# Nonuniform Fast Discrete Curvelet Transform

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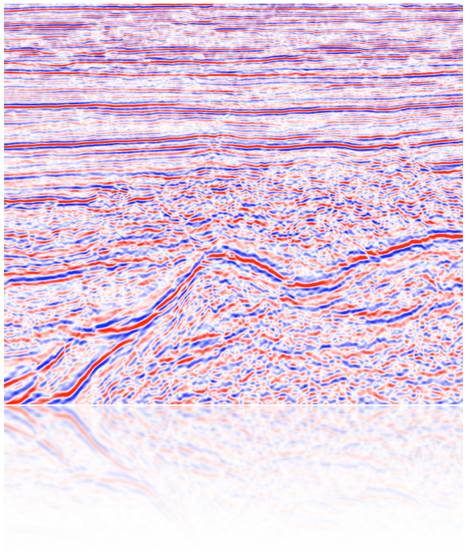
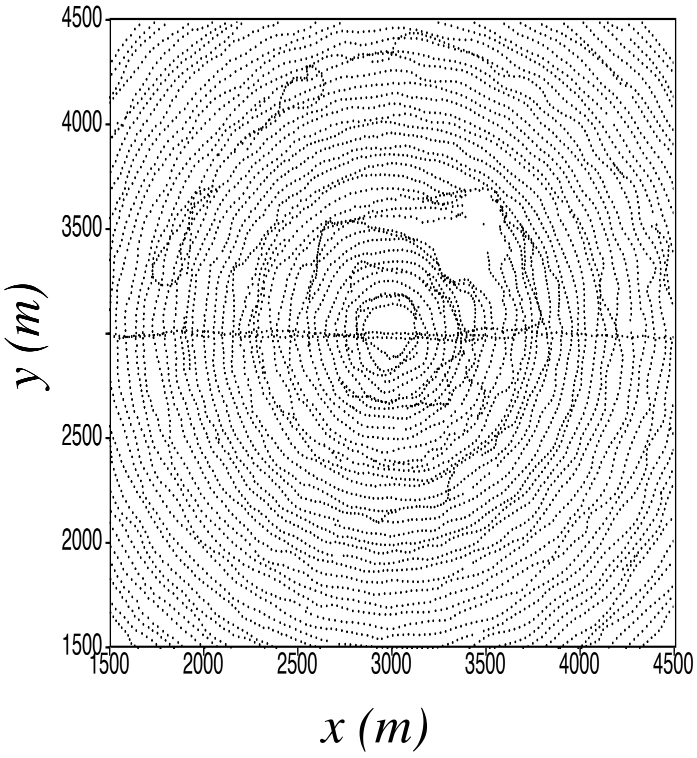


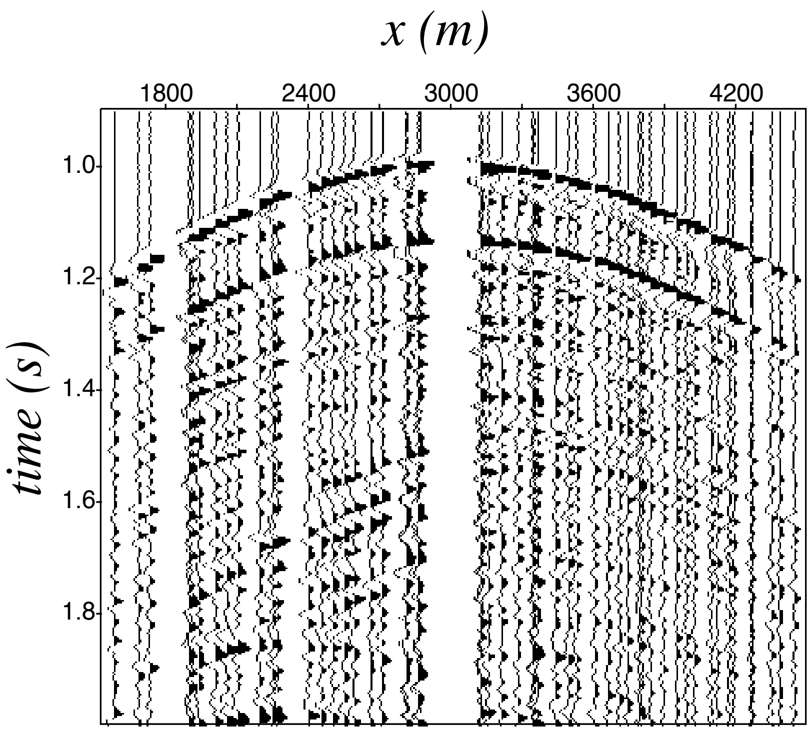
SINBAD meeting: Recent results & future directions
August 29, 2006

