Recent developments in primary-multiple separation

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Motivation

Motivation

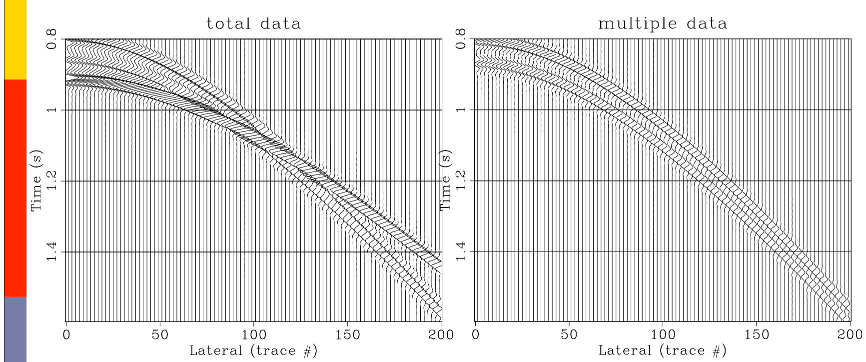
Primary-multiple separation step is crucial
- moderate prediction errors
- 3-D complexity & noise

Inadequate separation leads to
- remnant multiple energy
- deterioration primary energy

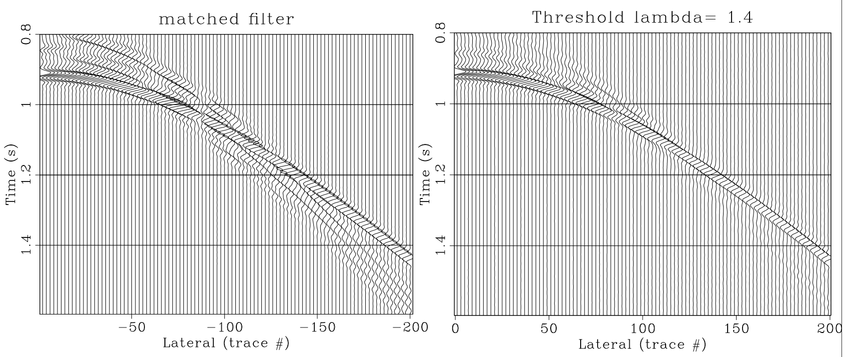
Introduce a transform-based technique
- stable
- insensitive to moderate shift, phase rotations

Exploit sparsity and parameterization transformed domain

Move-out error



Move-out error



The problem

Sparse signal model:

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \mathbf{n},$$

with

$$\mathbf{A} = [\mathbf{A}_1 \quad \mathbf{A}_2] \quad \text{and} \quad \mathbf{x}_0 = [\mathbf{x}_{01} \quad \mathbf{x}_{02}]^T$$

- augmented synthesis and sparsity vectors
- index 1 <-> primary
- index 2 <-> multiple

The solution

The weighted norm-one optimization problem:

$$\mathbf{P_w}: \begin{cases} \min_{\mathbf{x}} \|\mathbf{x}\|_{\mathbf{w},1} \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \varepsilon \\ \hat{\mathbf{s}}_1 = \mathbf{A}_1\hat{\mathbf{x}}_1 \quad \text{and} \quad \hat{\mathbf{s}}_2 = \mathbf{A}_2\hat{\mathbf{x}}_2 \\ \text{given:} \quad \breve{\mathbf{s}}_2 \quad \text{and} \quad \mathbf{w}(\mathbf{y}, \breve{\mathbf{s}}_2) \end{cases}$$

with

$$\begin{aligned} \mathbf{w} &:= [\mathbf{w}_1, \mathbf{w}_2]^T \\ \mathbf{A} &:= [\mathbf{C}^T, \mathbf{C}^T] \\ \breve{\mathbf{s}}_2 &:= \text{predicted multiples} \\ \breve{\mathbf{s}}_1 &:= \mathbf{S} - \breve{\mathbf{S}}_2 \end{aligned}$$

Solution cont'd

The weights

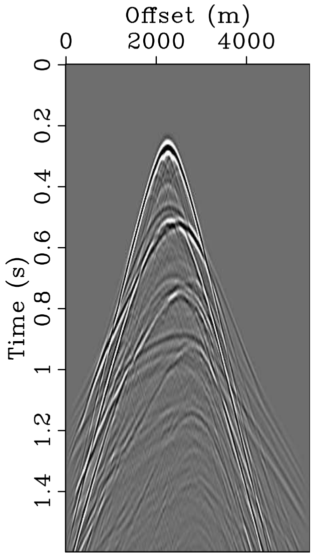
$$\begin{cases} \mathbf{w}_1 := \max\left(\sigma \cdot \sqrt{2\log N},\, C_1|\breve{\mathbf{u}}_1|\right) \\ \mathbf{w}_2 := \max\left(\sigma \cdot \sqrt{2\log N},\, C_2|\breve{\mathbf{u}}_2|\right) \end{cases}$$

with

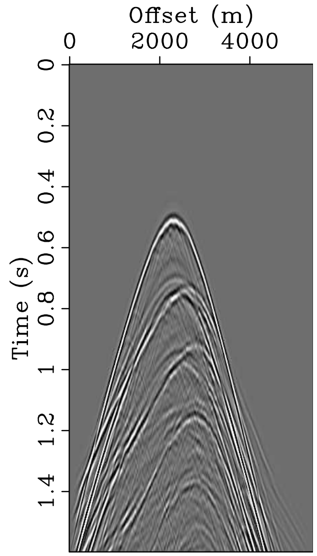
$$\begin{aligned} \breve{\mathbf{u}}_1 &\approx \mathbf{C}\breve{\mathbf{s}}_1 \\ \breve{\mathbf{u}}_2 &\approx \mathbf{C}\breve{\mathbf{s}}_2 \end{aligned}$$

- during minimization signal components are driven apart
- curvelet compression helps
- separates on the basis of position, scale and direction

Synthetic example

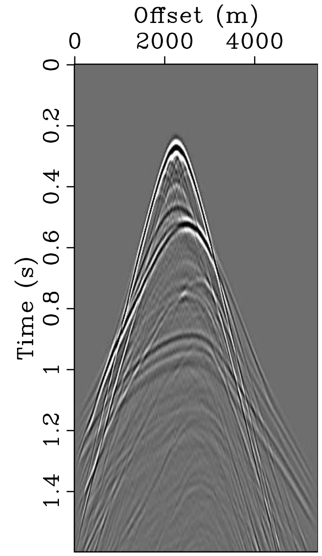


total data

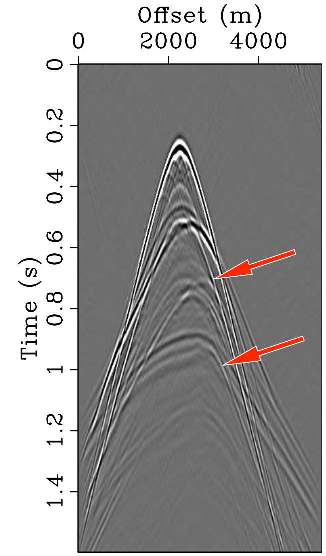


SRME predicted multiples

Synthetic example

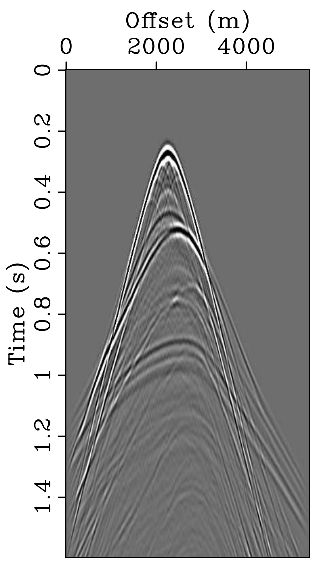


SRME predicted primaries

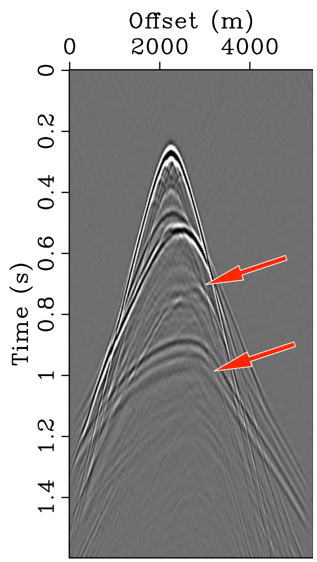


curvelet-thresholded

Synthetic example



SRME predicted primaries



estimated

Problem

Single thresholding is too aggressive

Iterative method tends to drive all energy to the primaries

- nontrivial weighting
- nontrivial stop criterion

Solutions:

- Non-adaptive Bayesian framework motivated by results from "Blind source deconvolution"
  - sparsity & decorrelation in the curvelet domain
- Adaptive curvelet-domain matched filtering
  - smoothness in the phase space <=> curvelet domain

Outline

Non-adaptive curvelet domain primary-multiple separation

- formulation of the primary-multiple separation problem
- the curvelet transform
- Bayesian formulation, taking inaccurate predictions into account
- Solution with iterative thresholding algorithm

Adaptive curvelet-domain matched filtering

- formulation of the SRME-base primary-multiple separation problem
- Phase space formulation taking nonstationary amplitude variations into account
- Curvelet-base matched filtering by imposing symbol smoothness in phase space

Non-adaptive curvelet domain primary-multiple separation

Joint work with Eric Verschuur, Deli Wang, Rayan Saab and Ozgur Yilmaz.




Problem formulation

Consider measurements as the sum of primaries, multiples and noise

$$\mathbf{b}_1 = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n}$$

Given a possibly erroneous prediction for the multiples (e.g. via SRME)

$$\mathbf{b}_2 = \mathbf{s}_2 + \mathbf{n}_2$$

Required: To estimate and hence separate primaries and multiples.

