Multiple prediction from incomplete data

Context

Claerbout Daylight imaging

Weglein's (inverse) series expansions

Verschuur and Berkhout's SRME and data inverse

Wapenaar's reciprocity relations

Motivation

Prediction of multiples requires **complete** data.

Success of primary-multiple **separation** depends on **quality** multiple prediction.

So far our interpolation algorithms lacked "physics" ....

We know from the wave equation

- reciprocity relations (convolution/correlation)
- primaries can be mapped to multiples and *vice versa*

Physics is often "naive" and non robust.

Can we use sparsity arguments?

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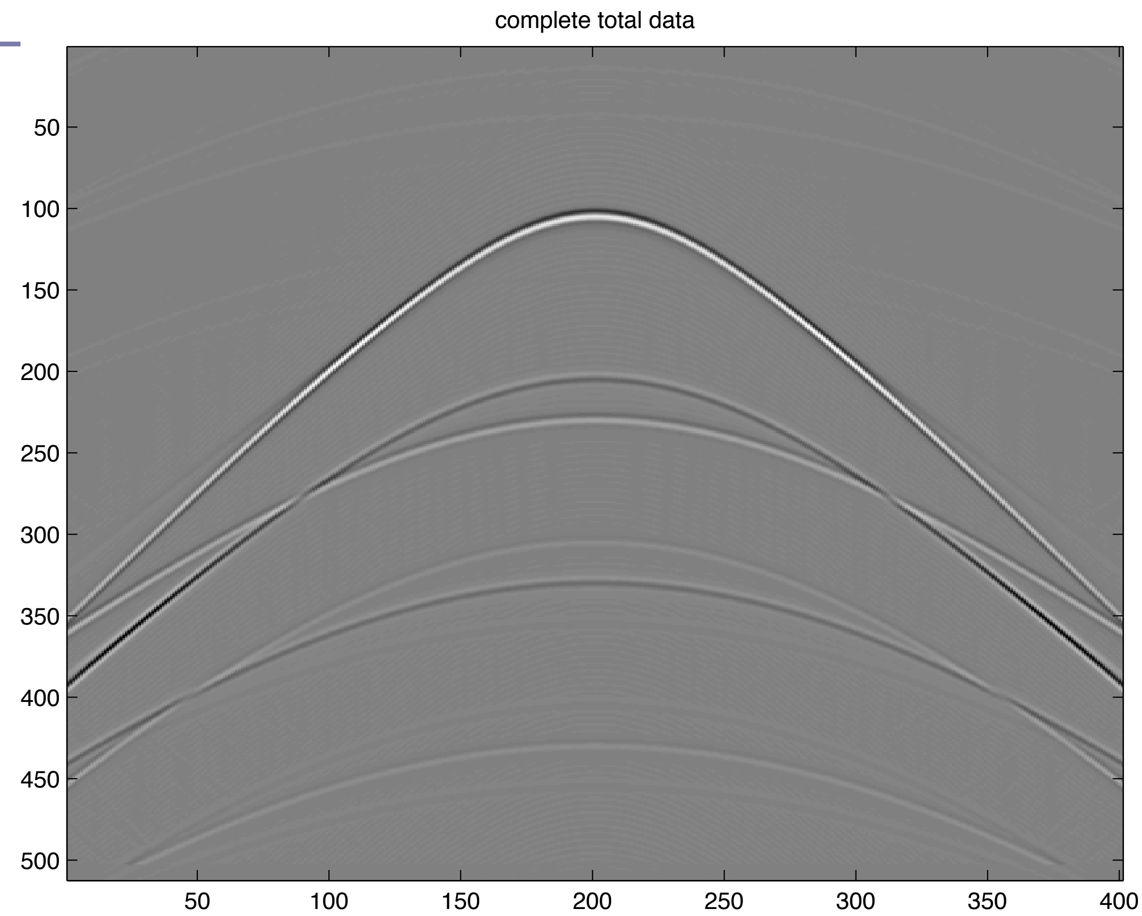
Total data



complete total data
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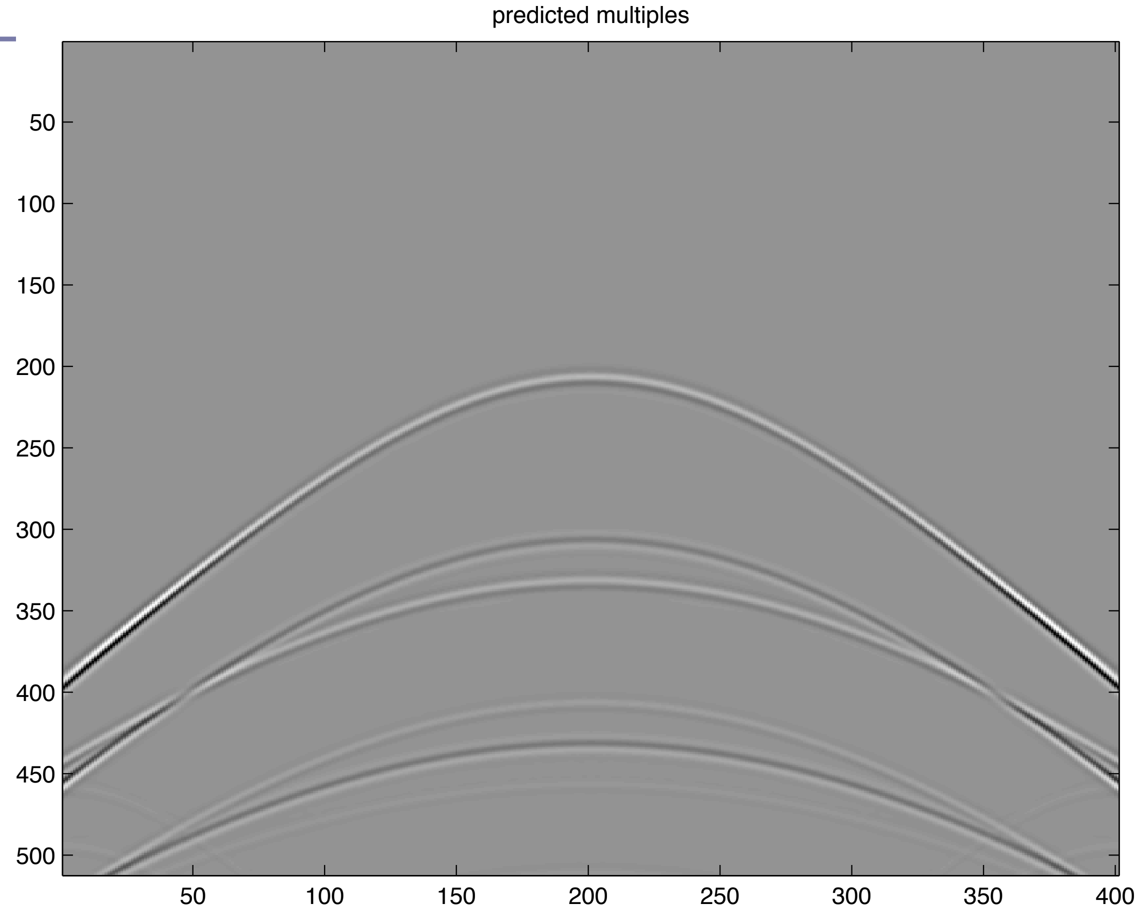
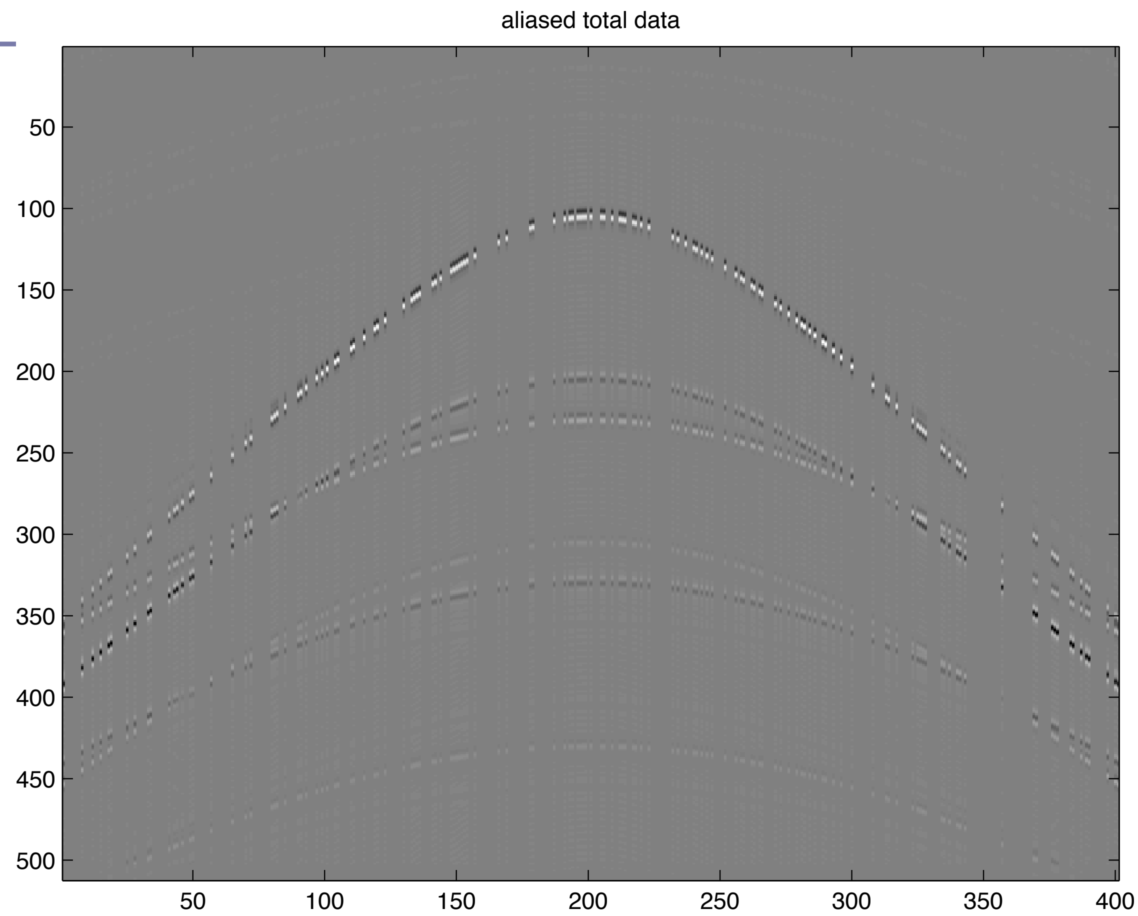
SMRE predicted multiples



Incomplete data



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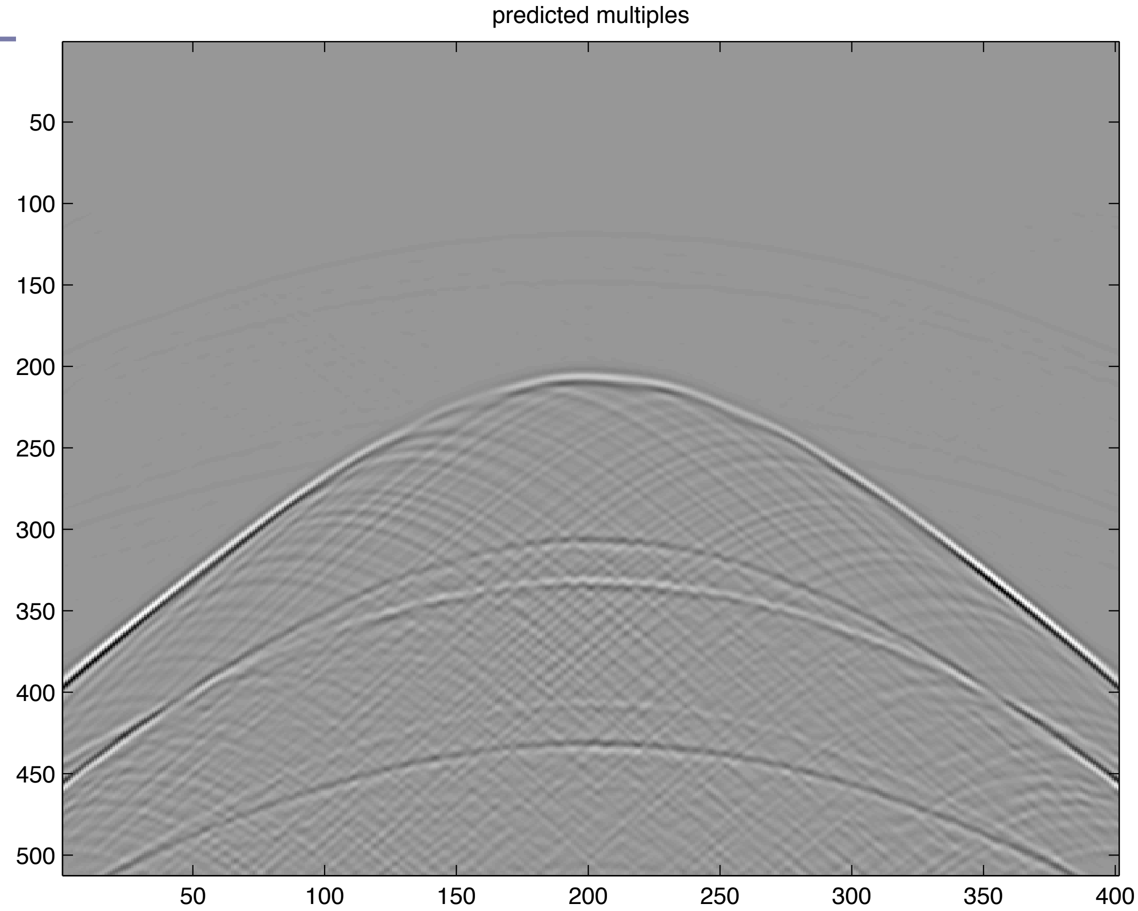
Erroneous prediction



The problem of missing data

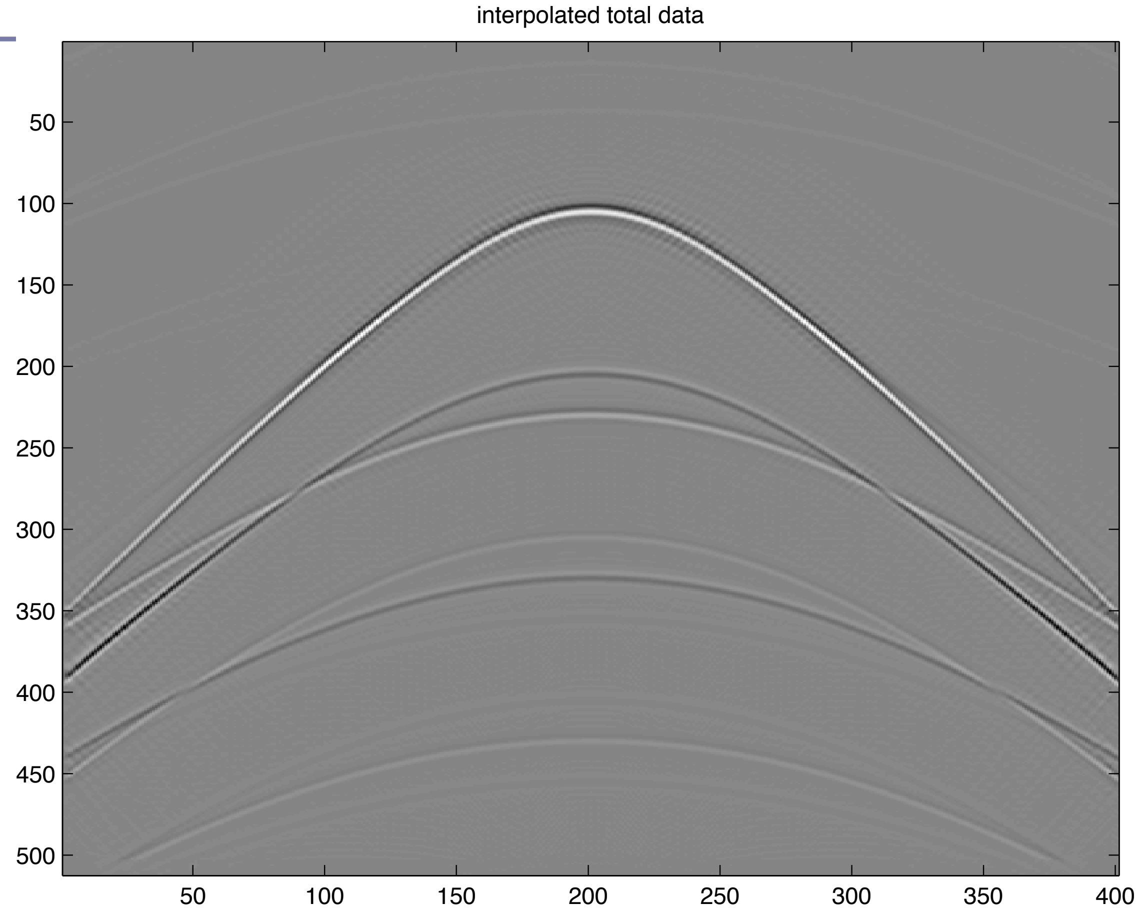
Recover data prior to multiple prediction.

Proceed with SRME prediction.

