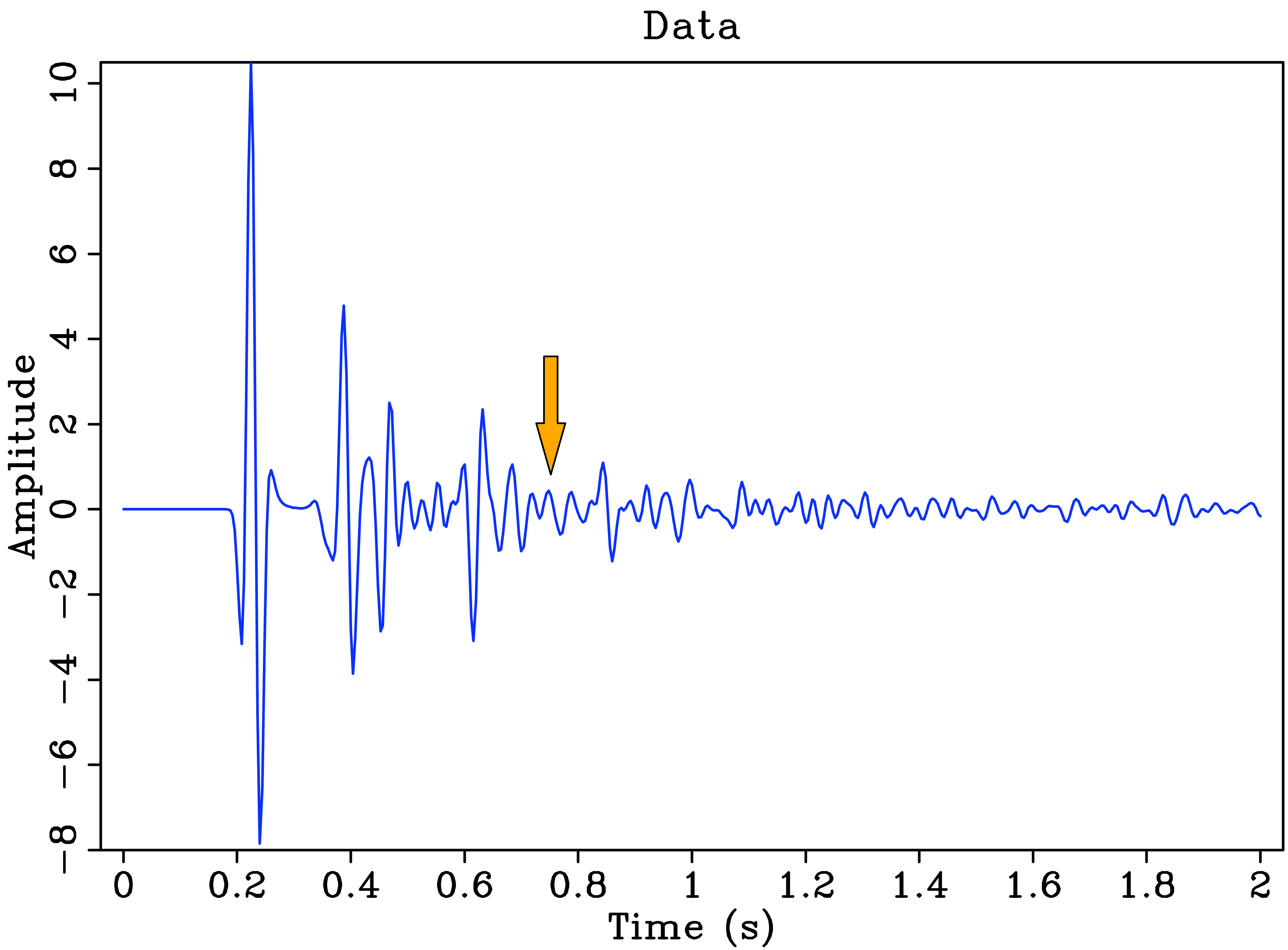
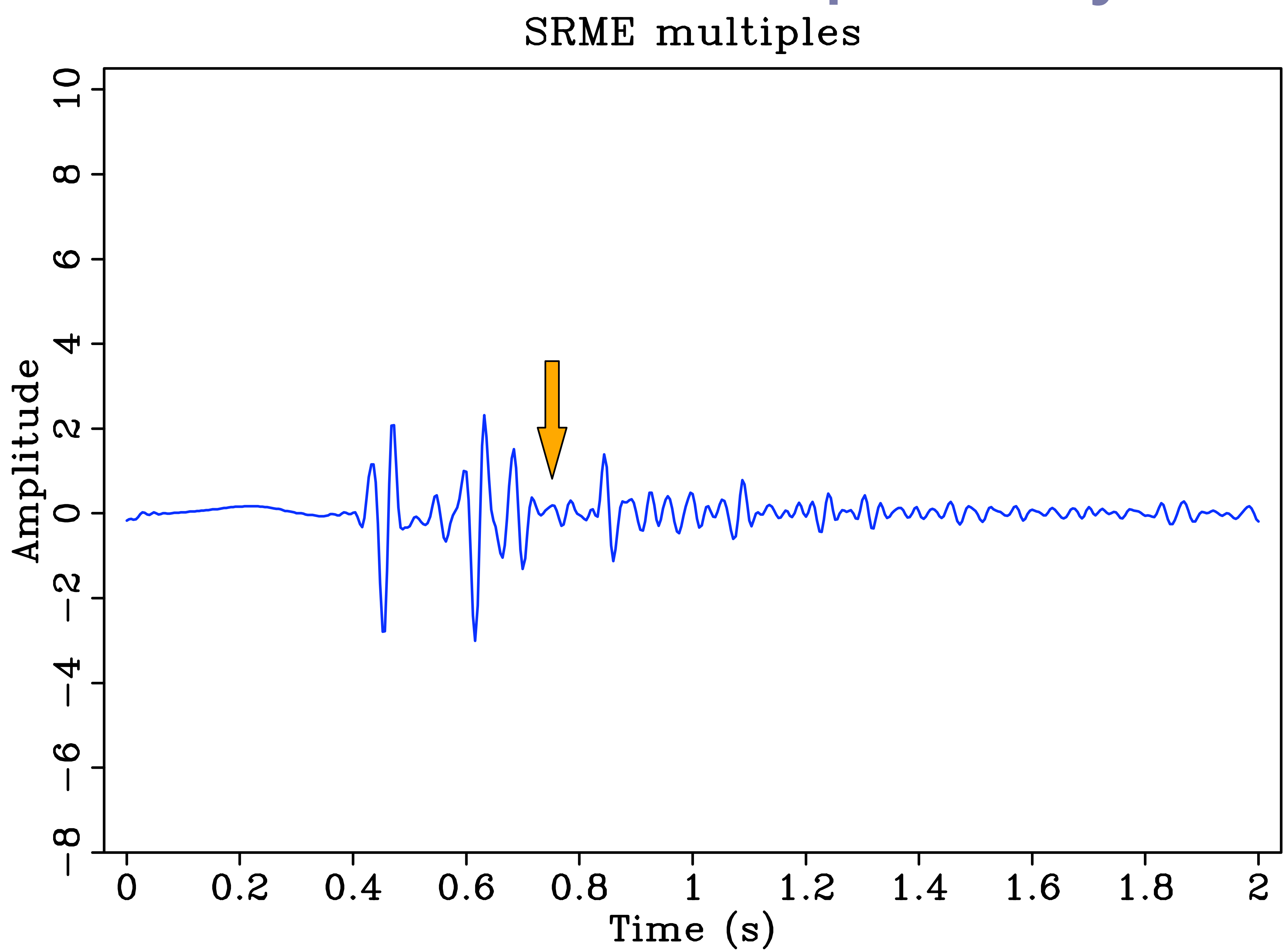
Smoothness penalty