Figures courtesy P. Zwartjes

# Motivation

- improve processing of non-uniformly sampled seismic data
  - binning
  - interpolation
  - multiple prediction & removal





# Approach

- look at seismic data in a **geometrically correct** way
  - high dimensional
    - typically 5D - i.e. time × source location × receiver location
  - typically non-uniformly sampling only along spatial axes
  - very strong geometrical structure (i.e. wavefronts)

# References

- G. Hennenfent, F. Herrmann, “*Seismic denoising with non-uniformly sampled curvelets*”, 2006.
- H. Rauhut, S. Kunis, “*Random Sampling of Sparse Trigonometric Polynomials*”, 2006.
- D. Potts, G. Steidl, M. Tasche. “*Fast Fourier transforms for nonequispaced data: A tutorial*”, 2001.

# Agenda

- Nonuniform Fast Fourier Transform (NFFT)
  - successful applications to interpolation by A. Duijndam, M. Sacchi, M. Schonewille, and P. Zwartjes among others
  - Fourier not best suited for seismic data

- Fast Discrete Curvelet Transform (FDCT)
  - successful applications to interpolation by G. Hennenfent, and F. Herrmann
  - assumes regular sampling

- Nonuniform FDCT (NFDCT)
  - NFDCT 1.0
  - NFDCT 2.0

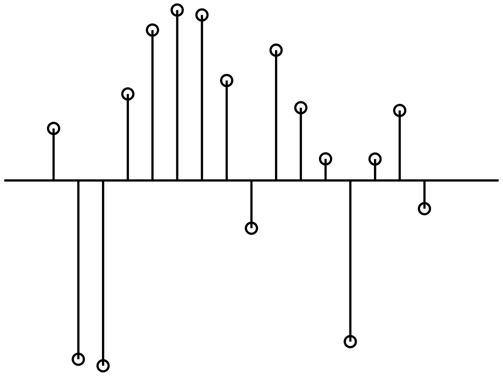
Figures courtesy S. Kunis

# Direct & adjoint NDFT

- direct NDFT
  - given the Fourier coefficients $\hat{f}_k$

$$f_j = f(x_j) := \sum_k \hat{f}_k \mathrm{e}^{-2\pi i k x_j}$$

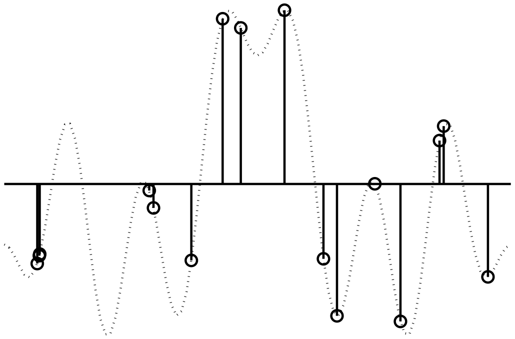
$$\mathbf{f} = \mathbf{A}\hat{\mathbf{f}}$$



