Just diagonalize: a curvelet-based approach to seismic amplitude recovery

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Motivation

Migration generally does not correctly recover the amplitudes.

Least-squares migration is computationally unfeasible.

Amplitude recovery (e.g. AGC) lacks robustness w.r.t. noise.

Existing diagonal amplitude-recovery methods

- do not always correct for the order (1 - 2D) of the Hessian [see Symes ‘07]
- do not invert the scaling robustly

Moreover, these (scaling) methods assume that there

- are no conflicting dips (conormal) in the model
- is infinite aperture
- are infinitely-high frequencies
- etc.

Curvelets & seismology

Wish list

A transform that

- detects the reflectors without ***prior*** information on the ***geologic*** dips
- is ***sparse***, i.e. the magnitude-sorted coefficients decay ***fast***
- is relative ***invariance*** under the ***demigration-migration***, i.e. sparse on migrated images

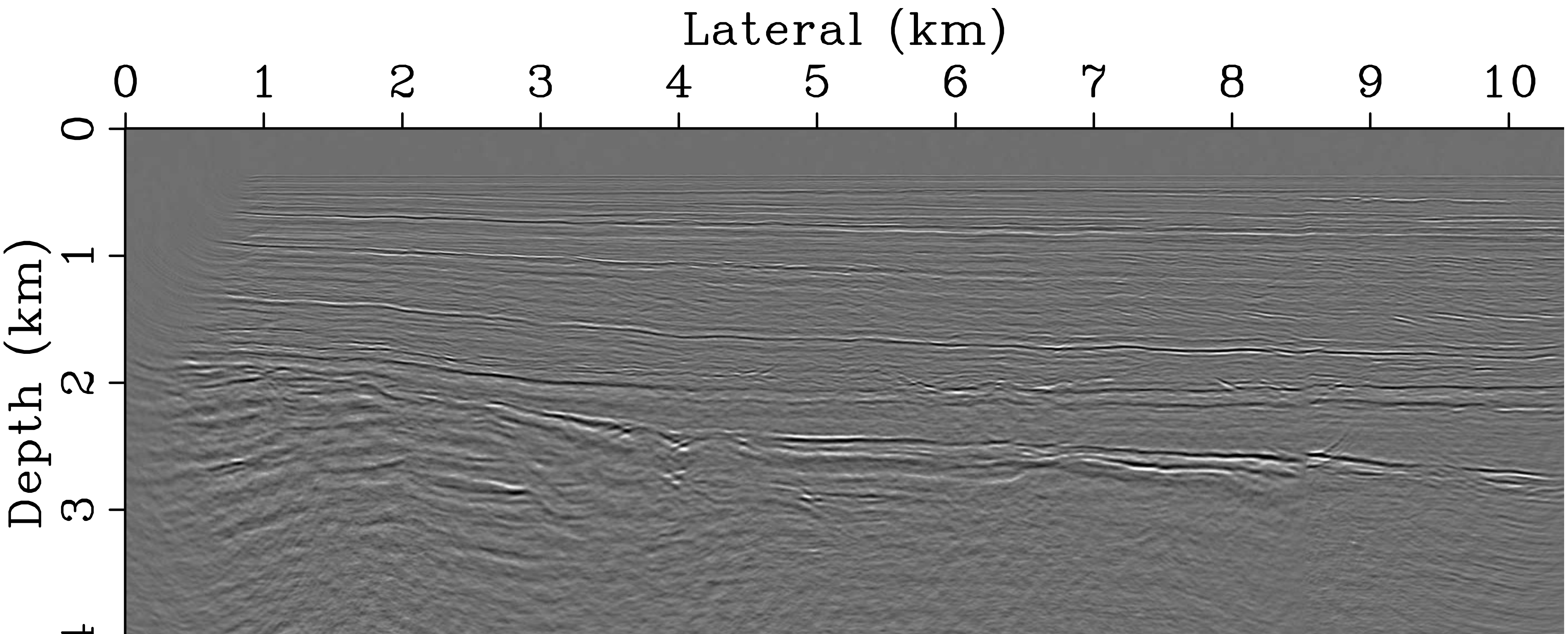
Curvelets

- were "born" from studying high-frequency solution operators for wave propagation*
- diagonalization of migration operators**

