Time-domain Wavefield Reconstruction Inversion for large-scale seismic inversion

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SLIM

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Agenda

- Motivations:
  robustness and computation
- Theory:
  FWI vs WRI vs WRI*
- Numerical experiments:
  acoustic, TTI, small 3D
- Discussion and conclusions

Full Waveform Inversion

$$\min_{\mathbf{m}} \mathcal{J}_{\mathrm{FWI}}(\mathbf{m}) = \frac{1}{2}||F(\mathbf{m})\mathbf{q} - \mathbf{d}||^2, \quad F(\mathbf{m}) = RA(\mathbf{m})^{-1}$$

[Tarantola, A., ‘84; Haber, E., et al, 2000; Epanomeritakis, I., et al, 2008]

✔ **3D** computations are affordable
via large HPC systems (or cloud computing) [Witte et al., 2019]

✗ (Effectively) **multimodal** problem:
it needs a good starting model!

Wavefield Reconstruction Inversion

$$\min_{\mathbf{m}} \mathcal{J}_{\mathrm{WRI}}(\mathbf{m}, \bar{\mathbf{u}}(\mathbf{m})) = \frac{1}{2}||R\bar{\mathbf{u}} - \mathbf{d}||^2 + \frac{\lambda^2}{2}||A(\mathbf{m})\bar{\mathbf{u}} - \mathbf{q}||^2$$

[van den Berg, P. M., and Kleinman, R. E., 1997; van Leeuwen, T. and Herrmann, F. J., 2013]

✔ Better **conditioning**

Wavefield Reconstruction Inversion

$$\min_{\mathbf{m}} \mathcal{J}_{\mathrm{WRI}}(\mathbf{m}, \bar{\mathbf{u}}(\mathbf{m})) = \frac{1}{2}||R\bar{\mathbf{u}} - \mathbf{d}||^2 + \frac{\lambda^2}{2}||A(\mathbf{m})\bar{\mathbf{u}} - \mathbf{q}||^2$$

[van den Berg, P. M., and K ... and Herrmann, F. J., 2013]

$$\begin{bmatrix} R \\ \lambda A(\mathbf{m}) \end{bmatrix} \bar{\mathbf{u}} = \begin{bmatrix} \mathbf{d} \\ \lambda \mathbf{q} \end{bmatrix}$$

augmented wave equation

✔ Better **conditioning**

✗ Augmented solver: hard to scale to **3D**
(explicit time-marching schemes?)

Prior art: extended-source formulation

$$\min_{\mathbf{m}} \mathcal{J}_{\mathrm{WRI}}(\mathbf{m}, \bar{\mathbf{u}}) = \frac{1}{2}||R\bar{\mathbf{u}} - \mathbf{d}||^2 + \frac{\lambda^2}{2}||A(\mathbf{m})\bar{\mathbf{u}} - \mathbf{q}||^2$$

[van den Berg, P. M., and Kleinman, R. E., 1997; van Leeuwen, T. and Herrmann, F. J., 2013]

=

$$\min_{\mathbf{m}} \mathcal{J}_{\mathrm{WRI-q}}(\mathbf{m}, \bar{\mathbf{q}}) = \frac{1}{2}||F(\mathbf{m})\bar{\mathbf{q}} - \mathbf{d}||^2 + \frac{\lambda^2}{2}||\bar{\mathbf{q}} - \mathbf{q}||^2$$

[Wang et al, 2016, Huang et al, 2018]

Prior art: augmented state approximation

$$\min_{\mathbf{m}} \mathcal{J}_{\mathrm{WRI-q}}(\mathbf{m}, \bar{\mathbf{q}}) = \frac{1}{2}||F(\mathbf{m})\bar{\mathbf{q}} - \mathbf{d}||^2 + \frac{\lambda^2}{2}||\bar{\mathbf{q}} - \mathbf{q}||^2$$

[Wang et al, 2016]

$$\begin{bmatrix} F(\mathbf{m}) \\ \lambda I \end{bmatrix} \bar{\mathbf{q}} = \begin{bmatrix} \mathbf{d} \\ \lambda \mathbf{q} \end{bmatrix} \quad \Longrightarrow \quad \bar{\mathbf{q}} \approx \mathbf{q} + F(\mathbf{m})^{\mathrm{H}}(\mathbf{d} - F(\mathbf{m})\mathbf{q})/\lambda^2$$

✔(Approximated) augmented solver: scale to **3D**

Prior art: augmented state approximation

$$\min_{\mathbf{m}} \mathcal{J}_{\mathrm{WRI-q}}(\mathbf{m}, \bar{\mathbf{q}}) = \frac{1}{2}||F(\mathbf{m})\bar{\mathbf{q}} - \mathbf{d}||^2 + \frac{\lambda^2}{2}||\bar{\mathbf{q}} - \mathbf{q}||^2$$

[Wang et al, 2016]

$$\nabla_{\mathbf{m}}\mathcal{J}_{\mathrm{WRI-q}} = \nabla_{\mathbf{m}}\mathcal{J}_{\mathrm{WRI-q}}(\mathbf{m}, \bar{\mathbf{q}}) + \cancel{\frac{\partial \bar{\mathbf{q}}}{\partial \mathbf{m}}^H \nabla_{\mathbf{q}}\mathcal{J}_{\mathrm{WRI-q}}(\mathbf{m}, \bar{\mathbf{q}})}$$

✘ Gradient computation inconsistency

WRI*: denoising reformulation

$$\min_{\mathbf{m},\mathbf{u}} \frac{1}{2}||A(\mathbf{m})\mathbf{u} - \mathbf{q}||^2 \quad \text{s.t.} \quad ||R\mathbf{u} - \mathbf{d}|| \leq \varepsilon$$

[Wang, R., and Herrmann, F. J., 2017]

WRI*: Lagrangian

$$\min_{\mathbf{m},\mathbf{u}} \frac{1}{2}||A(\mathbf{m})\mathbf{u}-\mathbf{q}||^2 \quad \text{s.t.} \quad ||R\mathbf{u}-\mathbf{d}|| \leq \varepsilon$$

[Wang, R., and Herrmann, F. J., 2017]

$$\max_{\mathbf{y}} \min_{\mathbf{m},\mathbf{u}} \mathcal{L}(\mathbf{m},\mathbf{u},\mathbf{y}) = \frac{1}{2}||A(\mathbf{m})\mathbf{u}-\mathbf{q}||^2 + \mathbf{y}\cdot(R\mathbf{u}-\mathbf{d}) - \varepsilon||\mathbf{y}||$$

WRI*: Lagrangian

$$\min_{\mathbf{m},\mathbf{u}} \frac{1}{2}||A(\mathbf{m})\mathbf{u}-\mathbf{q}||^2 \quad \text{s.t.} \quad ||R\mathbf{u}-\mathbf{d}|| \leq \varepsilon$$

[Wang, R., and Herrmann, F. J., 2017]

$$\max_{\mathbf{y}} \min_{\mathbf{m}} \bar{\mathcal{L}}(\mathbf{m},\mathbf{y}) = \frac{1}{2}||A(\mathbf{m})\boxed{\bar{\mathbf{u}}}-\mathbf{q}||^2 + \mathbf{y}\cdot(R\boxed{\bar{\mathbf{u}}}-\mathbf{d}) - \varepsilon||\mathbf{y}||$$

$$A(\mathbf{m})\bar{\mathbf{u}} = \mathbf{q} + F(\mathbf{m})^H\mathbf{y}$$

WRI*: augmented state approximation

$$\min_{\mathbf{m},\mathbf{u}} \frac{1}{2}||A(\mathbf{m})\mathbf{u}-\mathbf{q}||^2 \quad \text{s.t.} \quad ||R\mathbf{u}-\mathbf{d}|| \leq \varepsilon$$

