Dimensionality Reduction in FWI

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Lessons learned from a realistic complex blind synthetic...

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Wave-equation based *inversion*

Industry has difficulty *replenishing* produced *resources*

- have to find oil & gas in *complex* geological areas
- *low* hit rates (1 out of 10) w/ drill costs $.25 B / well
- reduce *risks* of *subsalt* production (BP horizon)

Big drive for *transformative wave*-equation based technology

- *PDE* constrained optimization or full-waveform inversion (FWI)

Challenges

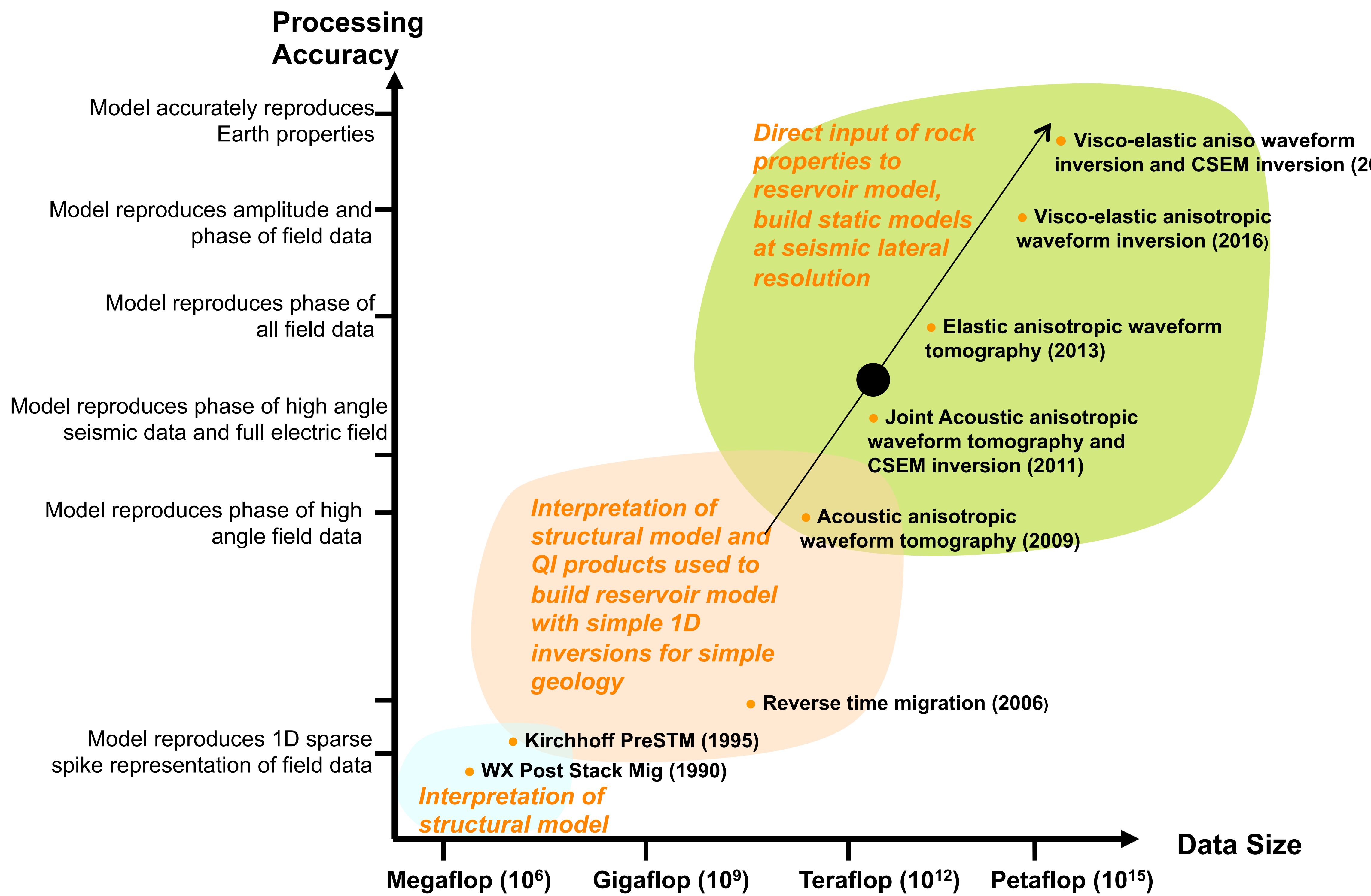
Computational costs

- proportional w/ # of sources
- sky rocket (X 1000) when move to elastic

FWI:

- *nonconvex*, i.e., *local* minima
  (cycle skipping)
- *incomplete* data
  (missing low frequencies, 'Marine' acquisition)

Computational costs



Today's agenda

Brief overview of dimensionality *reduction*

- *randomized* source aggregates/subsets
- *batching* for Quasi-Newton (l-BFGS)
- *CS & AMP* for Gauss-Newton (SPG)

Comparison on *realistic* blind GOM synthetic

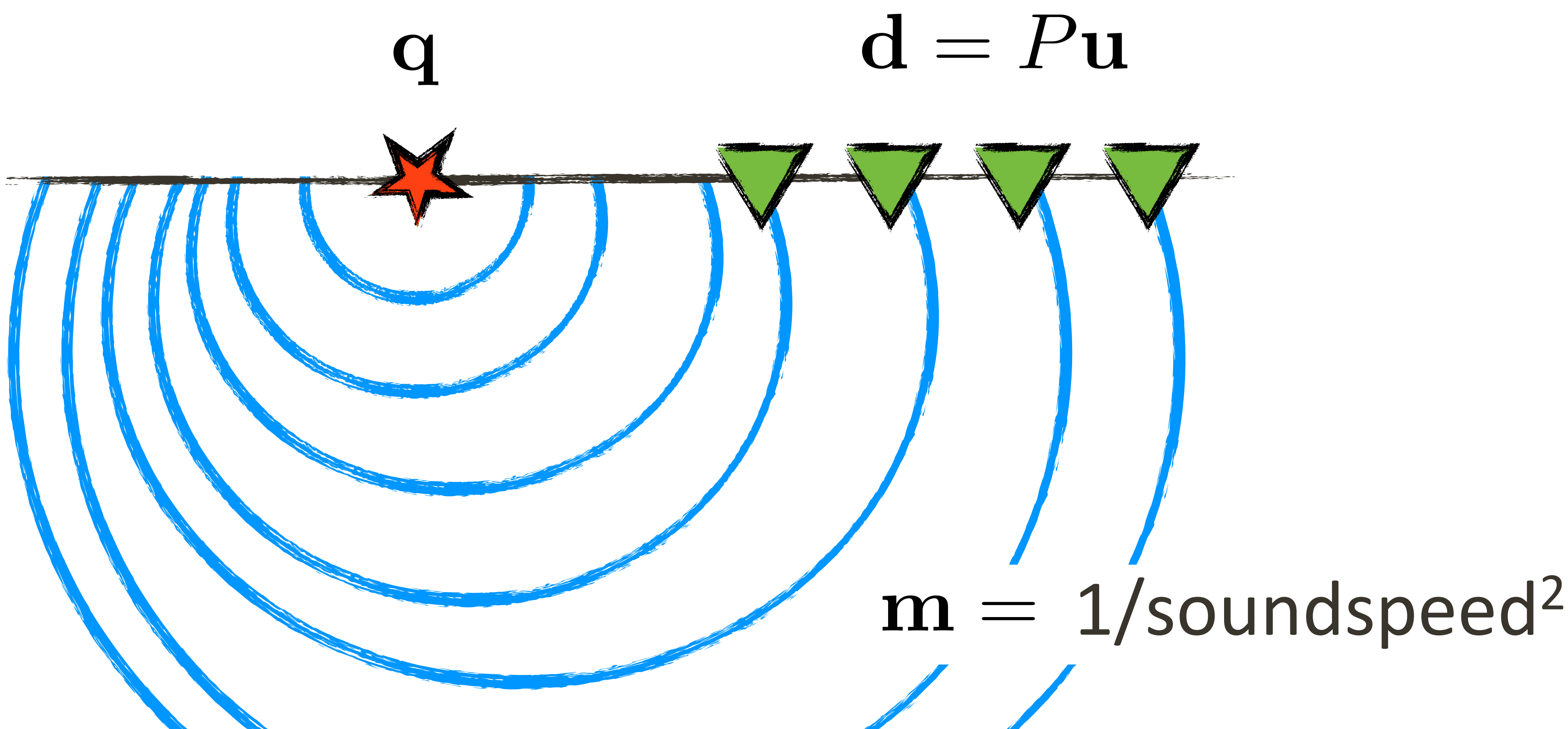
- *data*-side *regularization* & *linear* gradient *smoothings*
- *model*-side *regularization* & *nonlinear* gradient *approximations*

Strategies

- *separable* structure
  (*linearity* w.r.t. *sources*)
- use data *redundancy* to speed up *progress* towards *solution*
  (form *independent* randomized source *subsets*)
- *convex-composite* structure
  (*convex* on the outside & *smooth* within)
- *multiscale* structure
  (*curvelet*-domain *sparsity* GN updates)

Wave-equation based inversion

We model the data in the *acoustic* approximation $(\omega^2\mathbf{m} + \nabla^2)\mathbf{u} = \mathbf{q}$



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Adjoint-state/ *reduced* formulation

non-linear least-squares problem:

$$\min_{\mathbf{m}} \Phi(\mathbf{m}) = \sum_{i=1}^{M} ||\mathbf{d}_i - P_i \mathbf{u}_i||_2^2$$

gradient:

$$\frac{\partial \Phi}{\partial m_k} = \sum_{i=1}^{M} \mathbf{u}_i^H \left(\frac{\partial H(\mathbf{m})}{\partial m_k} \right)^H \mathbf{v}_i$$

where:

$$H(\mathbf{m})\mathbf{u}_i = \mathbf{q}_i$$

$$H(\mathbf{m})^H \mathbf{v}_i = P_i^T (\mathbf{d}_i - P_i \mathbf{u}_i)$$

Inversion of very large sparse linear systems

Batched optimization

$$\min_{\mathbf{m}} \Phi[\mathbf{m}] = \frac{1}{K}\sum_{i=1}^{K} \phi_i[\mathbf{m}]$$

Quasi-Newton approach

$$\mathbf{s}_k = -B_k \nabla\Phi[\mathbf{m}_k]$$
$$\mathbf{m}_{k+1} = \mathbf{m}_k + \lambda_k \mathbf{s}_k$$

But: evaluation of *full* misfit and gradient is very expensive.

Batched optimization

The gradient is the *average*

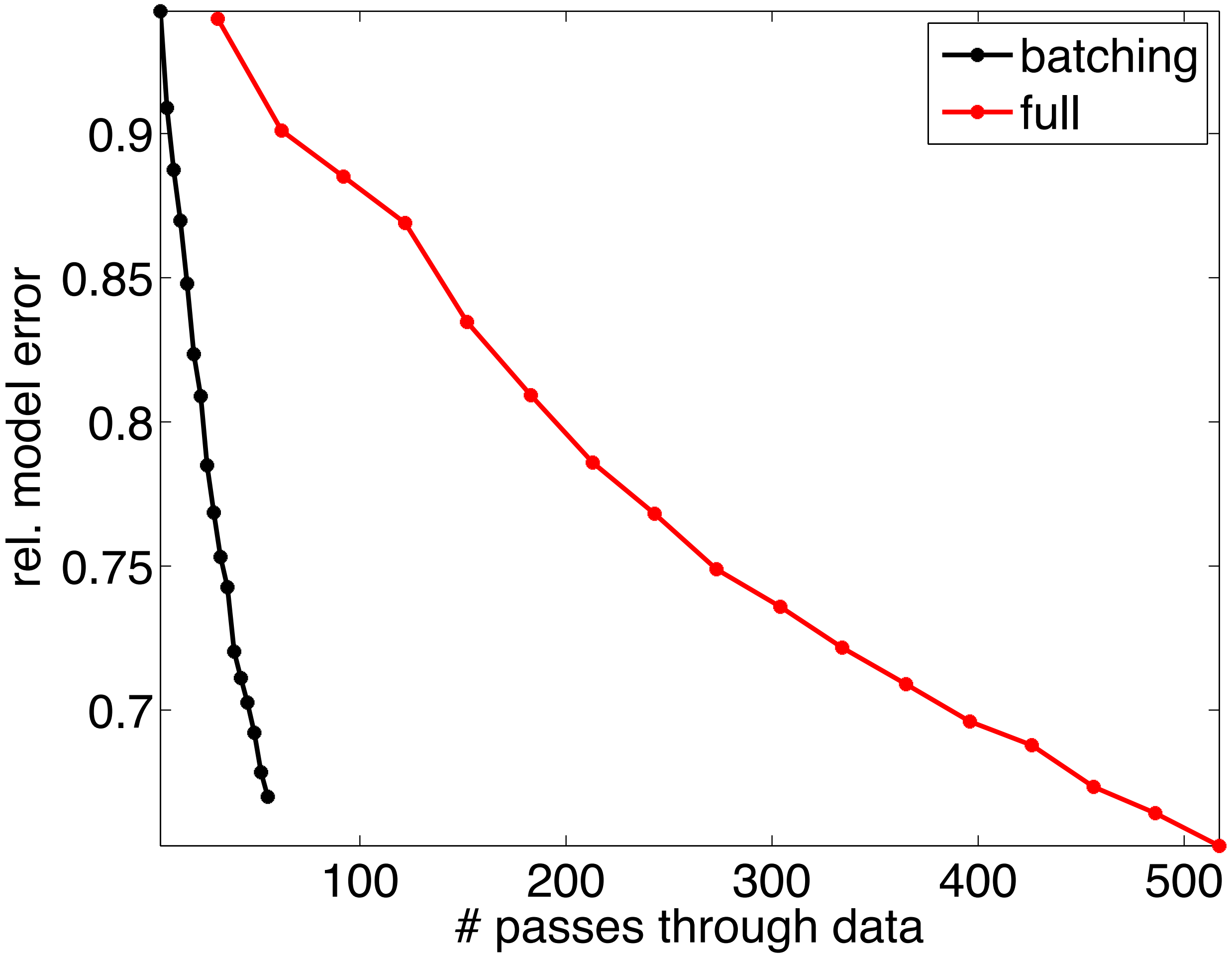
$$\nabla\Phi = \frac{1}{K}\sum_{i=1}^{K}\nabla\phi_i$$