***See work by Stein, Smit, Donoho, Candes & Demanet**
**** Main motivation for Douma & de Hoop and Chauris**

Nonlinear approximation

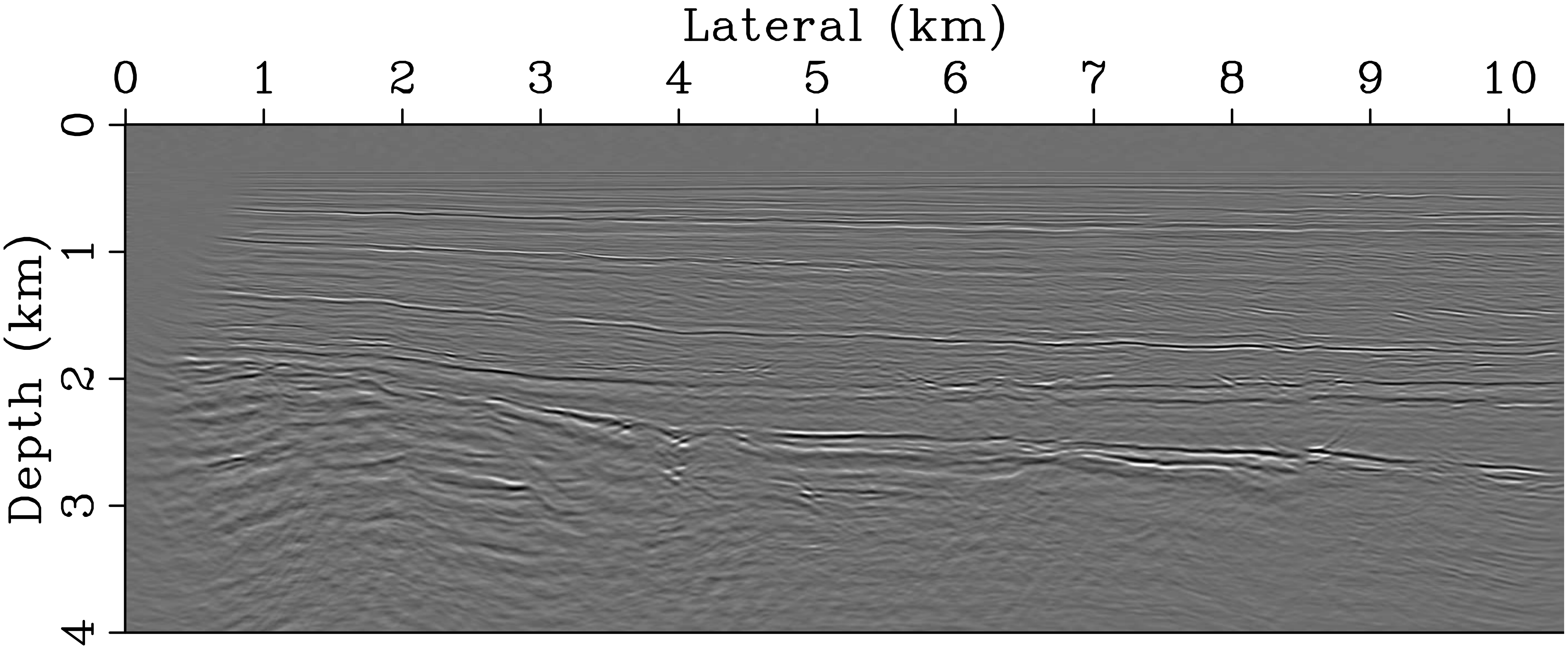
Migrated mobil data set



reconstructed data with p=99

Nonlinear approximation

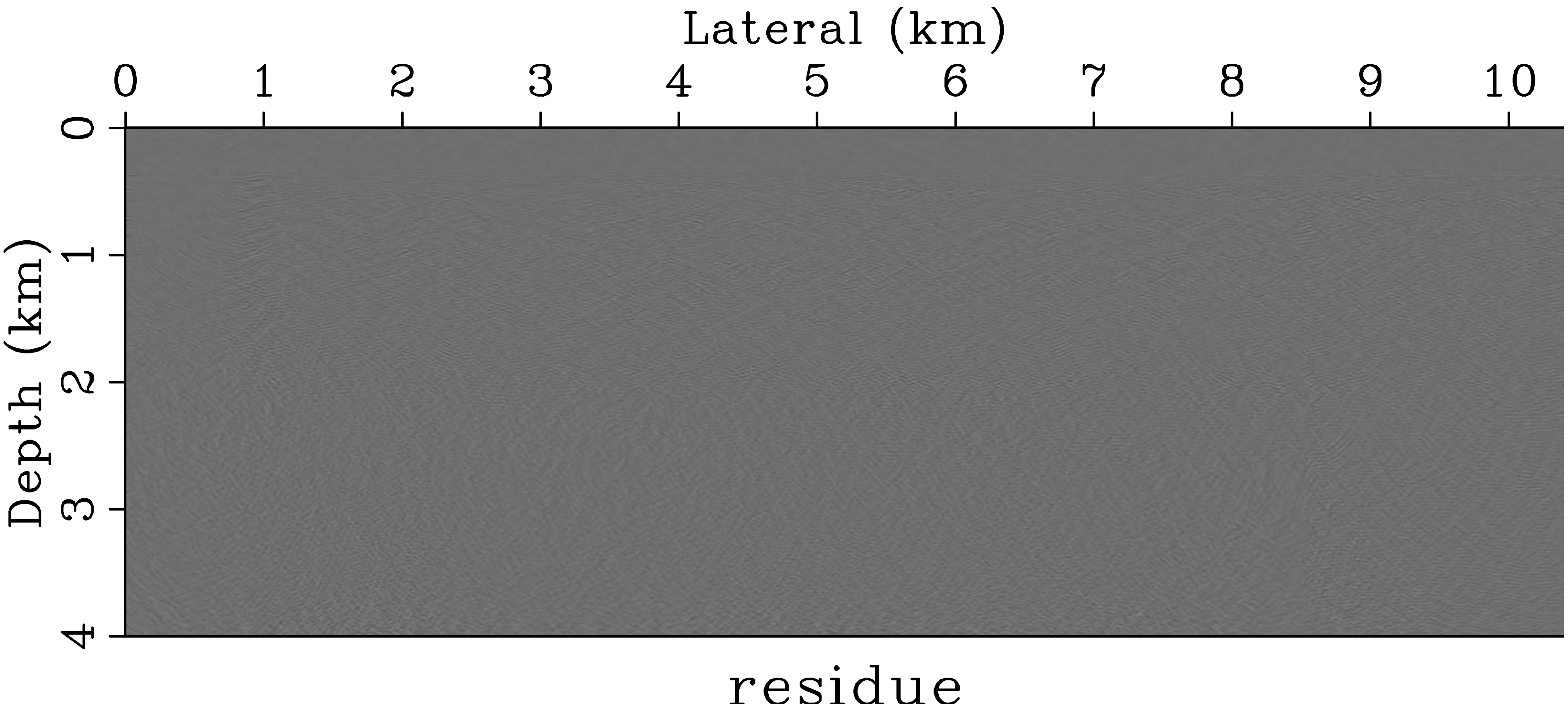
Recovery from largest 3 %



reconstructed data with p=3

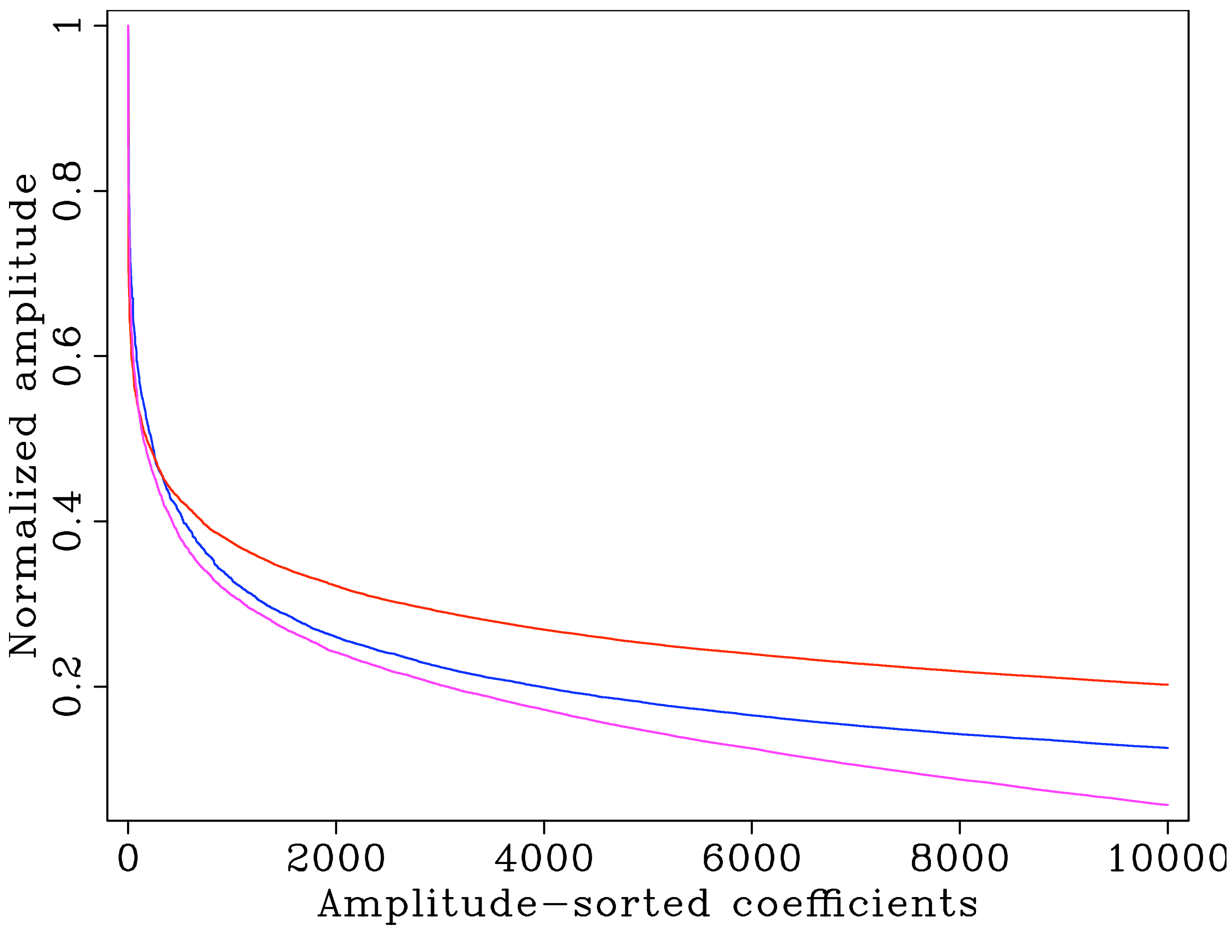
Nonlinear approximation

Difference

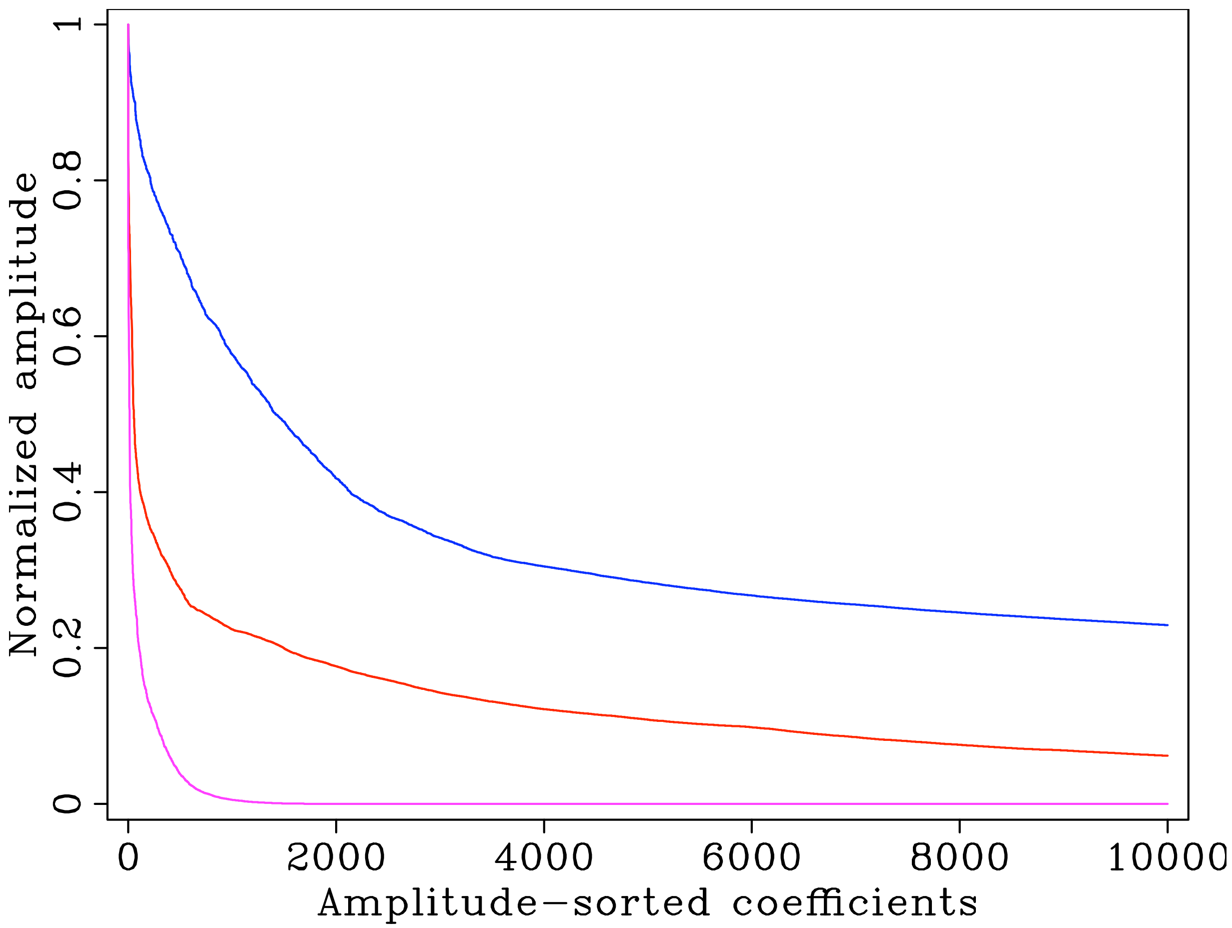


Nonlinear approximation rates

Imaged Mobil data



Reflectivity SEG AA'



Curvelets & wave propagation

Theoretical results that claim that curvelets near diagoanalize migration operators [Demanet et. al, de Hoop]

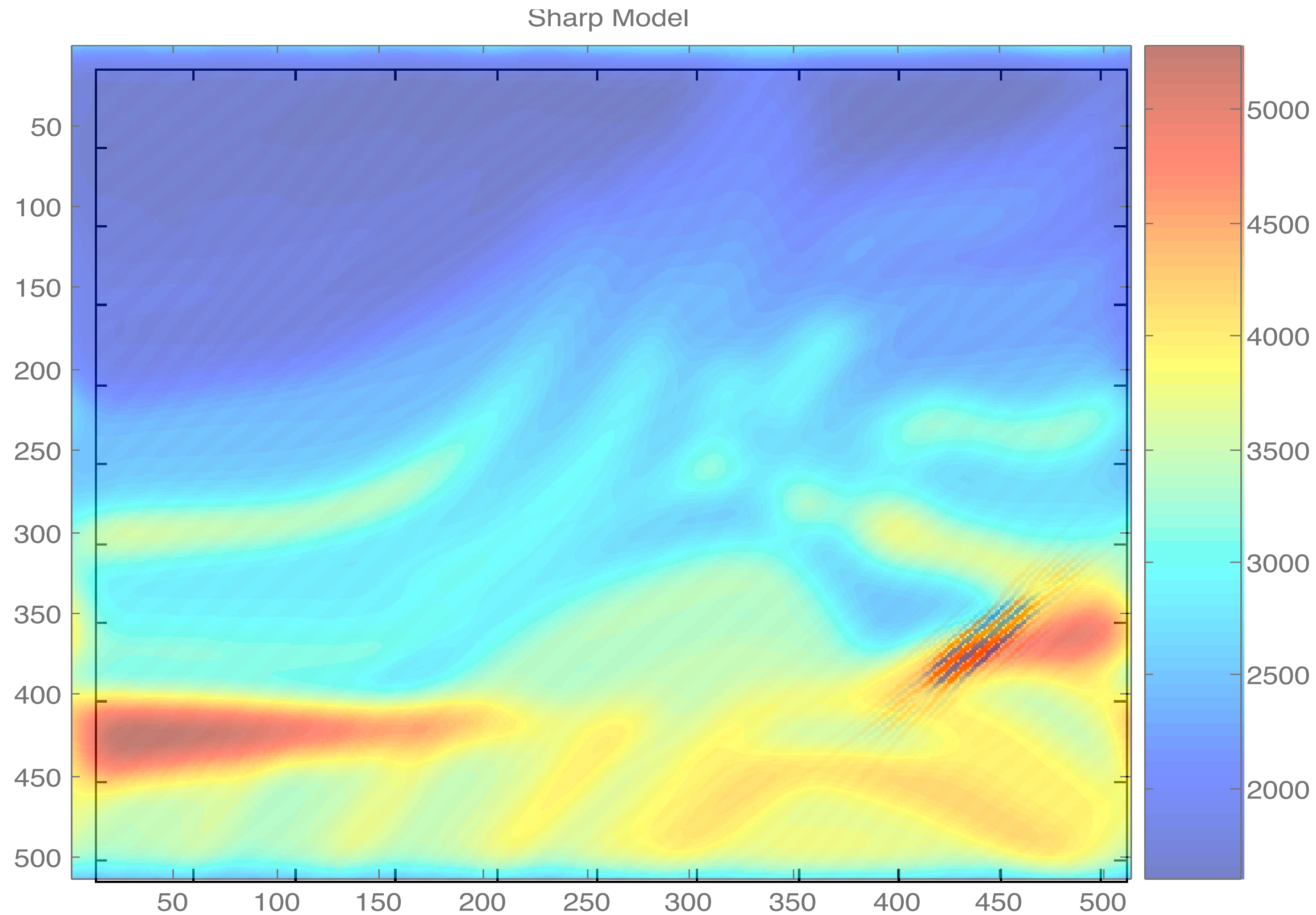
Encouraging results for constant velocity media [Douma & de Hoop; Chauris]

Challenge: discrete curvelets move off the grid

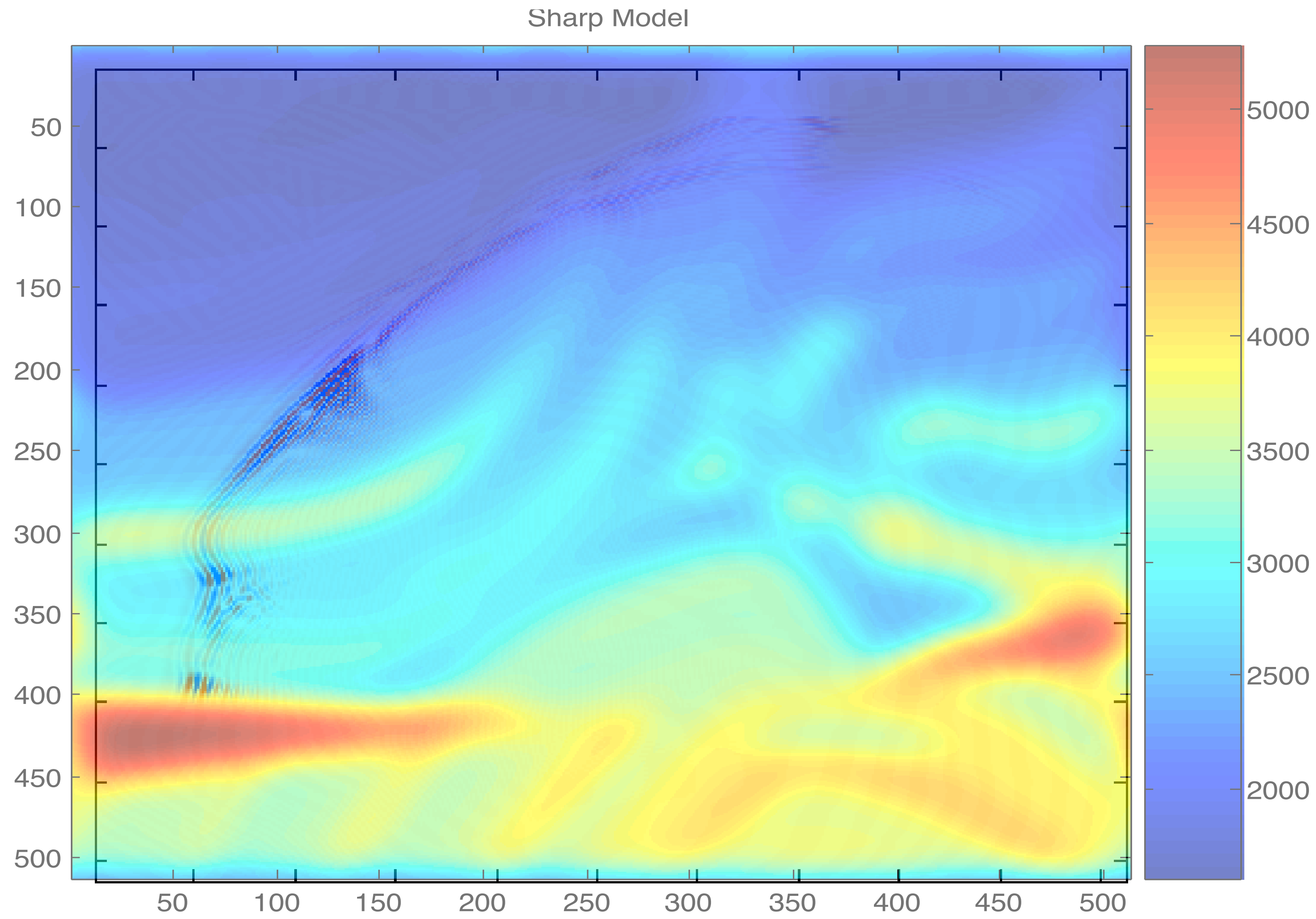
- interpolation
- definition of curvelet molecules [Demanet et. al, de Hoop]

In not so smooth media curvelets spread significantly ....