[Wang, R., and Herrmann, F. J., 2017]

$$\min_{\mathbf{m}} \tilde{\bar{\mathcal{L}}}(\mathbf{m}) = \frac{1}{2}||A(\mathbf{m})\bar{\mathbf{u}}-\mathbf{q}||^2 + \boxed{\tilde{\mathbf{y}}} \cdot (R\bar{\mathbf{u}}-\mathbf{d}) - \varepsilon||\boxed{\tilde{\mathbf{y}}}||$$

$$\tilde{\mathbf{y}} \propto F(\mathbf{m})\mathbf{q}-\mathbf{d}$$

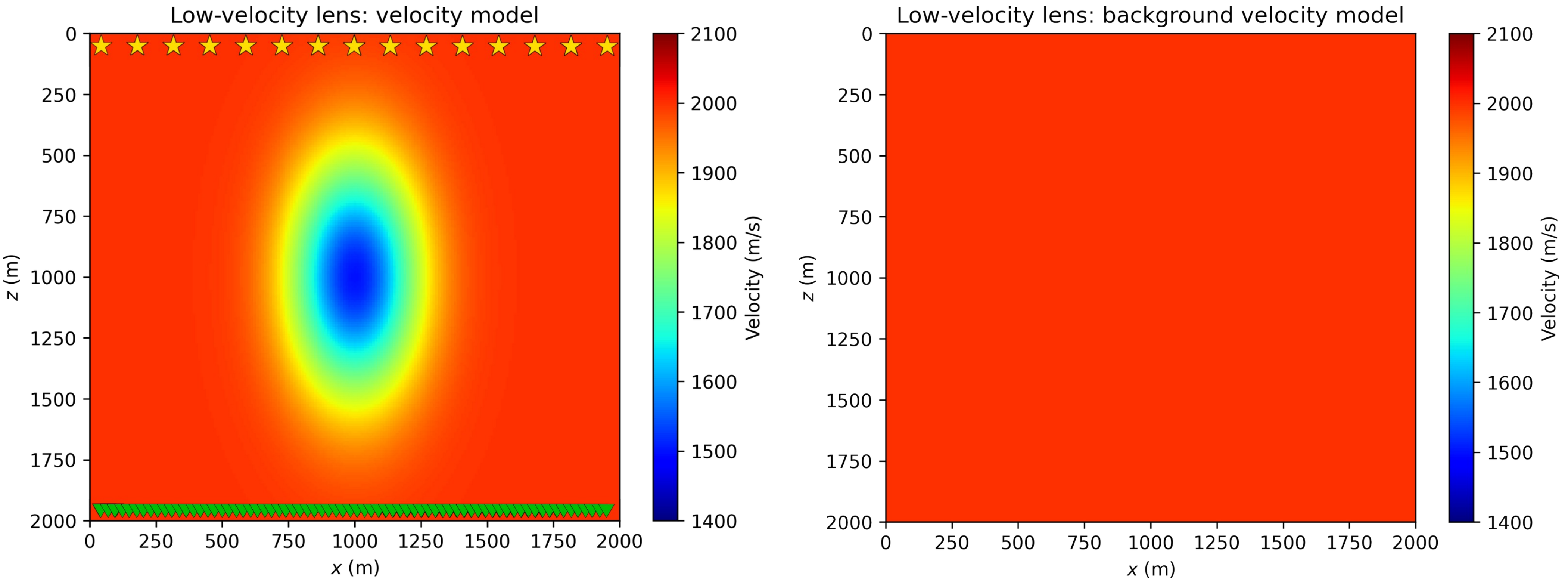
can be differentiated through...

WRI*: reduced formulation

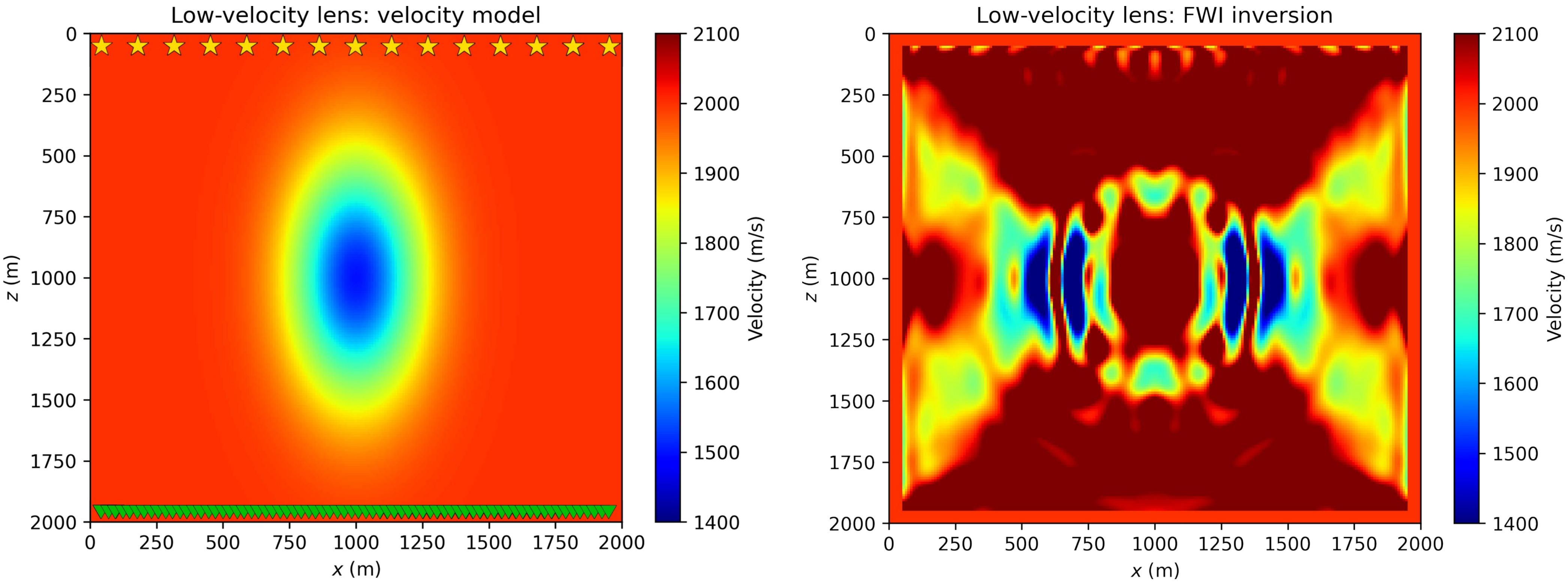
$$\min_{\mathbf{m}} \tilde{\tilde{\mathcal{L}}}(\mathbf{m}) = \frac{1}{2}||A(\mathbf{m})\bar{\mathbf{u}} - \mathbf{q}||^2 + \tilde{\mathbf{y}} \cdot (R\bar{\mathbf{u}} - \mathbf{d}) - \varepsilon||\tilde{\mathbf{y}}||$$

✔ Requires standard wave equation solver: scale to 3D
✔ No gradient computation inconsistency
✔ Retains WRI robustness
✔ Roughly equivalent to 2x the computational cost of FWI
✘ Roughly equivalent to 2x the computational cost of FWI
✘ Model resolution generally inferior to WRI