- adjoint NDFT
  - given the value of $f$ at the non equispaced points $x_j$

$$\hat{f}_k = \sum_j f_j \mathrm{e}^{2\pi i k x_j}$$

$$\hat{\mathbf{f}} = \mathbf{A}^H \mathbf{f}$$



# Inverse NDFT

- equispaced sampling points

$$\mathbf{A}\mathbf{A}^H = \mathbf{A}^H\mathbf{A} = \mathbf{I}$$

- non-uniformly sampled points
  - given the value of $f$ at the non equispaced points $x_j$ , find $\hat{f}_k$ such that

$$\mathbf{A}\hat{\mathbf{f}} \approx \mathbf{f}$$

- standard method: Moore-Penrose pseudo-inverse $\mathbf{A}^\dagger$
  - over-determined: weighted approximation problem (compensate for sampling clusters)

$$\|\mathbf{f} - \mathbf{A}\hat{\mathbf{f}}\|_{\mathbf{W}} = \sqrt{\sum_j w_j |f_j - f(x_j)|^2}$$

  - under-determined: damped minimization problem (favour smooth solution)

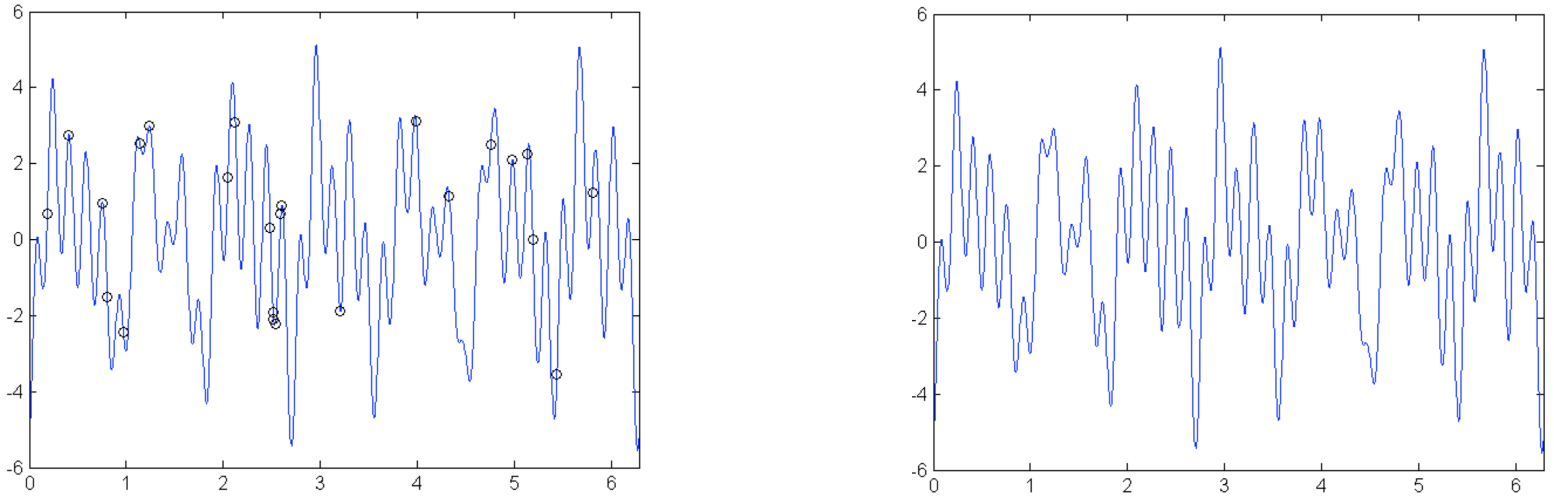
$$\min \|\hat{\mathbf{f}}\|_{\hat{\mathbf{W}}^{-1}} = \sqrt{\sum_k \hat{w}^{-1} |\hat{f}_k|^2} \quad \text{s.t.} \quad \mathbf{A}\hat{\mathbf{f}} = \mathbf{f}$$

Figures courtesy H. Rauhut

# Inverse NDFT by $l_1$ minimization

- assume $f$ has a sparse Fourier representation...

