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Recent Results on Seismic Deconvolution

Mohammad Maysami

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Outline

- Introduction & Background
  - Motivation
  - Detection-Estimation Algorithm [ChaRM]
- Recent results
  - Spiky Deconvolution
    - Stagewise Orthogonal Matching Pursuit(StOMP)
  - Multiscale Newton Method and Estimation
- Future work
  - Sensitivity analysis
  - Higher Dimensions
  - Well tie via percolation & IAM

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Credits

- Sparse spike Decon.[Mallat'05]:
  - Stéphane Jaffard (Univ. of Paris)
  - Béatrice Vedel (Univ. De Picardie)
  - Ozgur Yilmaz (Math. Dept.,UBC)

- Percolation model[H&B'04] & well tie :
  - Yves BernabÉ (MIT)

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Introduction to Previous works

ChaRM Project

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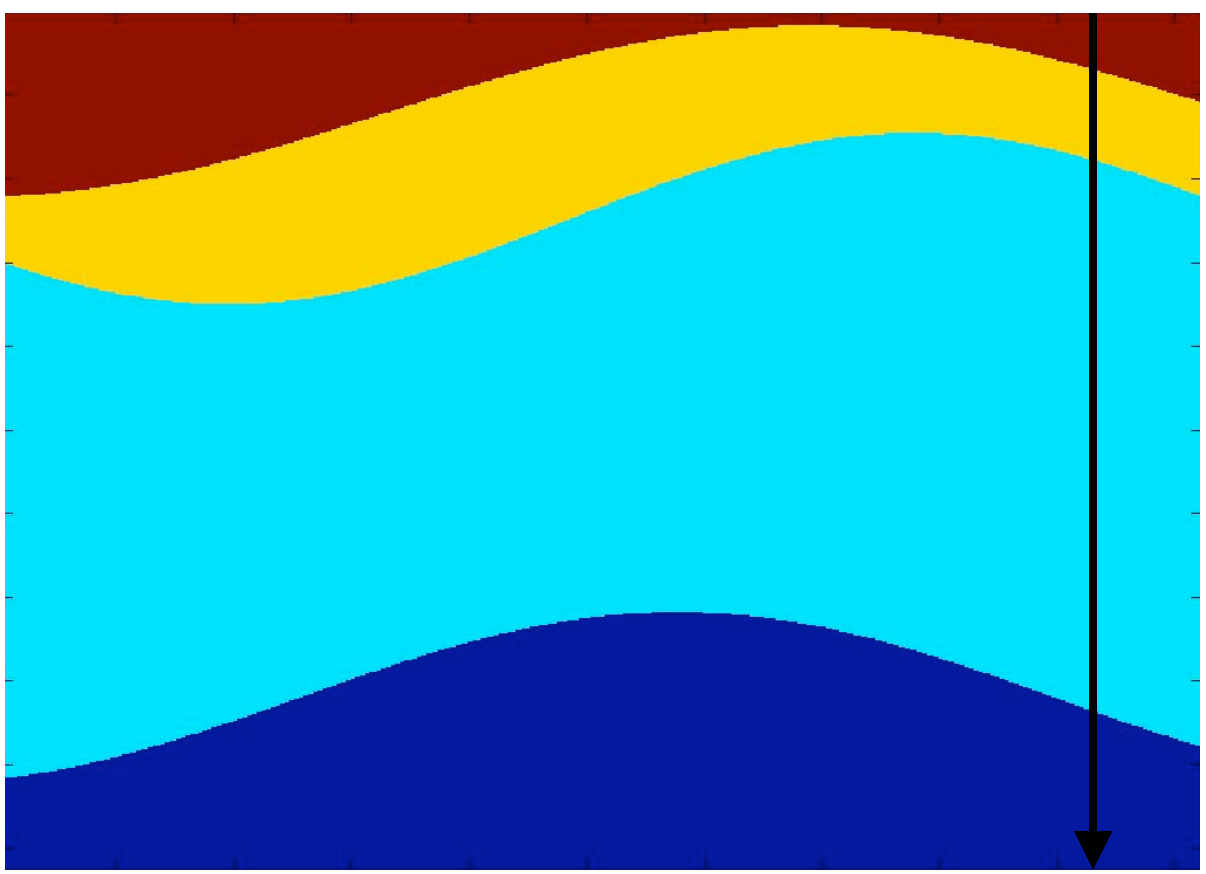
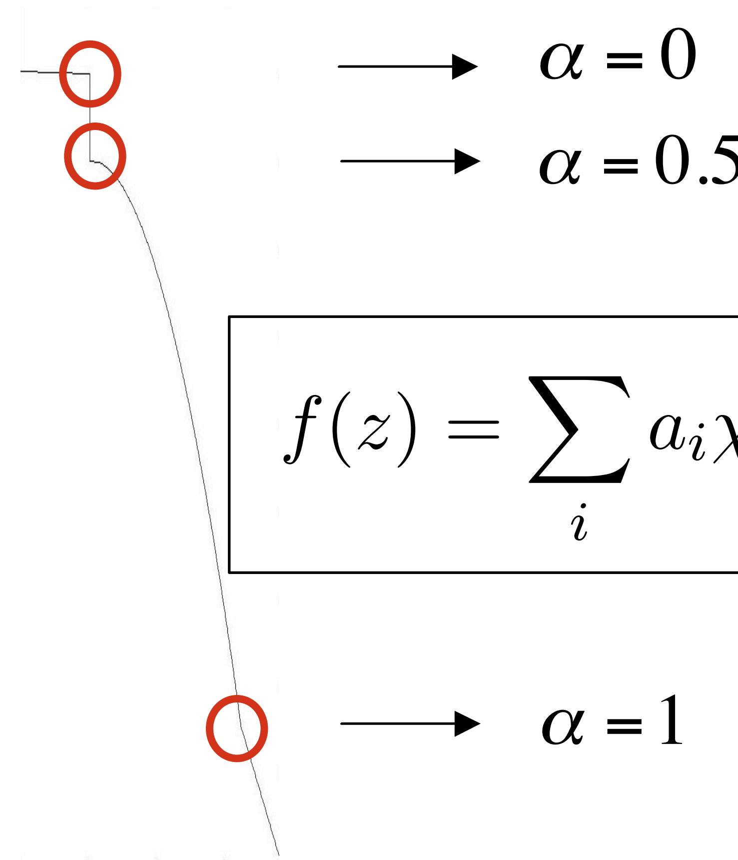
Model for Seismic Transition

Causal: $\chi_+^\alpha(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x^\alpha}{\Gamma(\alpha+1)} & \text{if } x \geq 0 \end{cases},$

Anticausal: $\chi_-^\alpha(x) = \begin{cases} \frac{(-x)^\alpha}{\Gamma(\alpha+1)} & \text{if } x \leq 0 \\ 0 & \text{if } x > 0 \end{cases}$





$$f(z) = \sum_i a_i \chi_\pm^{\alpha_i}(z - z_i),$$

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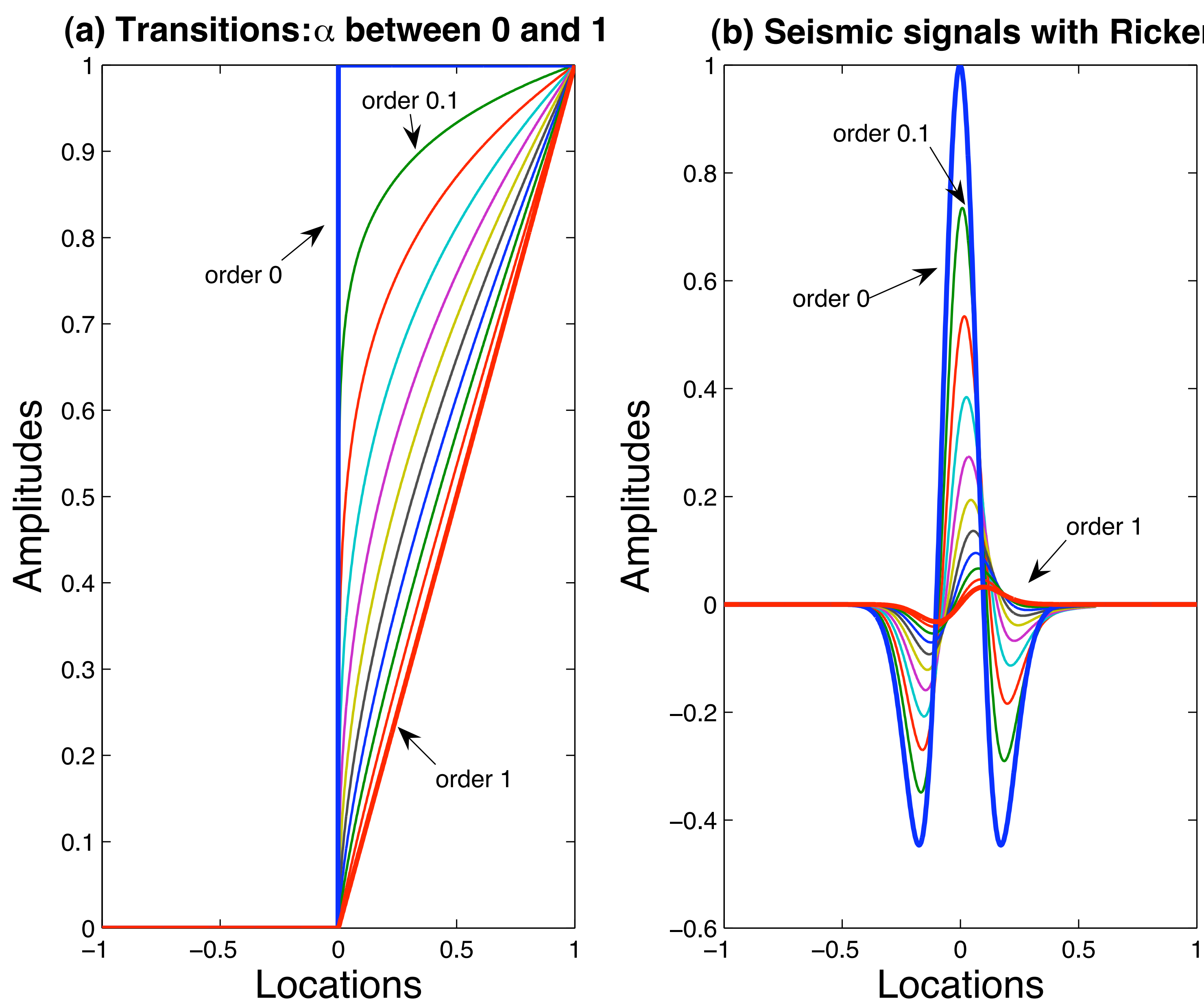
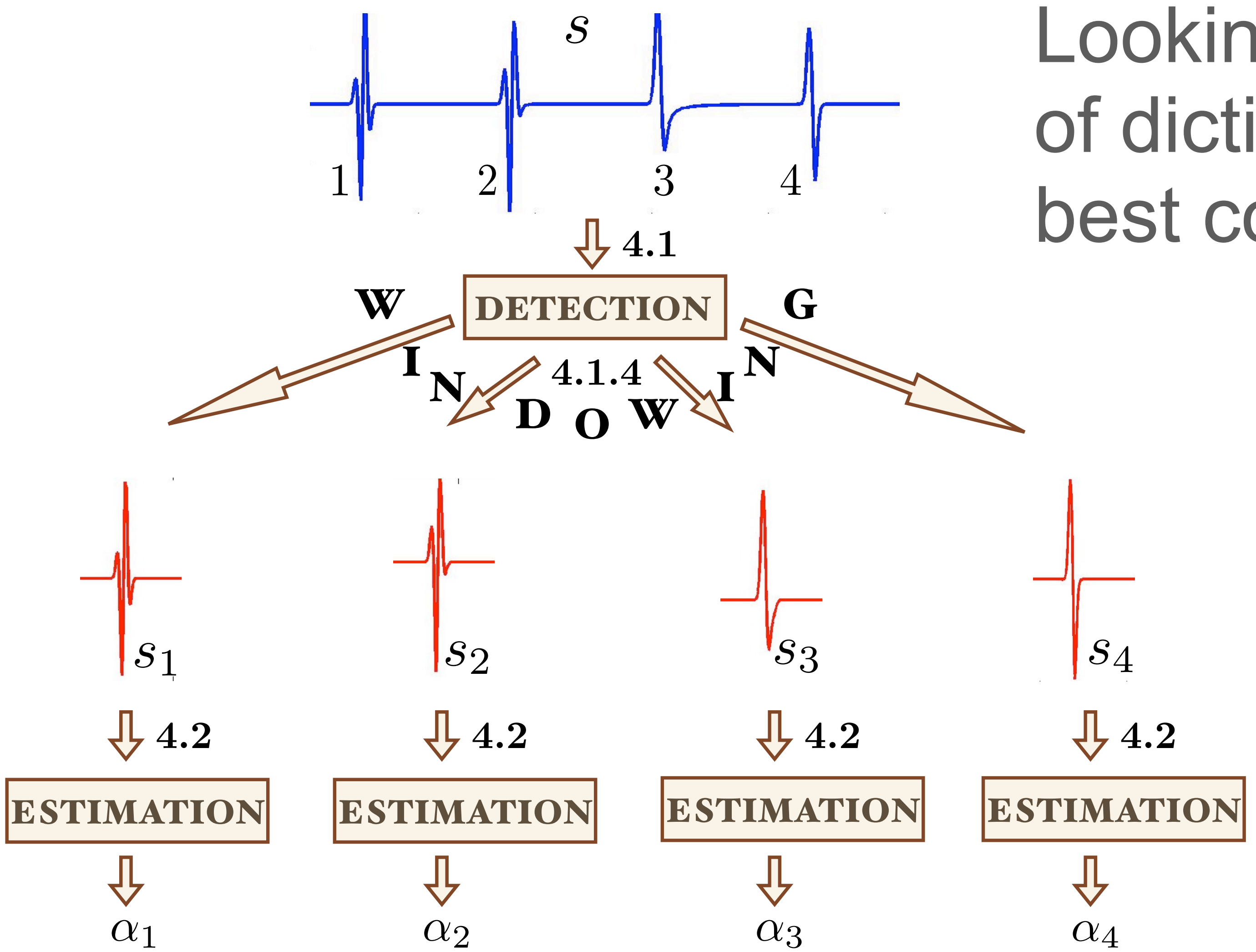
Reflectivity Models

$$r(z) = \sum_{i \in \Lambda_C} K_i \chi_+^{\alpha_i - 1}(z - z_i) - \sum_{i \in \Lambda_A} K_i \chi_-^{\alpha_i - 1}(z - z_i),$$

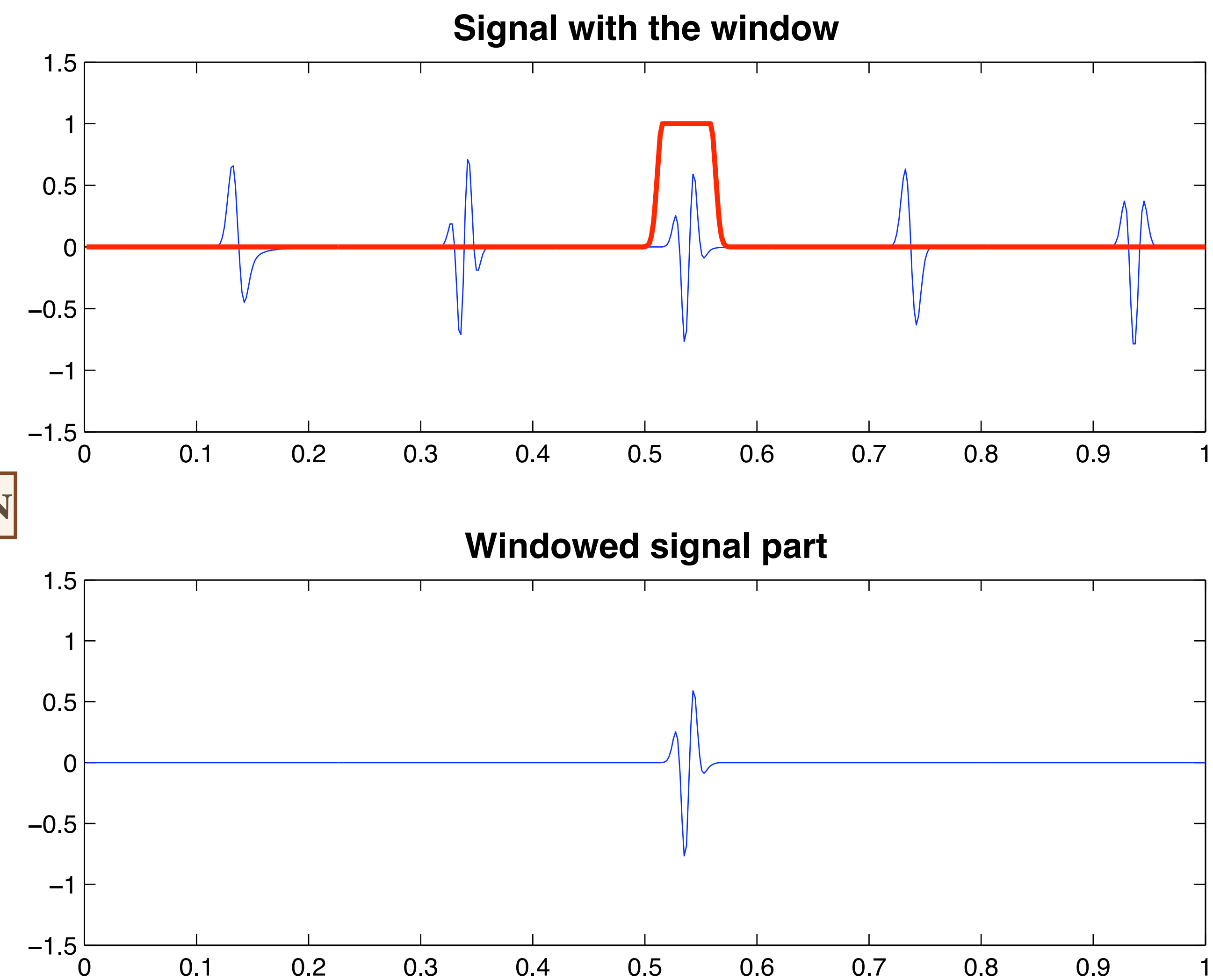


$$s(t) \quad = \quad (r * \psi)(z)$$

Detection-Estimation method



Looking for components of dictionary that are best correlated to data



- Detect the events (D_D)
- Isolate the events
- Characterize windowed events (D_E)

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Brief history

- Introduced a two-stage detection-estimation approach [C.M.Dupuis & F. Herrmann'05]
  - Detection ⇔ spiky decon. for non-spiky reflectivity
    - detect and ***isolate*** the main reflection events
  - Estimation ⇔ characterization of reflectors
    - scale exponents
    - elastic properties end-members binary members
    - percolation threshold and exponent
- Worked on new *estimation* methods to characterize the fine-structure of reflectors

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Sparse Spike Deconvolution

[Dossal and Mallat '05]

for Detection

[ChaRM]

