Sparsity- and continuity promoting norms for seismic images

Peyman P. Moghaddam
Joint work with Felix Herrmann

SLIM Seismic Laboratory for Imaging and Modeling

References

- Curvelets and Denoising,
  - Sparsity, Candes and Donoho (2000)
  - TV norm, Candes and Guo (2002)
  - Stable signal recover, Candes and Romberg (2005)
- Continuity promoting and Anisotropic diffusion,
  - Regularization for denoising, Scherzer (2003)
  - In seismic imaging, Fehmer (2003)
- Optimization method,
  - Soft-thresholding, Donoho (1995)
  - Iterative thresholding, Daubechies (2005)
  - Gradient based optimization, Nocedal (2001)

Overview

- Definition of problem
- Denoising as an inversion problem
- Curvelets and their properties
- Curvelets sparsity norm
- Continuity enhancing norm
- Problem reformulation
- Optimization method
- Results
- Conclusion

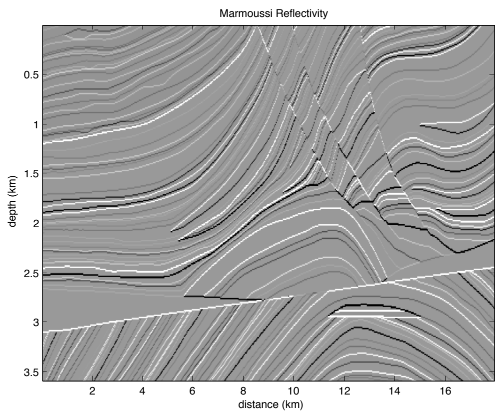
Definition of the Problem

- Basic denoising problem

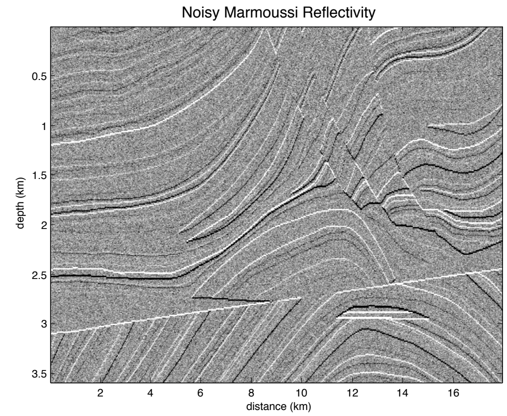
$$\mathbf{d} = \mathbf{m} + \mathbf{n}$$

- Desired seismic image characteristics:
  - Broad-band
    - Sharp, high resolution
    - 2D curves/3D sheets
  - Continuity along the reflectors
- Noise in seismic images
  - Random noise
    - Instruments distortion
    - Ambient
  - Imaging operator imperfections

Example (Marmoussi Reflectivity)



Examples (Noisy Marmoussi, SNR=5 dB)



Denoising as Inversion Problem

- Following inversion problem is introduced

$$\min_{m} J(\mathbf{m}) \text{ subject to } ||\mathbf{d}-\mathbf{m}||_{\ell_2} \leq \epsilon$$

J(m) is the norm or penalty function

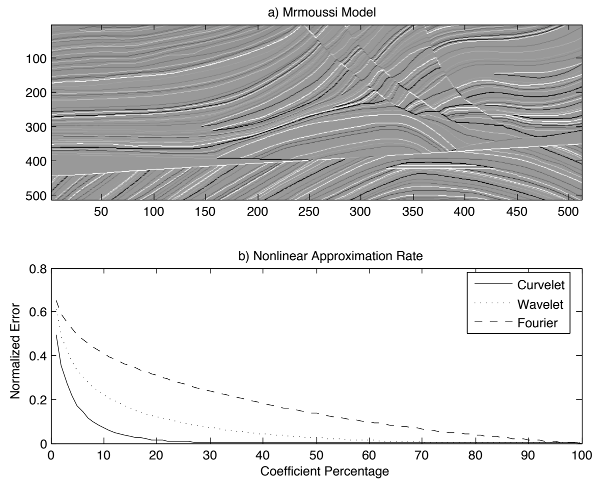
- This norm has to
  - Explore the continuity along the reflectors
  - Explore the sparsity of image in the Curvelet domain
  - Reduce the artifacts from image
  - Enhance the reflectors
  - Remove the noise from seismic image

Curvelets and their properties

Curvelets:

- are multi-scale and multi-directional
- sparsely present a seismic image
- are invariant under the action of idealize normal operator
- are constructed as tight frames
- transformation is fast
- are reliably used for denoising in image processing applications

Approximation Rate Comparison



Curvelet Sparsity Norm

- Properties
  - Theorem states that Curvelet transform of a 2D singularity is sparse (Candes 2001)
  - It is Lp norm on the Curvelet coefficients of an image
- Formulation

$$J_c(\mathbf{x}) = ||W\mathbf{x}||_{\ell_1}$$

x is the Curvelet coefficients vector and W is the weighting matrix which controls