Total data

Smoothness penalty

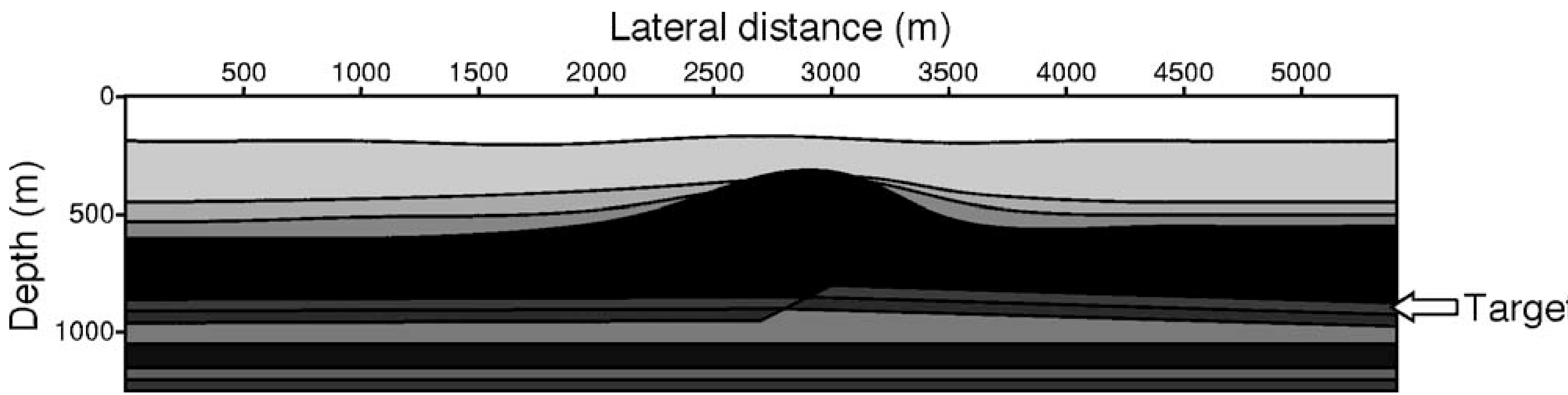


SRME windowed amplitude matched multiples

Synthetic-data example

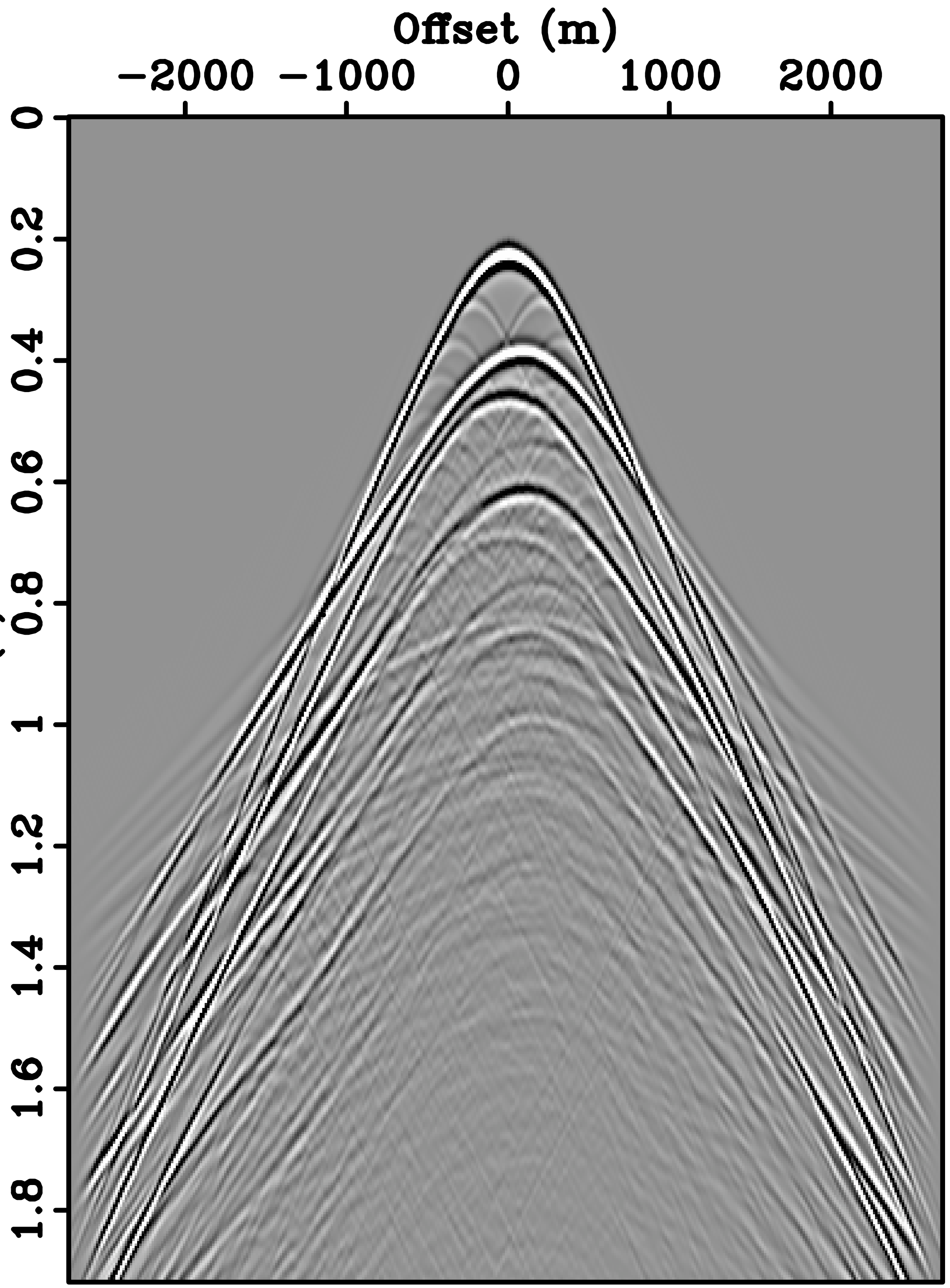
Figure 1

Synthetic-data example

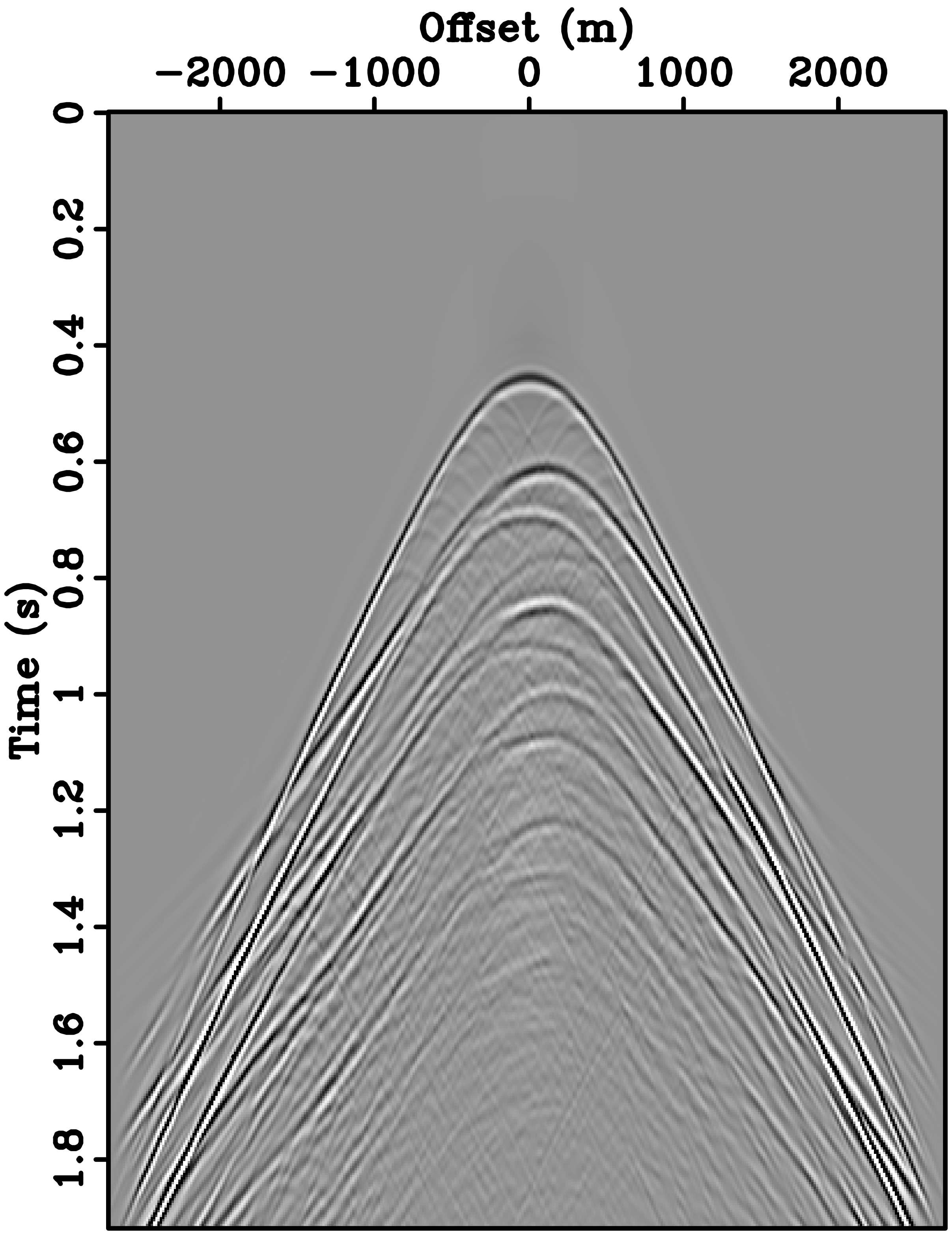


Velocity model used in the synthetic data examples

Synthetic-data example

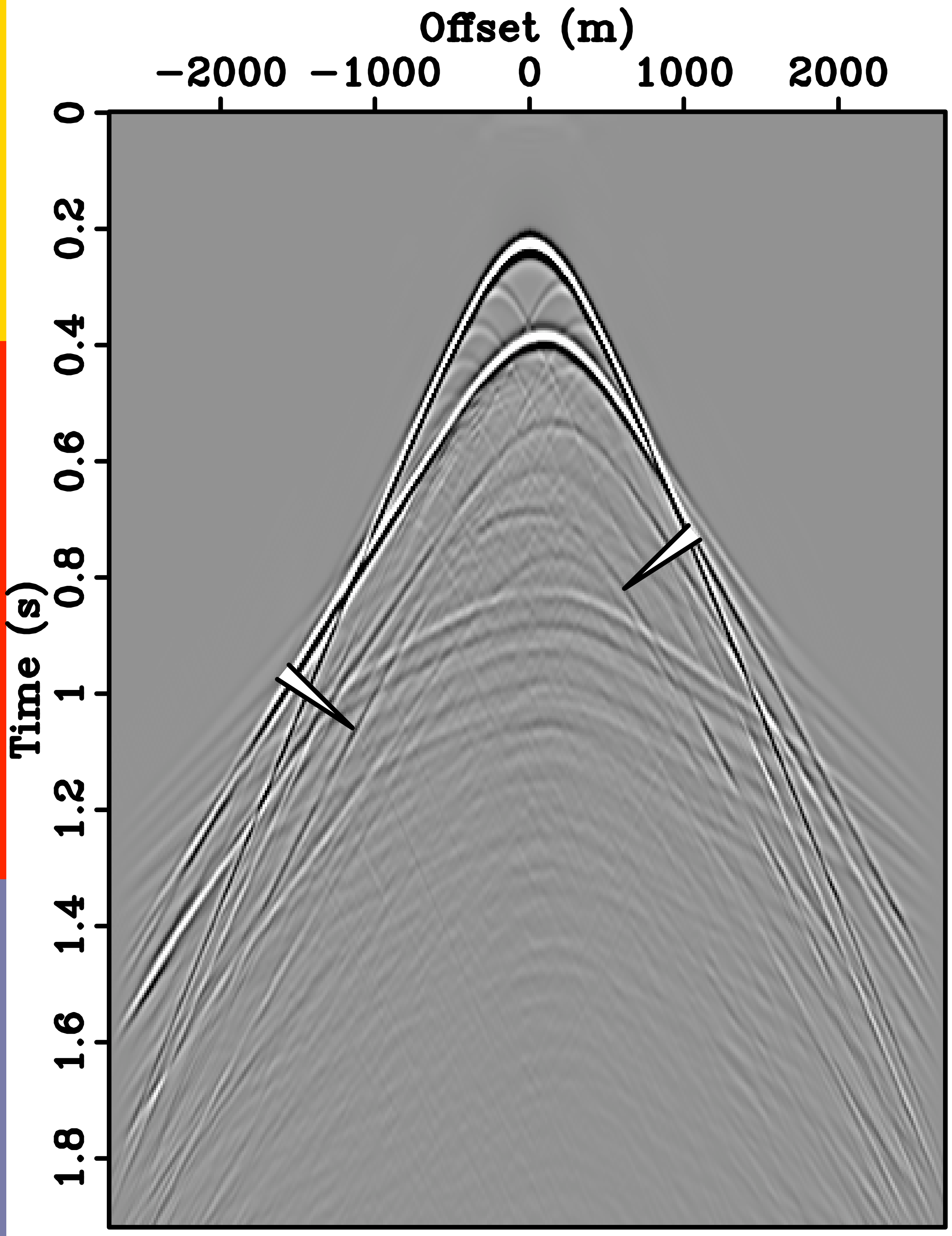


Total data

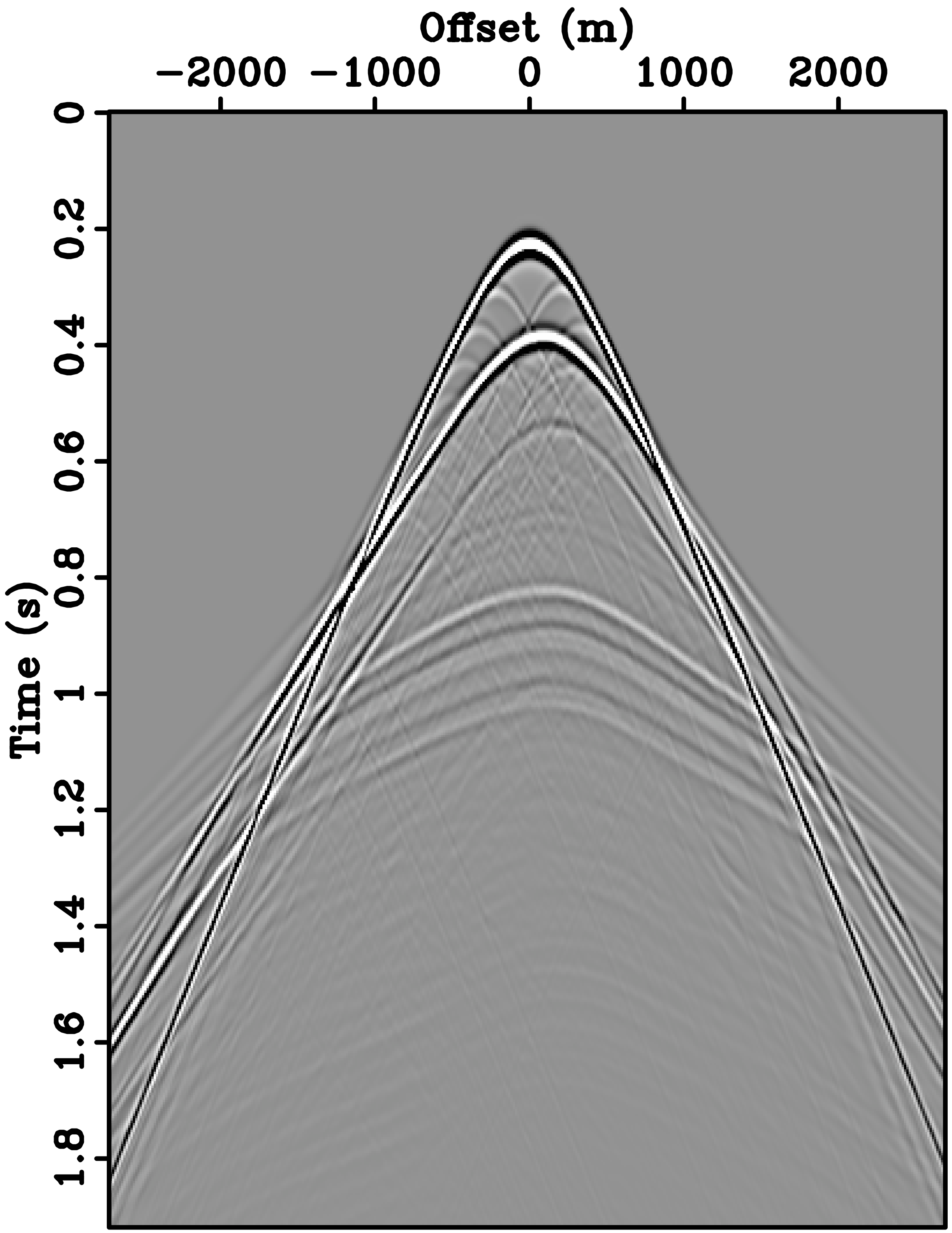


Single-term SRME multiples