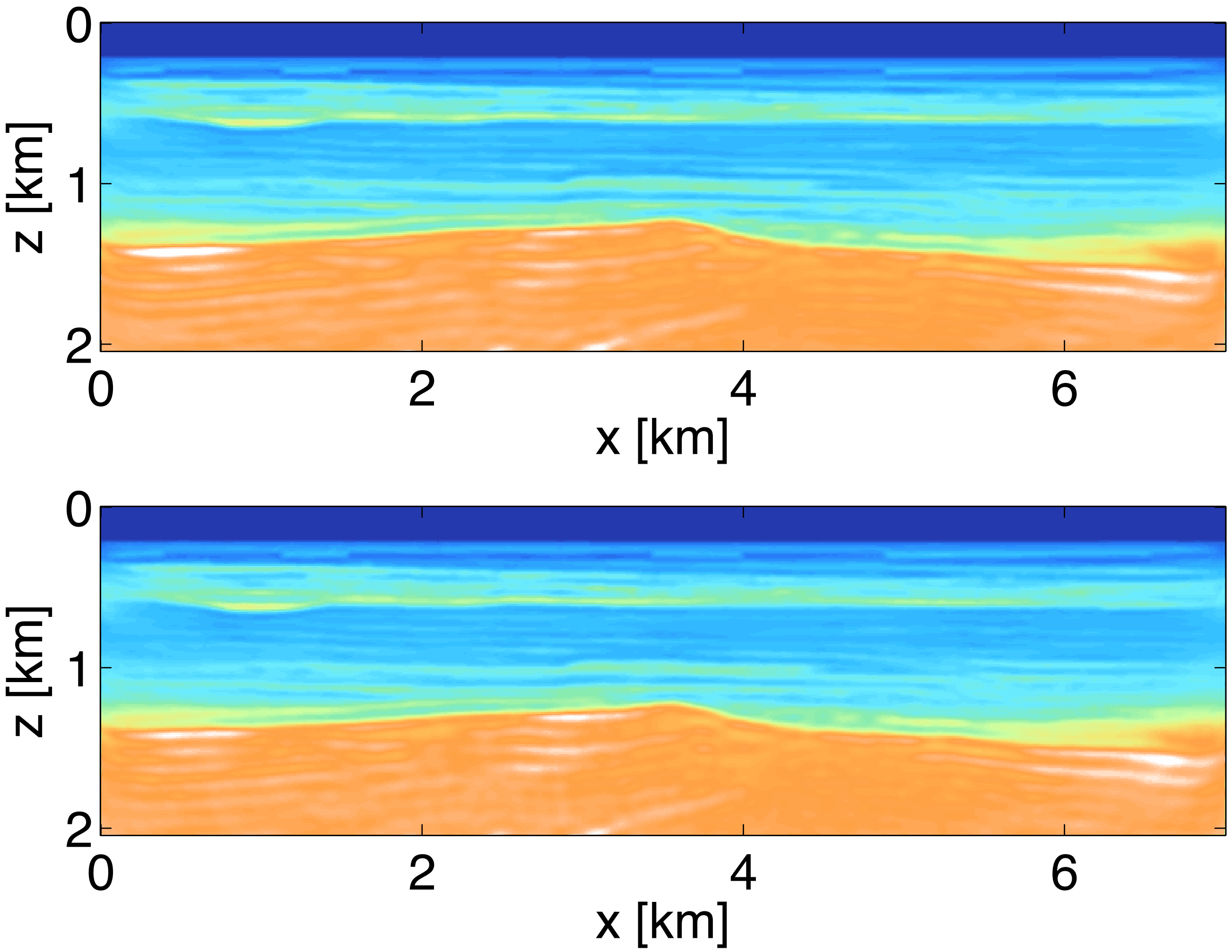
which we can approximate by

$$\nabla\Phi \approx \nabla\widetilde{\Phi} = \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\nabla\phi_i$$

Optimization

10 x speedup





[van Leeuwen & FJH ’11]

Observations

Our *batching* strategy controls *sampling* and/or *simulation* errors

- by *growing* the *batch* size in *accordance* w/ *convergence* rate
- best of both worlds: *stochastic* versus *deterministic*
- removes noise *sensitivity* of *stochastic* gradients

Can we exploit *sparse* structure of gradient *updates*

- *dimensionality* reduction w/ Compressive Sensing
- *acceleration* w/ Approximate Message Passing

Convex composite structure [Burke & Ferris, '95.]

FWI:

$$\min_{\mathbf{m}} \quad \phi(\mathbf{m}) := \underbrace{\frac{1}{2}\| \overbrace{\mathbf{D} - \boldsymbol{\mathcal{F}}[\mathbf{m};\mathbf{Q}]}^{\text{smooth}} \|_F^2}_{\text{convex}}$$

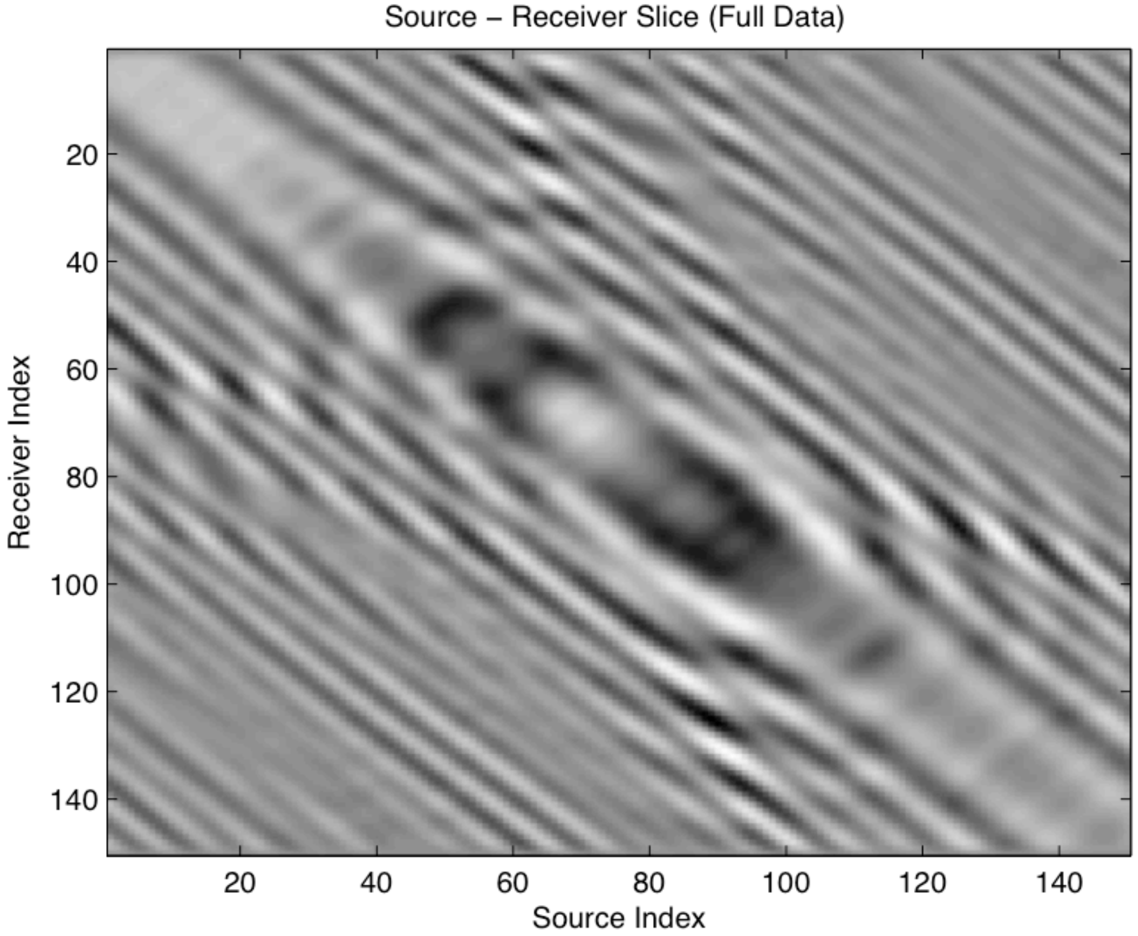
- exploit *convexity* by linearizing *within*

$$\min_{\boldsymbol{\delta}\mathbf{m}} \quad \phi(\boldsymbol{\delta}\mathbf{m}) := \frac{1}{2}\|\mathbf{D} - \boldsymbol{\mathcal{F}}[\mathbf{m};\mathbf{Q}] - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m};\mathbf{Q}]\boldsymbol{\delta}\mathbf{m}\|_F^2$$

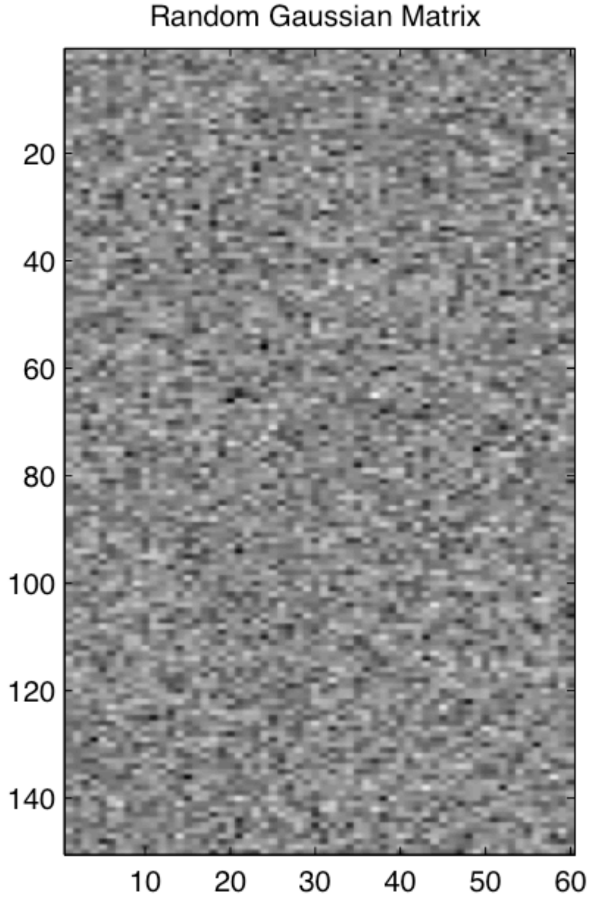
- control the norm of the updates to *guarantee* convergence

Randomized source aggregates

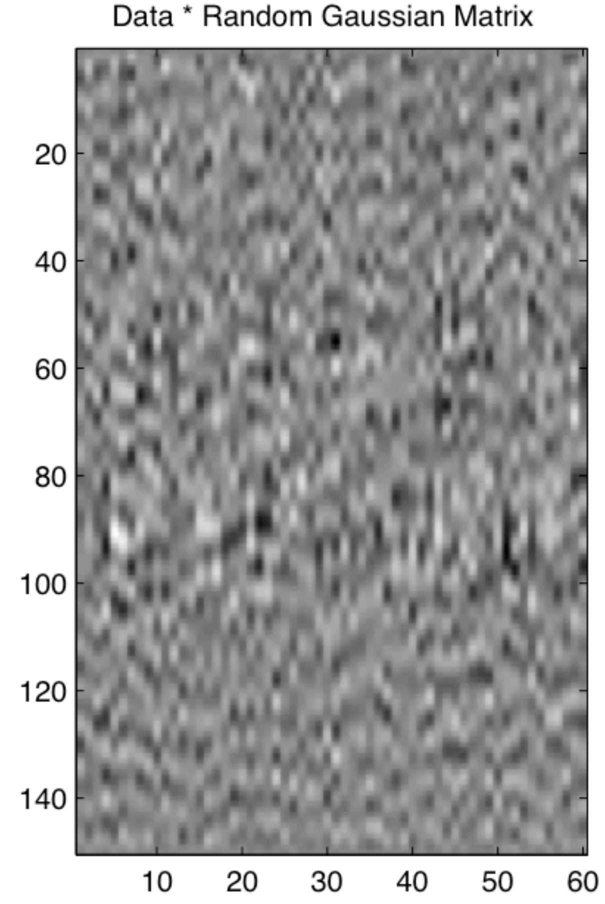
$$\mathbf{D}$$



$$\color{red}{\mathbf{W}}$$



$$\color{red}{\underline{\mathbf{D}}} = \mathbf{D}\color{red}{\mathbf{W}}$$



Convex optimization

[p=2 or p=1]

Linearized inversion with *randomized supershots:*

$$\delta\widetilde{\mathbf{m}} = \mathbf{S}^* \operatorname*{arg\,min}_{\delta\mathbf{x}} \|\delta\mathbf{x}\|_{\ell_p} \quad \text{subject to} \quad \| \underbrace{\delta\underline{\mathbf{d}}}_{\mathbf{b}} - \underbrace{\overbrace{\nabla\boldsymbol{\mathcal{F}}[\mathbf{m}_0;\underline{\mathbf{Q}}]}^{\text{Jacobian}}\mathbf{S}^*}_{\mathbf{A}} \delta\mathbf{x}\|_2 \leq \sigma$$

$$\begin{aligned}
\delta\mathbf{x} &= \text{Sparse curvelet-coefficient vector}\\
\mathbf{S}^* &= \text{Curvelet synthesis}\\
\underline{\mathbf{Q}} &= \text{Simultaneous sources}\\
\delta\underline{\mathbf{d}} &= \text{Super shots}
\end{aligned}$$

Fast Gauss-Newton step
[*via* stochastic optimization]

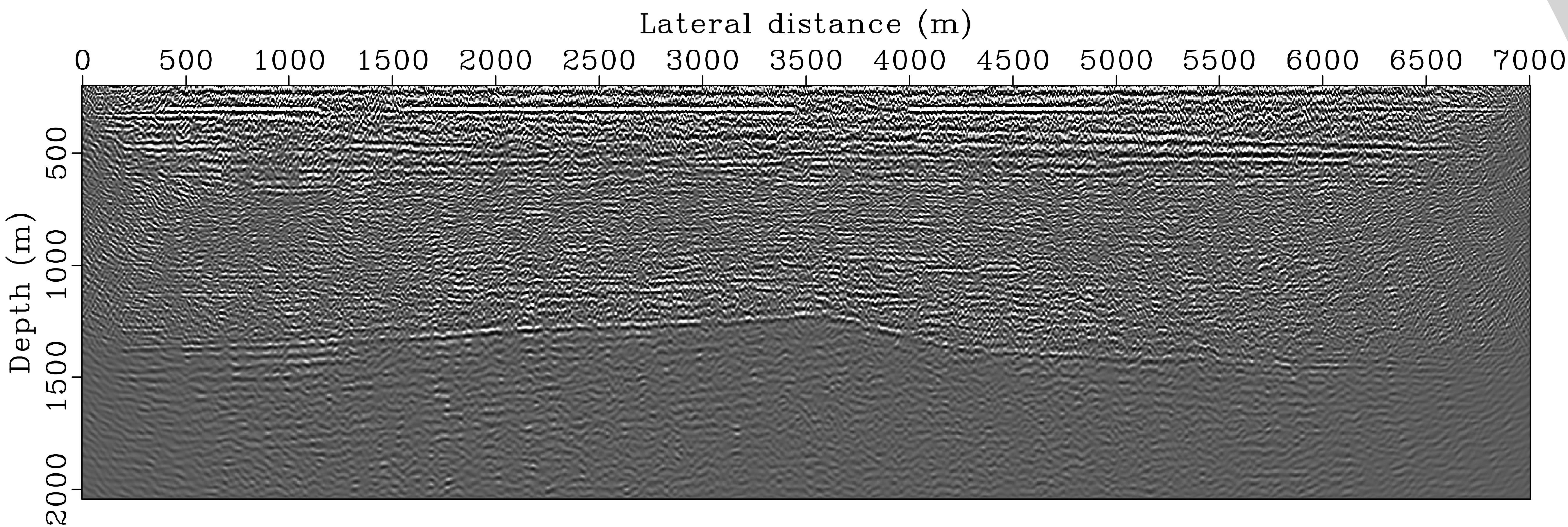
*E*xploit multi-experiment *redundancy* of seismic data volumes by *re*randomized sampling

- *regularly* draw *independent* source *aggregates*
- cancels *crosstalk/interference* by *rerandomization*