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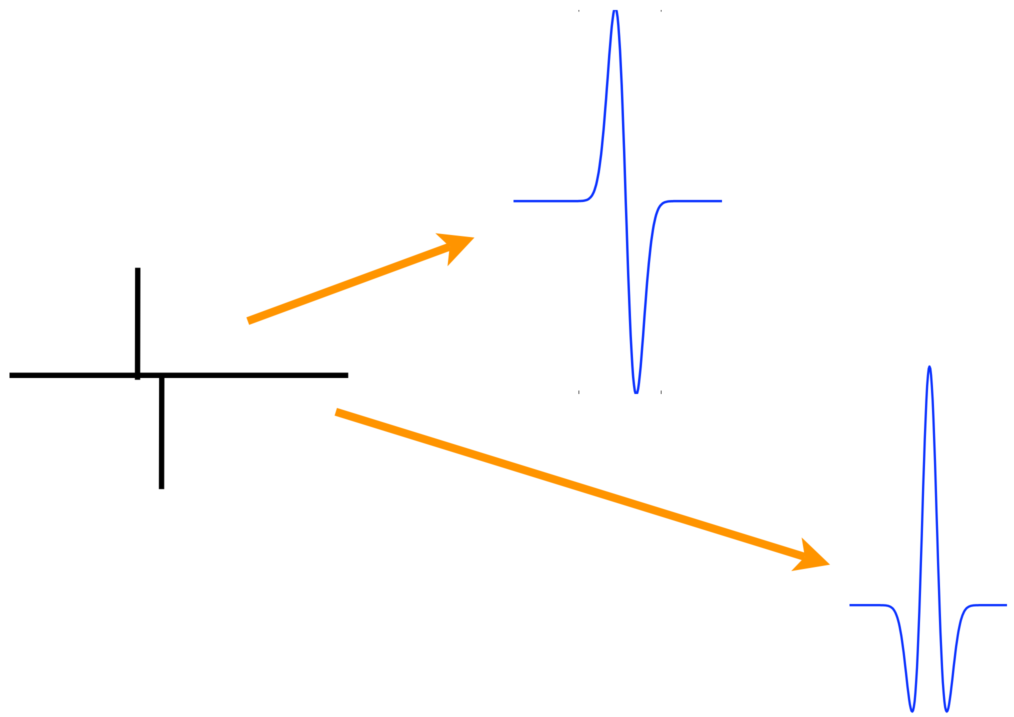
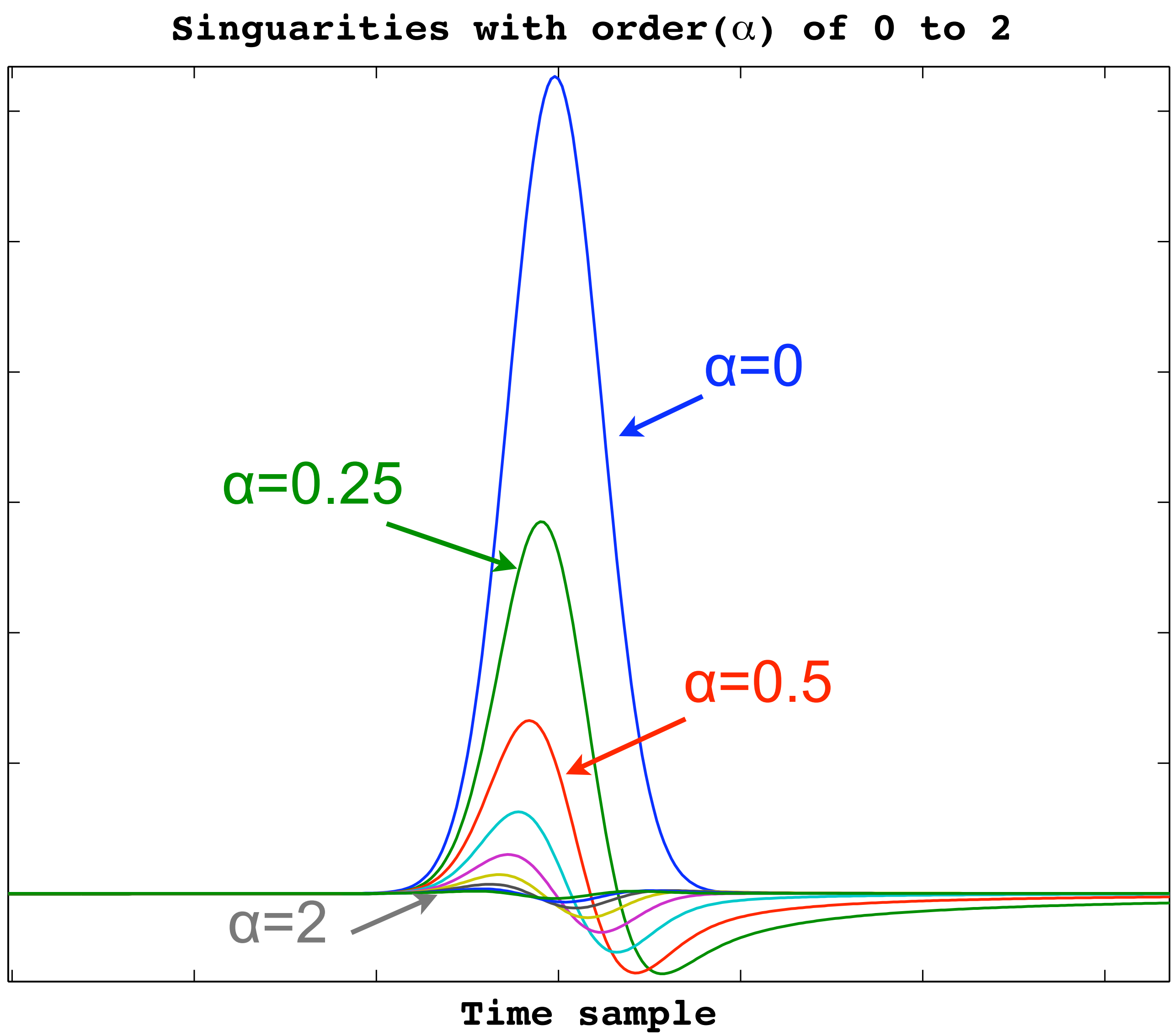
Utilizing Spike Decon.

- Used as a part of our Detection-Estimation approach
- Need of ***accurate (not exact)*** recovery
  - Detecting major events (main cluster)
- fractional order of differentiation
  - two wavelet next to each other
  - one derivative of wavelet





Deconvolution Method

- Widely used in geophysical inversion
- Singularity order of one ($\alpha=0$)
- Efficiency analysis for seismic data [Dossal-Mallat]

$$Y = \psi \star R + W.$$

$$R = \sum_{i \in S} a_i \delta_i$$

$$R = \arg\min_f \frac{1}{2} \|Y - \psi \star f\|_2^2 + \gamma \|f\|_1.$$

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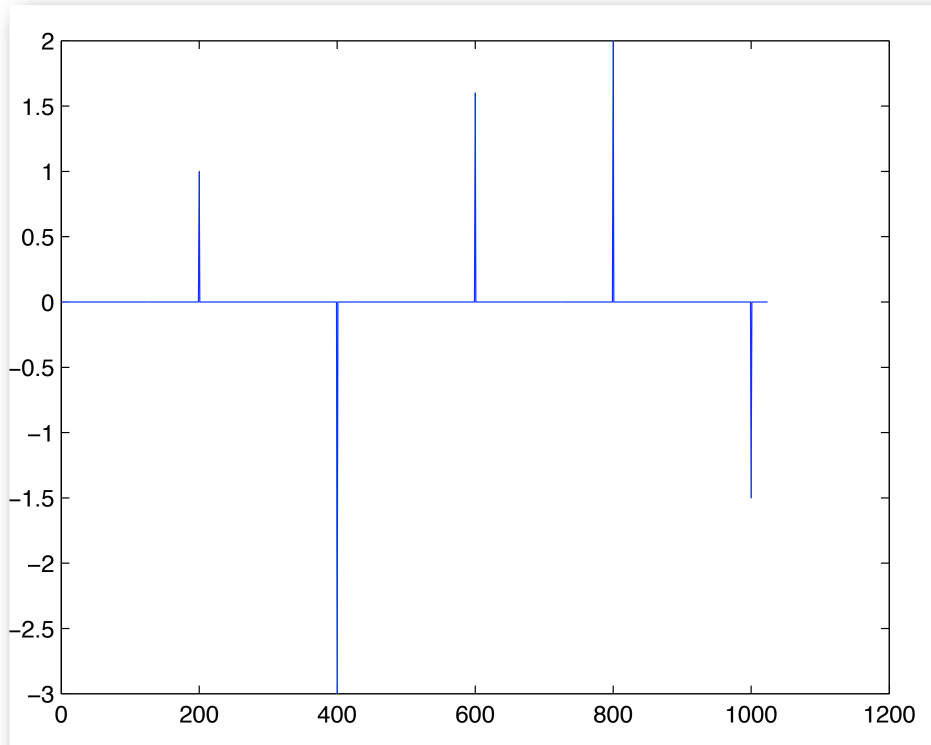
Efficiency Analysis

- Deconvolution without noise
  - Similar analysis for noisy data
- Minimum scale

$$\Delta = \min_{(i,j)\in S^2} \|i - j\|.$$

$$R = \arg\min \|f\|_1 \quad with \quad \psi \star f = Y.$$



Efficiency Analysis (cont'd)

- *Dictionary* = Matrix whose Columns are:

$$\mathbf{D} = [g_i = \psi \star \delta_i \quad for \quad 1 \leqslant i \leqslant N].$$

- *Weak Exact Recovery Coefficient* (***WERC***)

$$WERC(S) = \frac{\beta}{1-\alpha} \qquad , where \quad S \subset \{1, ..., N\}.$$

$$\alpha(S) = \sup_{i \in S} \sum_{k \in S, k \neq i} |< g_k, g_i >| \leqslant 2 \sum_k \phi(k\Delta_0)$$

$$\beta(S) = \sup_{j \notin S} \sum_{k \in S} |< g_k, g_j >| \leqslant \max_{j \leqslant \Delta_0} (\phi(j) + \phi(\Delta_0 - j)) + \alpha(S)$$

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StOMP:
a fast L1 solver

[Donoho et. al. 06]

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L0 - L1 Equivalency

- Strong equivalence ofPƒ$_0$ andPƒ$_1$
  - for given **A**, $\forall x_0$ P1(y,**A**)→Unique sparsest Solution
- Weak equivalence ofPƒ$_0$ andPƒ$_1$
  - equivalence holds for the typical sparse x_0

$$\|x\|_0 < \frac{1}{2}\sqrt{N}$$

StOMP Solver

- For (under)determined systems of equations
- Assumes additive Gaussian noise for non-zero entries
- Numerous terms enter at each thresholding stage and have fixed number of staged.
- Approximation to the sparsest solution over a region of the sparsity/indeterminacy plane

- Our Case
  - Determined System : $A_{(N \times N)}$
  - Mixing by random spike train
    - Random locations
    - Random amplitudes

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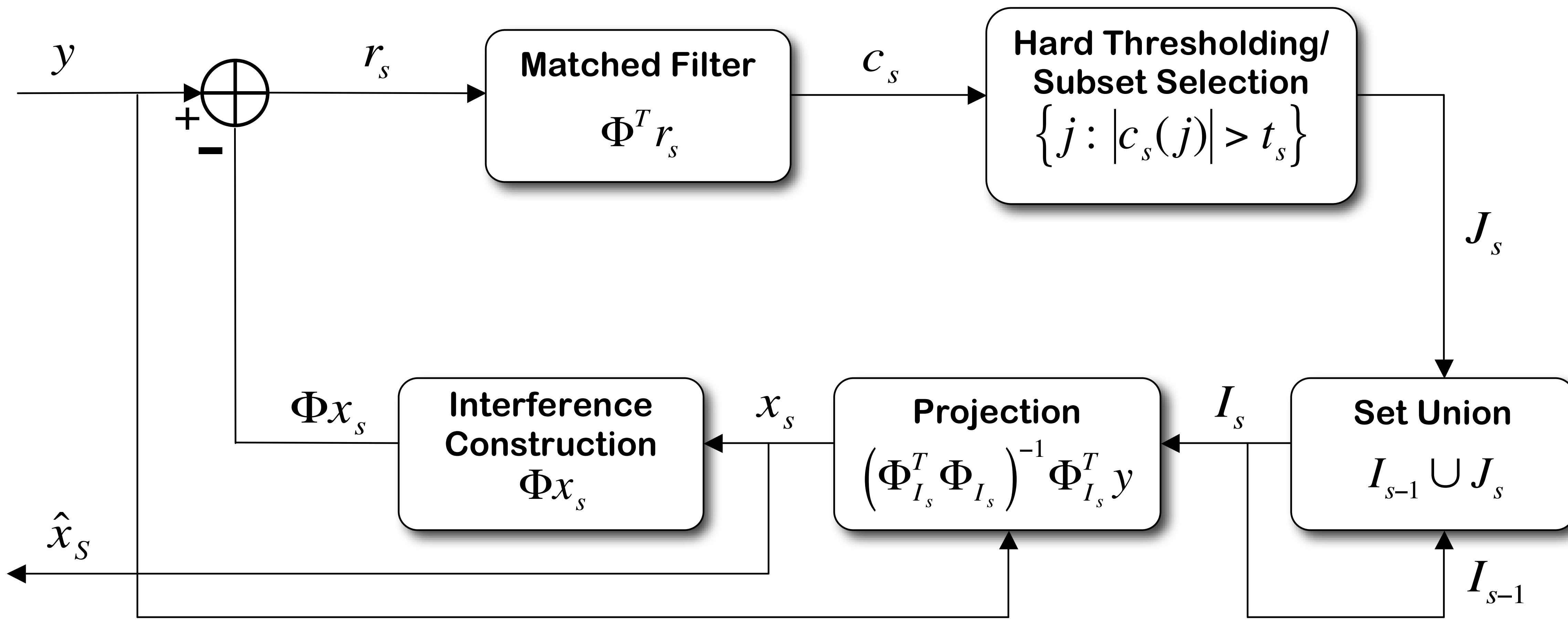
Algorithm Flowchart



$$\mathbf{J}_s = \{j : |c_s(j)| > t_s\sigma_s\}$$

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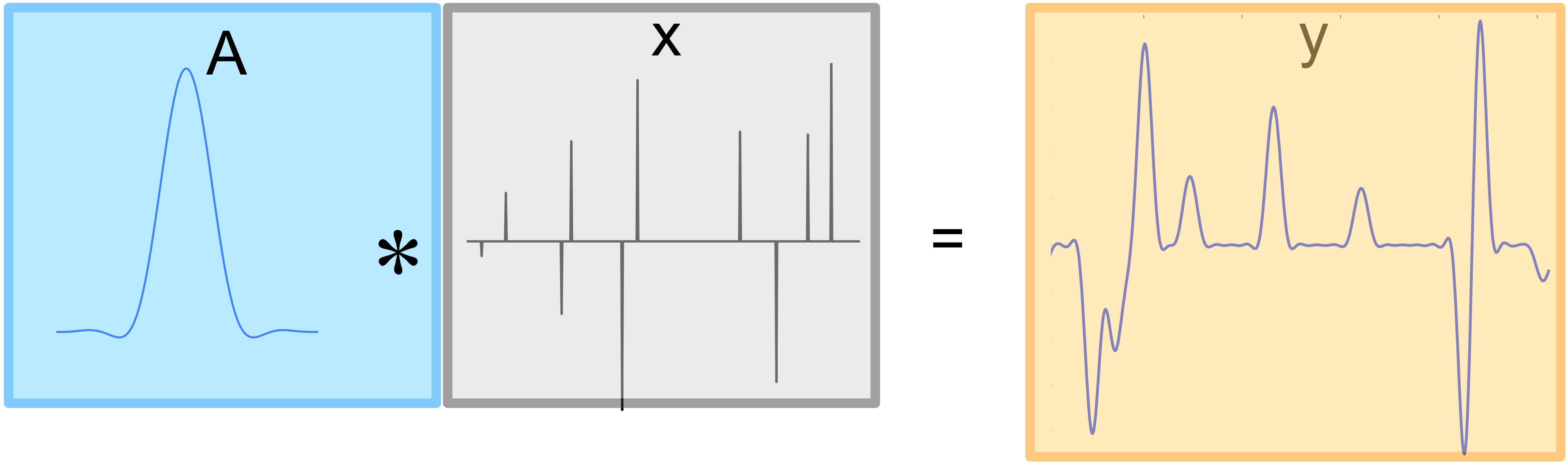
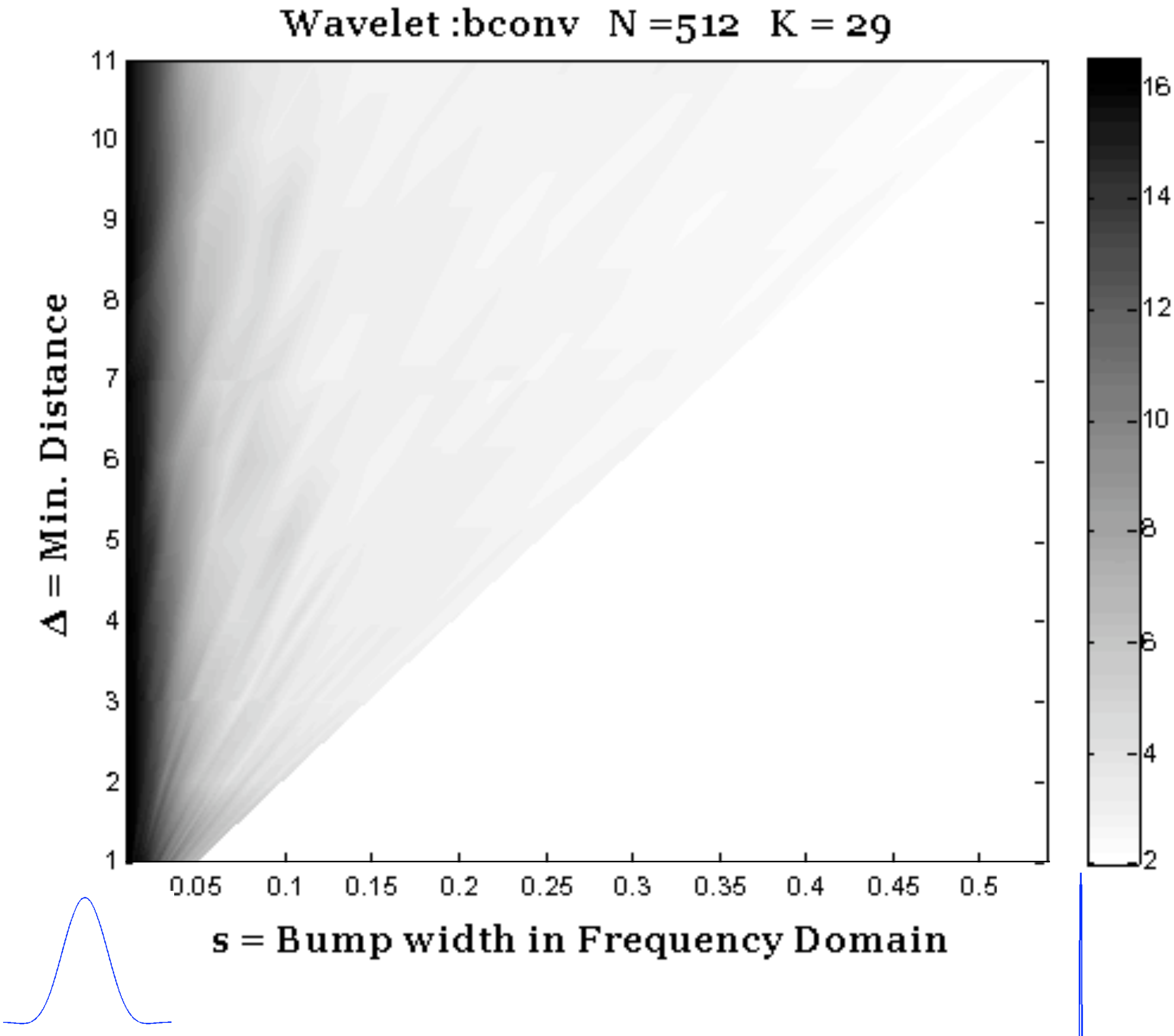
Recovery Phase Diagram

- Test Settings:
  - A: Convolution with cosine bump
  - Signal length : 512
  - No. of spikes (K) : 20
  - Dynamic range setting
  - Δ values : 13
  - scale values : 30
- L0 norm to show the error
  - L2 could also be used