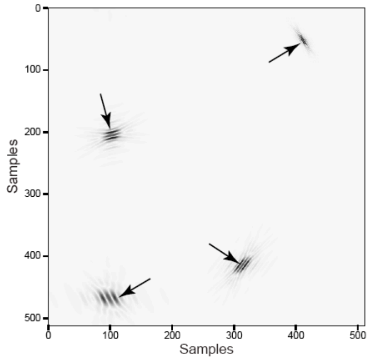
Curvelet transform

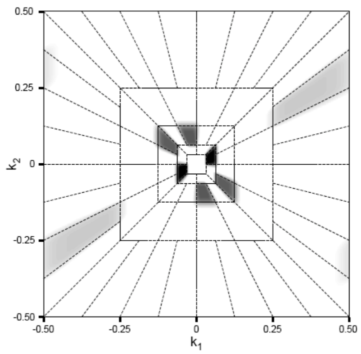
Formulate the separation problem in the curvelet domain where both the primaries and multiples can be modeled as ***sparse*** vectors.

Curvelets:

- little plain waves, multiscale and multidirectional, optimal for detecting wavefronts
- sparse on primaries and multiples
- parameterized by position, angle and scale (frequency band)

Curvelet transform





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Problem formulation revisited

Let $\mathbf{A} = \mathbf{C}^T$ be the inverse Curvelet transform.

Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be the curvelet coefficients of the primaries and multiples, respectively.

Write for the forward model

$$\mathbf{b}_2 = \mathbf{s}_2 + \mathbf{n}_2 \quad \Longrightarrow \quad \mathbf{b}_2 = \mathbf{A}\mathbf{x}_2 + \mathbf{n}_2$$

$$\mathbf{b}_1 = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n} \quad \Longrightarrow \quad \mathbf{b}_1 = \mathbf{A}\mathbf{x}_1 + \mathbf{n} - \mathbf{n}_2.$$

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Problem formulation revisited

Objective:

To estimate the primaries and multiples by estimating their curvelet coefficients.

Method:

- derive variational problem for $\mathbf{x}_1$ and $\mathbf{x}_2$ with a Bayesian formulation.
- impose sparsity as a ***prior*** on the coefficients
- solve for the coefficients to fit the data and the prior

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Maximum *A posteriori* estimation

We want to maximize $P(\mathbf{x}_1,\, \mathbf{x}_1 | \mathbf{b}_1,\, \mathbf{b}_2)$

Apply Bayes' rule

$$\begin{aligned} P(\mathbf{x}_1, \mathbf{x}_2 | \mathbf{b}_1, \mathbf{b}_2) &= \frac{P(\mathbf{x}_1,\mathbf{x}_2) P(\mathbf{b}_1 | \mathbf{x}_1,\mathbf{x}_2) P(\mathbf{b}_2 | \mathbf{b}_1,\mathbf{x}_1,\mathbf{x}_2)}{P(\mathbf{b}_1,\mathbf{b}_2)} \\ &= \frac{P(\mathbf{x}_1,\mathbf{x}_2) P(\mathbf{n}) P(\mathbf{n}_2)}{P(\mathbf{b}_1,\mathbf{b}_2)}. \end{aligned}$$

.... bottom line worry about the numerator:

$$\arg\max_{\mathbf{x}_1,\,\mathbf{x}_2} P(\mathbf{x}_1, \mathbf{x}_2 | \mathbf{b}_1, \mathbf{b}_2) = \arg\max_{\mathbf{x}_1,\,\mathbf{x}_2} P(\mathbf{x}_1, \mathbf{x}_2) P(\mathbf{n}) P(\mathbf{n}_2).$$

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Key assumptions

Curvelet coefficients of seismic data are sparse, i.e., mostly close to zero with few important non-zero coefficients,

- reasonable distribution to impose as a ***prior*** on the curvelet coefficients is a Laplacian/Cauchy distribution $\Longleftrightarrow \|\mathbf{x}\|_1$
- noise and prediction errors are modeled as Gaussian noise $\Longleftrightarrow \|\mathbf{b}-\mathbf{A}\mathbf{x}\|_2$

Optimization problem

Rewrite,

$$\arg\max_{\mathbf{x_1},\mathbf{x_2}} P(\mathbf{x}_1,\mathbf{x}_2|\mathbf{b}_1,\mathbf{b}_2) = \arg\min_{\mathbf{x_1},\mathbf{x_2}} f(\mathbf{x}_1,\mathbf{x}_2)$$

We want to minimize:

$$\begin{aligned} f(\mathbf{x}_1,\mathbf{x}_2) &= \|\mathbf{x}_1\|_{1,\mathbf{w}_1}+\|\mathbf{x}_2\|_{1,\mathbf{w}_2}+\|\mathbf{A}\mathbf{x}_2-\mathbf{b}_2\|_2^2 \\ &+\eta\|\mathbf{A}(\mathbf{x}_1+\mathbf{x}_2)-(\mathbf{b}_1+\mathbf{b}_2)\|_2^2, \end{aligned}$$

with the weights:

$$\begin{aligned} \mathbf{w}_1 &= \lambda_1|\mathbf{A}^T\mathbf{b}_2| \\ \mathbf{w}_2 &= \lambda_2|\mathbf{A}^T\mathbf{b}_1| \end{aligned}$$

Optimization problem properties

Control parameters:

- η controls the ***tradeoff*** between fitting the total data and fitting the predicted multiples

$$\begin{aligned} \eta\to 0 &\Longleftrightarrow \text{denoise multiples} \\ \eta\to\infty &\Longleftrightarrow \text{old formulation} \end{aligned}$$

- η controls trust in the prediction versus total data
- λ_1 and λ_2 control the ***sparsity*** of the coefficient vectors of the primaries and multiples
- ratio $\lambda_{1,2}$ versus η controls **sparsity** *versus* **data mismatch**
-

Separation algorithm

Minimize the objective function with the iterative algorithm:

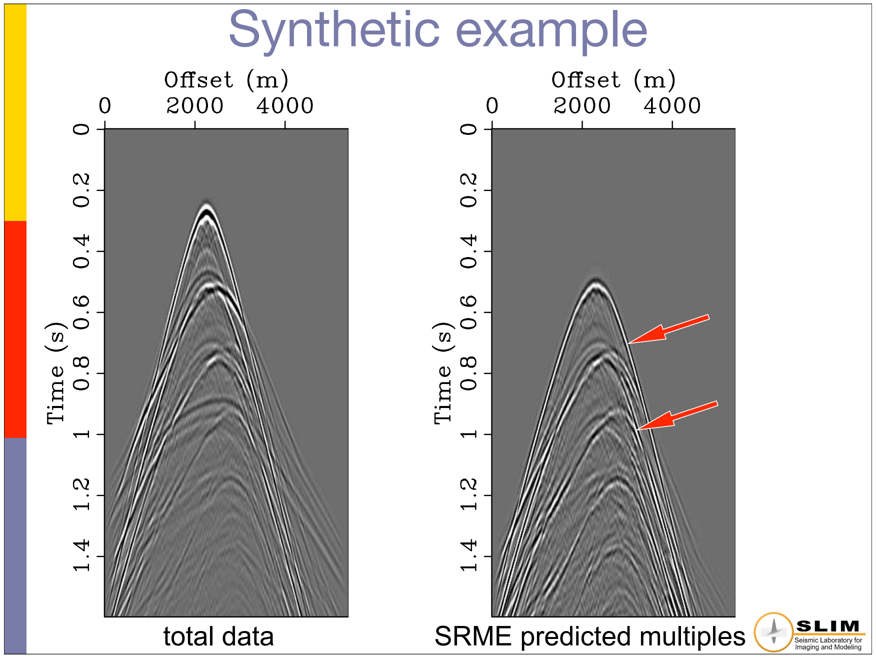
$$\begin{aligned} \mathbf{x_1^{n+1}} &= \mathbf{S}_{\frac{\mathbf{w_1}}{2\eta}}\left(\mathbf{A}^T\mathbf{b}_2-\mathbf{A}^T\mathbf{A}\mathbf{x_2^n}+\mathbf{A}^T\mathbf{b}_1-\mathbf{A}^T\mathbf{A}\mathbf{x_1^n}+\mathbf{x_1^n}\right) \\ \mathbf{x_2^{n+1}} &= \mathbf{S}_{\frac{\mathbf{w_2}}{2(1+\eta)}}\left[\mathbf{A}^T\mathbf{b}_2-\mathbf{A}^T\mathbf{A}\mathbf{x_2^n}+\mathbf{x_2^n}+\frac{\eta}{\eta+1}(\mathbf{A}^T\mathbf{b}_1-\mathbf{A}^T\mathbf{A}\mathbf{x_1^n})\right] \end{aligned}$$

where

$$S_{\alpha_\mu}(v_\mu)=\mathrm{sgn}(v_\mu)\cdot\max(0,|v_\mu|-|\alpha_\mu|)$$

is the **elementwise** soft thresholding operator.

Provably this converges for positive weights.


