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Just diagonalize: a curvelet-based approach to seismic amplitude recovery

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SUMMARY

In this presentation, we present a nonlinear curvelet-based sparsity-promoting formulation for the recovery of seismic amplitudes. We show that the curvelet's wavefront detection capability and invariance under wave propagation lead to a formulation of this recovery problem that is stable under noise and missing data.

INTRODUCTION

In this paper, a recent application of the discrete curvelet transform (see e.g. Candes et al., 2006; Hennenfent and Herrmann, 2006) is presented presented that involves the restoration of migration amplitudes. Our approach derives from two properties of curvelets, namely

- **detection of wavefronts** (see e.g. Candès and Donoho, 2005; Candes et al., 2006; Hennenfent and Herrmann, 2006), without prior information on the positions and local dips;
- relative **invariance** (see e.g. Candès and Demanet, 2005, and the ancillary electronic material) of curvelets under wave propagation.

These properties render this transform suitable for a robust formulation of data regularization the migration-amplitude recovery extending earlier work (Herrmann, 2003; Herrmann et al., 2005, and the technical report ”Sparsity- and continuity-promoting seismic image recovery with curvelet frames”, submitted for publication by the same authors and Chris Stolk). This method derives from sparsity in the curvelet domain that is a consequence of the above properties. This sparsity corresponds to a rapid decay for the magnitude-sorted curvelet coefficients and admits a separation of (coherent) ’noise’ and ’signal’. This separation underlies the successful application of this transform to exploration seismology (see e.g. Hennenfent and Herrmann, 2006; Herrmann et al., 2007).

Sparsity promoting inversion

To exploit curvelets, (in)complete and noisy measurements are related to a sparse curvelet coefficient vector, $\mathbf{x}_0$, according to

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \mathbf{n} \tag{1}$$

with $\mathbf{y}$ a vector with noisy and possibly incomplete measurements of $\mathbf{f}$; $\mathbf{A}$ the synthesis matrix that includes the inverse curvelet transform; and $\mathbf{n}$, zero-centered white Gaussian noise. The $\mathbf{A}$ is a wide rectangular matrix, so the vector $\mathbf{x}_0$ can not readily be calculated from the measurements, because there exist infinitely many vectors that match $\mathbf{y}$.

Recent work in ’compressive sensing’ or ’stable signal recovery’ has shown that rectangular matrices can stably be inverted by solving a nonlinear sparsity promoting program (Elad et al., 2005; Candès et al., 2006).

Following this approach, the vector $\mathbf{x}_0$ can be recovered from noise-corrupted and incomplete data. New in this approach is (i) the sparsity promoting multiscale and multi-angular curvelet transform that obtains near optimal theoretical (see e.g. Candès and Donoho, 2000) and empirical (Candes et al., 2006; Hennenfent and Herrmann, 2006) compression rates on seismic data and images; (ii) the theoretical understanding of the conditions for a successful recovery. This work applies these recent developments and involves the solution of the norm-one nonlinear program:

$$\mathbf{P}_\epsilon : \qquad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases} \tag{2}$$

in which $\mathbf{S}^T$ is the inverse sparsity transform and ϵ, a noise-dependent tolerance level. This constrained optimization problem is solved to within ϵ. The $\arg\min_{\mathbf{x}}$ stands for the argument of the minimum, i.e., the value of the given argument for which the value of the expression attains its minimum value. The vector $\widetilde{\mathbf{m}}$ stands for the recovered image with the symbol $\widetilde{}$ reserved for quantities obtained by optimization.

Curvelet-based seismic image recovery by sparsity and continuity promoting inversion

Following an extension of earlier work (Herrmann, 2003; Herrmann et al., 2005) and amplitude scaling (Rickett, 2003) – dating back to ideas by William Symes and recently reported in the technical report "TR 06-19: Optimal Scaling for Reverse Time Migration" technical report by William Symes – issues with remaining unknown clutter and deteriorated migration amplitudes are addressed. After preprocessing, seismic data is given by the linearized Born modeling operator, $\mathbf{d} = \mathbf{K}\mathbf{m} + \mathbf{n}$, with the nonlinear signal components and measurement errors modeled by Gaussian noise. In this data representation, $\mathbf{K}$ is the demigration operator. By applying the adjoint (matched filter) of the modeling operator to the data, a migrated image is created. This image, $\mathbf{y} = \mathbf{K}^T\mathbf{d}$, serves as input to our amplitude recovery scheme, whose task it is to recover the reflectivity, $\mathbf{m}$, from 'noisy' measurements $\mathbf{y} = \mathbf{\Psi}\mathbf{m} + \mathbf{e}$ that include clutter, $\mathbf{e}$, and the imprint of the discrete normal (demigration-migration) operator, $\mathbf{\Psi} := \mathbf{K}^T\mathbf{K}$. To solve this problem, the following approximate identity is used

