Compressive least squares migration with on-the-fly Fourier transforms

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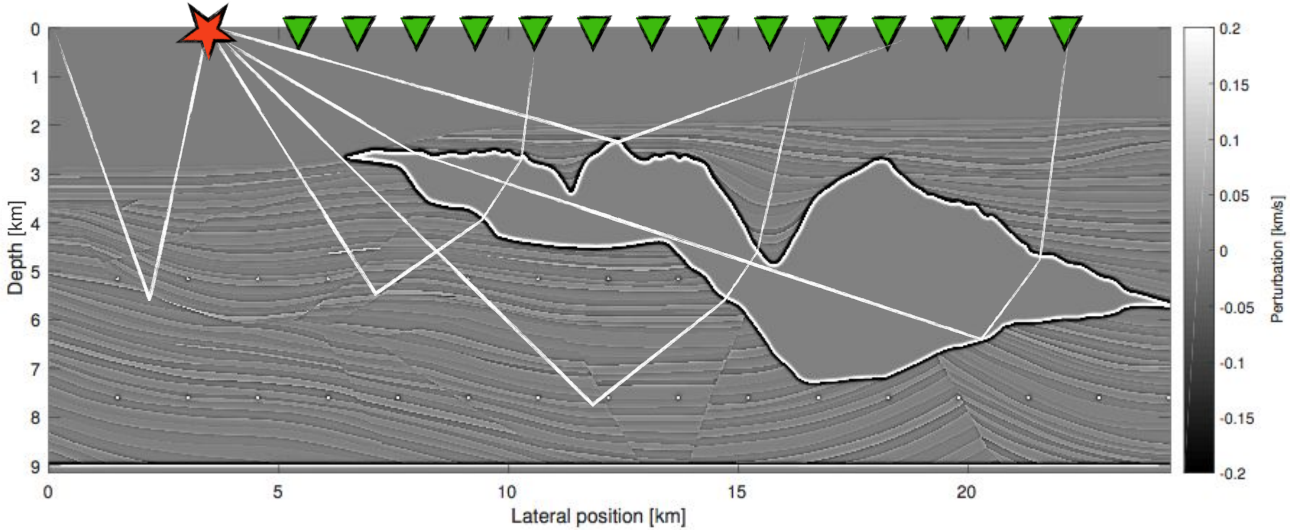
Motivation

Geophysical exploration using seismic imaging:

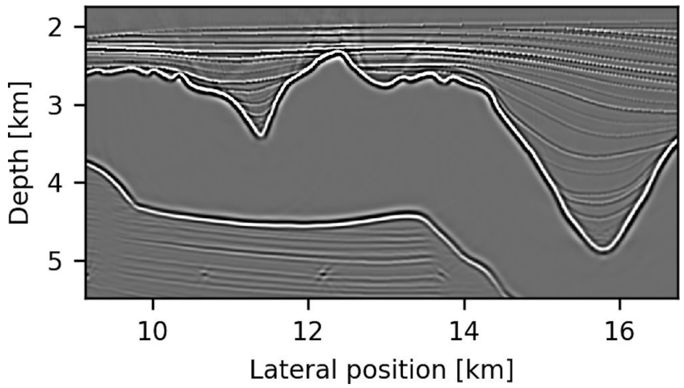
- Image velocity/impedance contrasts in the subsurface

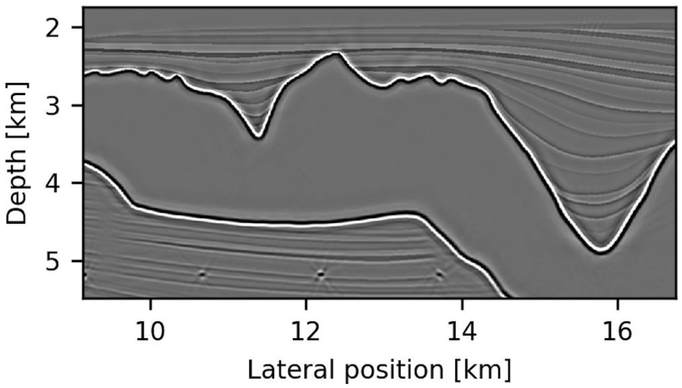


Motivation

Reverse time migration (RTM) vs. least squares imaging (LS-RTM):

- RTM: adjoint of Born scattering operator (1 data pass)
- LS-RTM: iterative inversion (many data passes, usually ~20)