Synthetic example
Offset (m)
0 2000 4000
Time (s)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
total data
SRME predicted multiples
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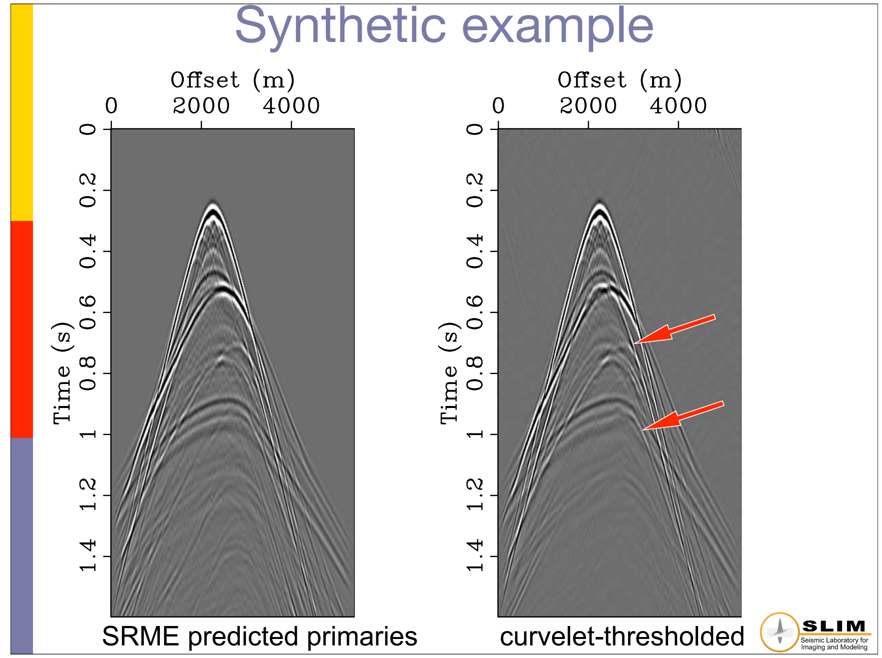
Synthetic example
Offset (m)
0 2000 4000
Time (s)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
SRME predicted primaries
curvelet-thresholded
SLIM
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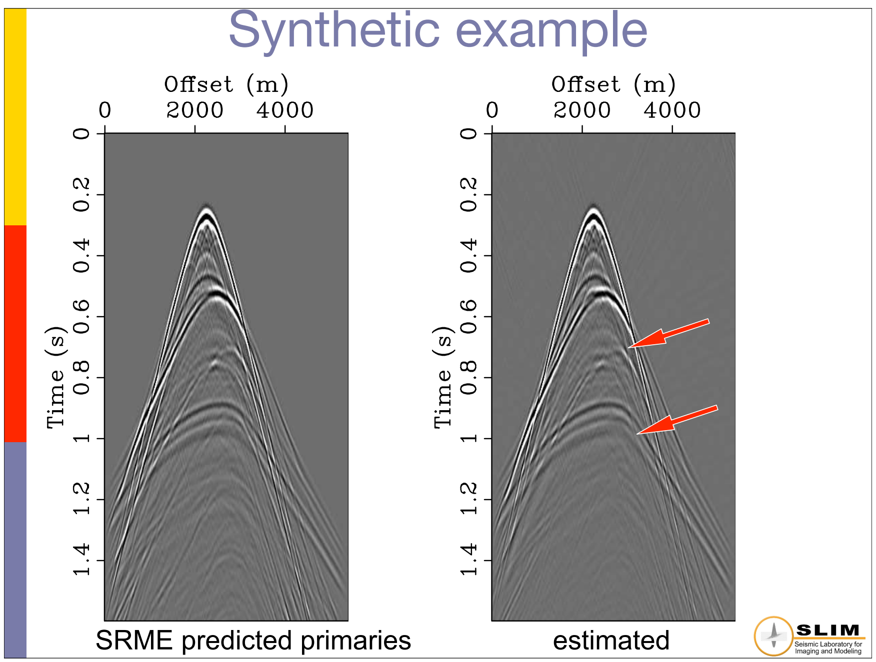
Synthetic example
Offset (m)
0 2000 4000
Time (s)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
SRME predicted primaries
estimated
SLIM
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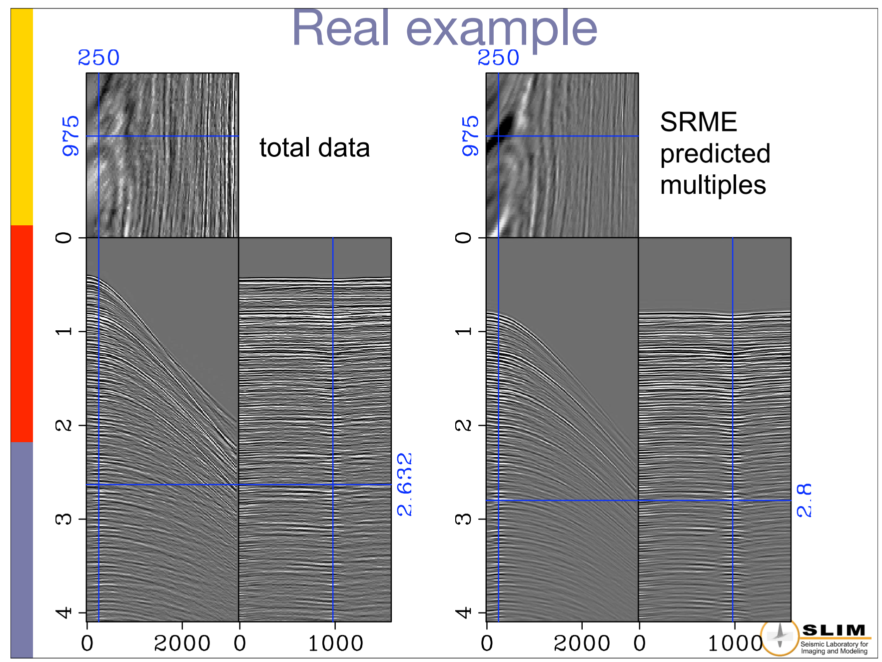
Real example
250
975
total data
SRME
predicted
multiples
0 1 2 3 4
2.632
2.8
0 2000 0 1000
SLIM
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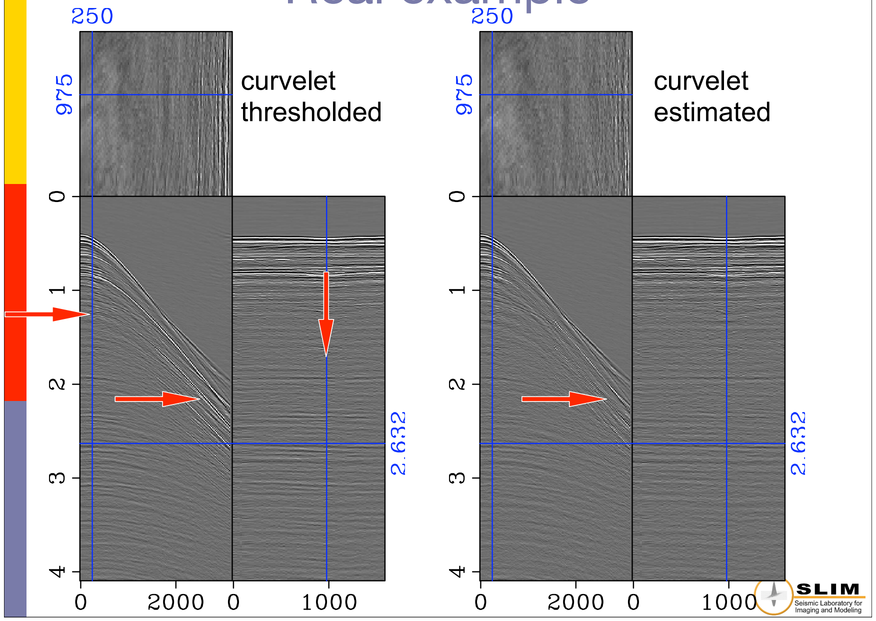
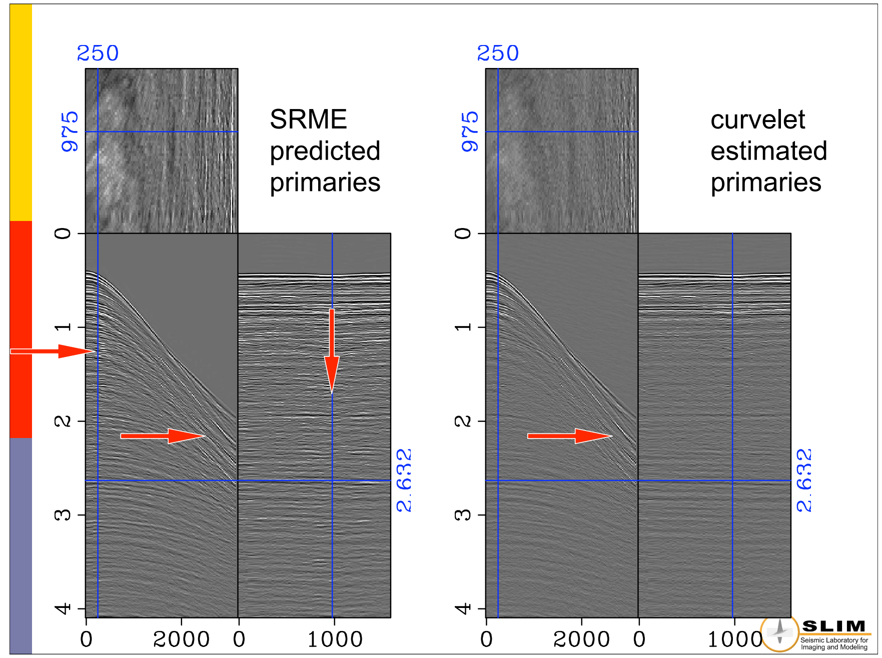
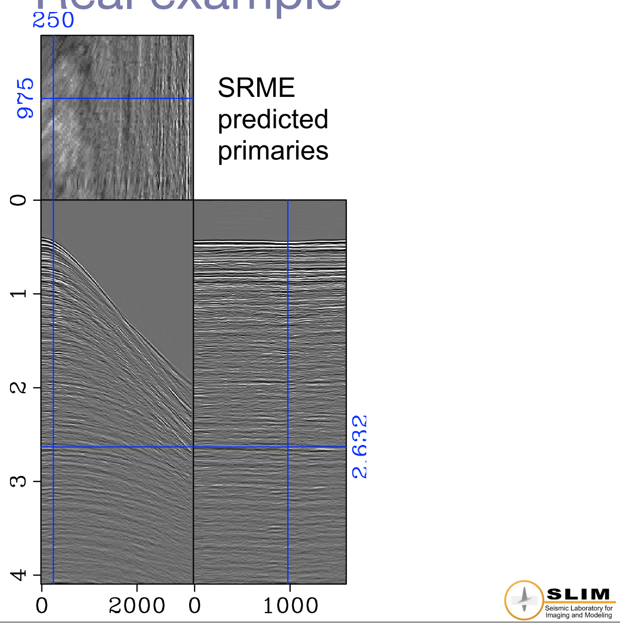
Real example