  - Scaling of the denoised image
  - Phase space of the denoised image

Continuity Enhancing Norm

- Properties
  - Minimize L2 norm on a specific function of the image gradient
  - Penalize the amplitude between reflectors and smooth along them
  - Enhance the singularities in an image
  - Linear and non-linear
- Formulation

$$J_a(m) = |\Lambda[\bar{\mathbf{b}}]^{\frac{1}{2}} m|_{\ell_2}$$

$$\Lambda[\bar{\mathbf{b}}] = \frac{1}{\|\nabla_d \bar{\mathbf{b}}\|_2^2} \left\{ \begin{pmatrix} D_2\bar{\mathbf{b}} \\ -D_1\bar{\mathbf{b}} \end{pmatrix} \begin{pmatrix} D_2\bar{\mathbf{b}} & -D_1\bar{\mathbf{b}} \end{pmatrix} \right\}.$$

$\bar{\mathbf{b}}$ is a smooth or primary estimate of image

Problem Reformulation

$$\begin{cases} \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{d} - \mathbf{C}^T\mathbf{x}\|_2 \le \epsilon \\ \mathbf{m} = \mathbf{C}^T\mathbf{x} \end{cases}$$

$$J(\mathbf{x}) = \sum_{j=1}^{N} |w_j x_j| + \|\mathbf{\Lambda}^{1/2}\mathbf{C}^T\mathbf{x}\|_2,$$

- This problem solve joint Curvelet sparsity and anisotropic diffusion while preserving the misfit less than the small control parameter
- It is required an optimization method to solve above problem
- A computationally simple method is introduced

Optimization Method

Step 1: Update of the Jacobian of $\frac{1}{2}\|d - C^T x\|_2^2$:

x← $x + C\left(d - C^T x\right)$;

Step 2: projection onto the ℓ_1 ball $S = \{\|x\|_1 \leq \|x_0\|_1\}$ by soft thresholding

x← $S^s_{\lambda w}(x)$;

Step 3: projection onto the anisotropic diffusion ball $C = \{x : \ J(x) \leq J(x_0)\}$ by

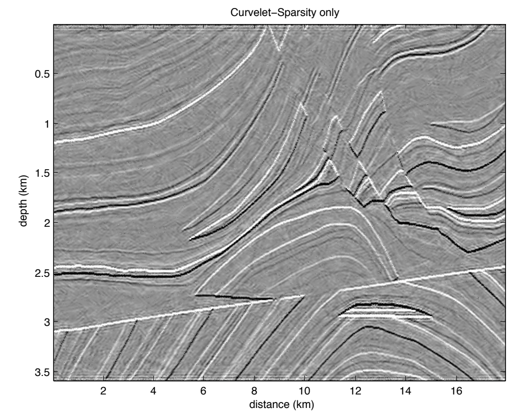
x← $x - \mu \nabla_x J_c(x)$

with

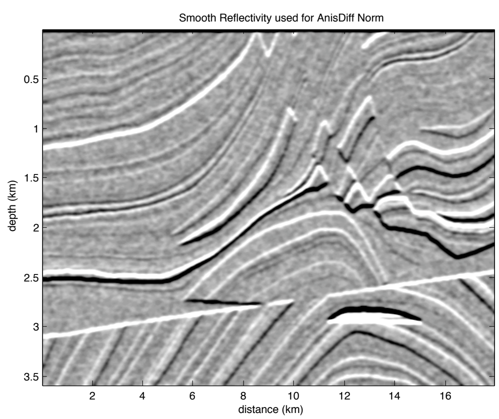
$\nabla_x J_c(x) = 2C\nabla \cdot \left(\Lambda \nabla \left(C^T x\right)\right)$.

The μ is found through a line search.

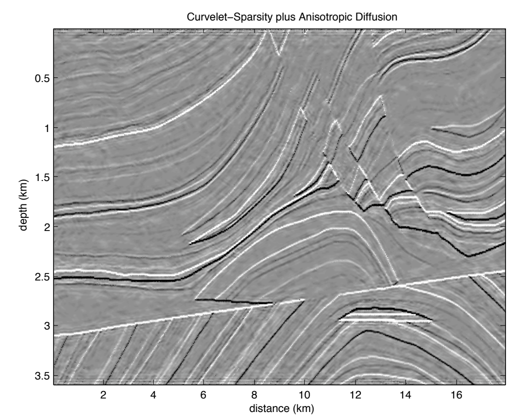
Results SNR=20 dB



Smooth Reflectivity used for Continuity norm



Results SNR= 28 dB



Conclusion

- New technique is introduced for denoising seismic images
- A sparsity penalty function is introduced which exploit the Curvelet properties
- A continuity promoting penalty function is utilized for denoising
- A solver is introduced which is reasonably fast and robust
- A python interface for this method is under development

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