Curvelet propagation

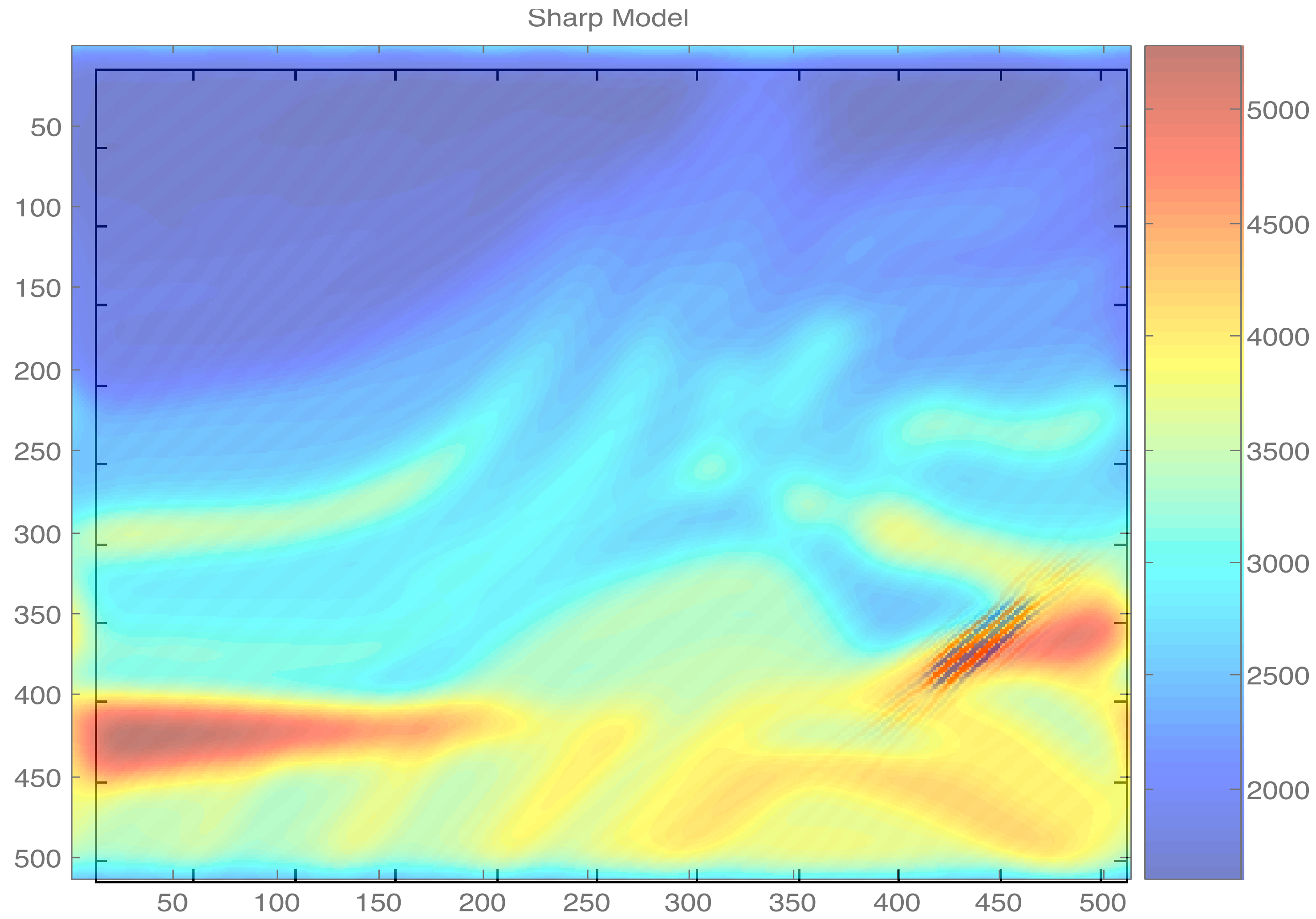


Curvelet propagation



Major challenge. Limit ourselves to migration amplitude recovery!

“Imaged” curvelet



Hessian/Normal operator

[Stolk 2002, ten Kroode 1997, de Hoop 2000, 2003]

Alternative to expensive least-squares migration.

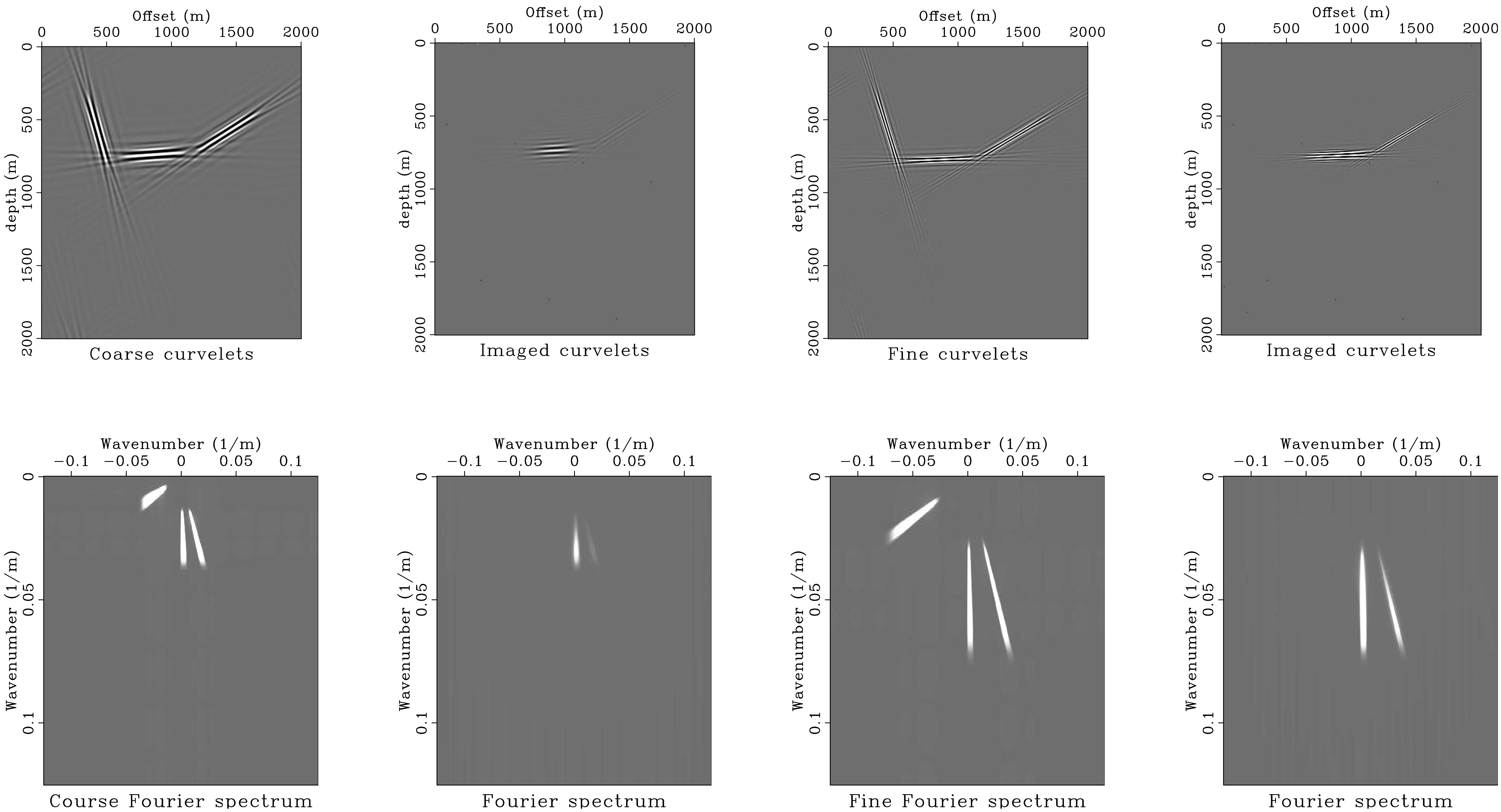
In high-frequency limit Ψ is a PsDO

$$\big(\Psi f\big)(x) := \big(K^T K f\big)(x) = \int_{\mathbb{R}^d} e^{-ix\cdot\xi} a(x,\xi)\hat{f}(\xi)\mathrm{d}\xi$$

- pseudolocal
- singularities are preserved

Corresponds to a spatially-varying dip filter after appropriate preconditioning (=> zero order).

Invariance under Hessian matrix



- curvelets remain invariant
- approximation improves for higher frequencies

Diagonal approximation of the Hessian

Existing scaling methods

Methods are based on a diagonal approximation of Ψ.

- Illumination-based normalization (Rickett '02)
- Amplitude preserved migration (Plessix & Mulder '04)
- Amplitude corrections (Guitton '04)
- Amplitude scaling (Symes '07)

We are interested in an 'Operator and image adaptive' scaling method which

- estimates the action of Ψ from a reference vector close to the actual image
- assumes a smooth symbol of Ψ in space and angle
- does not require the reflectors to be conormal <=> allows for conflicting dips
- stably inverts the diagonal

Approximation

Theorem 1. *The following estimate for the error holds*

$$\|(\Psi(x,D) - C^T \mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C'' 2^{-|\mu|/2},$$

where C'' *is a constant depending on* Ψ.

Allows for the decomposition

$$\begin{aligned}\big(\Psi\varphi_\mu\big)(x) &\simeq \big(C^T \mathbf{D}_\Psi C \varphi_\mu\big)(x) \\ &= \big(AA^T \varphi_\mu\big)(x)\end{aligned}$$

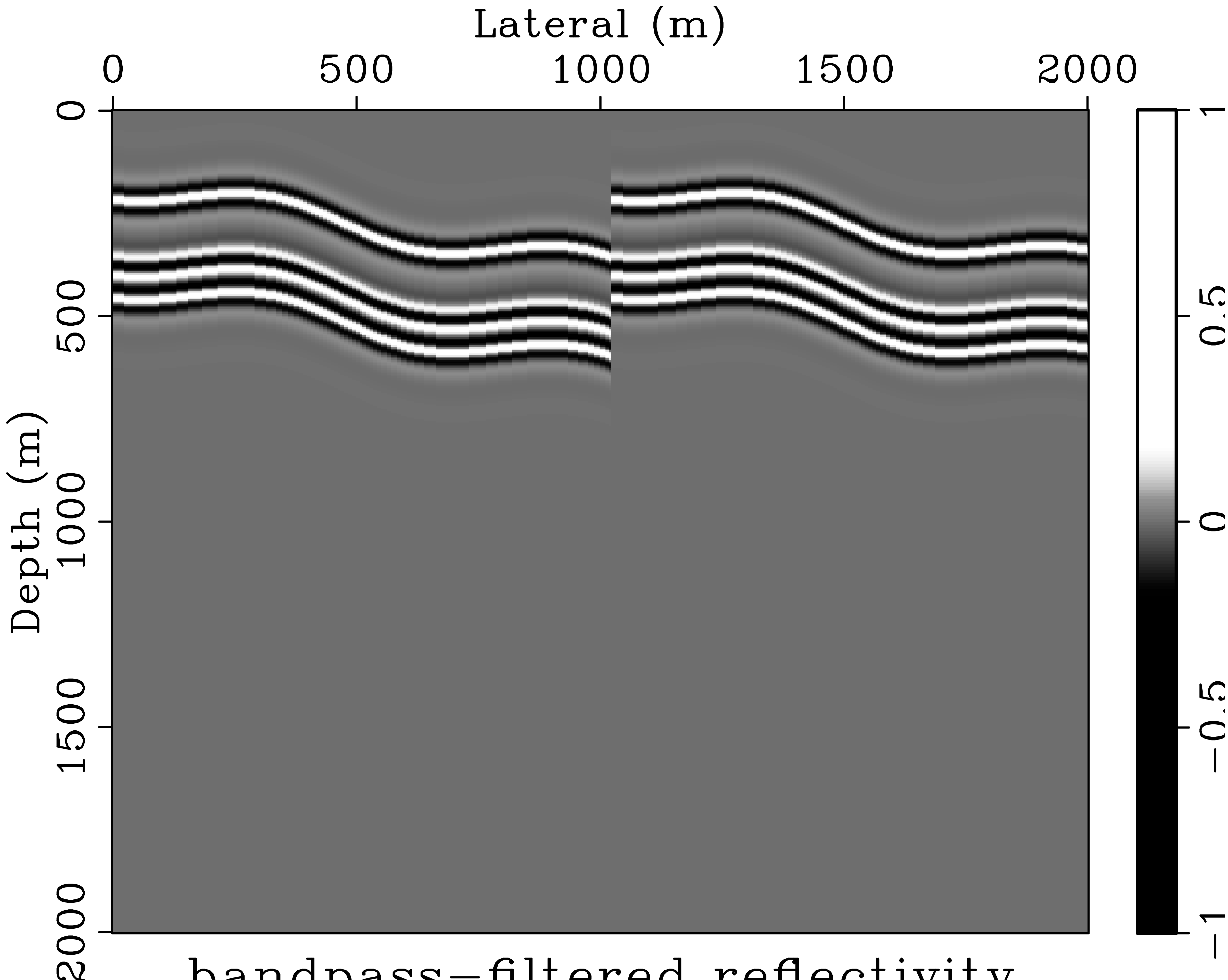
with $A := \sqrt{\mathbf{D}_\Psi} C$ and $A^T := C^T \sqrt{\mathbf{D}_\Psi}$.