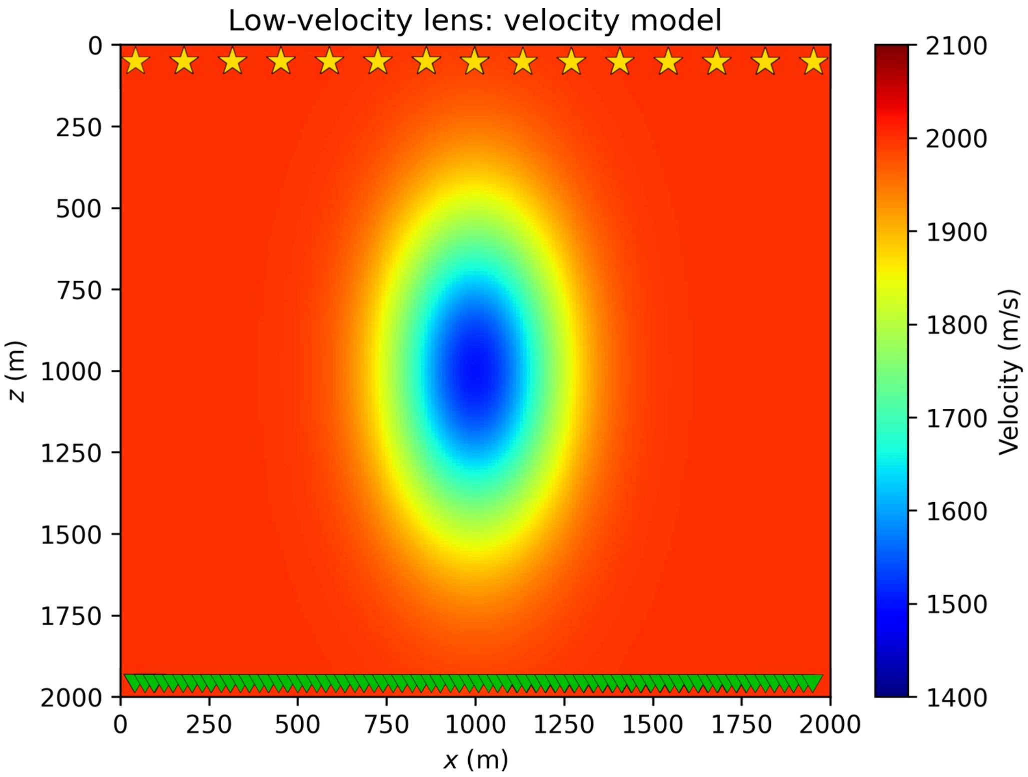
Numerical examples: transmission

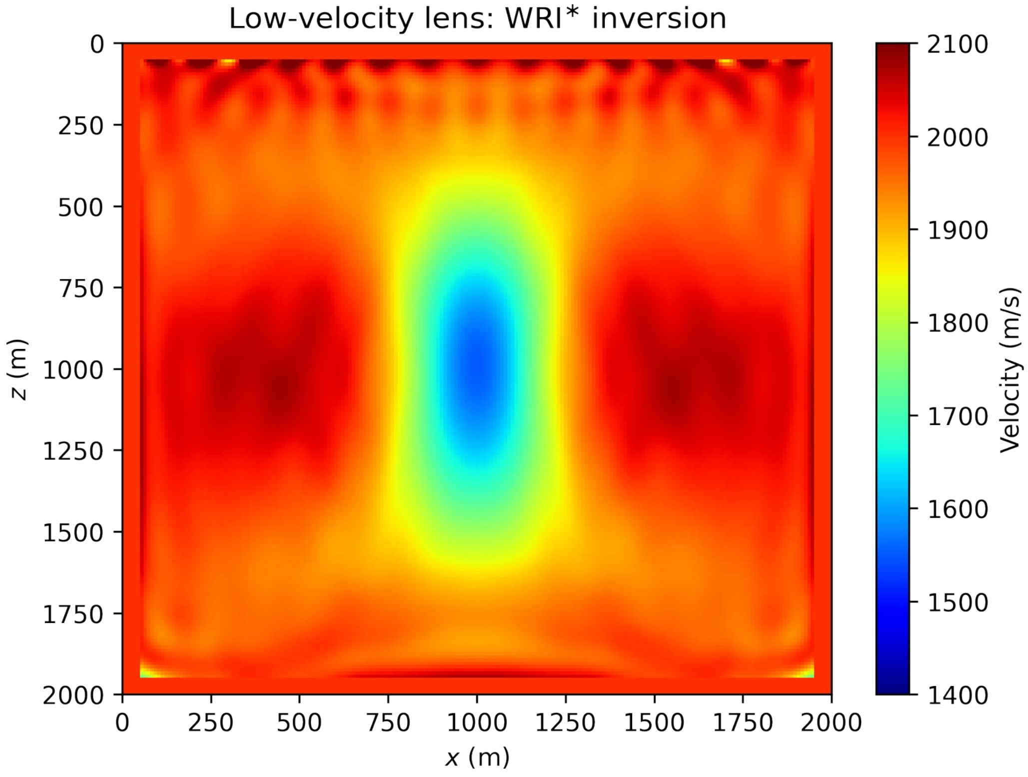


Numerical examples: transmission

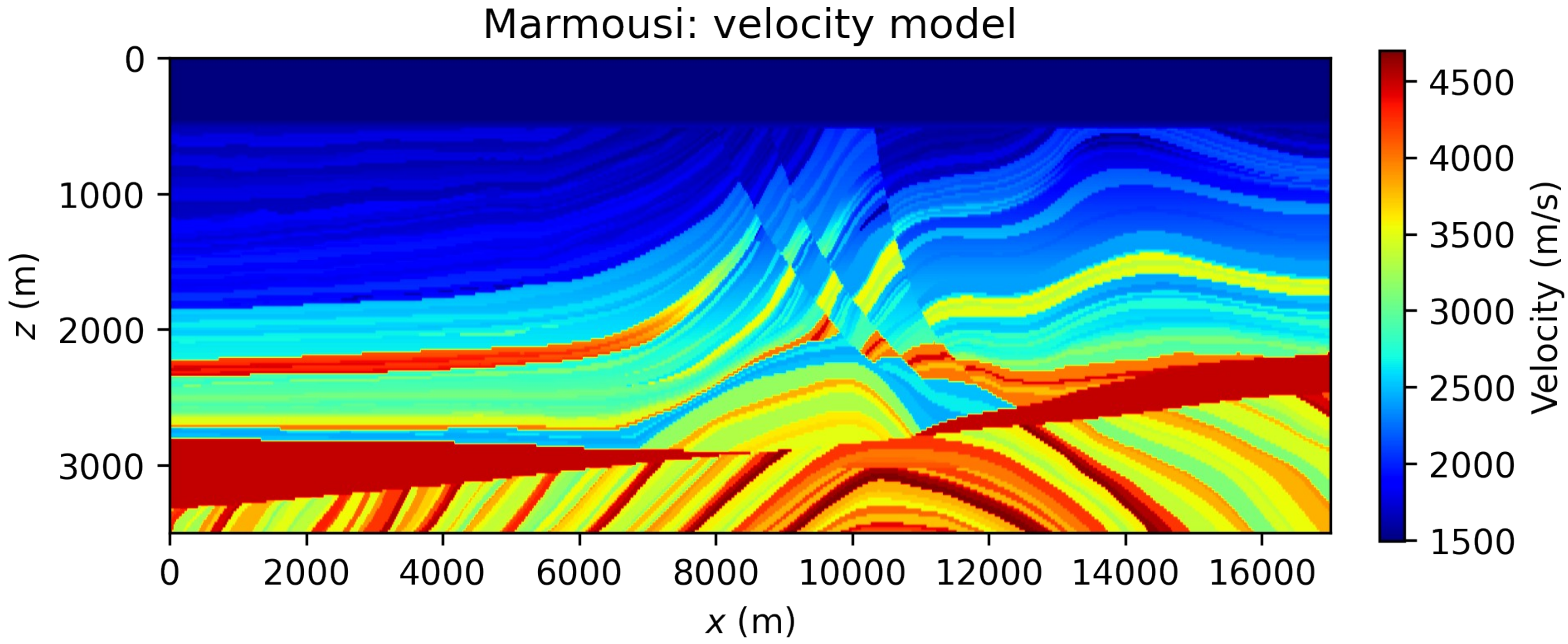


Numerical examples: transmission





Numerical examples: Marmousi (freq. domain)