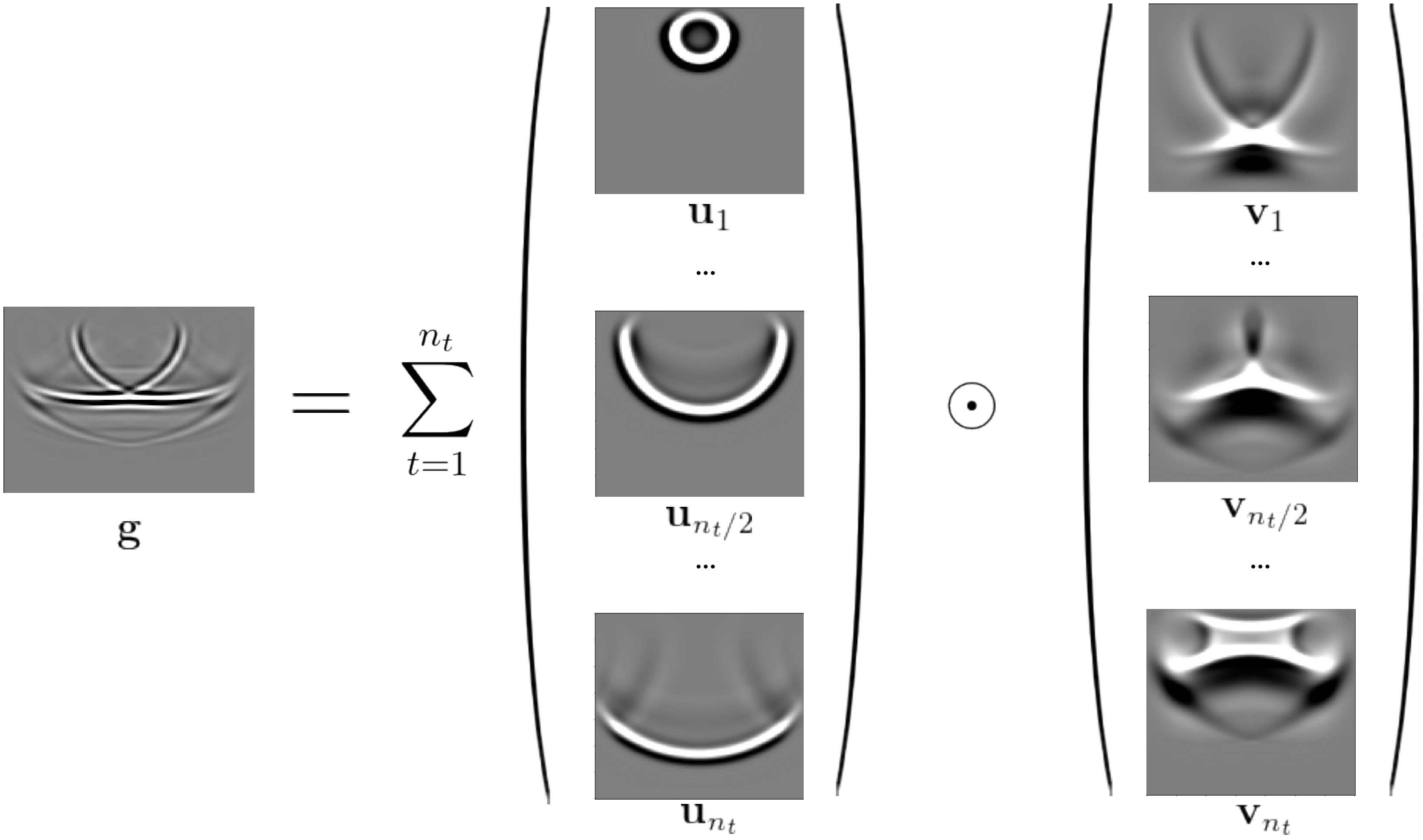


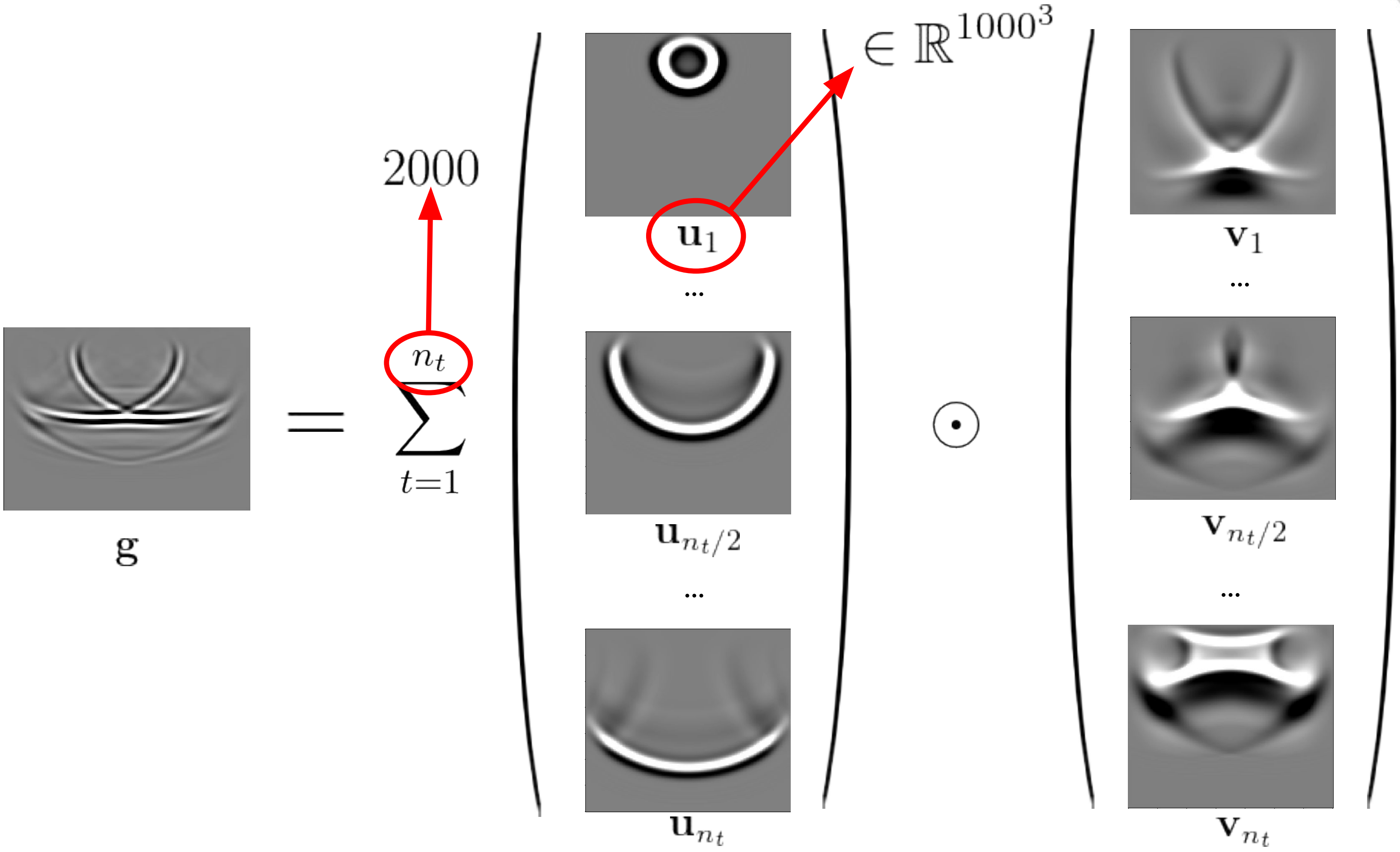


Motivation

Computational challenges of seismic imaging:

- LS-RTM: Need to solve large number of PDEs at every iteration
- Expensive PDE solves: propagate waves over ~20,000 time steps
- Large data sets (easily up to ~100 TB)
- Large number of variables (between 1e6 and 1e10)
- Large memory requirements for backpropagation


$\mathbf{g}$
$=$
$\sum_{t=1}^{n_t}$
$\mathbf{u}_1$
...
$\mathbf{u}_{n_t/2}$
...
$\mathbf{u}_{n_t}$
$\odot$
$\mathbf{v}_1$
...
$\mathbf{v}_{n_t/2}$
...
$\mathbf{v}_{n_t}$



$\mathbf{g} = \sum_{t=1}^{n_t} \left(\mathbf{u}_1 \ldots \mathbf{u}_{n_t/2} \ldots \mathbf{u}_{n_t} \right) \odot \left(\mathbf{v}_1 \ldots \mathbf{v}_{n_t/2} \ldots \mathbf{v}_{n_t} \right)$
2000
$\in \mathbb{R}^{1000^3}$

Motivation

Strategies for backpropagation:

- Wavefield subsampling
- Wavefield reconstruction from boundary (e.g. McMechan, 1983)
- Optimal checkpointing (Griewank and Walther, 2000)
- Domain decomposition (MPI based)
- Elasticache, memory optimized instances (AWS)
- Frequency domain imaging (solve Helmholtz)
- Time-to-frequency conversion (e.g. DFTs) (Overview in Furse, 1998)

Motivation

Low cost least squares RTM with on-the-fly DFTs:

- Time domain modeling w/ on-the-fly DFTs
- Work w/ random subsets of frequencies/sources
- Benefits of LS-RTM at a fraction of the cost
- Sparsity-promoting minimization to address subsampling artifacts

Time-to-frequency conversion

Least squares RTM objective function in frequency domain:

$$\underset{\delta\mathbf{m}}{\text{minimize}} \quad \sum_{j=1}^{n_s}\sum_{k=1}^{n_f}\frac{1}{2}\left\|\mathbf{J}(\mathbf{m}_0,\bar{q}_{jk})\,\delta\mathbf{m}-\bar{\mathbf{d}}_{jk}^{\text{obs}}\right\|_2^2.$$

With:

$\mathbf{J} \in \mathbb{C}^{n_r \times n}$	linearized Born scattering operator
$\delta\mathbf{m} \in \mathbb{C}^{n}$	unknown image
$\mathbf{m}_0 \in \mathbb{C}^{n}$	migration velocity (assumed to be known)
$\bar{\mathbf{d}}_{\text{jk}}^{\text{obs}} \in \mathbb{C}^{n_r}$	observed seismic data
$\bar{q}_{jk} \in \mathbb{C}$	source wavelet (assumed to be known)

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Time-to-frequency conversion

Model linearized data with time modeling code:

$$\bar{\mathbf{d}}_{jk}^{\mathrm{pred}} = -\mathbf{P}_r \mathbf{R}_k \mathbf{F} \mathbf{A}(\mathbf{m}_0)^{-1} \mathrm{diag}\left[\frac{\partial \mathbf{A}(\mathbf{m}_0)}{\partial \mathbf{m}} \mathbf{A}(\mathbf{m}_0)^{-1} \mathbf{F}^* \mathbf{R}_k^* \mathbf{p}_s^* \bar{q}_{jk} \right] \delta \mathbf{m}$$

With:

$\mathbf{P}_r \in \mathbb{C}^{n_r \times n}$ receiver restriction

$\mathbf{R}_k \in \mathbb{C}^{n \times n \cdot n_t}$ frequency restriction

$\mathbf{F} \in \mathbb{C}^{n \cdot n_t \times n \cdot n_t}$ discrete Fourier transform

$\mathbf{A}(\mathbf{m}_0) \in \mathbb{R}^{n \cdot n_t \times n \cdot n_t}$ discretized wave equation

Time-to-frequency conversion

Compute gradient of LS-RTM objective function:

$$\bar{\mathbf{g}}_{jk} = -\mathrm{Re}\Big[\mathrm{diag}\big(\omega_k^2 \bar{\mathbf{u}}_{jk}\big)^* \bar{\mathbf{v}}_{jk}\Big]$$

With: $\bar{\mathbf{u}}_{jk} = \mathbf{R}_k \mathbf{F} \mathbf{A}(\mathbf{m}_0)^{-1} \mathbf{F}^* \mathbf{R}_k^* \mathbf{p}_s^* \bar{q}_{jk}$

$$\bar{\mathbf{v}}_{jk} = \mathbf{R}_k \mathbf{F} \mathbf{A}(\mathbf{m}_0)^{-*} \mathbf{F}^* \mathbf{R}_k^* \mathbf{P}_r^* (\bar{\mathbf{d}}_{jk}^{\mathrm{pred}} - \bar{\mathbf{d}}_{jk}^{\mathrm{obs}})$$

$$\bar{\mathbf{u}}_{jk}, \bar{\mathbf{v}}_{jk} \in \mathbb{C}^n$$

Time-to-frequency conversion

So far: explicit DFTs in modeling expressions

- replace w/ on-the-fly DFTs
- during time stepping compute:

$$\bar{\mathbf{u}}_{jk}^{\text{real}} = \sum_{i=1}^{n_t} \cos(2\pi f_k i \Delta t)\mathbf{u}_i,$$

$$\bar{\mathbf{u}}_{jk}^{\text{imag}} = -\sum_{i=1}^{n_t} \sin(2\pi f_k i \Delta t)\mathbf{u}_i$$

$$\bar{\mathbf{u}}_{jk}^{\text{real}}, \bar{\mathbf{u}}_{jk}^{\text{imag}}, \mathbf{u}_i \in \mathbb{R}^n$$