Heuristic of *current* phase-encoding/*dimensionality* reduction for imaging/FWI

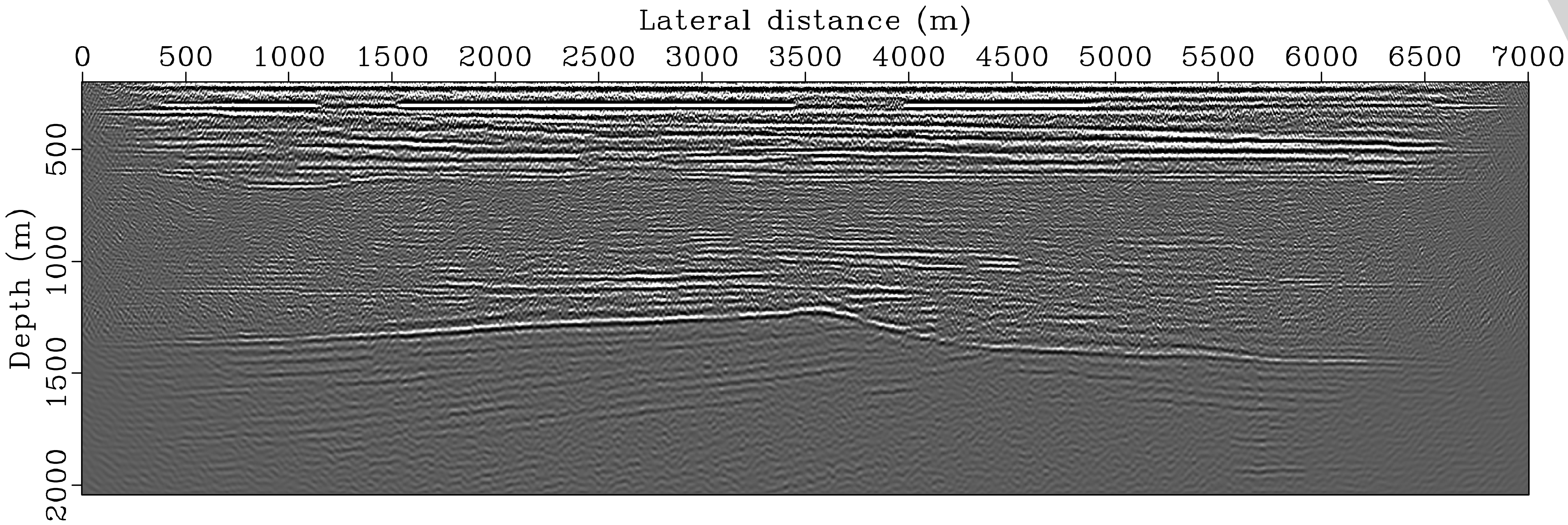
Fast Gauss-Newton step

[ℓ_2 *w/o rerandomization 3 super shots*]



Fast Gauss-Newton step

[ℓ_2 *w/ rerandomization* 3 *super* shots]



Fast Gauss-Newton step

[*via* compressive sensing]

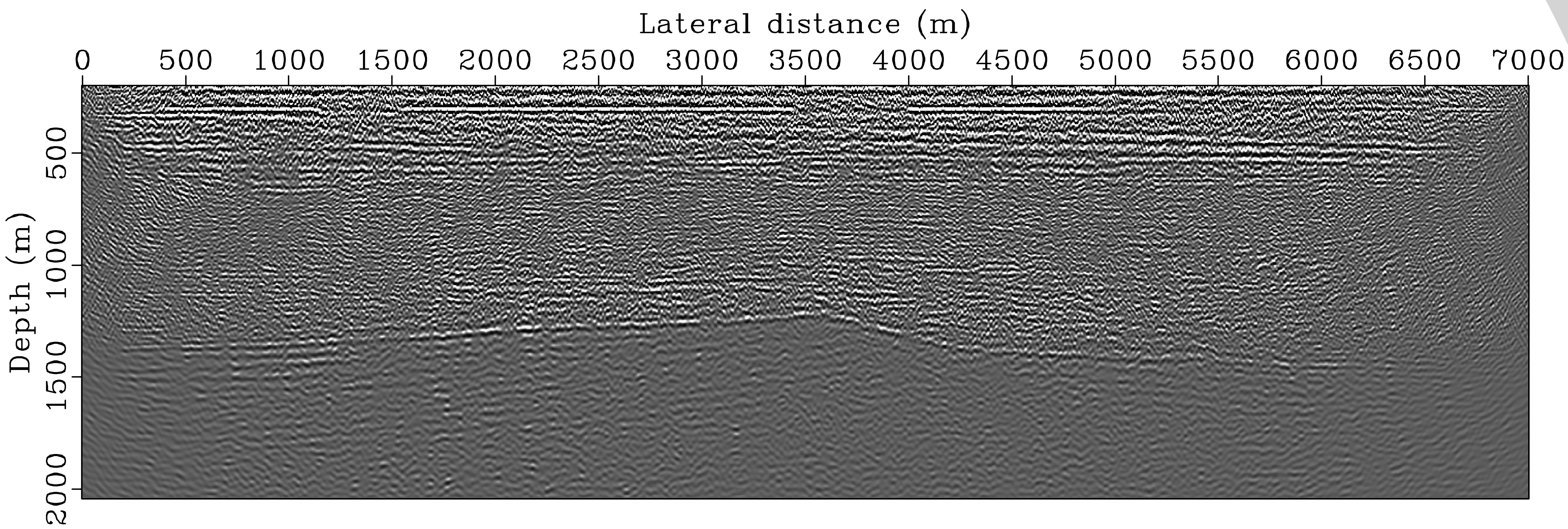
Randomized sampling turns *coherent* source crosstalk/ interferences into ***non-sparse*** *incoherent* noise.

Exploits transform-domain *structure* exhibited *within* GN *updates*

- leverage *curvelet*-domain ***sparsity*** promotion
- *map* "noisy" crosstalk/interferences to *coherent* reflectors

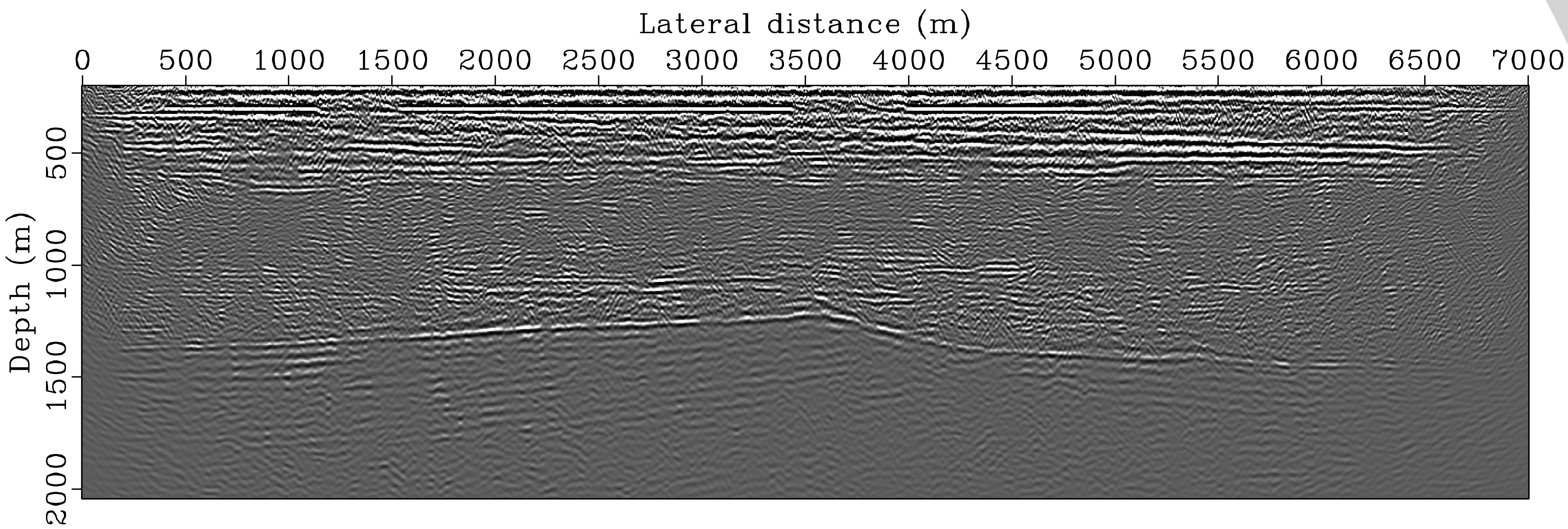
Fast Gauss-Newton step

[ℓ_2 3 *super* shots]



Fast Gauss-Newton step

[ℓ_1 3 *super* shots]



Observations
[w/ reasonable PDE solve budget]

Rerandomization and *curvelet*-domain *sparsity* promotion:

- *partly* eliminate "*noisy*" *crosstalk*
- *fail* to *remove* "small" *incoherent* crosstalk

Can we somehow combine these two methods?

- *continuation* method for large-scale *convex* optimization
- use *insights* from *approximate* message passing

Supercooling

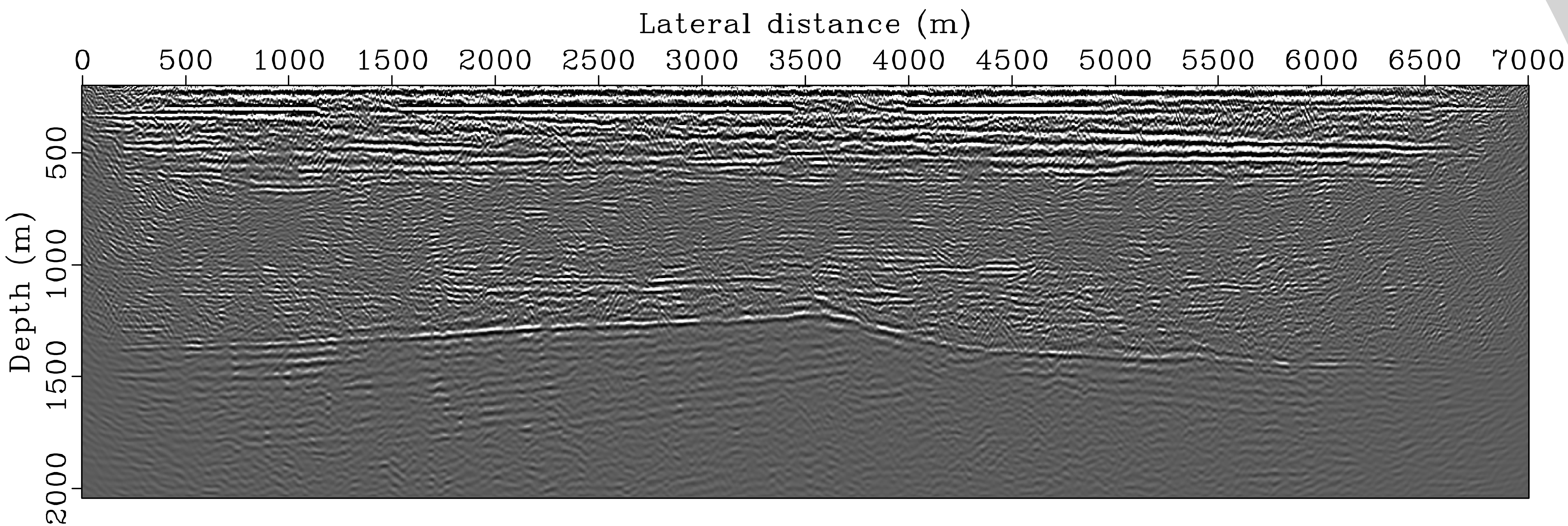
Break *correlations* between the model *iterate* and matrix **A** by *rerandomization*

- draw new *independent* $\{\mathbf{b}_t, \mathbf{A}_t\}$ after each subproblem is solved
- brings in "*extra*" information *without* growing the *system*
- ***minimal*** *extra* computational & memory cost

Progress one-norm solvers *no* longer *stalled*...

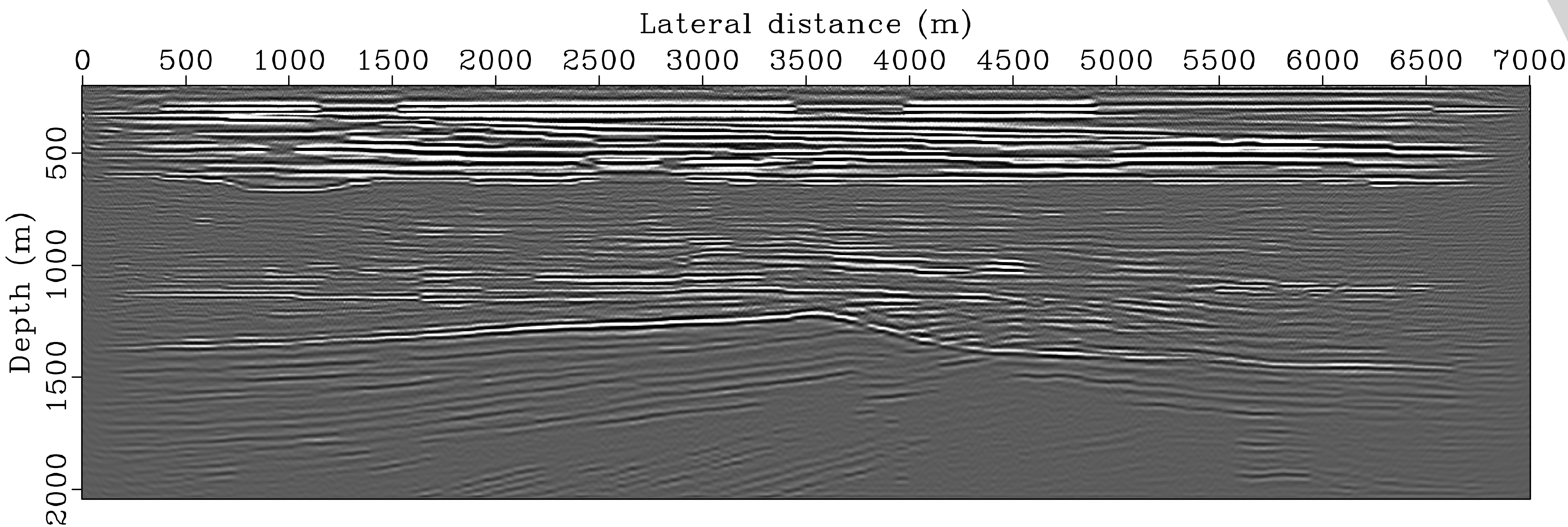
Fast Gauss-Newton step

[ℓ_1 *w/o* rerandomization 3 *super* shots]



Fast Gauss-Newton step

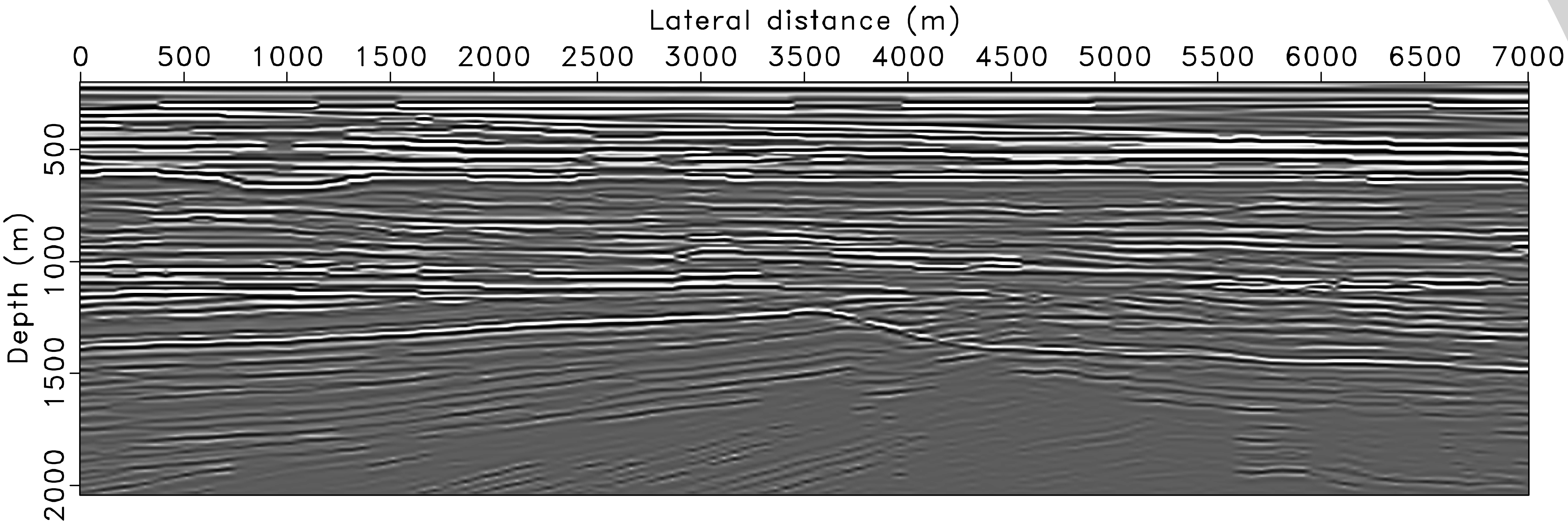
[ℓ_1 *w/* rerandomization 3 *super* shots]



cost of 1/2 *gradient* update w/ *all* data

Fast Gauss-Newton step

[*true* update]