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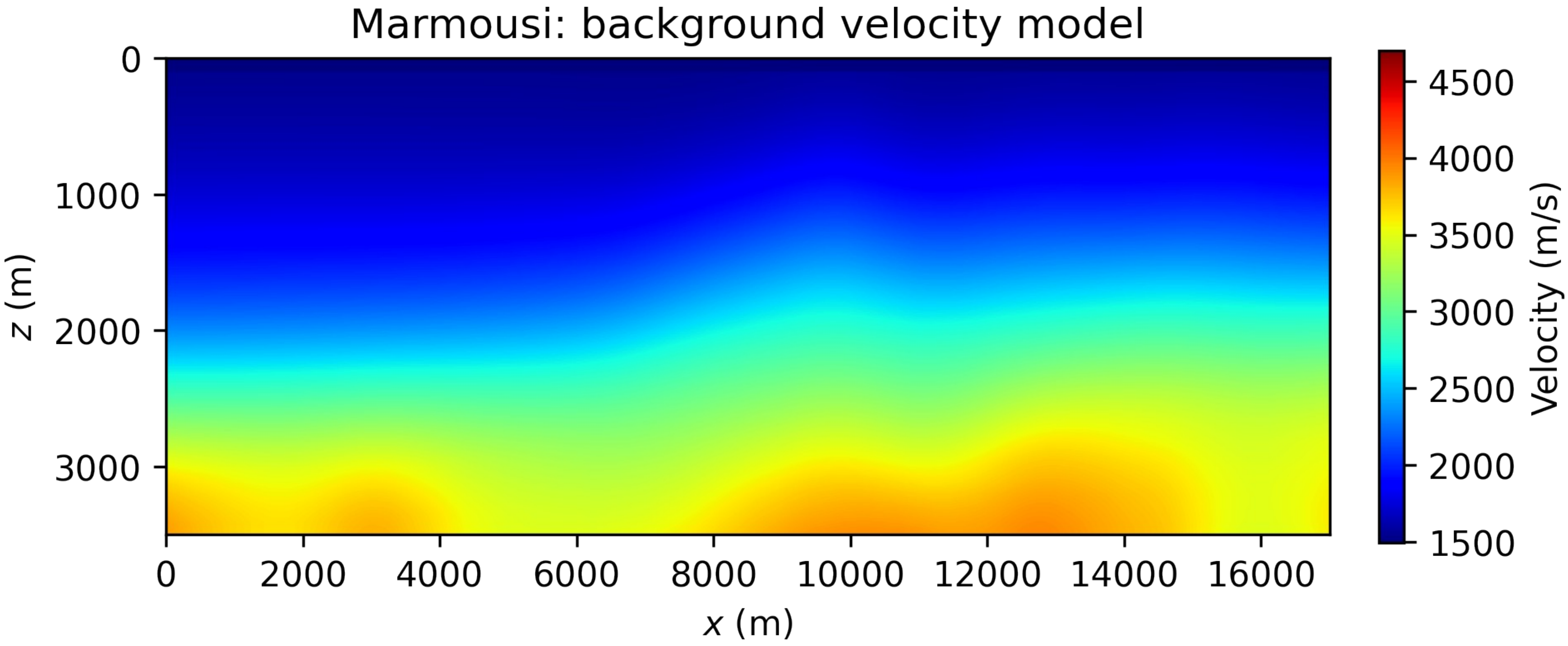

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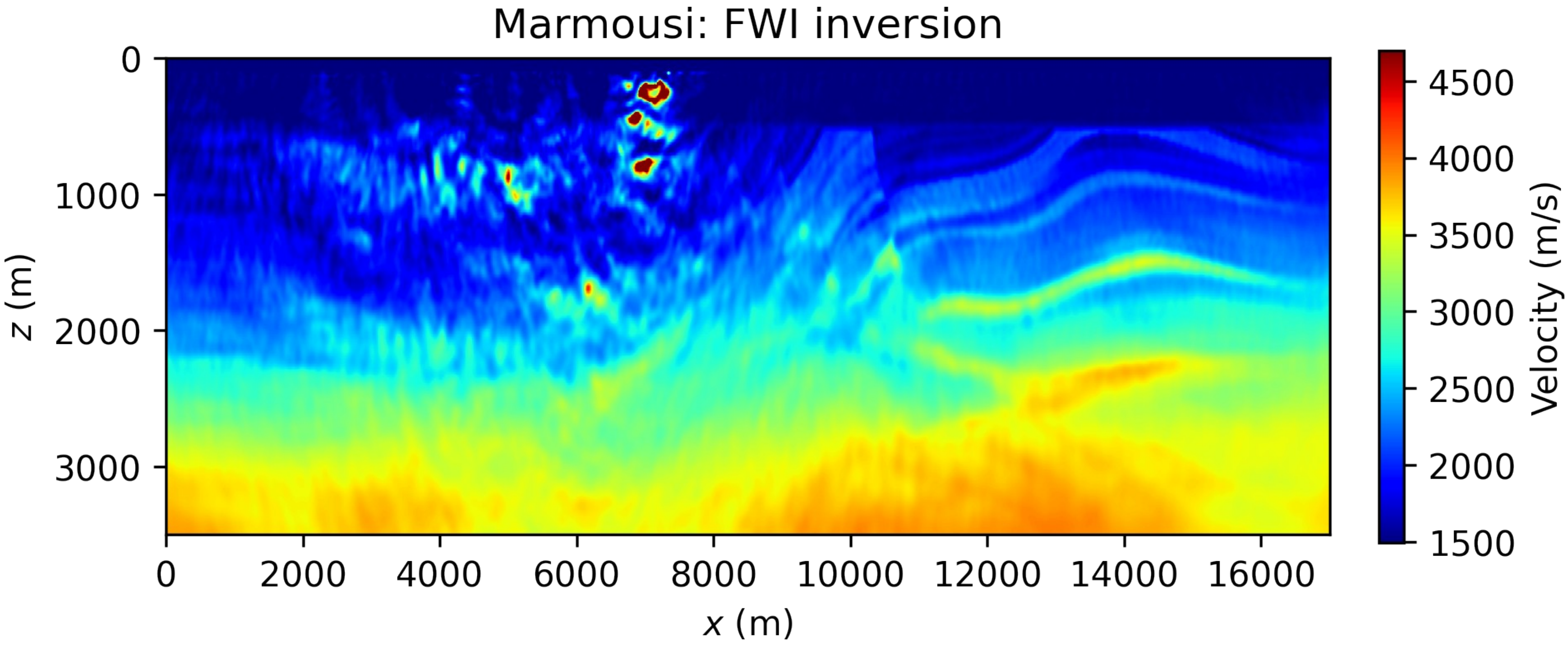
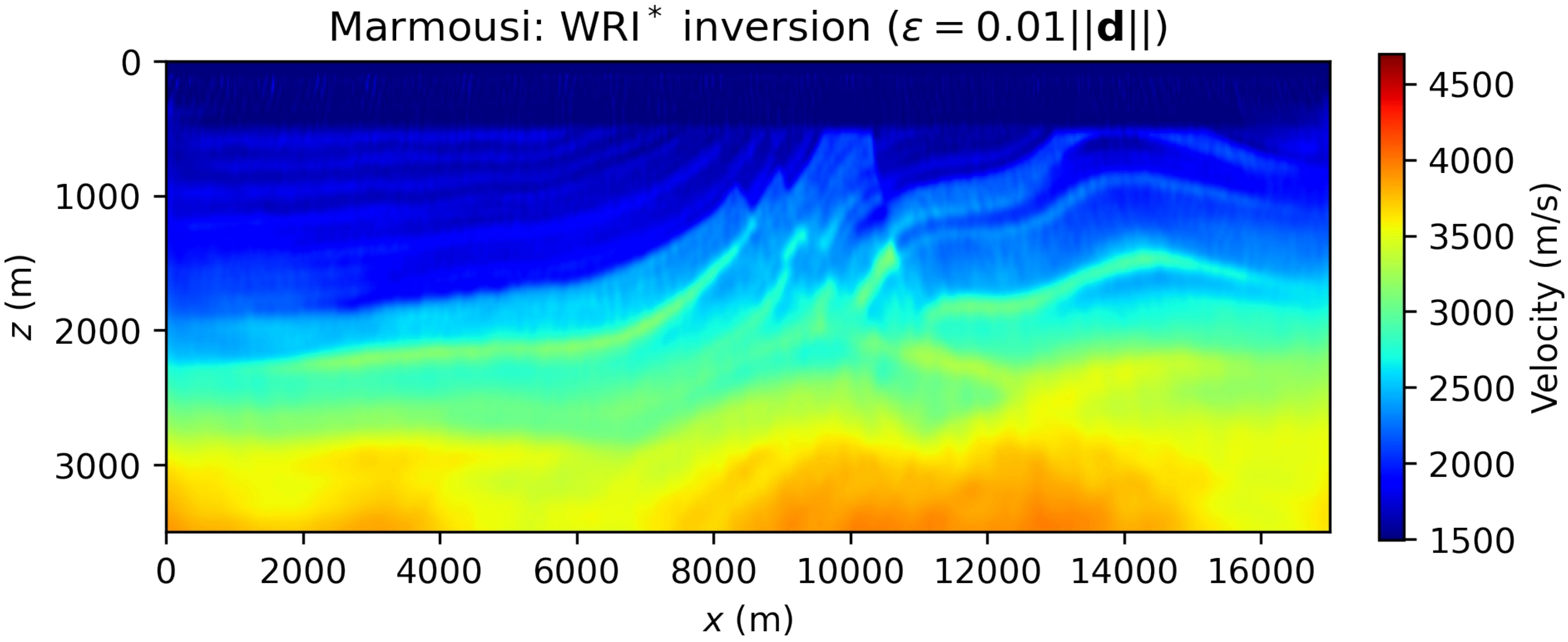