(Furse, 1998; Sirgue et al., 2010)

Time-to-frequency conversion

Computing gradients $\bar{\mathbf{g}}_{jk} \in \mathbb{R}^n$ w/ on-the-fly DFTs

- perform DFT in adjoint loop
- compute frequency-domain adjoint wavefield + gradient in single step:

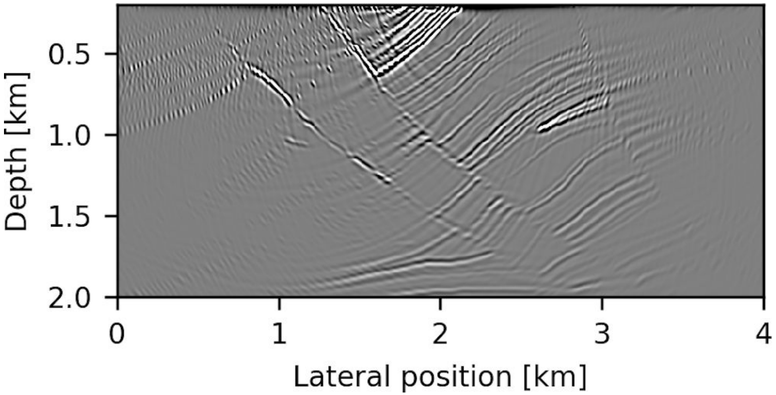
$$\bar{\mathbf{g}}_{jk} = -\sum_{i=1}^{n_t} (2\pi f_k)^2 \mathrm{diag}\Big[\bar{\mathbf{u}}_{jk}^{\mathrm{real}} \cos(2\pi f_k i \Delta t) - \bar{\mathbf{u}}_{jk}^{\mathrm{imag}} \sin(2\pi f_k i \Delta)\Big] \mathbf{v}_i$$

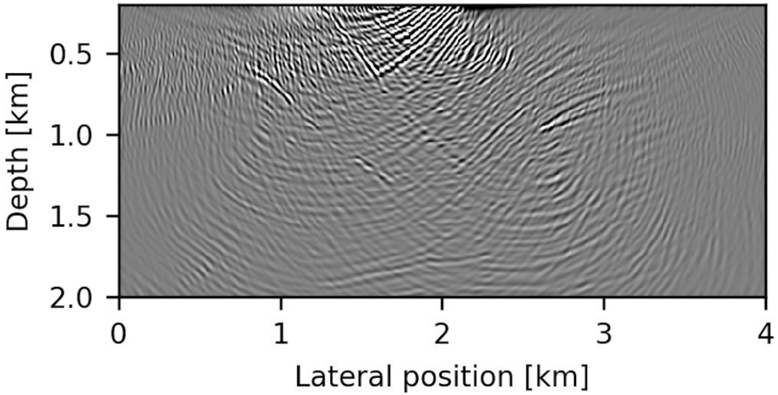
- alternative imaging conditions possible as well

Sparsity-promoting LS-RTM

So far: frequency-domain imaging w/ time modeling

- need fine frequency sampling for imaging
- periodic subsampling causes aliasing/coherent artifacts
- compressive sensing: random sampling

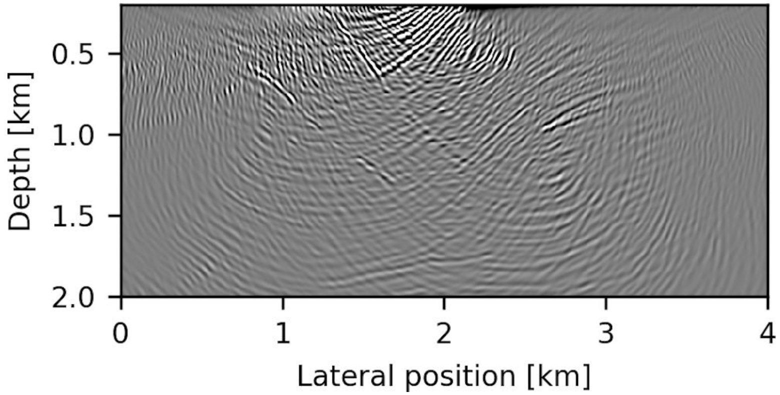


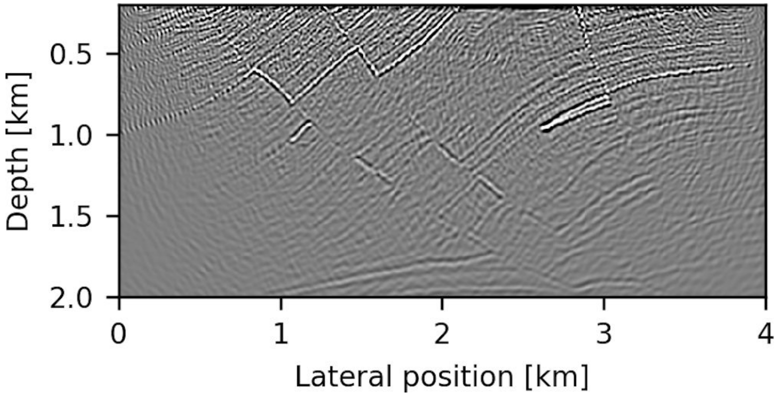


Sparsity-promoting LS-RTM

Compressive sensing (CS) inspired imaging:

- linear system becomes underdetermined
- different random frequencies for each experiment
- coherent reflectors + incoherent noise





Sparsity-promoting LS-RTM

Ingredients for CS inspired seismic imaging:

- random sampling to break coherency of noise
- sparsifying transform (e.g. curvelets)
- sparsity-promoting optimization to recover noise-free image

(Candes et al., 2006; Donoho, 2006)

(Tu et al., 2015; Herrmann et al., 2015)

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Sparsity-promoting LS-RTM

LS-RTM as sparsity-promoting minimization problem:

- elastic net objective function (strongly convex)

$$\underset{\delta\mathbf{z}}{\text{minimize}} \quad \lambda||\mathbf{C}\,\delta\mathbf{z}||_1 + \frac{1}{2}||\mathbf{C}\,\delta\mathbf{z}||_2^2$$

$$\text{subject to:} \quad \sum_{j=1}^{n_s}\sum_{k=1}^{n_f}\left\|\mathbf{M}_l^{-1}\mathbf{J}(\mathbf{m}_0,\bar{q}_{jk})\mathbf{M}_r^{-1}\delta\mathbf{z} - \mathbf{M}_l^{-1}\bar{\mathbf{d}}_{jk}^{\text{obs}}\right\|_2 \leq \sigma$$

$\mathbf{C} \in \mathbb{C}^{n_c \times n}$ curvelet transform

$\delta\mathbf{z} \in \mathbb{R}^n$ seismic impedance

$\mathbf{M}_l \in \mathbb{R}^{n_r \times n_r}$, $\mathbf{M}_r \in \mathbb{R}^{n \times n}$ left/right-hand preconditioners

$\lambda, \sigma \in \mathbb{R}$ penalty parameter and noise in ℓ_2-ball

Sparsity-promoting LS-RTM

Algorithm: the linearized Bregman method

1. Initialize $\mathbf{x}_1 = \mathbf{0}$, $\mathbf{z}_1 = \mathbf{0}$, q, λ, batch sizes $\hat{n}_s \ll n_s$ and $\hat{n}_f \ll n_f$
2. **for** $i = 1, ..., n$
3. Select subset of shots and frequencies $\mathcal{S} = (\int_{\text{shot}}, \int_{\text{freq}}), |\int_{\text{shot}}| = \hat{n}_s, |\int_{\text{freq}}| = \hat{n}_f$
4. $\bar{\mathbf{d}}_{\mathcal{S}}^{\text{pred}} = \mathbf{M}_l^{-1}\mathbf{J}_{\mathcal{S}}\mathbf{M}_r^{-1}\mathbf{x}$
5. $\bar{\mathbf{g}}_{\mathcal{S}} = \mathbf{M}_r^{-\top}\mathbf{J}_{\mathcal{S}}^{\top}\mathbf{M}_l^{-\top}\mathcal{P}_{\sigma}(\bar{\mathbf{d}}_{\mathcal{S}}^{\text{pred}} - \bar{\mathbf{d}}_{\mathcal{S}}^{\text{obs}})$
6. $\mathbf{z}_{i+1} = \mathbf{z}_i - t_i\bar{\mathbf{g}}_{\mathcal{S}}$
7. $\mathbf{x}_{i+1} = S_{\lambda}(\mathbf{z}_{i+1})$
8. **end**