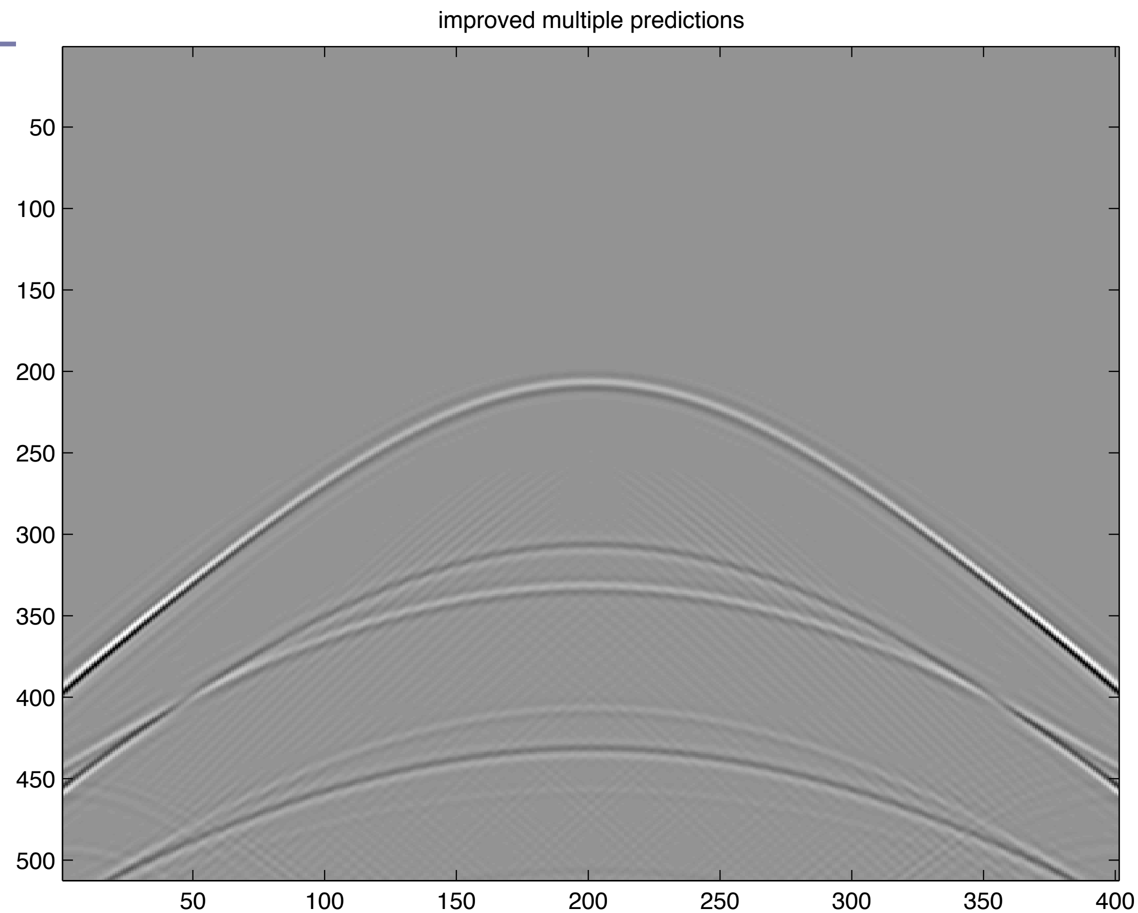
Will be applied

- higher dimensions
- real data

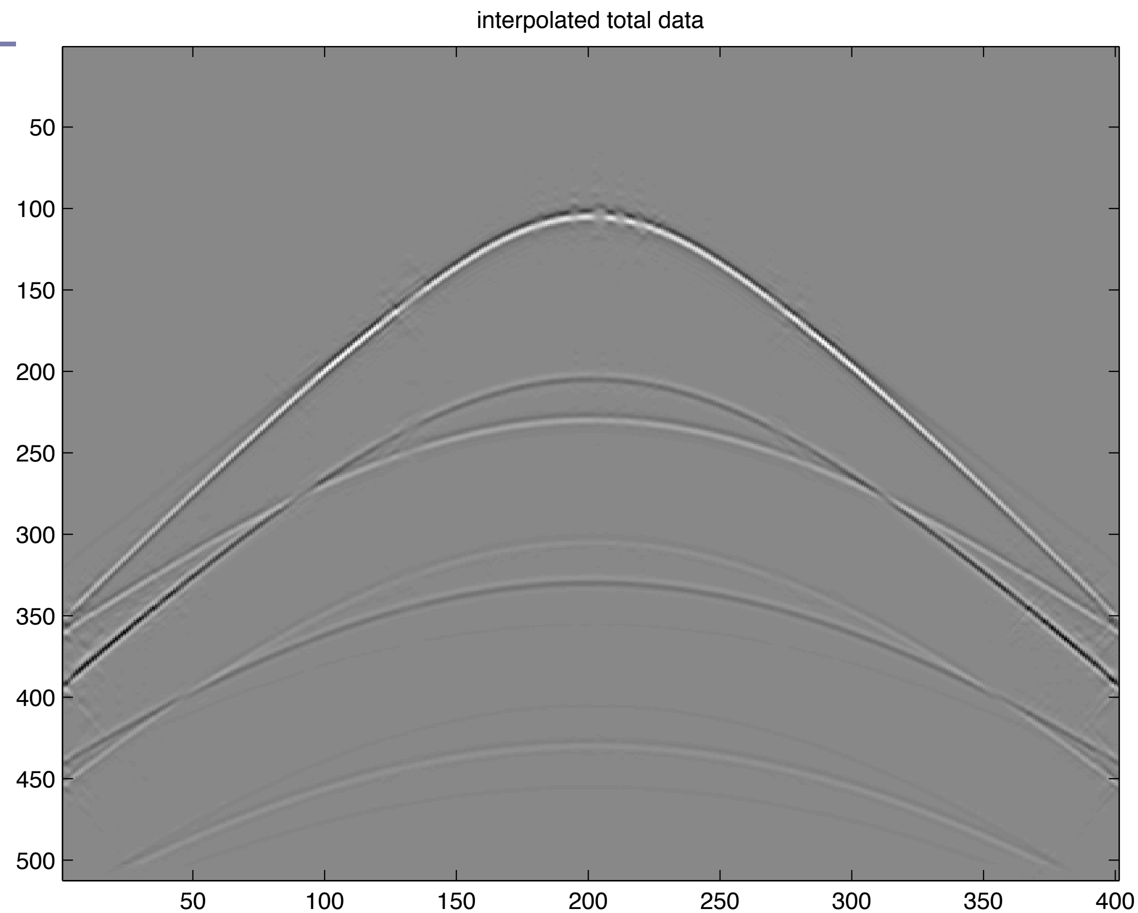
Recovered data



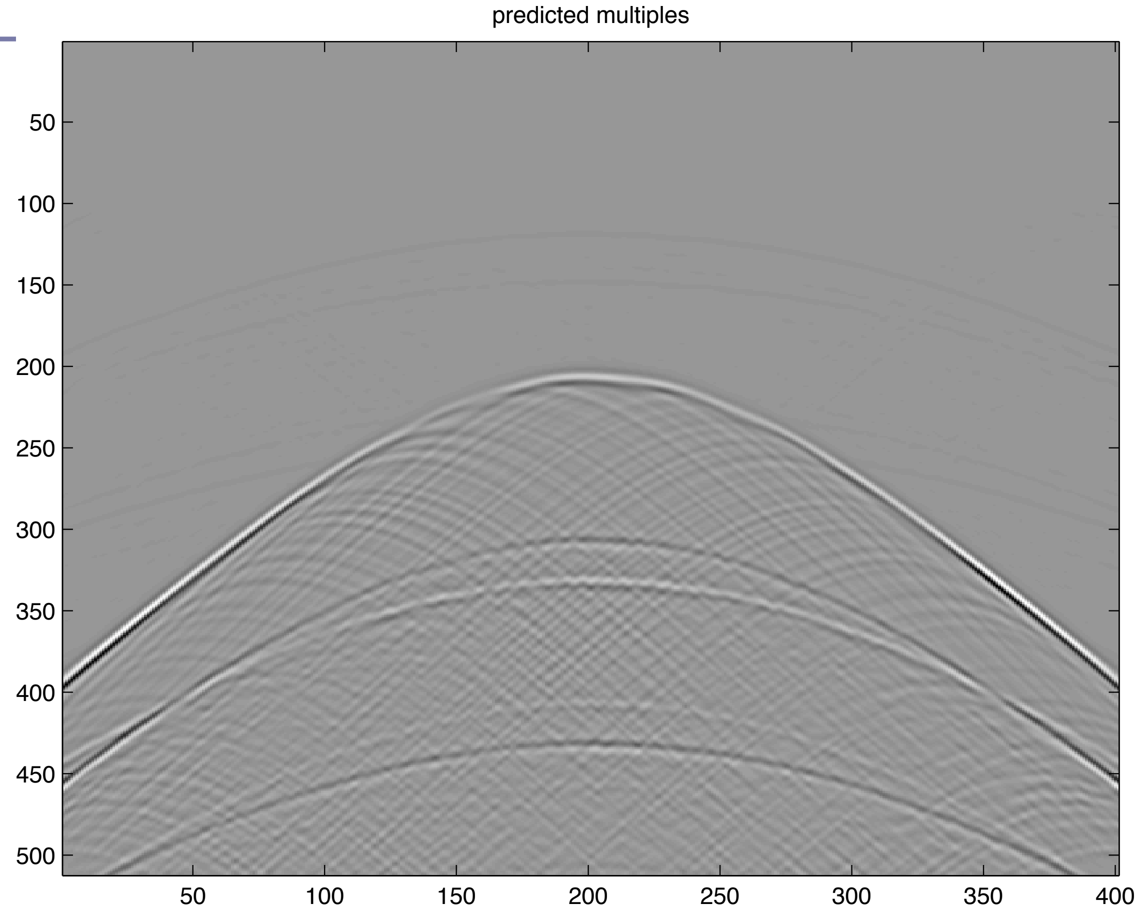
Improved prediction



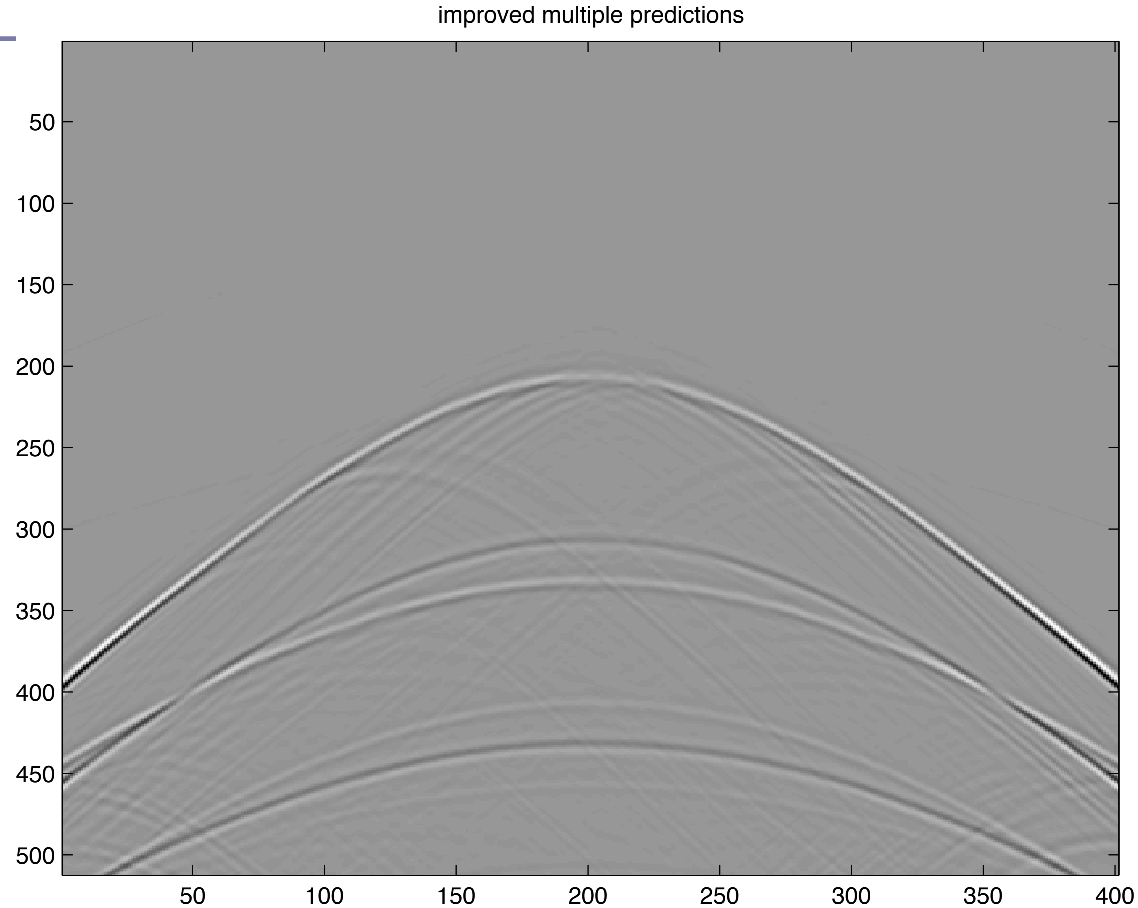
Recovered data



Erroneous prediction



Improved prediction



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Data mapping

Wave constituents are related by multidimensional

"cross-convolutions" mapping:

- primaries to first order multiples; first-order multiples to second order etc.
- spreads the energy & "increases" phase

"cross-correlations" mapping:

- first-order multiples to primaries; second-order multiples to first order etc.
- concentrate the energy and "decreases" phases

Consistent with different terms in the "forward" and "inverse" series, daylight imaging & focal transform.

Primaries 2 multiples references

Primaries into first-order multiples:

$$\Delta\mathbf{p} \mapsto \mathbf{m}^1 = (\Delta\mathbf{P}\boldsymbol{\mathcal{A}} *_{t,x} \Delta\mathbf{p})$$

First-order multiples into primaries:

$$\mathbf{m}^1 \mapsto \Delta\mathbf{p} = (\Delta\mathbf{P}\boldsymbol{\mathcal{A}} \otimes_{t,x} \Delta\mathbf{p})$$

with the acquisition matrix

$$\boldsymbol{\mathcal{A}} = \left(\boldsymbol{\mathcal{S}}^{\dagger}\mathbf{R}\boldsymbol{\mathcal{D}}^{\dagger}\right)$$

"inverting" for source and receiver wavelet wavelets geometry and surface reflectivity.

Primaries 2 multiples

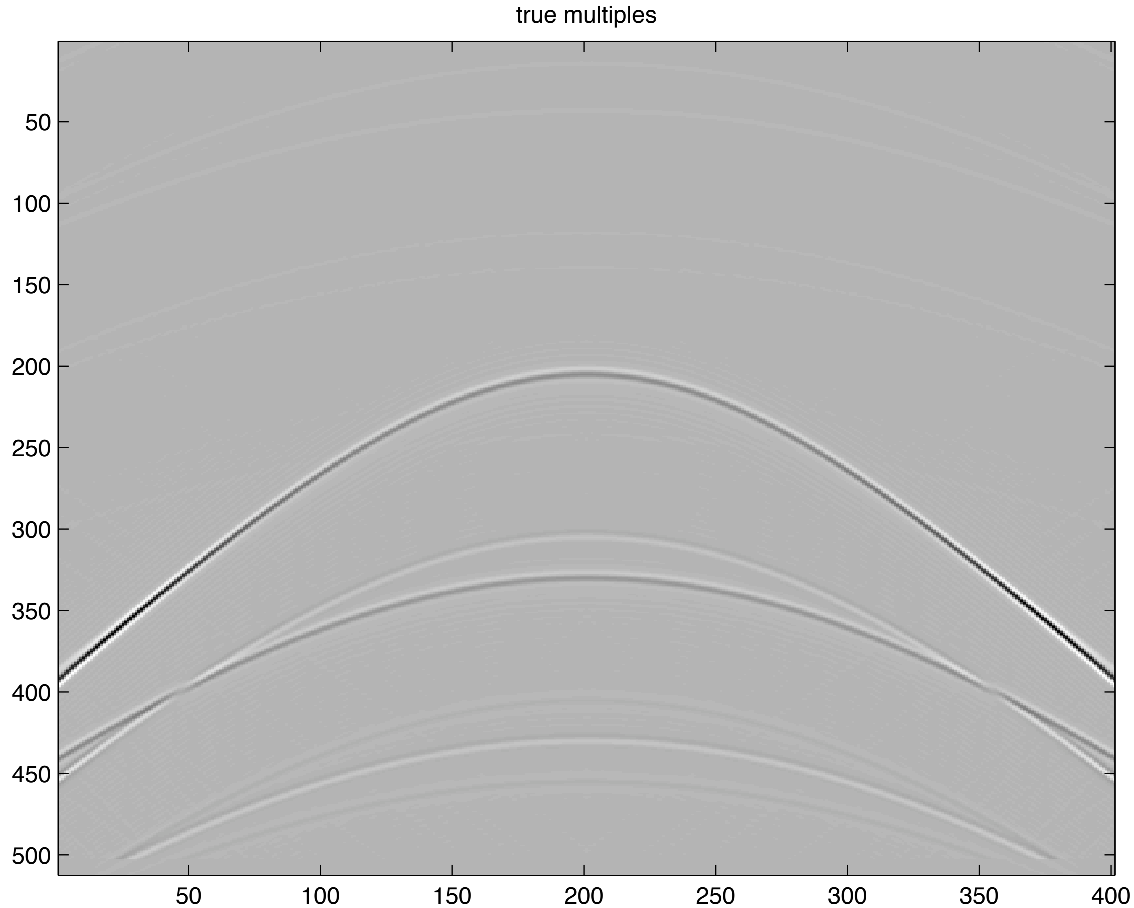
Multiple predictions need to be "deblurred".

Alternative formulation:

$$\begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \\ \mathbf{A} := \mathbf{\Delta P} \otimes \mathbf{S}^H \quad \text{(acquisition operator)} \\ \tilde{\mathbf{m}}^1 = \mathbf{S}^H \tilde{\mathbf{x}}. \end{cases}$$

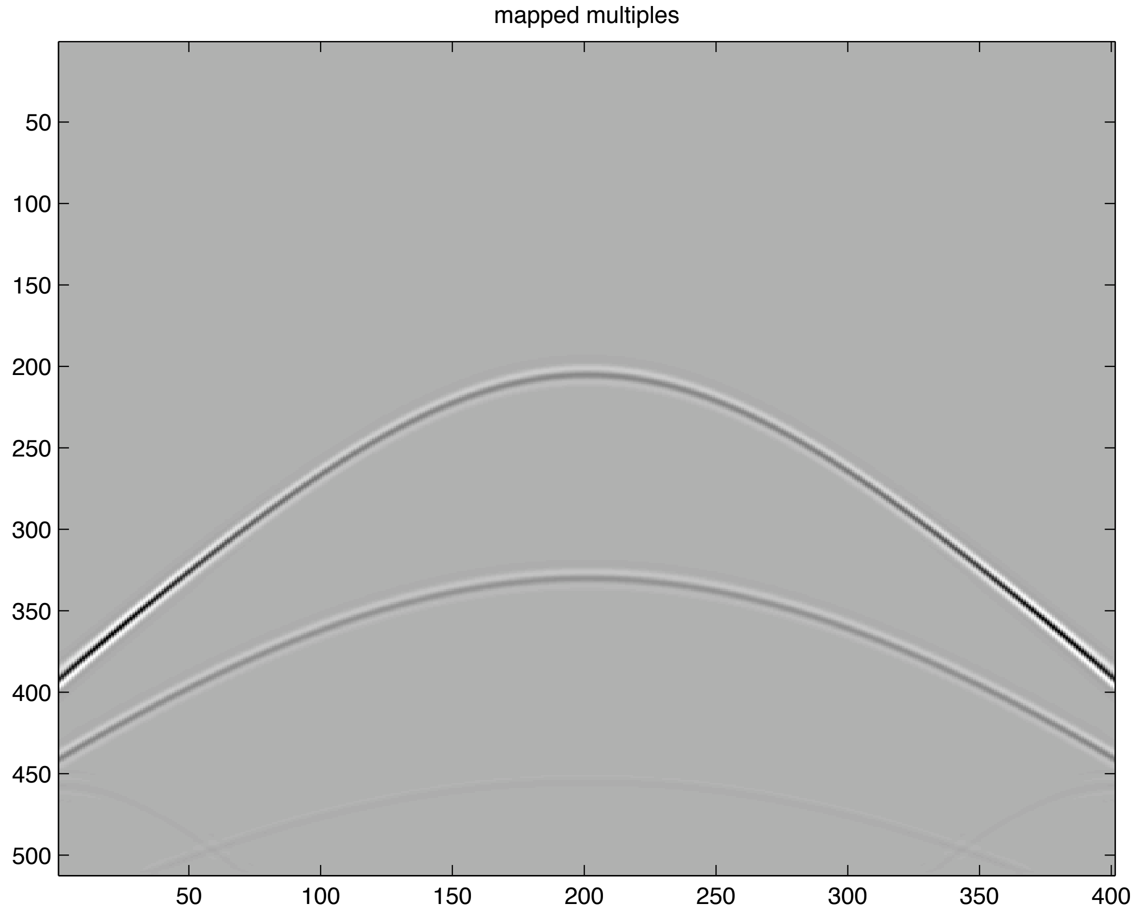
- find the sparsest set of coefficients that *cross-correlated* with the primaries yield the primaries **y**
- **no** acquisition operator necessary (tentative)
- acquisition operator is inverted intrinsically

True multiples
[surface + internal]