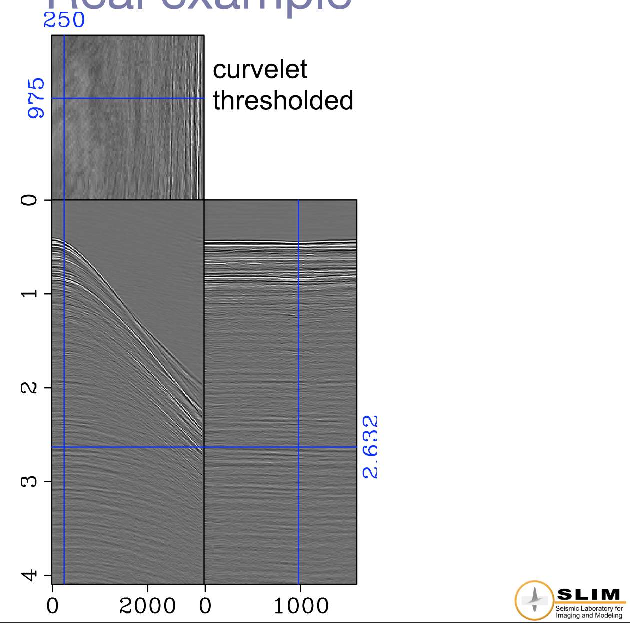




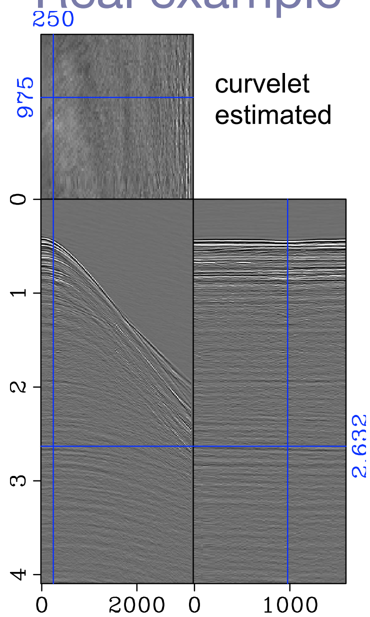
Real example



Real example



Real example



Observations

Inclusion of the additional equation for the predicted multiples prevents zero solutions for either signal component.

Formulation based on a solid Bayesian argument.

The algorithm provably converges.

Method is not adaptive but gives control over

- denoising the multiples versus solving the old problem
- sparsity versus data fit
- tradeoff trust in prediction versus trust in total data

Adaptive curvelet-domain matched filtering

Joint work with Deli Wang, Cody Brown and Peyman Moghaddam

Motivation

Kinematics are generally well predicted.

Non-adaptive curvelet-domain separation adds **robustness**.

Large errors in the location, dip and amplitude of the predicted multiples remain a problem.

Present an **adaptive** curvelet-domain separation based on **matched** filtering which assumes that

- the "seismic wavelet" has been removed
- the variations in the multiple predictions versus the true multiples vary **slowly** in **phase space**
- **kinematics** are roughly correct

Design a technique that exploits the invariance of curvelets under a certain class of operators.

Multiple prediction SRME

SRME-multiple prediction

$$\mathbf{\Delta p} \mapsto \breve{\mathbf{m}}^{(1)}(s,r,t) = \big(\mathbf{\Delta P} \boldsymbol{\mathcal{A}} *_{t,x} \mathbf{\Delta p}\big)(s,r,t)$$

with

$$\begin{aligned}\mathbf{\Delta p} &= \text{vector with the primaries}\\ \breve{\mathbf{m}}^{(1)} &= \text{vector with predicted first-order multiples}\\ \mathbf{\Delta P} &= \mathbf{F}^H \text{block diag}\{\Delta p\}\mathbf{F}\\ \mathbf{F} &= \text{temporal Fourier transform}\\ \boldsymbol{\mathcal{A}} &= \text{inverse wavelet.}\end{aligned}$$

In practice, $\mathbf{p} \mapsto \mathbf{\Delta p},\ \mathbf{P} \mapsto \mathbf{\Delta P},$ with $\mathbf{p}$ the total data, so

$$\breve{\mathbf{m}}^{(1)} \approx \mathbf{P}\boldsymbol{\mathcal{A}}\mathbf{p}$$

Multiple prediction SRME

Matched filter

$$\tilde{\mathbf{a}} = \arg\min_{\mathbf{a}} \|\mathbf{p} - \mathbf{a} *_t \breve{\mathbf{m}}^{(1)}\|_2$$

yielding

$$\begin{aligned}\tilde{\mathbf{\Delta p}} &= \mathbf{p} - \tilde{\mathbf{a}} *_t \breve{\mathbf{m}}^{(1)}\\ \boldsymbol{\mathcal{A}} &= \text{block diag}\{\tilde{\mathbf{a}}\}\end{aligned}$$

for each offset.

Problem

Assumes the filter to be ***stationary*** (diagonal in Fourier space)

Source characteristics may change with offset.

Wavelet changes as a function of (s,r,t).

Windowed matched-filtering techniques have been proposed

- window sizes arbitrary
- under fit (remnant primary energy)
- over fit (removal of primary energy)
- no control over the variations of the estimated filters amongst different windows

Propose a curvelet-domain matched filtering approach.

Curvelet-domain matched filtering

Naive solution,

$$\tilde{\mathbf{u}} = \arg\min_{\mathbf{u}} \|\mathbf{p} - \mathbf{P}[\mathbf{v}]\mathbf{u}\|_2$$

with

$$\begin{aligned}\tilde{\mathbf{u}} &= \text{curvelet coefficients of the matched multiples}\\ \mathbf{P}[\mathbf{v}] &= \text{operator dependent on } \mathbf{v}\\ \mathbf{v} &= \text{curvelet coefficients of } \breve{\mathbf{m}}^{(1)}\end{aligned}$$

- nonlinear is a problem
- underdetermined
- no control over estimated coefficients

Curvelet-domain matched filtering

Assume,

$$\mathbf{m}^{(1)} = \mathbf{B}\breve{\mathbf{m}}^{(1)}$$

Operator decomposition

$$\mathbf{B} = \boldsymbol{\mathcal{A}}\boldsymbol{\Psi}$$

with

$$(\Psi f)(x) = \int_{\mathbb{R}^d} e^{-ix\cdot\xi} a(x,\xi)\hat{f}(\xi)\mathrm{d}\xi$$

a zero-order Pseudodifferential operator.

Curvelet-domain matched filtering

Motivated by related work

- "Prediction of internal multiples" by F. ten Kroode who introduces an obliquity factor.
- "A method for inverse scattering based on the generalized Bremmer coupling series" by A. Malcolm and M. de Hoop who introduce introduce certain weighting factors
- "Amplitude and kinematic corrections of migrated images for nonunitary imaging operators" by A. Guitton & "Optimal Scaling for Reverse Time Migration" by Symes who introduce a diagonal smooth scaling.

Curvelet-domain matched filtering

Observe that "classical" matched filter

- absorbs the seismic wavelet
- deals with stationary phase rotations (Hilbert) and differential operators (derivatives)
- yields an inverse wavelet that is
  - of compact support in the time domain
  - smooth in the Fourier domain

Reasonable to assume

$$\begin{aligned}\mathbf{m}^{(1)} &= \mathbf{B}\breve{\mathbf{m}}^{(1)}\\ &= \boldsymbol{\mathcal{A}}\boldsymbol{\Psi}\breve{\mathbf{m}}^{(1)}\\ &= \boldsymbol{\Psi}\boldsymbol{\mathcal{A}}\breve{\mathbf{m}}^{(1)}\\ &= \boldsymbol{\Psi}\breve{\mathbf{m}}_0^{(1)}\end{aligned}$$

with $\boldsymbol{\Psi}$ a zero-order PsDO.

Curvelet-domain matched filtering

From work on migration amplitude recovery we have

Theorem 1. *The following estimate for the error holds*

$$\|(\Psi(x,D) - C^T\mathbf{D}_\Psi C)\varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C'' 2^{-|\mu|/2},$$

where C'' is a constant depending on Ψ.

Allows for a curvelet domain diagonalization of $\boldsymbol{\Psi}$,

$$\mathbf{m}^{(1)} = \mathbf{C}^T\mathrm{diag}\{\tilde{\mathbf{u}}\}\mathbf{C}\breve{\mathbf{m}}_0^{(1)}$$

Curvelet-domain matched filtering

Solve

$$\tilde{\mathbf{u}} = \arg\min_{\mathbf{u}} \|\mathbf{p} - \mathbf{P}[\mathbf{v}]\mathbf{u}\|_2$$

with

$$\begin{aligned} \tilde{\mathbf{u}} &= \text{curvelet coefficients of the matched multiples} \\ \mathbf{P}[\mathbf{v}] &= \mathbf{C}^T \text{diag}\{\mathbf{v}\} \\ \mathbf{v} &= \text{curvelet coefficients of } \breve{\mathbf{m}}_0^{(1)} \end{aligned}$$

remains

- underdetermined system
- no control on the smoothness of the coefficients

Assume symbol $a(x,\zeta)$ **to be smooth!**

Curvelet-domain matched filtering

Solve

$$\tilde{\mathbf{u}} = \arg\min_{\mathbf{u}} \frac{1}{2}\|\mathbf{b} - \mathbf{P}\mathbf{u}\|_2^2 + \eta^2 \|\mathbf{L}\mathbf{u}\|_2^2$$

with

$$\mathbf{L} = [\mathbf{D}_1 \quad \mathbf{D}_2 \quad \mathbf{D}_\theta]$$

equivalent to

$$\begin{bmatrix} \mathbf{P} \\ \eta\mathbf{L} \end{bmatrix} \mathbf{u} = \begin{bmatrix} \mathbf{p} \\ \mathbf{0} \end{bmatrix}$$

or

$$\mathbf{A}\mathbf{u} = \mathbf{b}$$

Curvelet-domain matched filtering

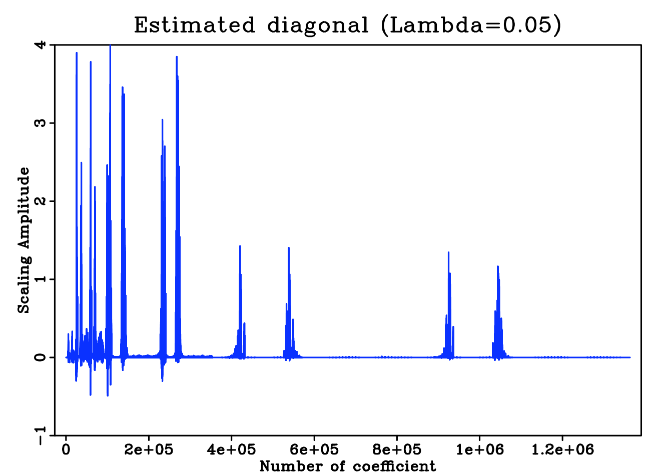
Procedure

Calculate: $\mathbf{v} = \mathbf{C}\breve{\mathbf{m}}_0^{(1)}$.
Set: $\eta = \eta_{min}$;
while $\exists (\tilde{u}_\mu)_{\mu \in \mathcal{M}} < 0$ **do**
Solve
$\tilde{\mathbf{u}} = \arg\min_{\mathbf{u}} \frac{1}{2}\|\mathbf{p} - \mathbf{P}\mathbf{u}\|_2^2 + \eta^2\|\mathbf{L}\mathbf{u}\|_2^2$
Increase the Lagrange multiplier
$\lambda = \eta + \Delta\eta$
end while