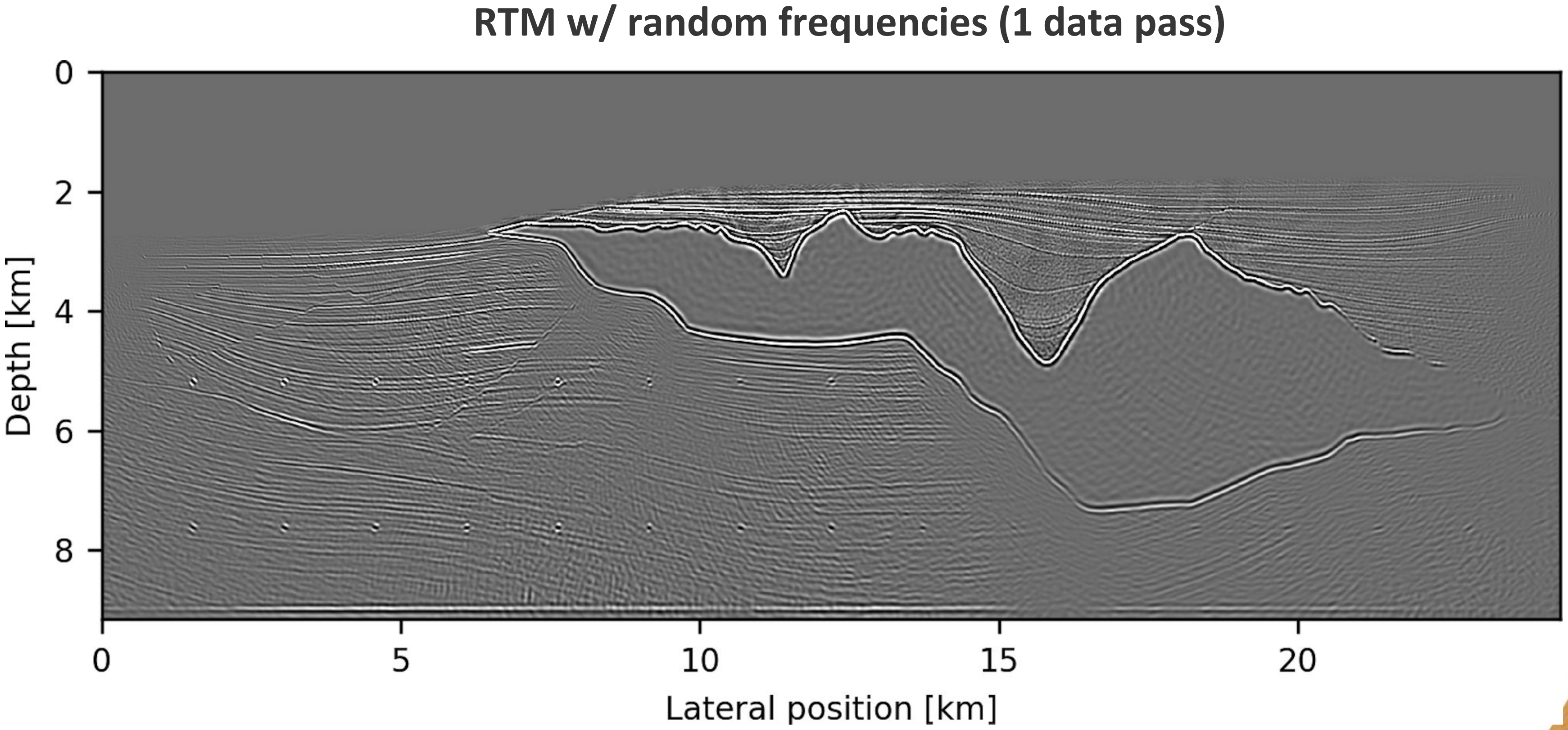
(Yin et al., 2008; Lorenz et al., 2014)

Sigsbee 2A example

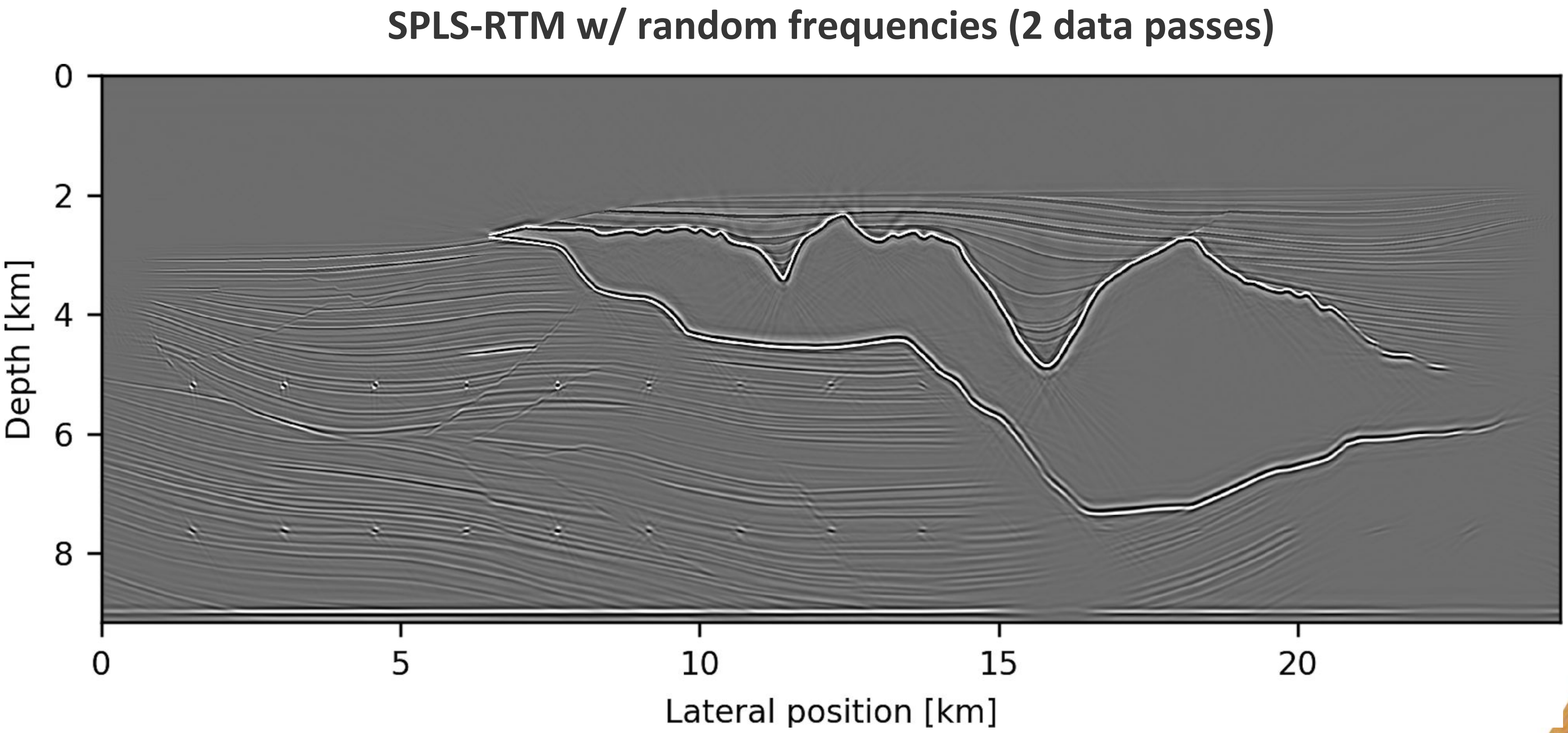
Sigsbee 2A example:

- 24.6 by 9.2 km 2D model (3201 x 1201 grid points)
- 14,095 time steps
- 935 experiments (shot locations)
- comparison of RTM, SPLS-RTM
- on-the-fly DFTs w/ subsampling vs optimal checkpointing
- 20 iterations w/ 100 shots and 20 frequencies per shot