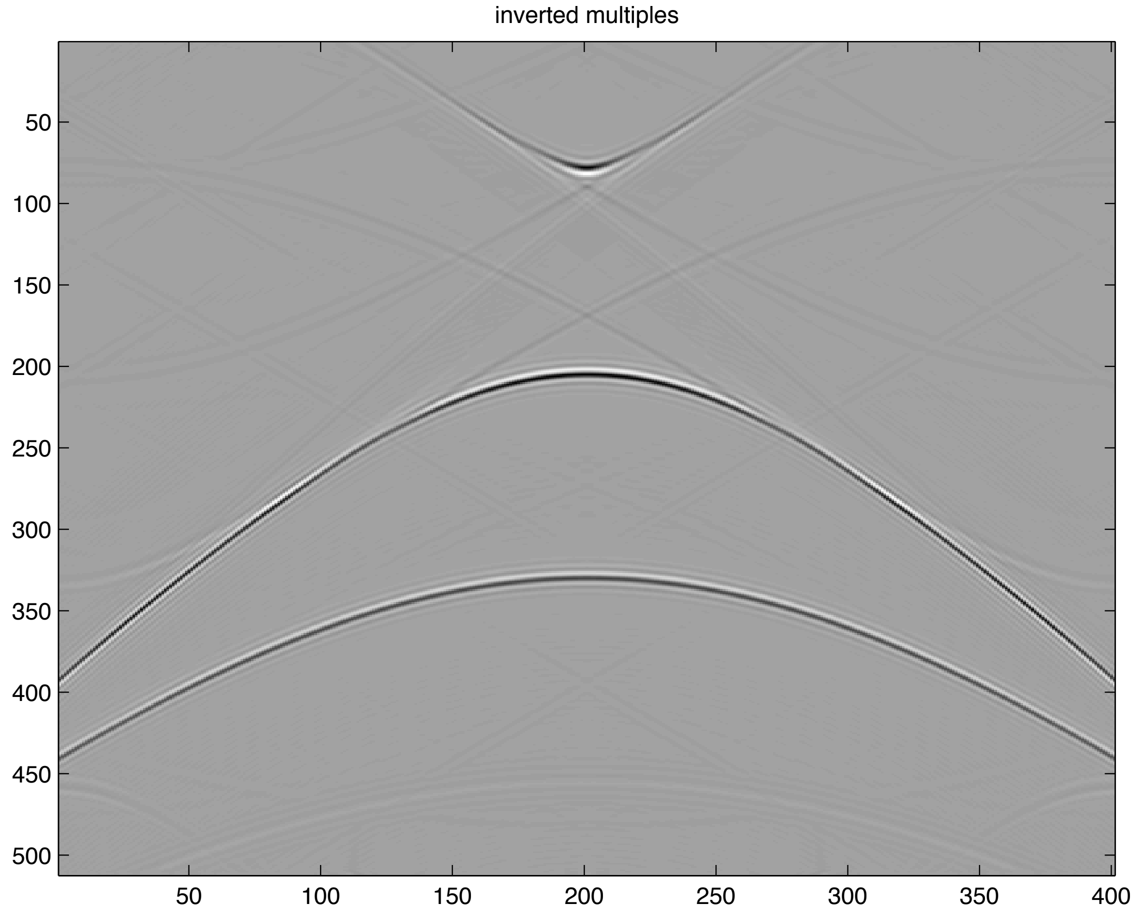
Mapped multiples

[surface]

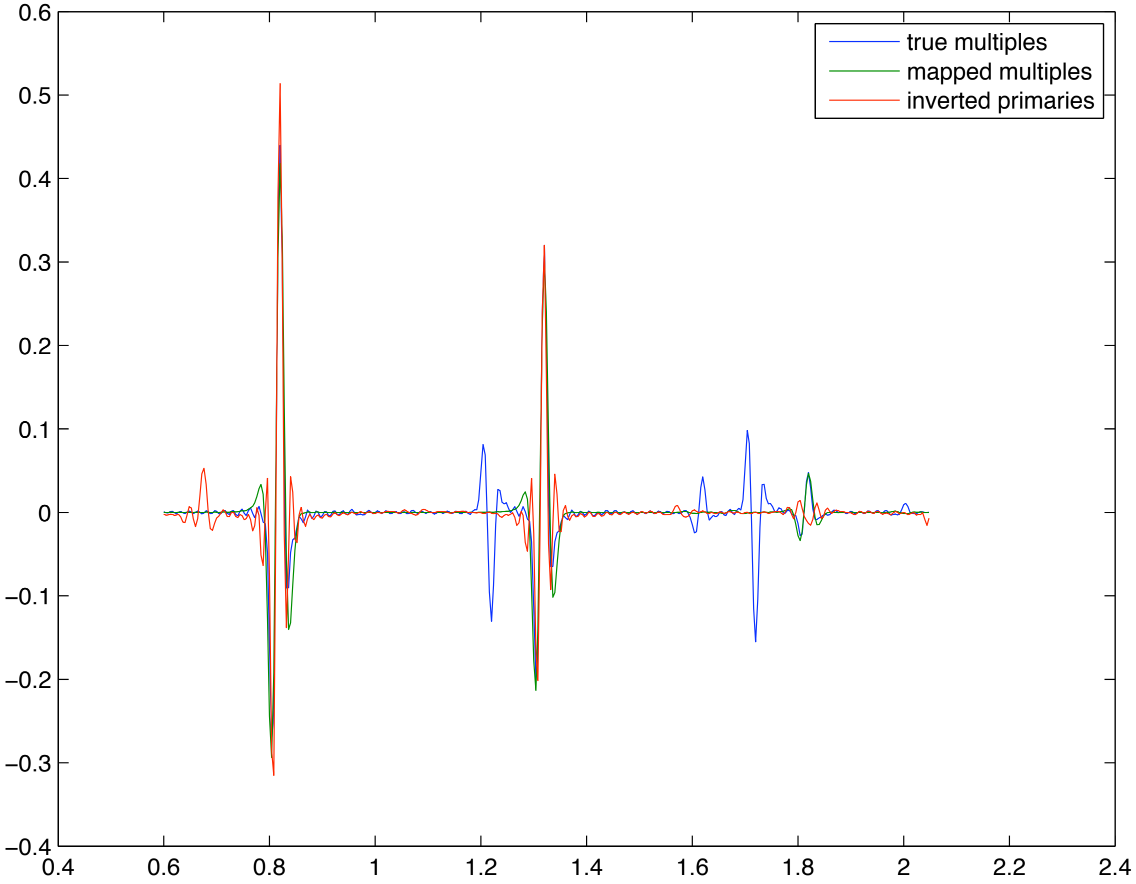


$$\mathbf{M}^1 = (\mathbf{\Delta P}\boldsymbol{\mathcal{A}} *_{t,x} \mathbf{\Delta P})$$

Inverted multiples
[surface]



Mapped vs inverted



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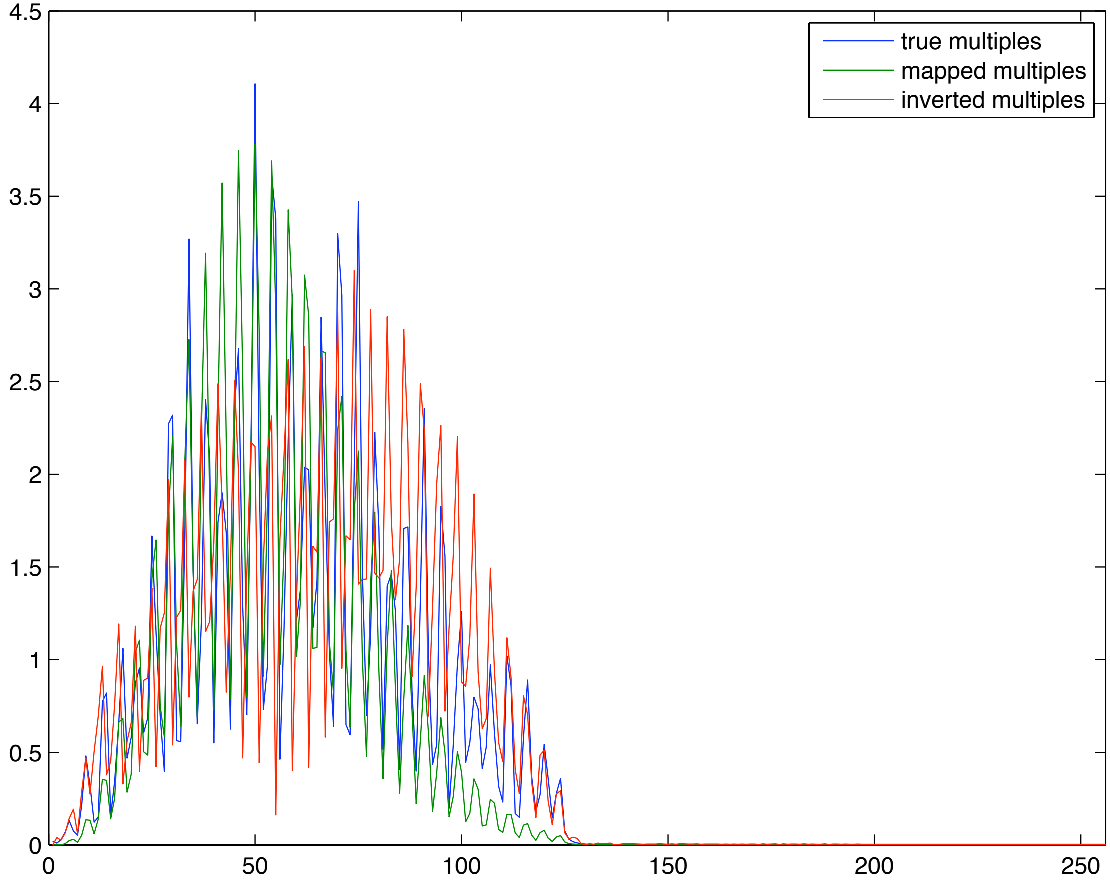
Mapped vs inverted



Multiples 2 primaries

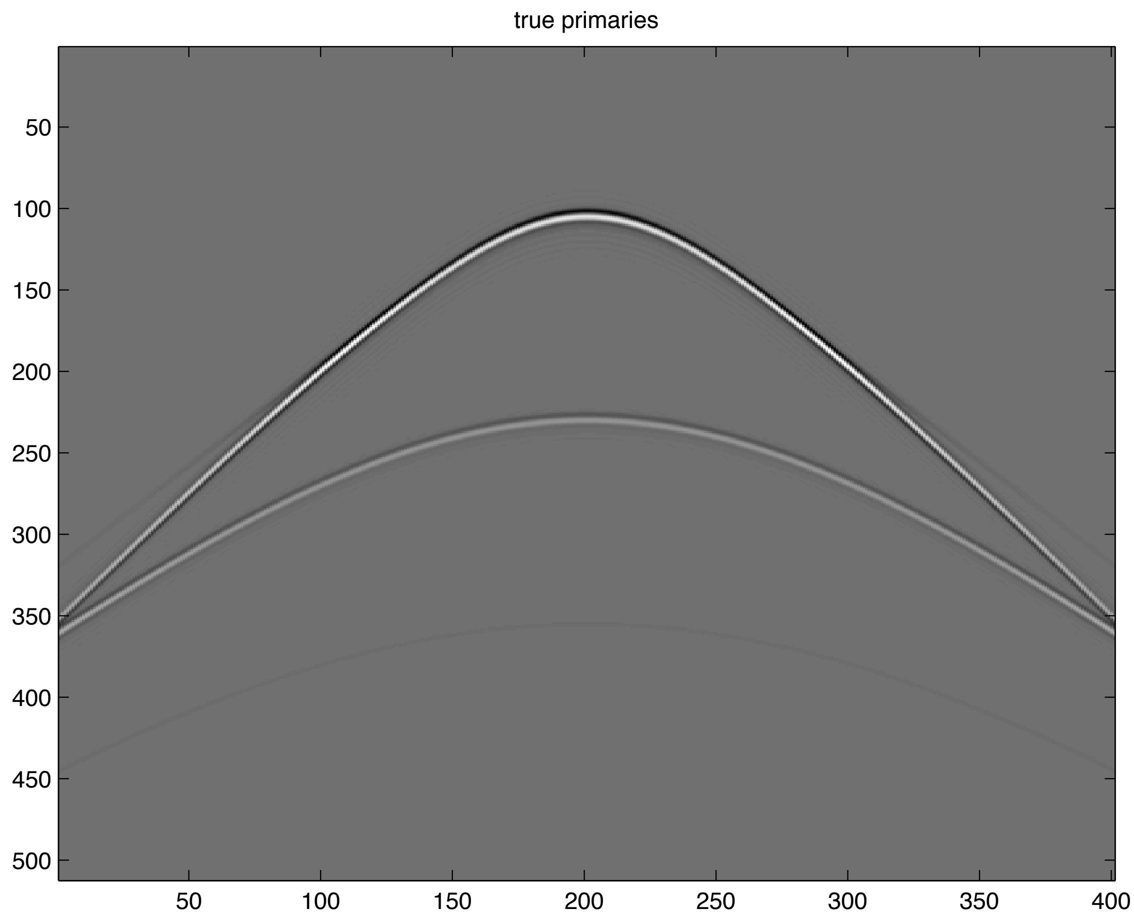
Primary predictions need to be "deblurred".

Alternative formulation:

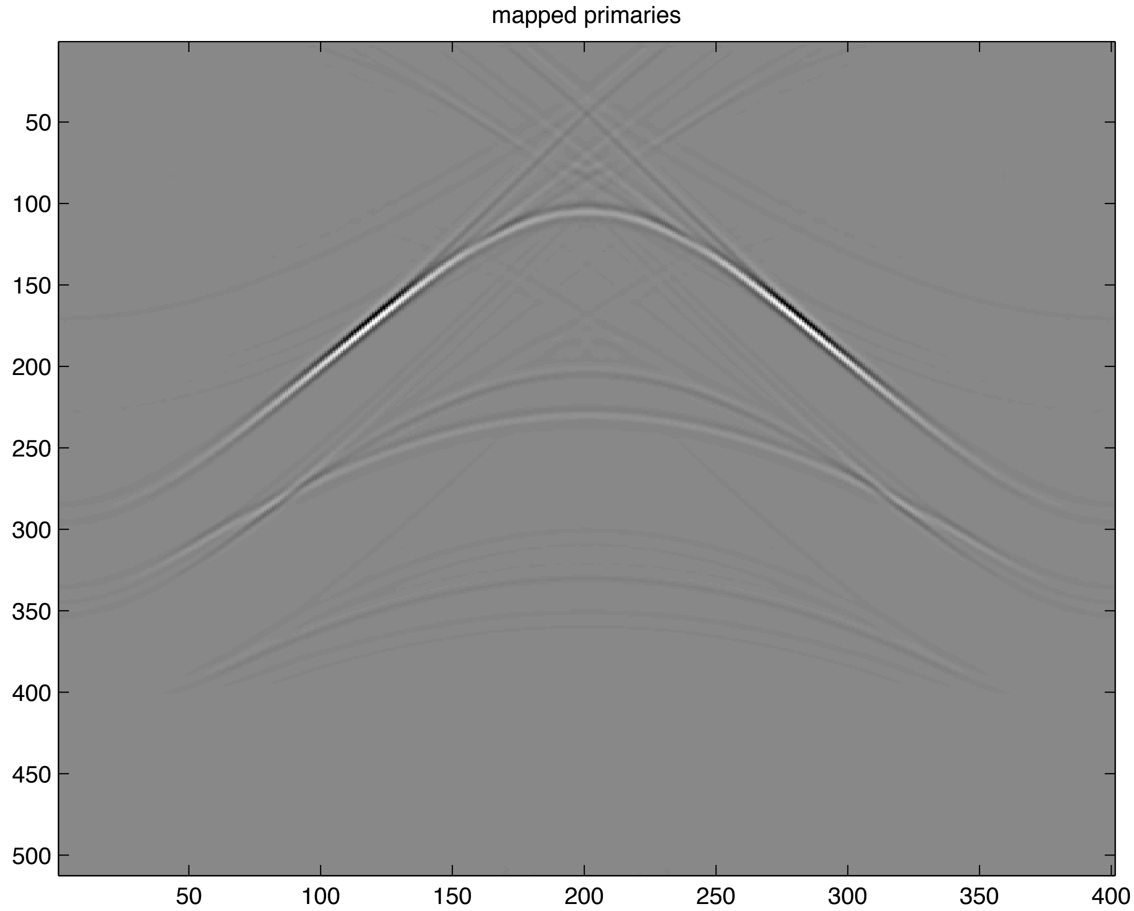
$$\begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \\ \mathbf{A} := \mathbf{\Delta P} * \mathbf{S}^H \\ \tilde{\mathbf{\Delta p}} = \mathbf{S}^H \tilde{\mathbf{x}}. \end{cases}$$

- find the sparsest set of coefficients that cross-convoluted with the primaries yields the multiples **y**
- **no** acquisition operator necessary
- acquisition operator is inverted intrinsically
- compare with the focal transform