$$\min \|\hat{\mathbf{f}}\|_1 \quad \text{s.t.} \quad \mathbf{A}\hat{\mathbf{f}} = \mathbf{f}$$



<u>key facts</u>:

- *S = 6*
- ~ *25* observations required to <u>perfectly</u> recover
- more than 1:3 underdetermined

# Curvelets

- defined in the FK domain

- FDCT
  - construction

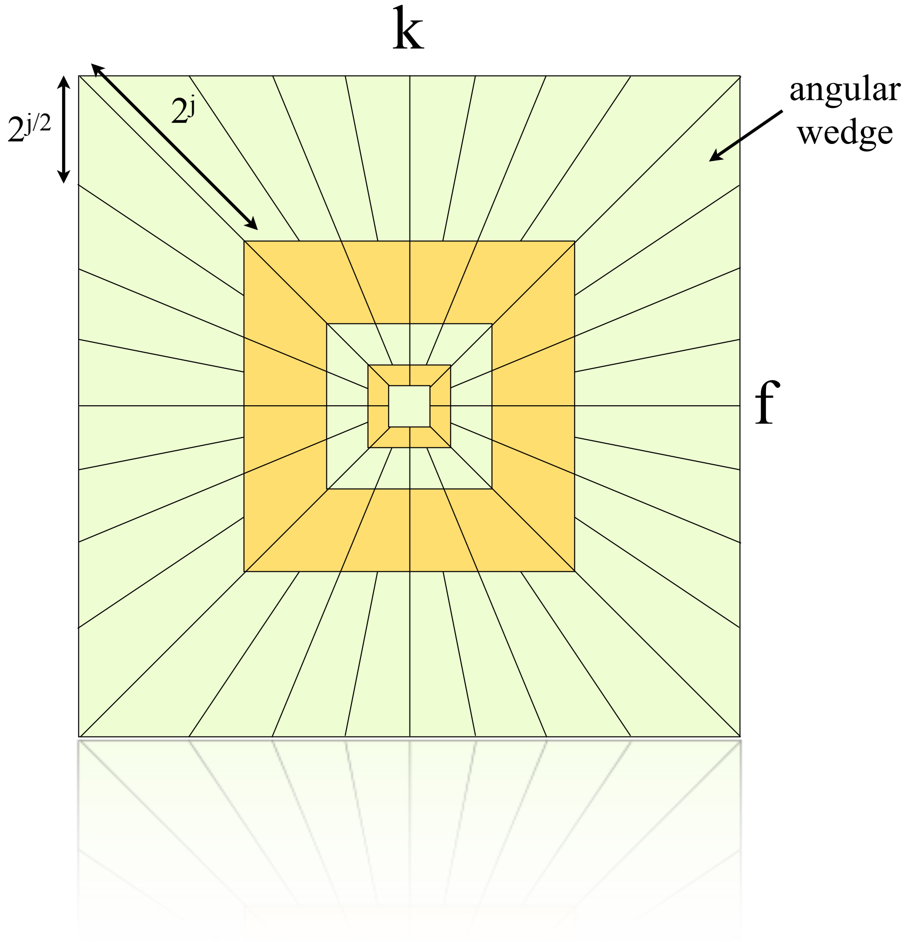
$$\mathbf{C} = \mathbf{TF}$$

*curvelet transform*

*curvelet tiling*

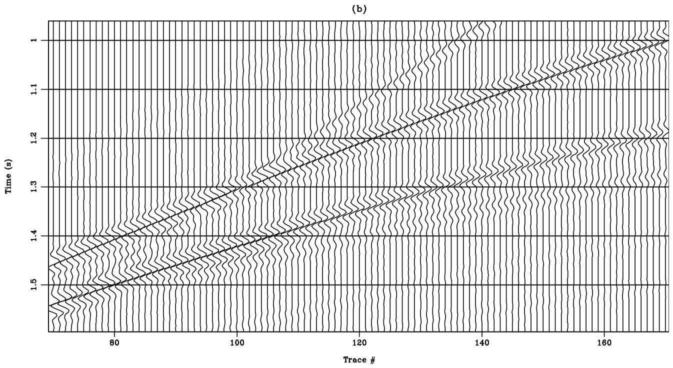
*Fourier transform*