Synthetic-data example

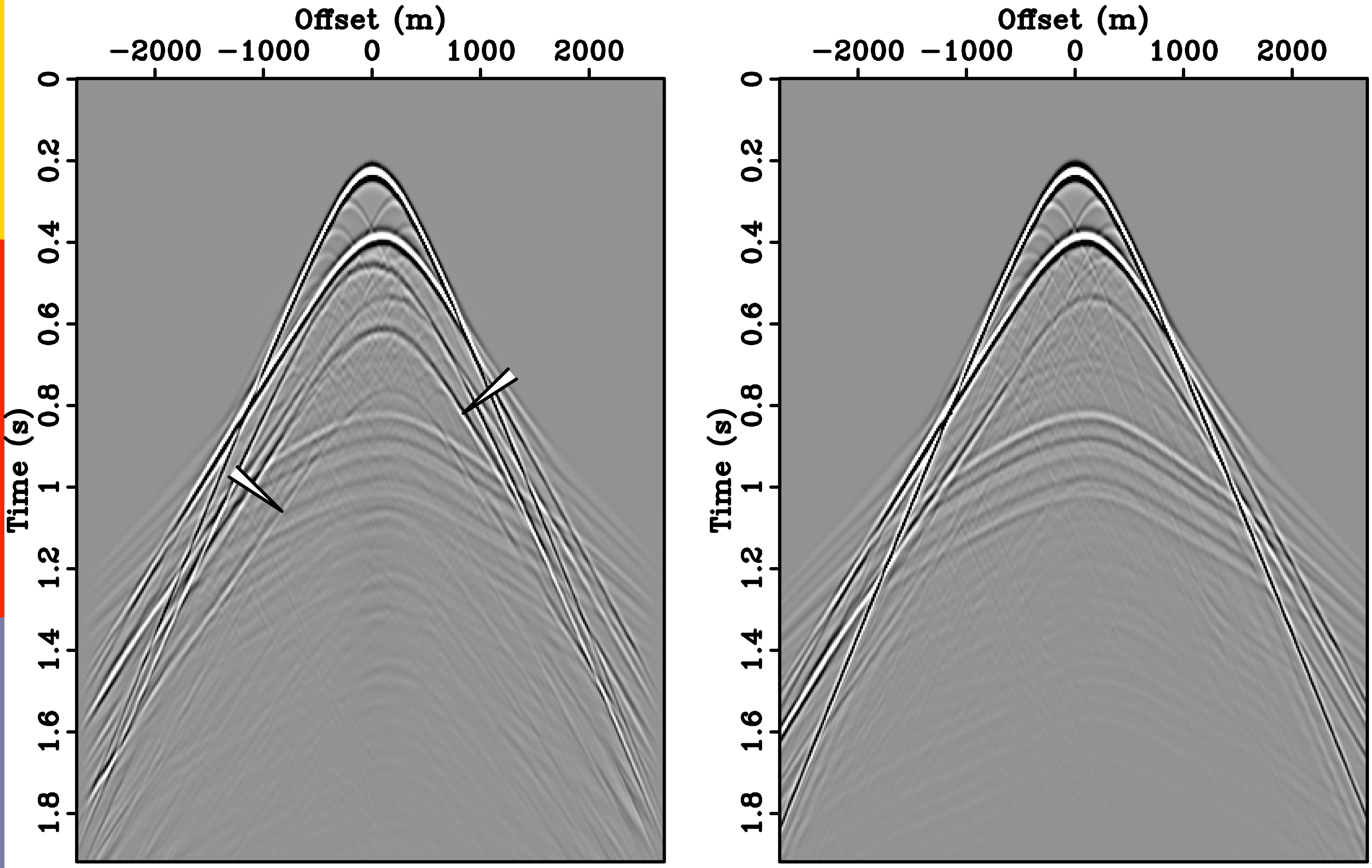


SRME primaries



Real primaries

Synthetic-data example



No matching Bayesian

Real primaries

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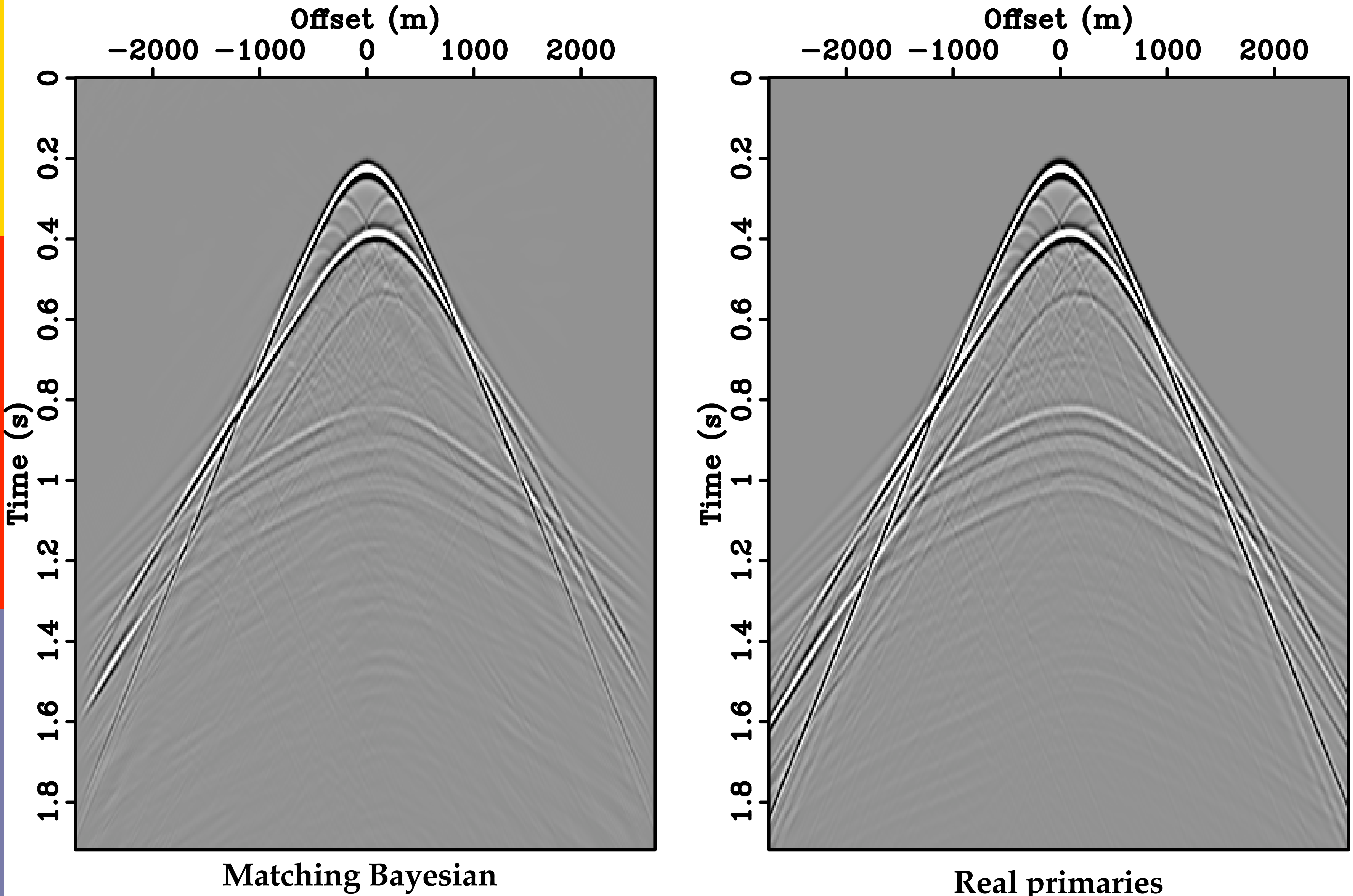
Synthetic-data example



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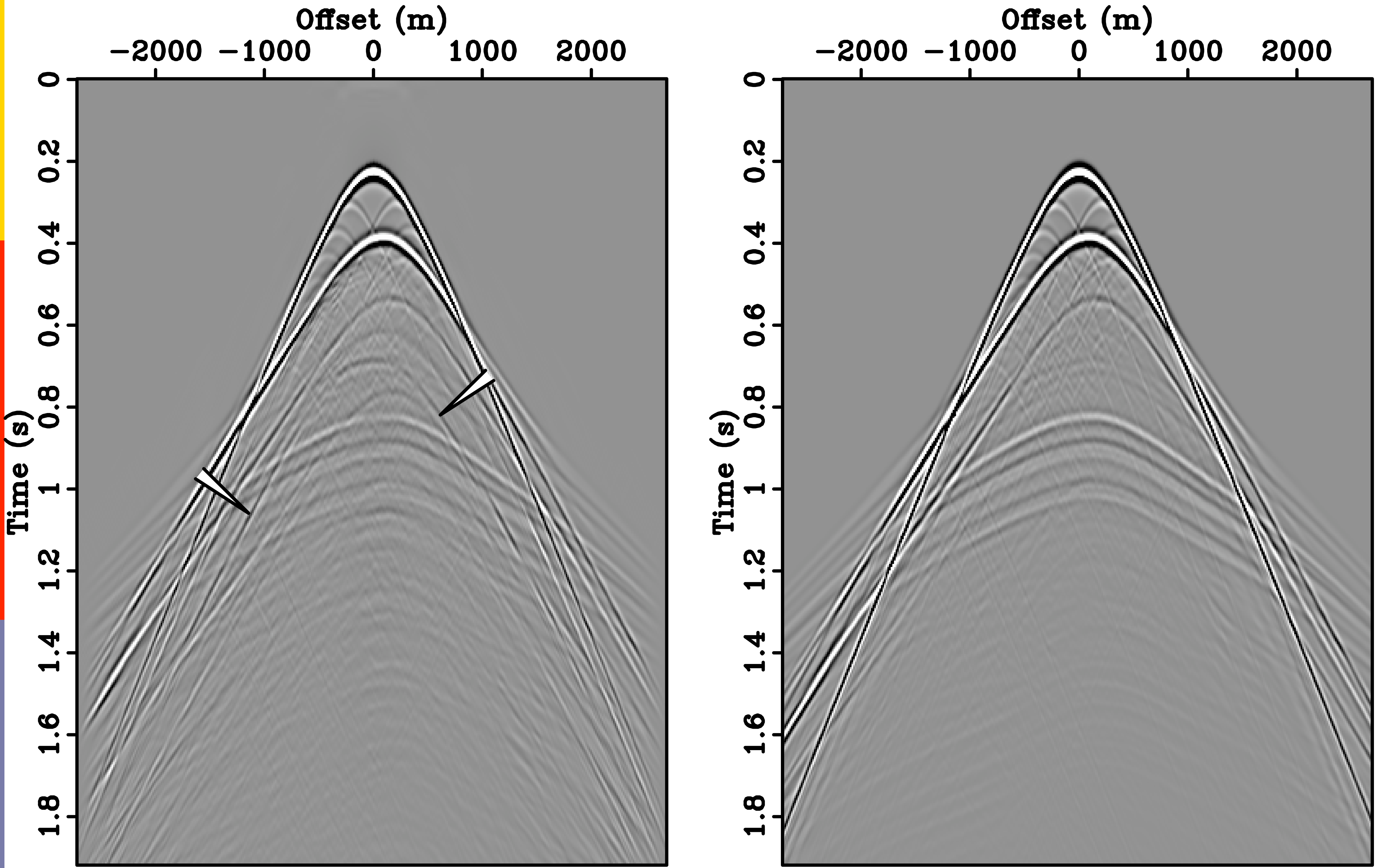
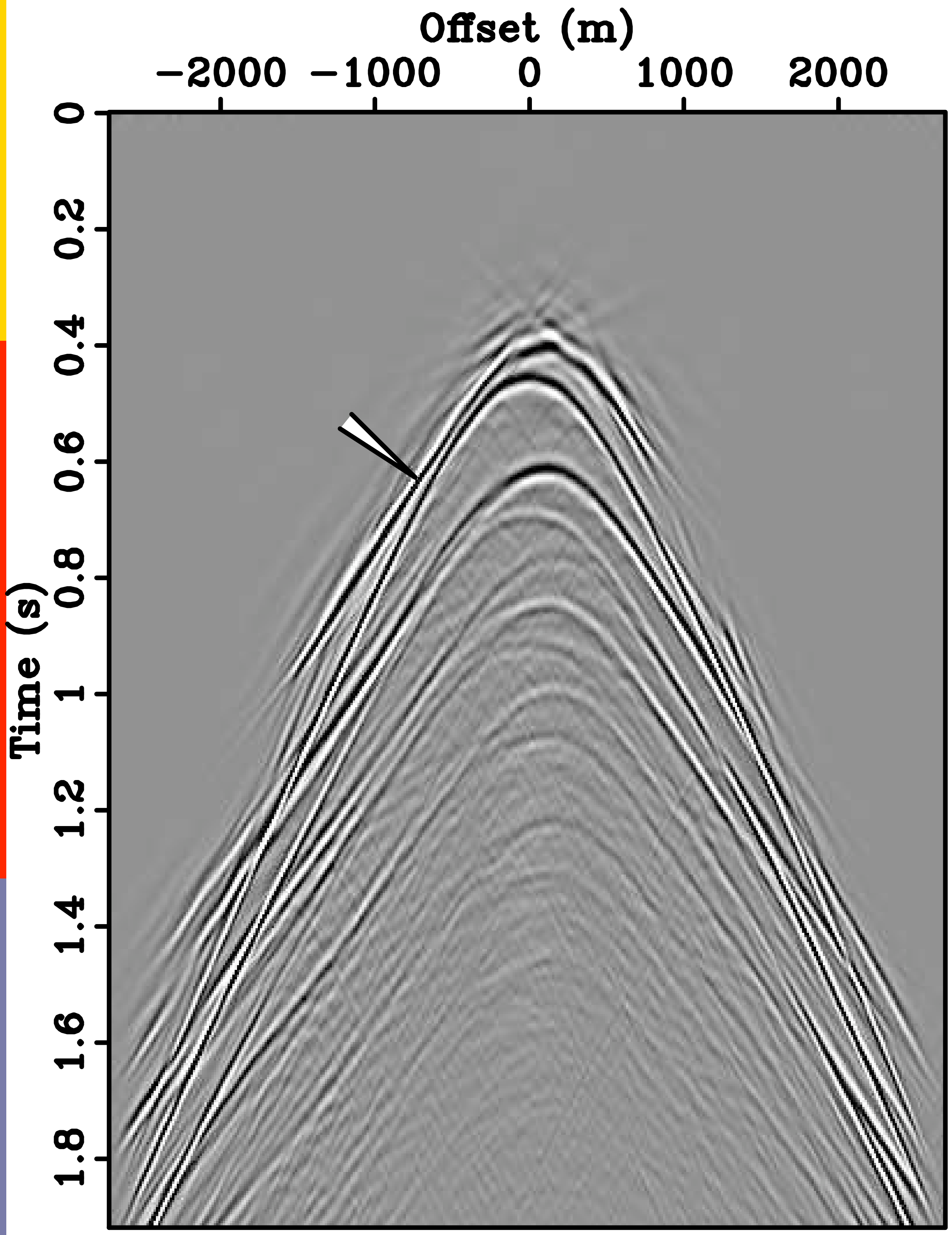
Synthetic-data example