Approximation

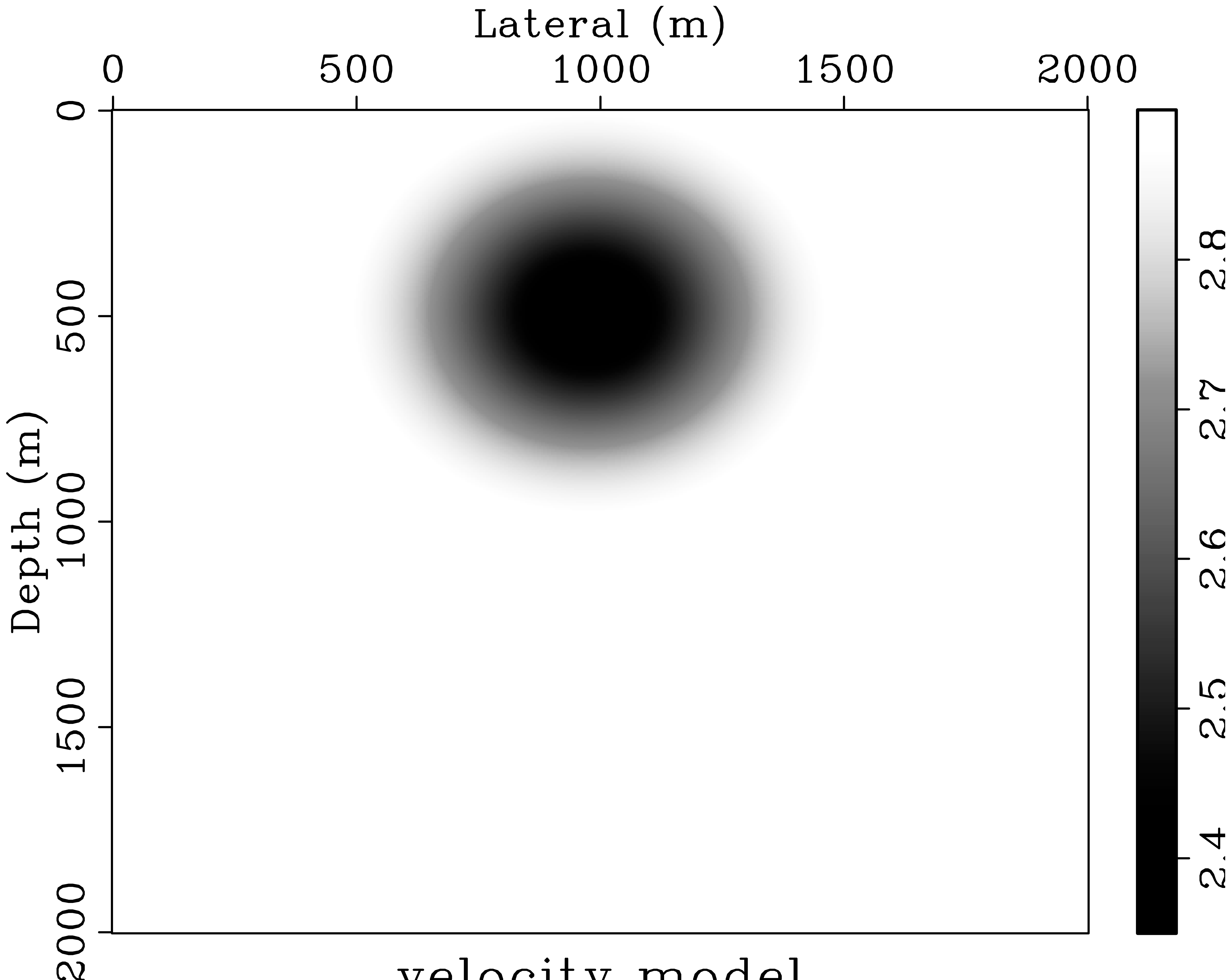
$$\begin{aligned} y(x) &= \big(\Psi m\big)(x) + e(x) \\ &\simeq \big(AA^T m\big)(x) + e(x) \\ &= Ax_0 + e, \end{aligned}$$

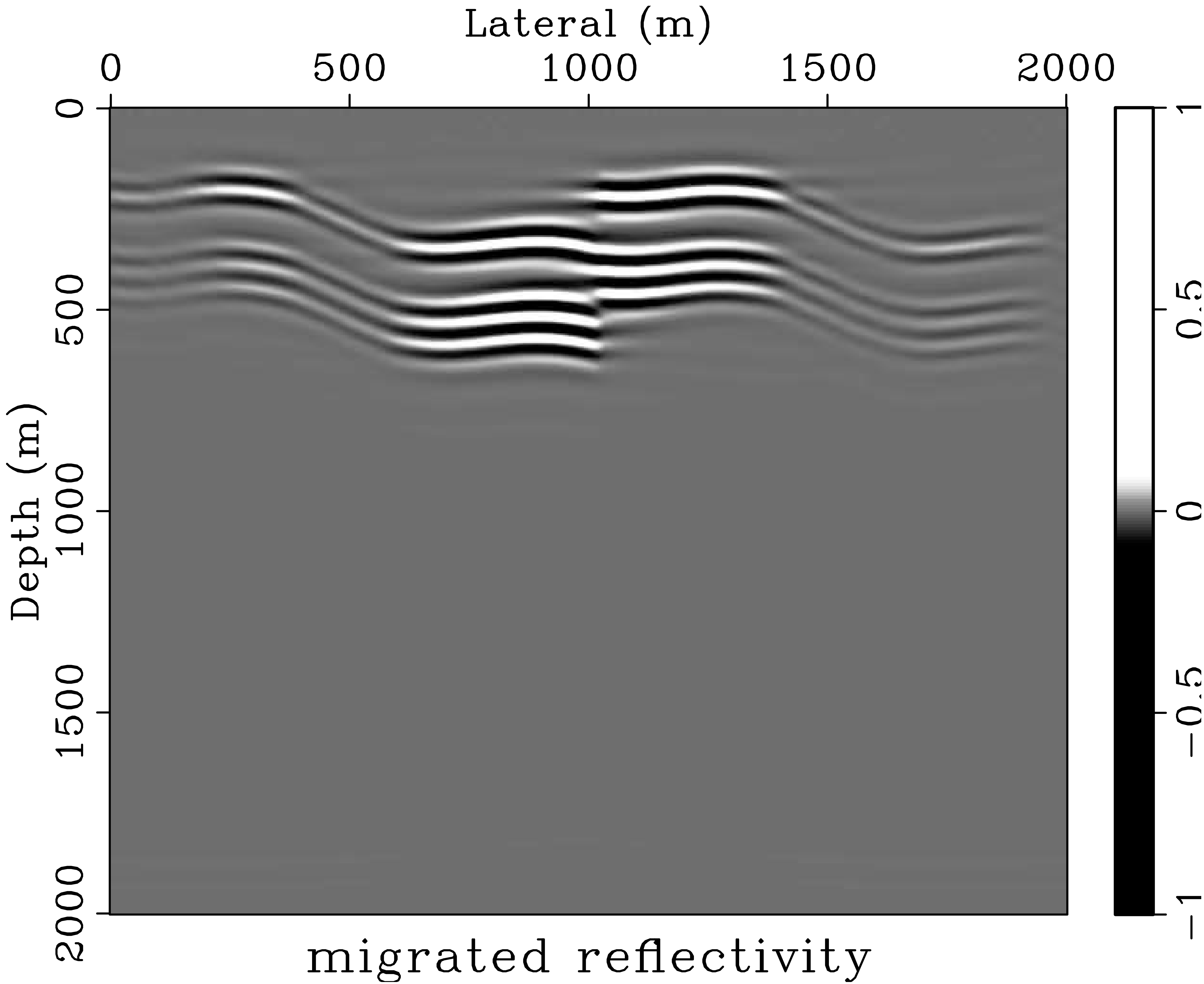
- Wavelet-vagulette like [Donoho, Candes]
- Amenable to nonlinear recovery