Sample Phase Diagram



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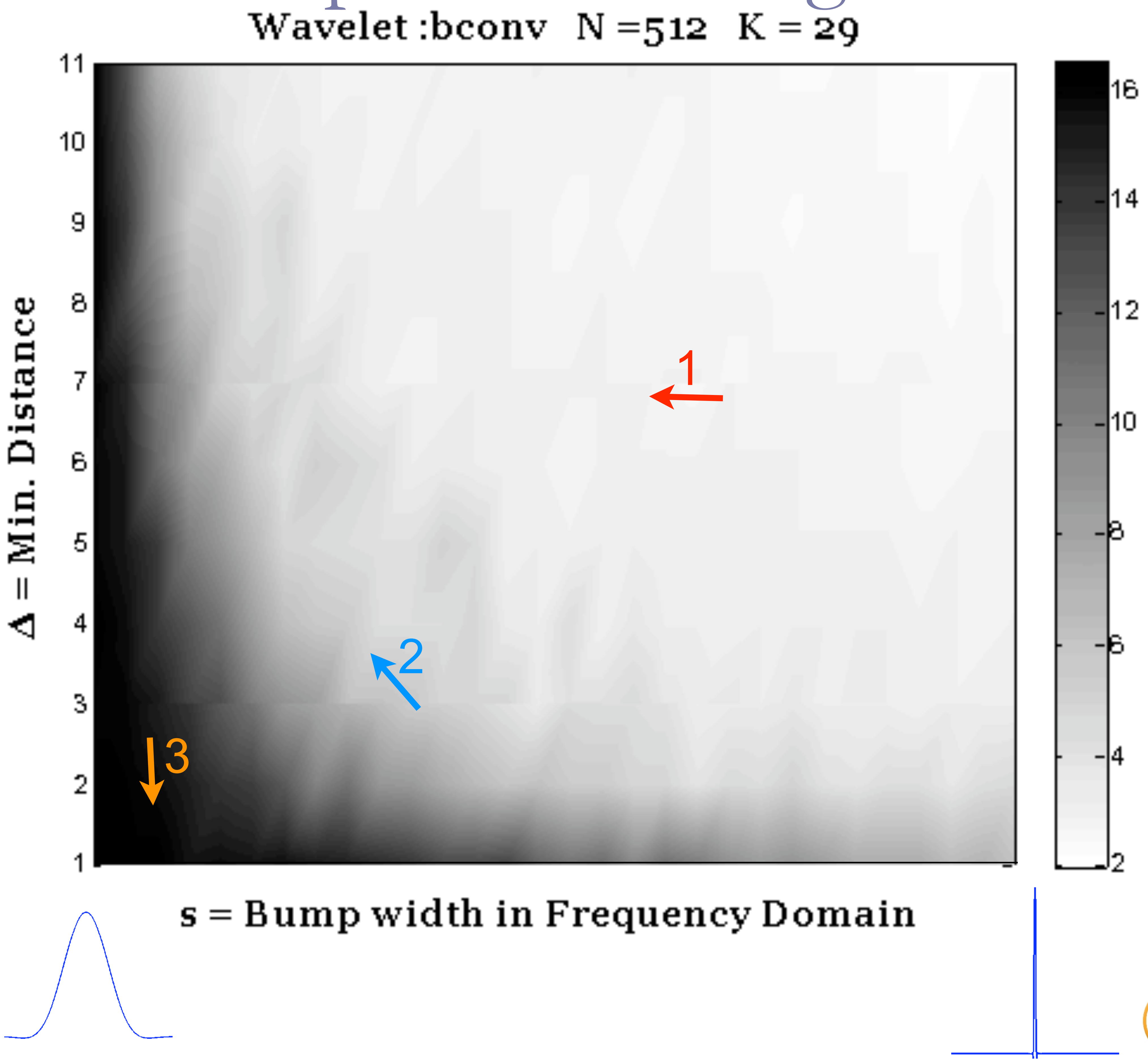
Sample Phase Diagram



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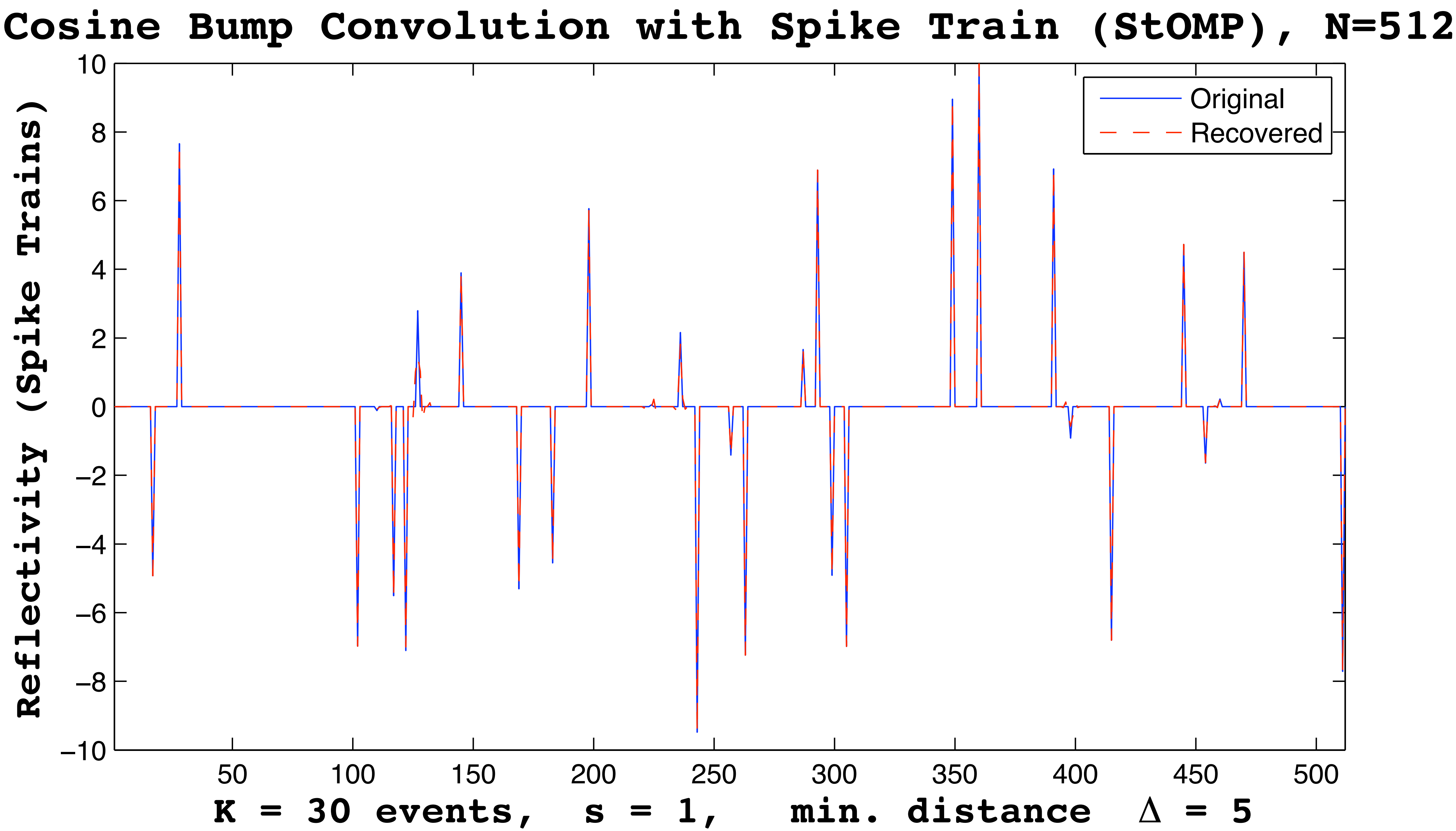
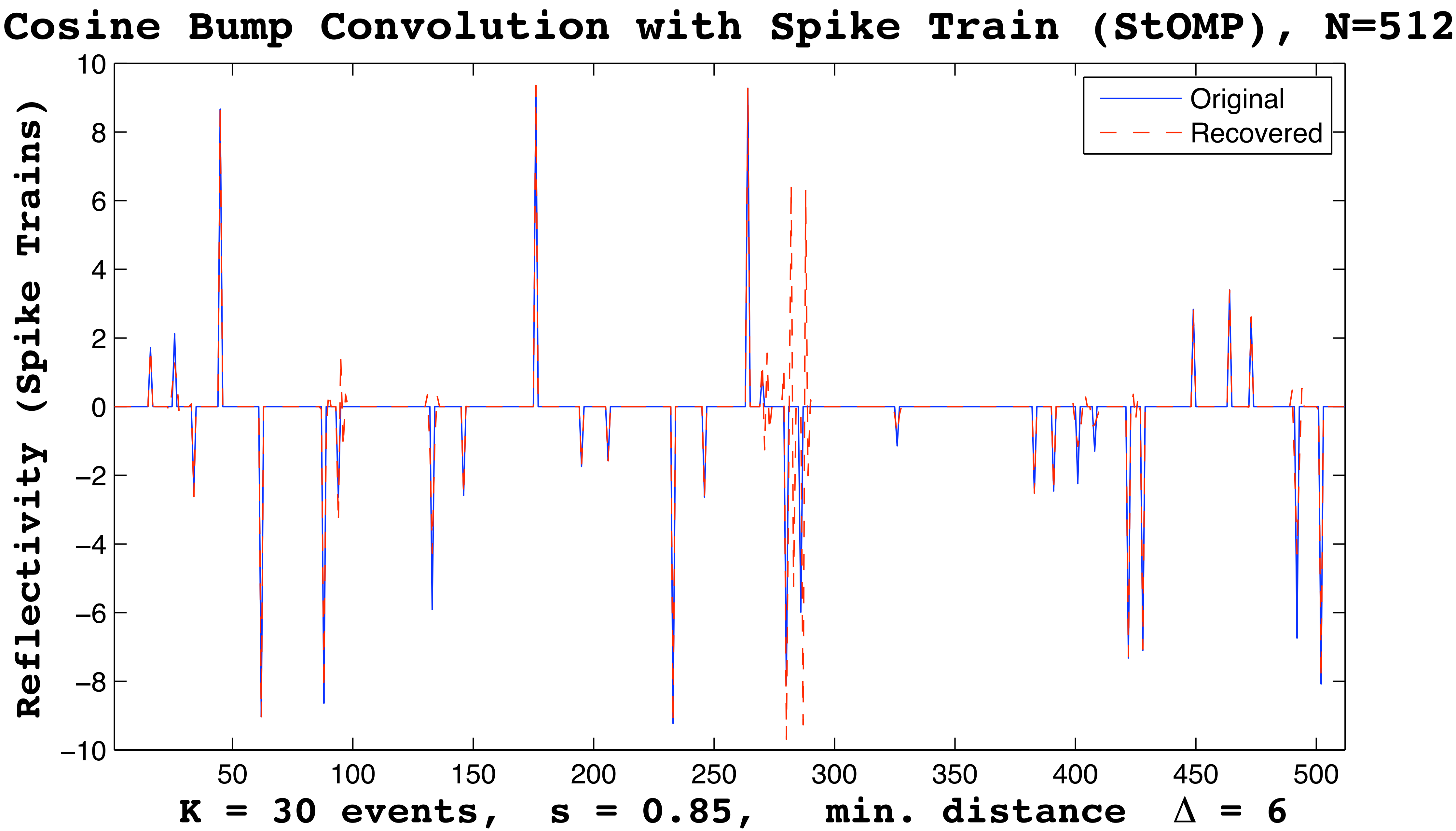
1: Accurate Recovery



Length of Signal : 512	StOMP Solving Time : 1.6
Number of Spikes : 30	StOMP Stage : 12
Min & Max Spacing : 5,74	L2 Nrom(x) : 31.13
Bump/Wavelet Scale : 1	% L0 Error : 1.867
Regular , Uniform : 0,0	% L2 Error : 0.07104

2 : Partial Recovery



Length of Signal : 512	StOMP Solving Time : 1.75
Number of Spikes : 30	StOMP Stage : 10
Min & Max Spacing : 6,57	L2 Nrom(x) : 30.47
Bump/Wavelet Scale : 0.85	% L0 Error : 2.3
Regular , Uniform : 0,0	% L2 Error : 0.5451

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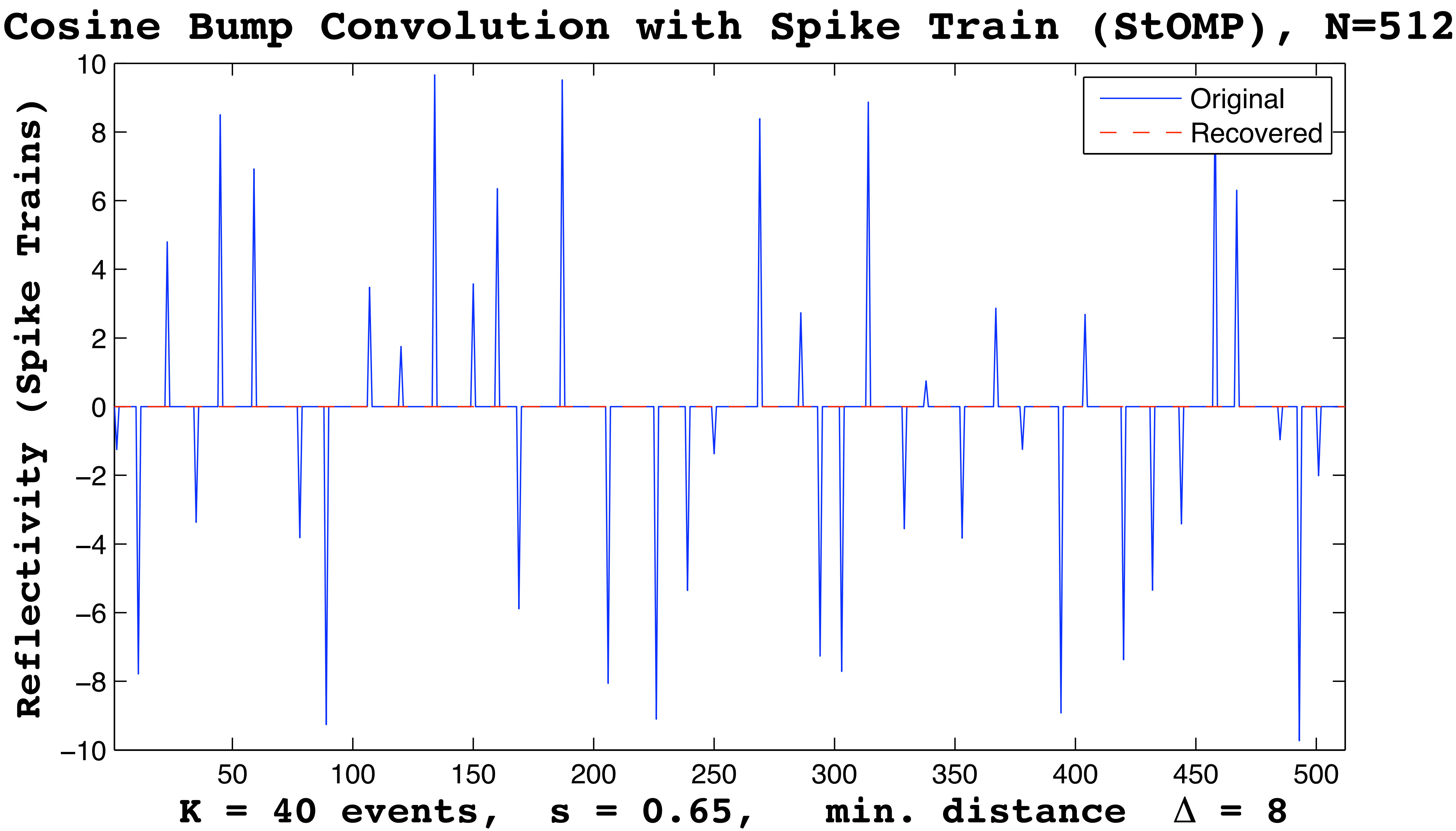
3: Unrecoveralbe



Length of Signal : 512 Number of Spikes : 40 Min & Max Spacing : 8,20 Bump/Wavelet Scale : 0.65 Regular , Uniform : 0,0	StOMP Solving Time : 0.01 StOMP Stage : 2 L2 Nrom(x) : 38.67 % L0 Error : 1 % L2 Error : 1

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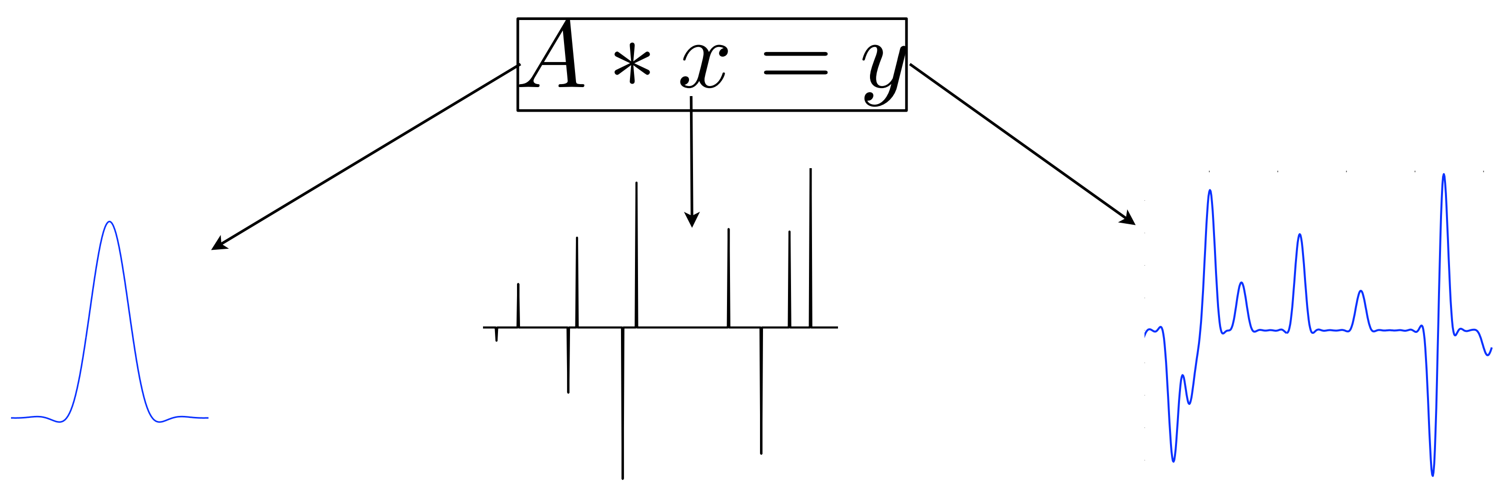
Spike Decon. Analysis Scheme

Analysis w.r.t.

- wavelet type
- wavelet width/scale
- Minimum distance (Δ)
- Solver
  - Stagewise Orthogonal Matching Pursuit (StOMP)
  - Basis Pursuit (BP)
- Different synthesis and analysis wavelets



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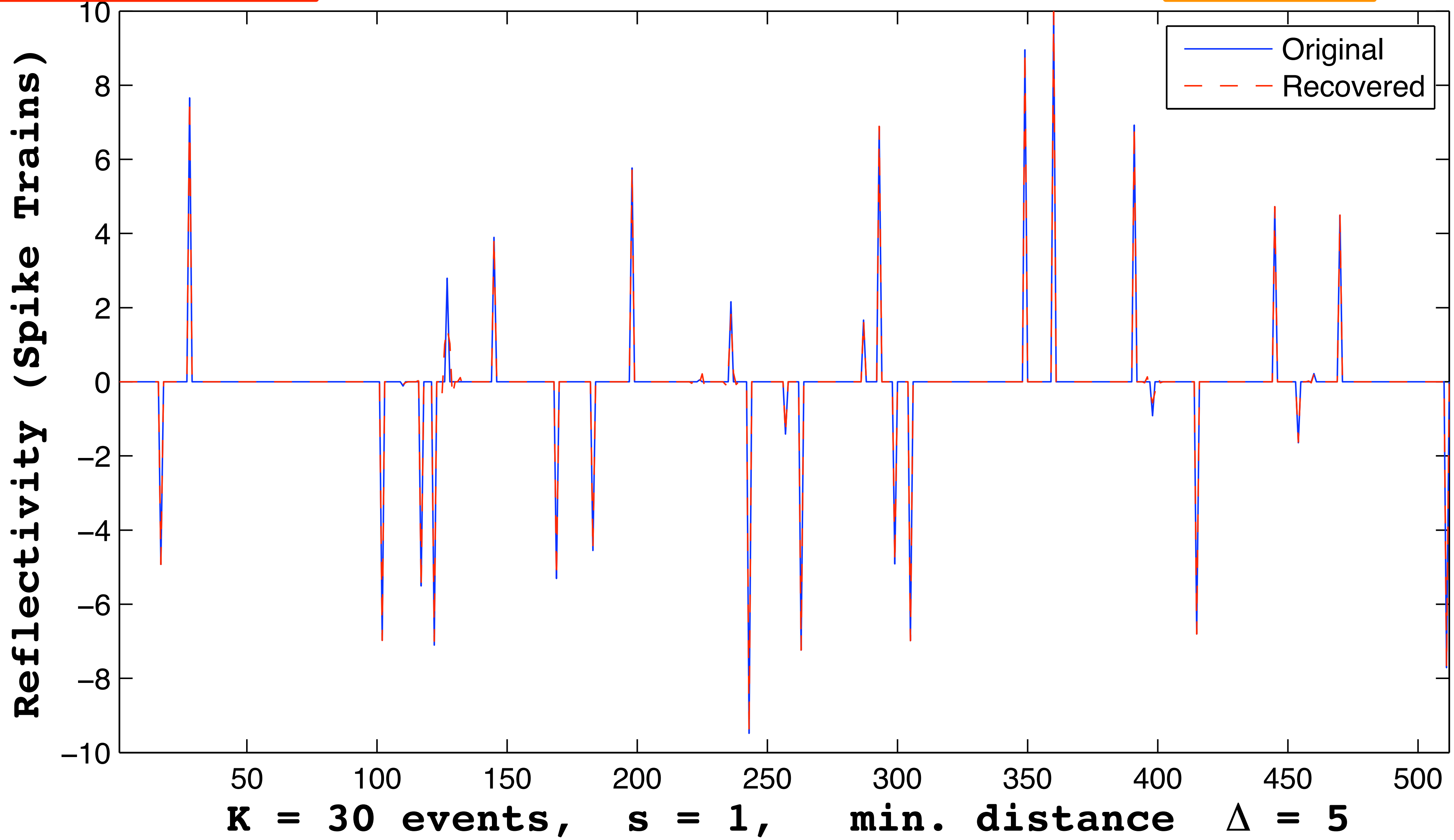
Cosine Bump Convolution with Spike Train (StOMP), N=512