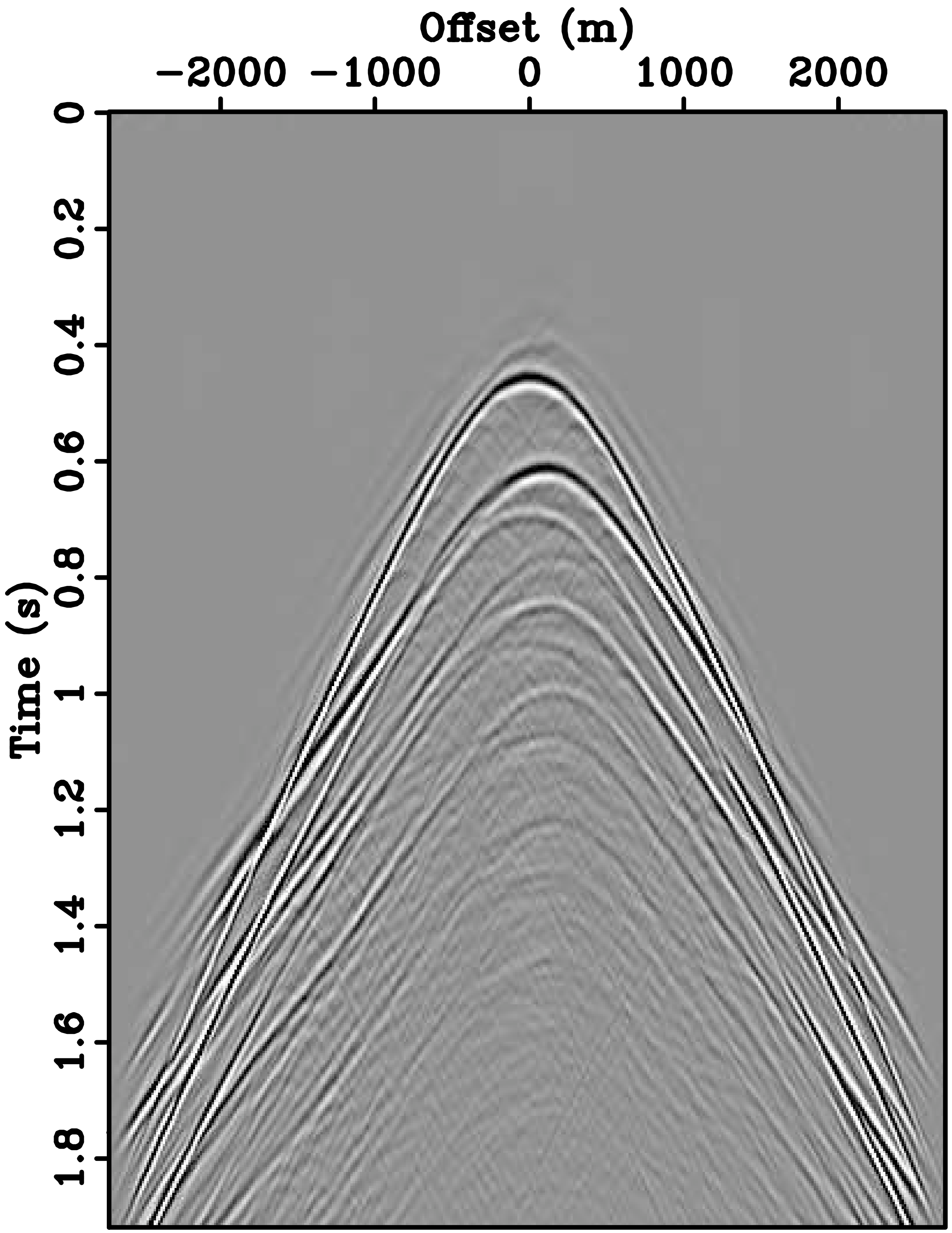
SRME primaries

Real primaries

Synthetic-data example



Over matched multiples



Correctly matched multiples

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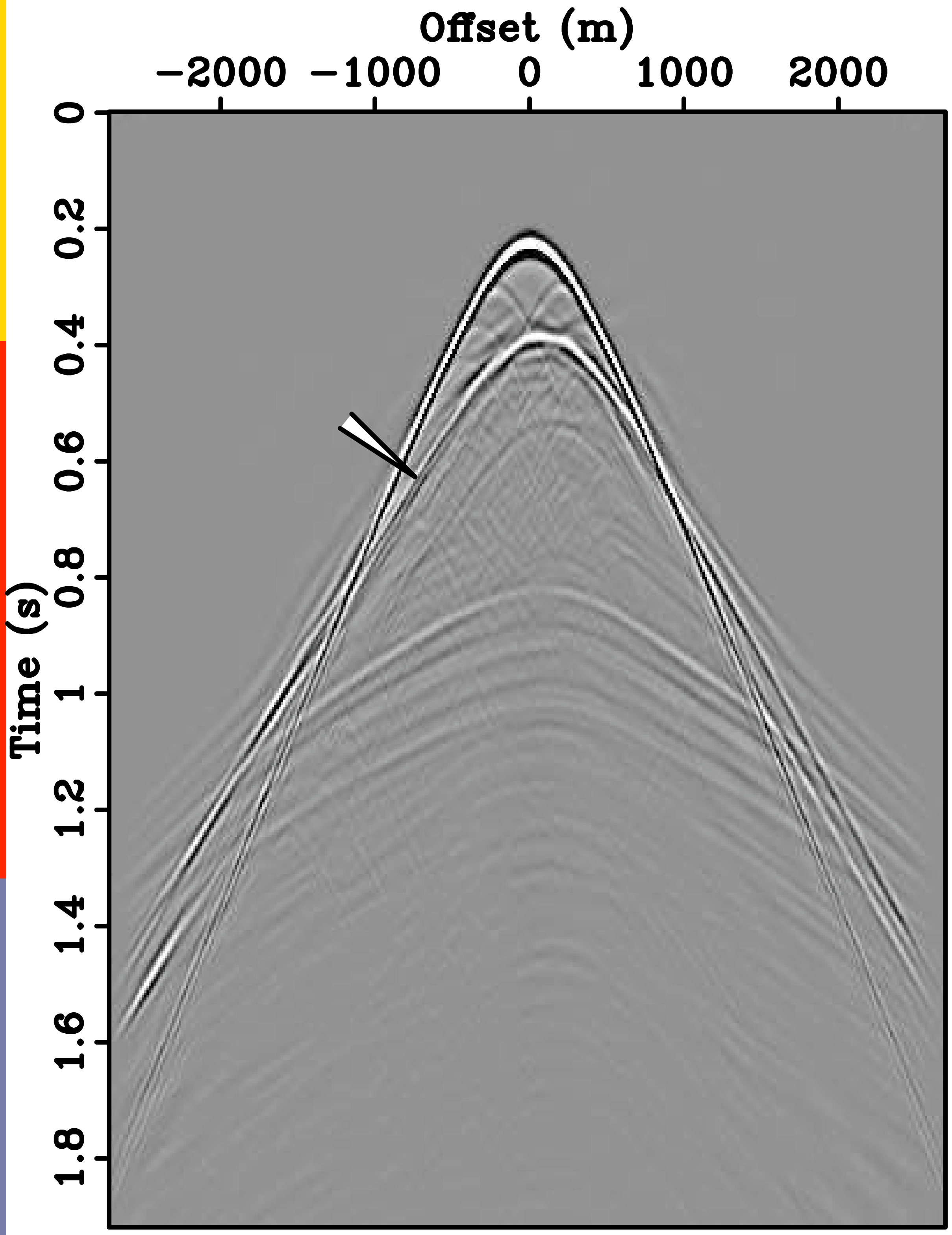
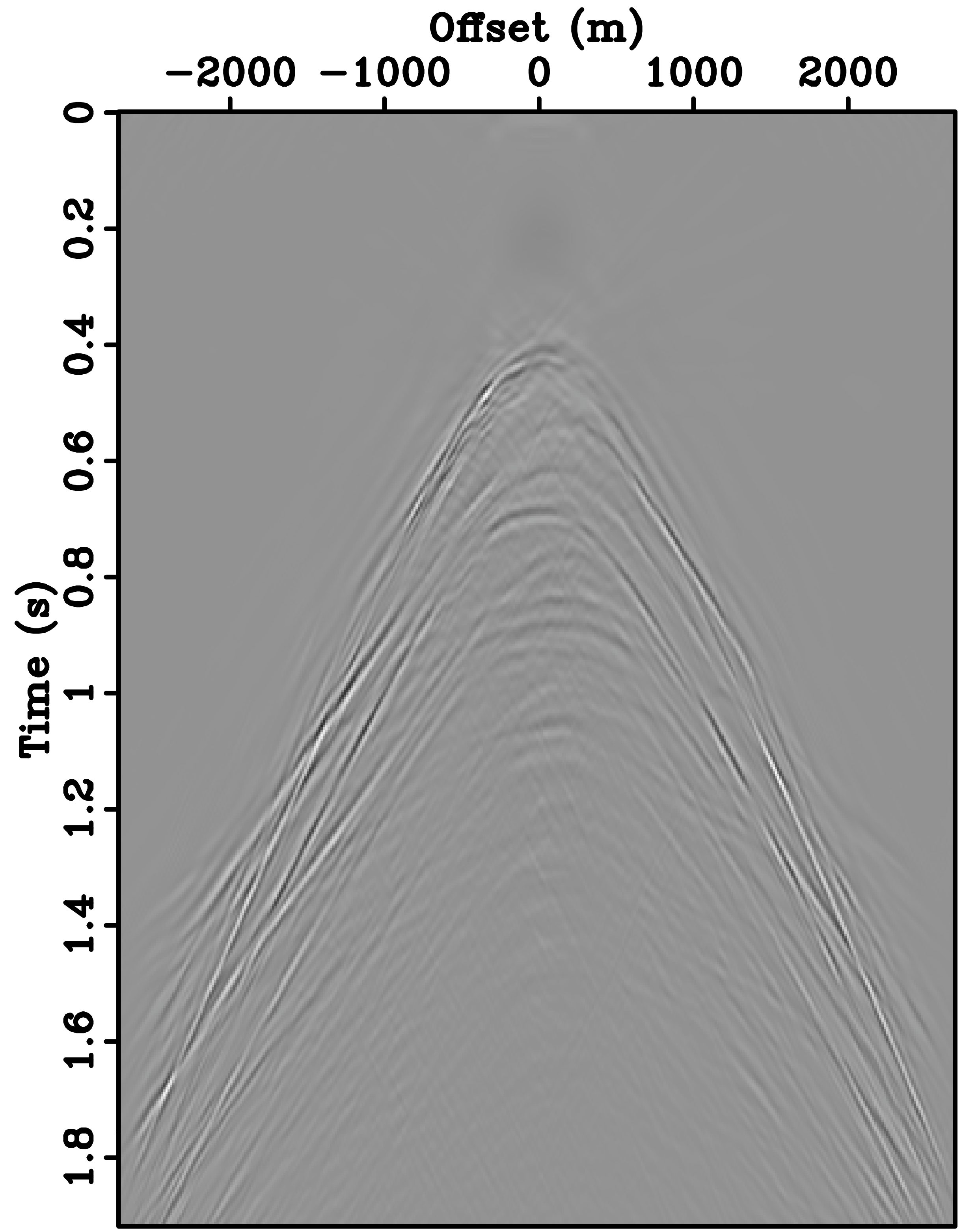
Synthetic-data example