  - **assumes regular sampling**

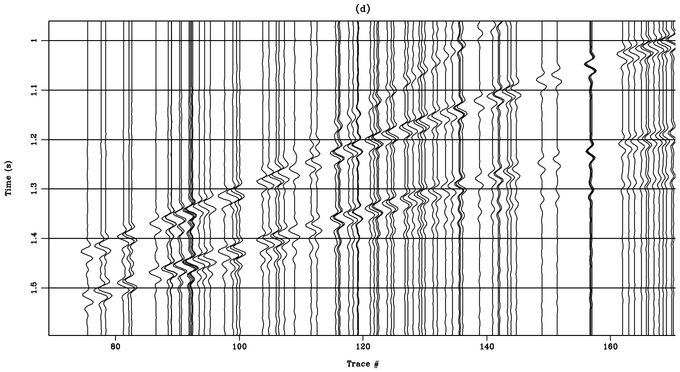


# Wavefronts & sampling

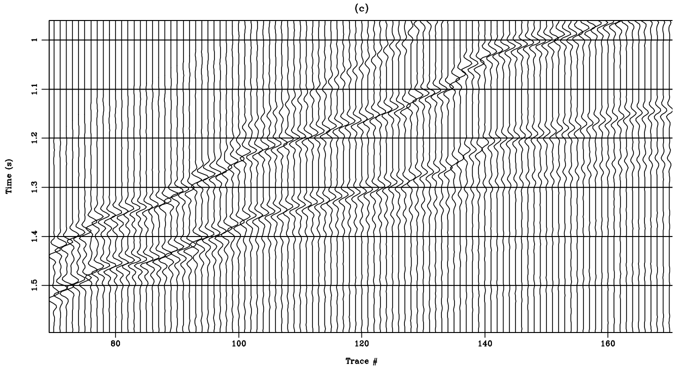
- regular sampling
  - continuity along wavefronts



- irregular sampling
  - continuity along wavefronts



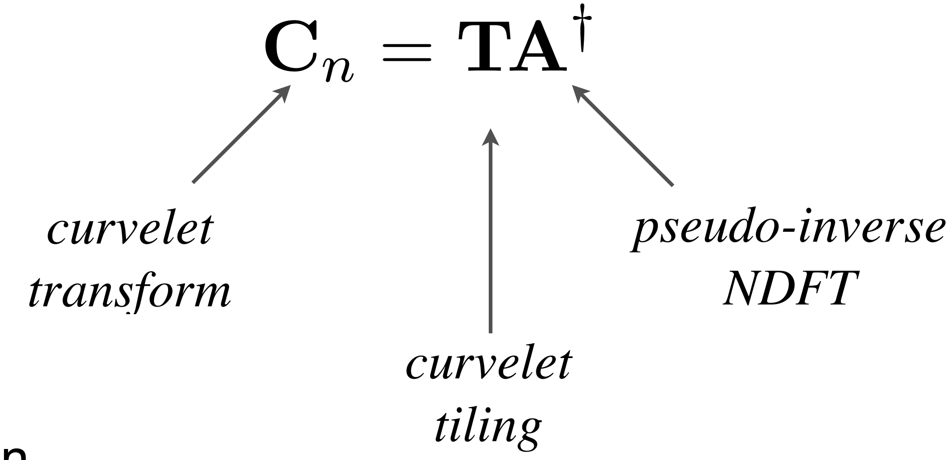
  - broken continuity along wavefronts



# NFDCT 1.0

- combination of FDCT & NFFT
  - handle irregular sampling (over-determined problem)
  - explore continuity along wavefronts