Estimation of the diagonal scaling

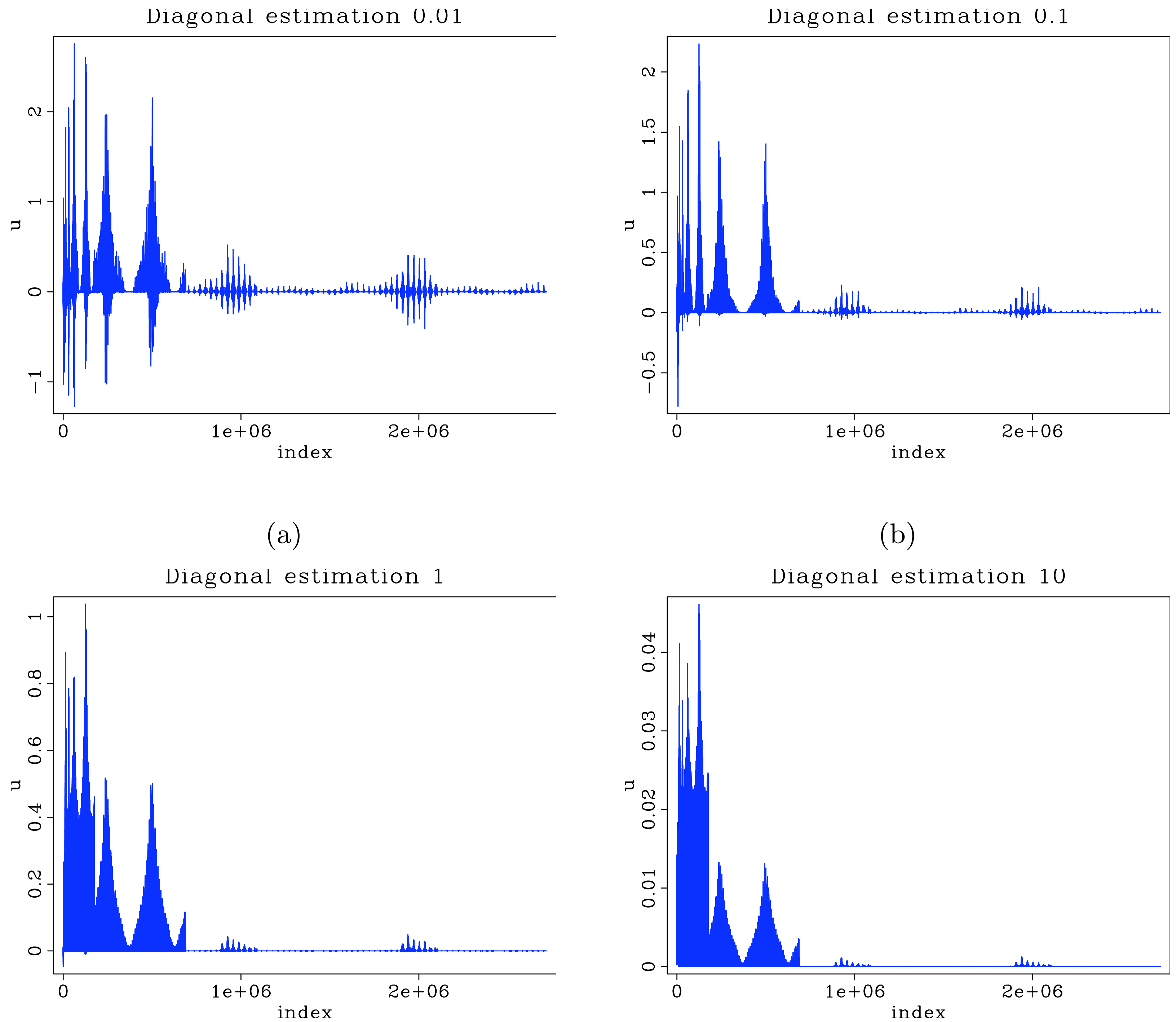


bandpass−filtered reflectivity

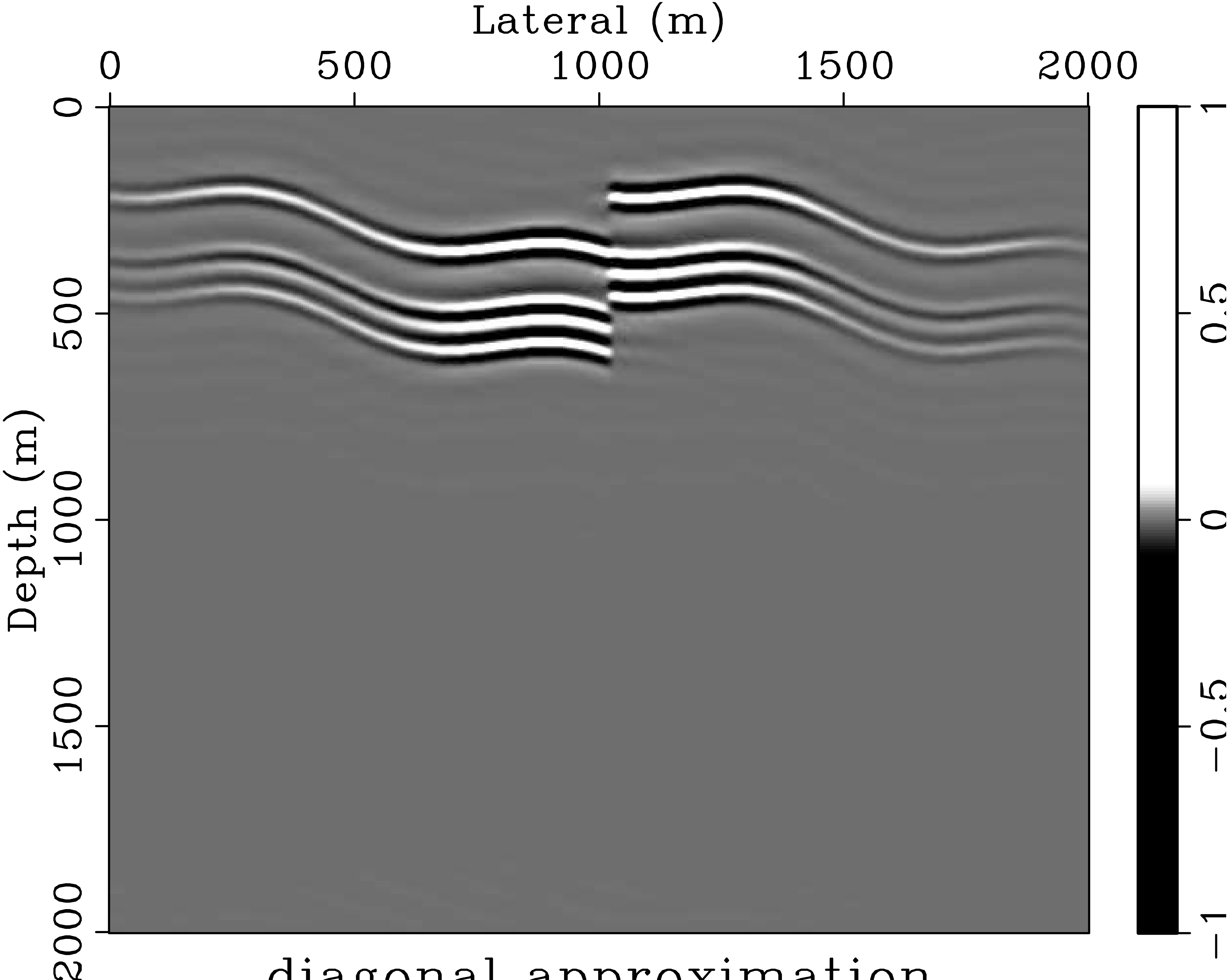

Lateral (m)
0
500
1000
1500
2000
Depth (m)
0
500
1000
1500
2000
2.4
2.5
2.6
2.7
2.8
velocity model


Lateral (m)
0
500
1000
1500
2000
Depth (m)
0
500
1000
1500
2000
1
0.5
0
−0.5
−1
migrated reflectivity

Diagonal estimation



(a) (b)



diagonal approximation

Seismic amplitude recovery

Recovery

Final form

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \boldsymbol{\varepsilon}$$

with $\mathbf{x}_0 = \mathbf{\Gamma C m}$ and $\boldsymbol{\epsilon} = \mathbf{A}\mathbf{e}$.

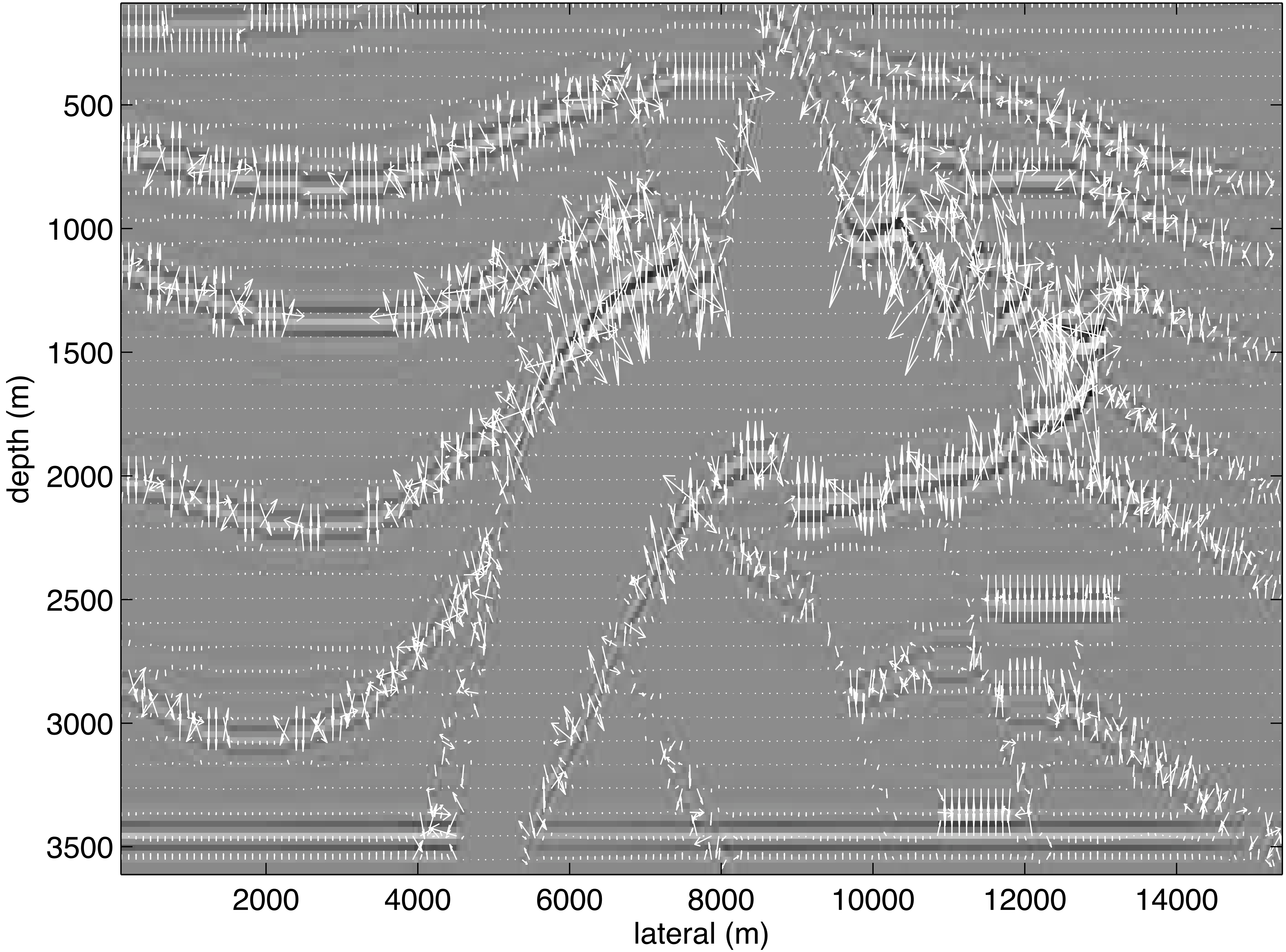
Solve

$$\mathbf{P}: \quad \begin{cases} \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \\ \tilde{\mathbf{m}} = (\mathbf{A}^{\mathbf{H}})^{\dagger}\tilde{\mathbf{x}} \end{cases}$$

with

$$J(\mathbf{x}) = \overbrace{\alpha\|\mathbf{x}\|_1}^{\text{sparsity}} + \beta \underbrace{\|\mathbf{\Lambda}^{1/2}\left(\mathbf{A}^{H}\right)^{\dagger}\mathbf{x}\|_p}_{\text{continuity}}.$$

Gradient of the reference vector



Application to the SEG AA' model

Example

SEGAA’ data:

- “broad-band” half-integrated wavelet [5-60 Hz]
- 324 shots, 176 receivers, shot at 48 m
- 5 s of data

Modeling operator

- Reverse-time migration with optimal check pointing (Symes ‘07)
- 8000 time steps
- modeling 64, and migration 294 minutes on 68 CPU’s