Estimate for the primaries with over matched multiples



Difference between SRME and curvelet matching

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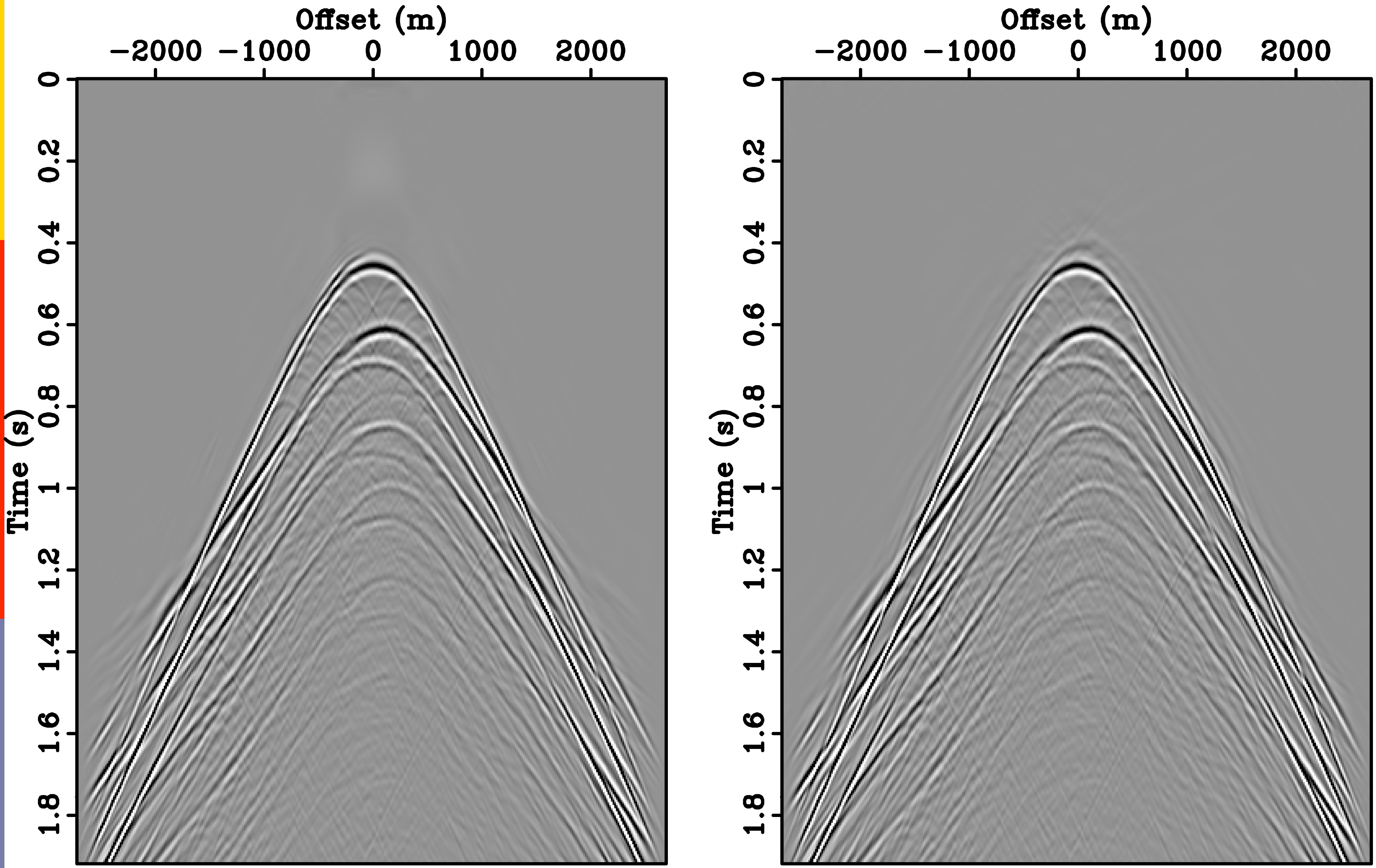
Synthetic-data example



Difference between data and SRME

Difference between data and curvelet matching

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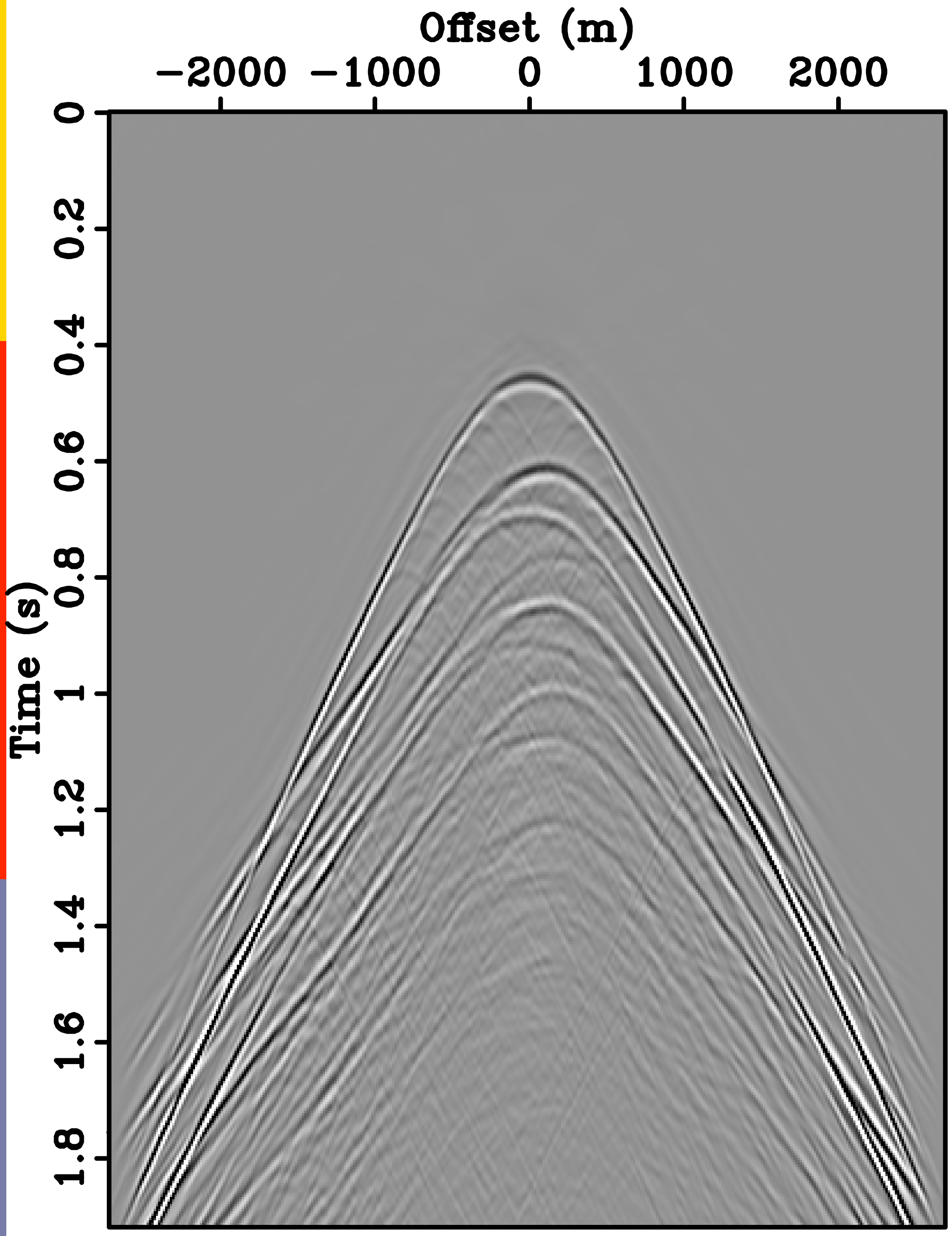
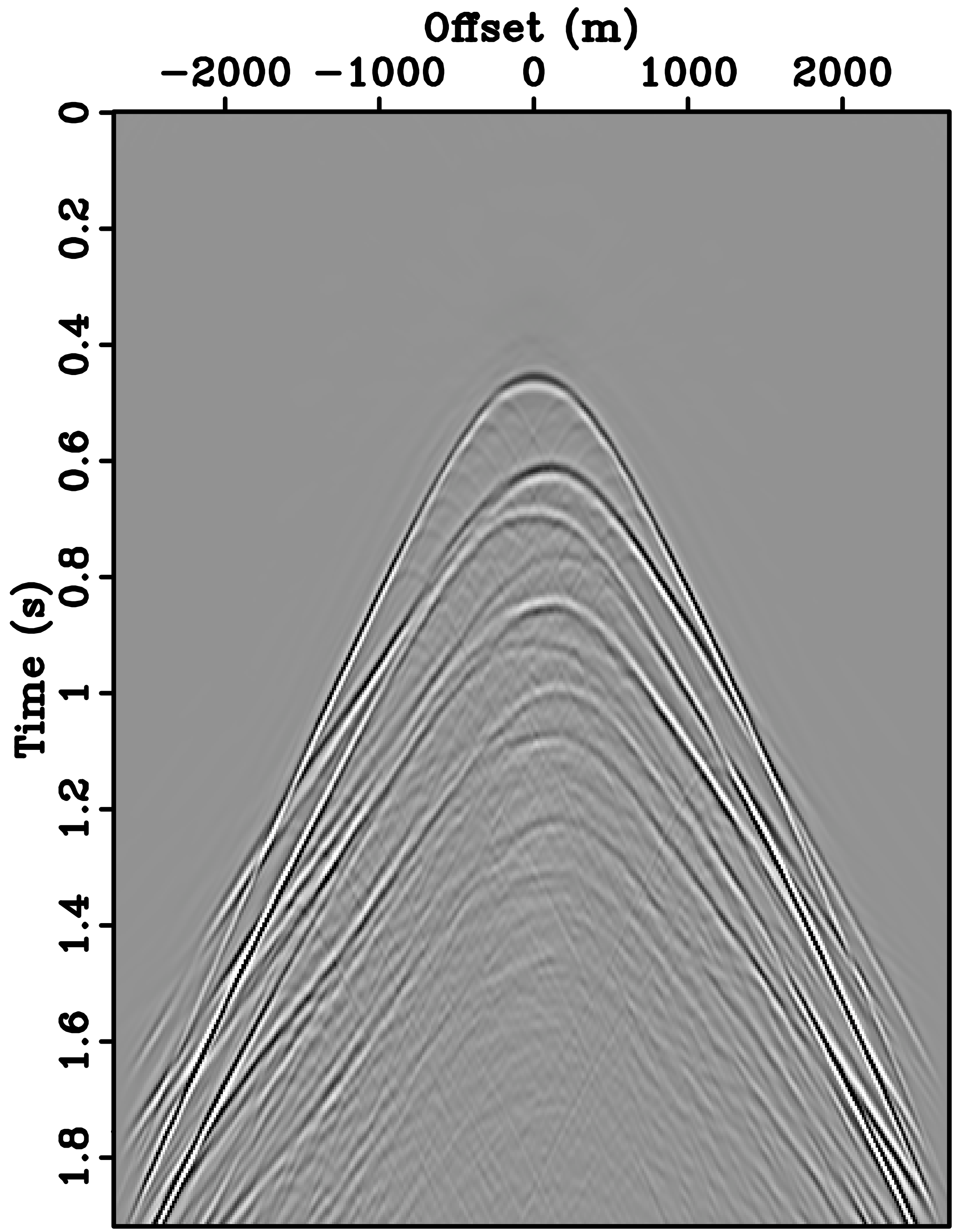
Synthetic-data example



Difference between data (Figure 1 a) and No matching (Figure 1 e)