Numerical examples: Marmousi (freq. domain)



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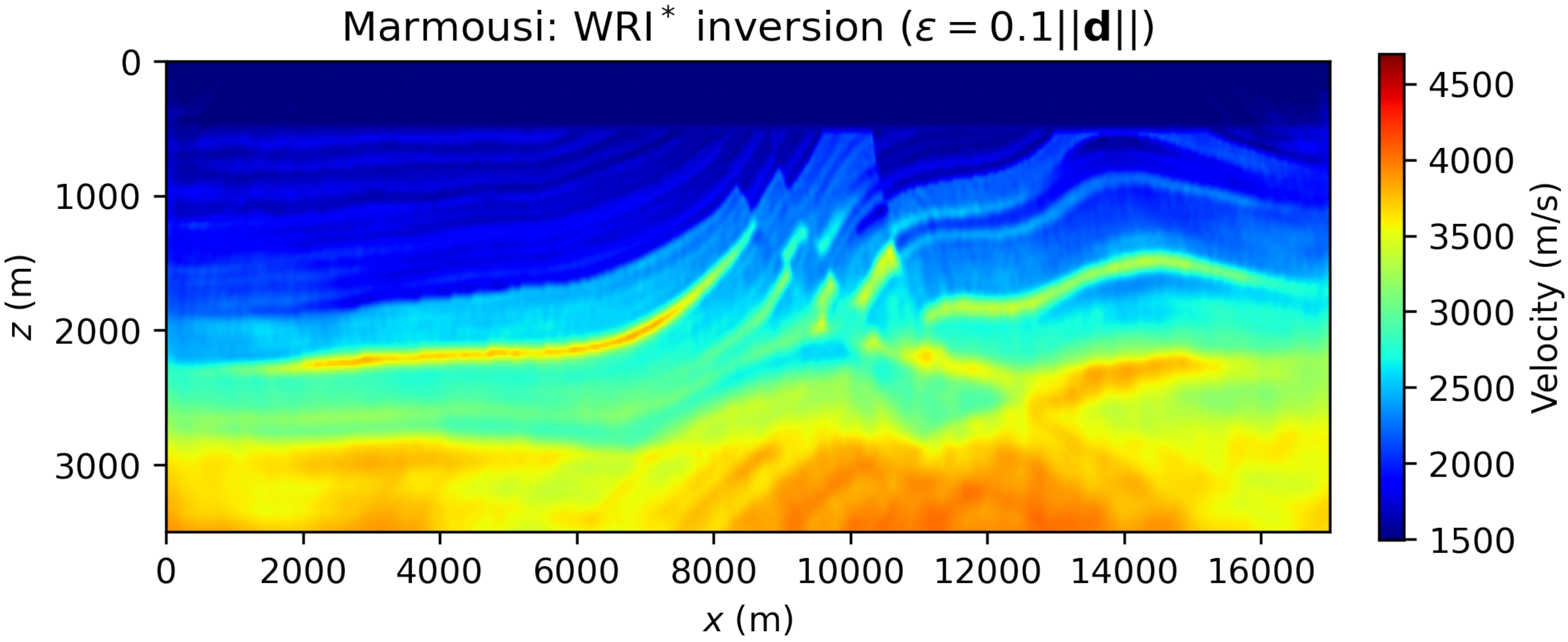
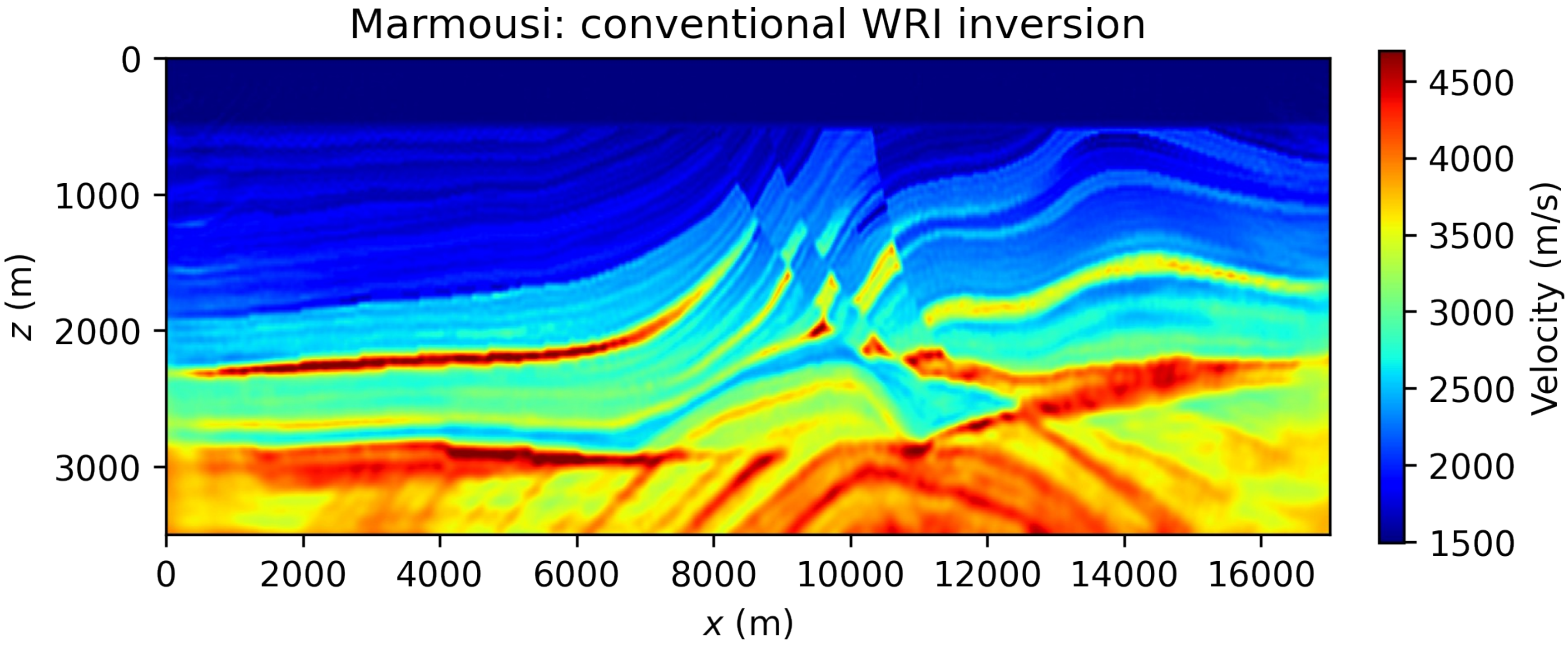
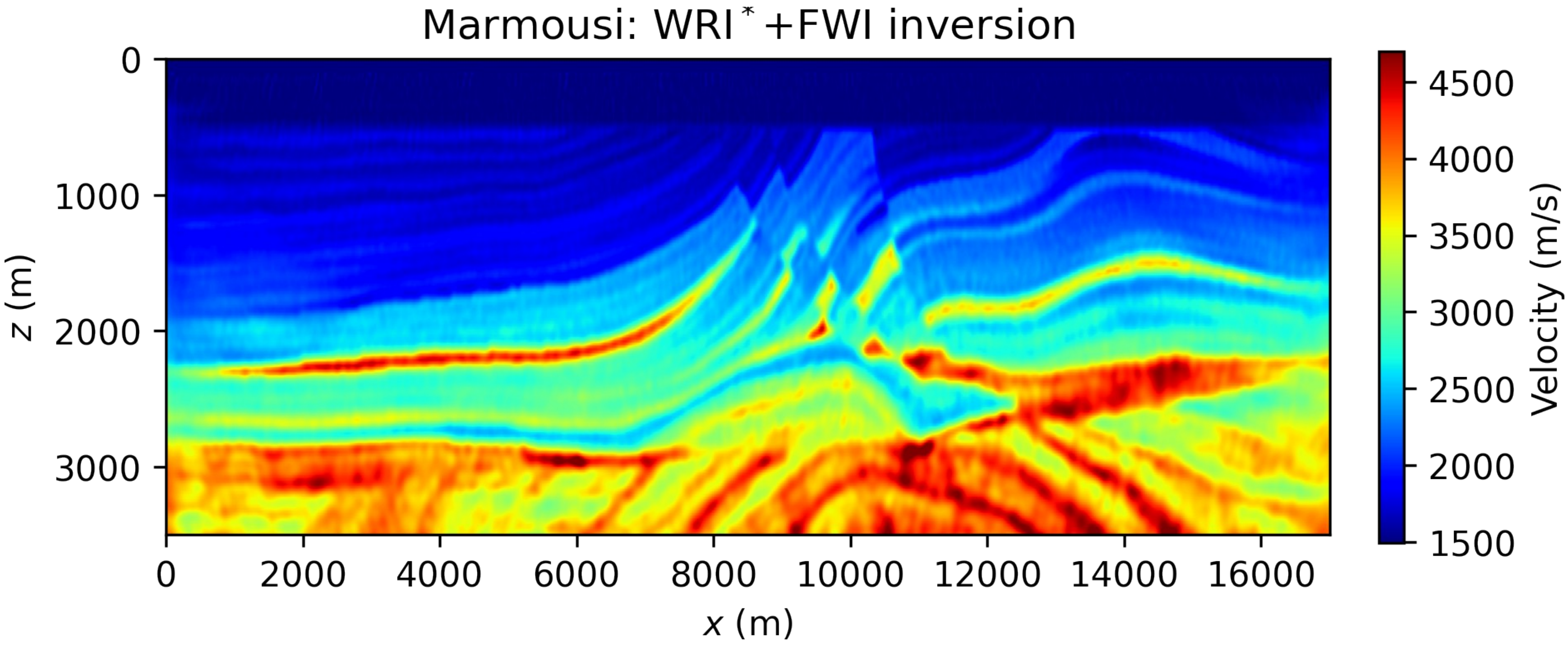