Length of Signal : 512	StOMP Solving Time : 1.6
Number of Spikes : 30	StOMP Stage : 12
Min & Max Spacing : 5,74	L2 Nrom(x) : 31.13
Bump/Wavelet Scale : 1	% L0 Error : 1.867
Regular , Uniform : 0,0	% L2 Error : 0.07104

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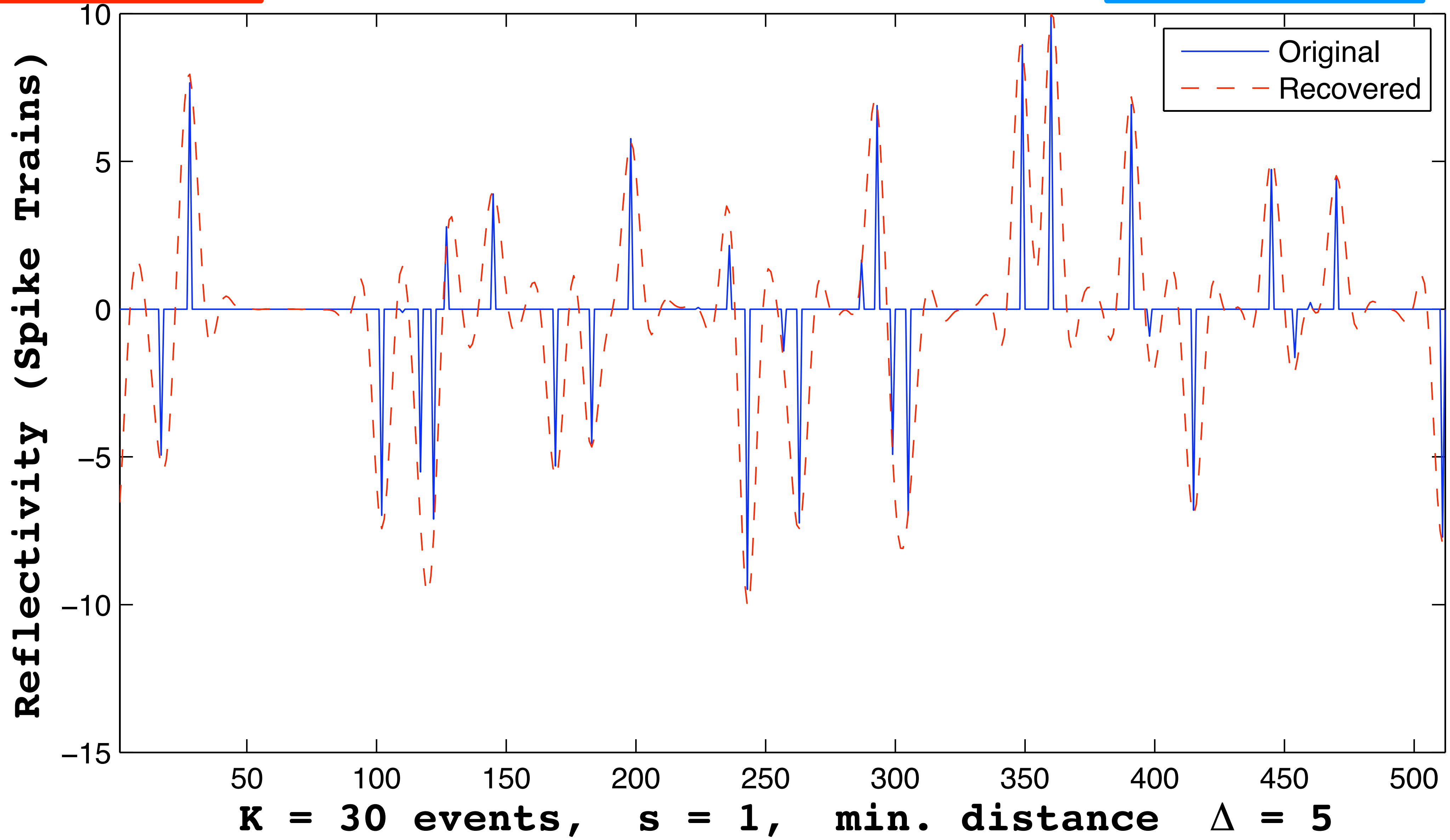
Cosine Bump Convolution with Spike Train (Scaled BP), N=512



Length of Signal : 512 Number of Spikes : 30 Min & Max Spacing : 5,74 Bump/Wavelet Scale : 1 Regular , Uniform : 0,0	BP Solving Time : 0.15 L2 Nrom(x) : 31.13 % L0 Error : 16.07 % L2 Error : 0.9992

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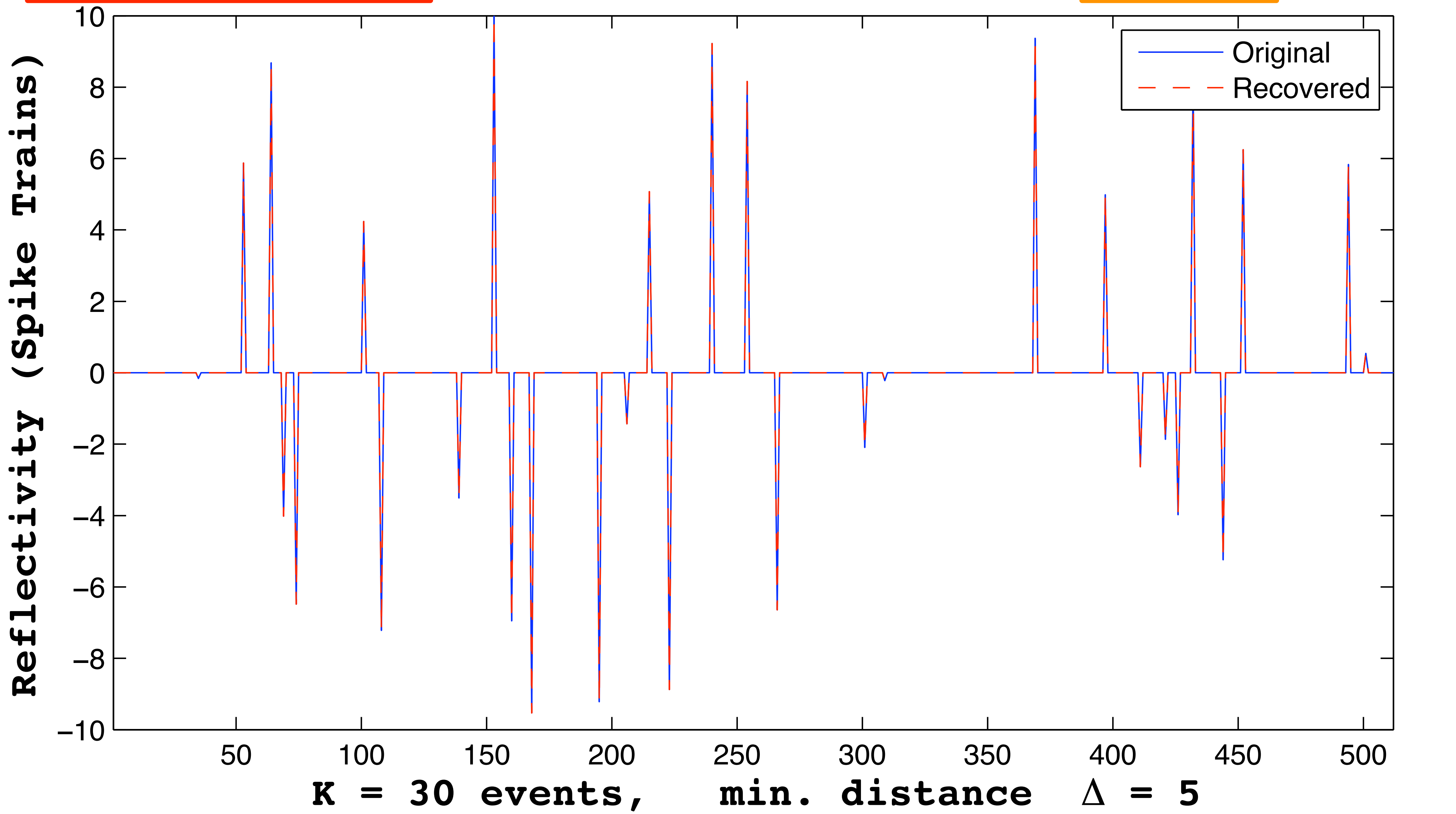
Ricker Wavelet Conv. with Spike Train (StOMP), N=512



Length of Signal : 512	StOMP Solving Time : 0.55
Number of Spikes : 30	StOMP Stage : 5
Min & Max Spacing : 5,60	L2 Nrom(x) : 34.48
Bump/Wavelet Scale : 1	% L0 Error : 0.8667
Regular , Uniform : 0,0	% L2 Error : 4.568e−16

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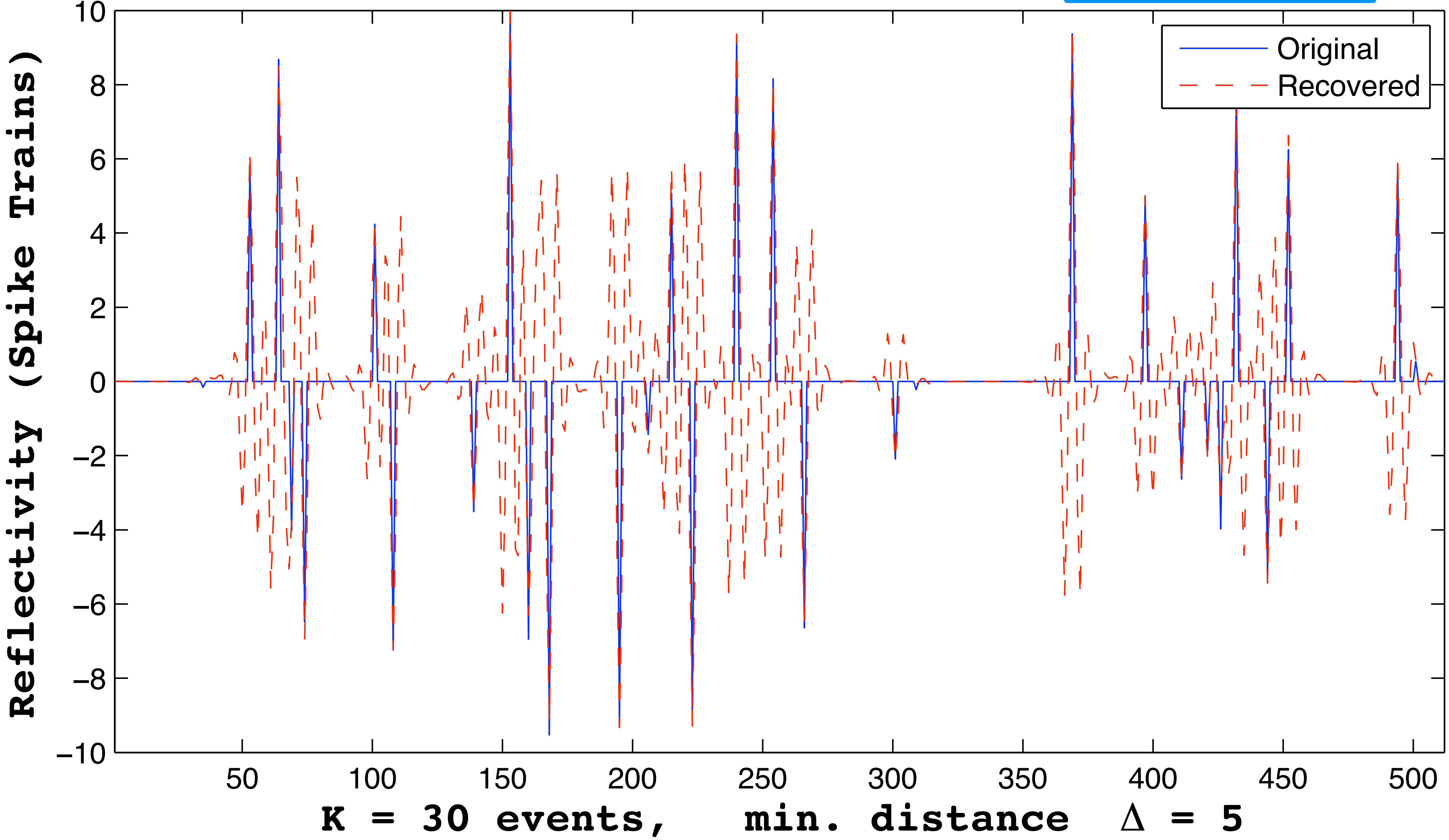
Ricker wavelet Conv. with Spike Train (Scaled BP), N=512



Length of Signal : 512	BP Solving Time : 0.34
Number of Spikes : 30	L2 Nrom(x) : 34.48
Min & Max Spacing : 5,60	% L0 Error : 16.07
Bump/Wavelet Scale : 1	% L2 Error : 0.9993
Regular , Uniform : 0,0	

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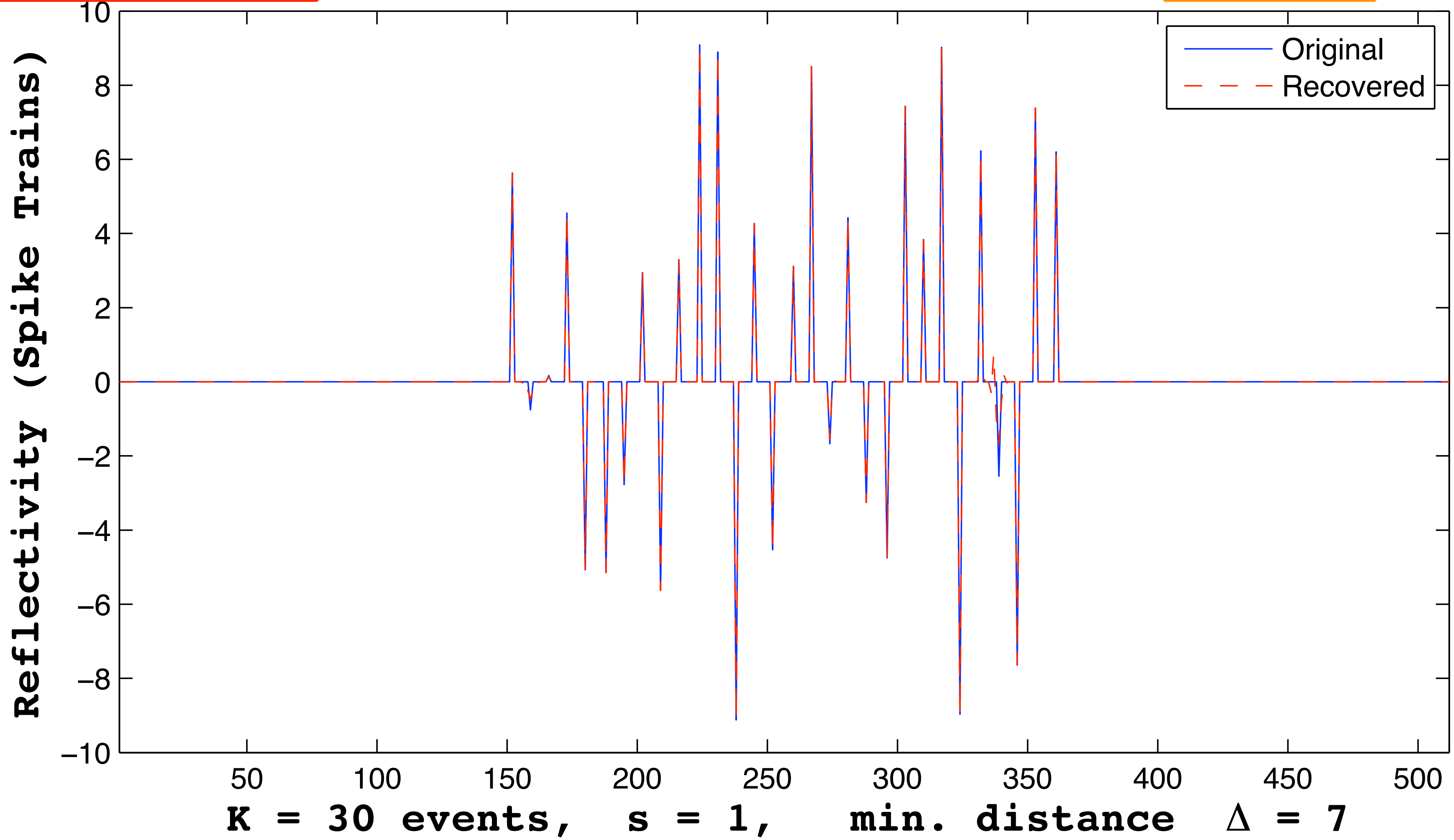
Cosine Bump Convolution with Spike Train (StOMP), N=512



Length of Signal : 512	StOMP Solving Time : 1.45
Number of Spikes : 30	StOMP Stage : 12
Min & Max Spacing : 7,8	L2 Nrom(x) : 31.83
Bump/Wavelet Scale : 1	% L0 Error : 1.467
Regular , Uniform : 1,0	% L2 Error : 0.05429

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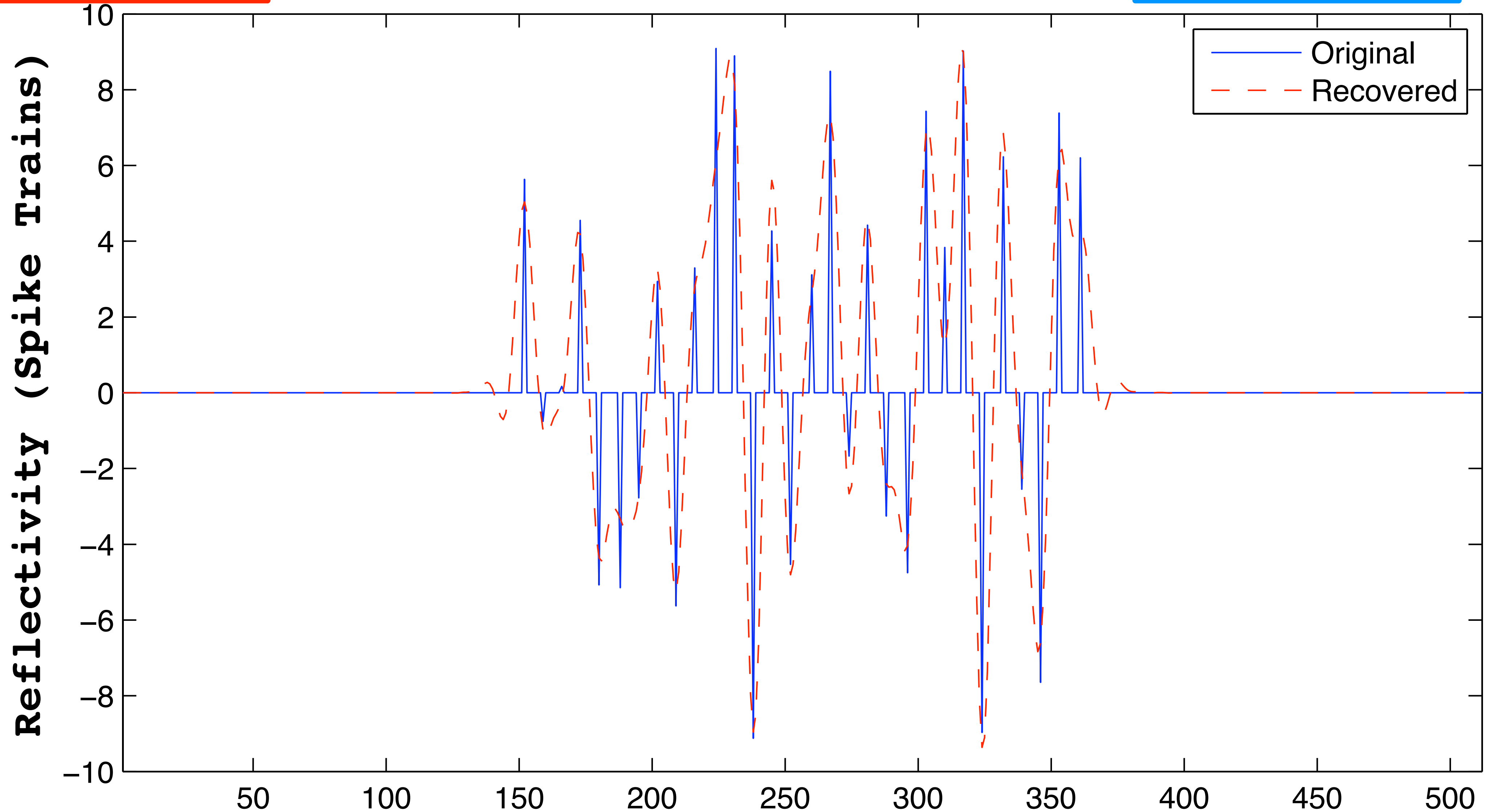
Cosine Bump Convolution with Spike Train (Scaled BP), N=512