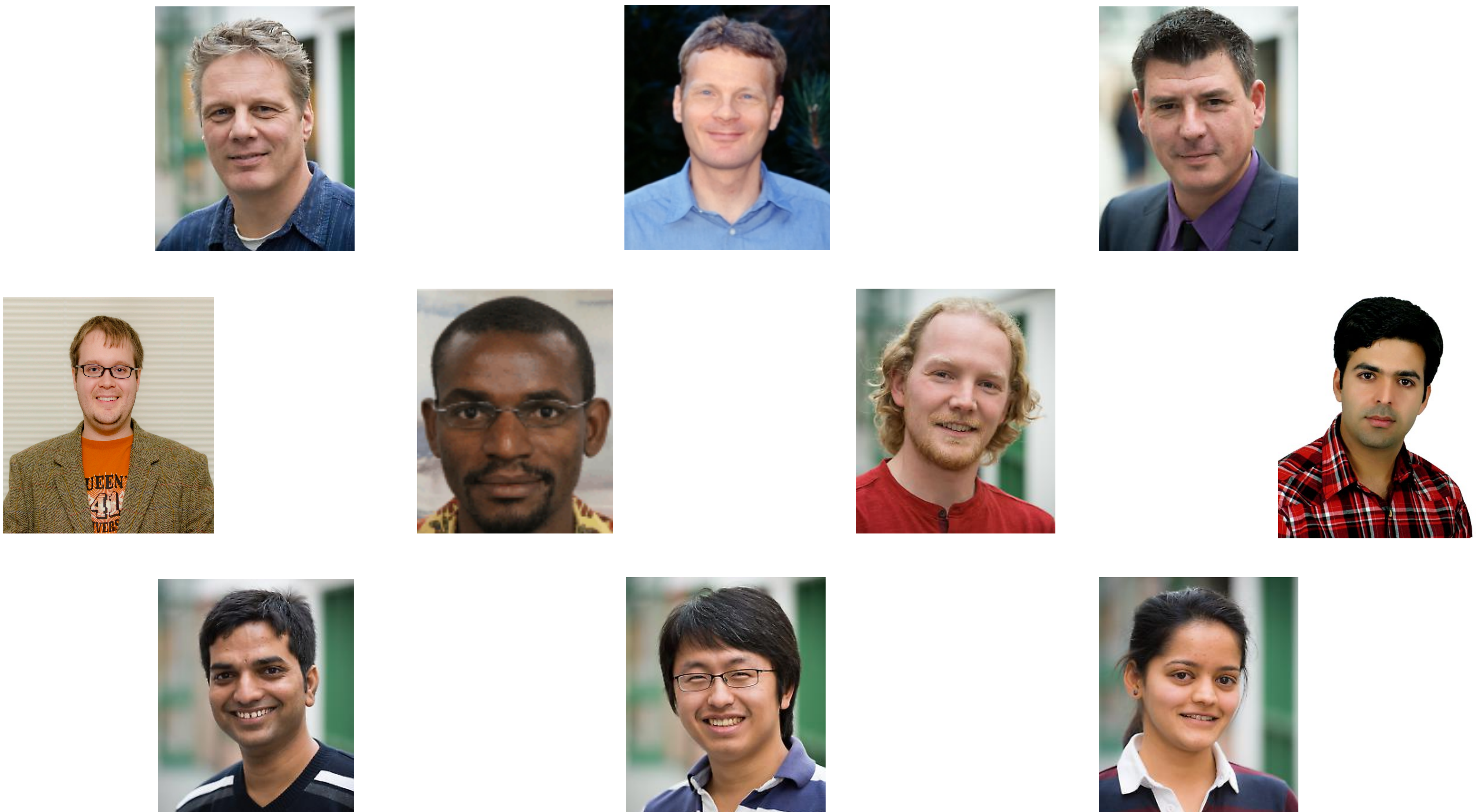


Blind GOM subsalt case study

Elastic synthetic data (3200 shots) for an *unknown* earth model was generated by a team from Chevron, ExxonMobil, and Schlumberger

Several contractor companies worked w/ large teams for weeks/months to get results w/ a lot of "hand holding"...

Dream team



Comparison

- nonlinear conjugate gradients
  - ***linear*** *anisotropic* smoothing of *gradients*
- *modified* Gauss-Newton
  - ***nonlinear*** "smoothing"/ *approximations* of *gradients* with *anisotropic* curvelets

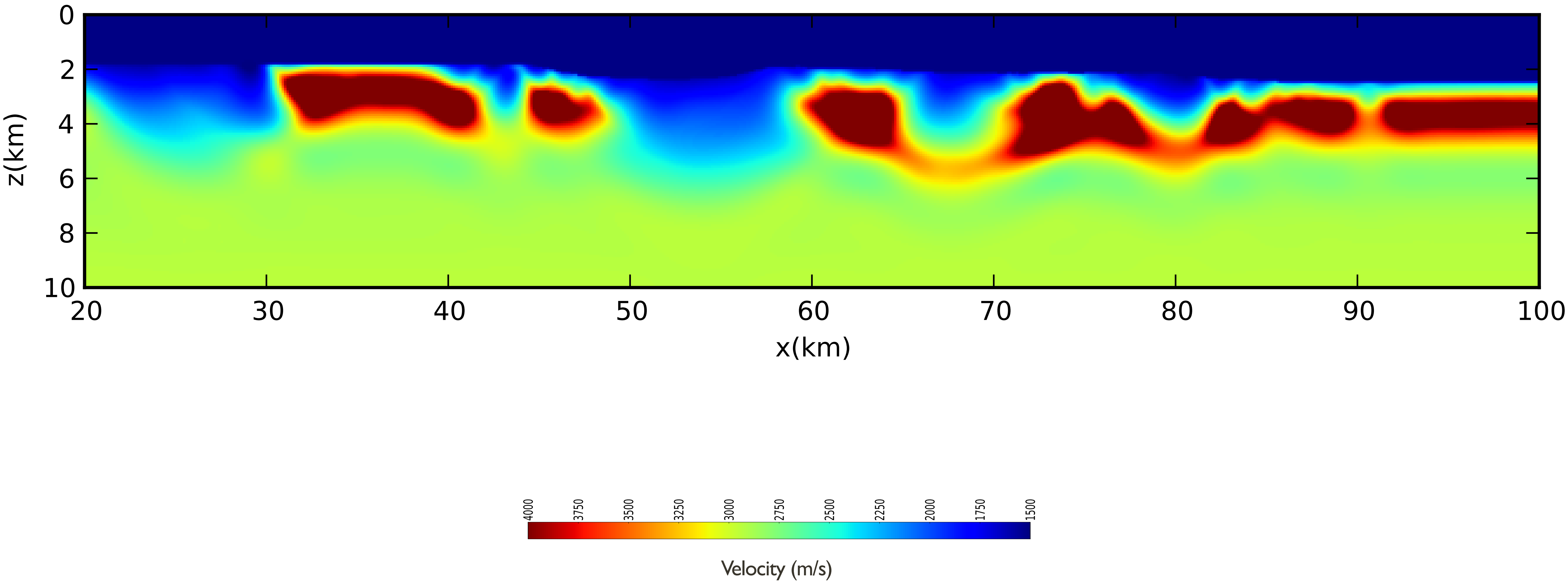
Acoustic FWI workflows with *minimal* parameters & user *intervention*...

Preprocessing

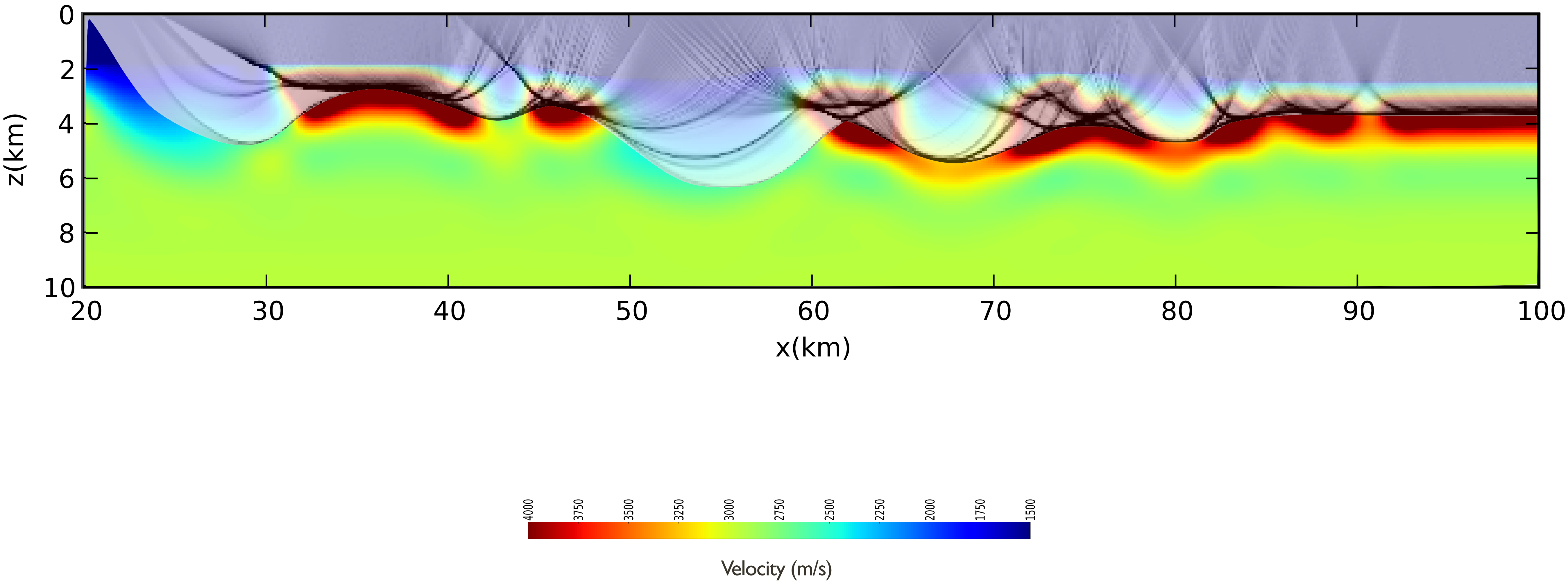
Travel-time *tomography* based on hand-picked *first* breaks ⟶ *initial velocity model*

Curvelet-denoising at selected *low*-frequencies ⟶ *improve SNR* for FWI

Initial model
[ray-based tomography]



Ray paths
[RMS traveltime misfit 11ms]



Denoising

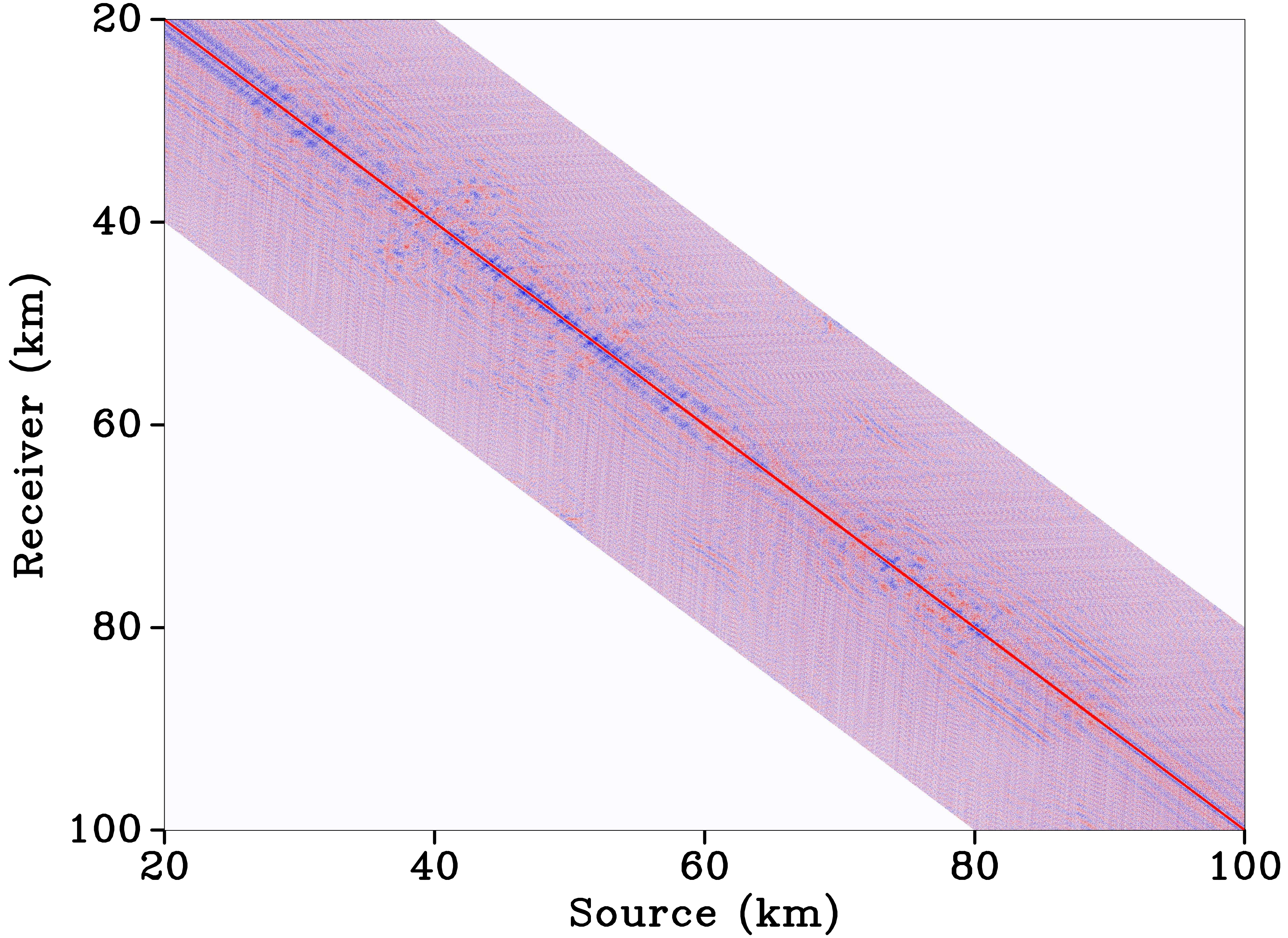
Work on “frequency slices” in the *source/ receiver* plane

Find *support* by *hand*-selected *thresholding* of *syntheses* curvelet coefficients

Followed by *debiasing* to restore *amplitudes*...

