Imaging Operator Approximation Using Curvelets

Peyman P. Moghaddam
Joint work with Chris. Stolk, Felix Herrmann

SLIM Seismic Laboratory for Imaging and Modeling

Overview

Definition of the Problem

- Linear map between image and data defined as

$$\mathbf{d} = \mathbf{K}\mathbf{m}$$

- Classical linear algebra inversion
  - Conjugate gradient based inversion
  - Approximation based inversion

$$\hat{\mathbf{m}} = (\mathbf{K}^*\mathbf{K})^{-1}\mathbf{K}^*\mathbf{d}$$

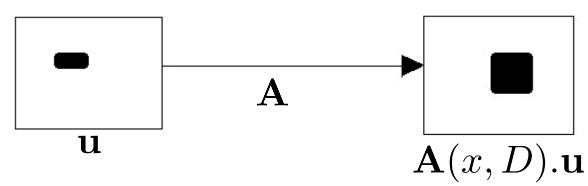
- Normal Operator

$$\mathbf{\Psi} = (\mathbf{K}^*\mathbf{K})$$

- In this talk, an approximation for this operator using Curvelets is discussed.

Normal Operator

- Normal operator is ideally a Pseudo-differential operator

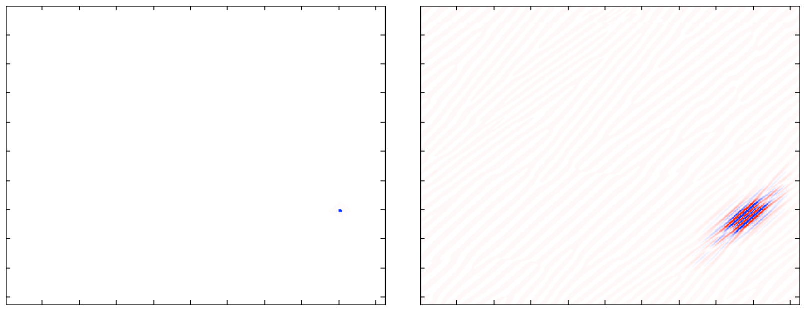


$$\mathbf{A}(x,D).\mathbf{u} = (2\pi)^{-n}\int e^{i\xi.x} a(x,\xi)U(\xi)d\xi$$

- a(.,.) is smooth for smooth background velocity

Invariance of Curvelets

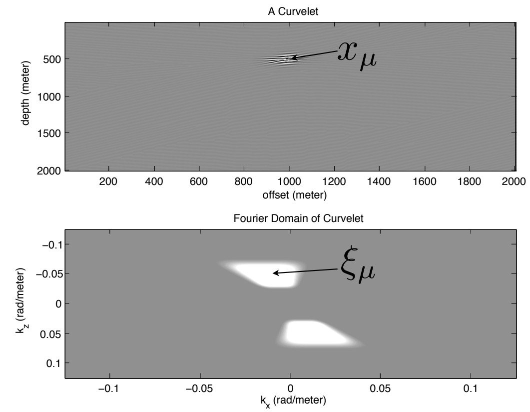
impulse response | **Curvelet response**