- construction



$$\mathbf{C}_n = \mathbf{T}\mathbf{A}^{\dagger}$$

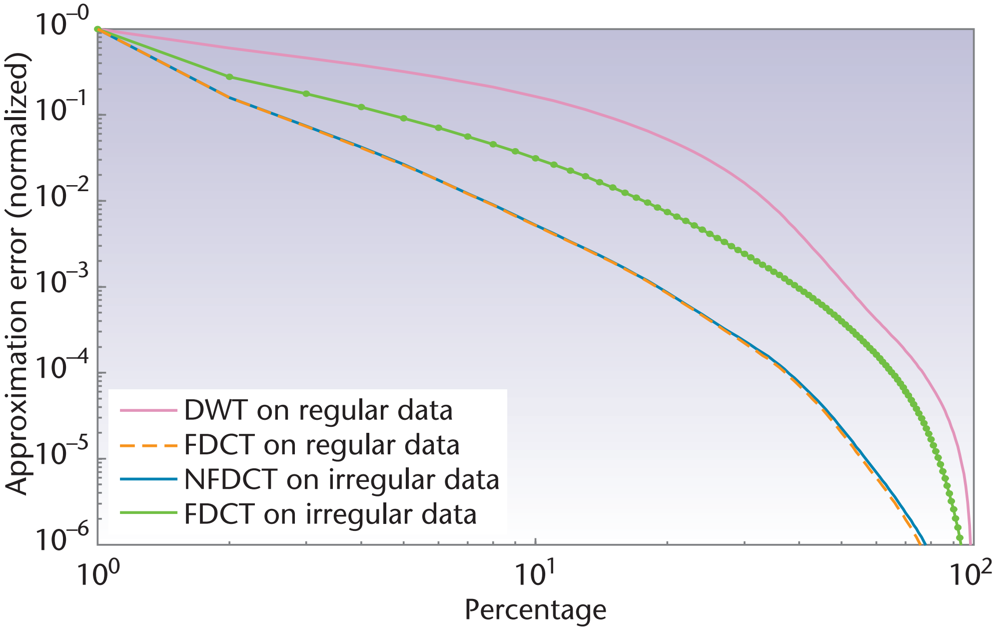
*curvelet transform*

*curvelet tiling*

*pseudo-inverse NDFT*

- regularization
  - in the Fourier domain

# NFDCT 1.0 - performance



- FDCT on regular data (reference)
    - curvelets explore continuity along wavefronts
    - best compression

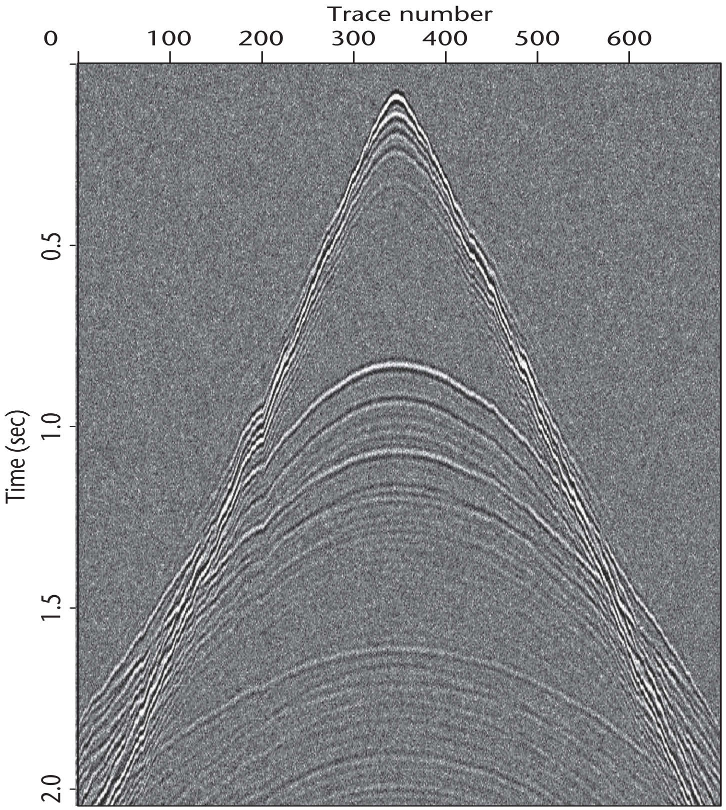
- FDCT on irregular data
    - continuity along wavefronts broken
    - loss in performance