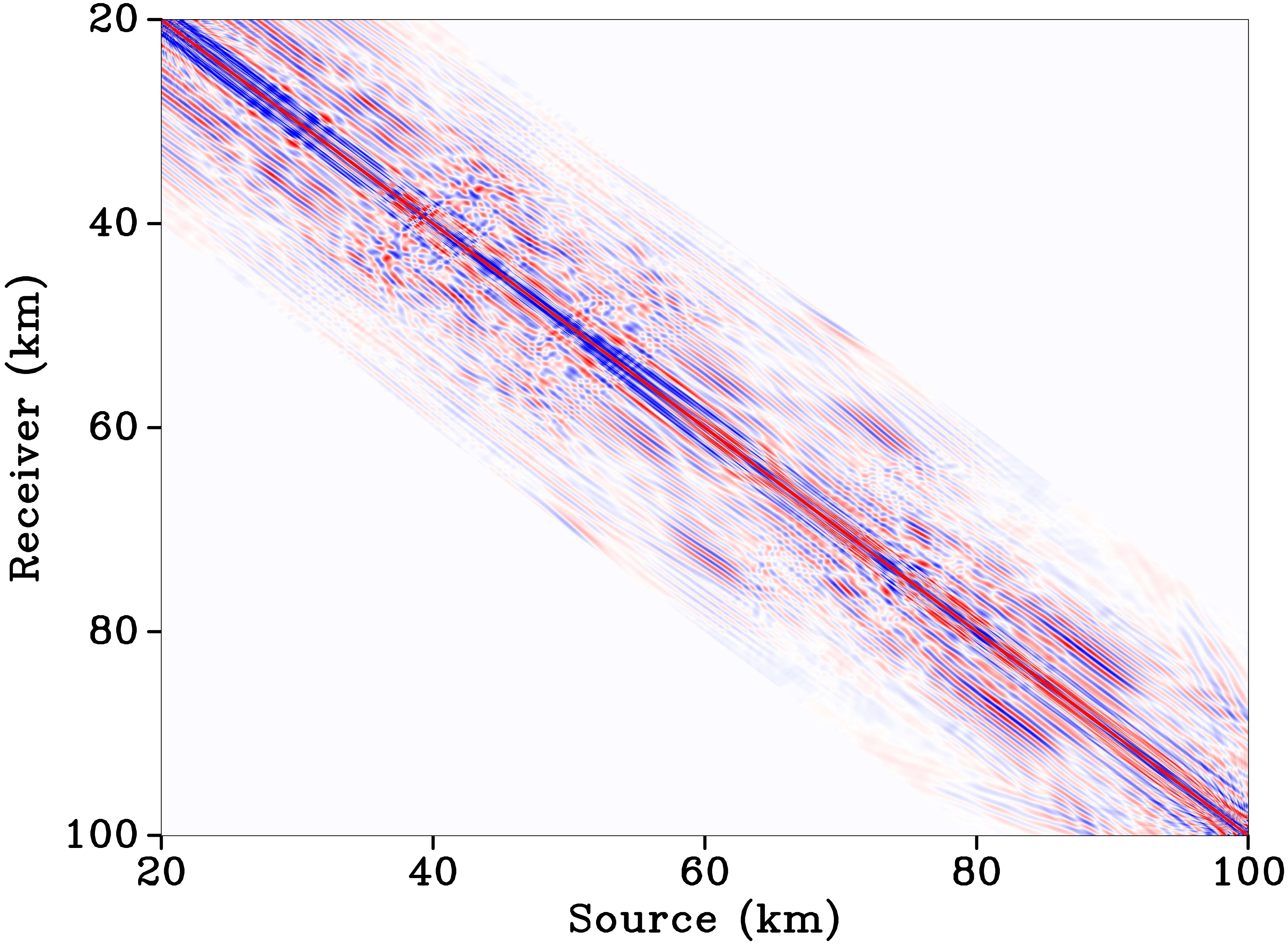
Before denoising

[real part at 2.0 Hz]



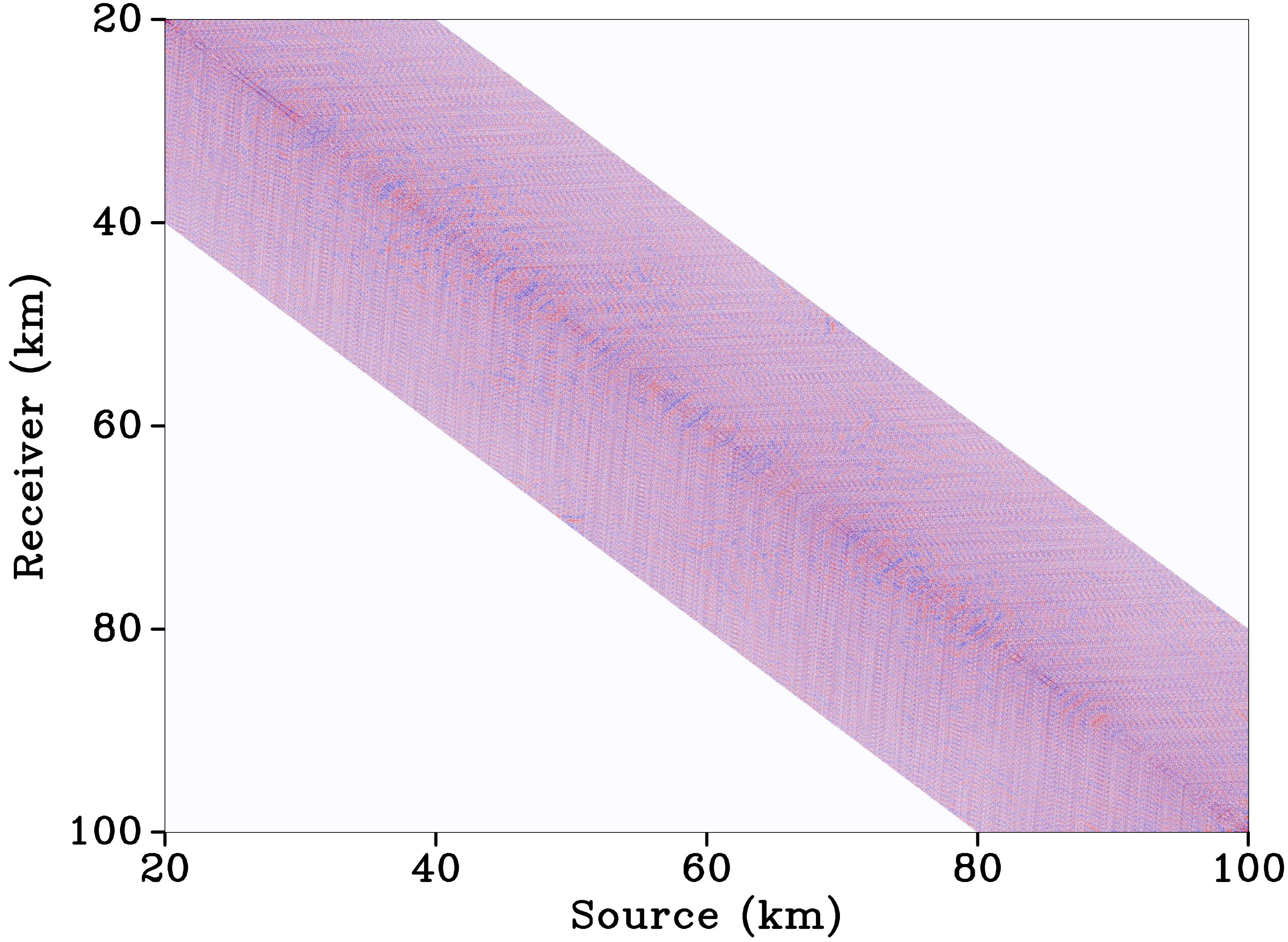
After denoising

[real part at 2.0 Hz]



Difference

[real part at 2.0 Hz]

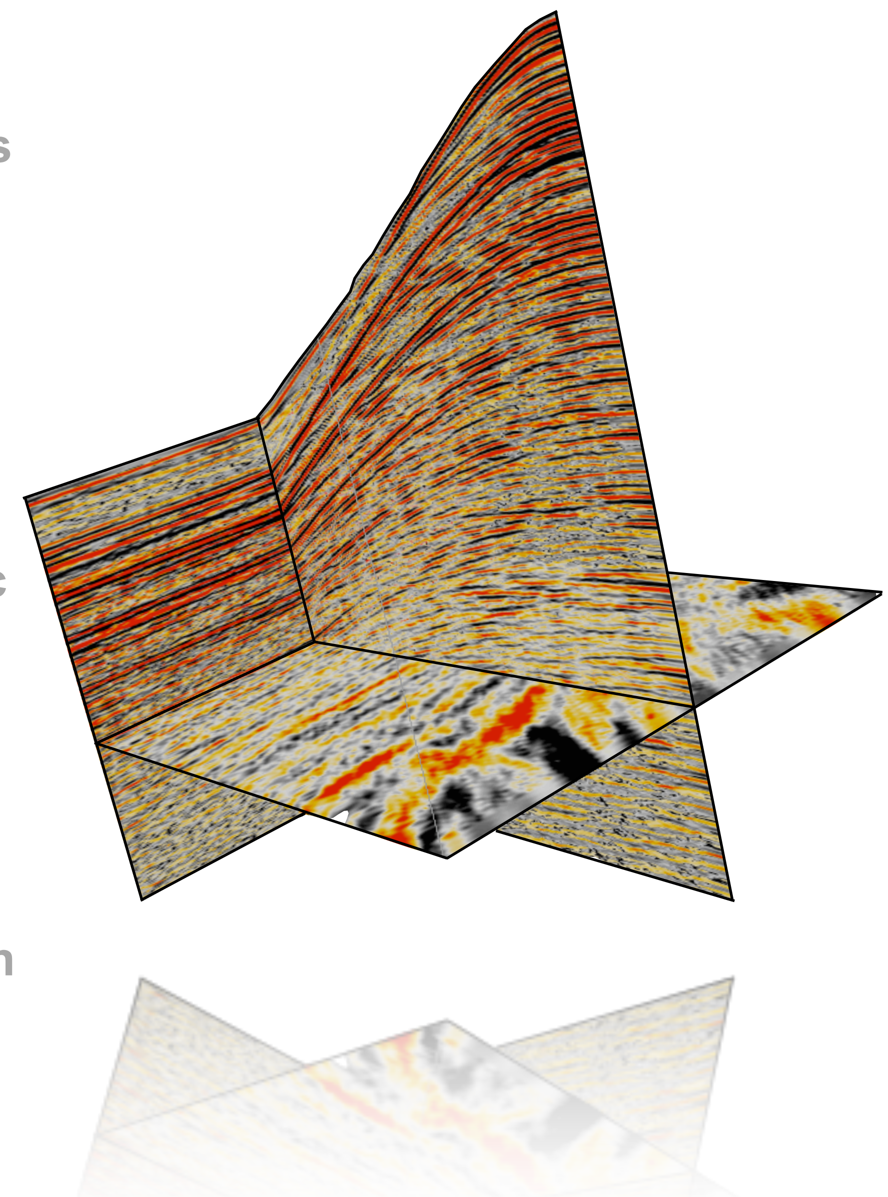


Nonlinear Conjugate Gradients

Pratt, R.G., Shin, Changsoo and Hicks, G.J., 1998. Gauss-Newton and full Newton methods in frequency domain seismic waveform inversion. Geophysical Journal International, 133, 341-362.

Brenders, A. J. and Pratt, R. G., 2007. Full waveform tomography for lithospheric imaging: results from a blind test in a realistic crustal model, Geophysical Journal International, 168, 133-151

Takam Takougang, E. M. and Calvert, A. J., 2011. Application of waveform tomography to marine seismic reflection data from the Queen Charlotte Basin of western Canada, Geophysics,76, B1-B16



Methodology

Non-linear conjugate gradients (first order)

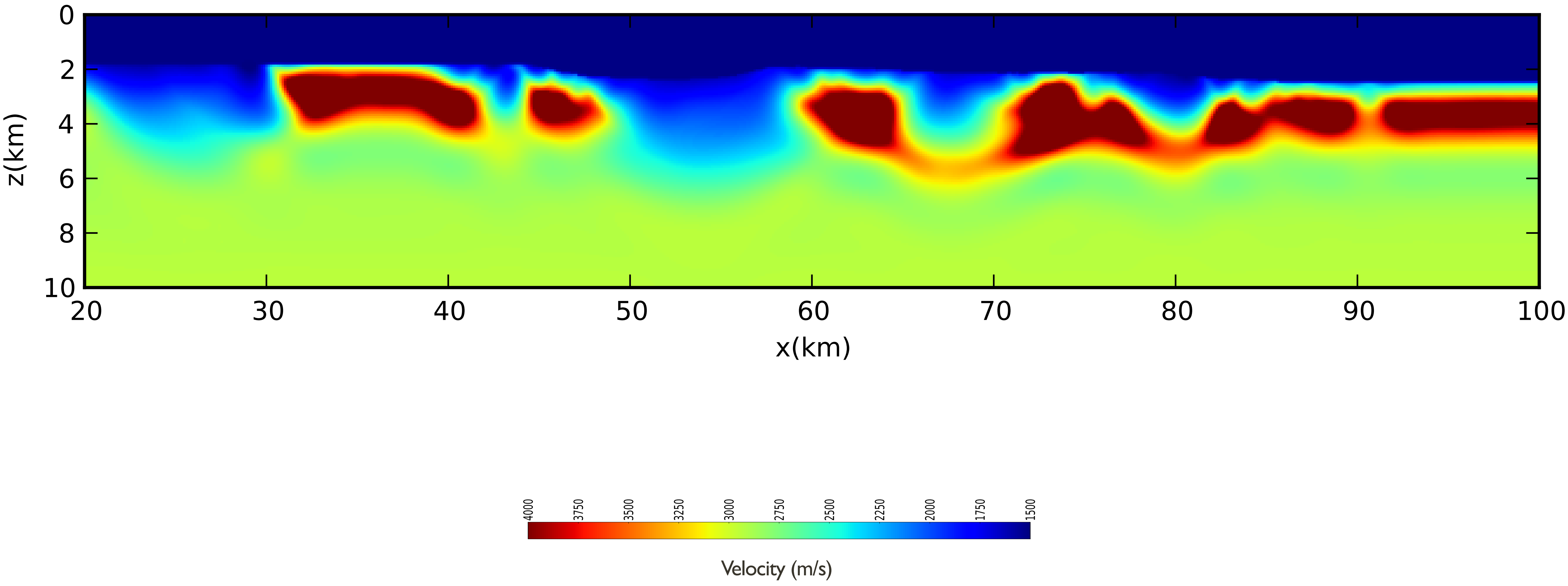
Scaling by *RMS* from (initial) *forward* model

Layer stripping with gradient & offset weights

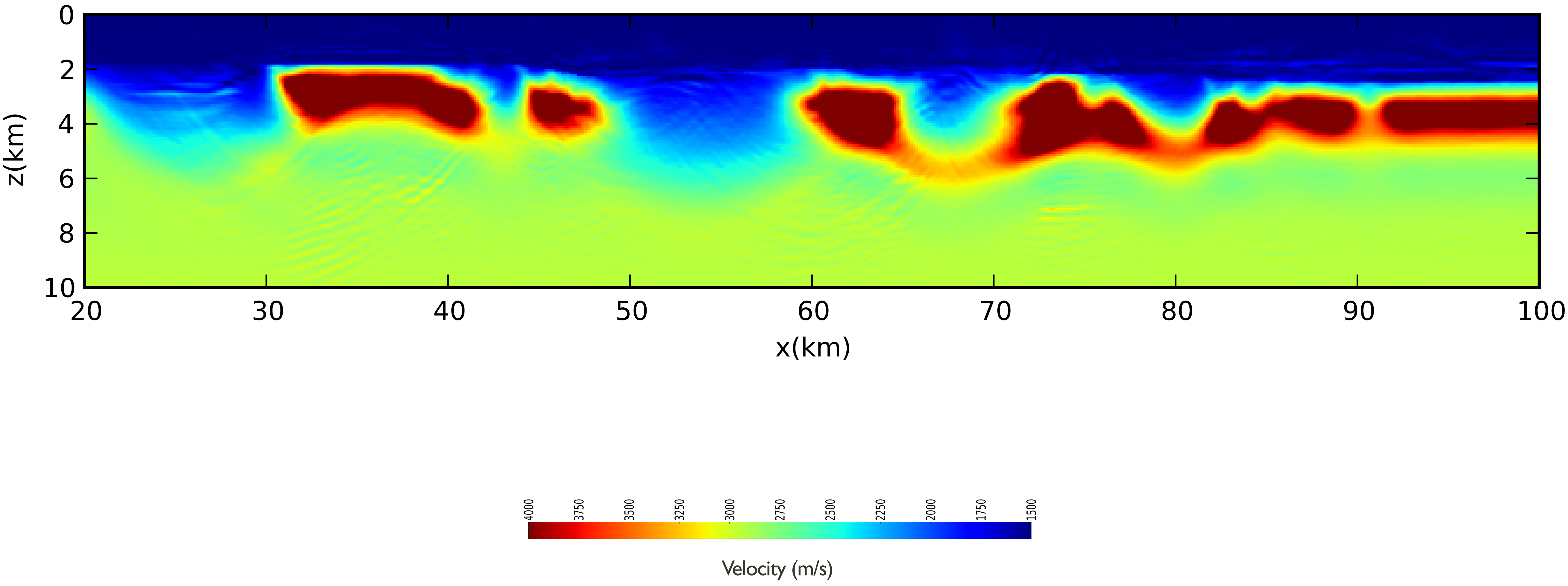
- *phase* only 2–7 Hz

Anisotropic *smoothing* on *gradients*...