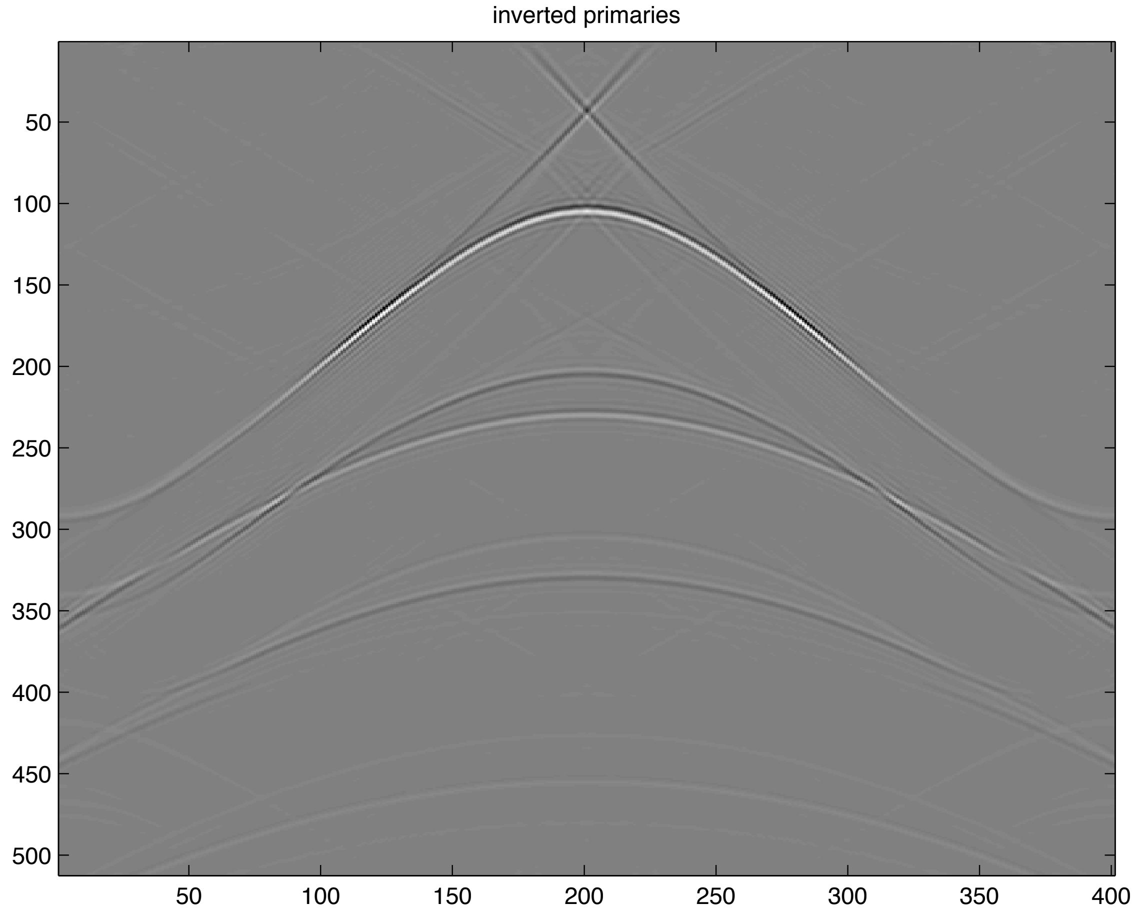
True primaries



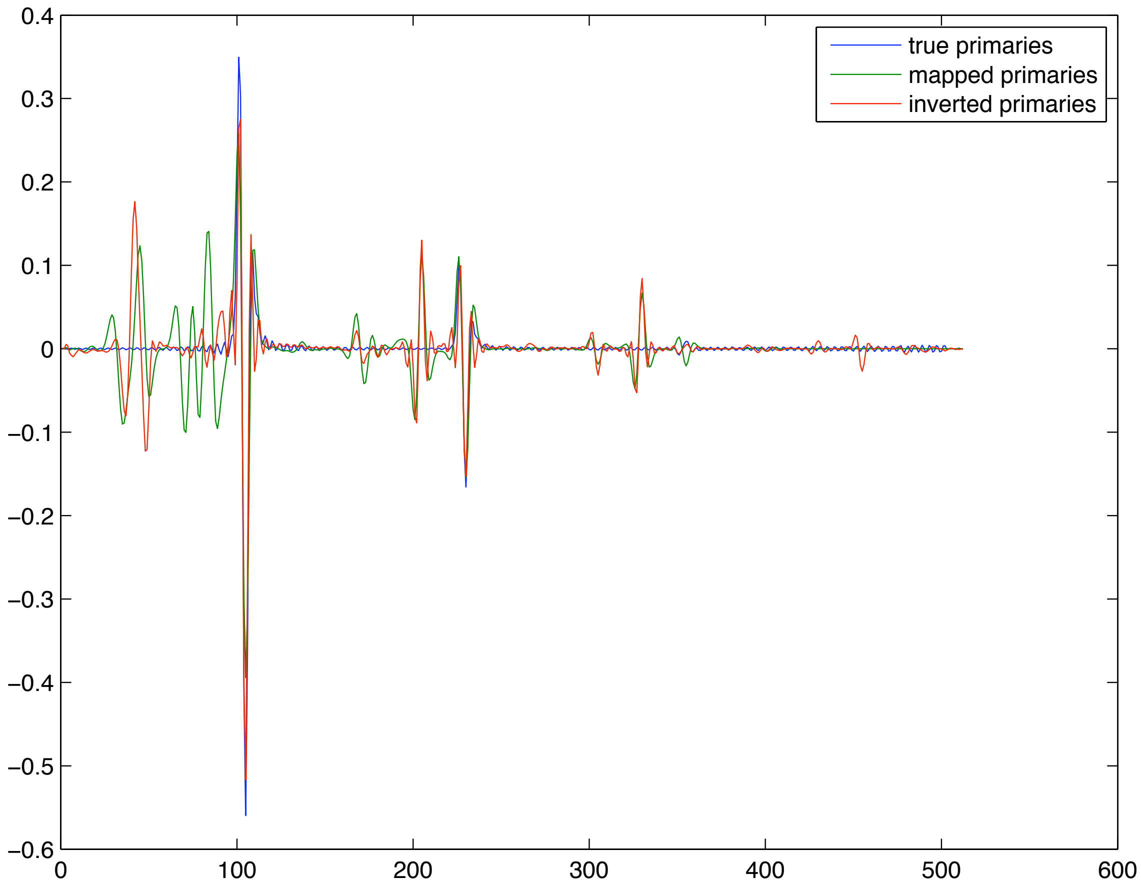
Mapped primaries



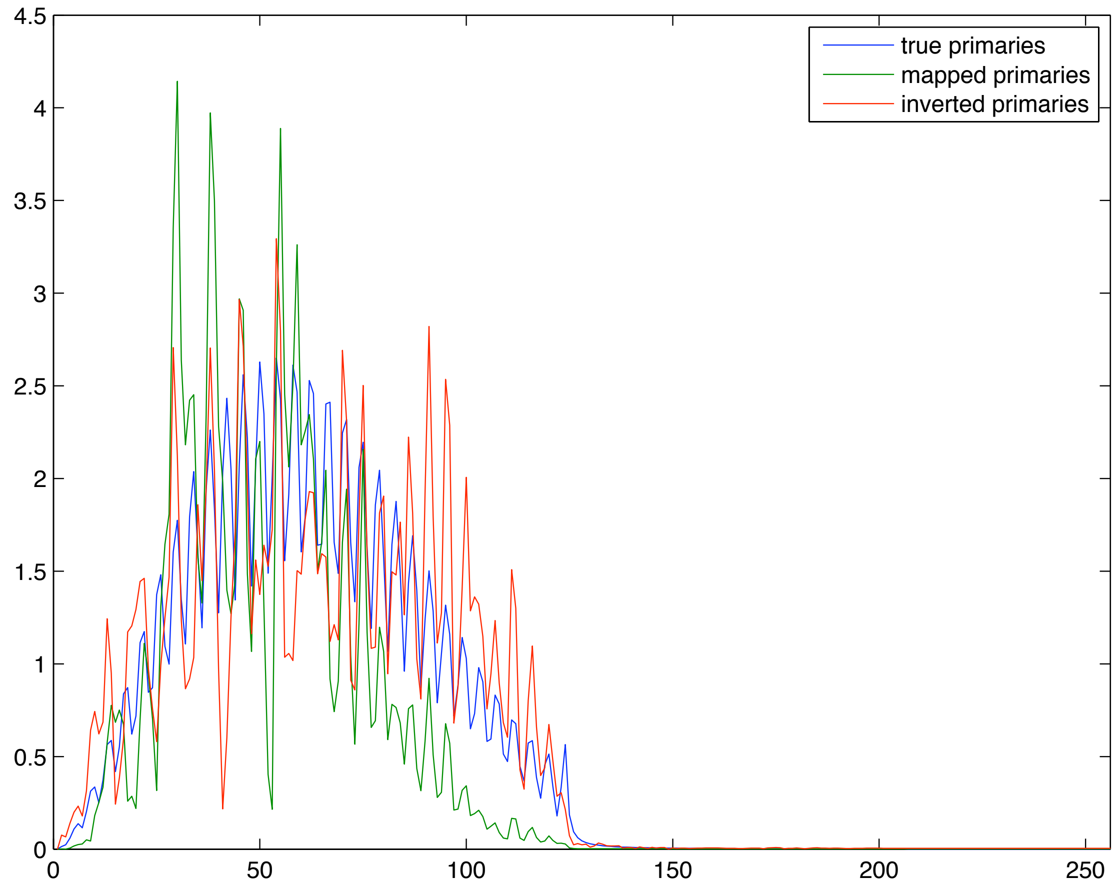
Inverted primaries



Mapped vs inverted



Mapped vs inverted



Observations

Primaries can be mapped to multiples and *vice versa.*

Formulation with sparsity constraints

- **does not require info on acquisition**
- **preservers frequency content**

Primaries 2 multiples *spreads* energy

Multiples 2 primaries *concentrates* energy

- Plots are *modulus* signs
- Needed a Hilbert transform + 3 sample shift

Same can be done with total data

Focussing can be used to do interpolation

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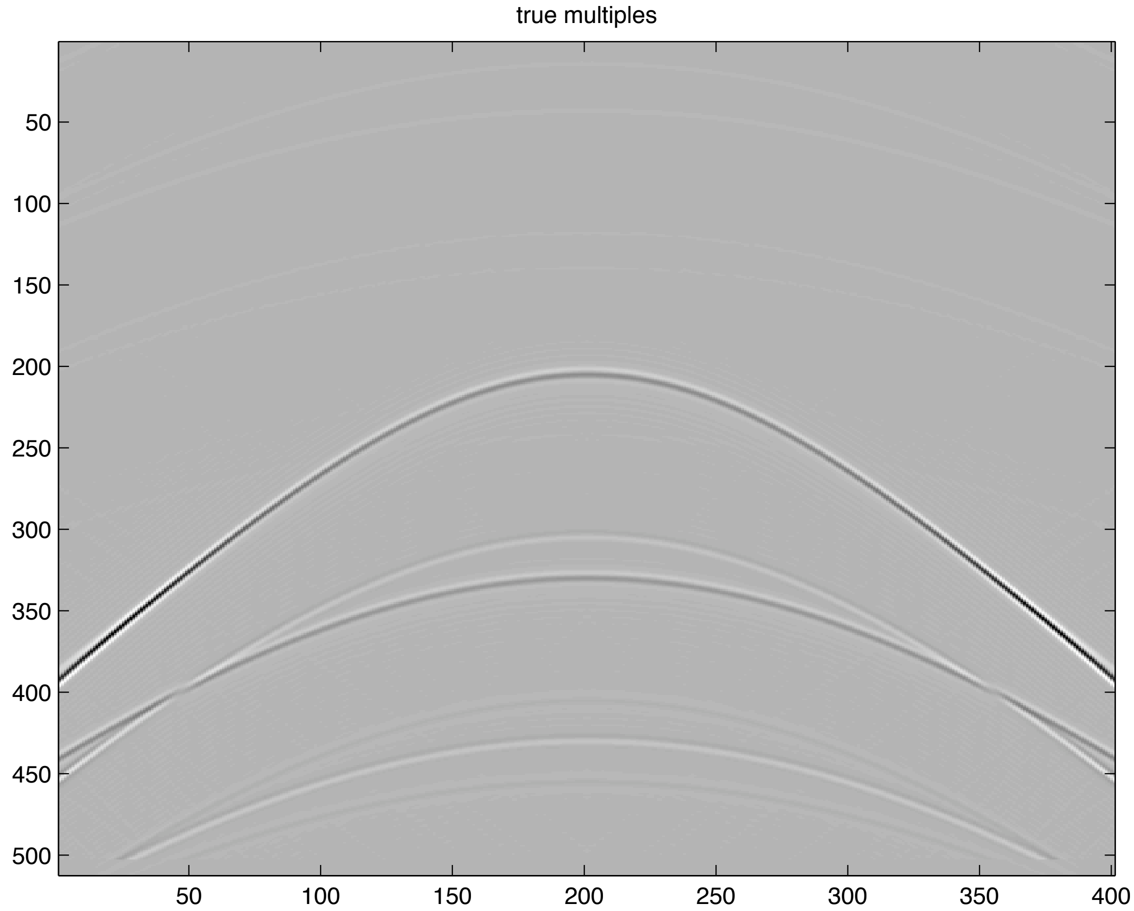
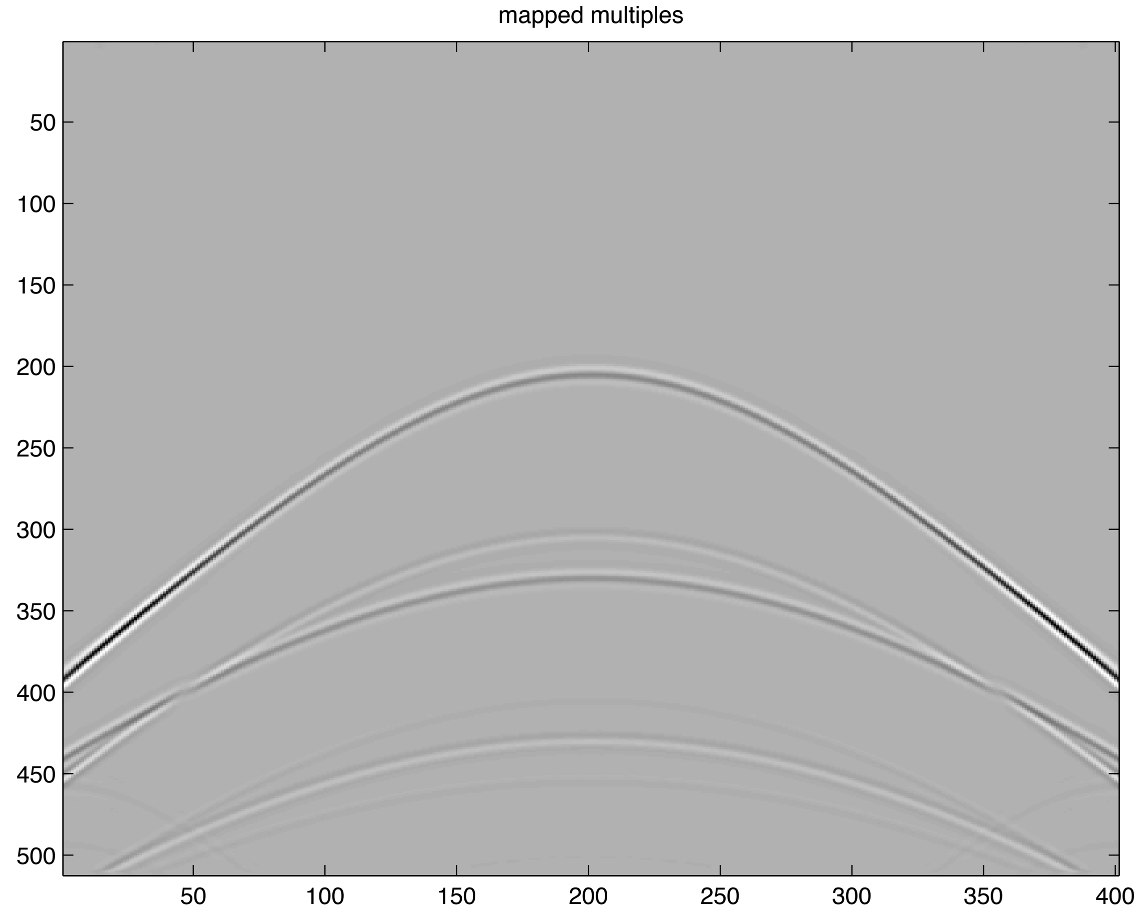
True multiples
[surface + internal]



Mapped multiples
[surface + internal]



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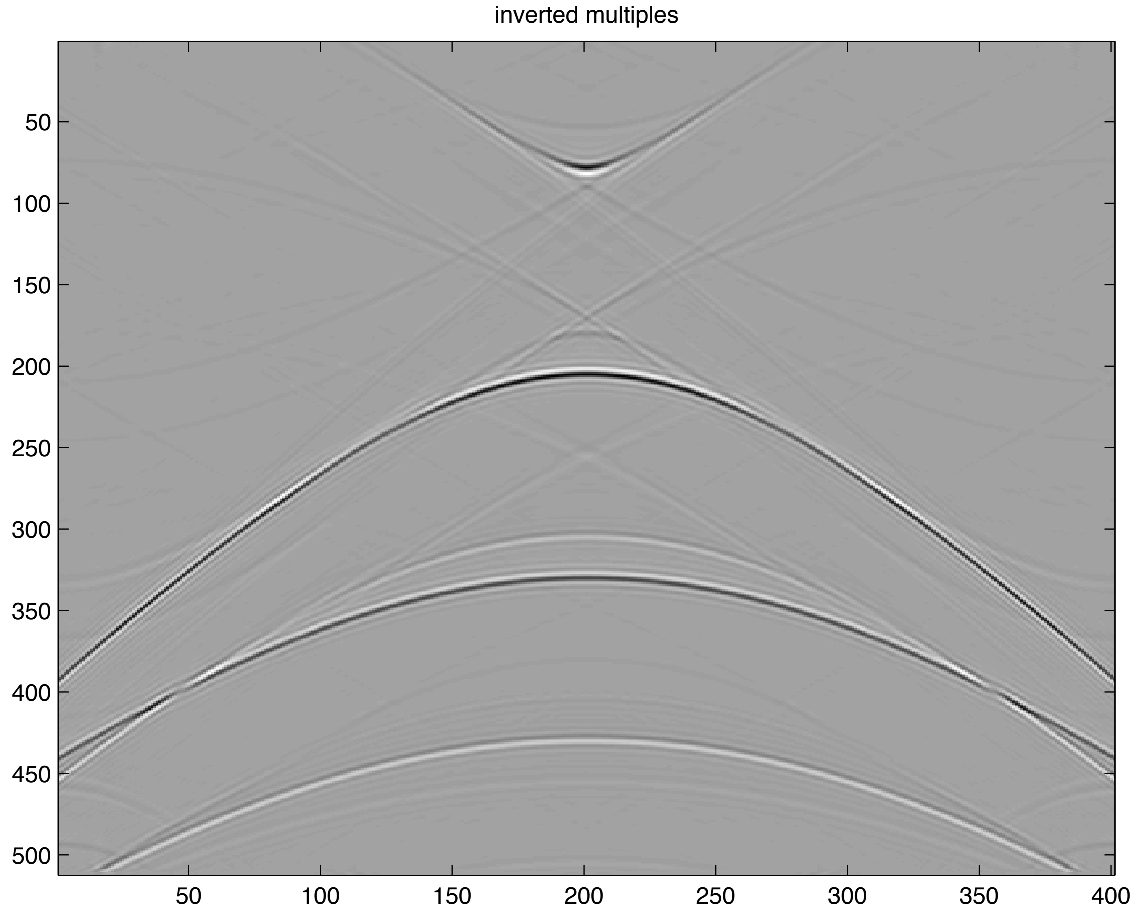
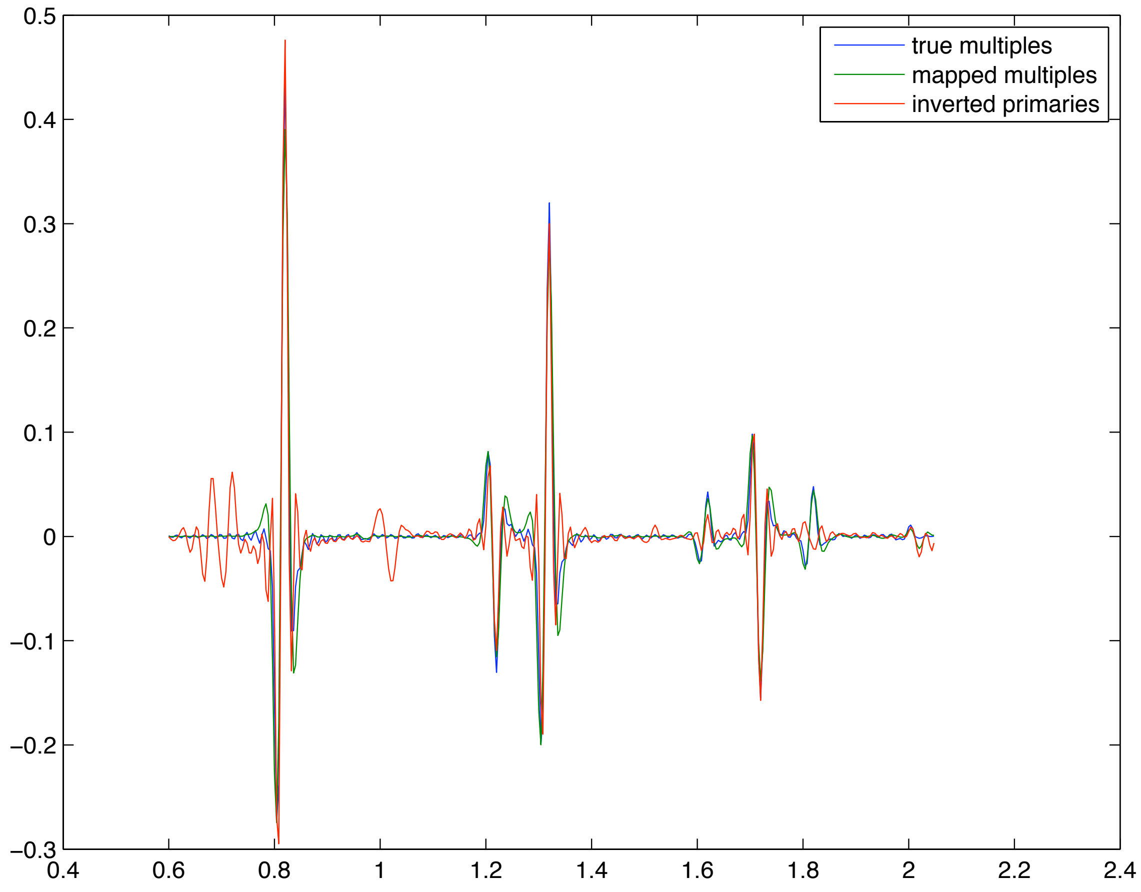
Inverted multiples
[surface and internal]