Normal Operator Approximation

This bound proved to be exist

$$||(A(x,D) - a(x_\mu,\xi_\mu))\phi_\mu||_{\ell_2} \leq M2^{|\mu|/2}$$



Normal Operator Approximation

- The result of theorem

$$||(A(x,D) - C^T D_A C)\phi_\mu||_{L^2} \leq K2^{-|\mu|/2}$$

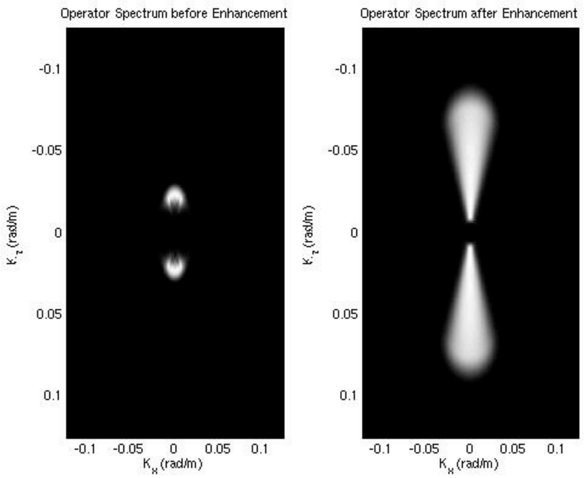
- The above bound holds for an ideal normal operator with
  - Constant background velocity
  - Infinite Aperture
  - Sufficiently smooth operator spectrum
  - Broad-band operator spectrum

Enhancing the Operator

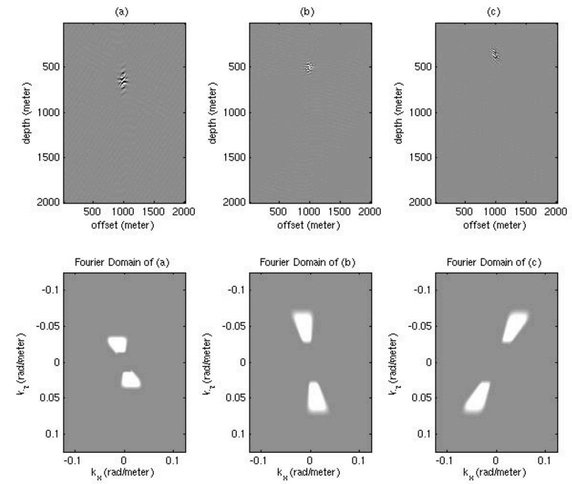
Necessary steps to make the normal operator ideal for decomposition

- Tapering: to flatten the operator spectrum
- Multiple-source: to widen the operator aperture
- Broad-band Sources: to widening the operator support
- Moving boundaries: to eliminating the effect of free surface
- Depth correction: to eliminating the effect of depth attenuation

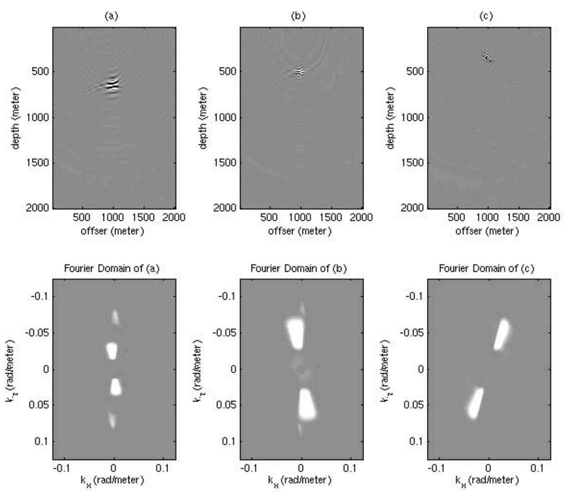
Comparisons



Examples (Three Curvelets)



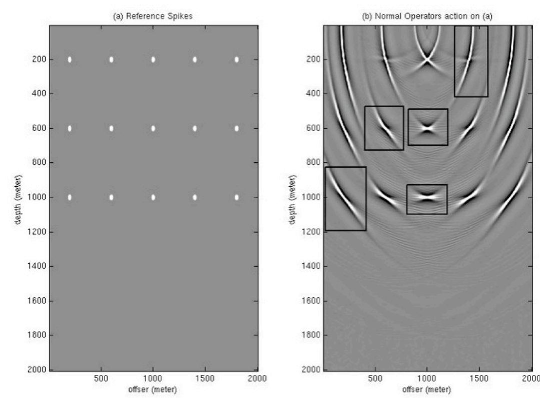
Examples(Normal Operator on three Curvelets)