SRME Bayesian multiples

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SNRs

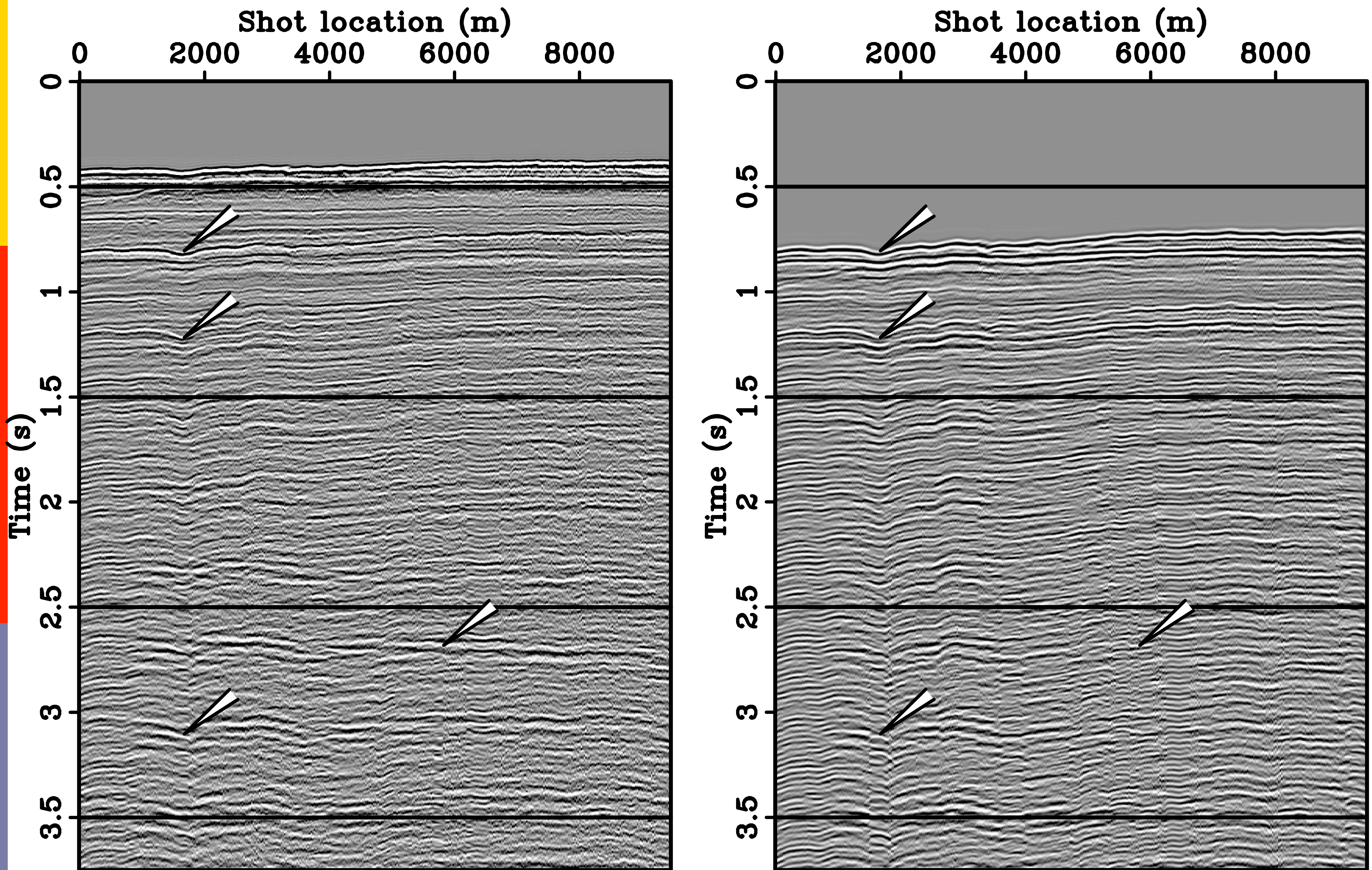
Comparison with “ground truth”