Sigsbee 2A example

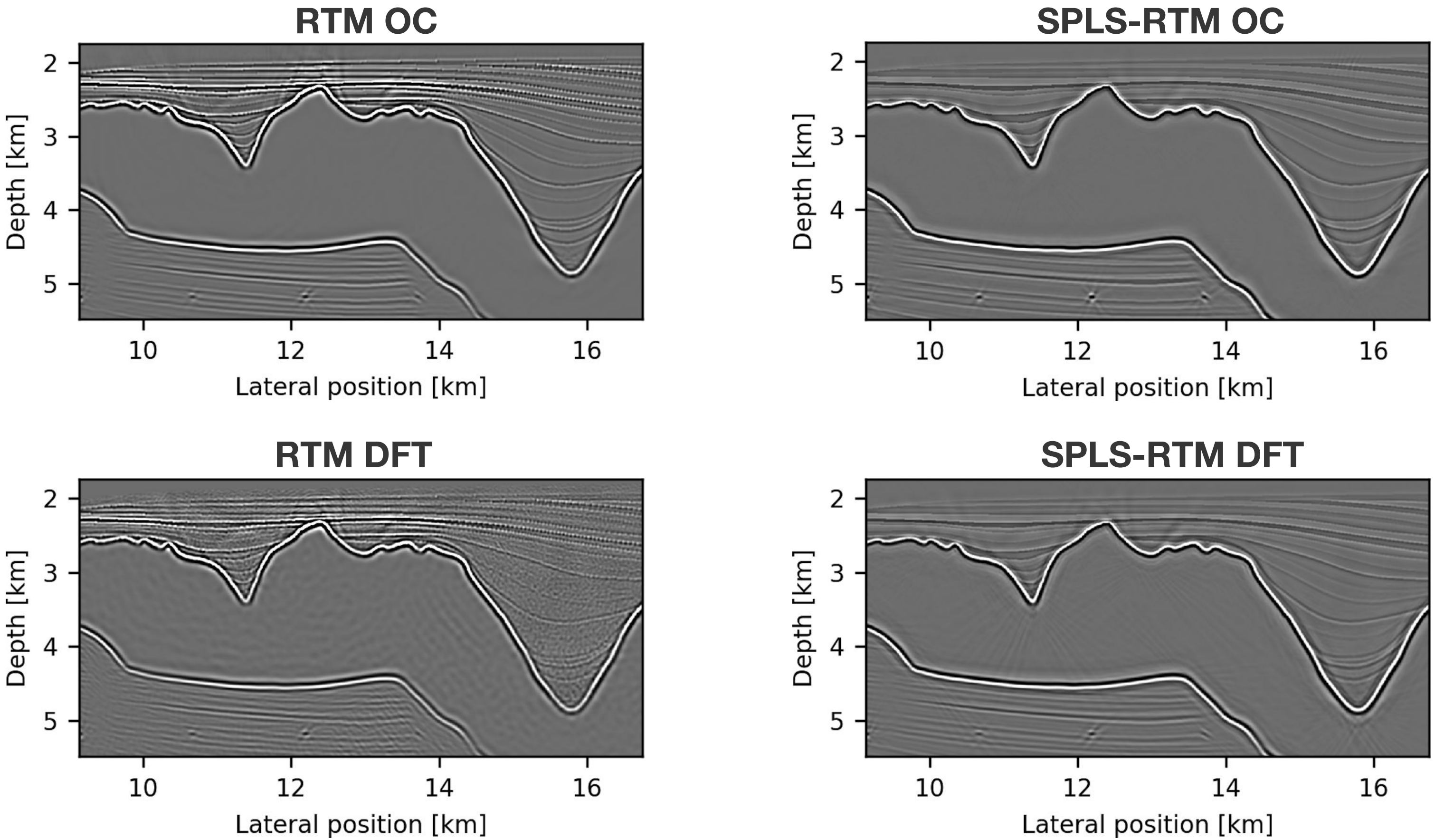


Sigsbee 2A example



Sigsbee 2A example

OC: optimal checkpointing
DFT: on-the-fly Fourier transforms



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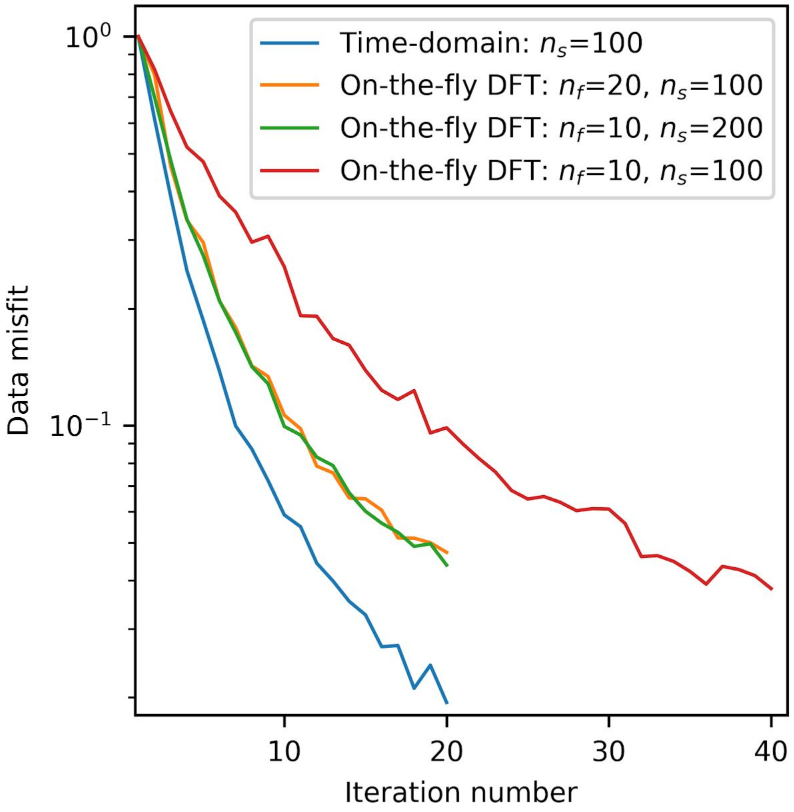

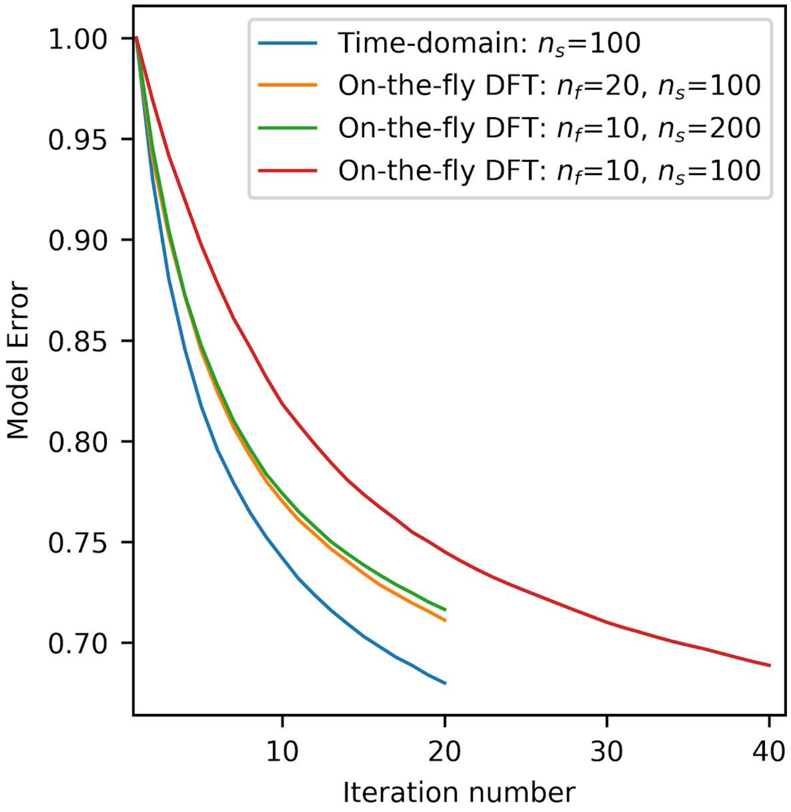

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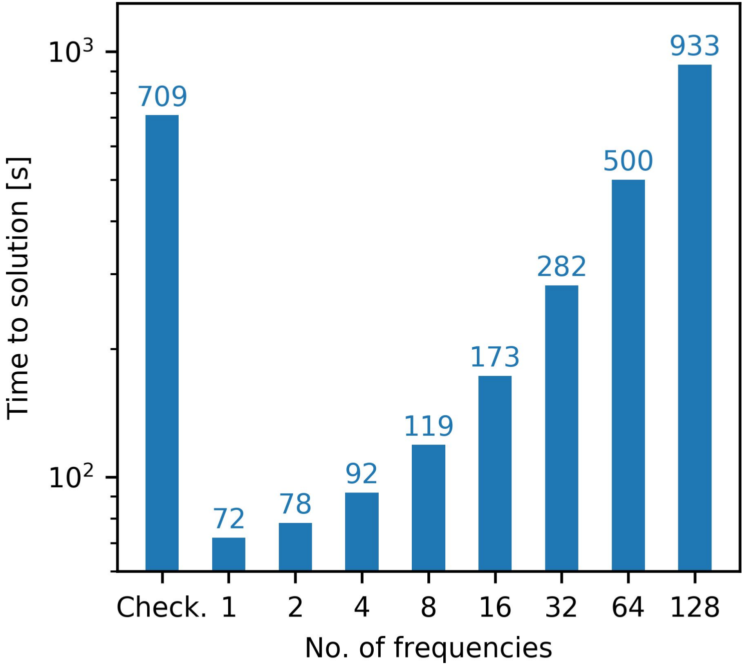