Scaling requires 1 extra migration-demigration

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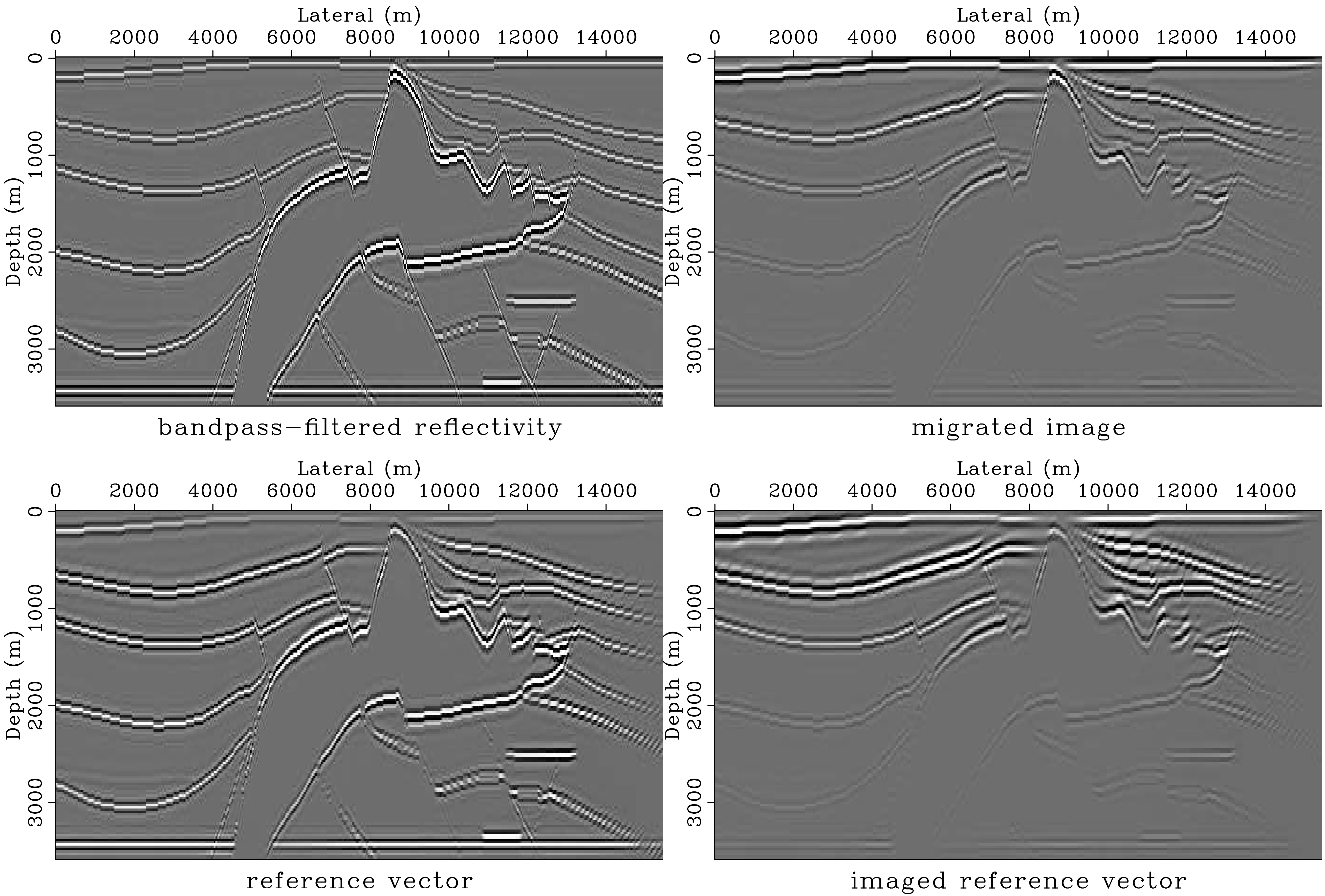
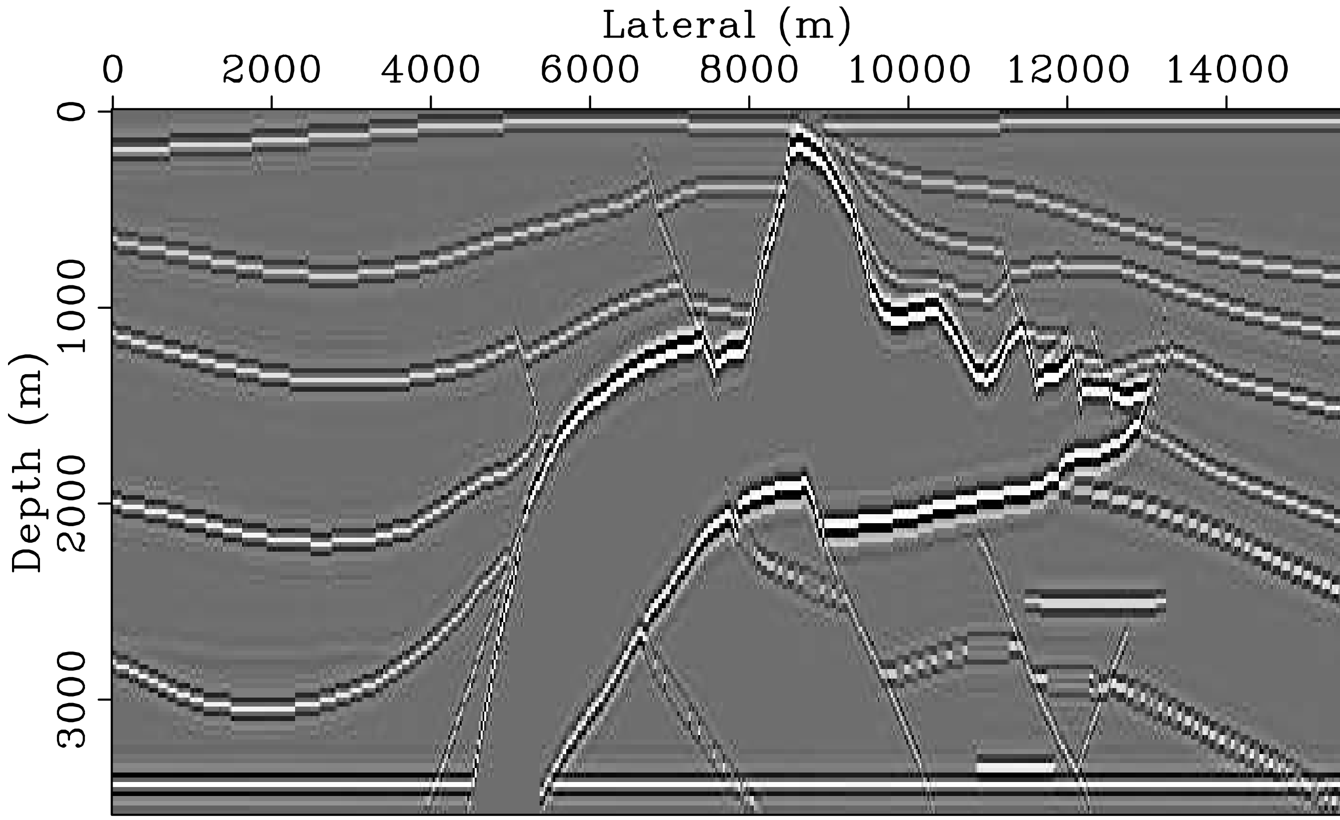
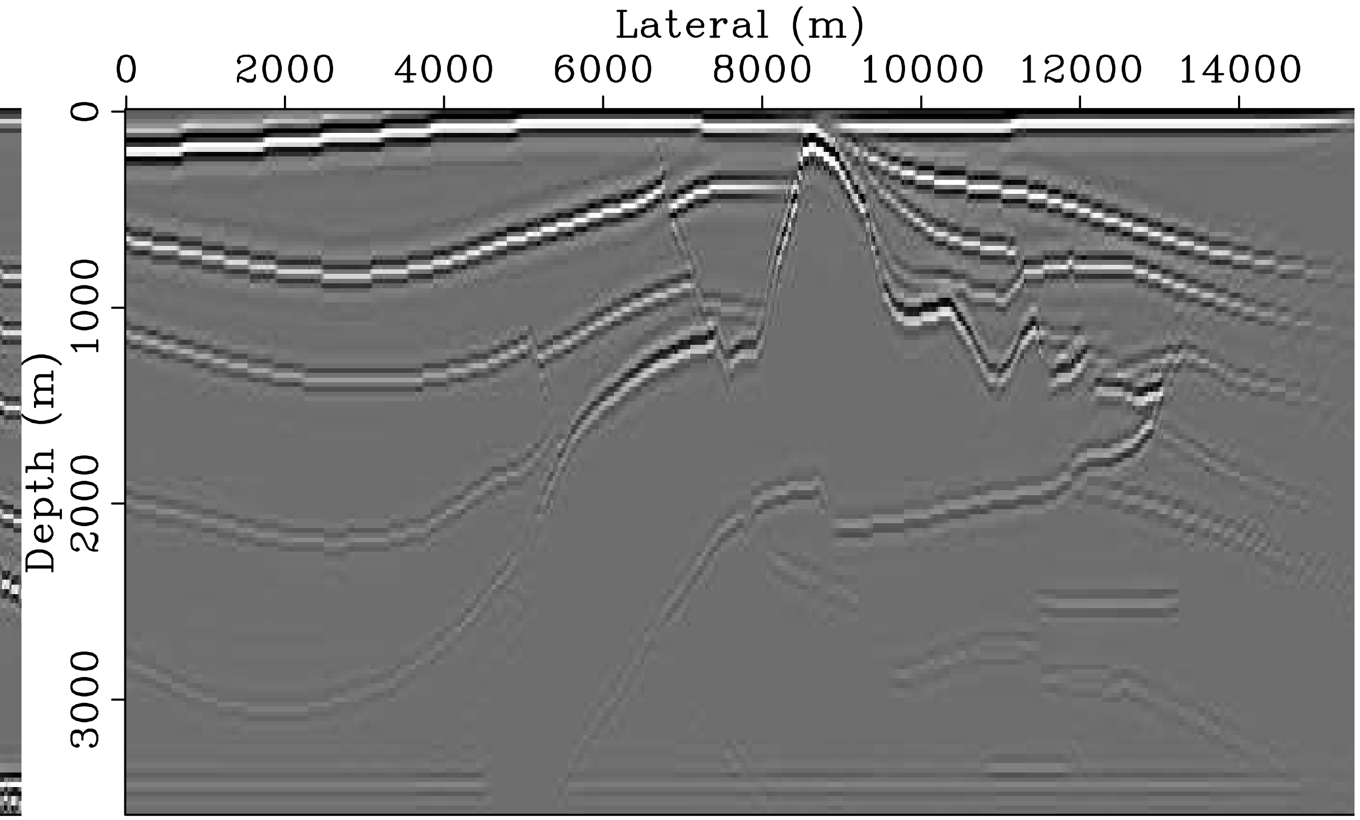
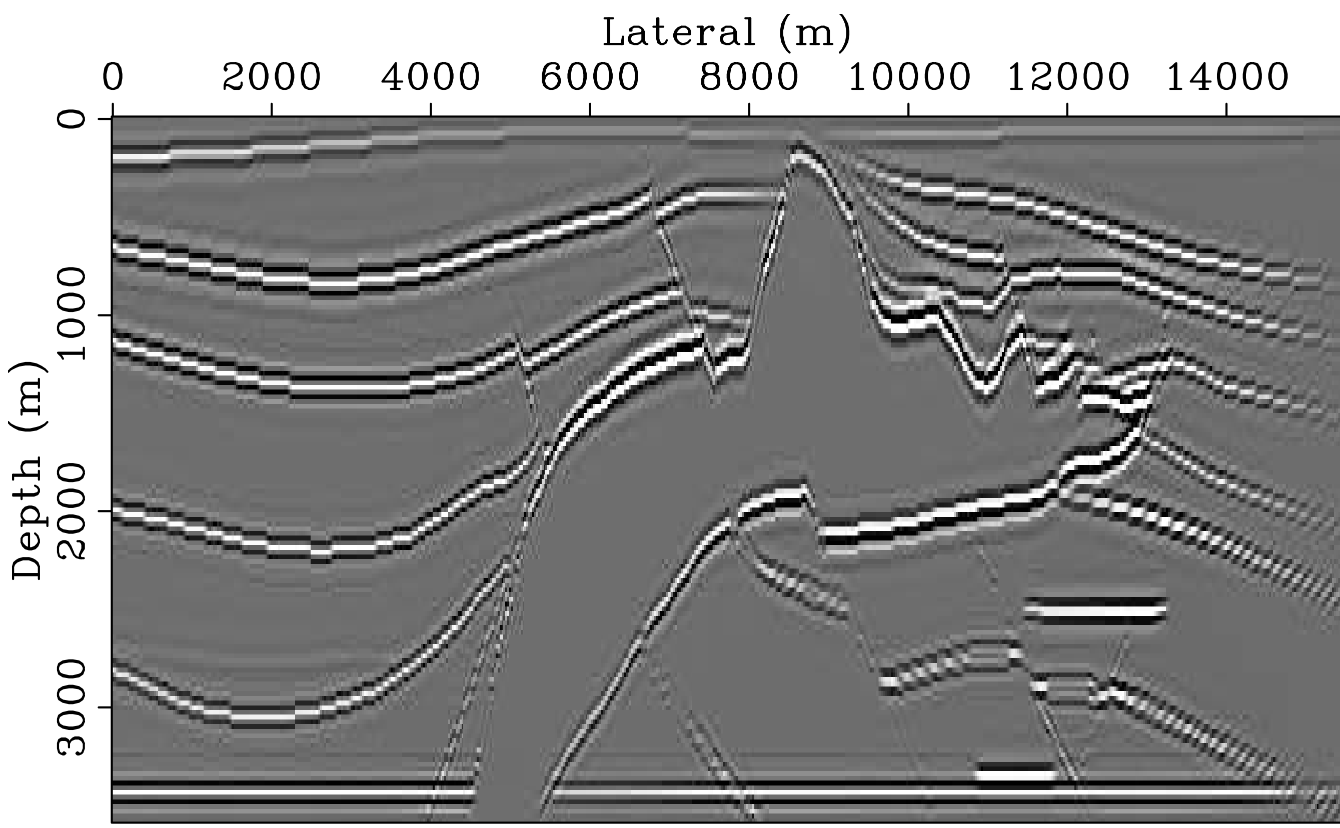

Lateral (m)
0
2000
4000
6000
8000
10000
12000
14000
Depth (m)
0
1000
2000
3000
bandpass–filtered reflectivity
migrated image
reference vector
imaged reference vector