K = 30 events, s = 1, min. distance Δ = 7

Length of Signal : 512 Number of Spikes : 30 Min & Max Spacing : 7,8 Bump/Wavelet Scale : 1 Regular , Uniform : 1,0	BP Solving Time : 0.1 L2 Nrom(x) : 31.83 % L0 Error : 16.07 % L2 Error : 0.9992

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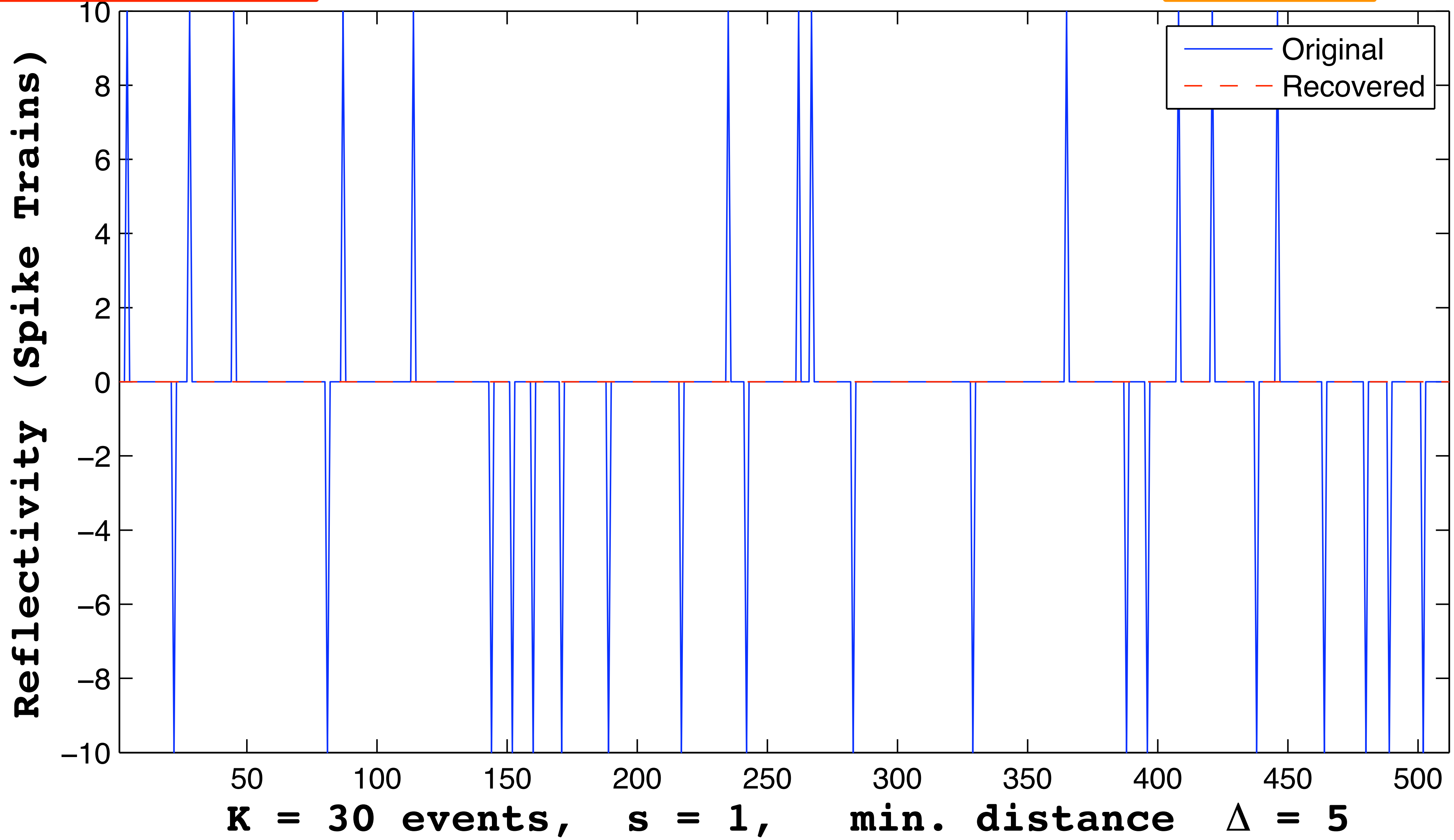
Cosine Bump Convolution with Spike Train (StOMP), N=512



Length of Signal : 512	StOMP Solving Time : 0.01
Number of Spikes : 30	StOMP Stage : 2
Min & Max Spacing : 5,46	L2 Nrom(x) : 54.77
Bump/Wavelet Scale : 1	% L0 Error : 1
Regular , Uniform : 0,1	% L2 Error : 1

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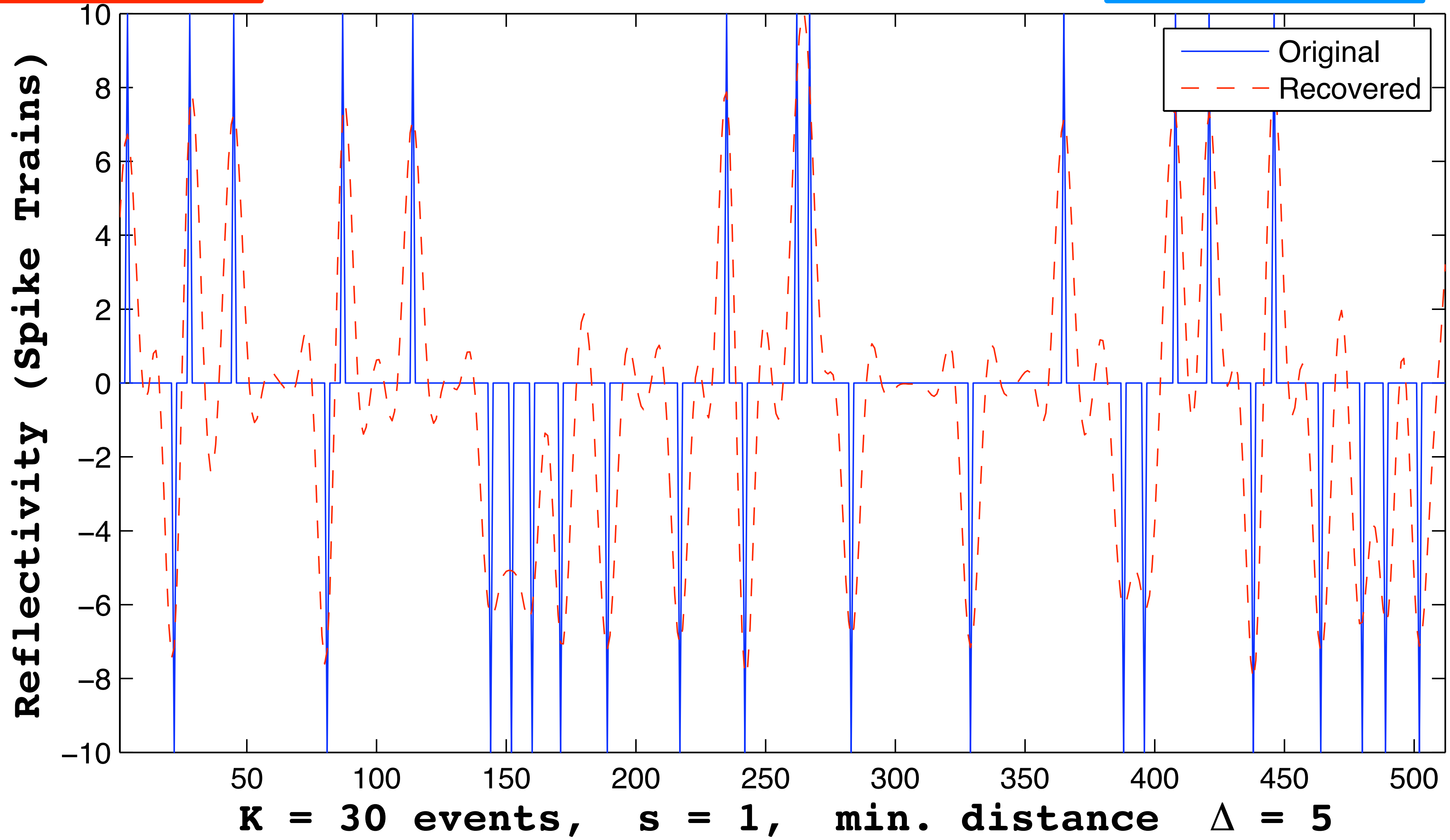
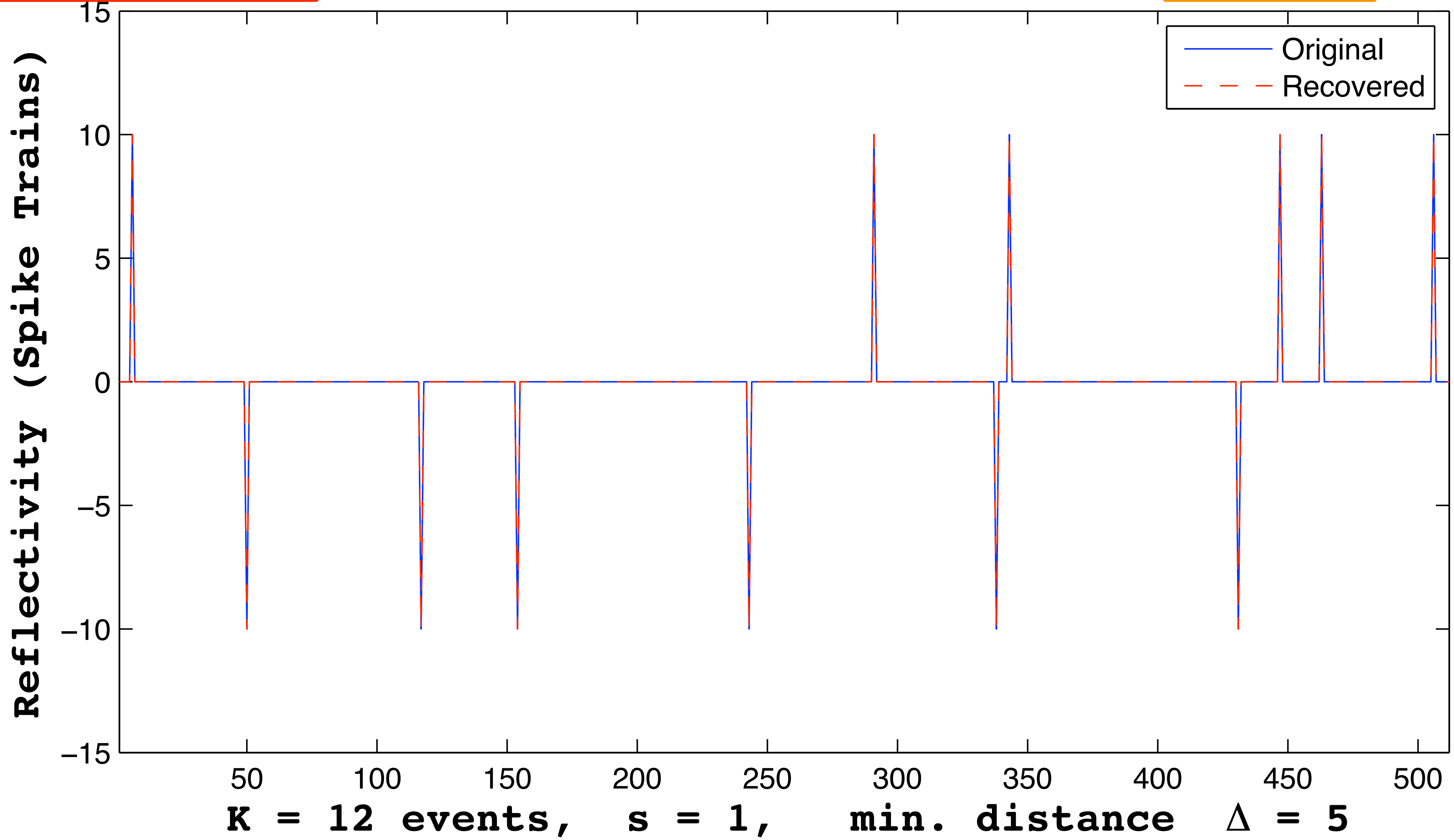
Cosine Bump Convolution with Spike Train (Scaled BP), N=512



Length of Signal : 512 Number of Spikes : 30 Min & Max Spacing : 5,46 Bump/Wavelet Scale : 1 Regular , Uniform : 0,1	BP Solving Time : 0.11 L2 Nrom(x) : 54.77 % L0 Error : 16.07 % L2 Error : 0.9992

Cosine Bump Convolution with Spike Train (StOMP), N=512



Length of Signal : 512	StOMP Solving Time : 0.08
Number of Spikes : 12	StOMP Stage : 3
Min & Max Spacing : 5,89	L2 Nrom(x) : 34.64
Bump/Wavelet Scale : 1	% L0 Error : 0.5
Regular , Uniform : 0,1	% L2 Error : 8.075e-14

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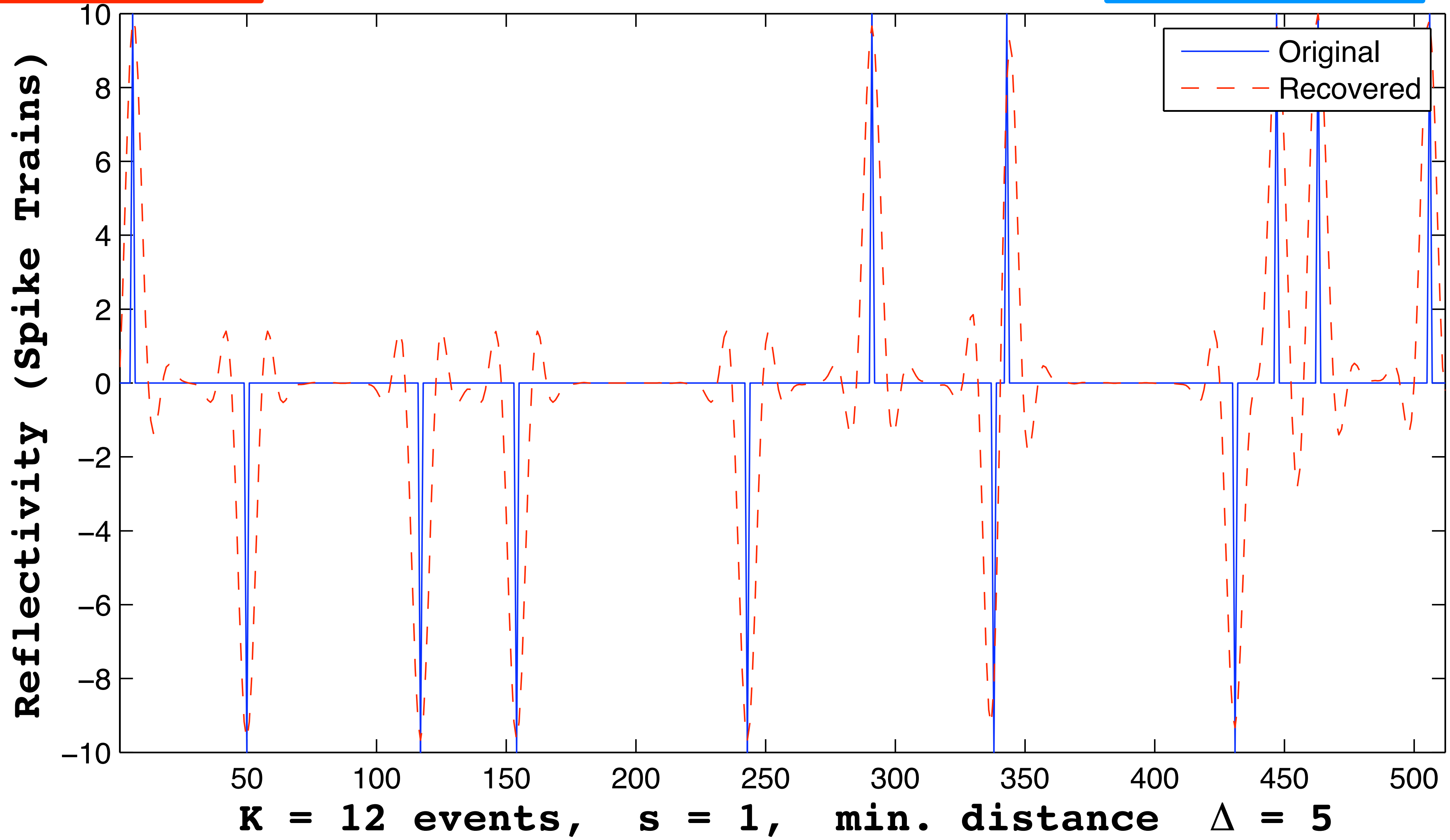
Cosine Bump Convolution with Spike Train (Scaled BP), N=512



Length of Signal : 512	BP Solving Time : 0.11
Number of Spikes : 12	L2 Nrom(x) : 34.64
Min & Max Spacing : 5,89	% L0 Error : 41.67
Bump/Wavelet Scale : 1	% L2 Error : 0.9992
Regular , Uniform : 0,1	

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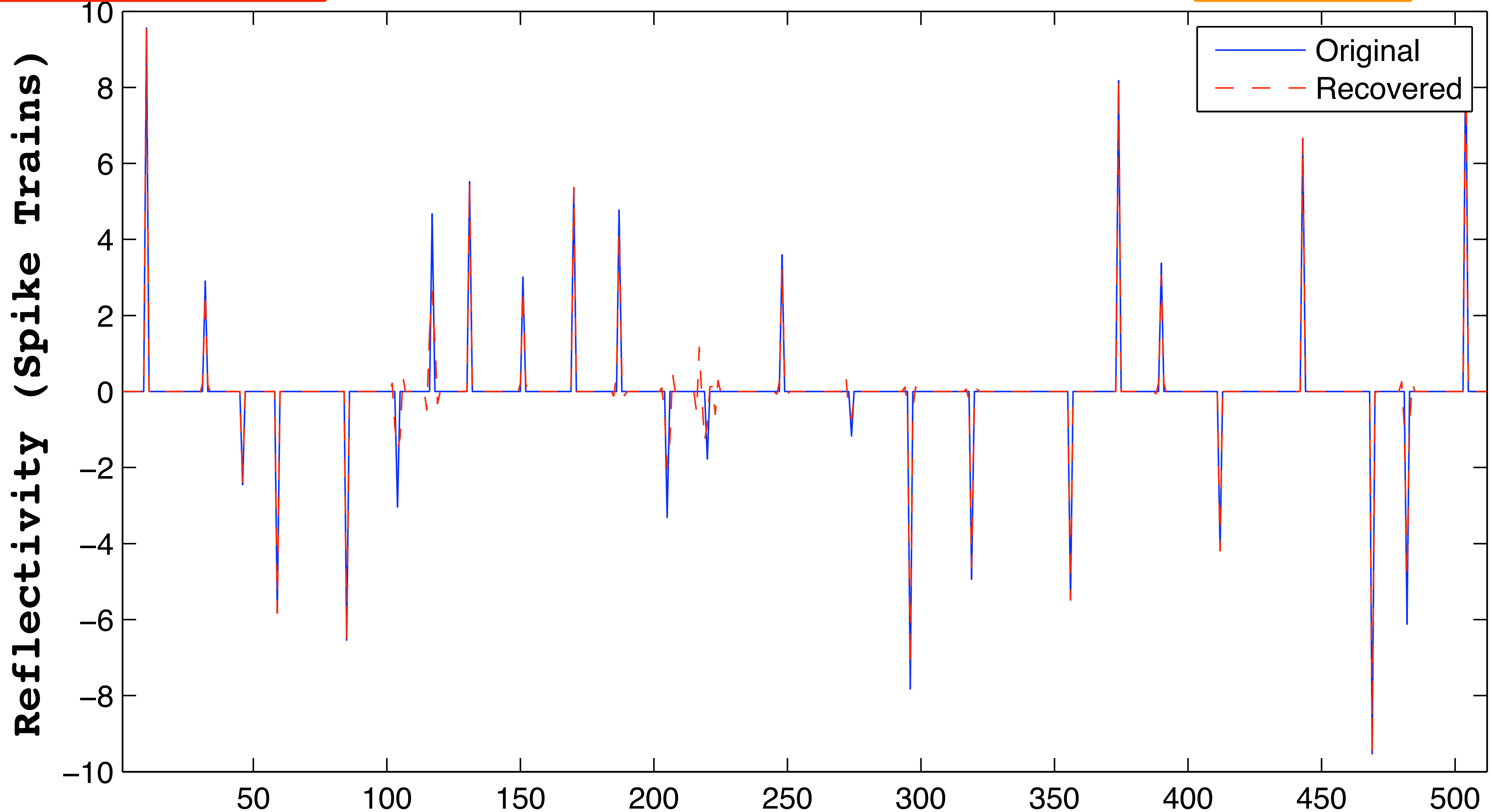
Cosine Bump Convolution with Spike Train (StOMP), N=512