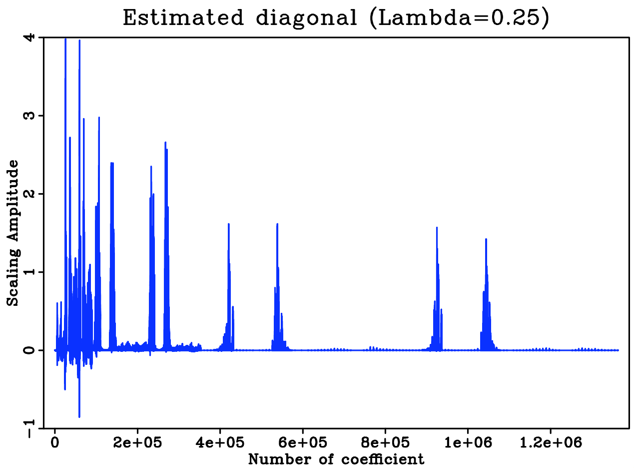
New estimate for the primaries

$$\begin{aligned} \tilde{\Delta \mathbf{p}} &= \mathbf{p} - \mathbf{\Psi}\mathcal{A}\breve{\mathbf{m}}^{(1)} \\ &\approx \mathbf{p} - \mathbf{P}\tilde{\mathbf{u}} \end{aligned}$$