Initial model
[ray-based tomography]



Final result
[w/o denoising]



Batched FWI with nuisance parameter estimation

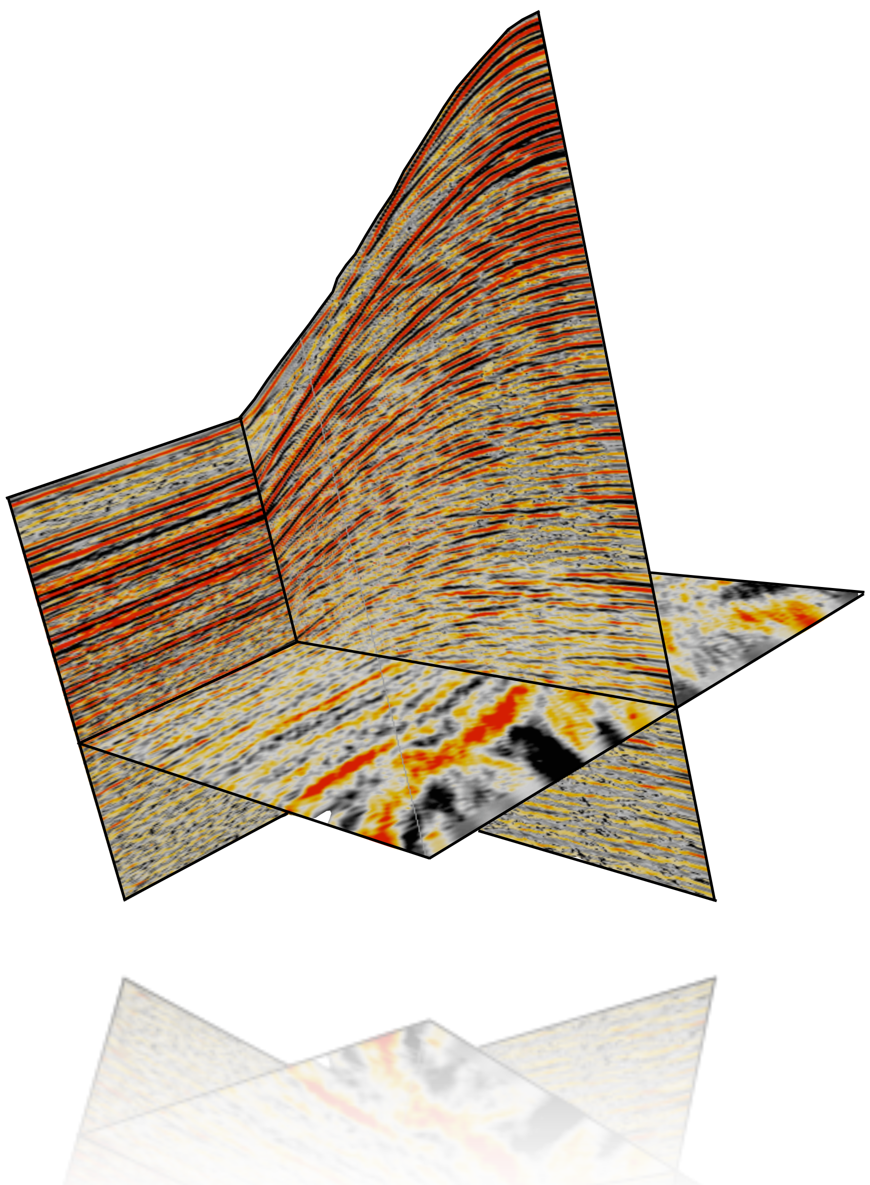
A.Y. Aravkin and T. van Leeuwen - Estimating nuisance parameters in inverse problems. Inverse Problems, 2012.

T. van Leeuwen and F.J. Herrmann - Fast waveform inversion without source encoding. Geophysical Prospecting, 2012

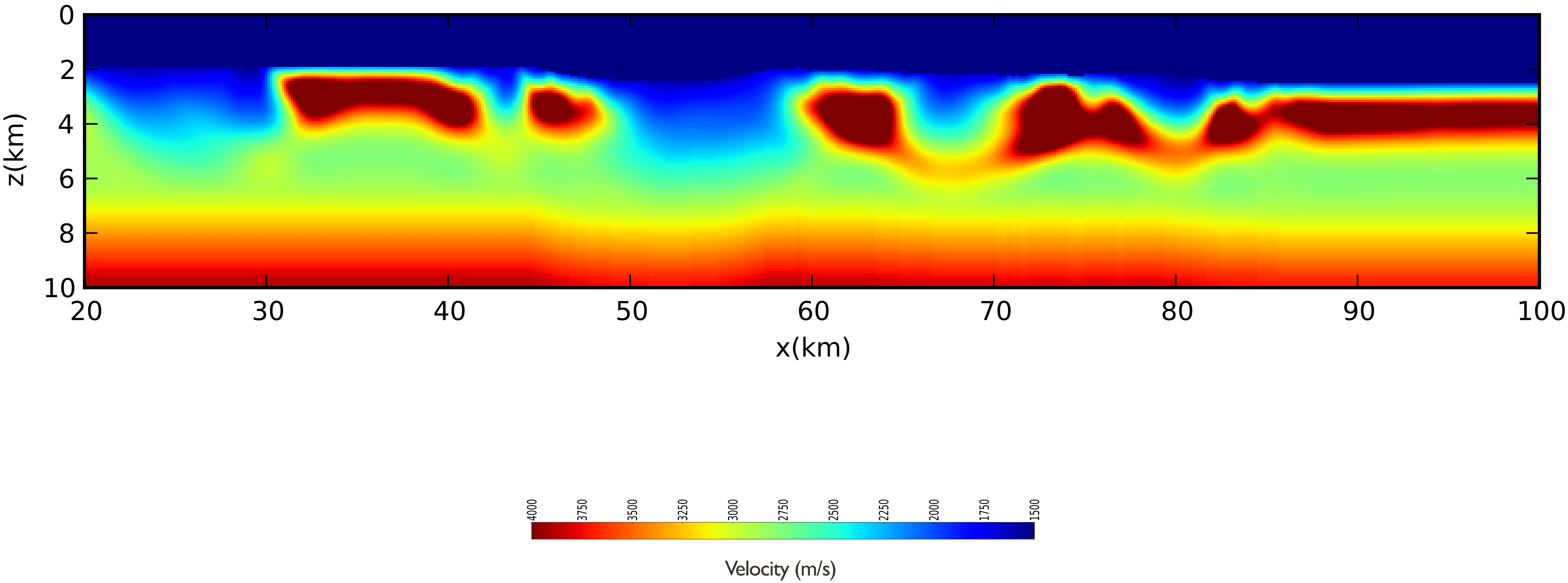
A. Aravkin, M.P. Friedlander, F.J. Herrmann and T. van Leeuwen - Robust inversion, dimensionality reduction and randomized sampling. Mathematical Programming, 2012.

Tristan van Leeuwen, “A parallel matrix-free framework for frequency-domain seismic modelling, imaging and inversion in Matlab”. 2012.

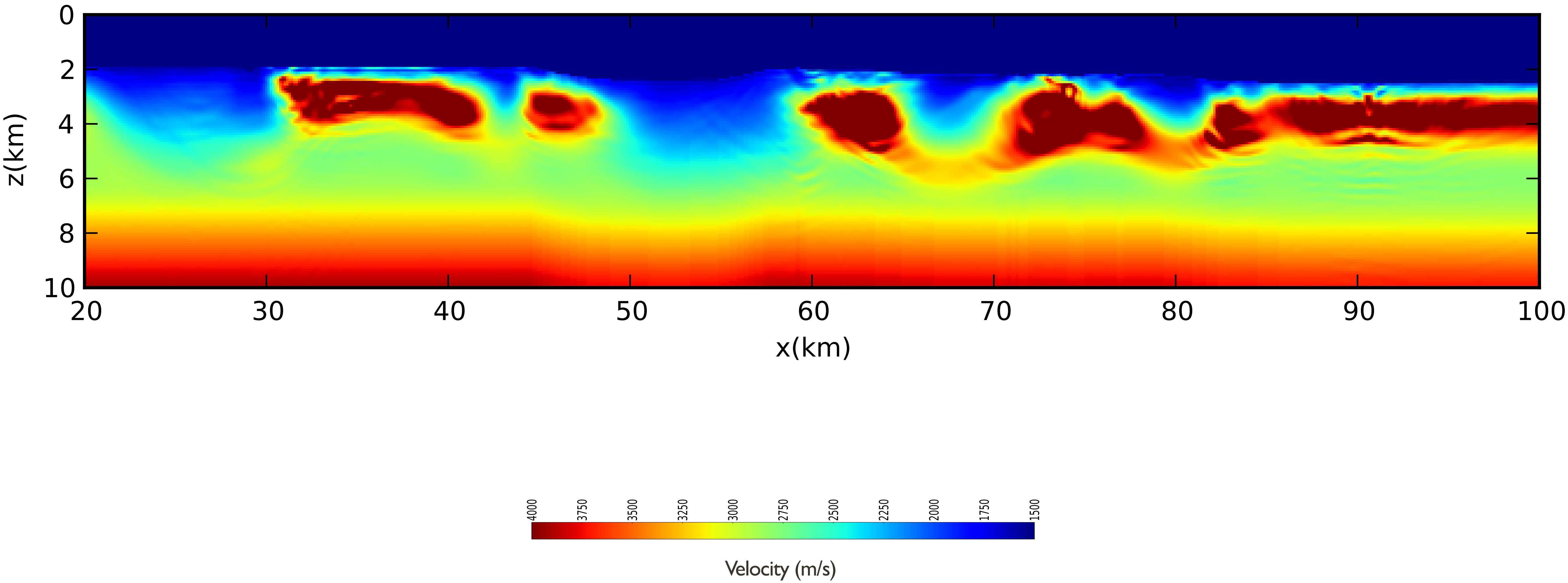
Tristan van Leeuwen and Felix J. Herrmann, “A parallel, object-oriented framework for frequency-domain wavefield imaging and inversion.”. 2012.



Input Model
[ray-based tomography + NMO]



Final result
[w/o denoising]

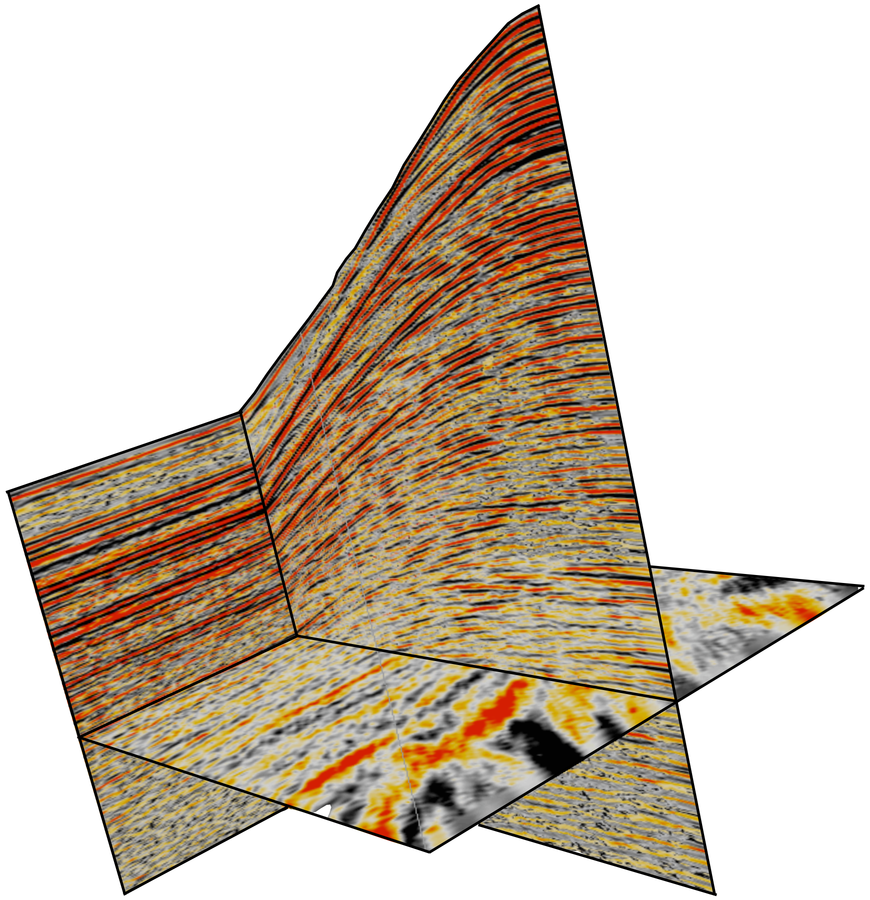


Modified Gauss-Newton

F. J. Herrmann, X. Li, A. Y. Aravkin, and T. van Leeuwen, “A modified, sparsity promoting, Gauss-Newton algorithm for seismic waveform inversion”, in Proc. SPIE, 2011, vol. 2011.

X. Li, F.J. Herrmann, A.Y. Aravkin and T. van Leeuwen - Fast randomized full waveform inversion with compressed sensing. Geophysics 77 (A13), 2012.