Strategy	Memory	Additional cost
TD: save all wavefields	$\mathcal{O}(n_t)$	–
TD: optimal checkpointing	$\mathcal{O}(\log n_t)$	$\mathcal{O}(\log n_t)$
TD: boundary reconstruction	$\mathcal{O}(n_t)$	$\mathcal{O}(n_t)$
FD: on-the-fly DFT	$\mathcal{O}(n_f)$	$\mathcal{O}(n_f)$

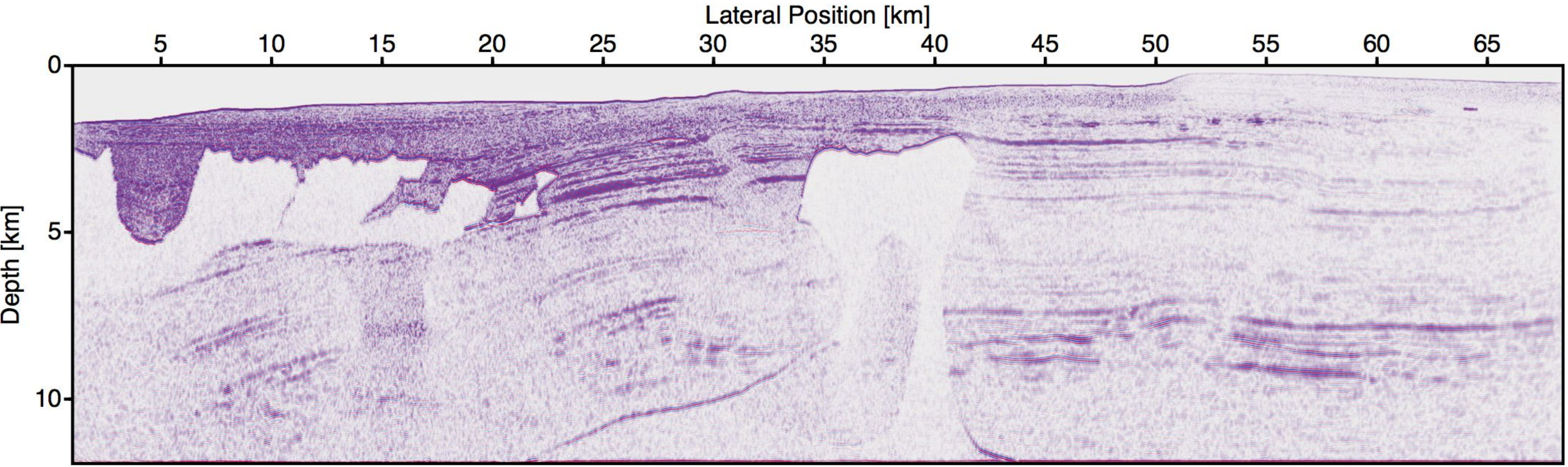
- DFT approach independent of number of time steps
- But: how many frequencies are required?

BP Synthetic 2004

SPLS-RTM enables imaging at large-scales

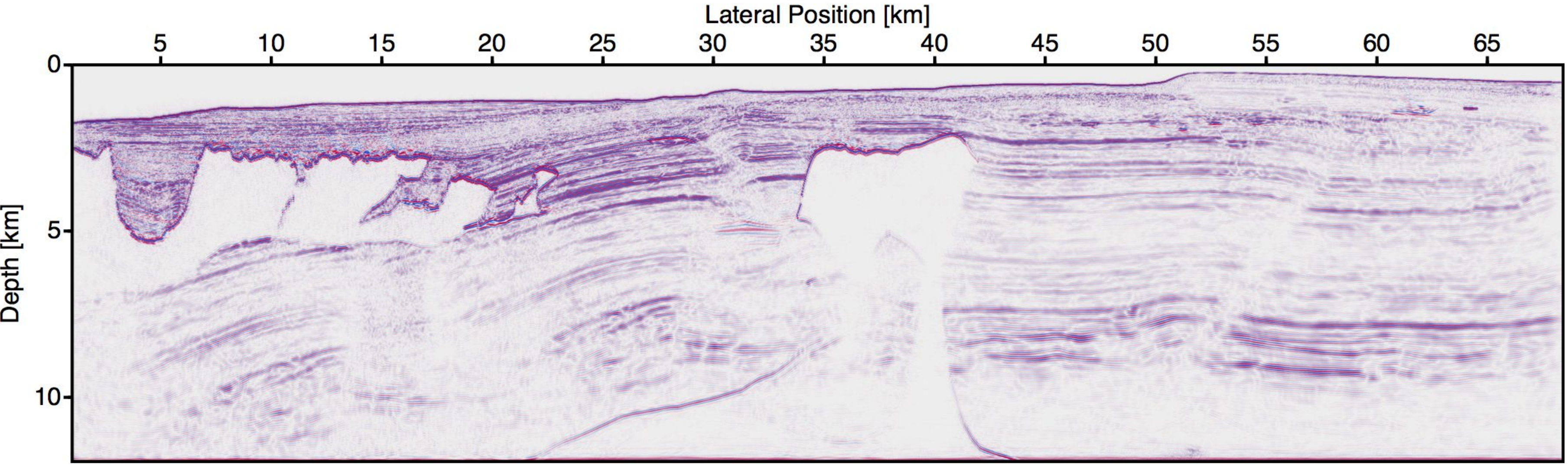
- 2D BP synthetic model 2004
- 67.4 by 11.9 km image (10,789 x 1,911 grid points)
- 1340 shot locations
- 16,920 time steps
- 20 iterations w/ 200 shots, 20 frequencies

BP Synthetic 2004



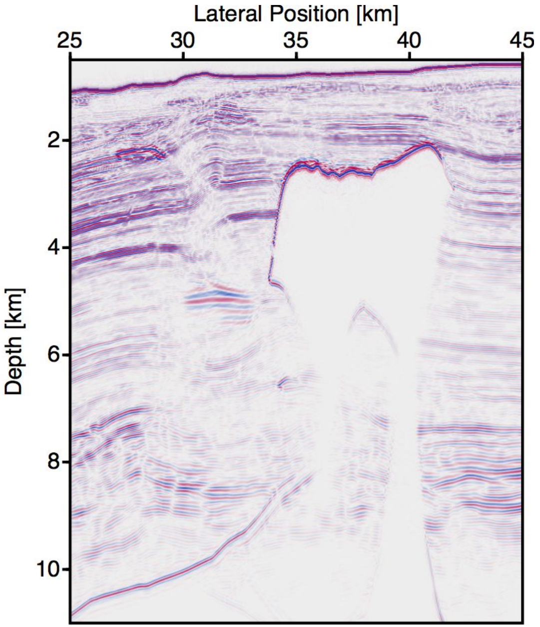
RTM w/ 20 random frequencies per shot

BP Synthetic 2004

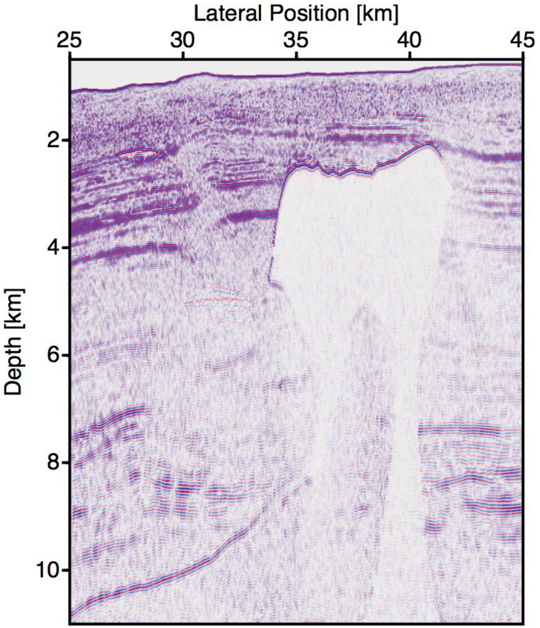


SPLS-RTM w/ 20 random frequencies per shot

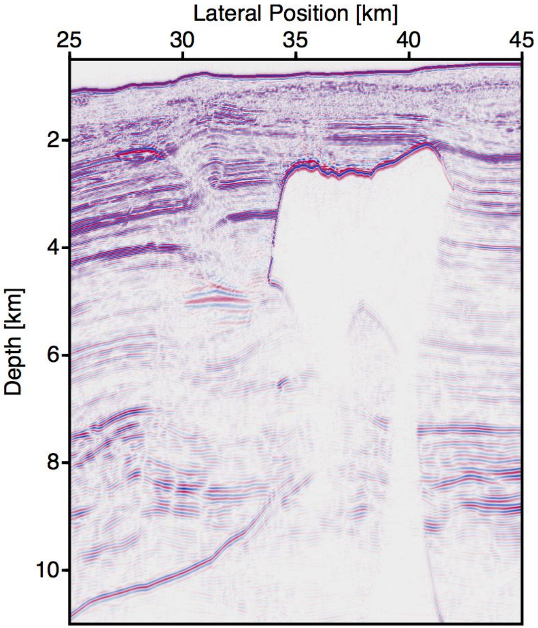
BP Synthetic 2004



SPLS-RTM (OC)