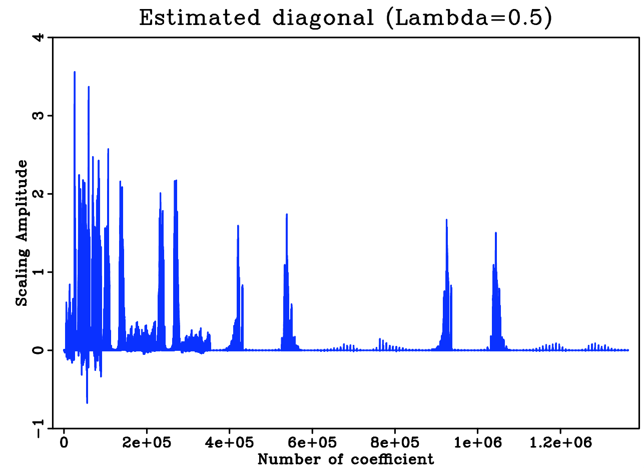
Curvelet-domain matched filtering



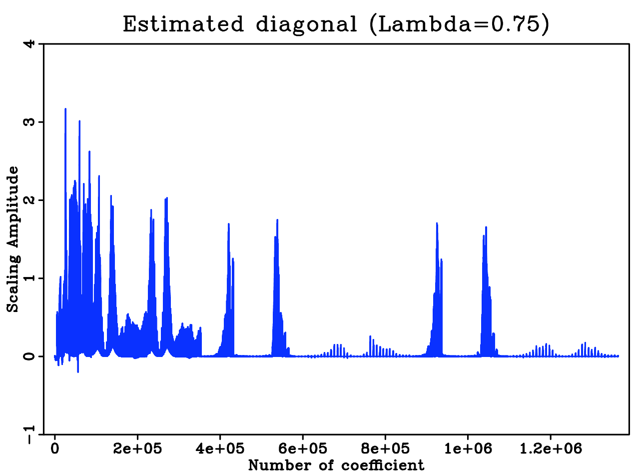
Curvelet-domain matched filtering



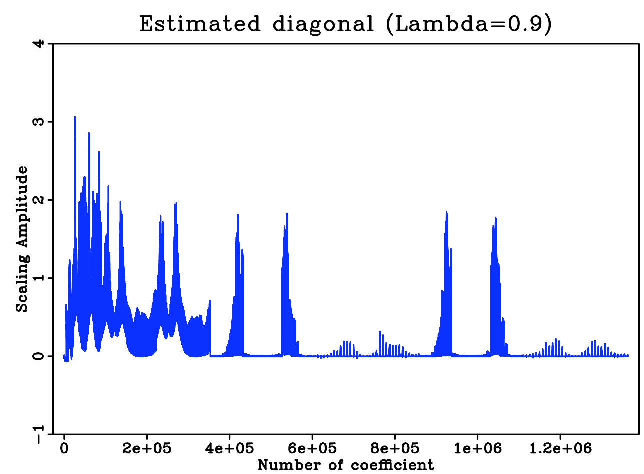
Curvelet-domain matched filtering



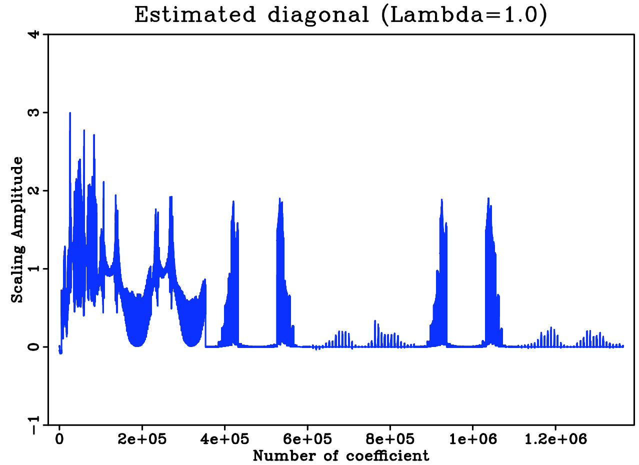
Curvelet-domain matched filtering

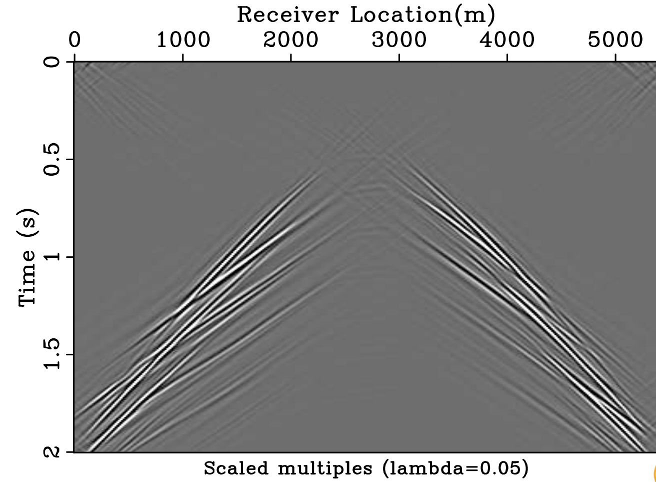


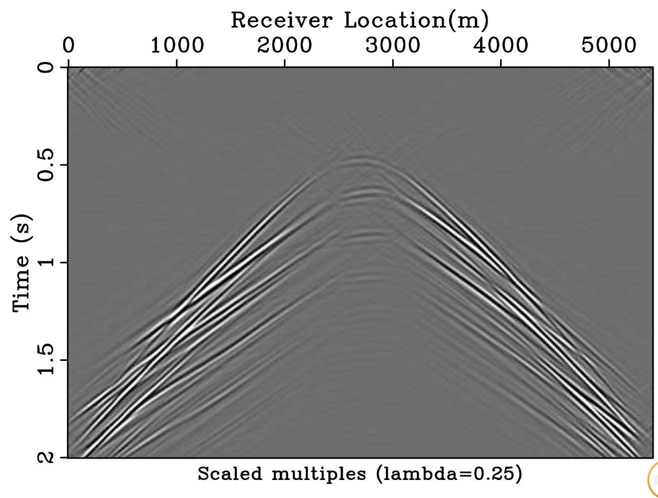
Curvelet-domain matched filtering

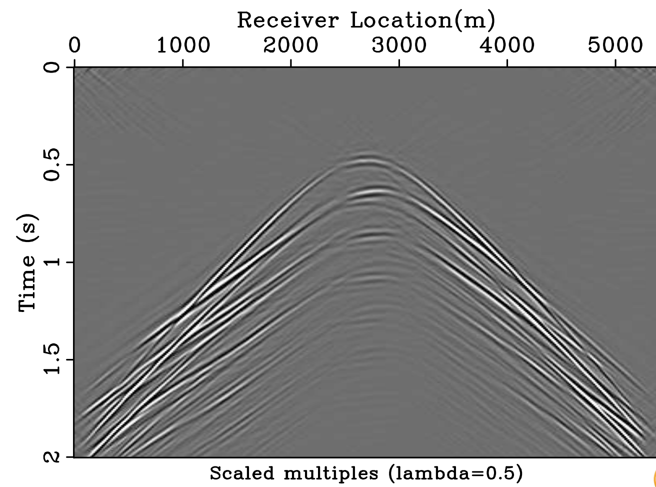


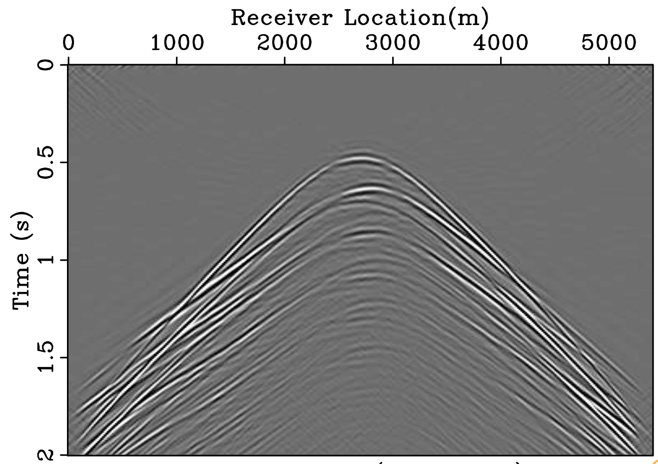
Curvelet-domain matched filtering



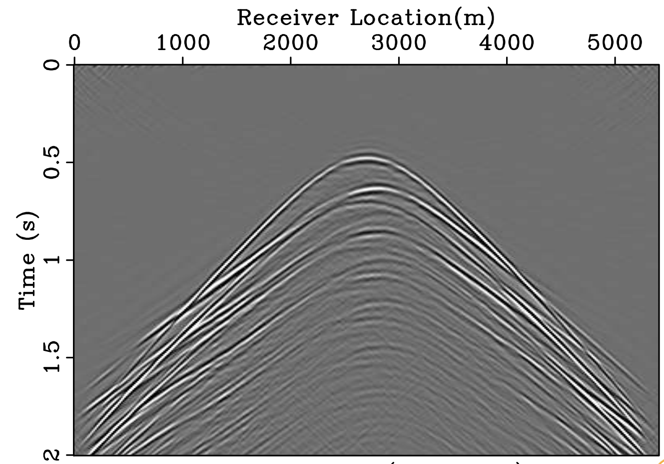




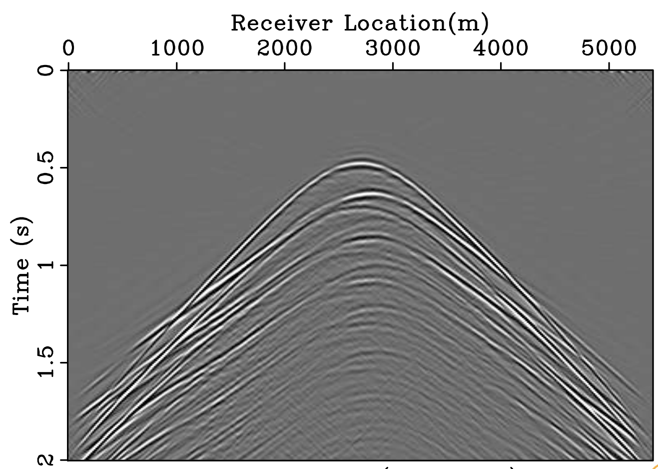




Scaled multiples (lambda=0.75)



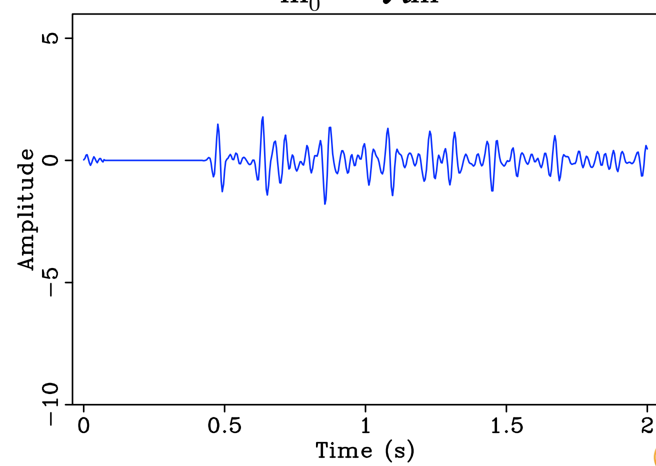
Scaled multiples (lambda=0.9)



Scaled multiples (lambda=1.0)

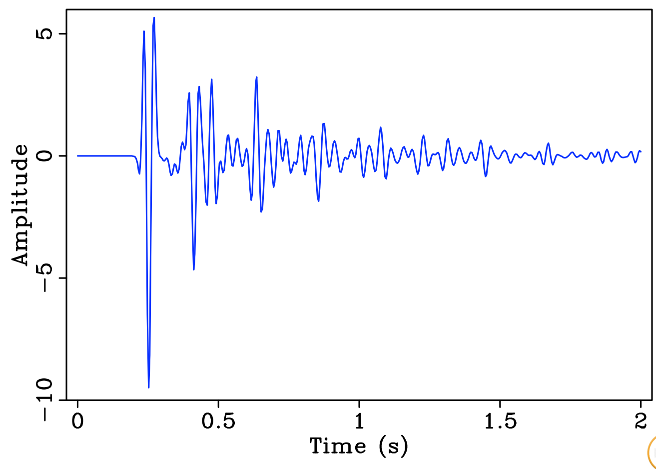
Synthetic example conventional matched filter

$$\breve{\mathbf{m}}_0^{(1)} = \mathcal{A}\breve{\mathbf{m}}^{(1)}$$



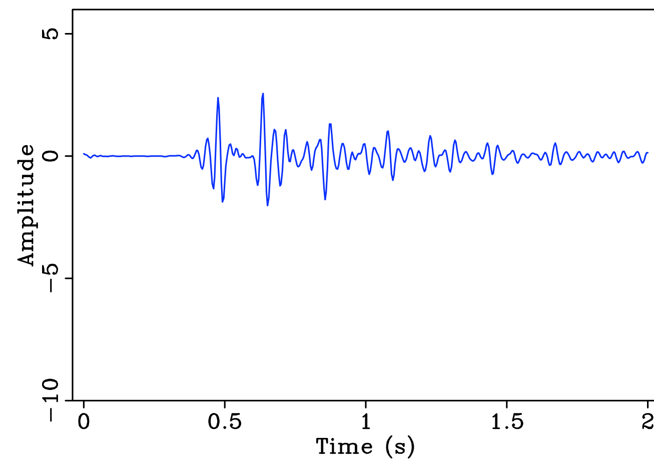
Synthetic example
total data

$\mathbf{p}$



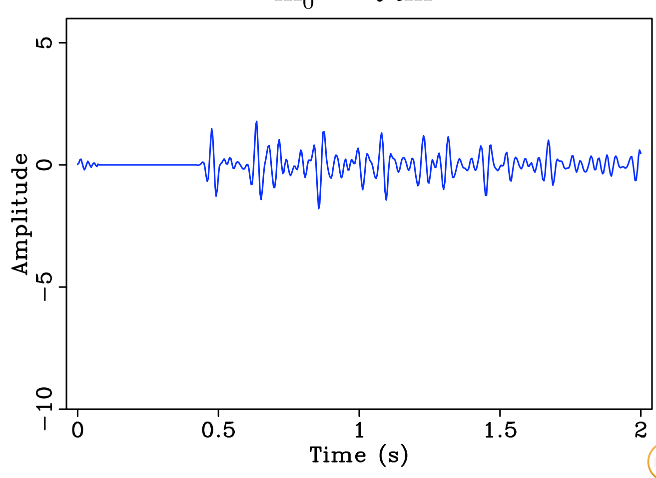
Synthetic example
curvelet-domain matched multiples

$$\tilde{\mathbf{m}}^{(1)} = \mathbf{P}\tilde{\mathbf{u}} \approx \boldsymbol{\Psi}\boldsymbol{\mathcal{A}}\breve{\mathbf{m}}^{(1)}$$



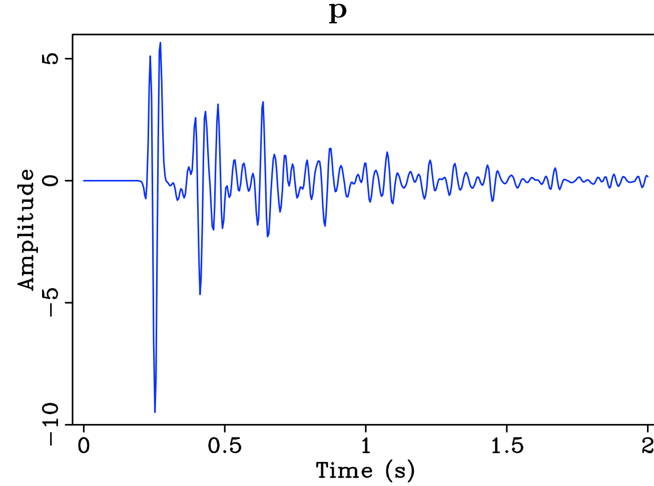
Synthetic example
conventional matched filter