K = 25 events, s = 0.66667, min. distance Δ = 13

Length of Signal : 512	StOMP Solving Time : 1.43
Number of Spikes : 25	StOMP Stage : 9
Min & Max Spacing : 13,37	L2 Nrom(x) : 28.35
Bump/Wavelet Scale : 0.667	% L0 Error : 3.16
Regular , Uniform : 0,0	% L2 Error : 0.1873

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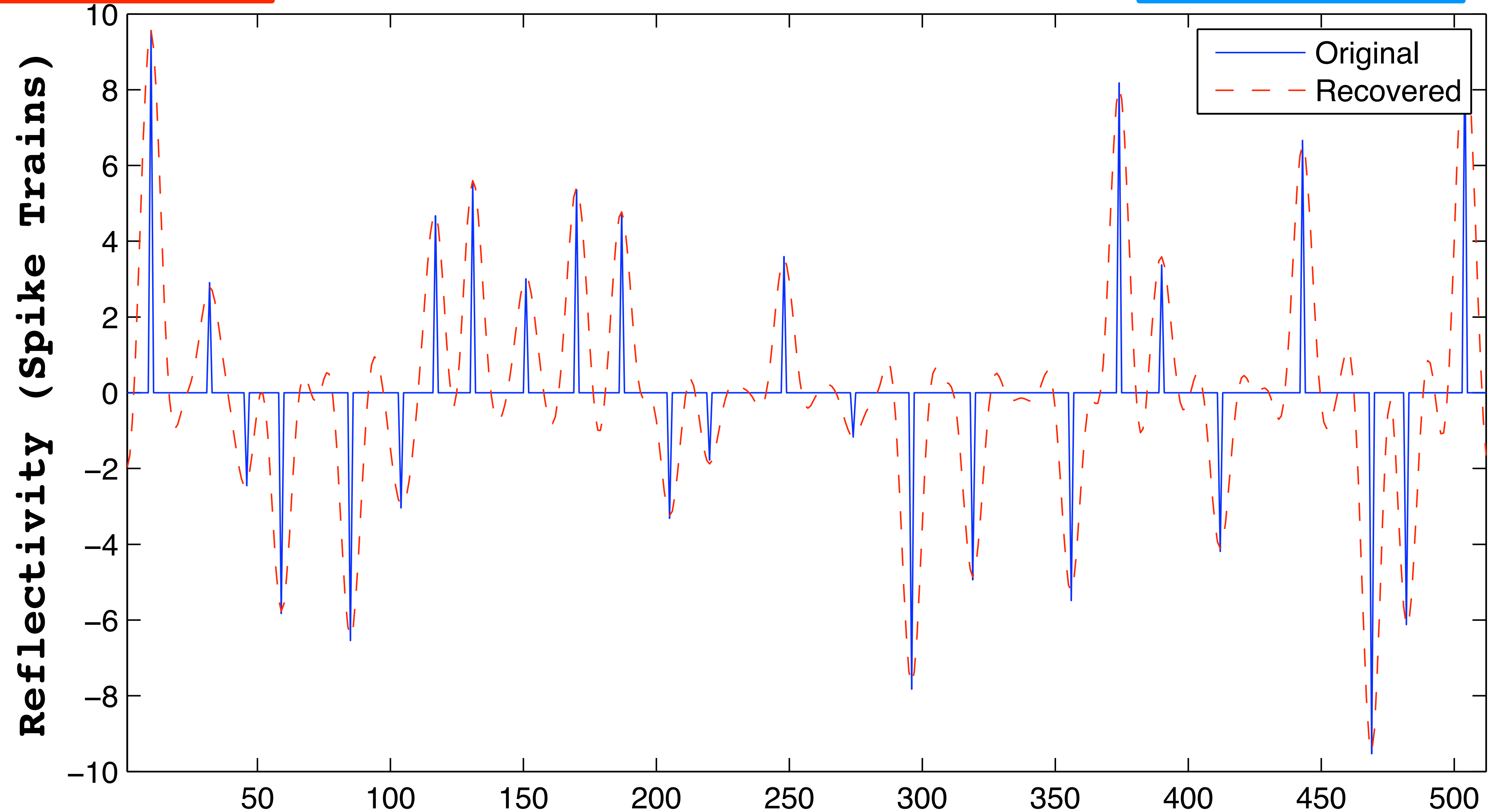
Cosine Bump Convolution with Spike Train (Scaled BP), N=512



K = 25 events, s = 0.66667, min. distance Δ = 13

Length of Signal : 512	BP Solving Time : 0.13
Number of Spikes : 25	L2 Nrom(x) : 28.35
Min & Max Spacing : 13,37	% L0 Error : 19.48
Bump/Wavelet Scale : 0.667	% L2 Error : 0.9992
Regular , Uniform : 0,0	

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Deconv. Summary

- Signal with length of 512 samples

Wavelet	Scale	K	Δ	Recovery
Cosine Bump	1	30	5	+
Ricker wavelet	1	30	5	+
Cosine Bump, Regularly spaced	1	30	7	+
Cosine Bump, Uniform Amp.	1	30	5	0
Cosine Bump, Uniform Amp.	1	12	5	+
Cosine Bump	2/3	25	13	0

Multiscale Newton Method

[ChaRM Estimation]

Mohammad Maysami

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Image Manifolds [Wakin'06]

- Varying Parameter: $\theta \in \Theta$ (Dimension: d)
- Image function model: $f_\theta : \mathbb{R}^d \mapsto \mathbb{R}$
- IAM : $\mathbf{F} = \{f_\theta : \theta \in \Theta\}$
  - 1-to-1 $\theta \mapsto f_\theta$ Relation
  - F is Square integrable: $\mathbf{F} \subset L^2(\mathbb{R}^2)$
- Non-Lipschitz relation → manifolds with Φ_s , s >0

$$F_s = \{ \Phi_s f_\theta : \theta \in \Theta, s > 0\}$$

$$\Phi_s f = \phi_s * f, \text{ where } \phi_s(x) = \frac{1}{2\pi s^2} \exp\{\frac{-\|x\|^2}{2s^2}\}$$

$$T(s, \theta^{(0)}; \mathbf{F}) = T_{f_{\theta^{(0)}}, s}(\mathbf{F}_s)$$

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Multiscale Newton Method

- Local tangent vectors on $\mathbf{F}_{\mathbf{s_k}}$

$$\tau^i_{\theta^{(k)},s_k} = \frac{\partial}{\partial \theta_i} f_{\theta,s_k}\Big|_{\theta=\theta^{(k)}}, \quad i = 0, 1, \ldots, d-1.$$

- Project estimation error

$$J_i = 2\langle f_{\theta^{(k)},s_k} - I_{s_k}, \tau^i_{\theta^{(k)},s_k}\rangle$$

- products of tangent vectors

$$H_{ij} = 2\langle \tau^i_{\theta^{(k)},s_k}, \tau^j_{\theta^{(k)},s_k}\rangle$$

$$\langle \tau^i_{s_0}, \tau^j_{s_1}\rangle = c_{s_0,s_1}\delta_{i,j}$$

- Update estimation

$$\theta^{(k+1)} \leftarrow \theta^{(k)} + H^{-1}J.$$

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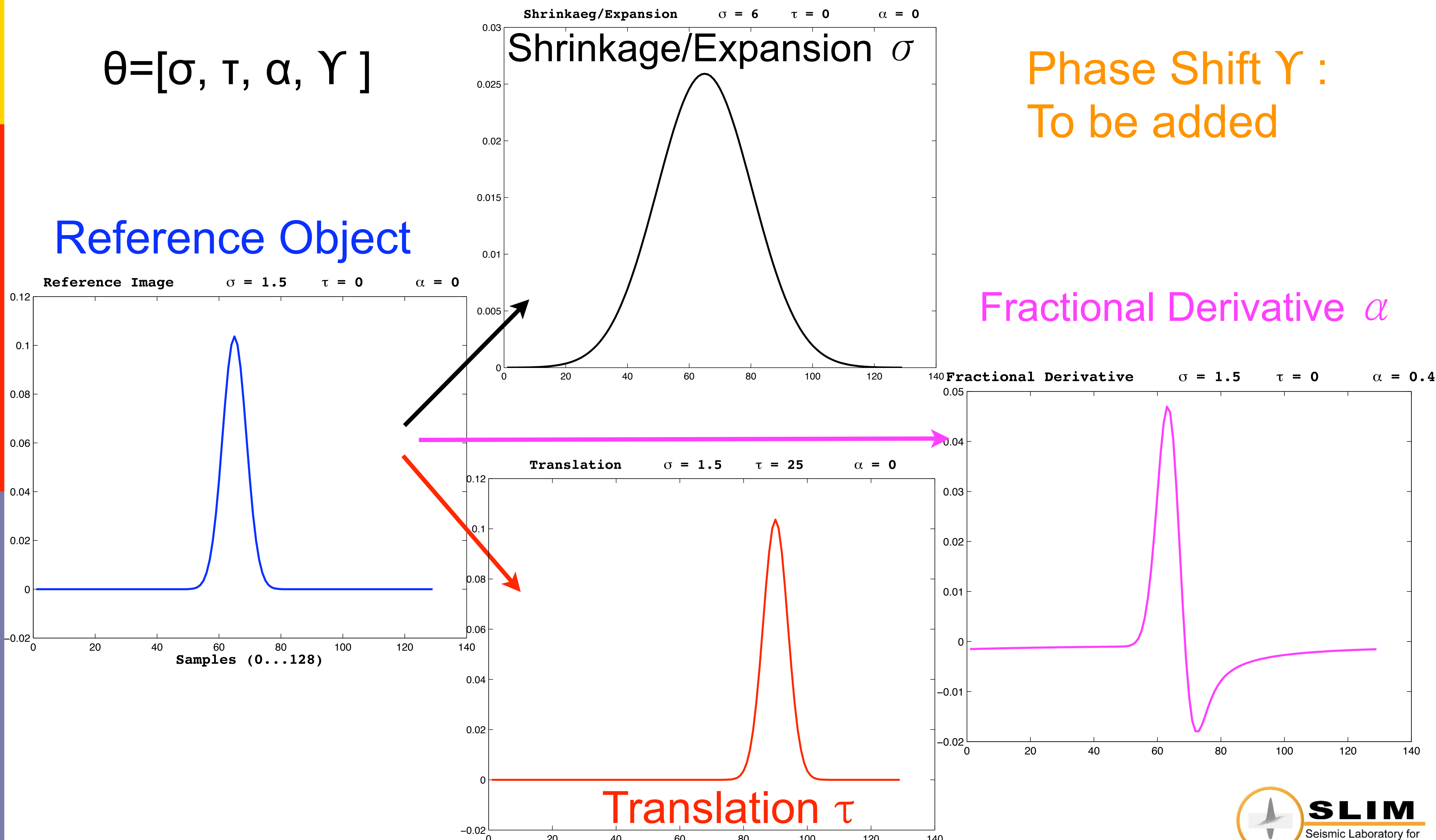
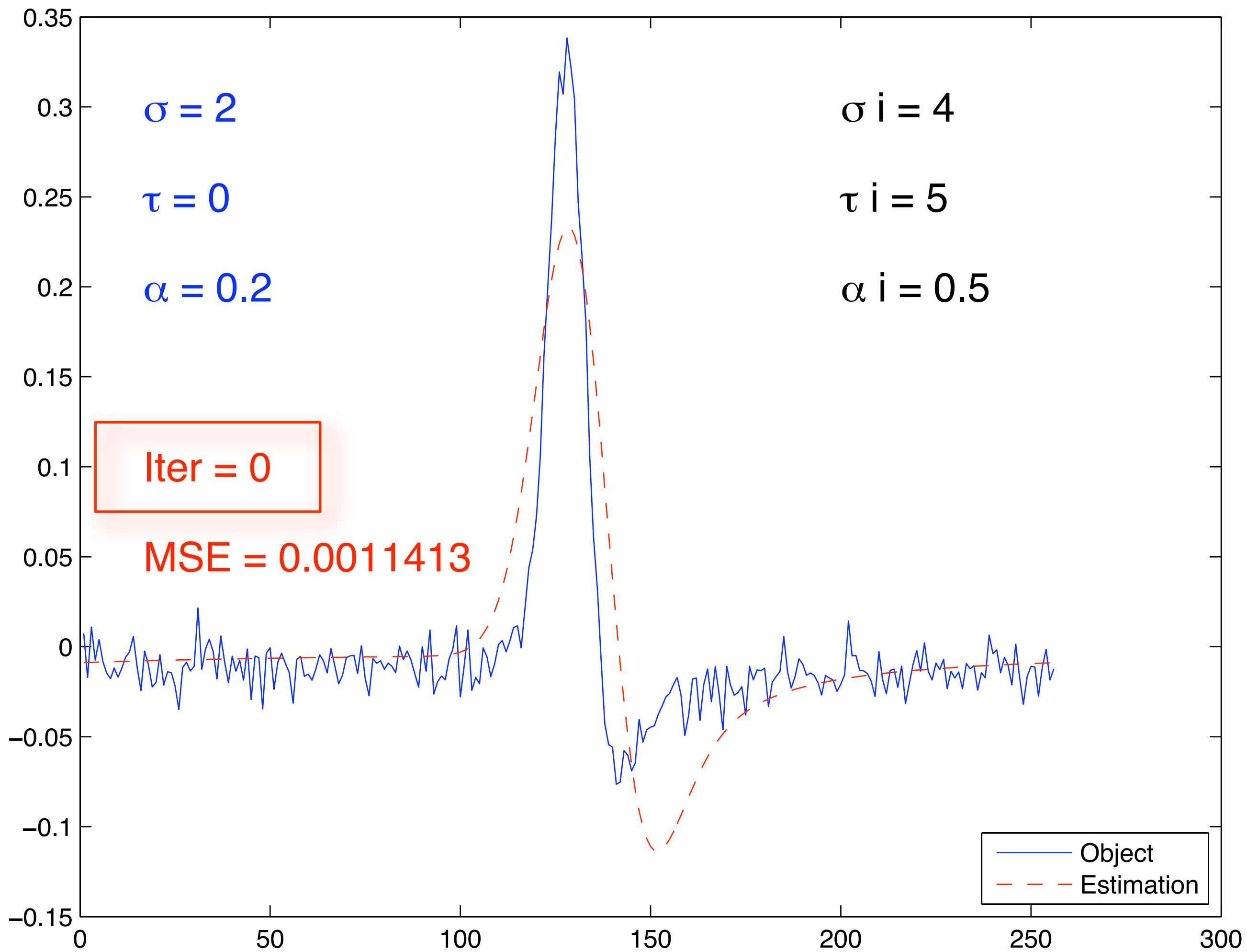
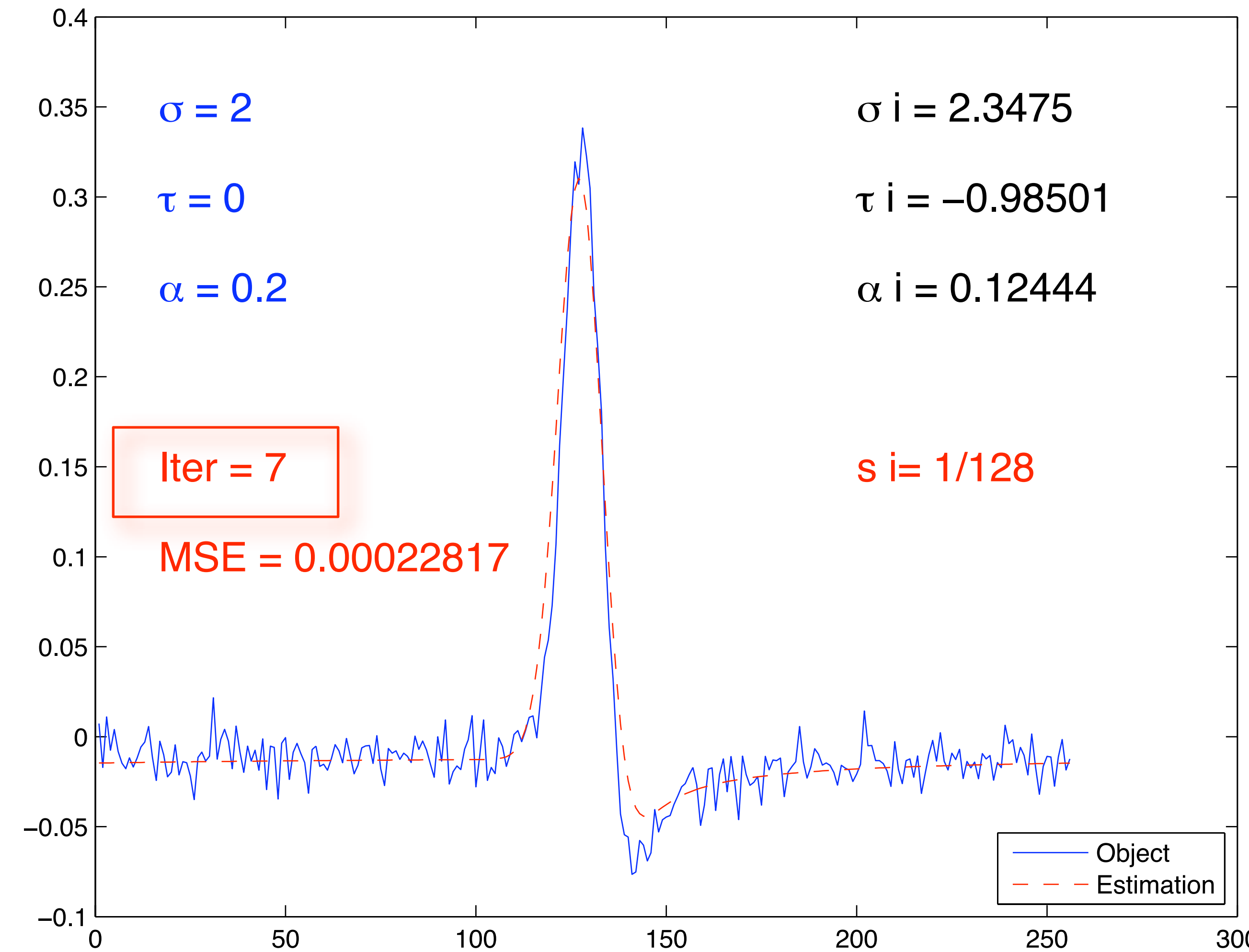
Form IAM for Gaussians