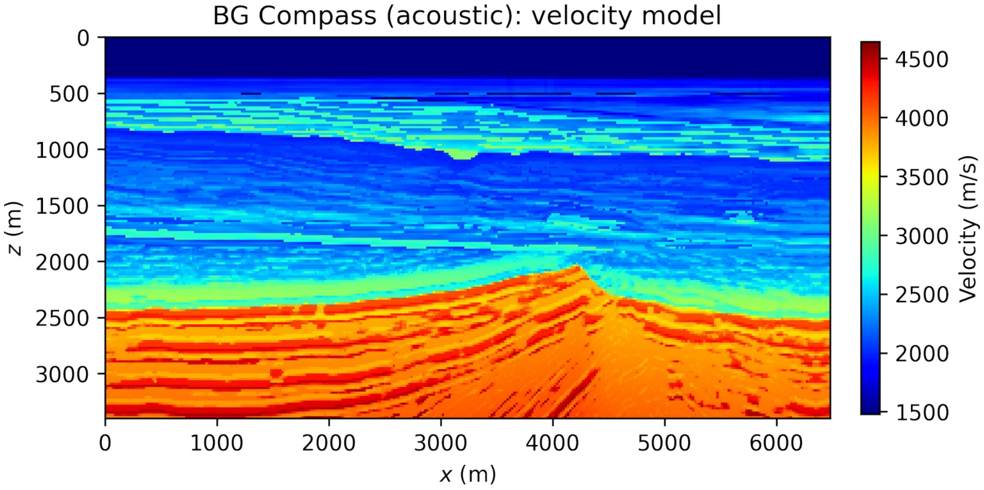
Numerical examples: Marmousi (freq. domain)

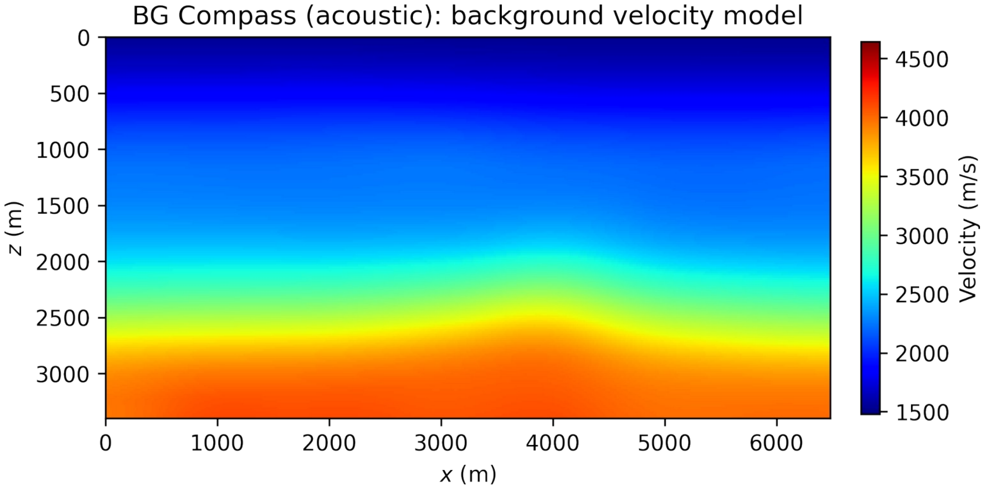


Numerical examples: Marmousi (freq. domain)