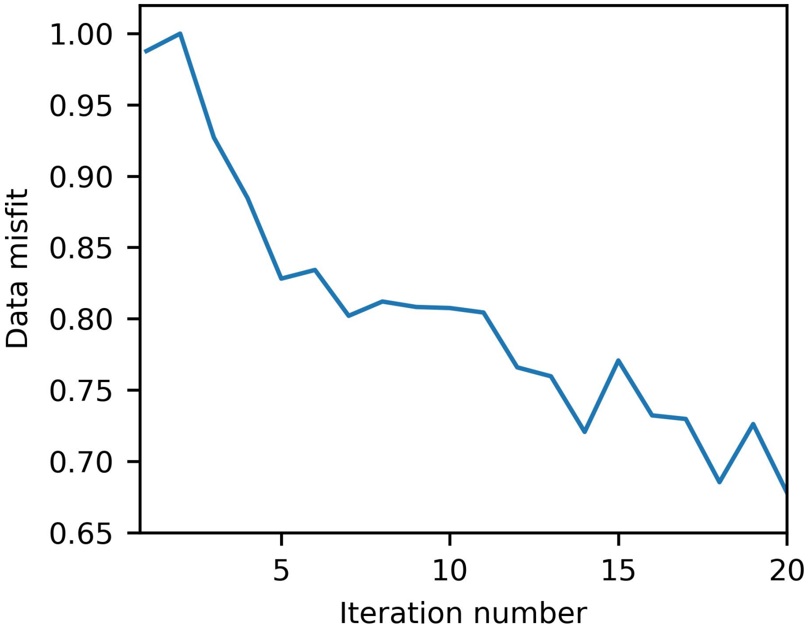


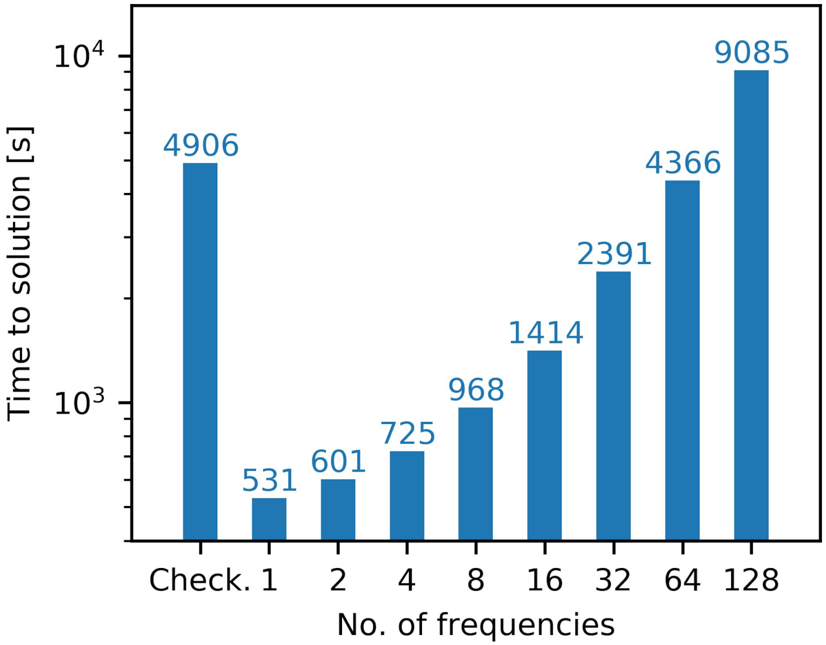
RTM (DFT)



SPLS-RTM (DFT)

BP Synthetic 2004





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Summary

Iterative least squares imaging at a fraction of the cost:

- on-the-fly DFTs
- comparable quality to time-domain **IF** formulated as sparsity-promoting LS problem
- memory independent of no. of time steps
- cost and memory depend of number of frequencies
- flexibility in terms of no. of iterations and batchsizes
- possible to trade cost for quality

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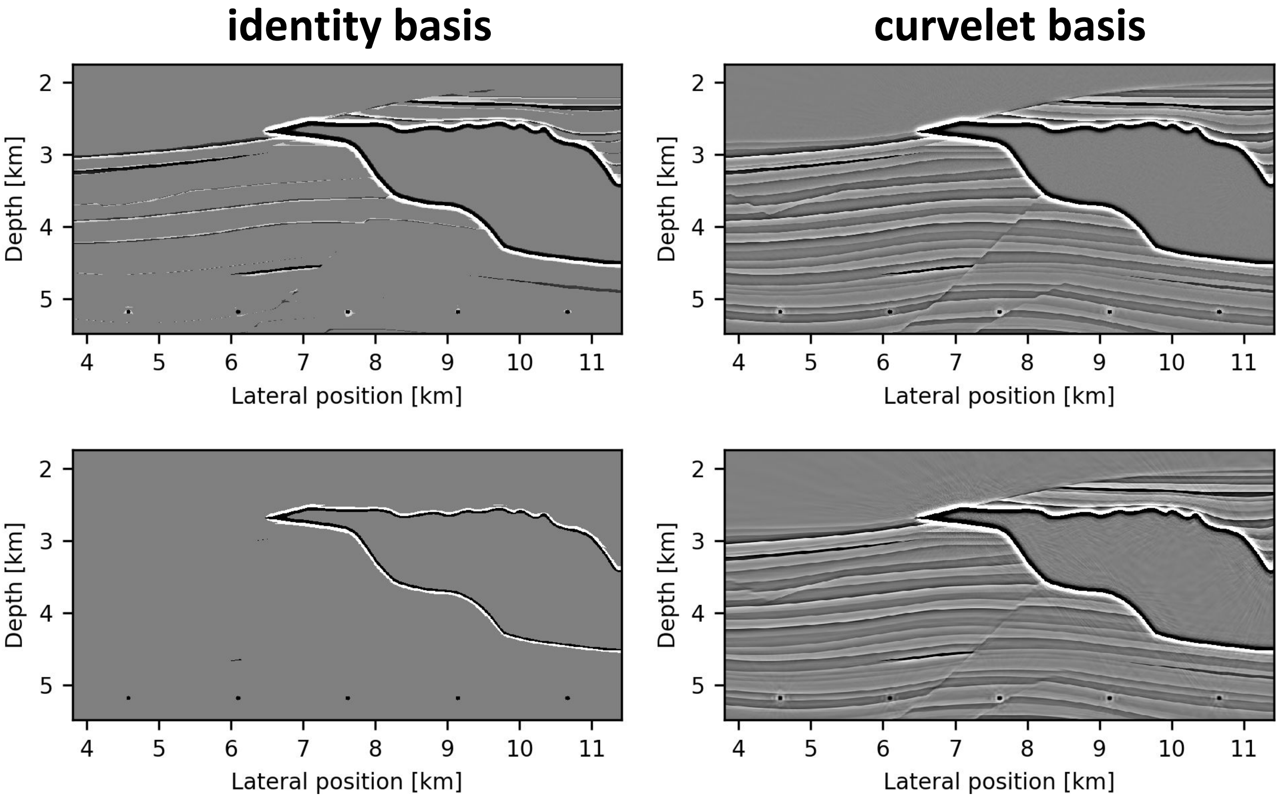
Sparsity-promoting LS-RTM