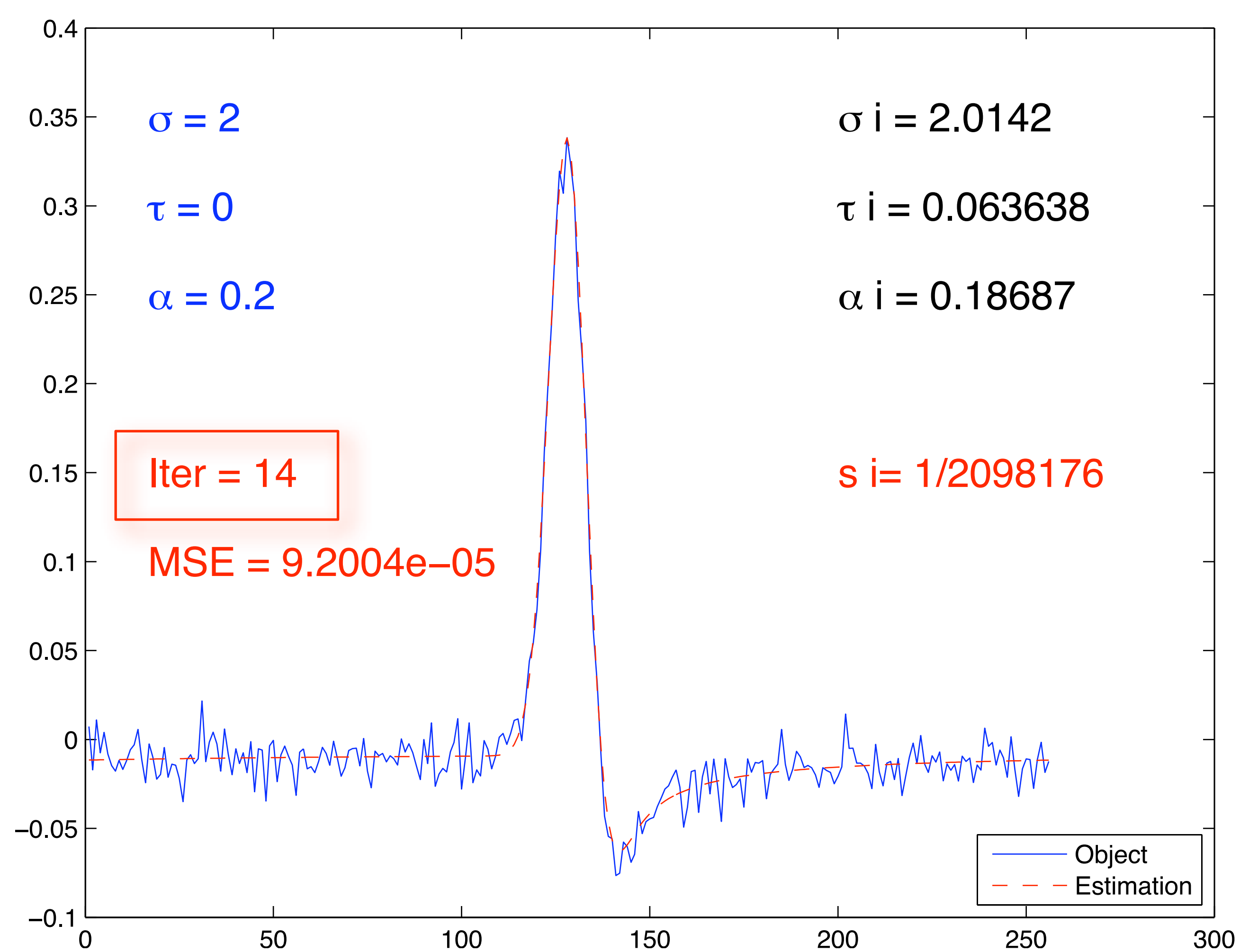
$\theta = [\sigma, \tau, \alpha, \Upsilon]$

Phase Shift ϒ : To be added




σ = 2
τ = 0
α = 0.2
σ i = 4
τ i = 5
α i = 0.5
Iter = 0
MSE = 0.0011413
Object
Estimation

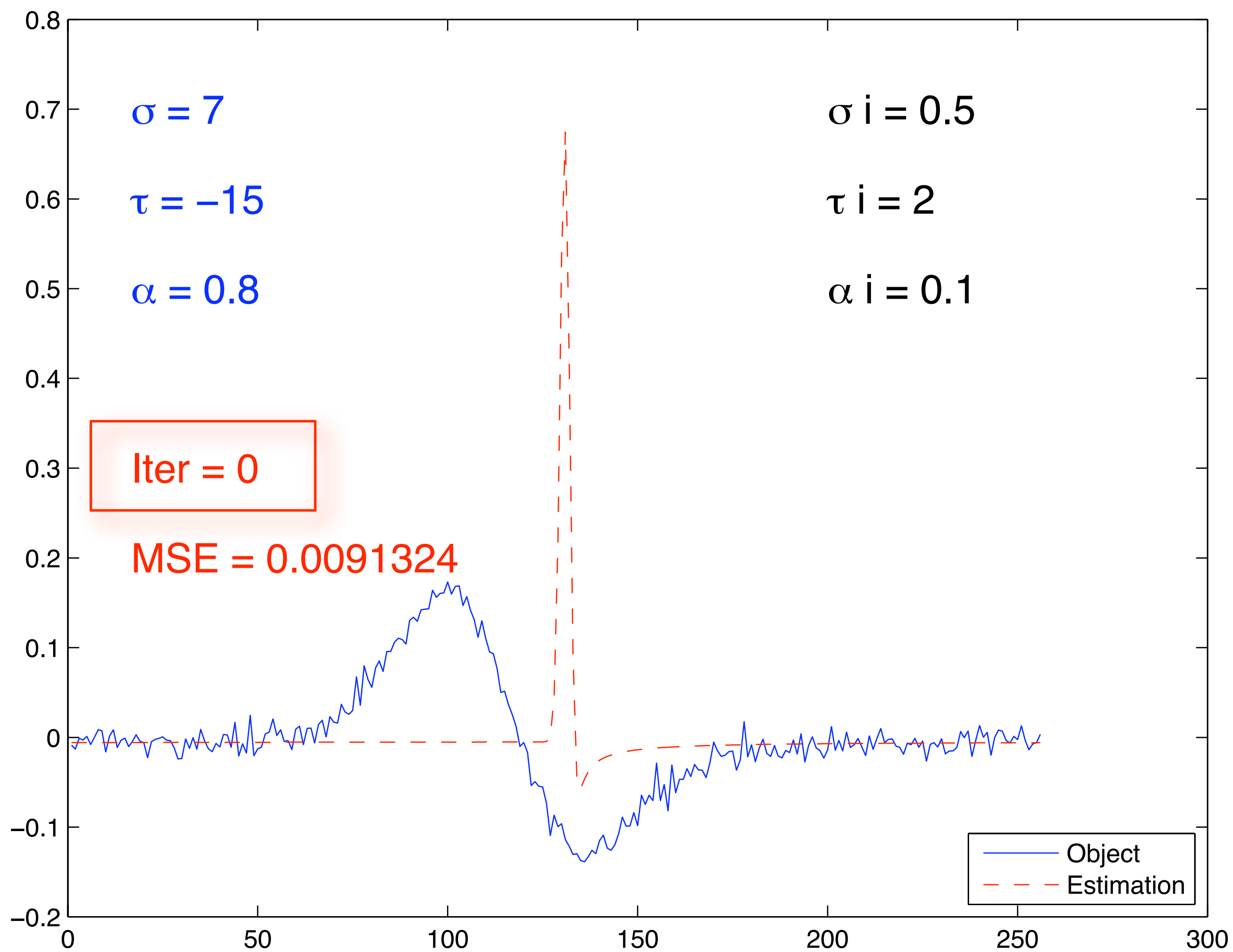

σ = 2
τ = 0
α = 0.2
σ i = 2.3475
τ i = −0.98501
α i = 0.12444
Iter = 7
s i= 1/128
MSE = 0.00022817
Object
Estimation


σ = 2
τ = 0
α = 0.2
σ i = 2.0142
τ i = 0.063638
α i = 0.18687
Iter = 14
s i= 1/2098176
MSE = 9.2004e−05
Object
Estimation

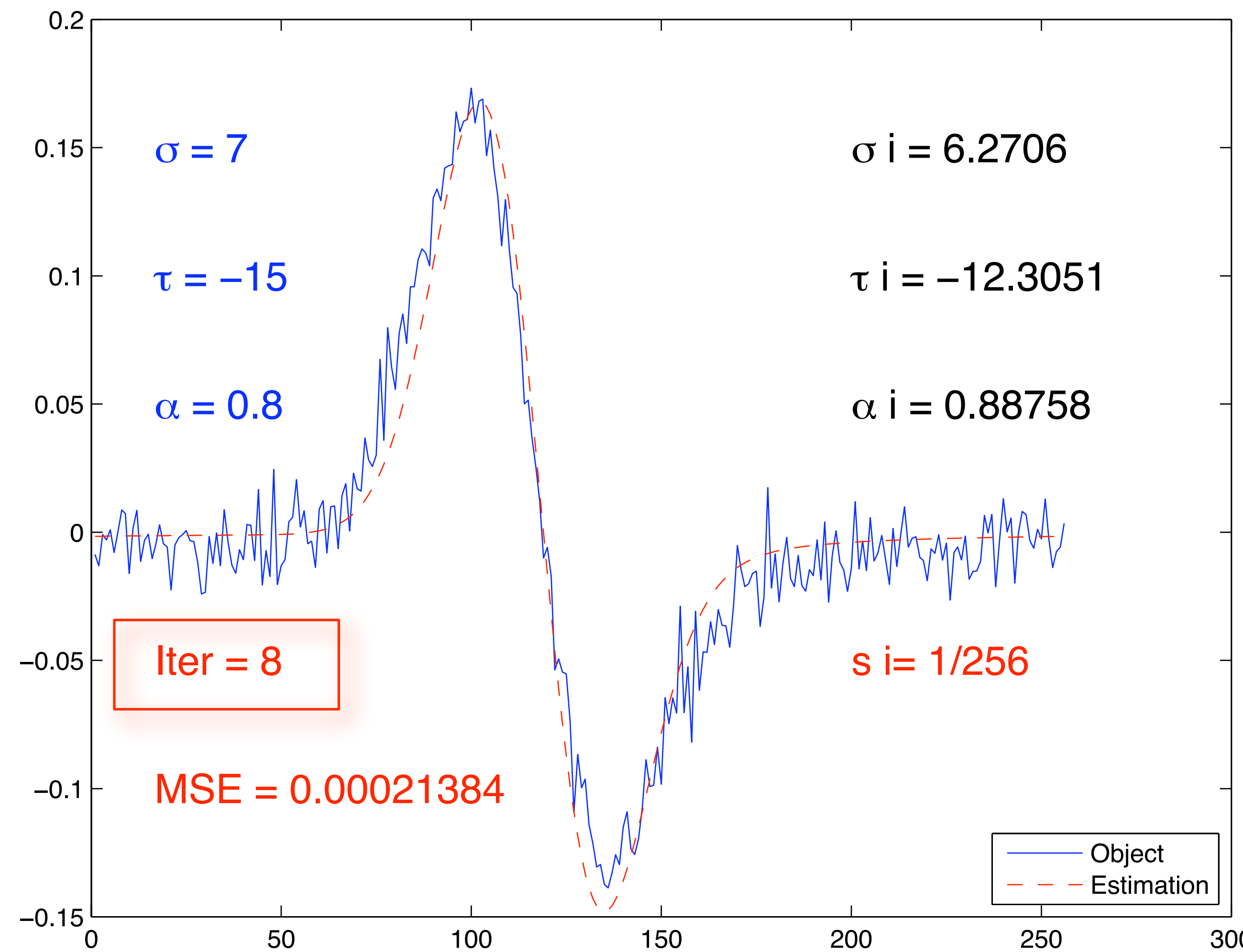
Result statistics - Case 1

	Actual	Initial Guess	Estimated
Sigma	2	4	2.0142
Tau	0	5	0.0636
Alpha	0.2	0.5	0.1869

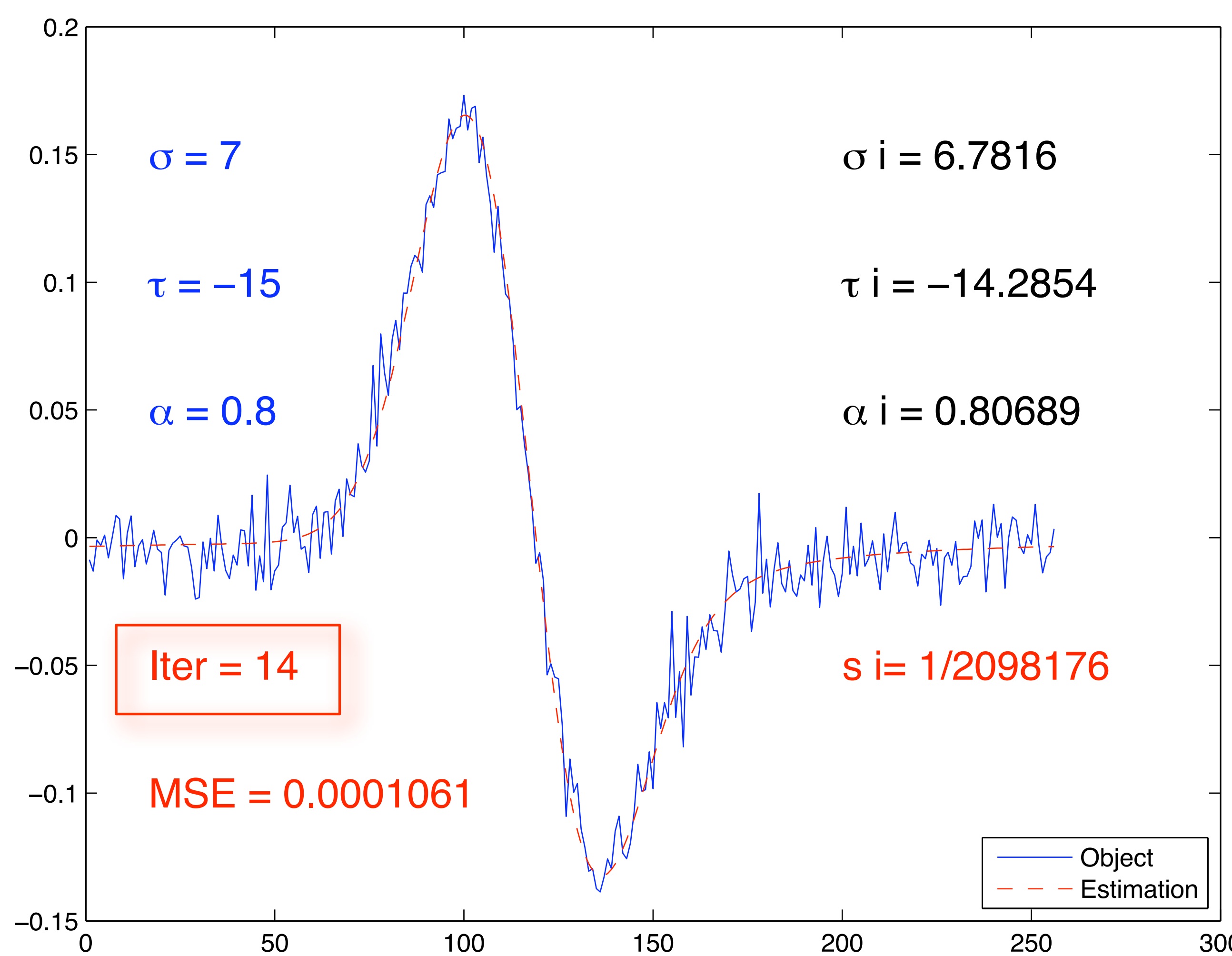
Noise Variance	8.01E-06
Iter. No.	14
MSE	9.20E-04
Elapsed Time	0.42



σ = 7
τ = −15
α = 0.8
σ i = 0.5
τ i = 2
α i = 0.1
Iter = 0
MSE = 0.0091324
Object
Estimation



σ = 7
τ = −15
α = 0.8
σ i = 6.2706
τ i = −12.3051
α i = 0.88758
Iter = 8
s i= 1/256
MSE = 0.00021384
Object
Estimation



σ = 7
τ = −15
α = 0.8
σ i = 6.7816
τ i = −14.2854
α i = 0.80689
Iter = 14
s i= 1/2098176
MSE = 0.0001061
Object
Estimation

Result statistics - Case 2

	Actual	Initial Guess	Estimated
Sigma	7	0.5	6.6911
Tau	-15	2	-14.025
Alpha	0.8	0.1	0.8066

Noise Level	3.522E-08
Iter. No.	14
MSE	1.06E-04
Elapsed Time	0.31

Multiscale Newton method

- Replace the ***slow*** redundant dictionary technique with a multiscale Newton method
- Resolve problem with the ***non-differentiability*** of the parametrization => stable
- Good initial guess provided by detection step
- Convergence rate
- robust under noise and incomplete data
- Smoothing Sensitivity

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Percolation model and Well tie

[Herrmann-Bernabe'05]

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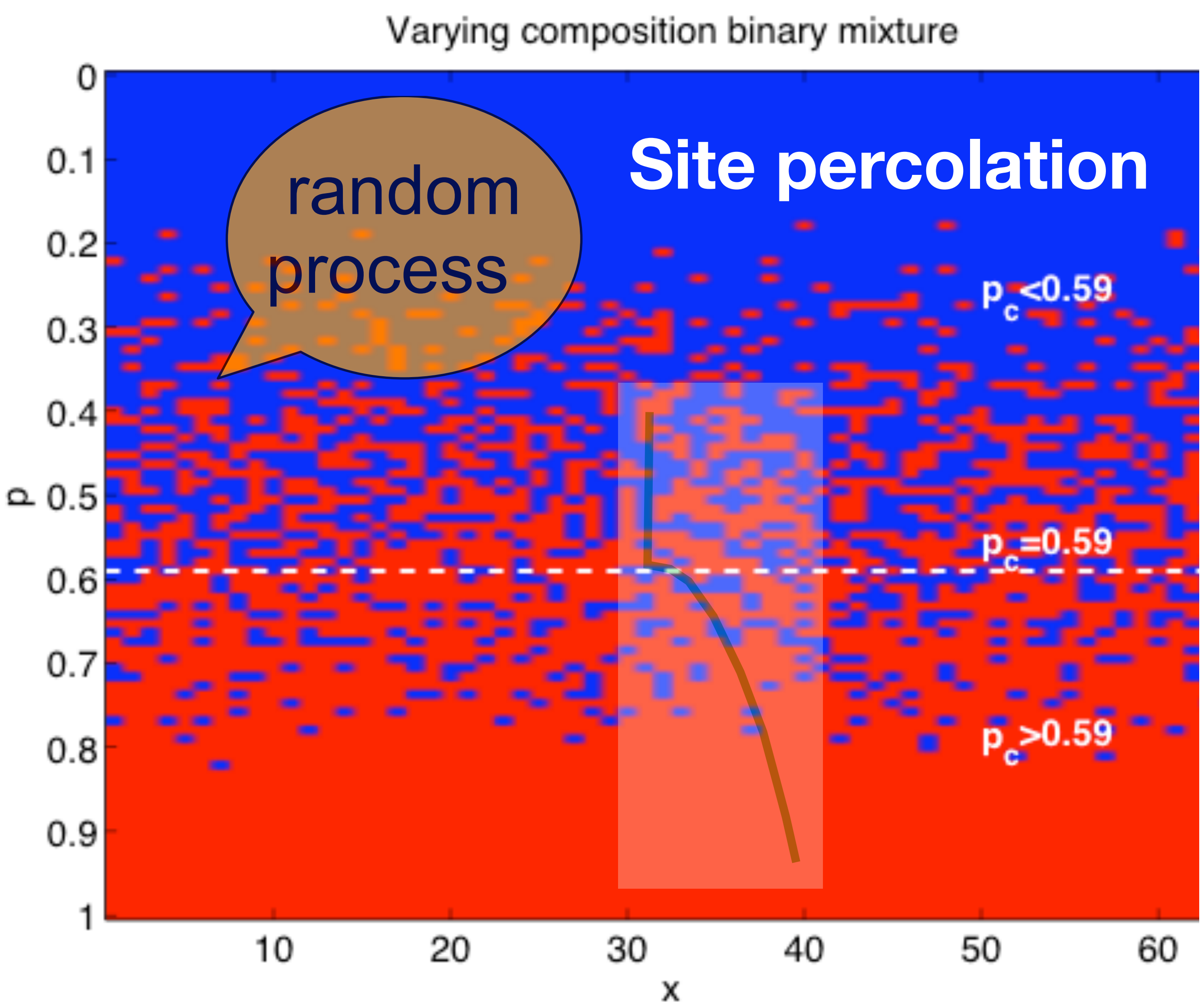
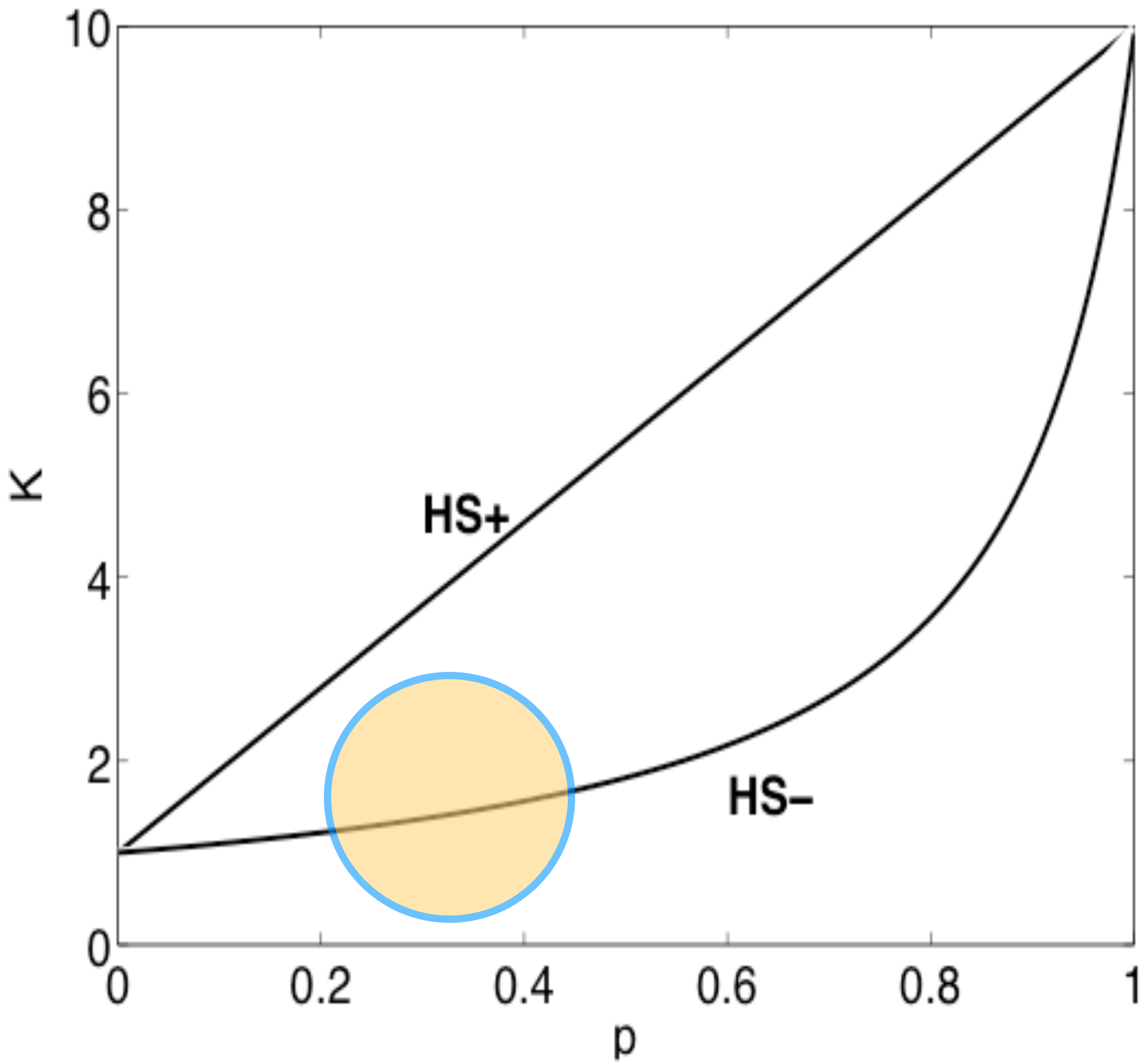
Discontinuity modeling