F.J. Herrmann, X. Li, A.Y. Aravkin, and T. van Leeuwen - A modified, sparsity-promoting, Gauss-Newton algorithm for seismic waveform inversion. Proc. SPIE 8138, 81380V (2011



Methodology

Modified Gauss-Newton:

- frequency continuation (7 bands 4 freqs, 2-5Hz)
- *rerandomized subsets* with *only* 600 shots
- 6 GN *iterations* per frequency *band*
- *preconditioning* of *Jacobian* by depth *weighting*
- *projection* of water *layer*

One-norm curvelet *regularized* gradients...

Algorithm
modified Gauss-Newton

Result: Output estimate for the model $\mathbf{m}$

$k \longleftarrow 0; \mathbf{m}^k \longleftarrow \mathbf{m}_0$

while not converged **do**

$\{\underline{\mathbf{D}}^k, \underline{\mathbf{Q}}^k\} \longleftarrow \{\mathbf{D}\mathbf{W}^k, \mathbf{Q}\mathbf{W}^k\}$ with $\mathbf{W}^k \subset [\mathbf{e}_1, \cdots, \mathbf{e}_{n_s}]$

$\underline{\delta\mathbf{D}}^k \longleftarrow \underline{\mathbf{D}}^k - \boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]$

$\tau^k \longleftarrow \|\underline{\delta\mathbf{D}}^k\|_F \,/\, \|\mathbf{C}_2 \nabla\boldsymbol{\mathcal{F}}^*[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\underline{\delta\mathbf{D}}^k\|_\infty$

$\delta\mathbf{x} \leftarrow \arg\min_{||\mathbf{x}||_1 \leq \tau_k} \|\underline{\delta\mathbf{D}}^k - \nabla\boldsymbol{\mathcal{F}}[\mathbf{m}^k; \underline{\mathbf{Q}}^k]\mathbf{C}_2^H \mathbf{x}\|_F^2$

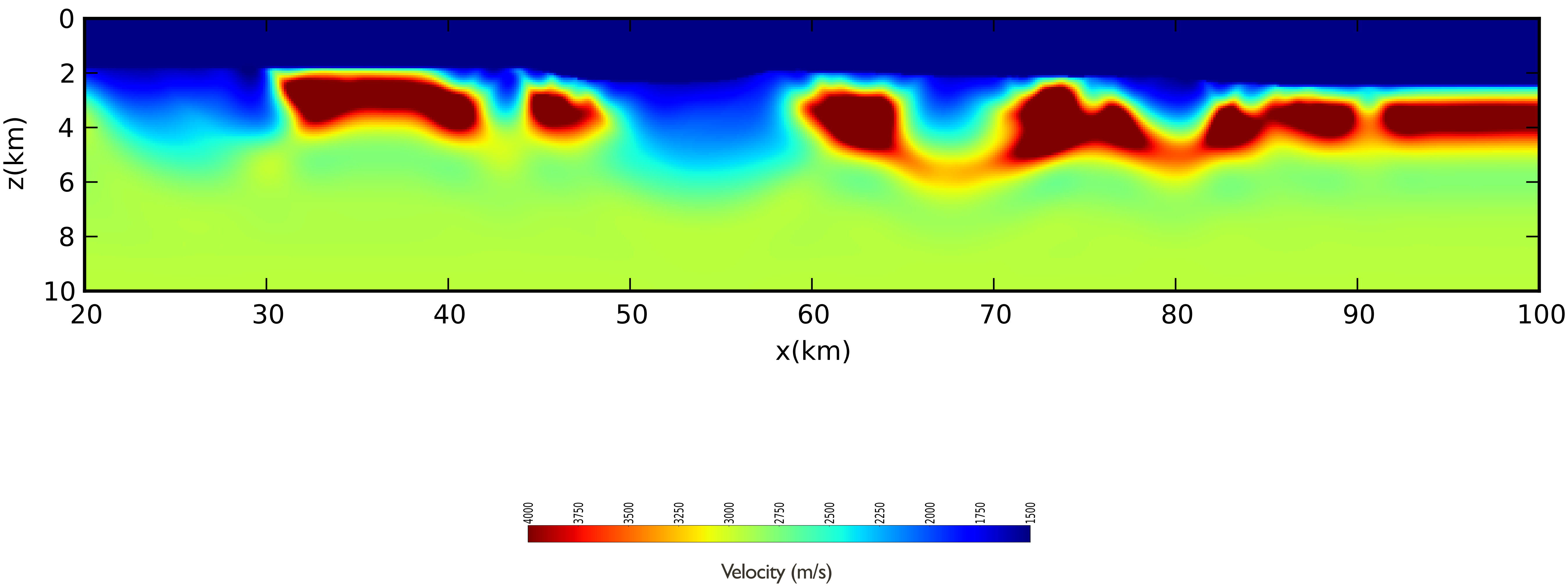
$\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \gamma^k \mathbf{C}_2^H \delta\mathbf{x}$

$k \longleftarrow k + 1;$

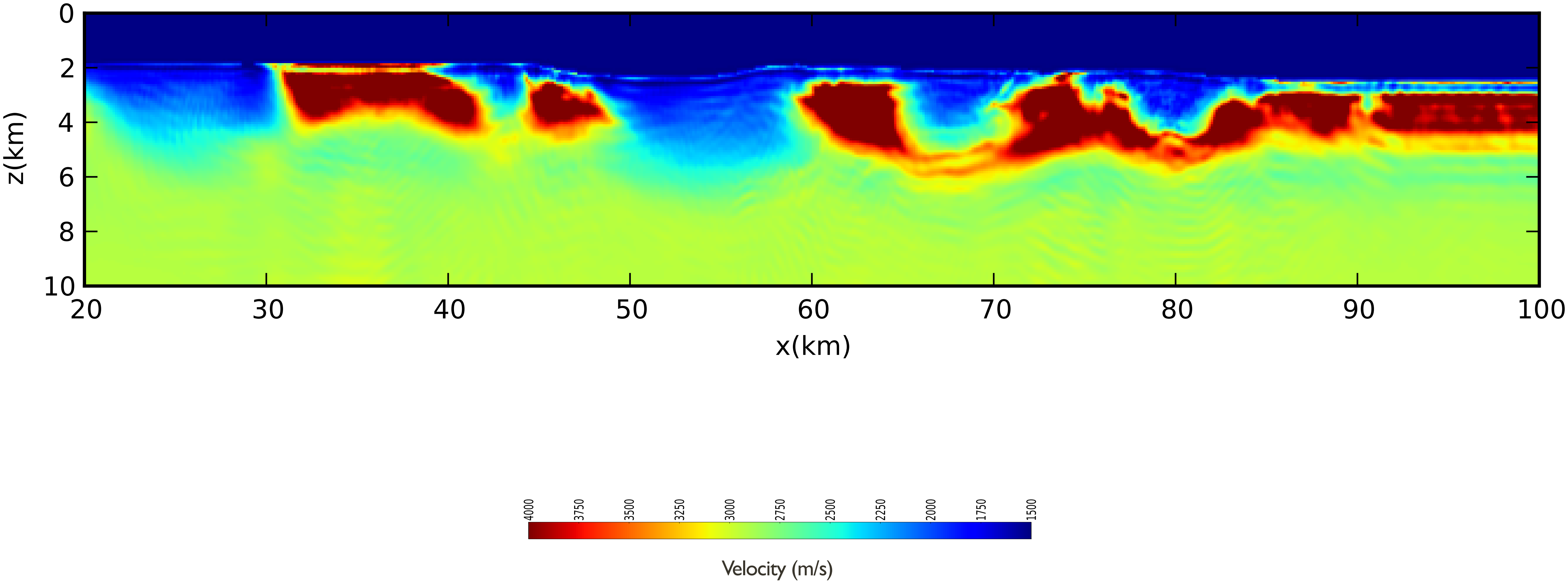
end

Algorithm 1: modified Gauss Newton with sparsity promotion

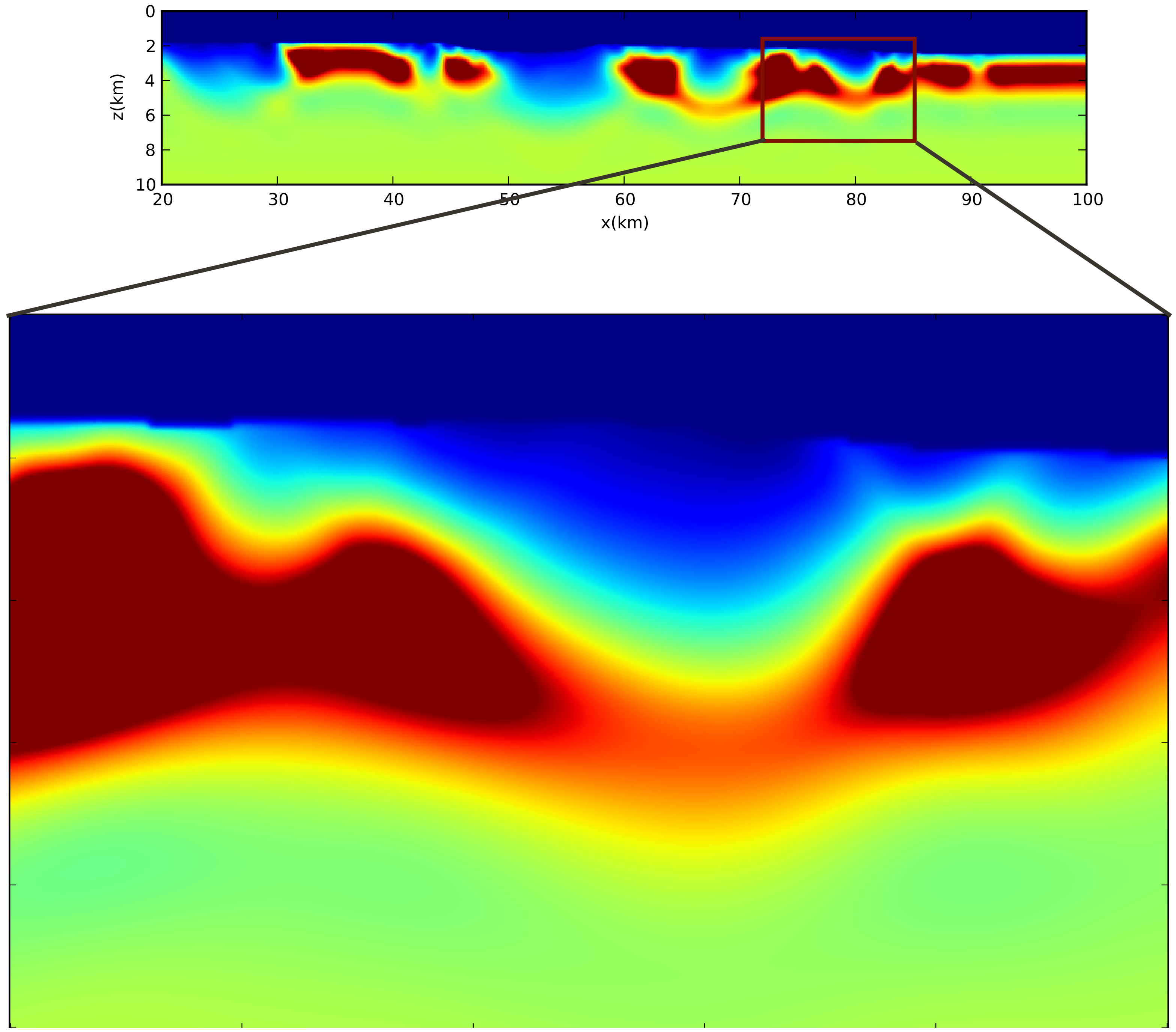
Initial model
[ray-based tomography]