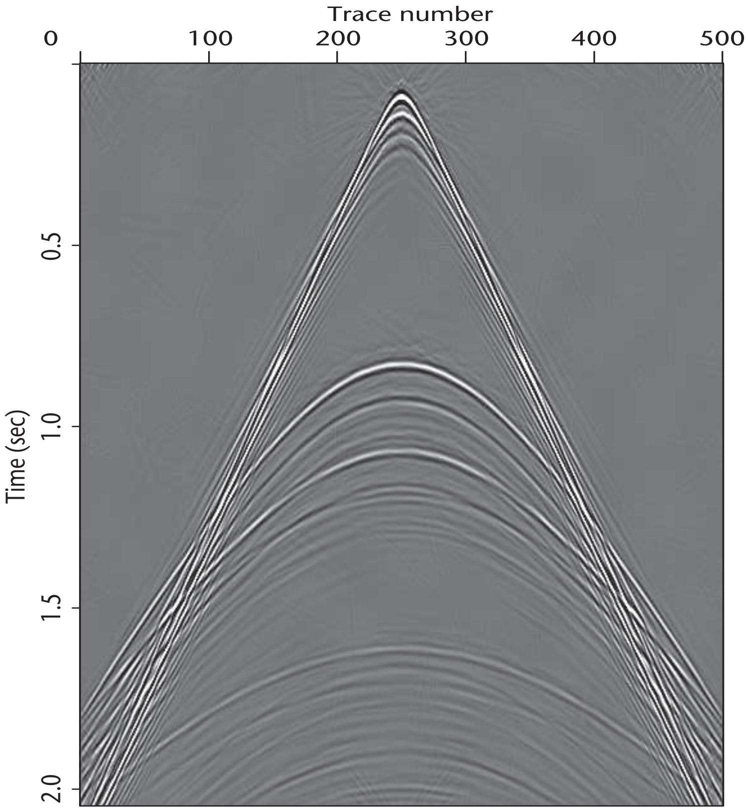
- NFDCT on irregular data
    - continuity along wavefronts restored
    - performance on irregular data to the same level as FDCT on regular data

# NFDCT 1.0 - binning & denoising

$$\tilde{\mathbf{m}} = \mathbf{C}^H S_{\mathbf{w}}(\mathbf{C}_n d)$$

| Operations performed | Signal-to-noise ratio (dB) |
|---|---|
| Linear binning | –1.96 |
| Nonequally sampled fast Fourier transform (NFFT) binning | 9.04 |
| Denoising | 13.35 |
| NFFT binning and denoising | 8 |





# NFDCT 2.0

- combination of FDCT & NFFT
  - handle irregular sampling
  - explore continuity along wavefronts

- construction

$$\mathbf{C}_n = \mathbf{T}\mathbf{A}^H$$

*curvelet transform*

*adjoint NDFT*

*curvelet tiling*

- regularization
  - in the curvelet domain

# CnRSI using NFDCT 2.0

- find the **sparsest set of curvelet coefficients** that explains the irregular sampled data

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{W}\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{C}_n\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

- construct the interpolated result from its curvelet representation

$$\tilde{\mathbf{f}} = \mathbf{C}^H\tilde{\mathbf{x}}$$

# Conclusions

- NFDCT is the combination of NFFT & FDCT
  - handle irregular sampling
  - explore continuity along wavefronts

- NFDCT has comparable performance on irregular data to FDCT on regular data

- NFDCT 2.0 opens the path to CnRSI

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