binary mixtures

LP olivine HP β-spinel

elastic properties





volume fraction

HP: high porosity

LP: low porosity

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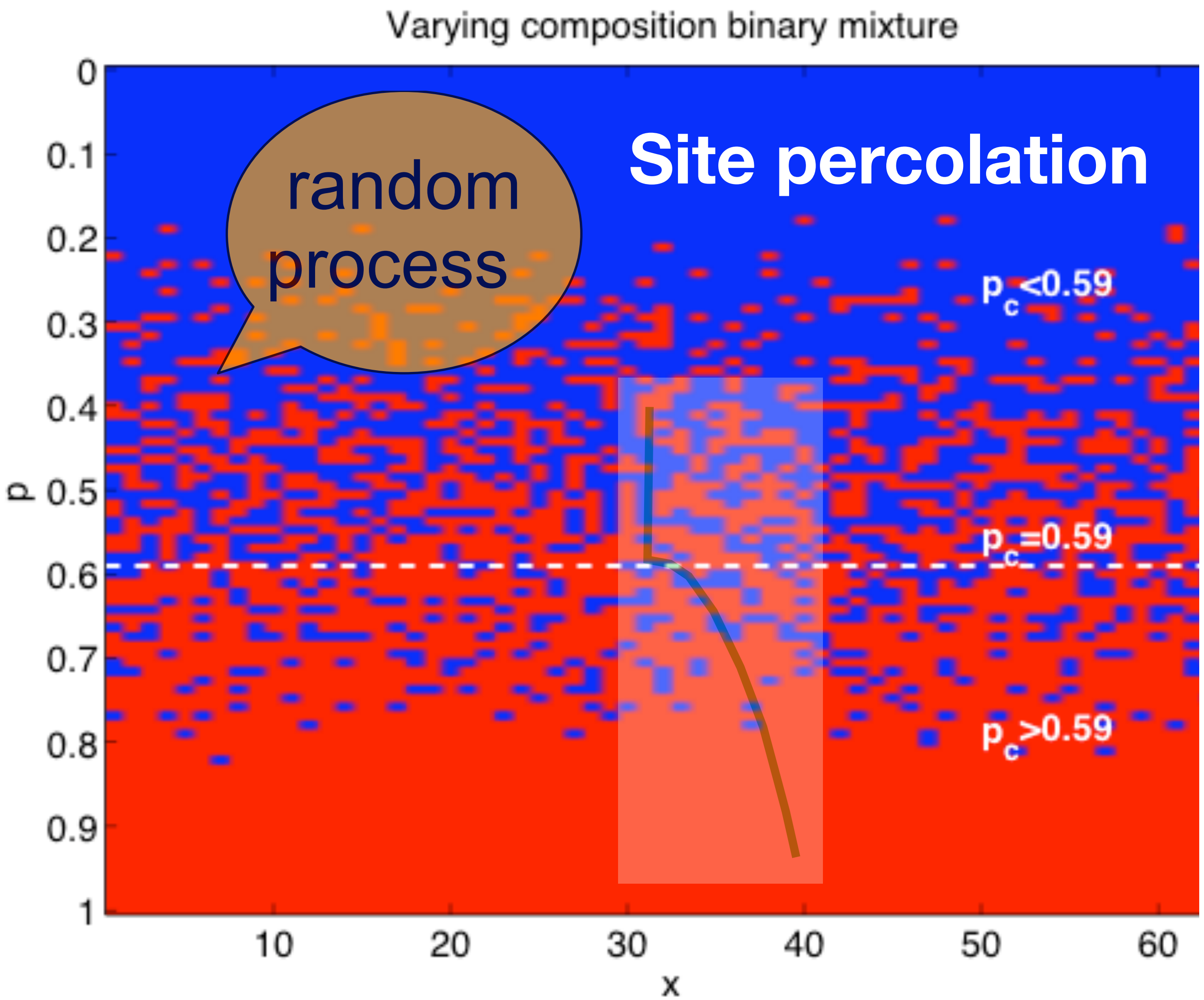
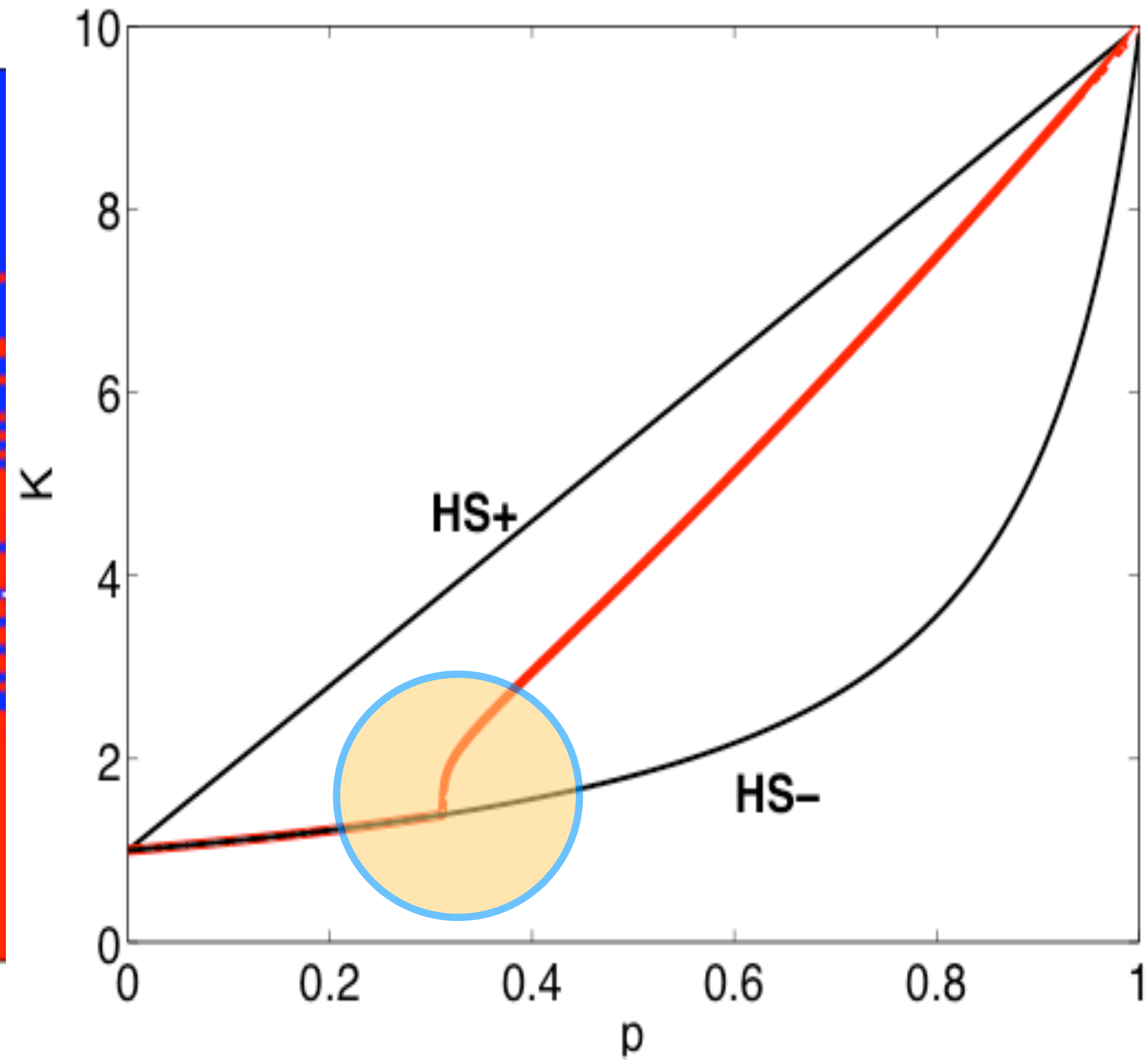
Discontinuity modeling

binary mixtures

LP olivine HP β-spinel



elastic properties

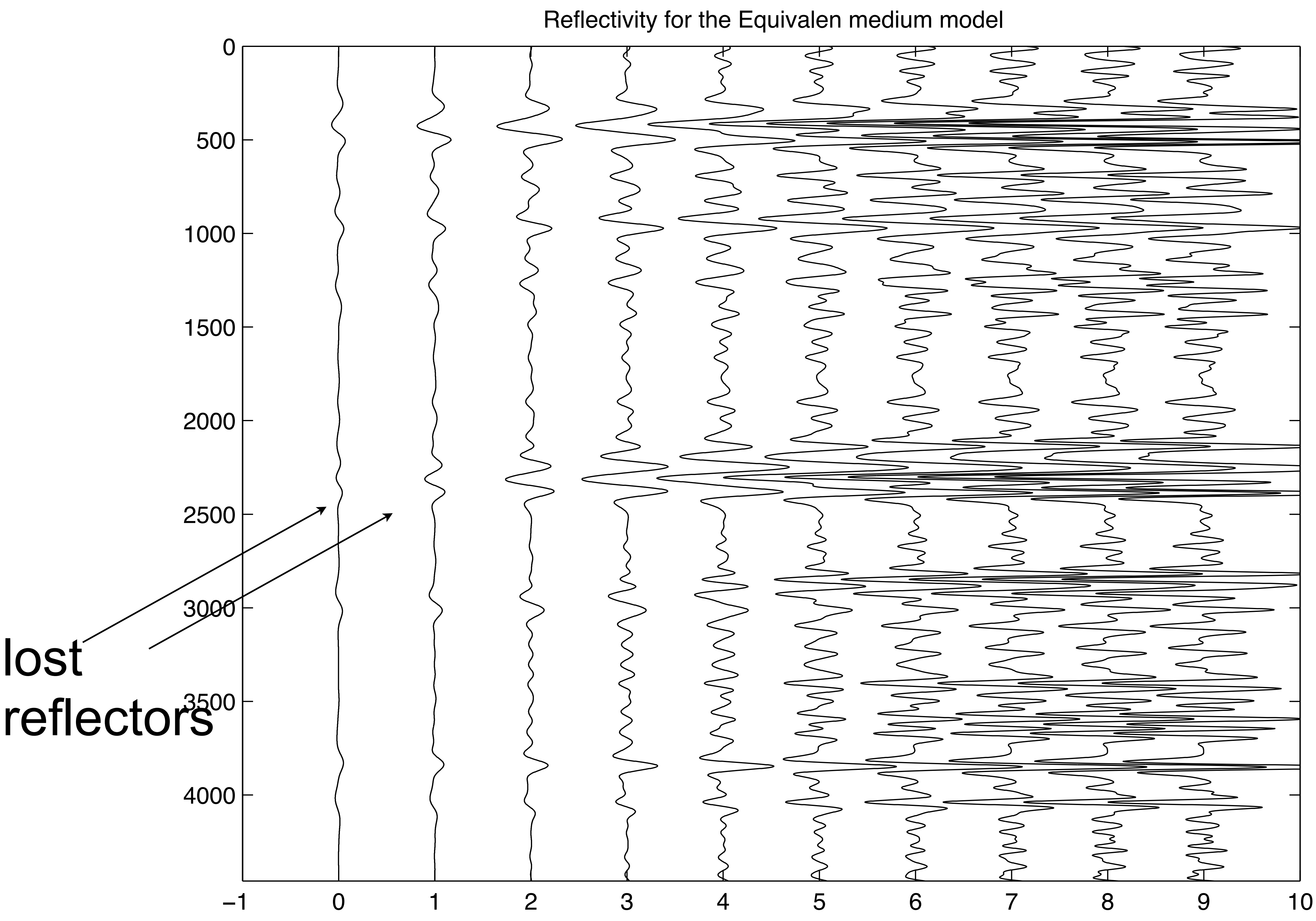


volume fraction

HP: high porosity

LP: low porosity

Upscaling: EM upscaled reflectivity



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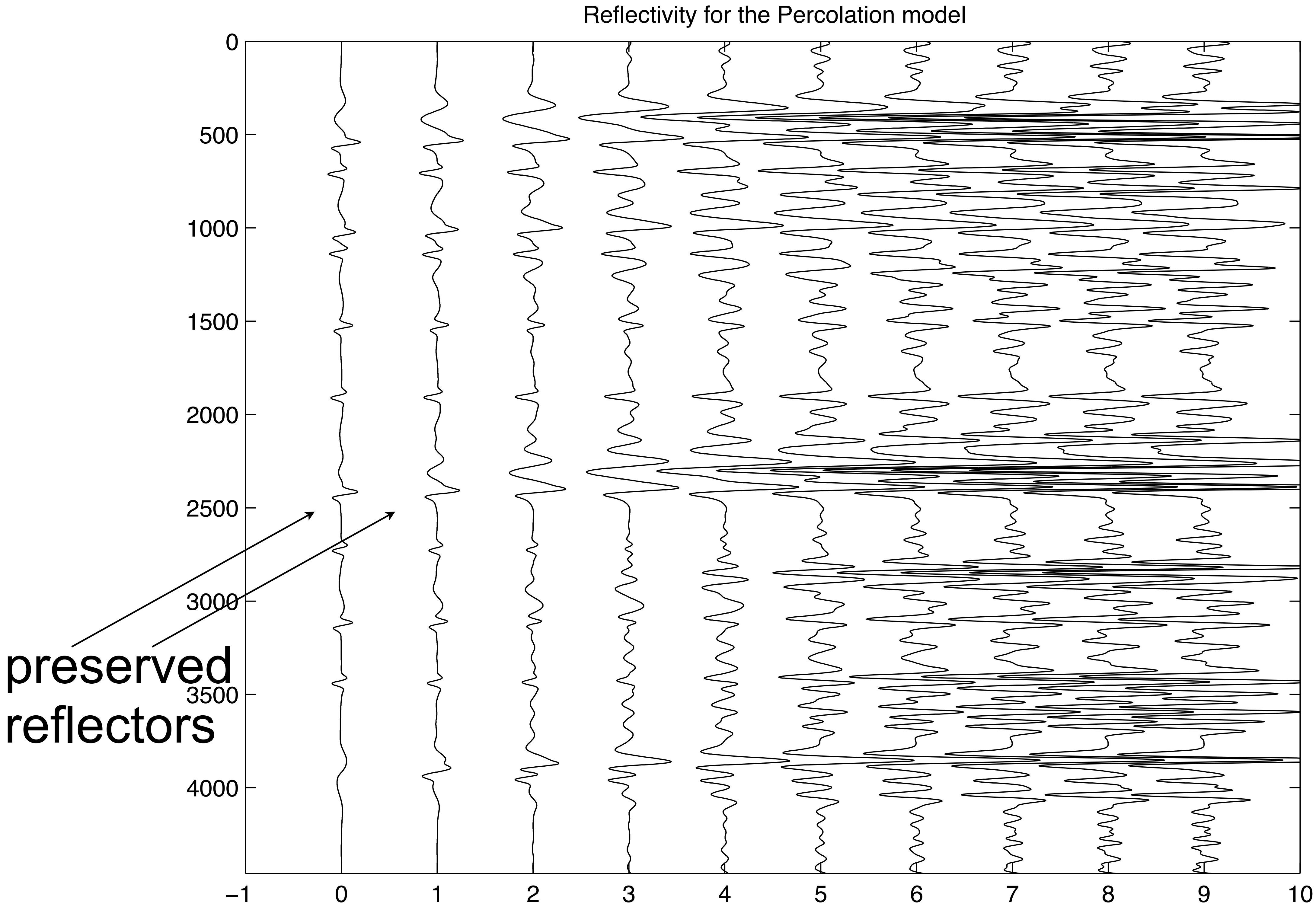
Upscaling: Perc. upscaled reflectivity



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Upscaling: lithology

- Use the volume fractions, p, to compute
  - ***density*** via a ***linear*** relation
  - ***velocity*** via a ***nonlinear*** & ***singular*** (switch) relation
- Upscaling by smoothing p and not the velocity
- preserves singularities because of the switch
- ***singularities*** from ***p*** or ***switch***

Using IAMs for Well tie

- Percolation Model
  - P_c
  - β
- Match well-log (P) to seismic reflectivity
  - Parametrizing by IAM
  - solve for inverse problem
- Model mixtures as binary mixtures

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Future plans

[ChaRM]

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ChaRM Detection

- Use recent results on spiky decon (Mallat) to make the detection unique given minor information on the seismic wavelet
  - insensitivity to estimated wavelet and spike assumption
- Extend these results to higher dimensions
  - multidimensional reflector detection
  - solve spike decon. for curvelet
    - Joint work with Yilmaz, Jaffard, Vedel
- Characterization depends on an sufficiently accurate event detection

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ChaRM Characterization

- Develop the multiscale Newton method
- Extend to higher dimensions
- Make robust under noise and missing data
- Extend the multiscale Newton technique to invert for (given the end members)
  - the percolation threshold and exponent
  - the extend of the lithological transition (the width of a layer in the composition)
- Joint work with Yves Bernabé

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ChaRM Upscaling and well tie

- Study well-defined binary mixtures (e.g. Opal, gas-hydrates etc.)
- Develop upscaling techniques that preserve singularities
- Provide new insights in the well-seismic tie
- Joint work with Yves Bernabé

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Questions ?

Comments ?

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