$$\breve{\mathbf{m}}_0^{(1)} = \boldsymbol{\mathcal{A}}\mathbf{m}^{(1)}$$

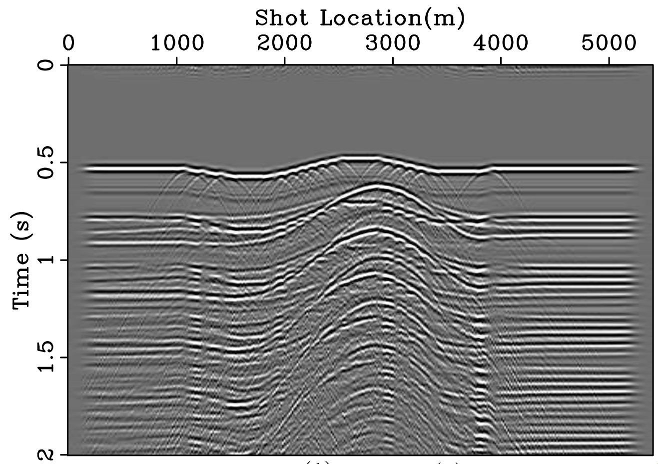


Synthetic example
total data

$\mathbf{p}$

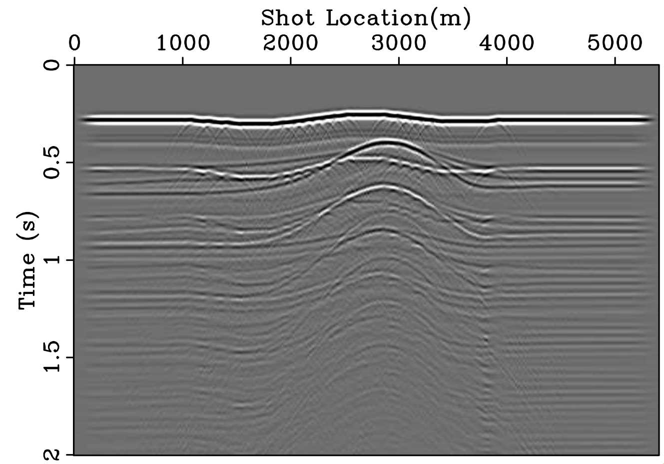


Synthetic example
conventional matched filter



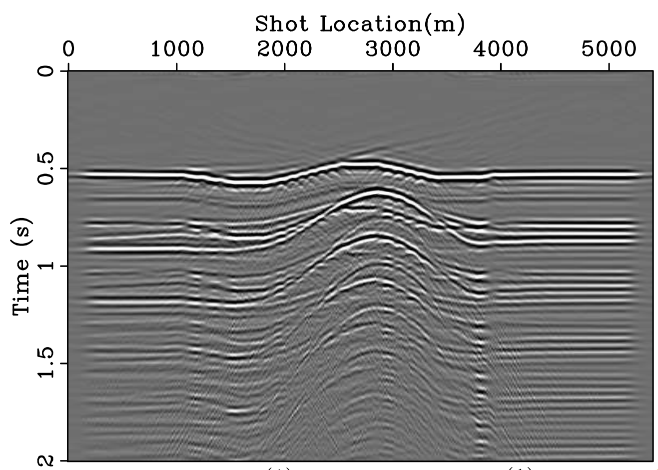
$$\breve{\mathbf{m}}_0^{(1)} = \mathcal{A}\breve{\mathbf{m}}^{(1)}$$

Synthetic example
total data



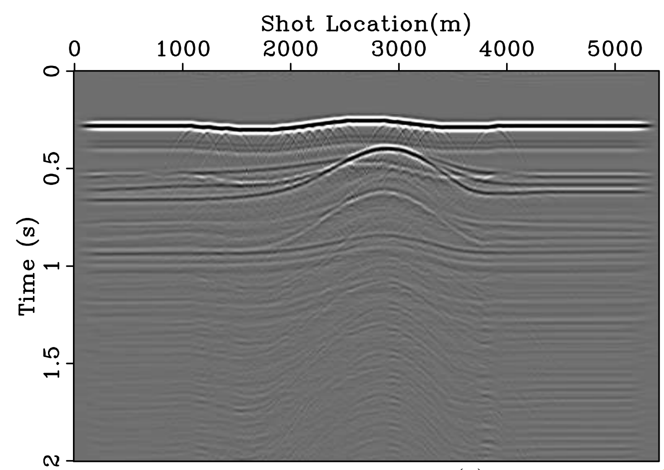
$$\mathbf{p}$$

Synthetic example
curvelet-domain matched multiples



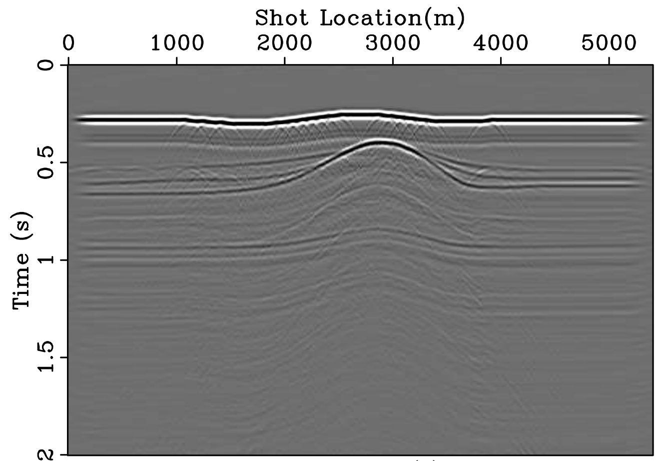
$$\tilde{\mathbf{m}}^{(1)} = \mathbf{P}\tilde{\mathbf{u}} \approx \Psi\mathcal{A}\breve{\mathbf{m}}^{(1)}$$

Synthetic example
conventional estimate for primaries

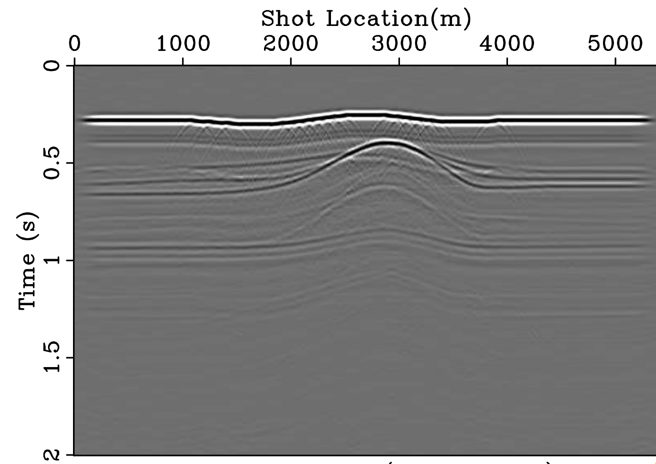


$$\tilde{\Delta \mathbf{p}} = \mathbf{p} - \mathcal{A}\breve{\mathbf{m}}^{(1)}$$

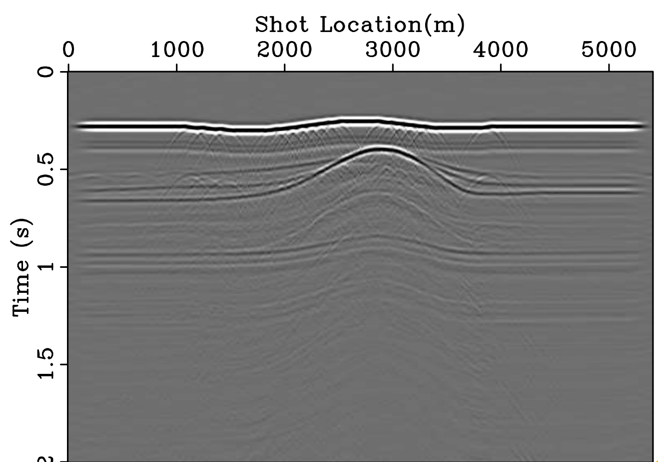
Synthetic example
curvelet-based estimate for primaries



$$\tilde{\Delta \mathbf{p}} = \mathbf{p} - \Psi \mathcal{A} \breve{\mathbf{m}}^{(1)} \approx \mathbf{p} - \mathbf{P}\tilde{\mathbf{u}}$$

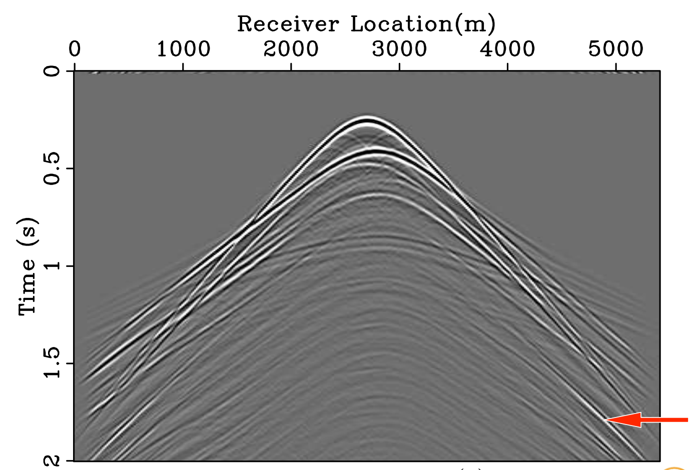


Bayesian Primaries (LSQR multiples)



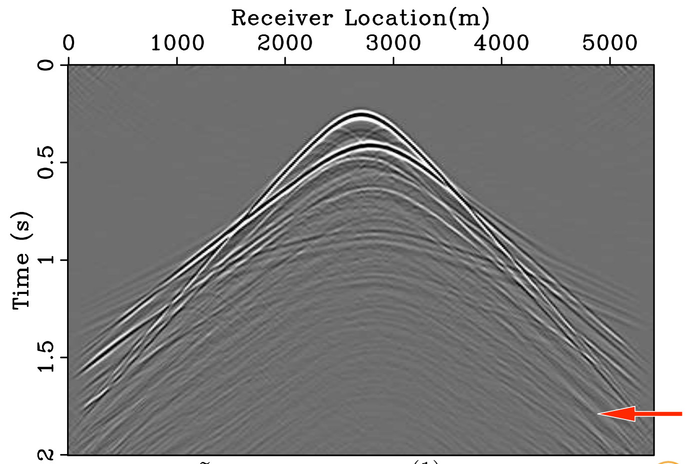
Bayesian Primaries (Scaled multiples)

Synthetic example
conventional estimate for primaries



$$\tilde{\Delta \mathbf{p}} = \mathbf{p} - \mathcal{A} \breve{\mathbf{m}}^{(1)}$$

Synthetic example
curvelet-based estimate for primaries



$$\tilde{\Delta \mathbf{p}} = \mathbf{p} - \mathbf{\Psi}\mathcal{A}\breve{\mathbf{m}}^{(1)} \approx \mathbf{p} - \mathbf{P}\tilde{\mathbf{u}}$$

Observations

Smoothness penalty on the curvelet coefficients

- behaves as expected (becomes positive)

Alternative to Bayesian-based separation.