$$\mathbf{A}\mathbf{A}^T\mathbf{r} \simeq \mathbf{\Psi}\mathbf{r} \tag{3}$$

with $\mathbf{r}$ a reference vector given by the migrated image, corrected for spherical spreading. Above expression, is derived from an eigen-function like decomposition of the normal operator, $\mathbf{\Psi r} \simeq \mathbf{C}^T\mathbf{D}_\Psi\mathbf{Cr}$ in terms of 'eigenfunctions', the rows of the curvelet transform matrix (individual curvelets), and 'eigenvalues', collected in the diagonal matrix $\mathbf{D}_\Psi$. This diagonal can be estimated from numerical implementations of the migration/modeling operators by solving a regularized least-squares problem that uses the reference and demigrated-migrated reference vector as input (refer to technical reports by William Symes and Herrmann). The factorization in Eq. 3 is accomplished by defining the synthesis matrix in terms of a diagonally-weighted inverse curvelet transform, i.e., $\mathbf{A} := \mathbf{C}^T\mathbf{\Gamma}$ with $\mathbf{\Gamma} := \sqrt{\mathbf{D}_\Psi}$. This factorization leads to $\mathbf{y} \simeq \mathbf{Ax}_0 + \mathbf{e}$ as the approximate image representation that is amenable to a nonlinear solution by sparse inversion. After solving for $\mathbf{x}_0$, the reflectivity is obtained by applying the synthesis operator, $\mathbf{S}^T := \left(\mathbf{A}^T\right)^\dagger$, with † the pseudo inverse.

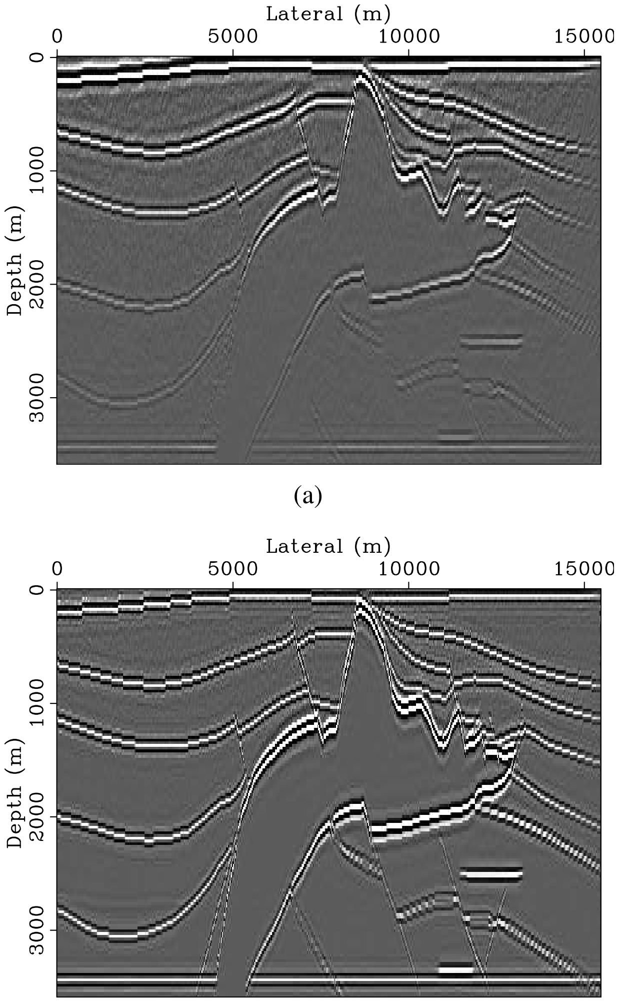
The diagonal approximation serves two purposes. It approximately corrects the amplitudes and it whitens the colored clutter. As the results in Fig. 1(b) indicate, a recovery with $\mathbf{P}_\epsilon$, leads to a stable recovery of the imaged reflectivity. These improvements are obtained by choosing a penalty functional that jointly promotes the curvelet sparsity and the continuity along the imaged reflectors, i.e., the penalty functional reads $J(\mathbf{x}) = J_s(\mathbf{x}) + J_c(\mathbf{x})$ with $J_s(\mathbf{x}) = \|\mathbf{x}\|_1$ and $J_c(\mathbf{m}) = \|\mathbf{\Lambda}^{1/2}\mathbf{\nabla m}\|_2^2$, an anisotropic diffusion (see e.g. Fehmers and Höcker, 2003). This second term penalizes fluctuations along the imaged reflectors and in regions where the length of the gradient vector of the reference vector is small. The rotation/weighting matrix is calculated with

$$\mathbf{\Lambda}[\mathbf{r}] = \frac{1}{\|\mathbf{\nabla r}\|_2^2 + 2\upsilon}\left\{\begin{pmatrix}+\mathbf{D}_2\mathbf{r}\\ -\mathbf{D}_1\mathbf{r}\end{pmatrix}\begin{pmatrix}+\mathbf{D}_2\mathbf{r} & -\mathbf{D}_1\mathbf{r}\end{pmatrix} + \upsilon\mathbf{Id}\right\}, \quad (4)$$

where $\mathbf{D}_{1,2}$ are the discrete first-order derivative operators in the $x_{1,2}$-directions and υ a control parameter (see Black et al., 1998, for details).