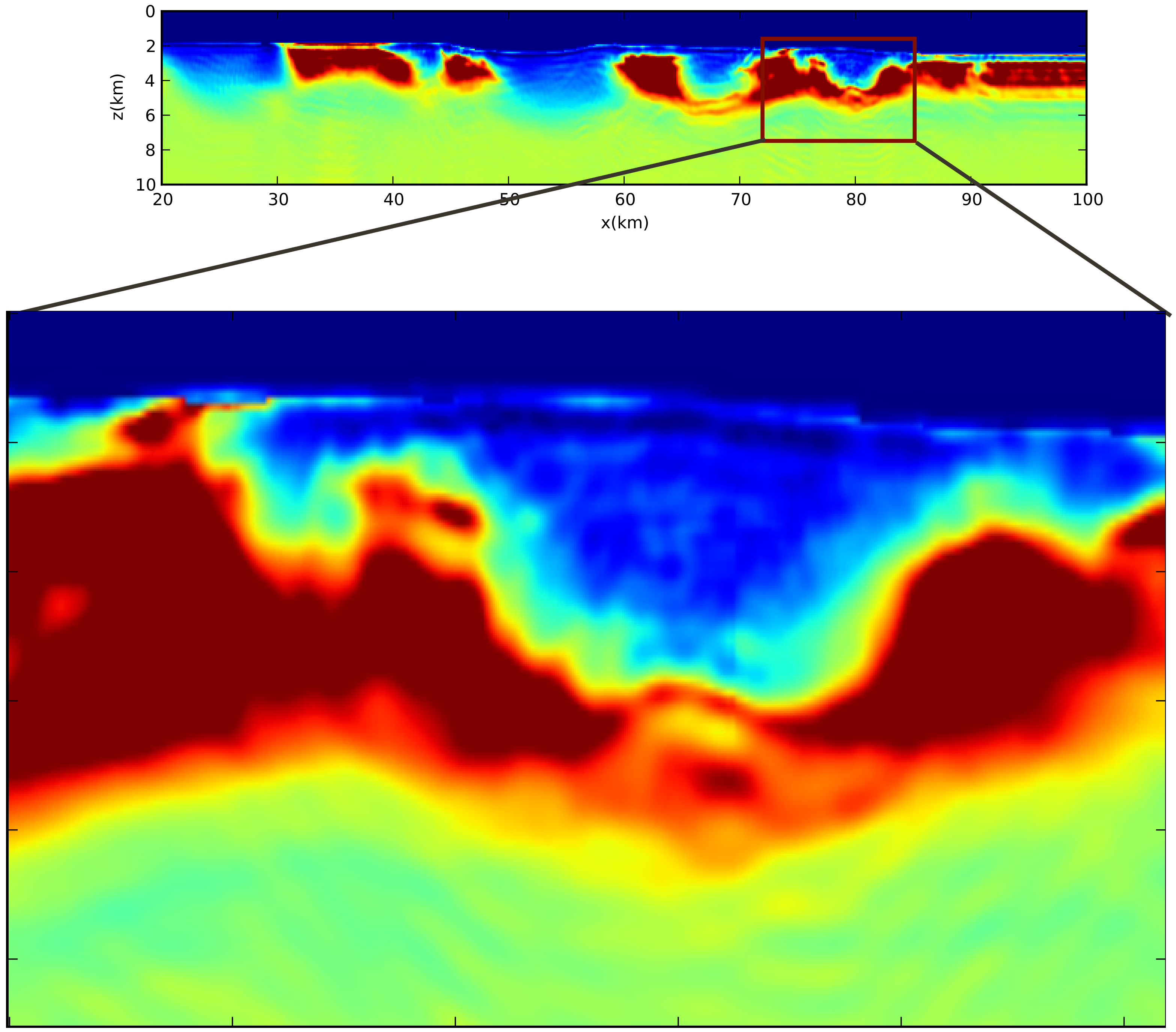
Final result



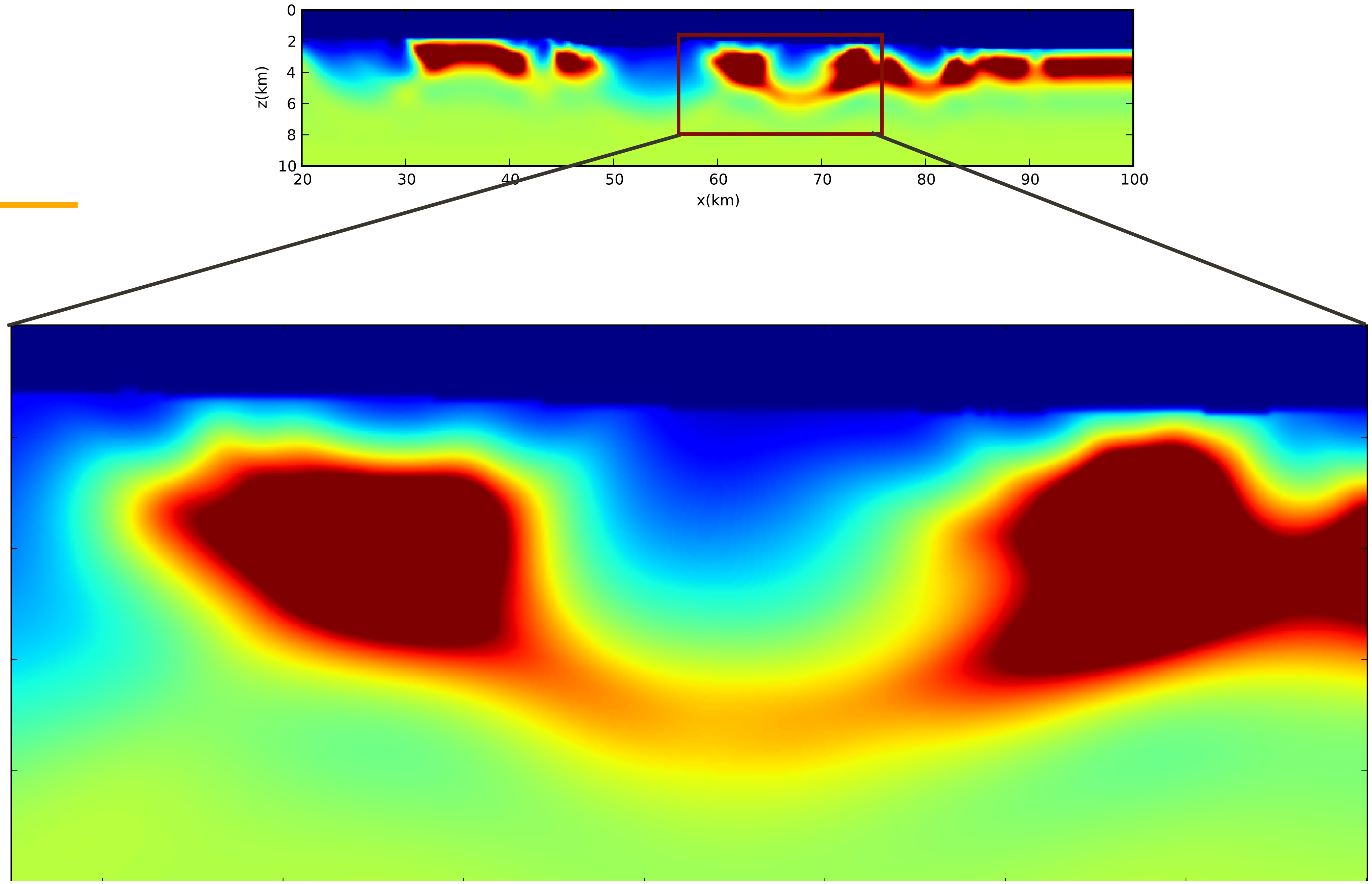
Initial Model



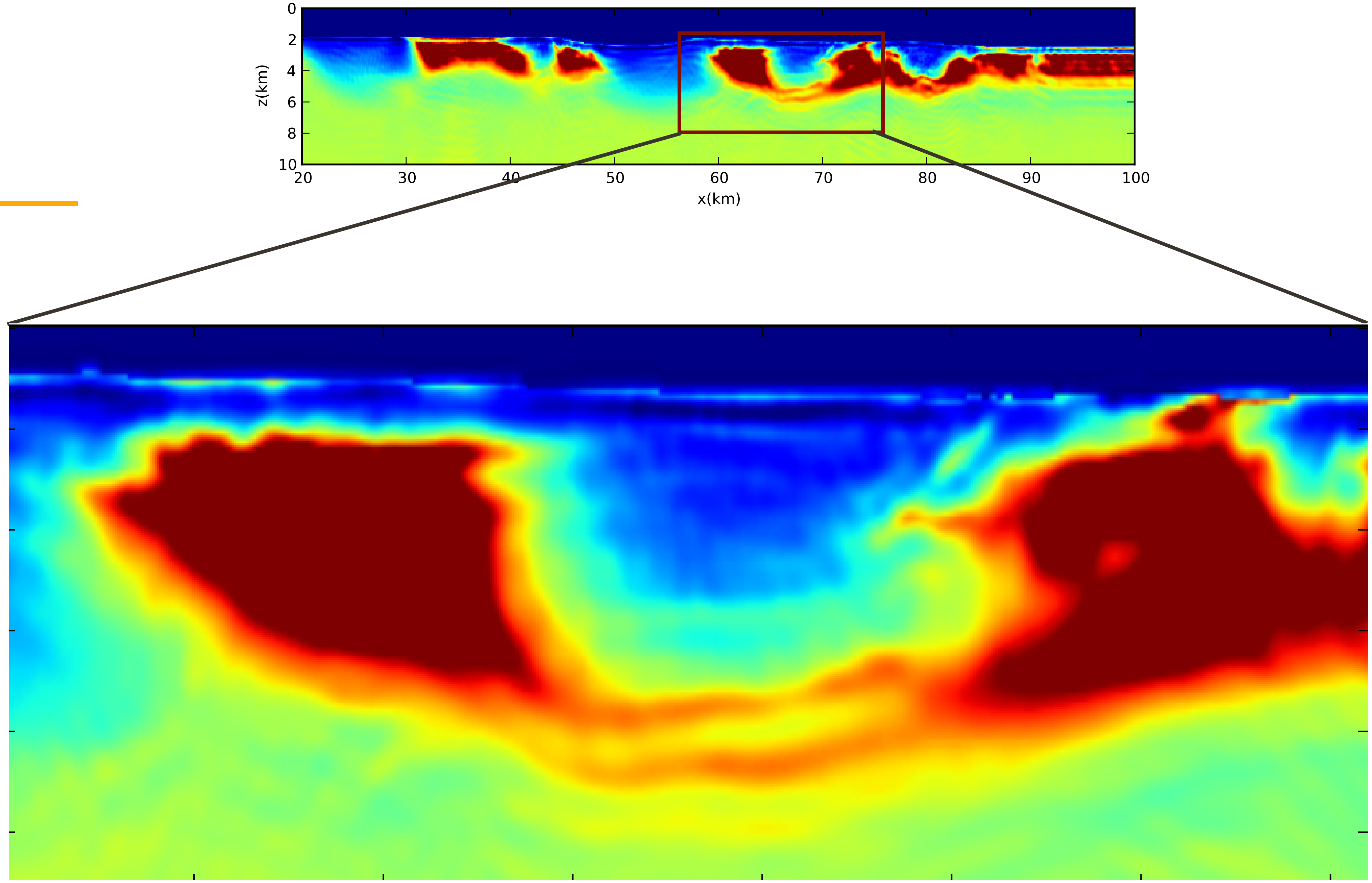
Final result
[w/ denoising & complex velocity]



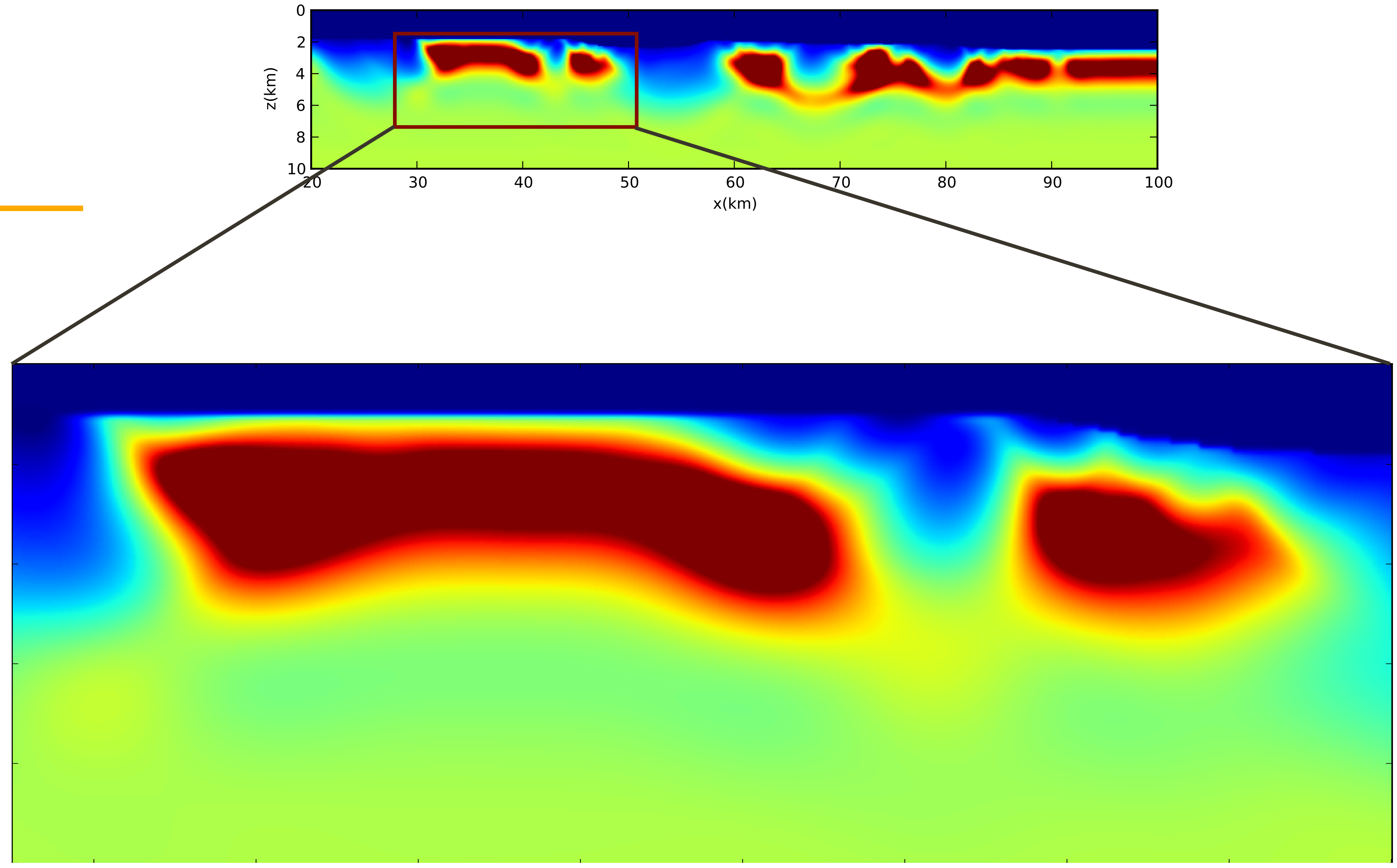
Initial Model



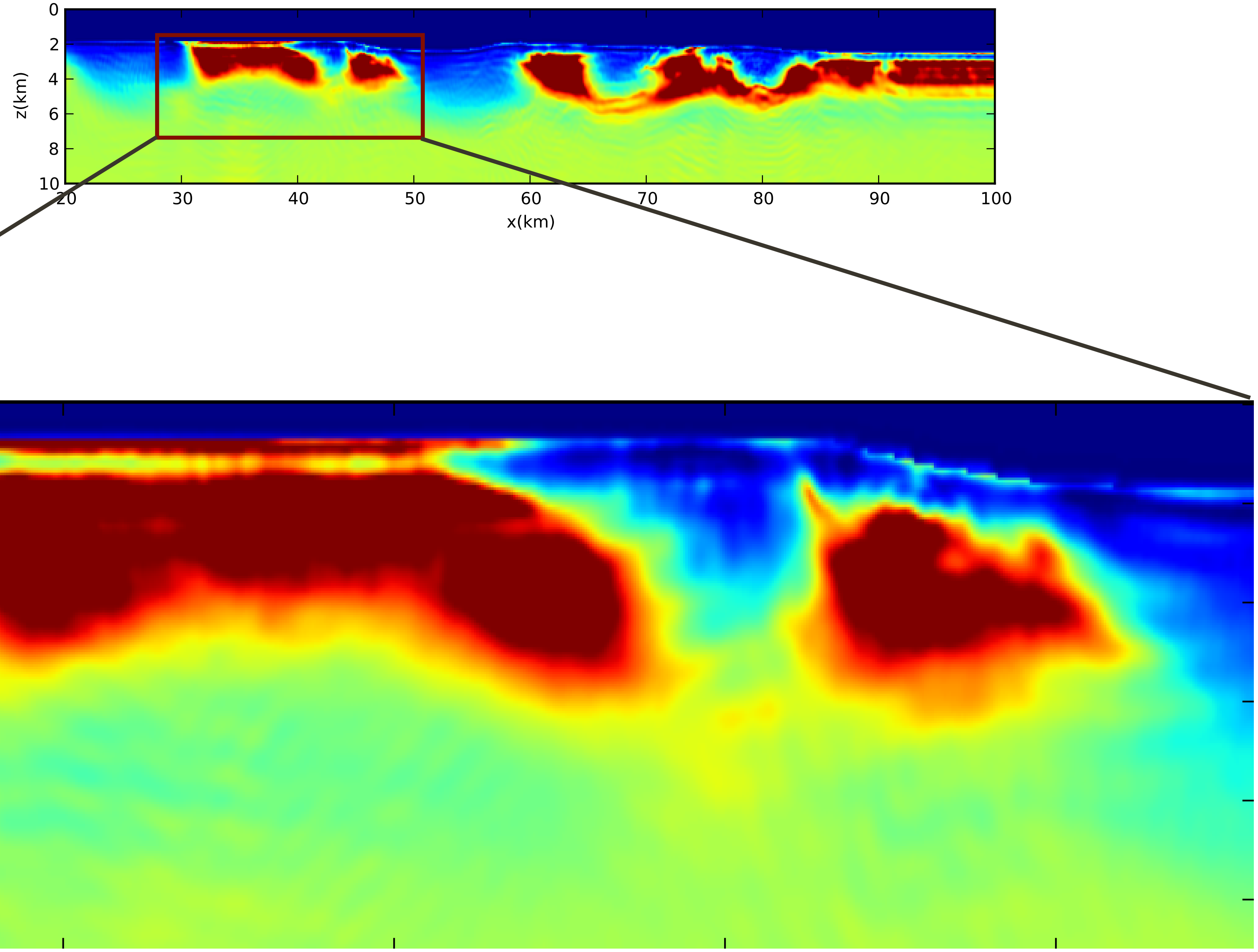
Final result
[w/ denoising & complex velocity]



Initial Model



Final result
[w/ denoising & complex velocity]



Observations

	Nonlin. CG [phase]	Modified GN [amp. & phase]
Acoustic	mixed results	encouraging
Noise	?	important
Conditioning	involved	“simple”
Quality	smooth	“sharp”

Conclusions

Dimensionality reduction techniques allow us to impose *sparsity* on the GN updates

GOM data set is *extremely* challenging

- *data*-space *regularization* is *insufficient*
- *model*-space regularization w/ *curvelets* is encouraging

Results *really* depend on *details*...

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Acknowledgments

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NSERC
CRSNG

Thank you

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