Direct Approximation

- Approximation with interpolation

$$a(x_\mu, \xi) = e^{i\xi x_\mu} \mathbf{F}\left(\mathbf{A}(x, D).\delta(x - x_\mu)\right)$$



Diagonal Approximation

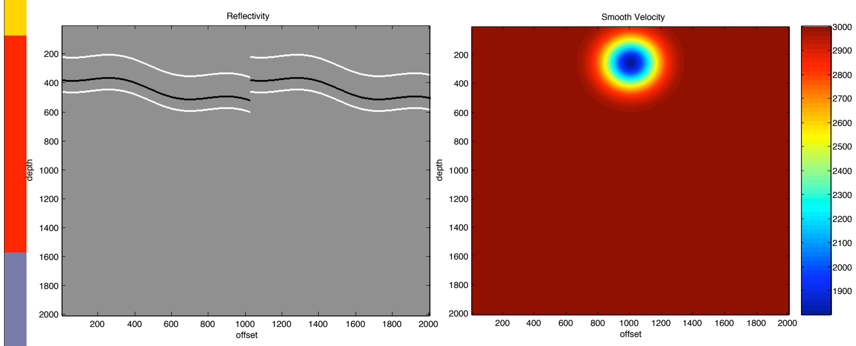
- Approximation with Curvelet Regularization

$$C^* D C r \approx K^* K r$$

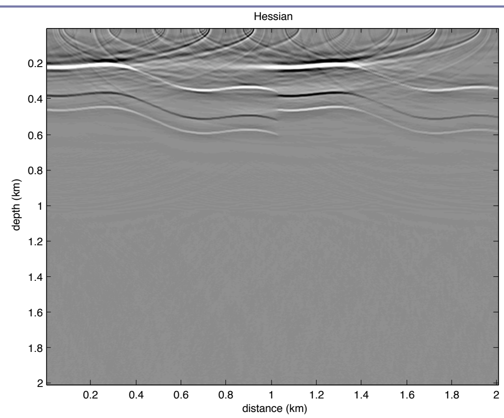
$$\begin{pmatrix} C^T diag(u) \\ \lambda \mathbf{Q} \end{pmatrix} d = \begin{pmatrix} K^* K r \\ \mathbf{Z} \end{pmatrix}$$

- Solve using LSQR method
- Explore smoothness along Curvelet's symbols
- Computationally cheap, Required only "one" evaluation of normal operator

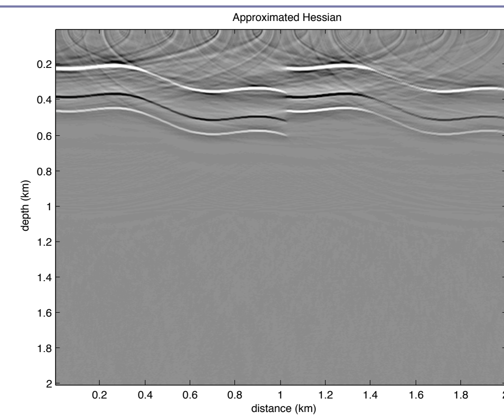
Evaluation Model



Result



Results



Further Extension

- Multiple Reference Vectors

$$\begin{pmatrix} C^T diag(u_1) \\ C^T diag(u_2) \\ \cdots \\ C^T diag(u_N) \\ \mathbf{Q} \end{pmatrix} d = \begin{pmatrix} K^* K r_1 \\ K^* K r_2 \\ . \\ K^* K r_N \\ \mathbf{Z} \end{pmatrix}$$

- Aperture limitation correction

Conclusion

We introduce a novel approach

- Employs Curvelets as essential elements in both approximation and estimation
- Employs an accurate decomposition of expensive operator
- Employs smoothness in Curvelets symbols
- Its Application to synthetic data generates promising results
- Further extension of this method is under development
- A python interface for this method is under investigation