Results for the SEG AA' dataset are summarized in Fig. 1 and 2. These results were obtained for data modeled with a linearized Born approximation and a two-way reverse-time migration operator, described in the technical report "TR 06-18: Reverse time migration with optimal checkpointing" by William Symes. The recovered images show a nice amplitude recovery and clutter removal for data with a signal-to-noise ratio of $3\,\mathrm{dB}$. Data generated from the estimated image, $\widetilde{\mathbf{d}} = \mathbf{K}\widetilde{\mathbf{m}}$

shows a significant improvement compared to the demigrated data from the noisy data (cf. Fig. 2(b)-2(c)). This visual improvements leads to an improvement for the SNR.



(a)

(b)

Figure 1: Image amplitude recovery for a migrated image calculated from noisy data (SNR $3\,\mathrm{dB}$, see Fig. 2(a)). **(a)** Image with clutter. **(b)** Image after nonlinear recovery with $\mathbf{P}_\epsilon$. The clearly visible non stationary noise in **(a)** is mostly removed during the recovery while the amplitudes are also restored.

DISCUSSION AND CONCLUSIONS

The presented methodology banks on two favorable properties of curvelets, namely their ability to detect wavefronts (the 'wavefront set') and their approximate invariance under wave propagation. By compounding the curvelet transform with certain matrices, each of the recovery and separation problems was cast into one and the same optimization problem.

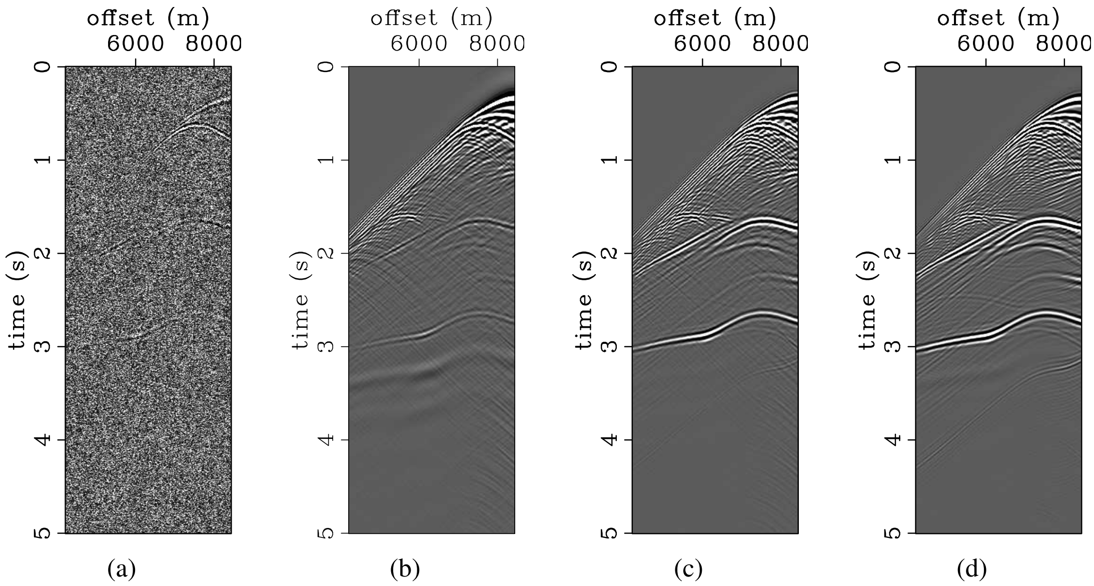


Figure 2: Seismic 'denoising'. **(a)** Noisy data with SNR 3 dB. **(a)** Image with clutter. **(b)** Data after demigrating the image with clutter (Fig. 1(a)). **(c)** Demigrated data data after amplitude recovery (Fig. 1(b)). **(d)** The original noise-free data. Observe the significant improvement in the data quality, reflected in an increase for the SNR.

The successful application of the curvelet transform, juxtaposed by sparsity-promoting inversion, opens a range of new perspectives on non-linear solution strategies for seismic data processing, wave propagation and imaging. Because of their singular wavefront detection capability, the curvelet transform represents in our vision the ideal domain for future seismic exploration.

ACKNOWLEDGMENTS:

The authors would like to thank the authors of CurveLab for making their codes available and William Symes for providing us with his reverse-time migration code. The examples presented in this paper were prepared with Madagascar (`rsf.sourceforge.net/`). SAGA and Exxon-Mobil are thanked for making test datasets available. This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and Collaborative Research and Development Grant DNOISE (334810-05) of Felix J. Herrmann and was carried out as part of the SINBAD project with support, secured through ITF (the Industry Technology Facilitator), from the following organizations: BG Group, BP, Chevron, ExxonMobil and Shell.

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