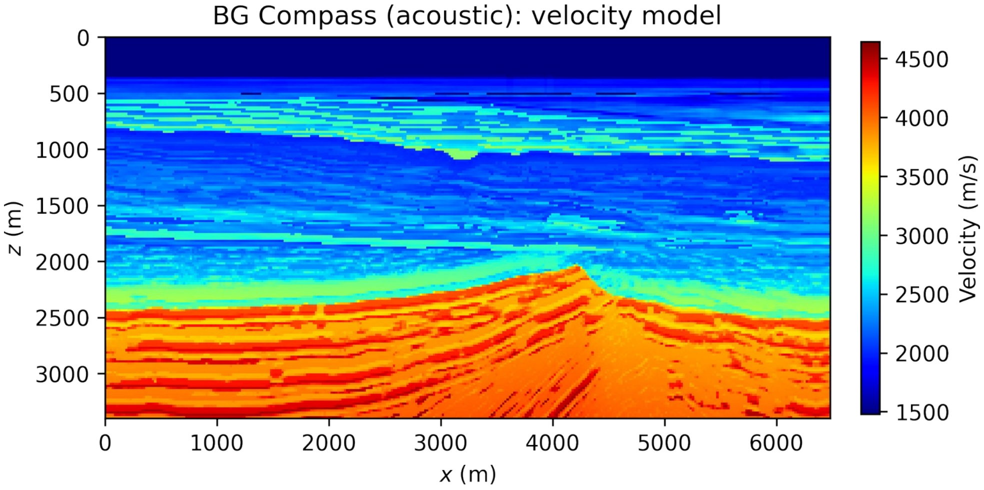


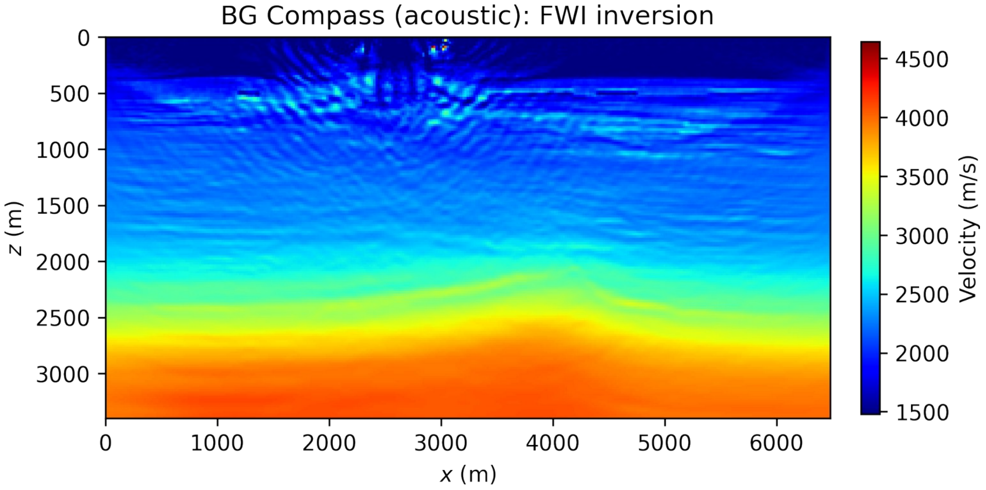
Numerical examples: BG Compass (acoustic)



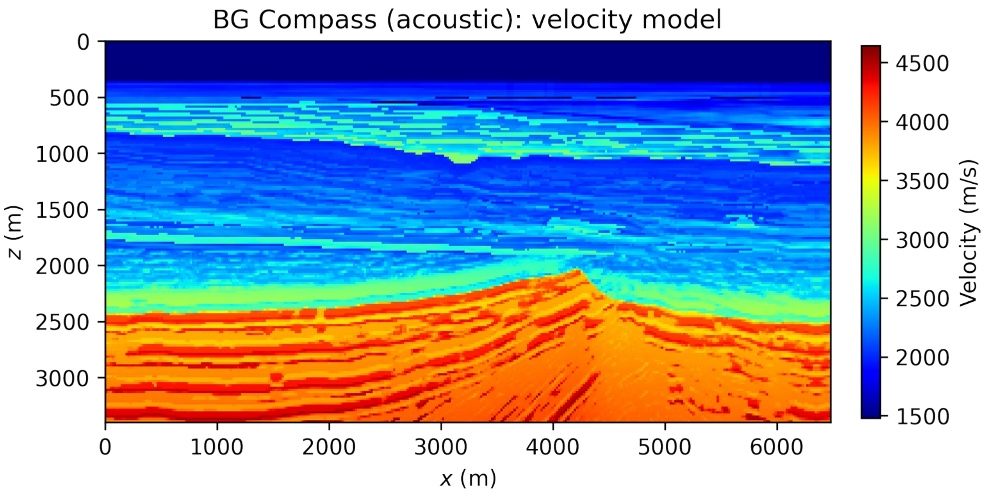


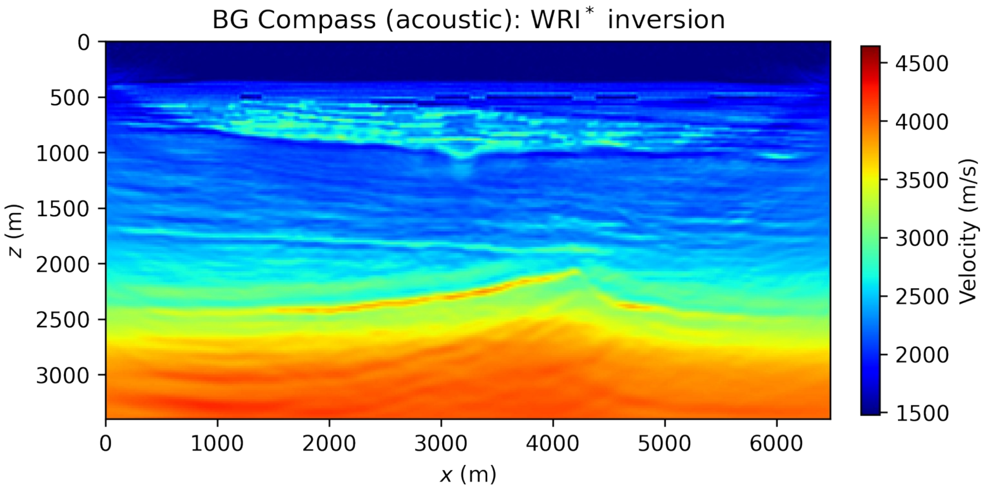
Numerical examples: BG Compass (acoustic)



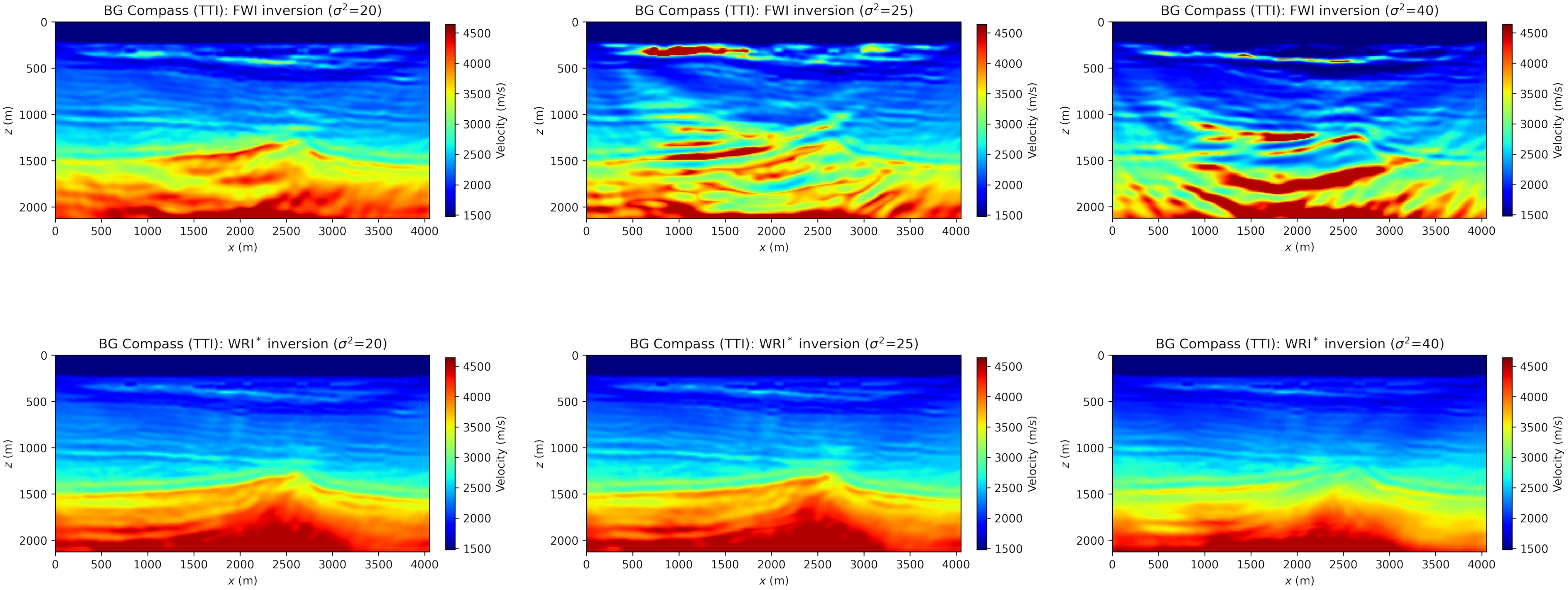


Numerical examples: BG Compass (acoustic)