Mapped vs inverted



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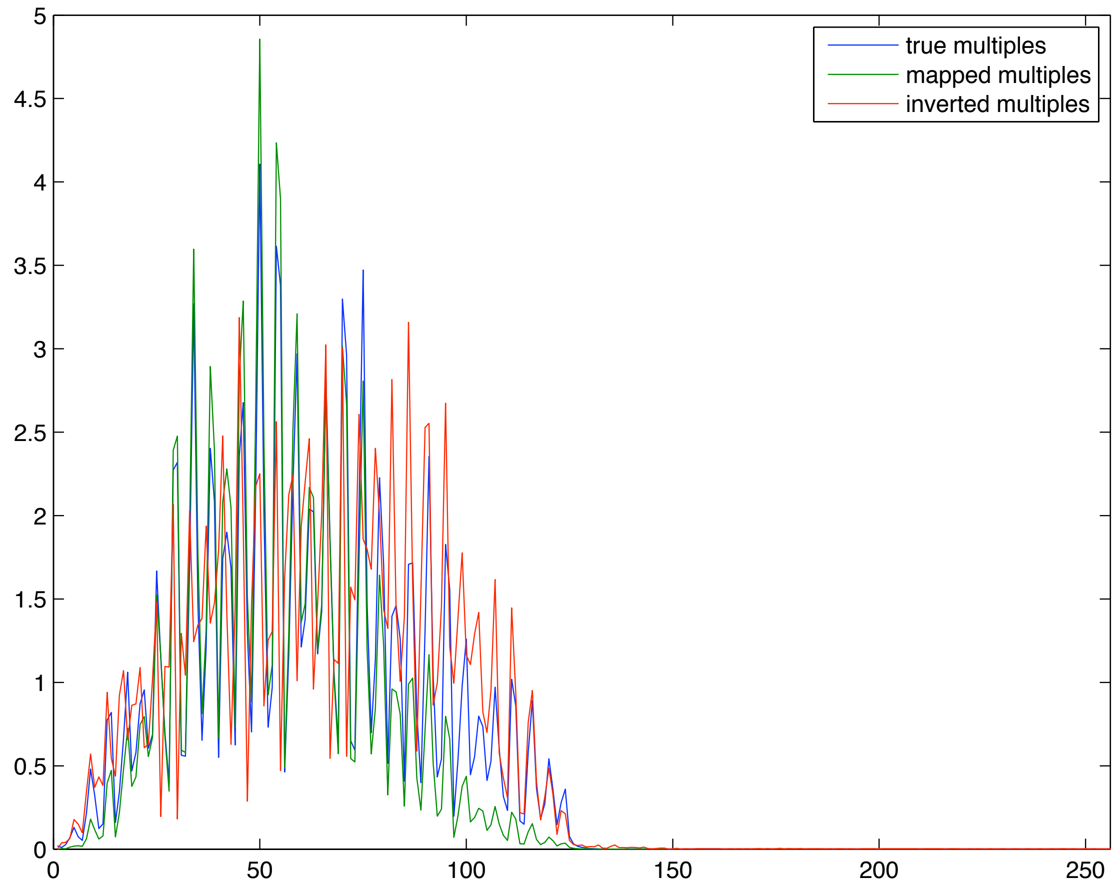
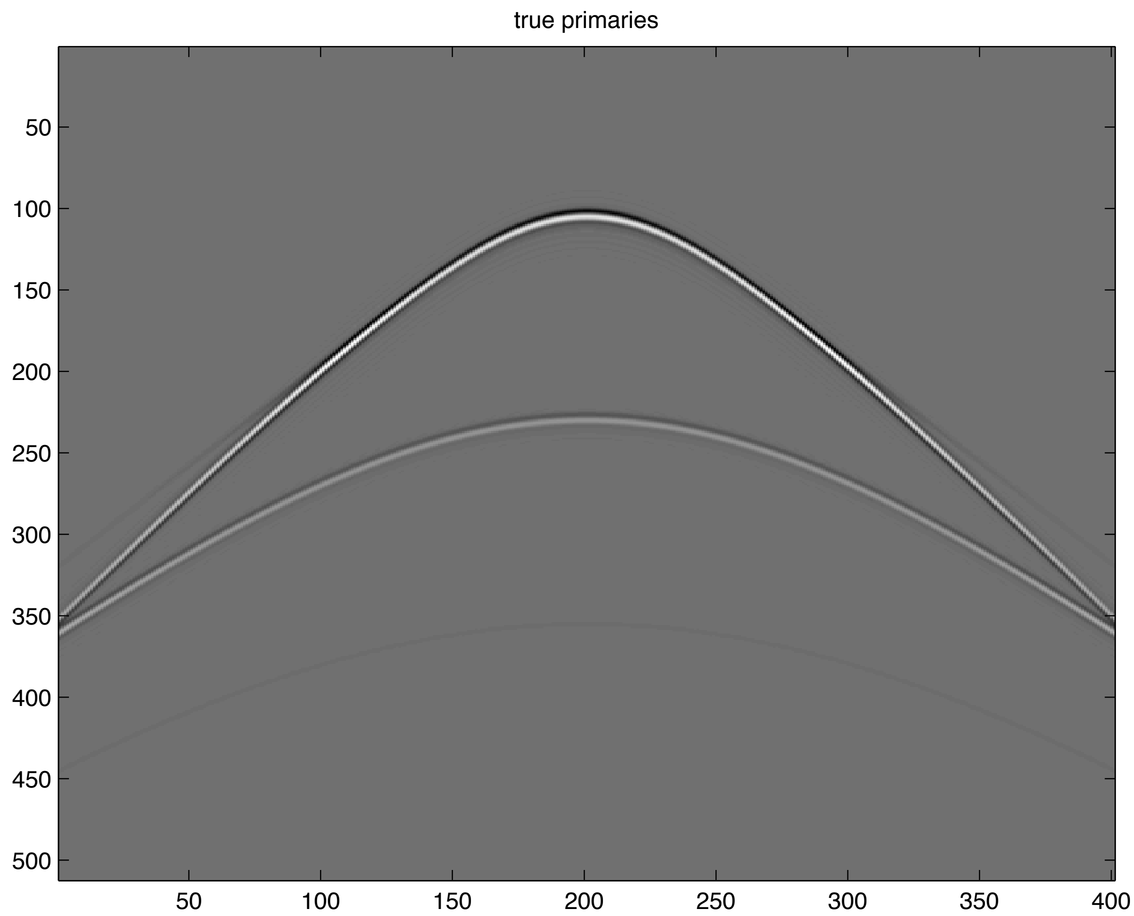
Mapped vs inverted



True primaries



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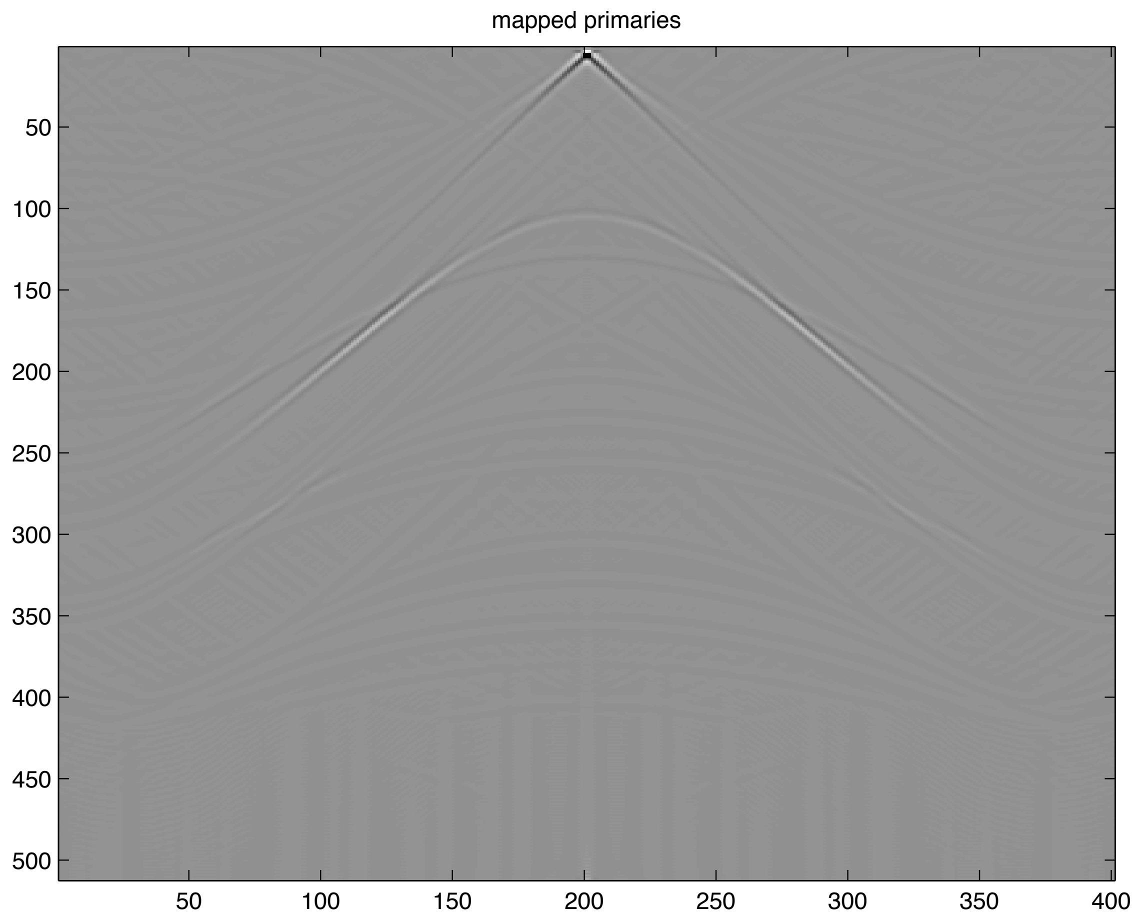
Mapped primaries



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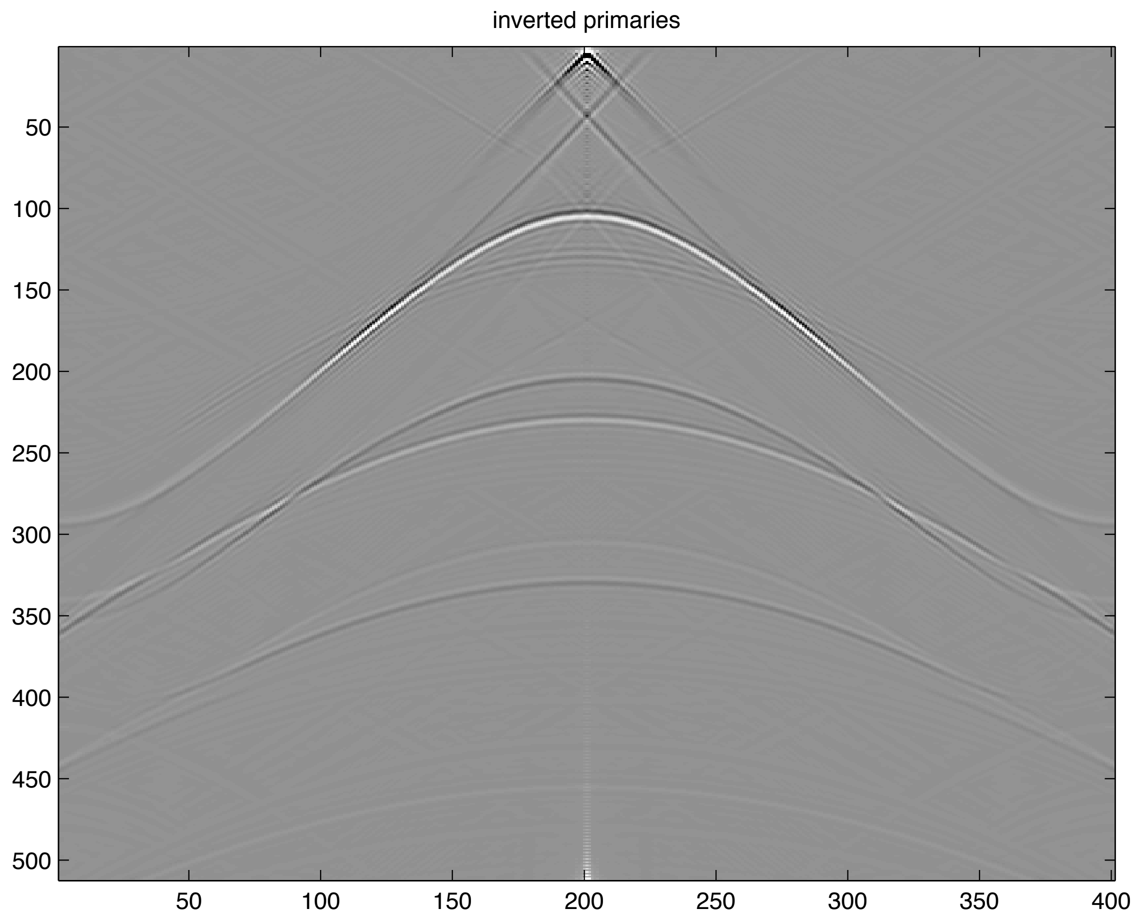
Inverted primaries



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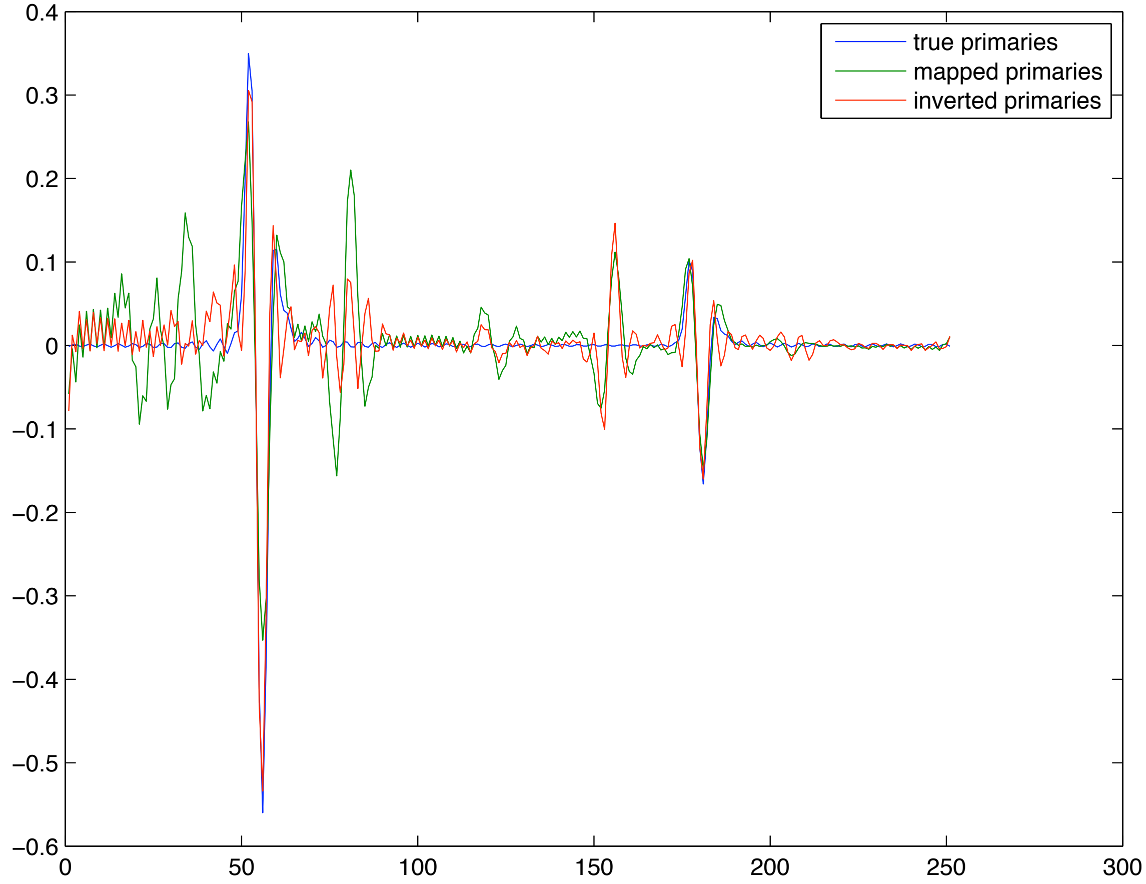
Mapped vs inverted



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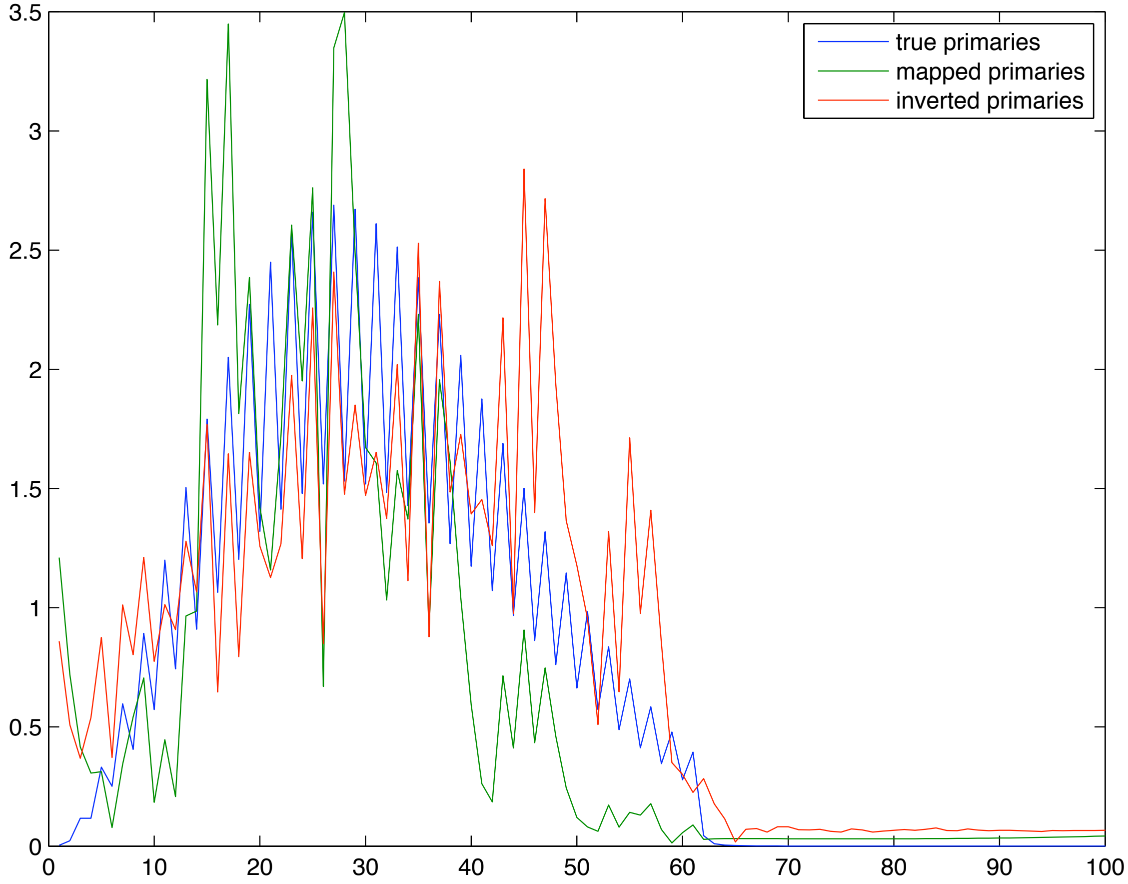
Mapped vs inverted



Recovery by focussing

Remember inverting cross-convolution focusses.