SRME	9.82	
Bayes	7.25	
matched Bayes	11.22	

Real-data example

Figure 2

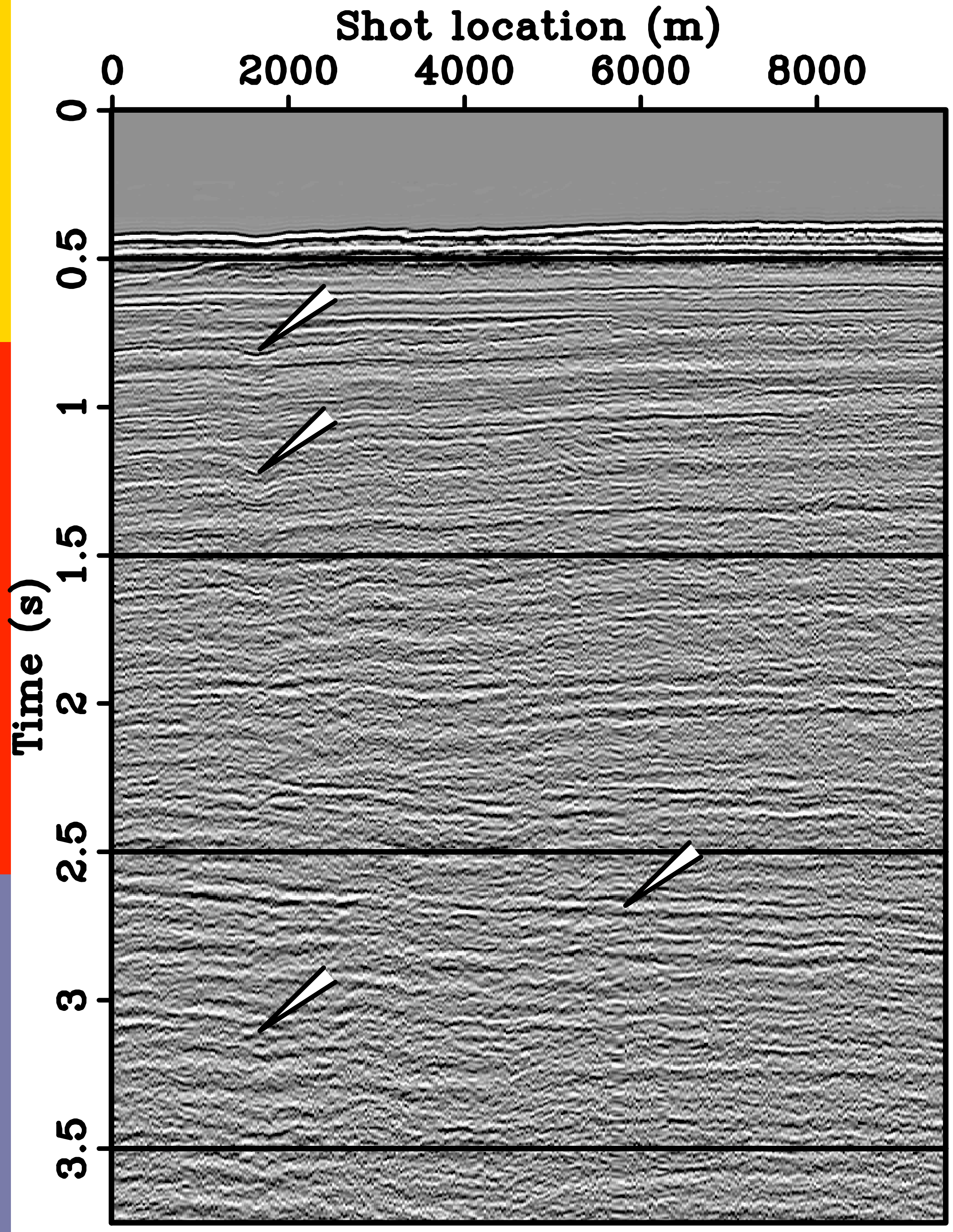
Real-data example



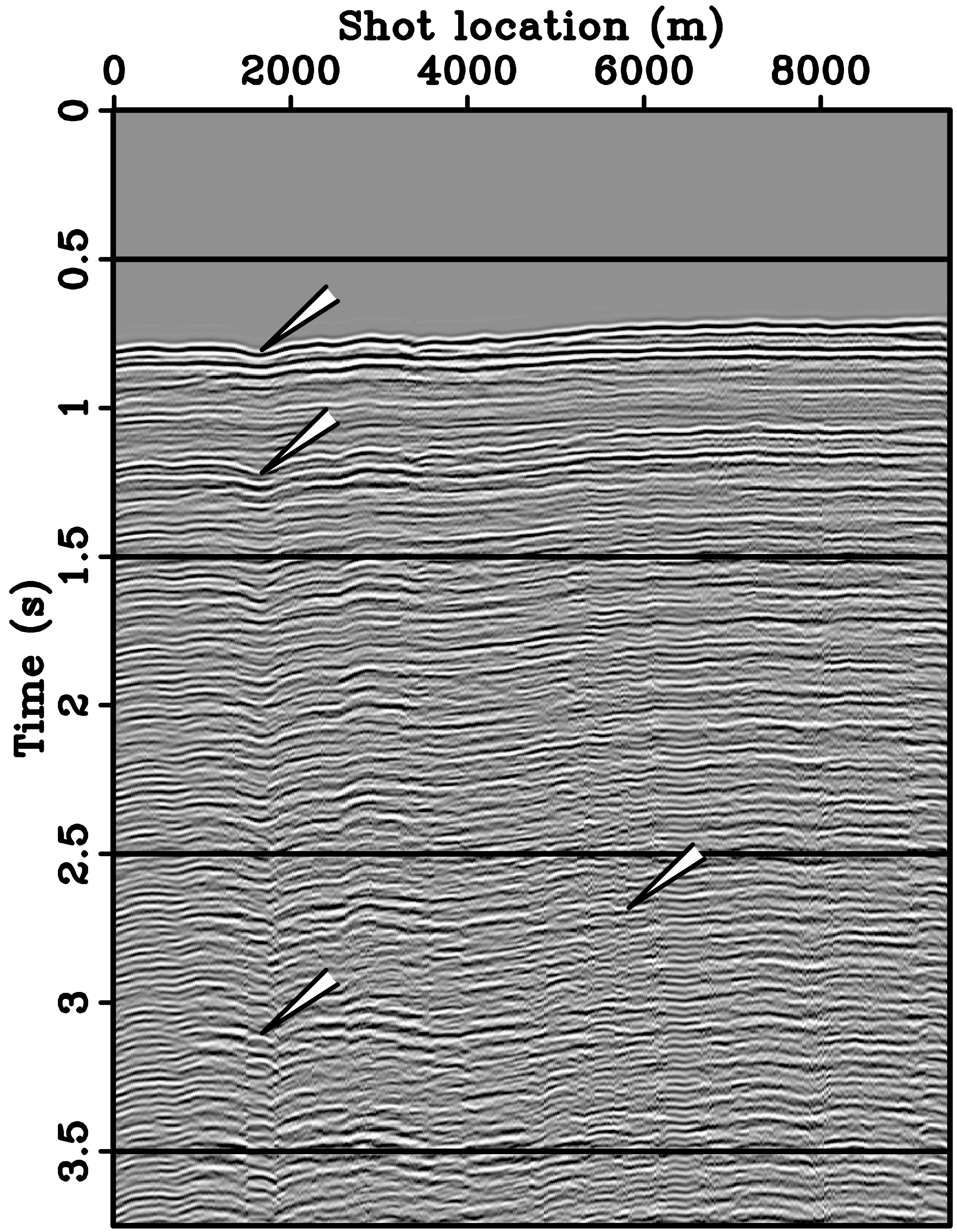
Total data

Predicted multiples

Real-data example



SRME primaries



Predicted multiples

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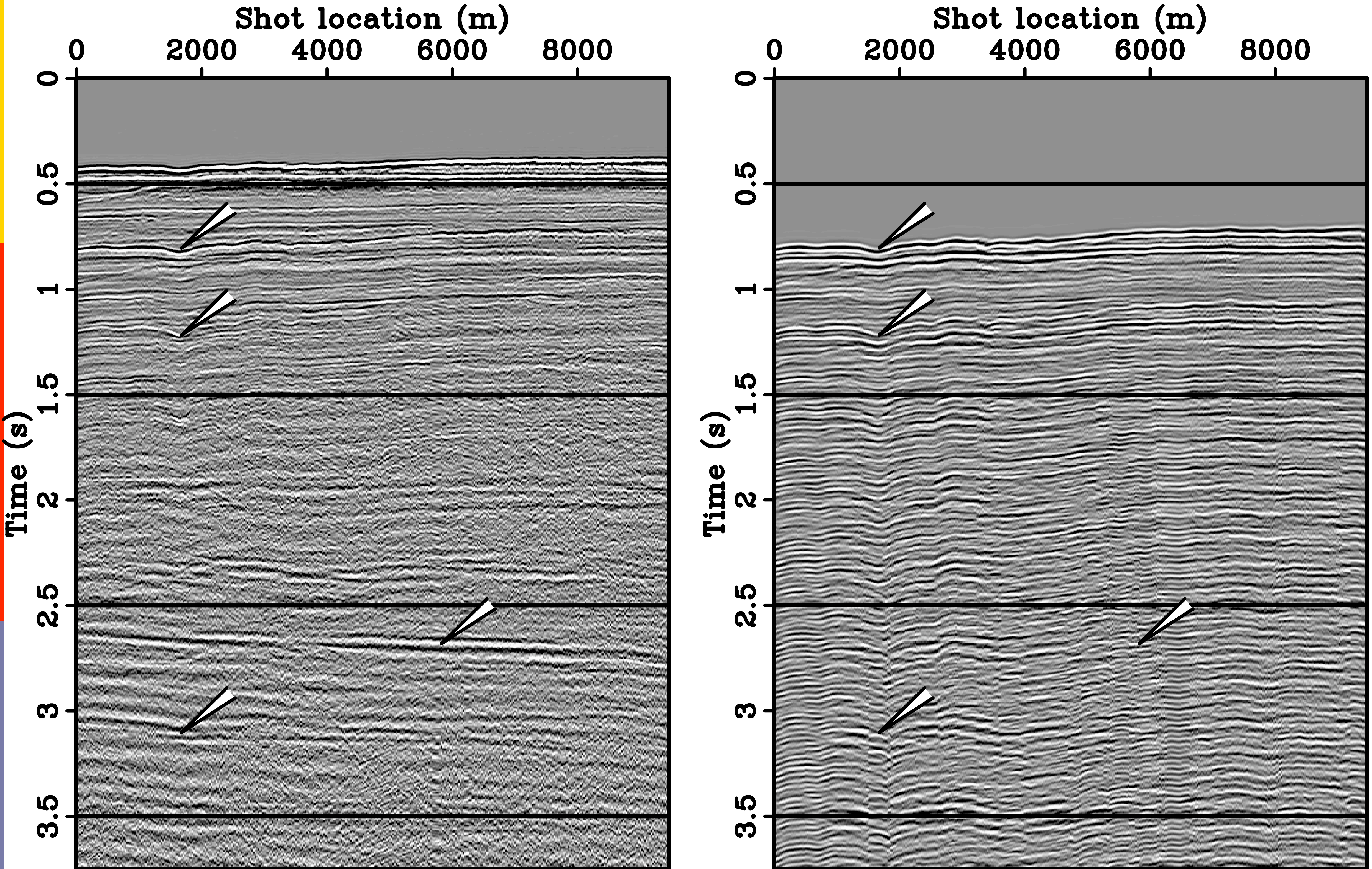
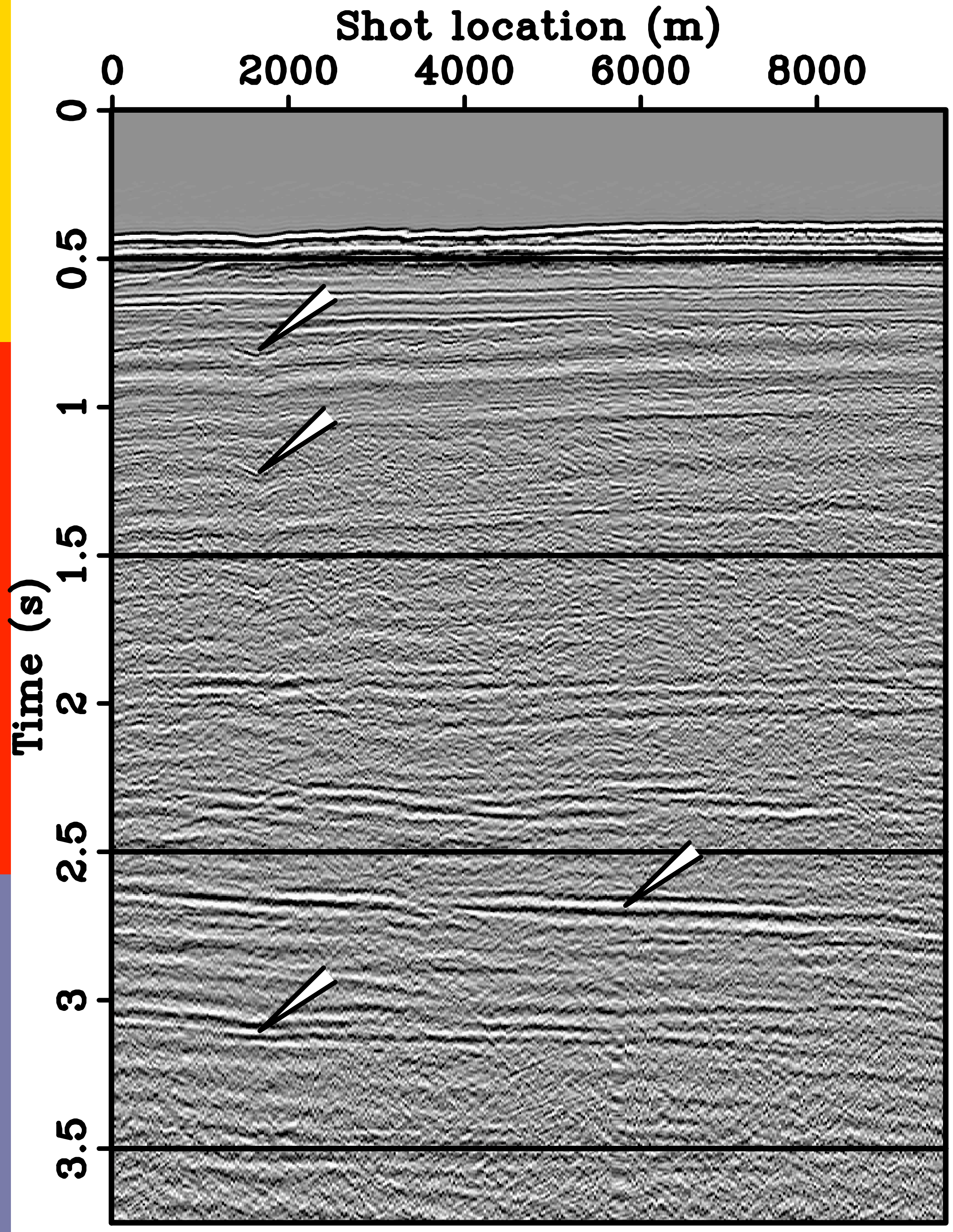
Real-data example