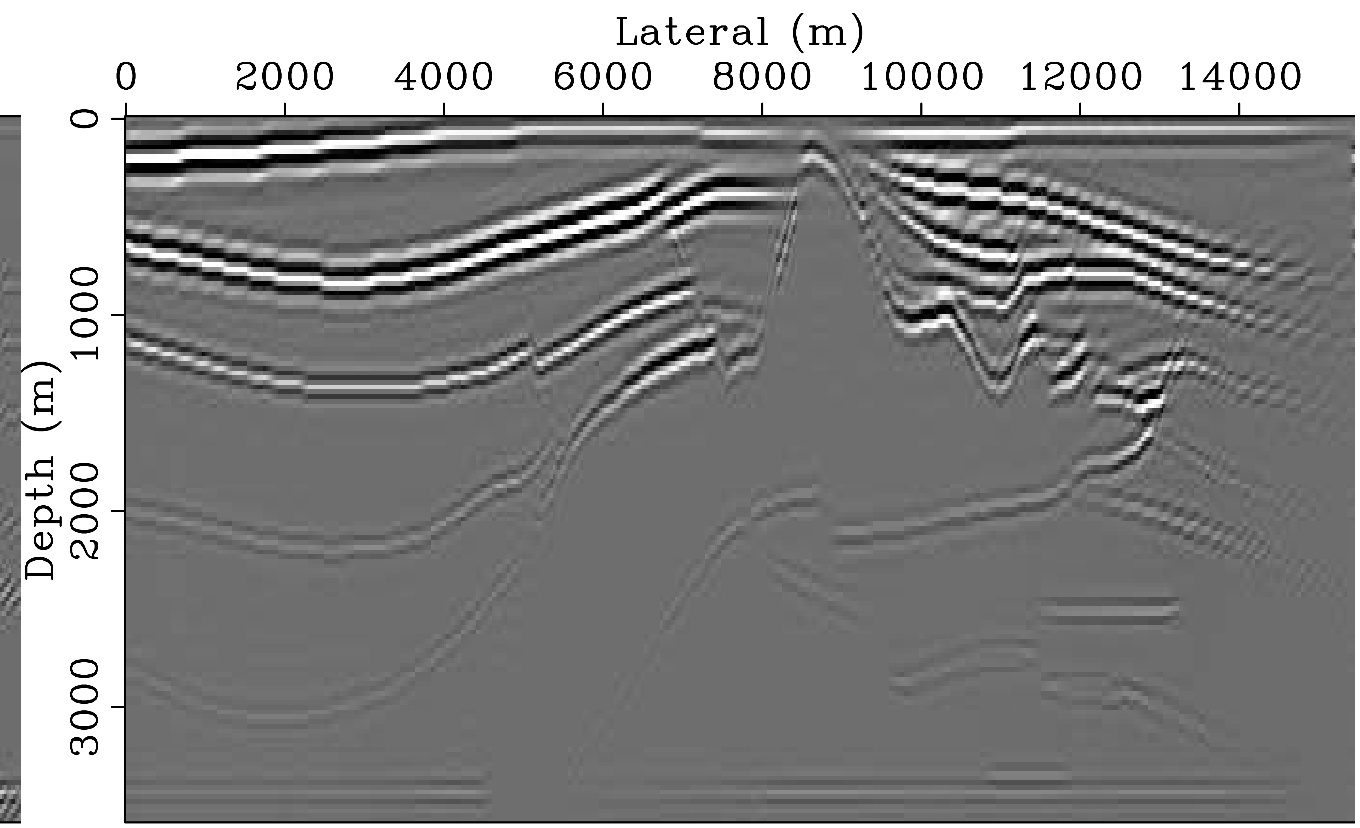
bandpass−filtered reflectivity



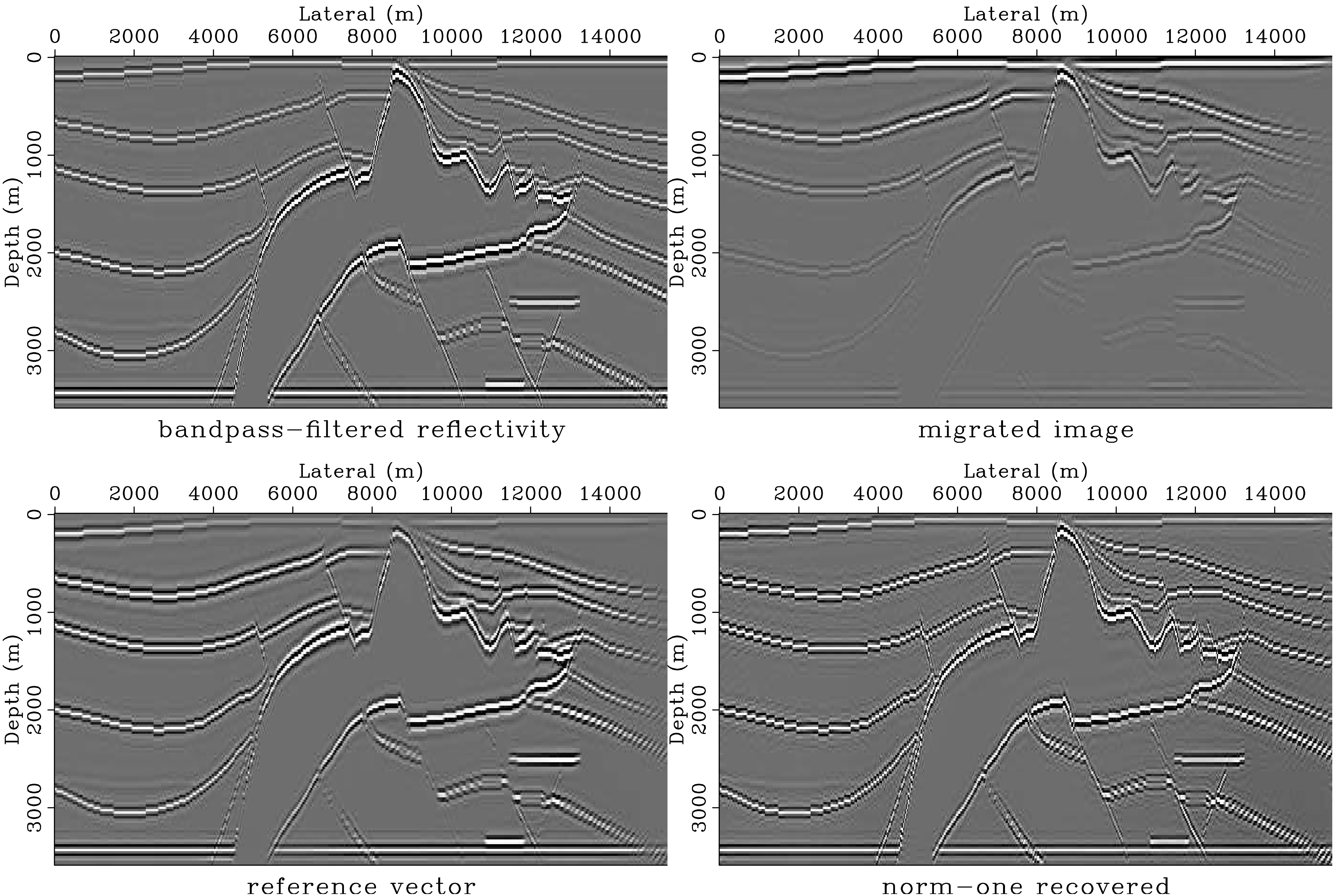
migrated image

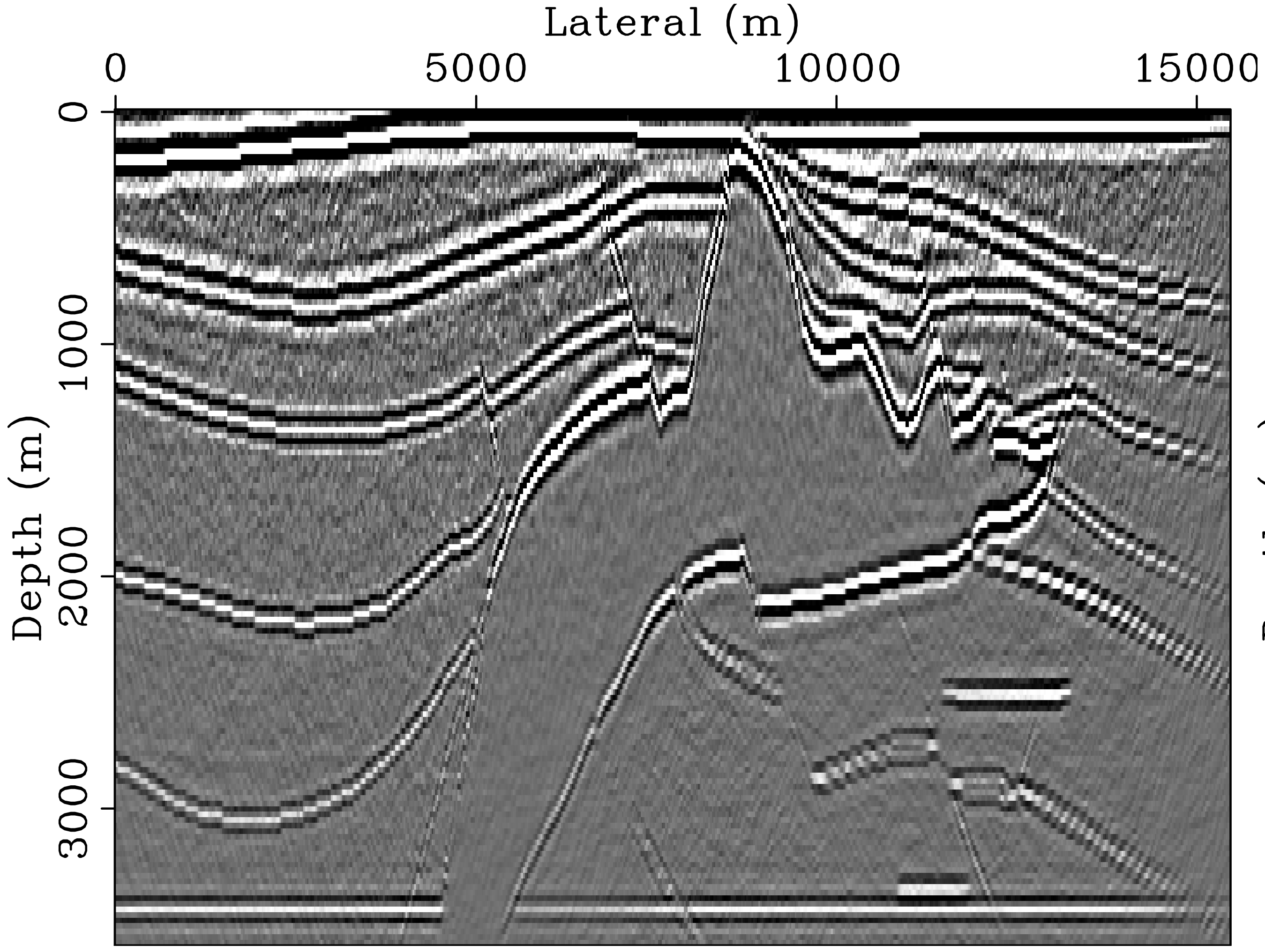


reference vector

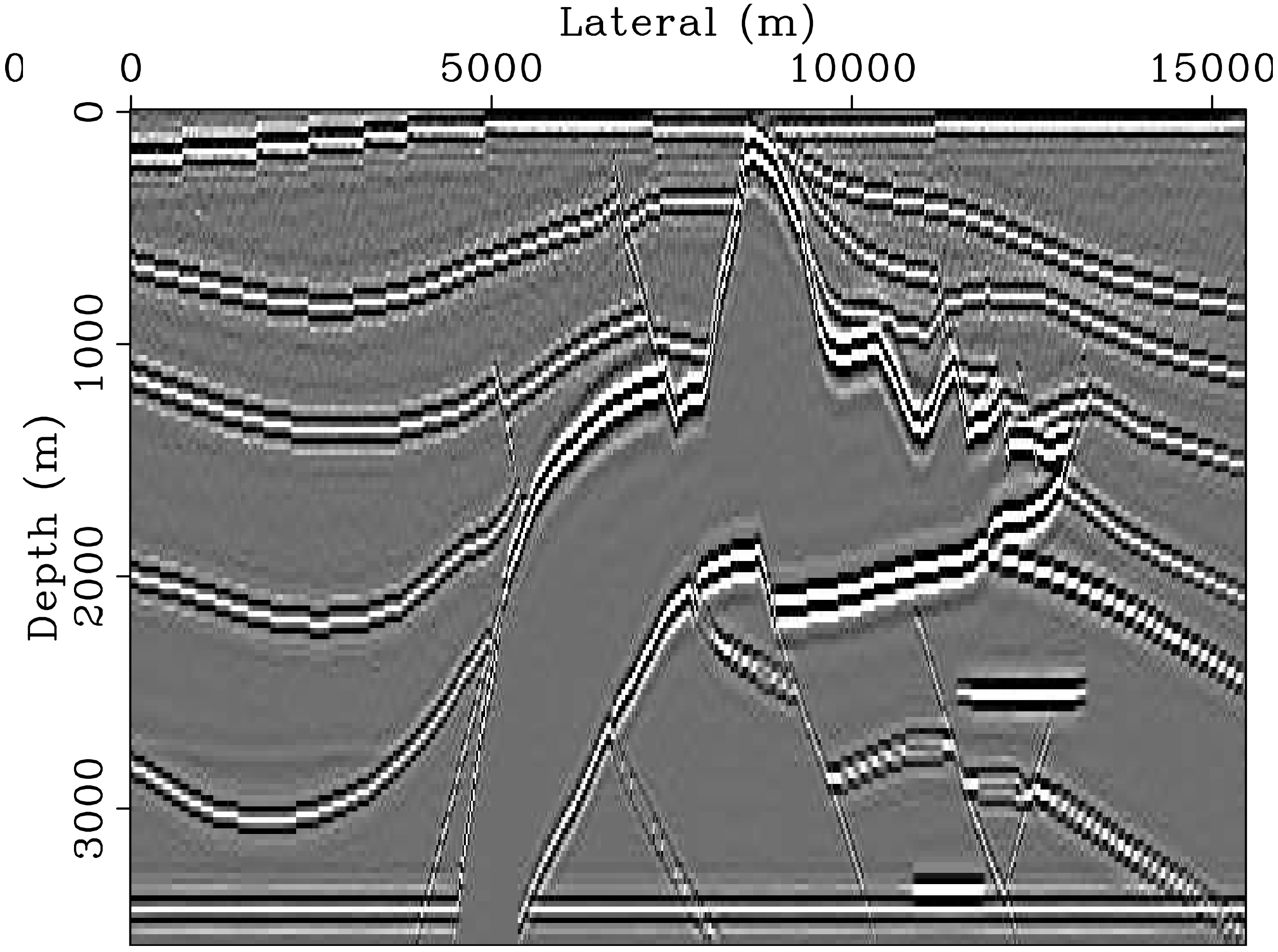


diagonal approximation

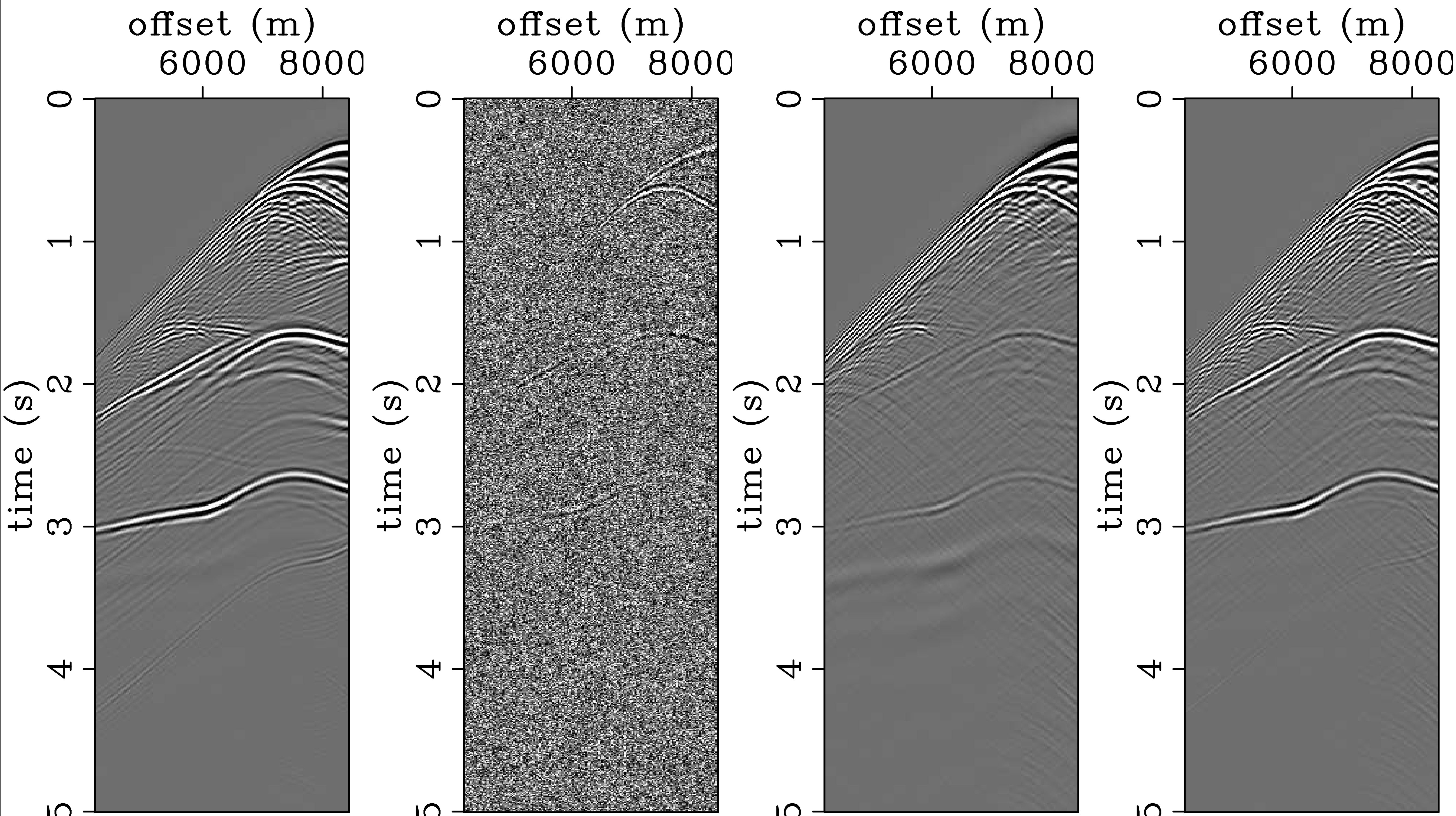

Lateral (m)
Depth (m)
bandpass−filtered reflectivity
migrated image
reference vector
norm−one recovered



Migrated data



Amplitude-corrected & denoised migrated data



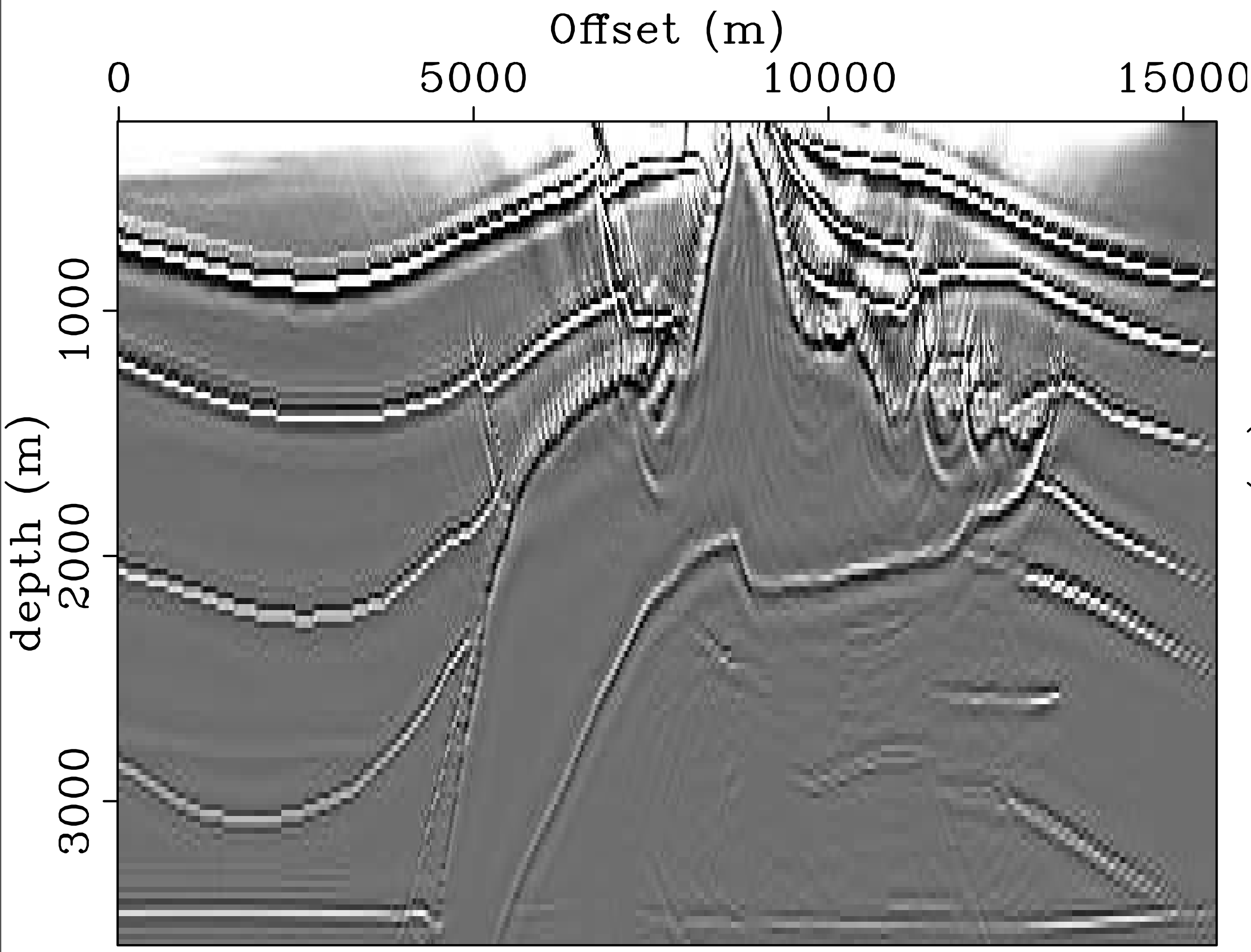
Noise-free data

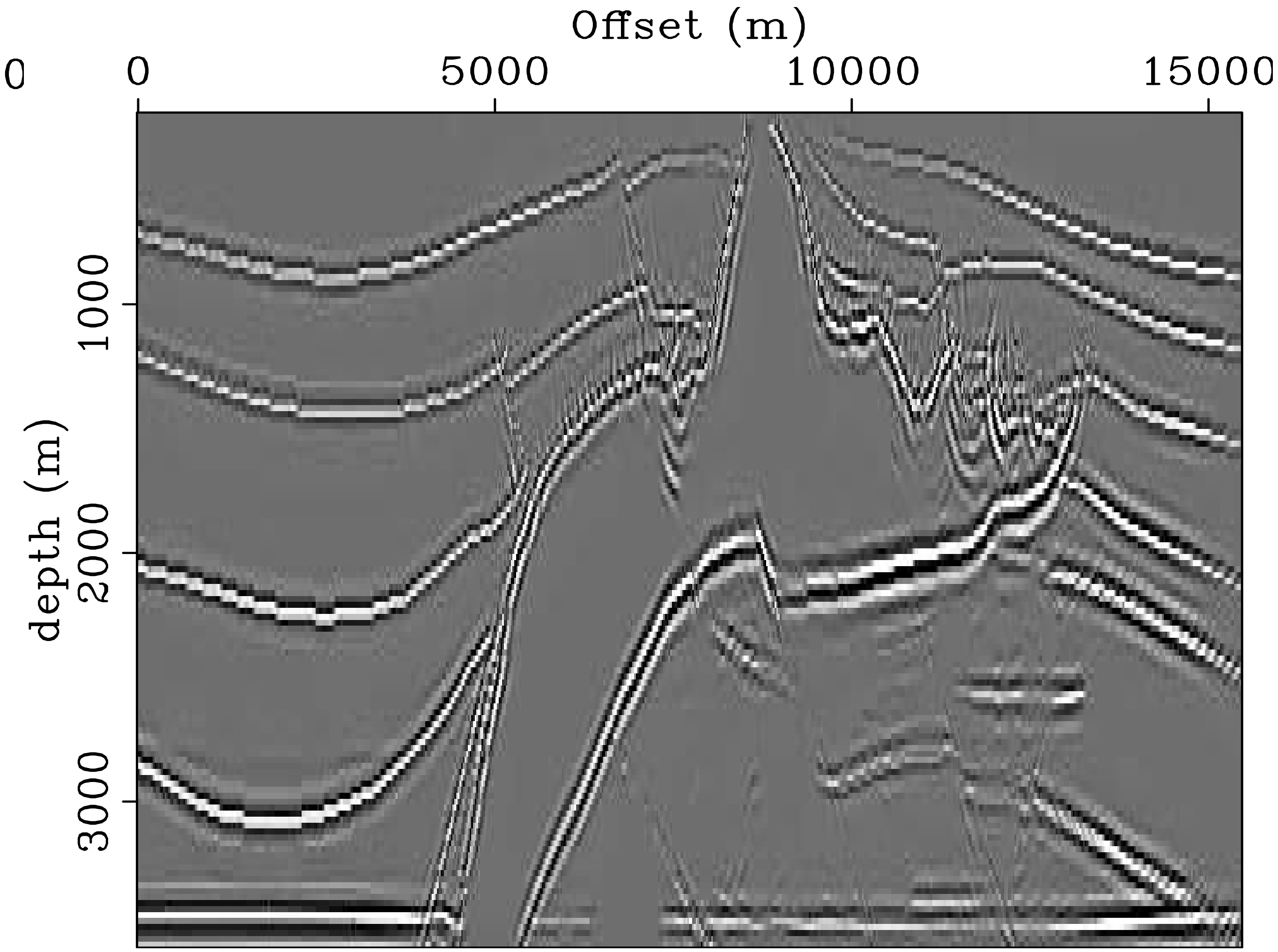
Noisy data (3 dB)

Data from migrated image

Data from amplitude-corrected & denoised migrated image

Nonlinear data





Conclusions

Curvelet-domain scaling

- handles conflicting dips (conormality assumption)
- exploits invariance under the PsDO
- robust w.r.t. noise

Diagonal approximation

- exploits smoothness of the symbol
- uses “neighbor” structure of the curvelet transform

Results on the SEG AA’ show

- recovery of amplitudes beneath the Salt
- successful recovery of clutter
- improvement of the continuity

Acknowledgments

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