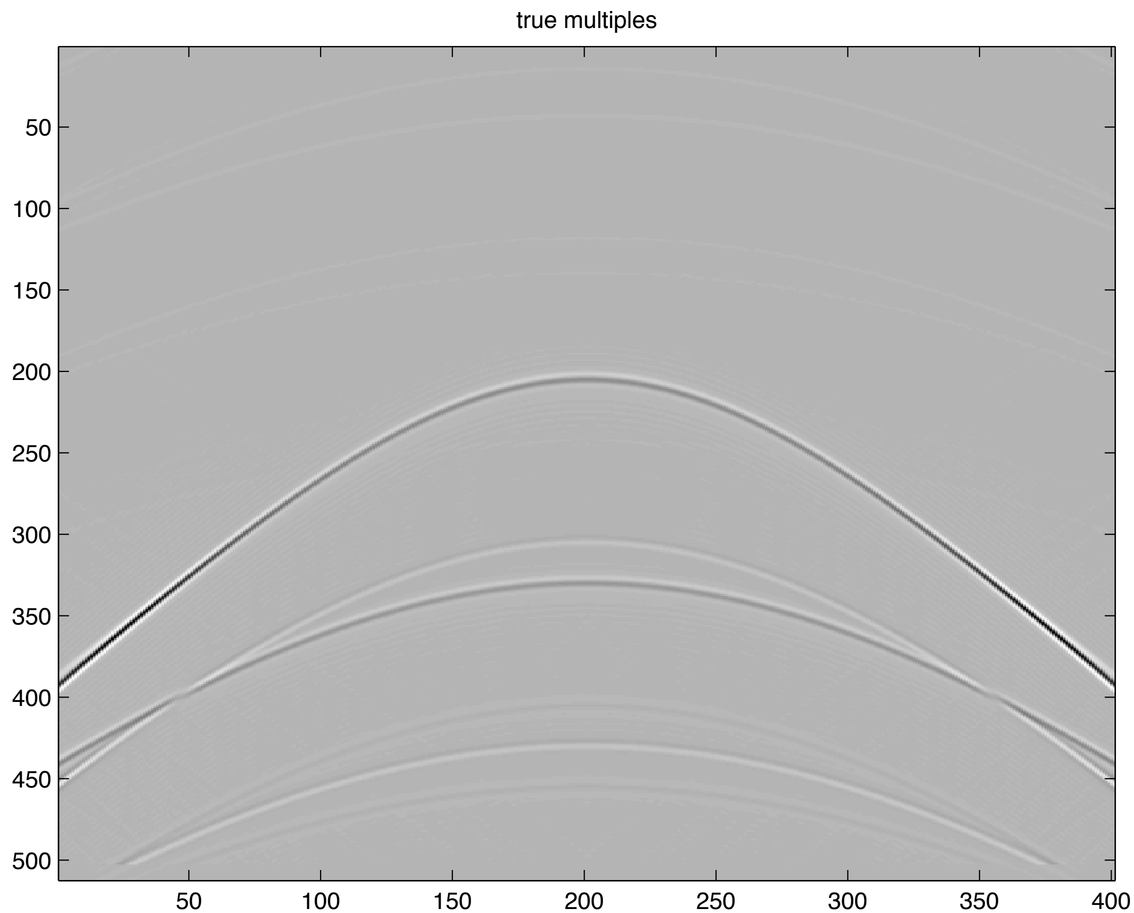
Interpolate incomplete multiple energy through

$$\begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \\ \mathbf{A} := \mathbf{R}\mathbf{I}\left(\boldsymbol{\Delta}\mathbf{P} * \mathbf{S}^H\right) \\ \tilde{\mathbf{m}}^1 = \mathbf{A}\tilde{\mathbf{x}}, \, \mathbf{y} = \mathbf{R}\mathbf{I}\mathbf{m}^1. \end{cases}$$

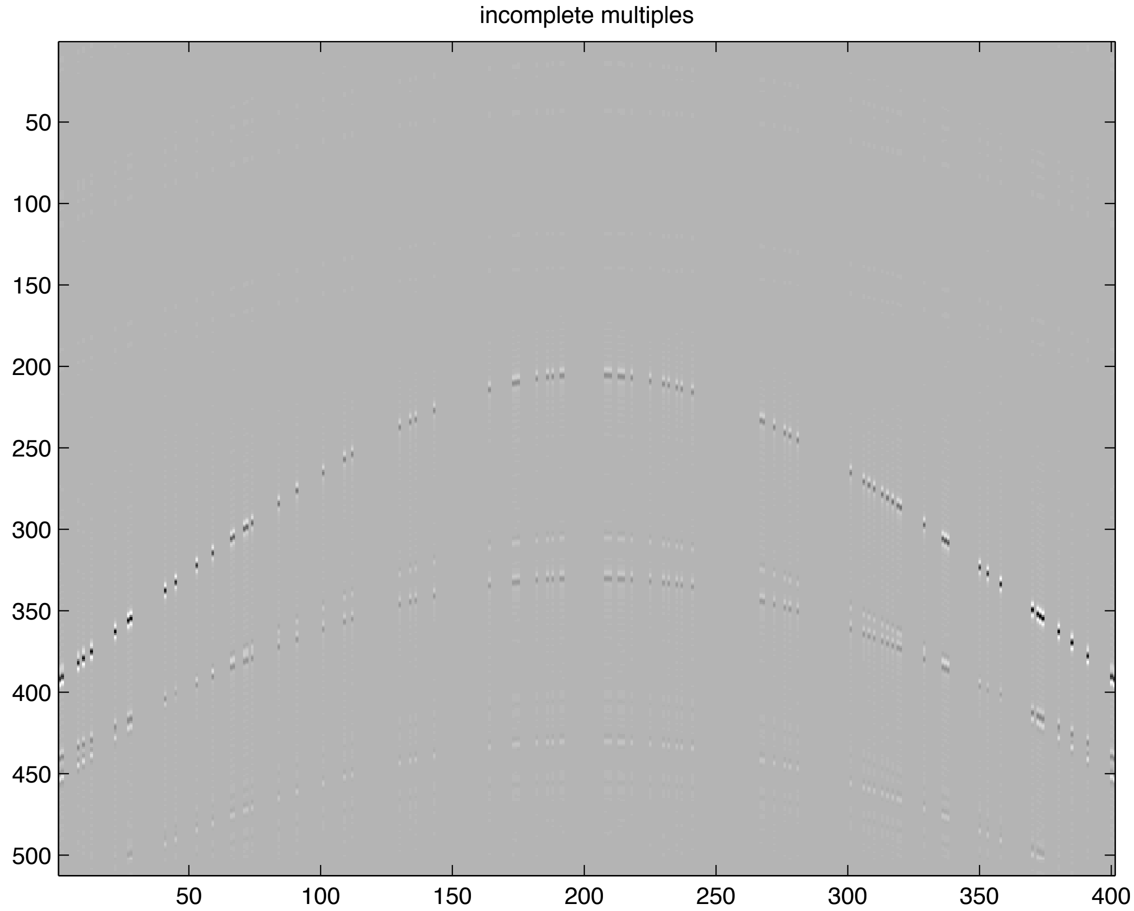
with **y** the incomplete multiples.

- **R** is the restriction operator
- **I** the Dirac measurement matrix
- multiples are recovered

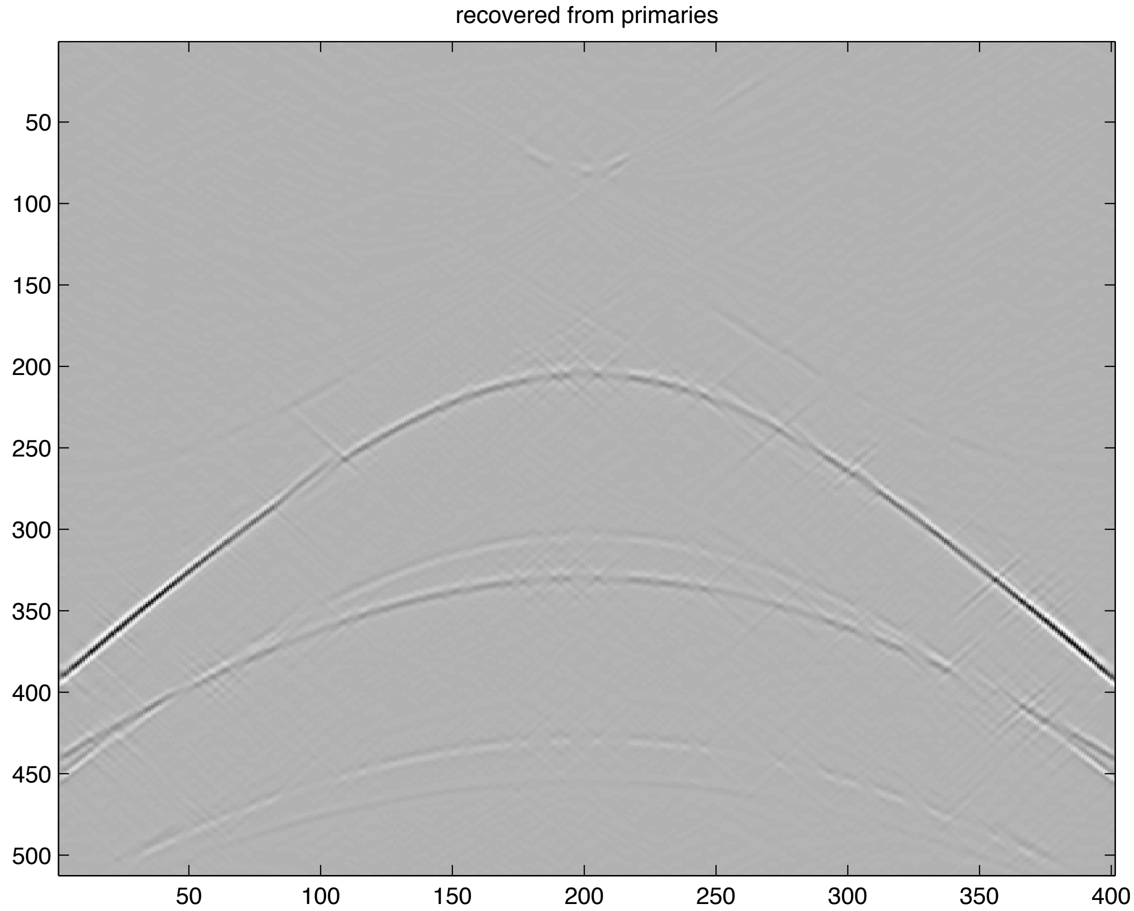
True multiples



Incomplete multiples



Recovered multiples



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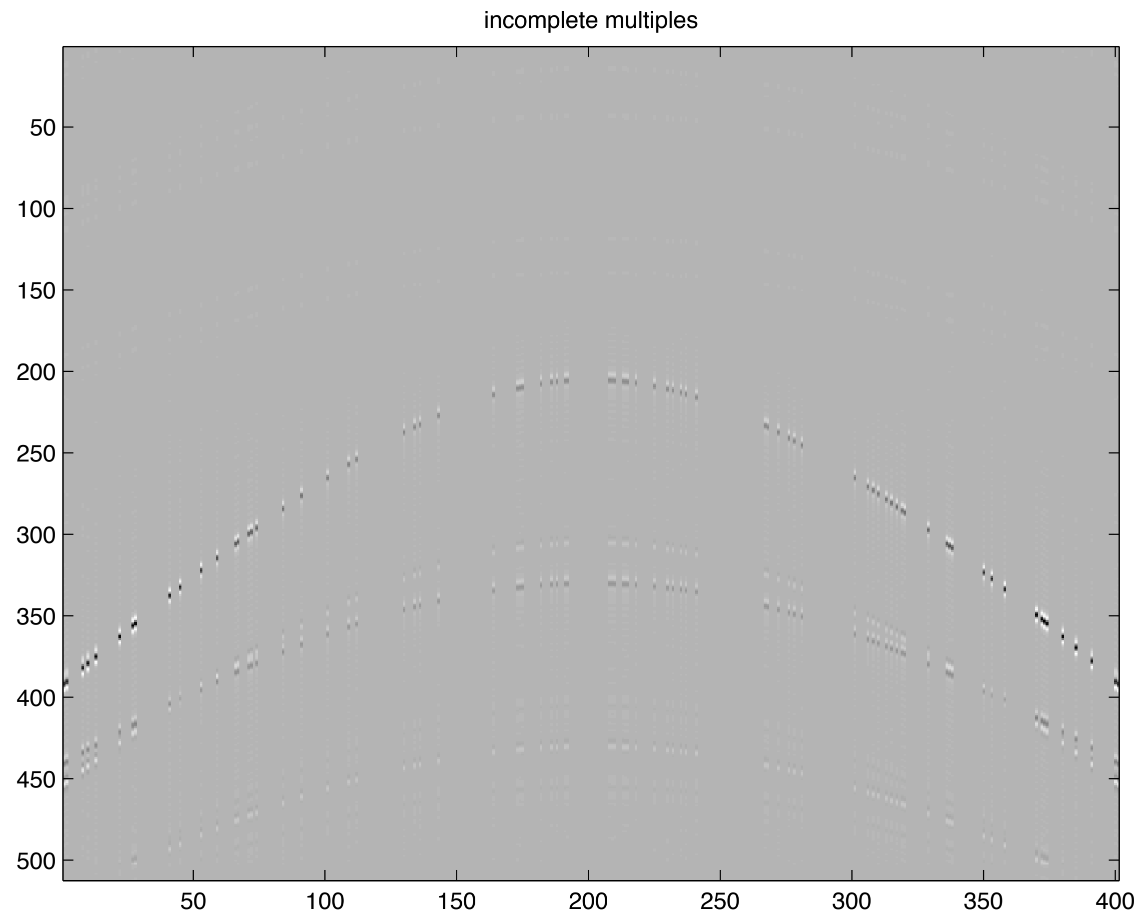
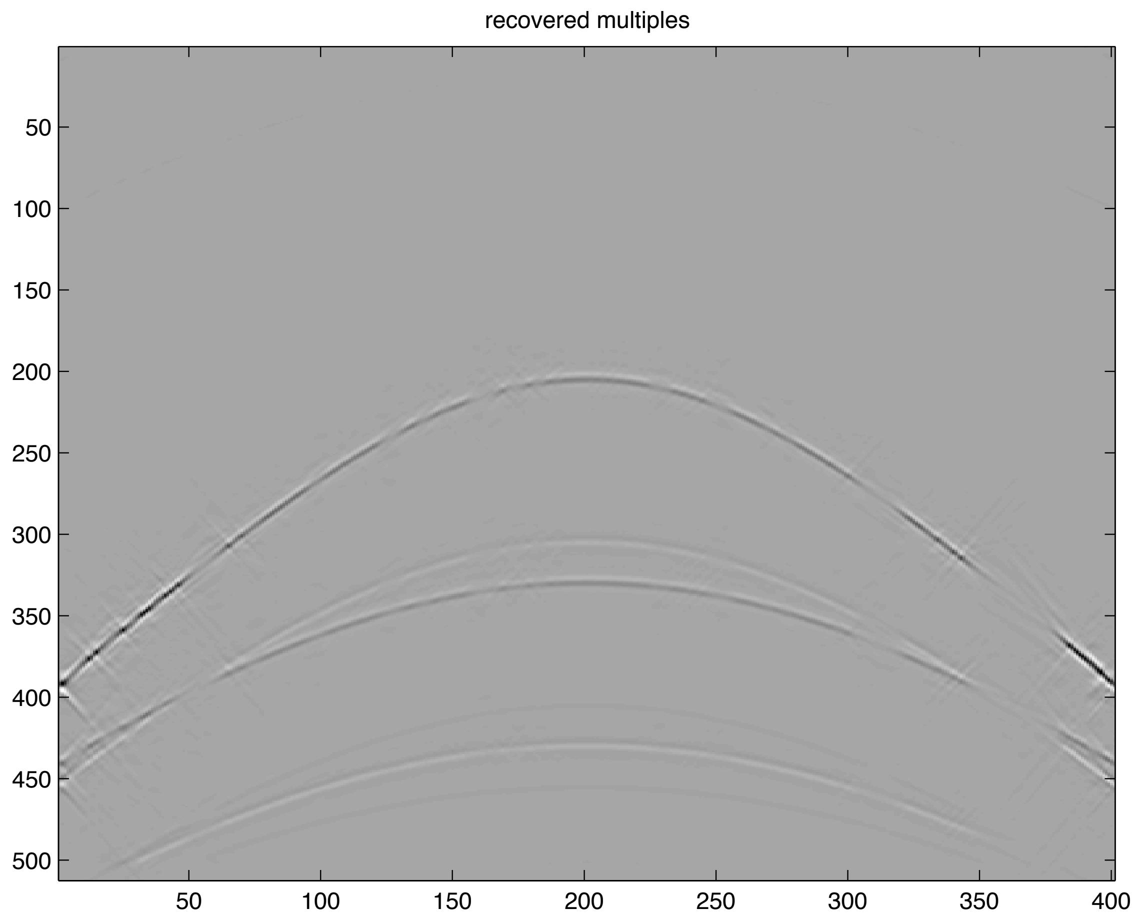
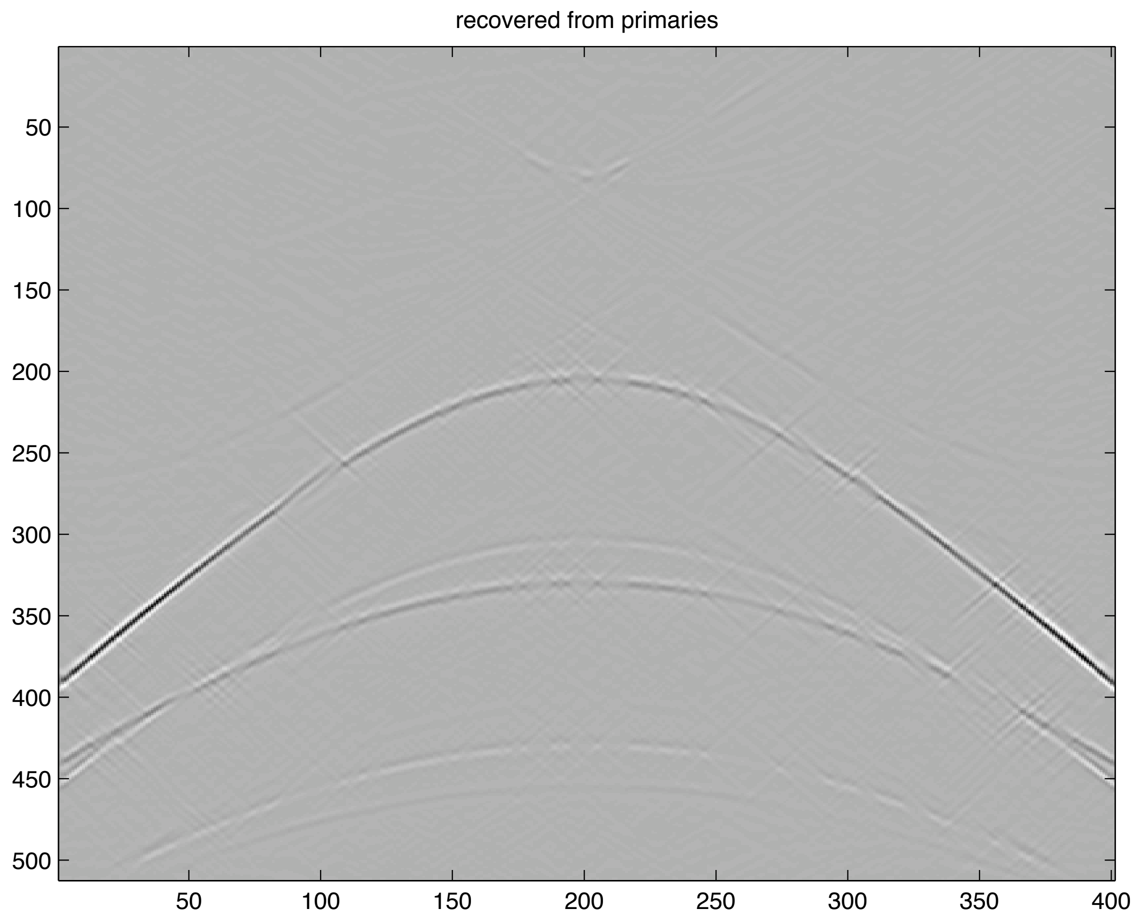
Incomplete multiples