Diffraction

Curvelet-based Multi-term Multiple subtraction

Most of the application of surface-related multiple attenuation are 2D

Most of 3D effects come from diffractive structures

Split the data into a specular and a diffraction part can handle 3D effects

Multiple prediction scheme can be rewritten as:

$$\mathbf{M} = [\Delta\mathbf{P}_r + \Delta\mathbf{P}_d][\mathbf{P}_r + \mathbf{P}_d]$$

and we get four terms multiples:

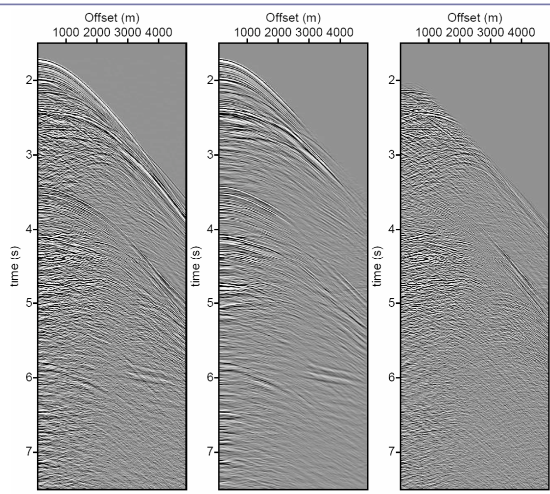
$$\mathbf{M}_{rr} = \Delta\mathbf{P}_r\mathbf{P}_r$$
$$\mathbf{M}_{rd} = \Delta\mathbf{P}_r\mathbf{P}_d$$
$$\mathbf{M}_{dr} = \Delta\mathbf{P}_d\mathbf{P}_r$$
$$\mathbf{M}_{dd} = \Delta\mathbf{P}_d\mathbf{P}_d$$

Where:

r : represent reflection part

d : represent diffraction part

Curvelet-based multi-term multiple separation



a) Total data *b) Specular reflections* *c) Diffractions*

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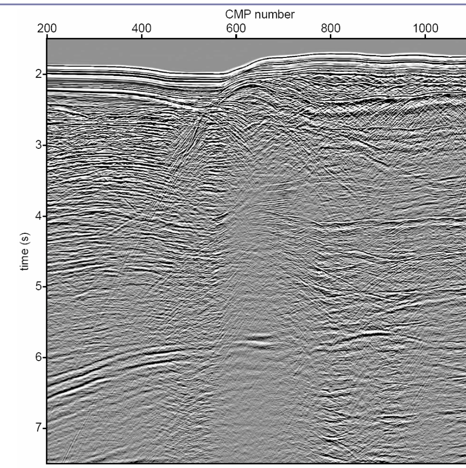
Curvelet-based multi-term multiple separation



Stack section with all multiples

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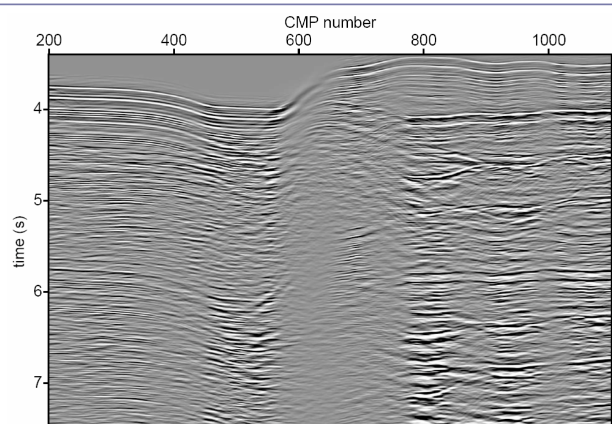
Curvelet-based multi-term multiple separation



a) $\mathbf{M}_{rr} = \mathbf{P}_r\mathbf{P}_r$

Stack section of the first-term multiples

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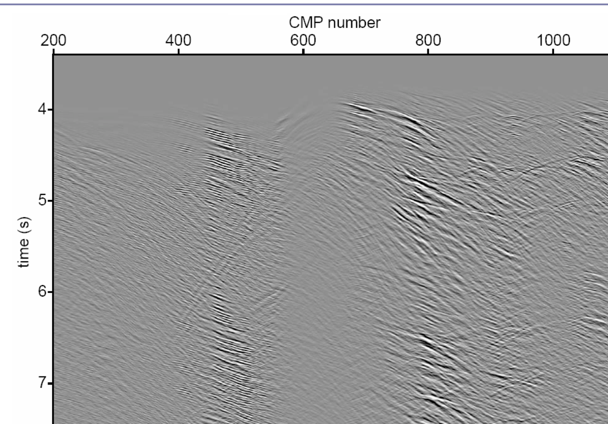
Curvelet-based multi-term multiple separation



b) $\mathbf{M}_{rd} = \mathbf{P}_r\mathbf{P}_d$

Stack section of the second-term multiples

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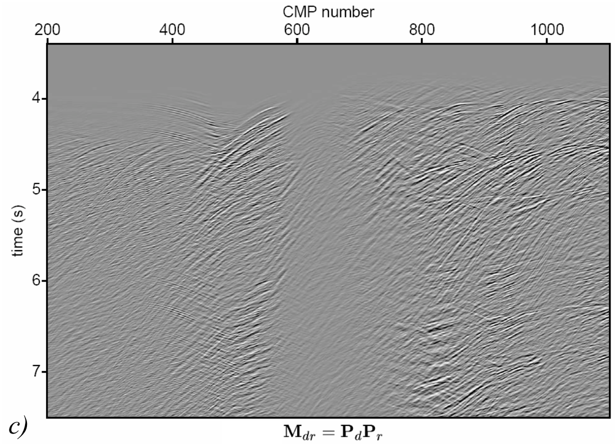
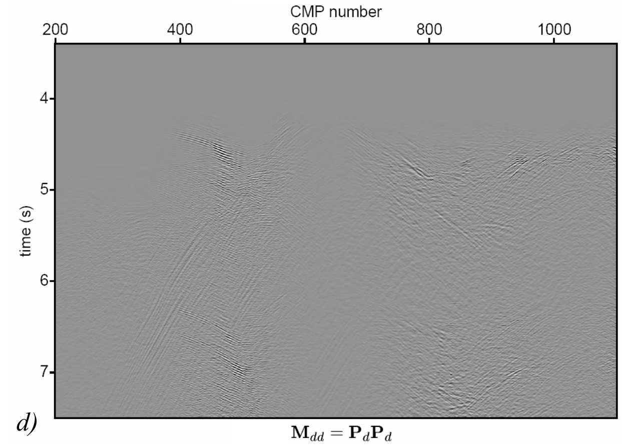
Curvelet-based multi-term multiple separation



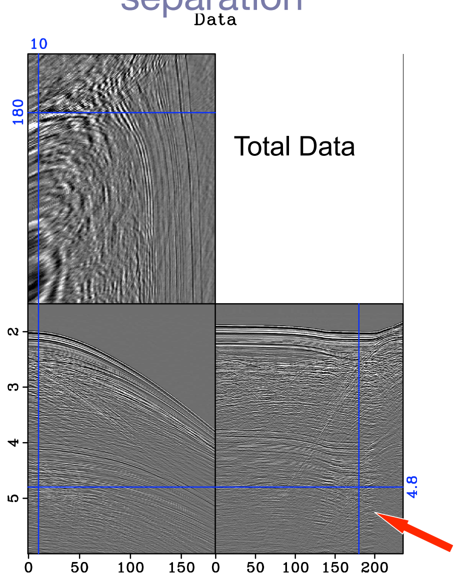
Stack section of the third-term multiples

Curvelet-based multi-term multiple separation

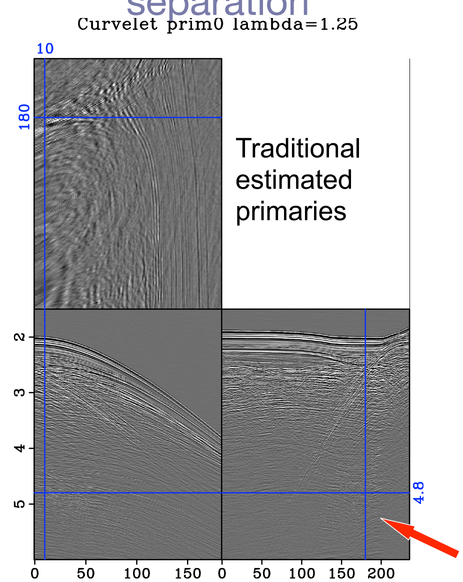


Stack section of the fourth-term multiples

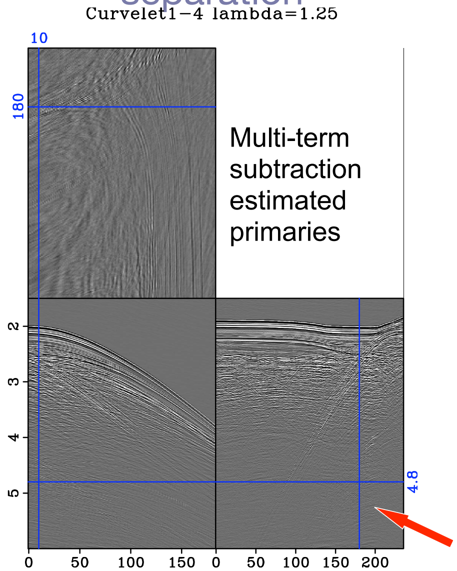
Curvelet-based multi-term multiple separation



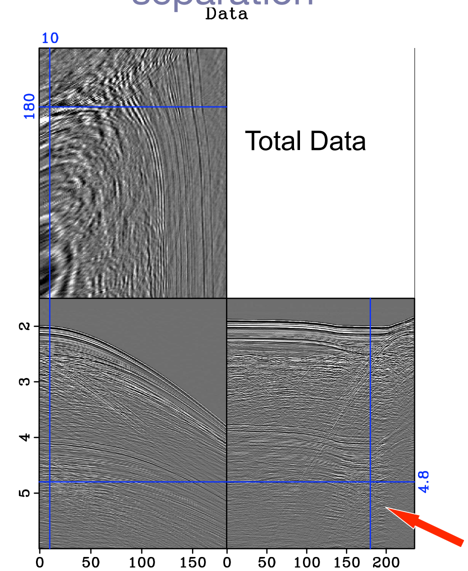
Curvelet-based multi-term multiple separation



Curvelet-based multi-term multiple separation



Curvelet-based multi-term multiple separation



Observations

Single-threshold based separation with multiterm prediction

Each term is handled separately

Improved multiple separation compared to the "all in one separation"

Increased flexibility to handle 3-D diffractions

Extensions

part of SINBAD II