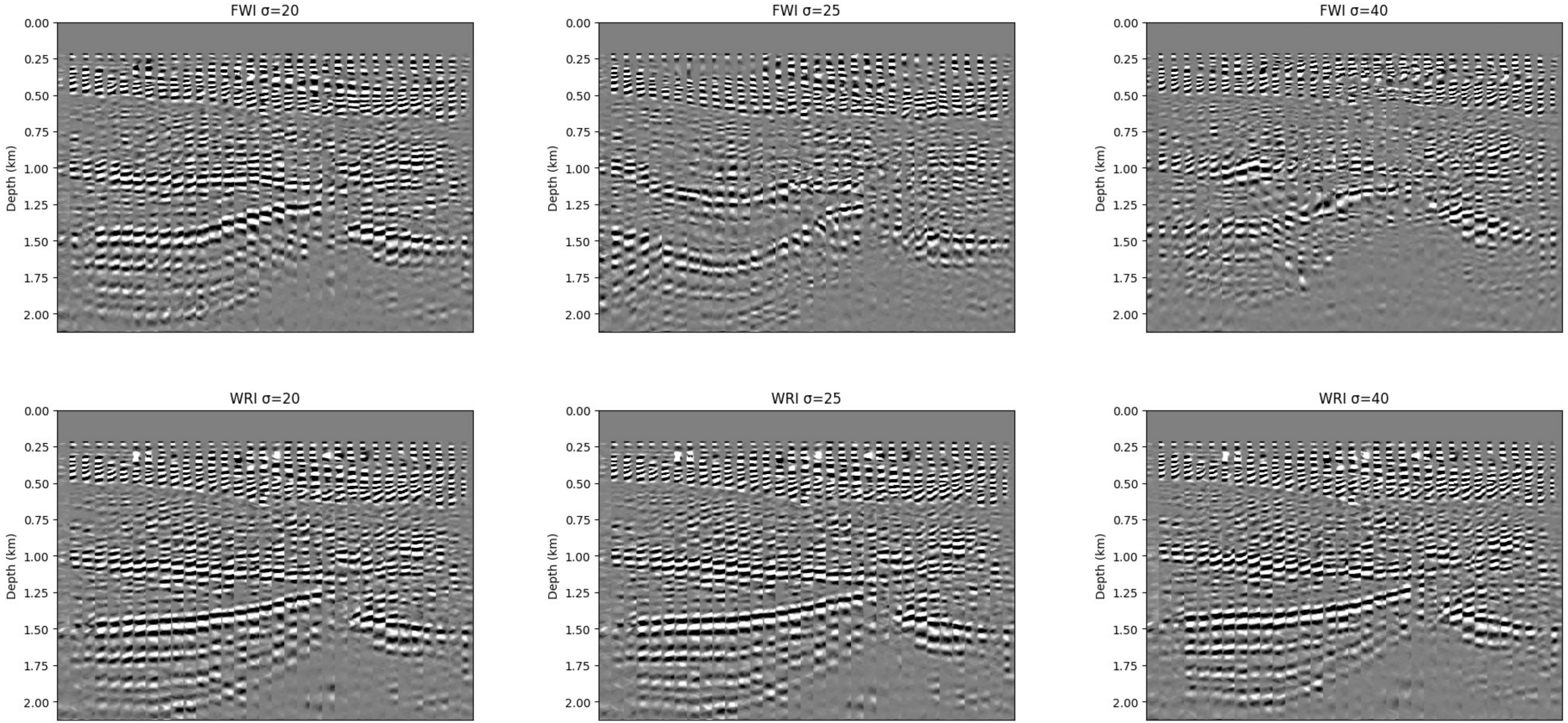




Numerical examples: BG Compass (TTI)

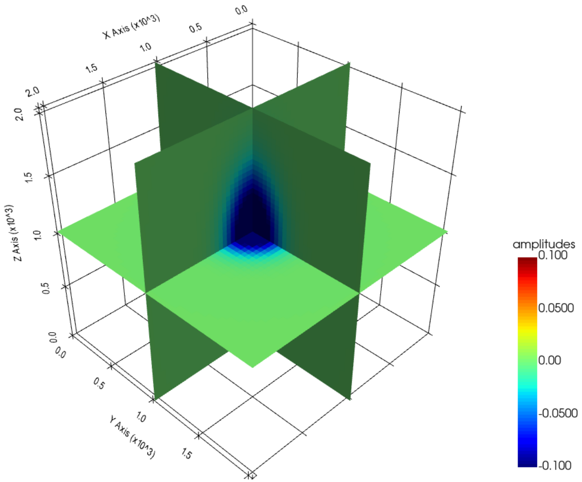


Numerical examples: BG Compass (TTI, CIGs)

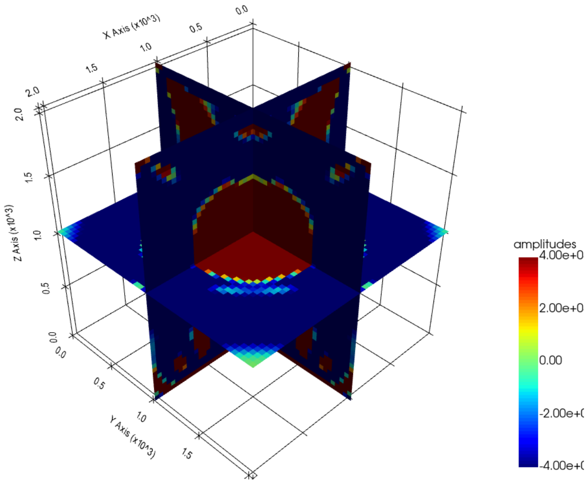


Numerical examples: small 3D

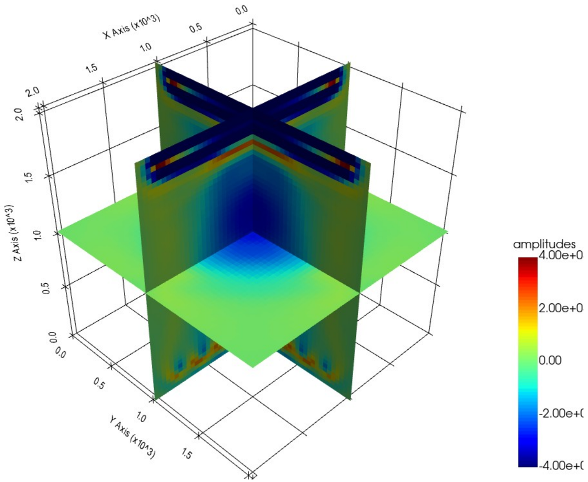
True



FWI



WRI*



Discussion

- WRI*:
  - affordable version of WRI
  - retains robustness of WRI
  - 2nd order methods/hybrid schemes to improve resolution
- Source-focusing annihilator might be necessary to avoid local minima [Symes, W. W., 2020]

Discussion

- Better approximation for augmented variable? Need for automatic differentiation! [Ablin et al., 2020]

$$\nabla_{\mathbf{m}} \tilde{\bar{\mathcal{L}}} = \nabla_{\mathbf{m}} \bar{\mathcal{L}} + \frac{\partial \tilde{\mathbf{y}}}{\partial \mathbf{m}}^{H} \nabla_{\mathbf{y}} \bar{\mathcal{L}}$$

- Some special choice for covariance of data/model error avoid the need for augmented variable approximation [van Leeuwen, T., 2019]

Open source implementation

- github.com/slimgroup/JUDI.jl
- devitoproject.org

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