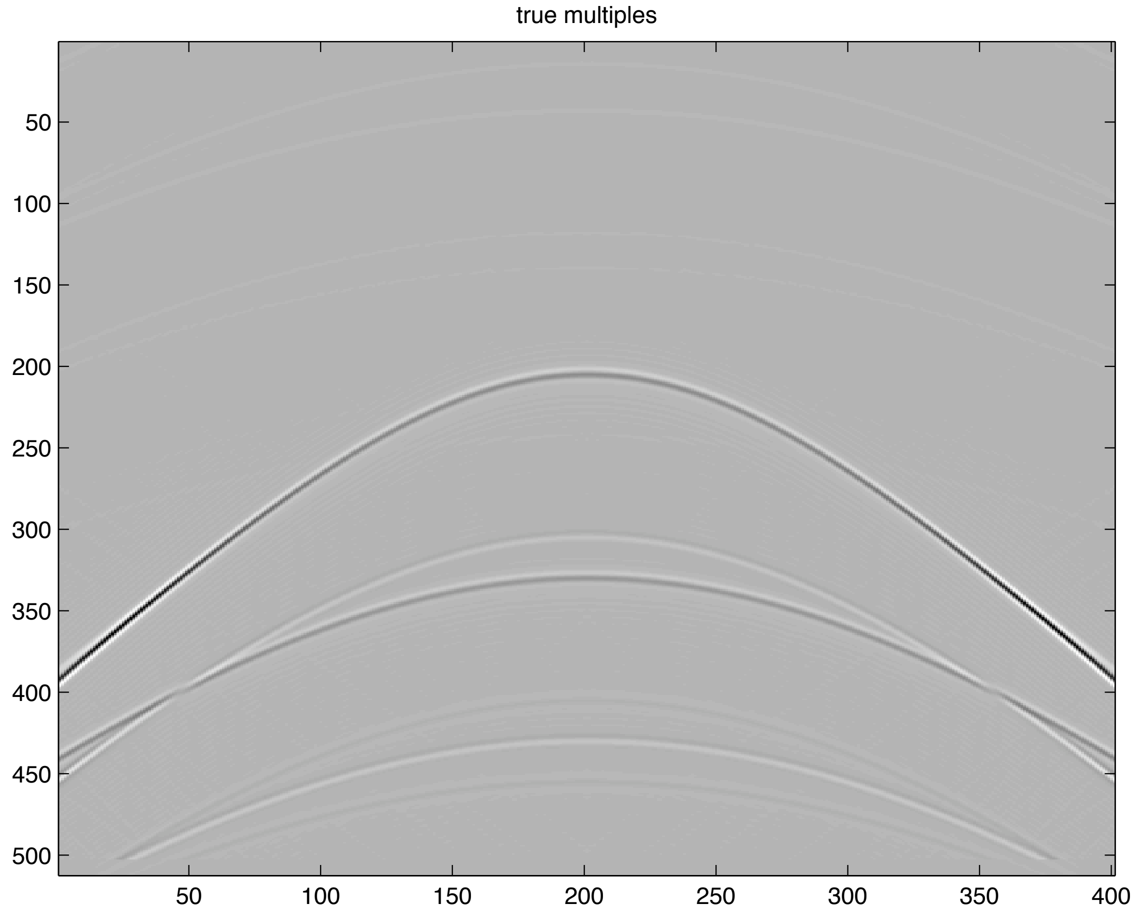
Interpolated multiples



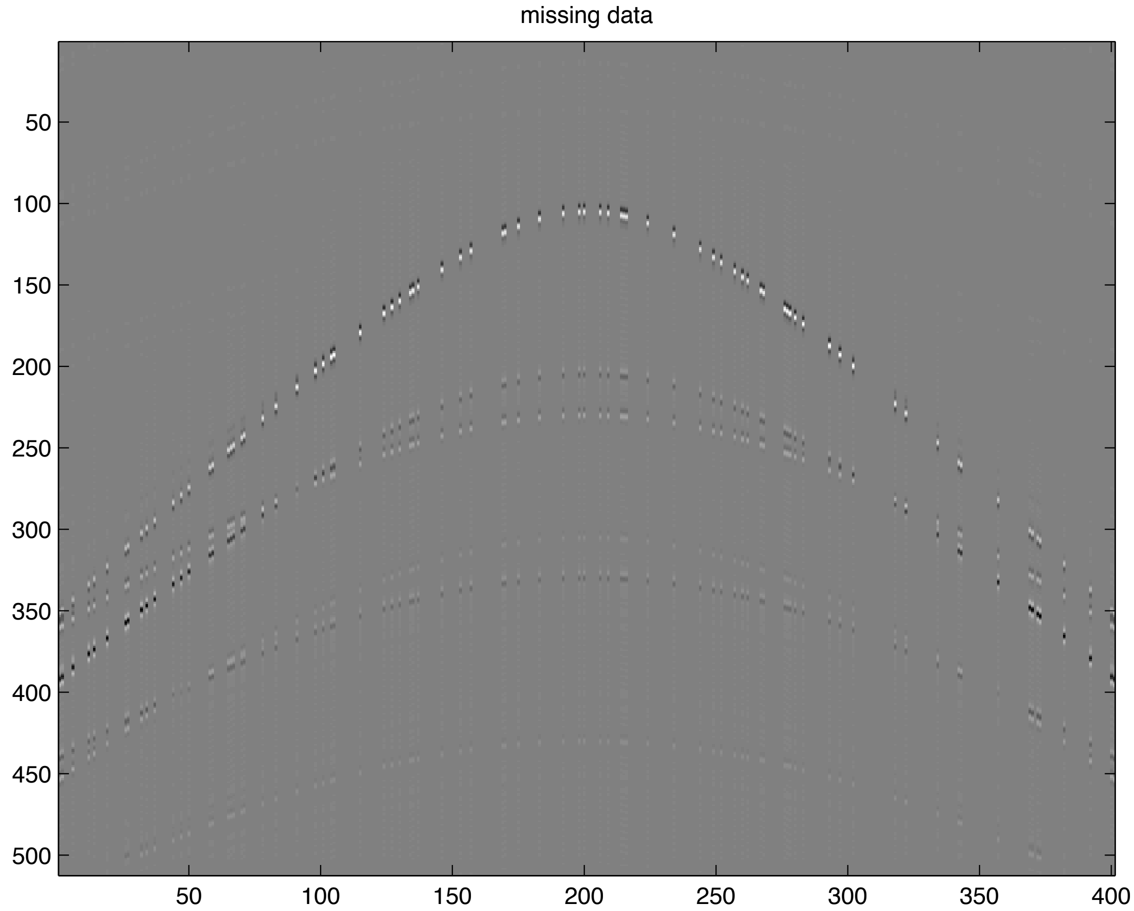
Recovered multiples



True multiples



Data



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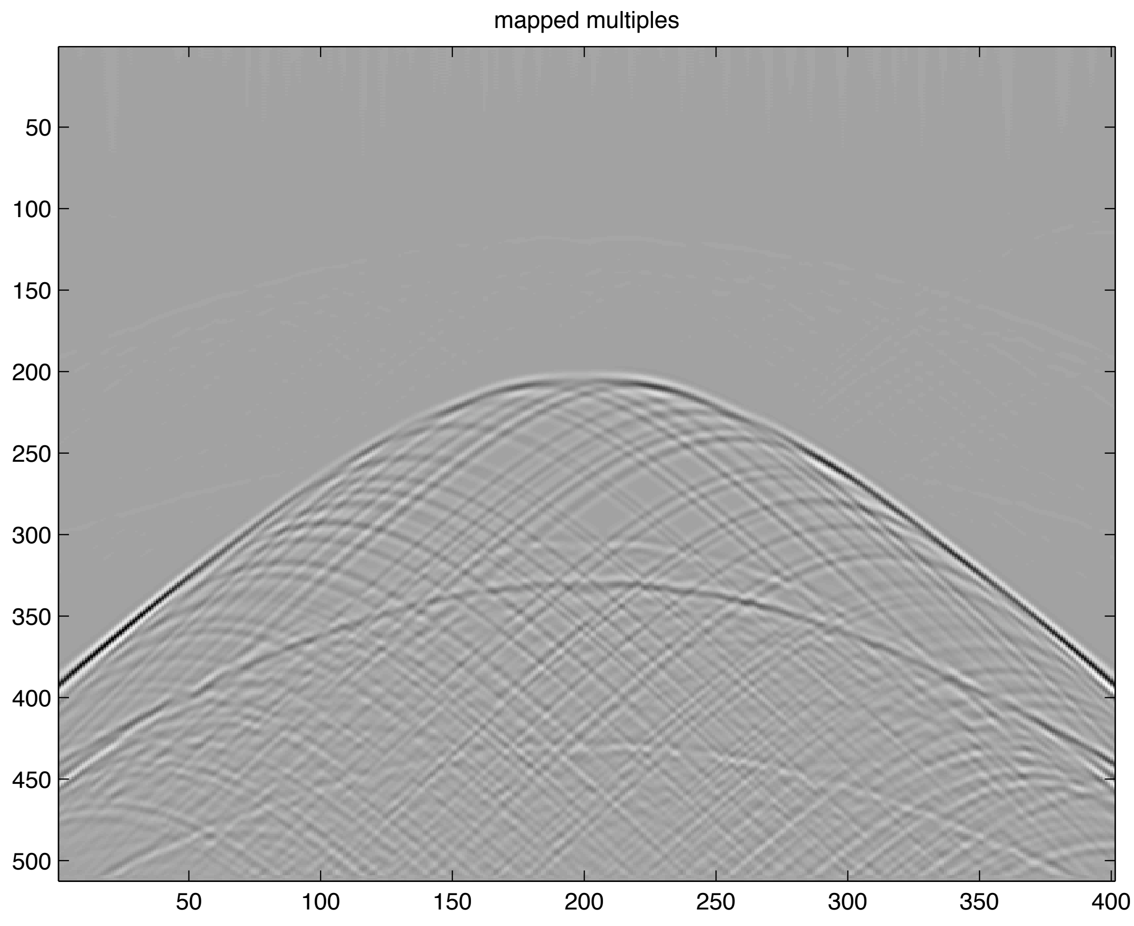

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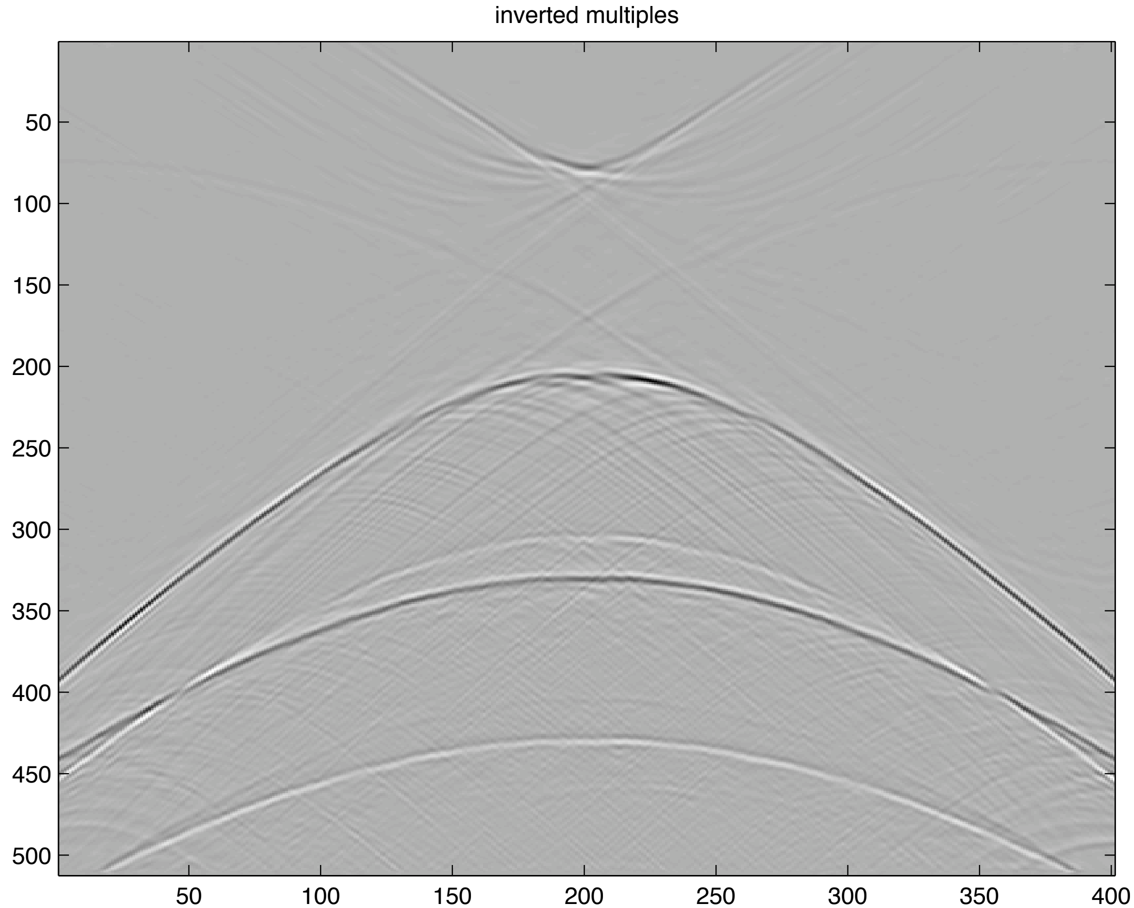

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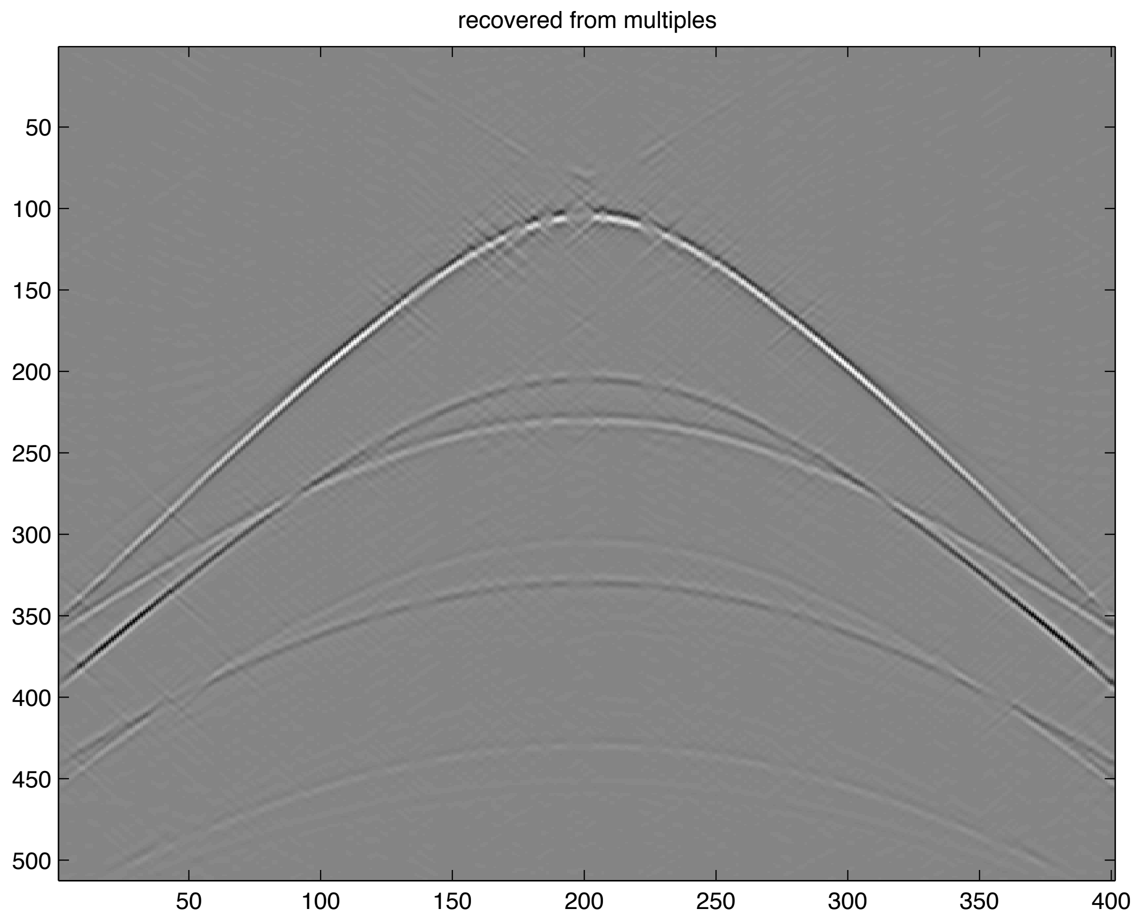
recovered from multiples
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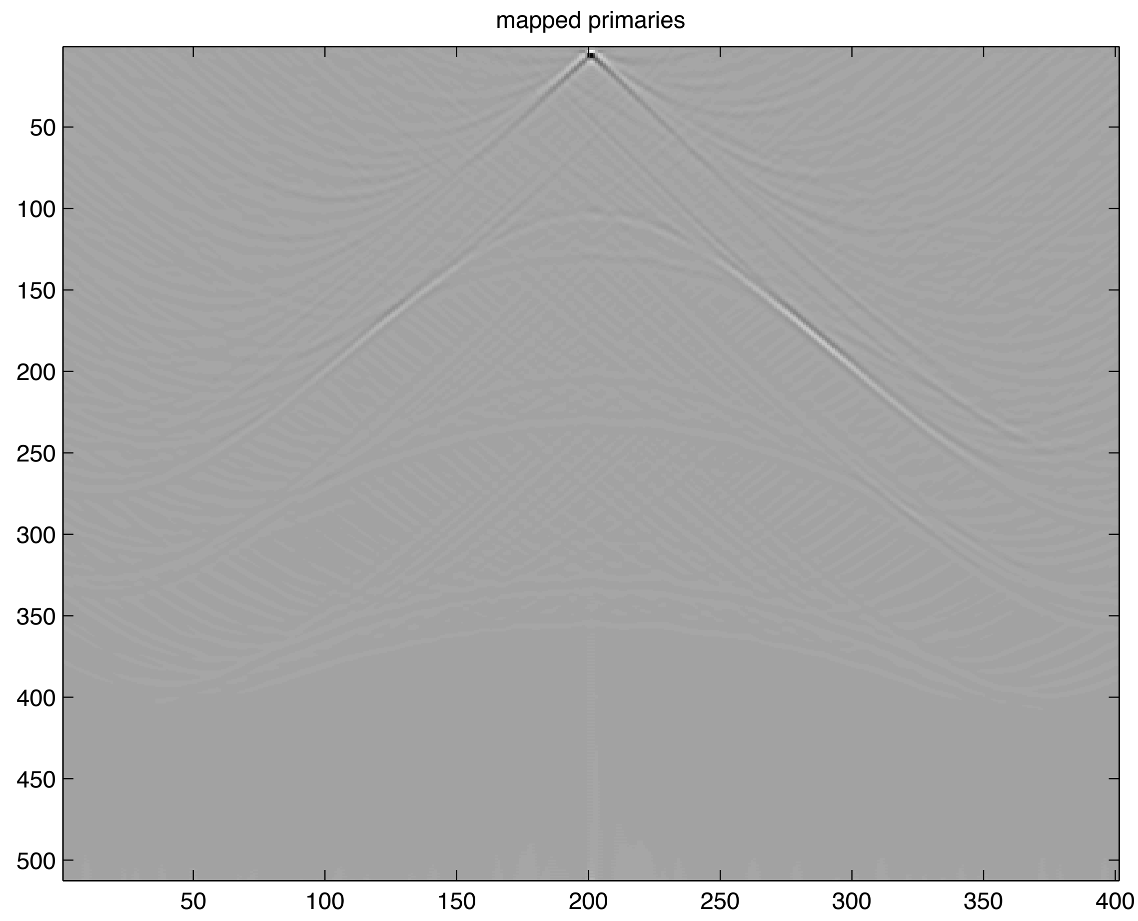