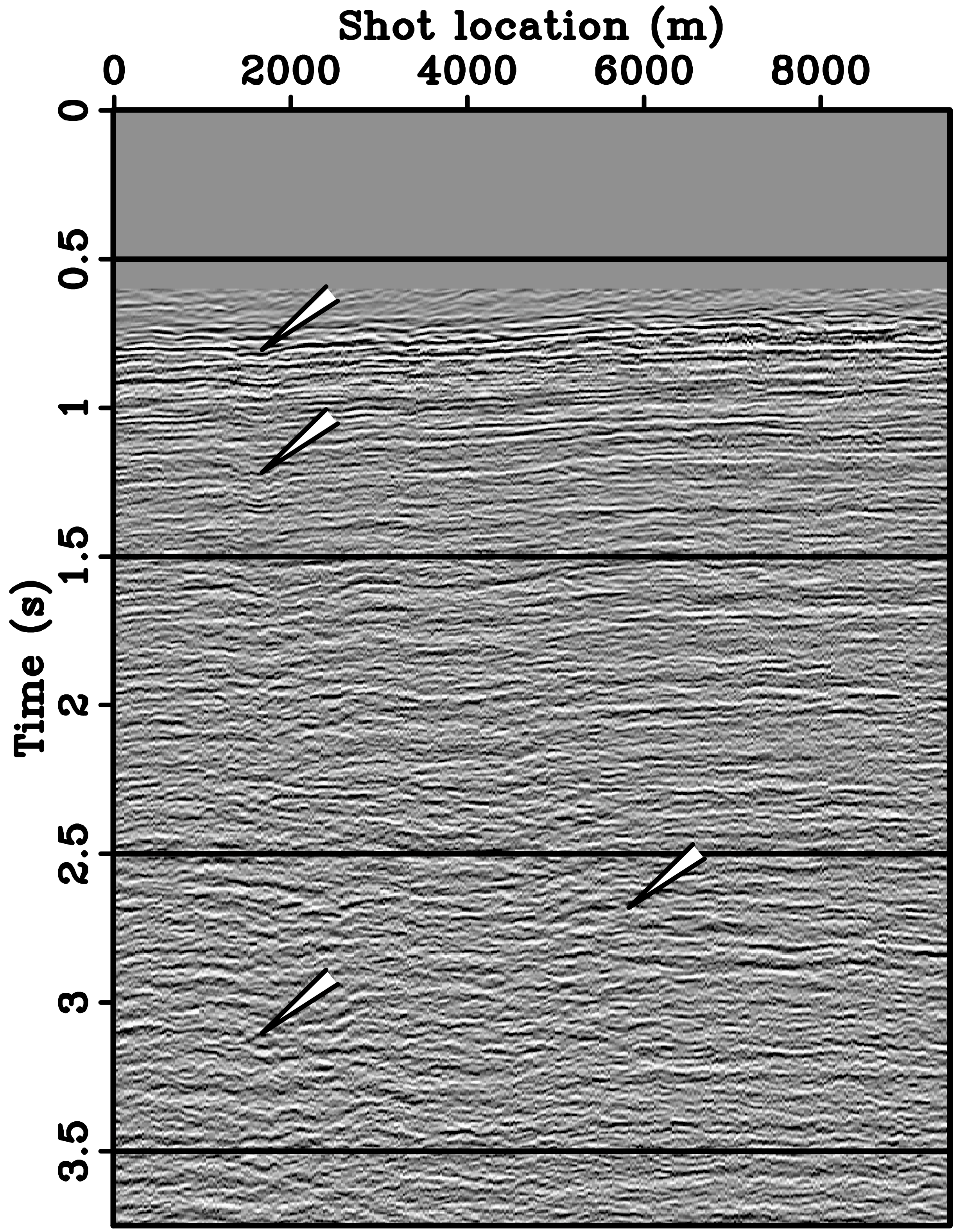
Not scaled Bayesian

Predicted multiples

Real-data example



Scaled Bayesian



Difference between SRME and scaled Bayesian

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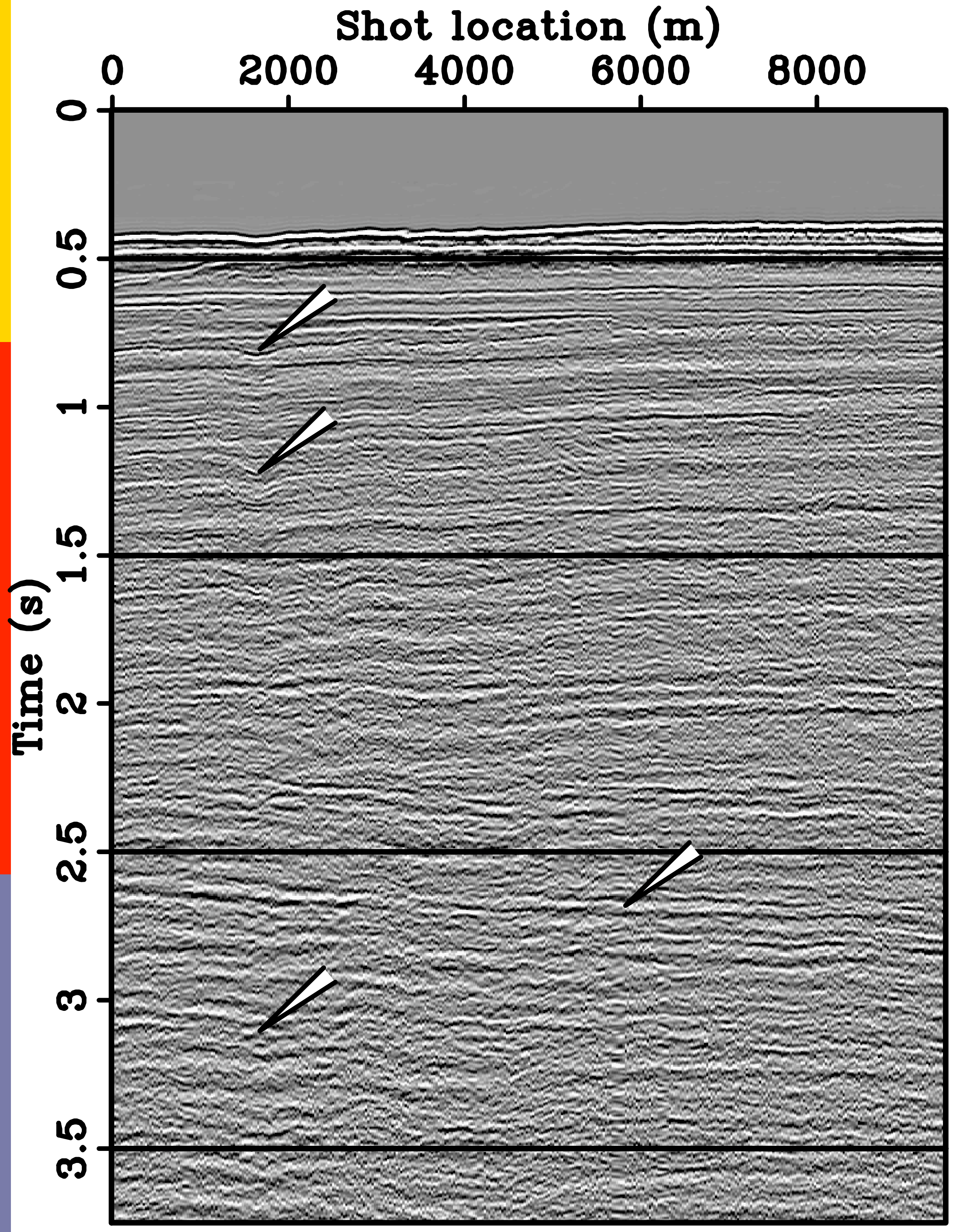
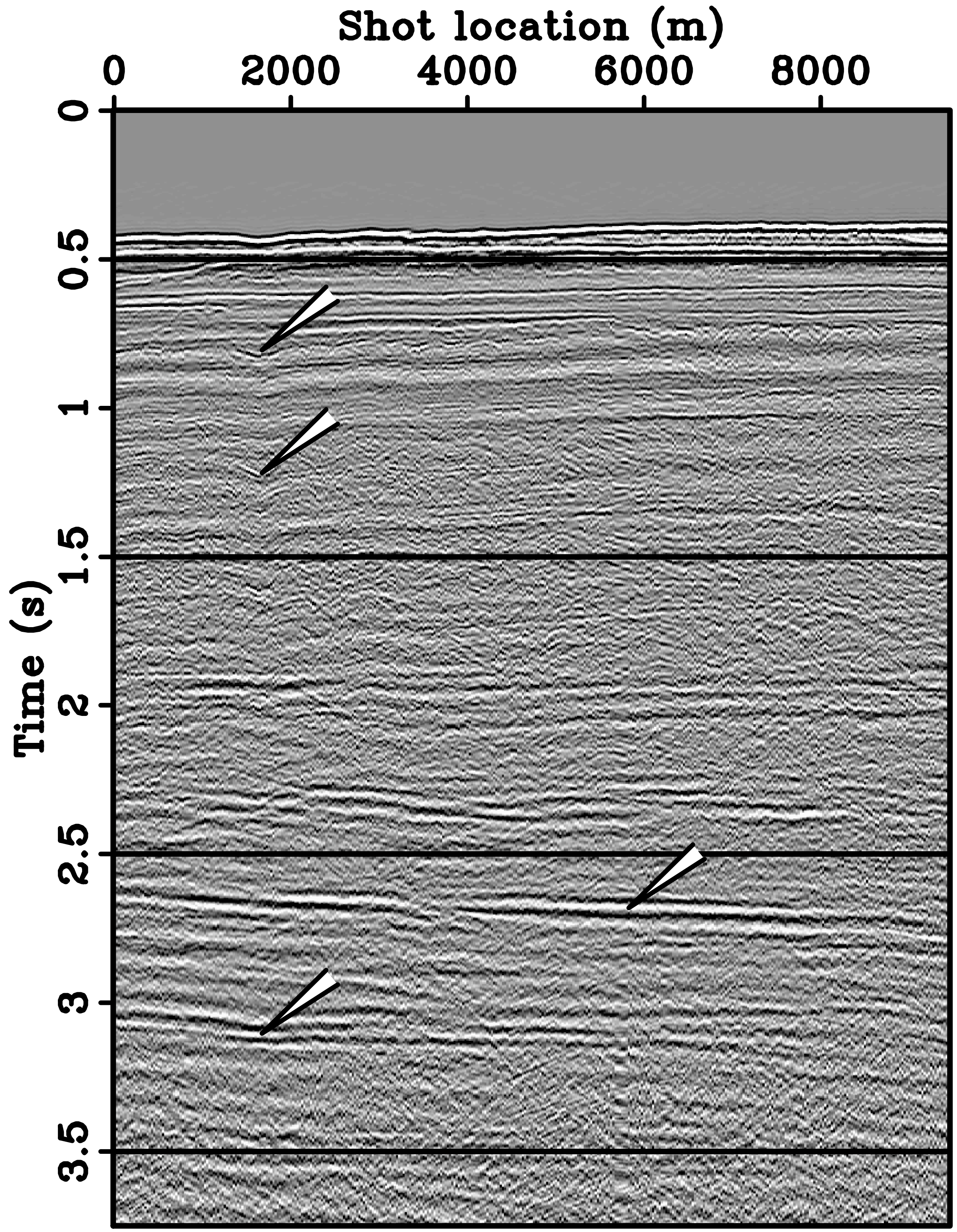
Real-data example



SRME primaries



Scaled Bayesian

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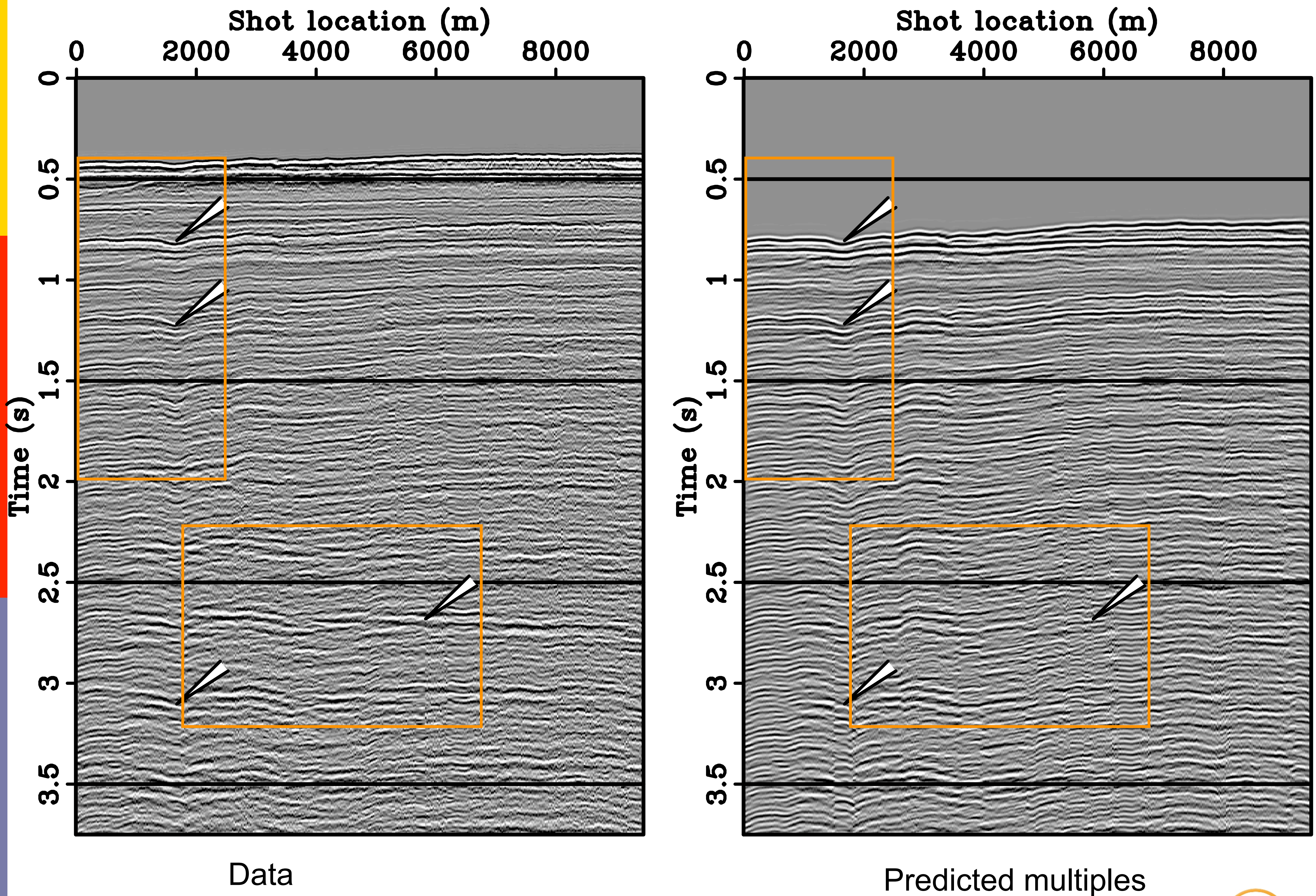
Real-data example



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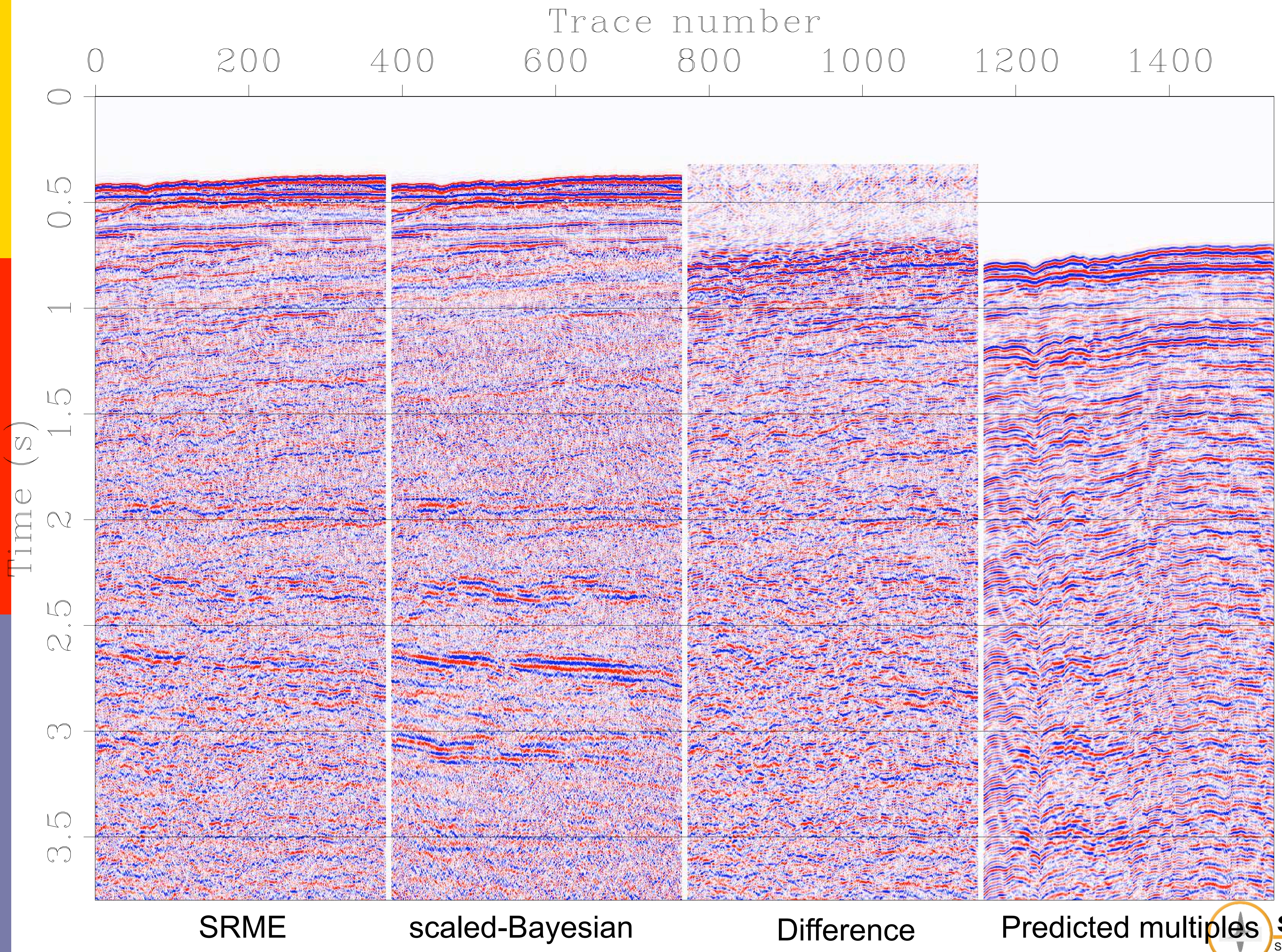
Real-data example



Conclusions

Adaptive curvelet-domain matched filter significantly improves results

- reflected in SNR
- “eye-ball” norm

Results nearly as good as iterative SRME

Appropriate for real data for which iterative SRME is often not an option.

Future plans:

- more case studies
- extension to 3-D

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