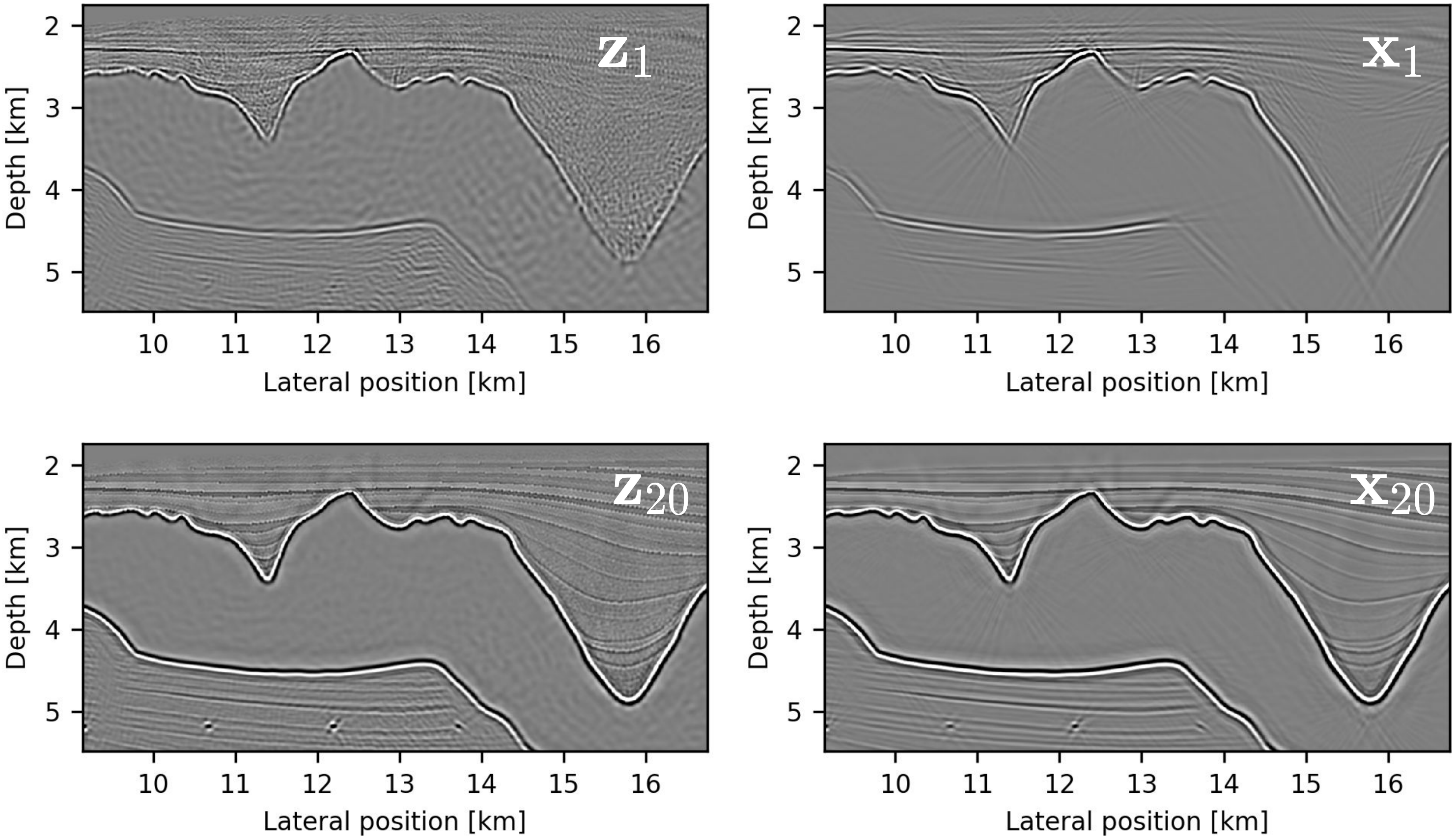
Why curvelets?



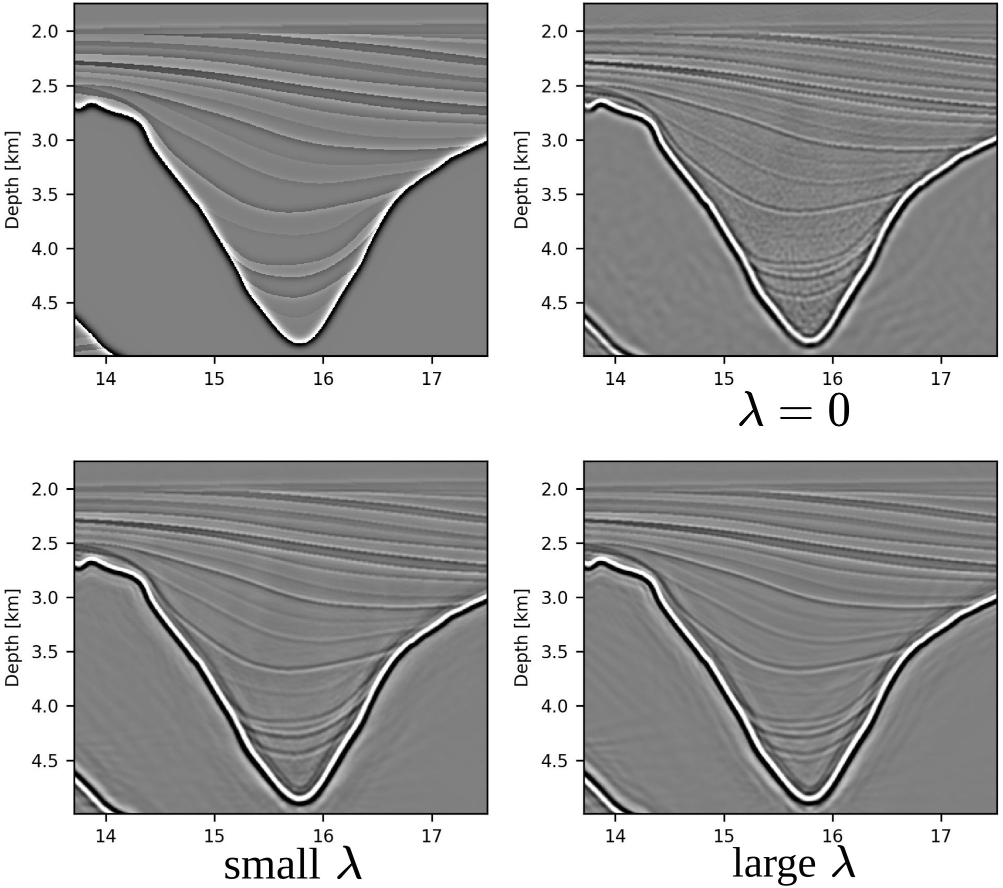
Largest 5% of coefficients

Largest 1% of coefficients

Sigsbee 2A example



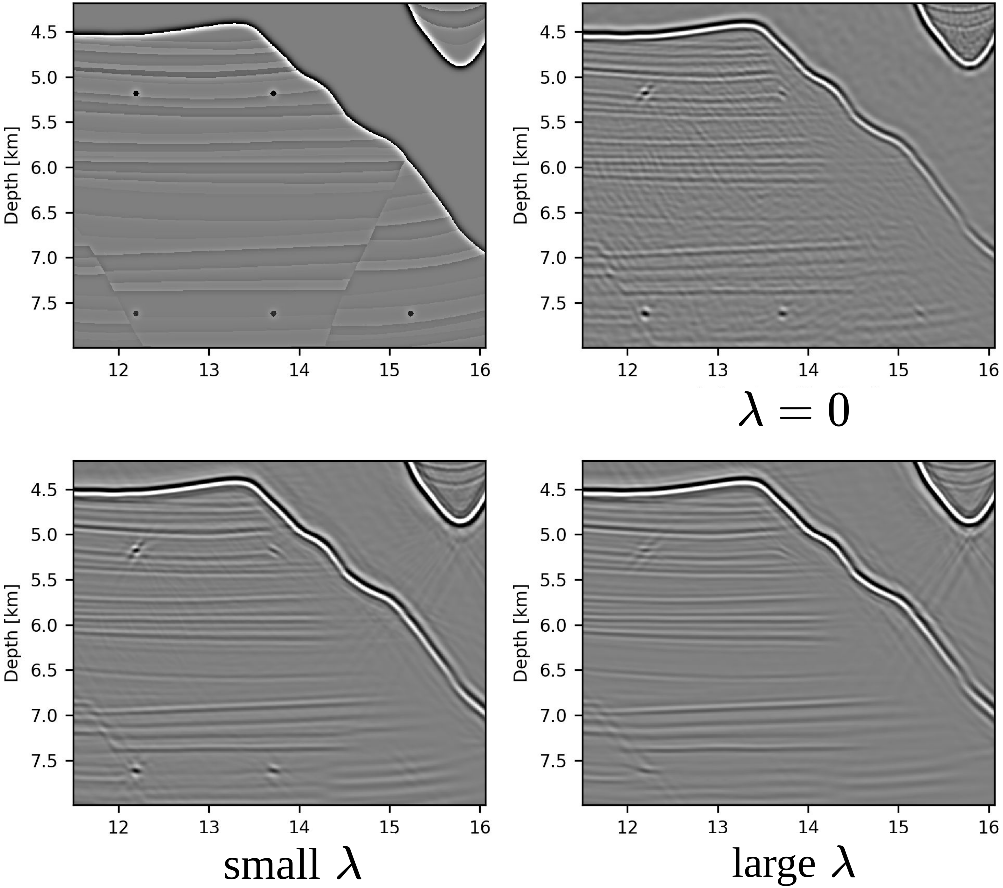
Supplementary slides



Role of the penalty parameter:

- determines the amount of thresholding
- areas of poor illumination more sensitive to its choice

Supplementary slides



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