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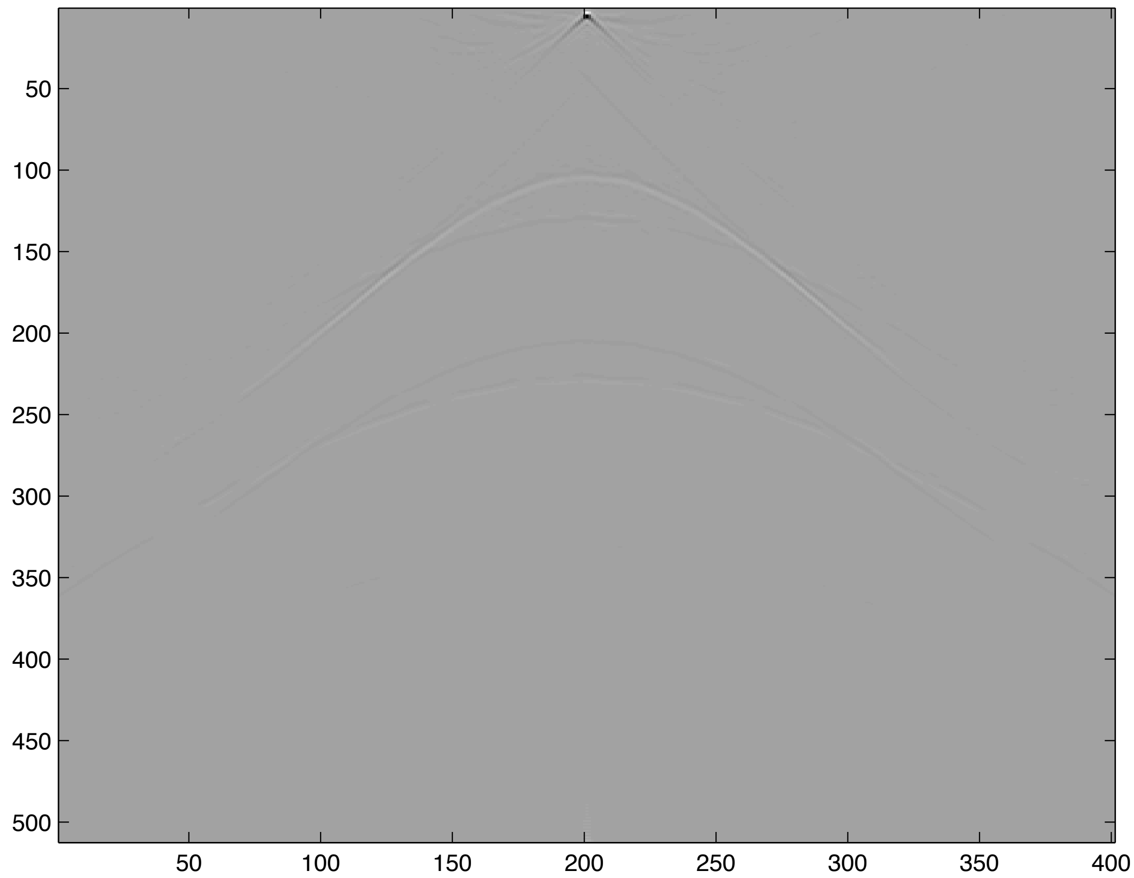
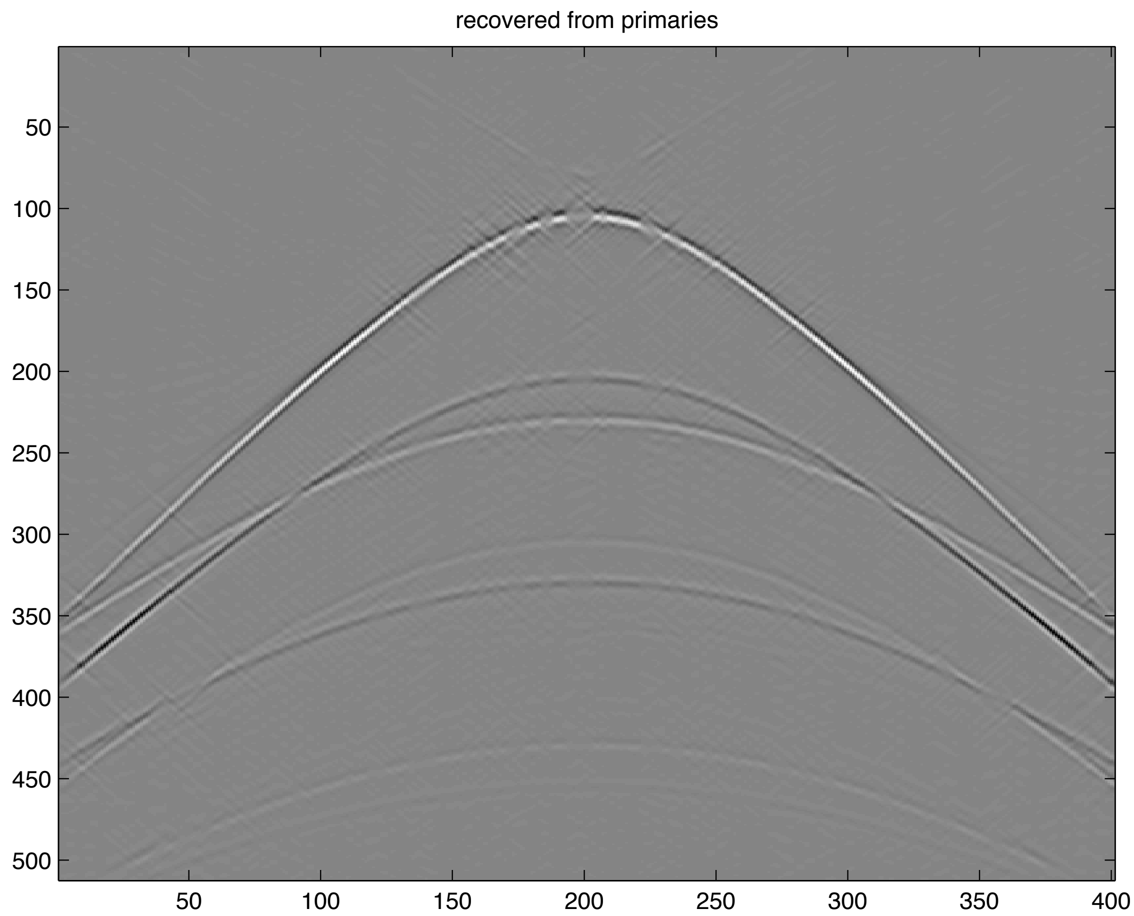
inverted primaries





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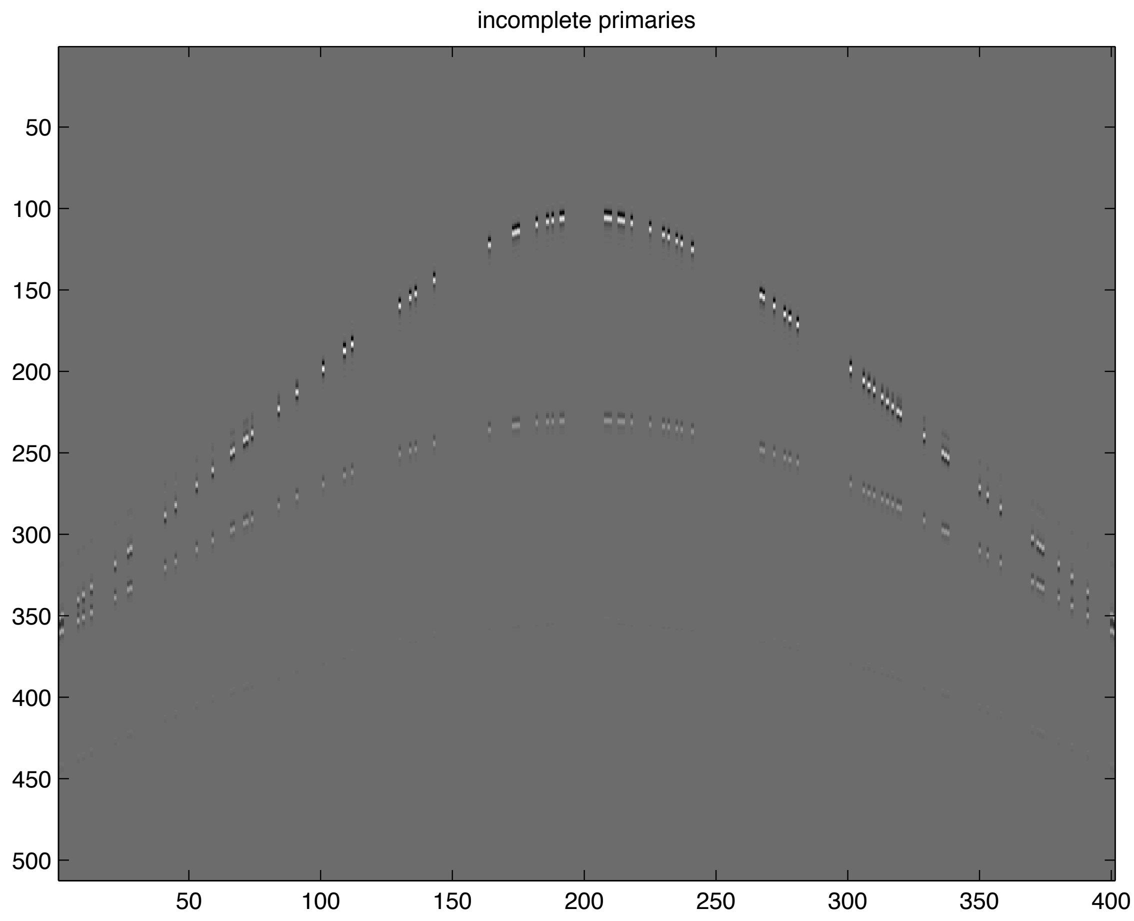

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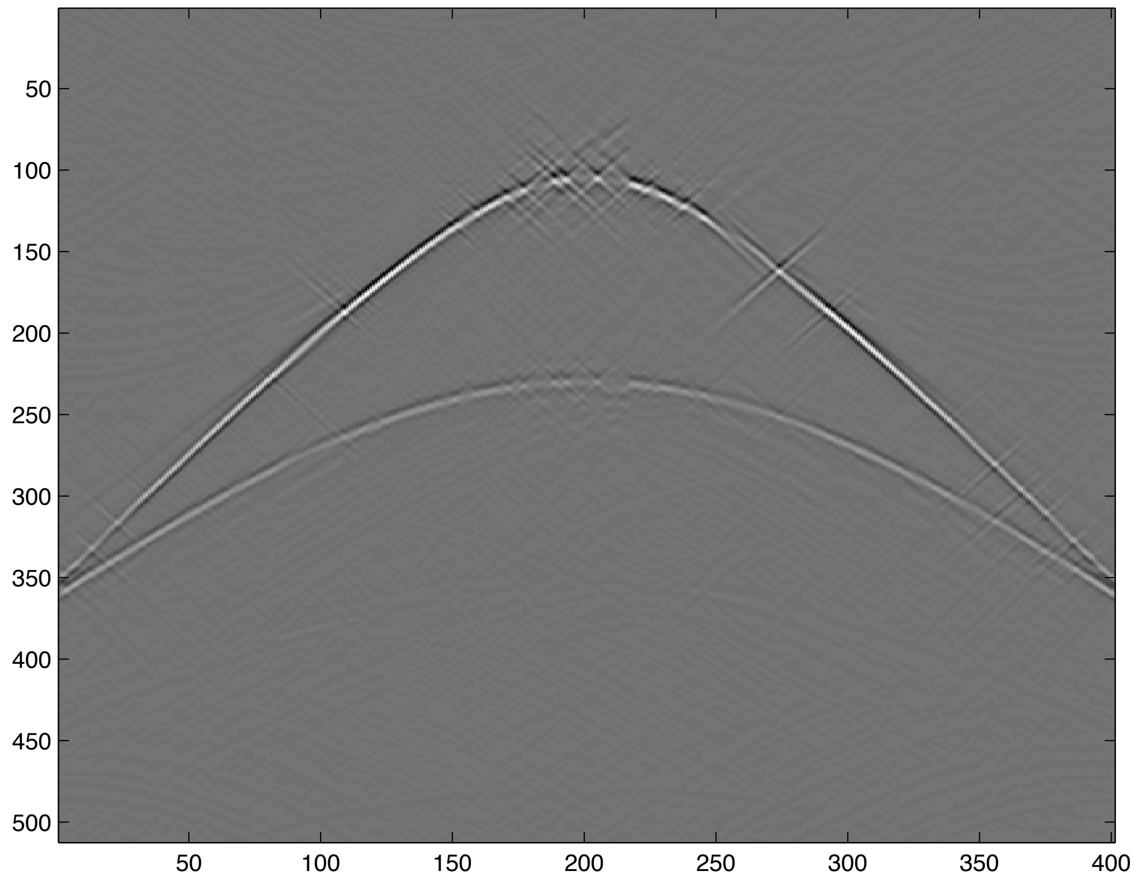
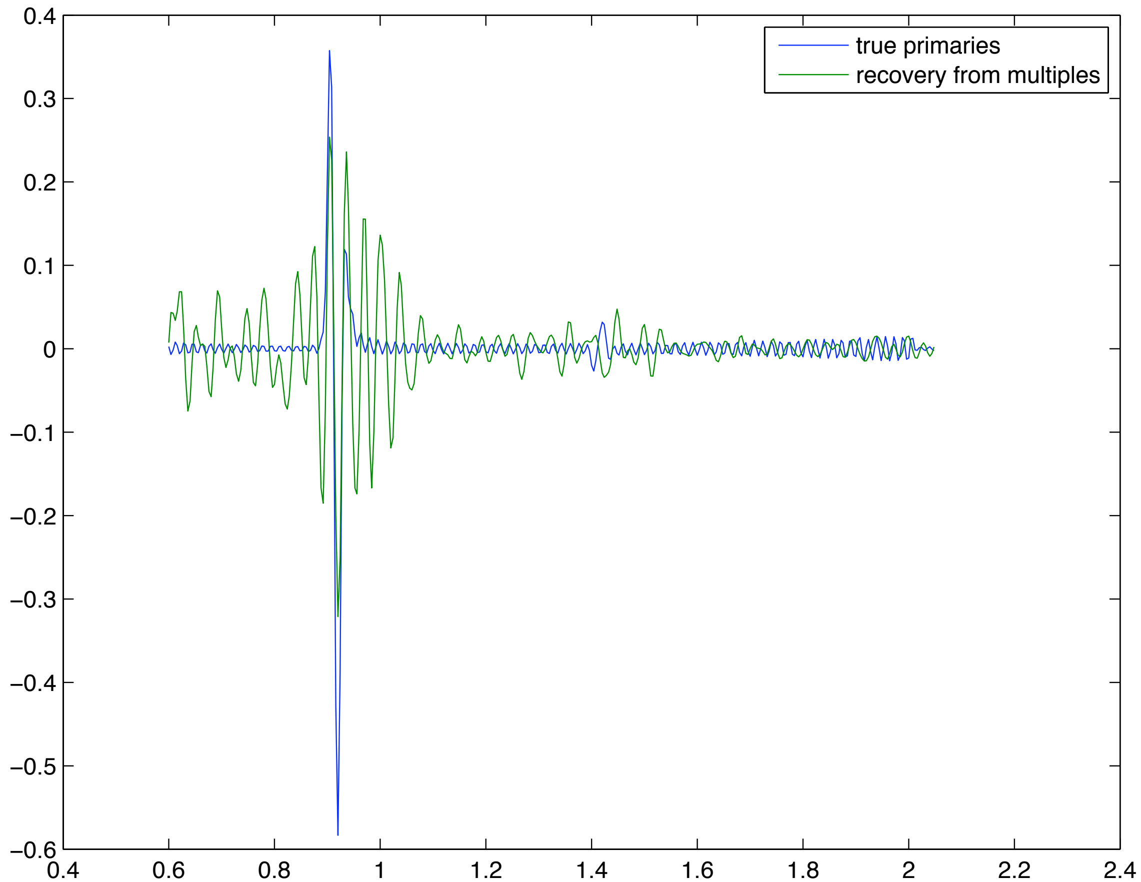
recovered from multiples





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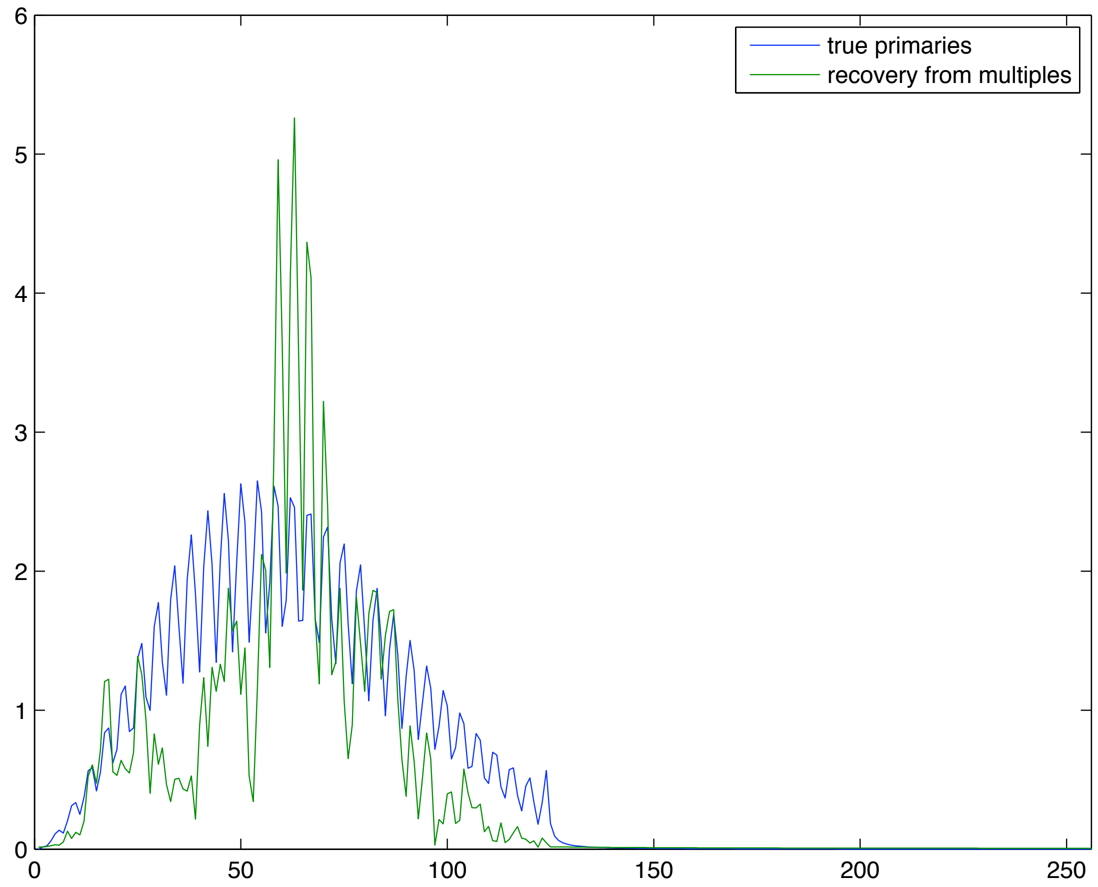

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Tentative observations

Exploiting sparsity omits "inversion" of acquisition.

Inverting the cross-convolution enhances sparsity.

Recovery is uplifted by the physics of focussing.

Opens the perspective of formulating a program

- stably separates data into its different constituents
- recovers missing data with transform-based techniques compounded with the physics of cross-correlations and cross-convolutions

Opens the pathway towards nonlinear inversion.

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