Multiscale aspects of waveform tomography

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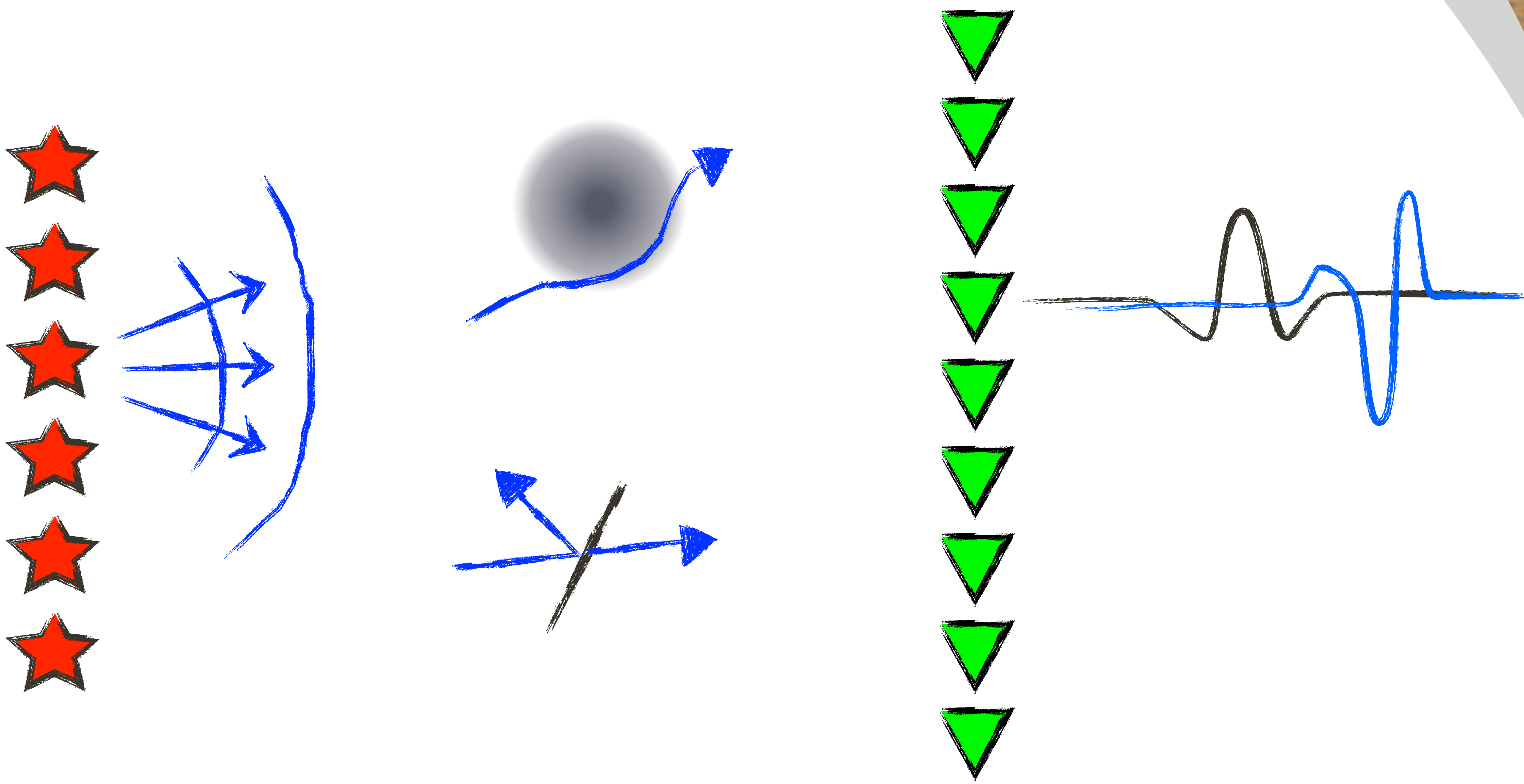
Waveform imaging



Overview

- Waveform tomography
- Wavefrontset detection
- Misfit criteria
- Numerical example
- Future work & Conclusions

Waveform tomography

Model the data as

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right] u = w(t)\delta(x-s),$$

$$d(t,s,r) = u(t,x,s)|_{x=r} \equiv F[c].$$

Goal is to find the velocity given the data and source signature

Waveform tomography

Such inverse problems have been extensively studied. Major findings:

- **recovery via LS is problematic for bandlimited data**
- **some form of traveltime fitting needed for `complete' reconstruction**

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Waveform Tomography

Sensitivity of the data to velocity perturbations can be studied in high-frequency asymptotic regime

$$\hat{d}(\omega, s, r) \simeq A(\omega, s, r) \exp[\imath\omega T(s, r)]$$

- amplitude

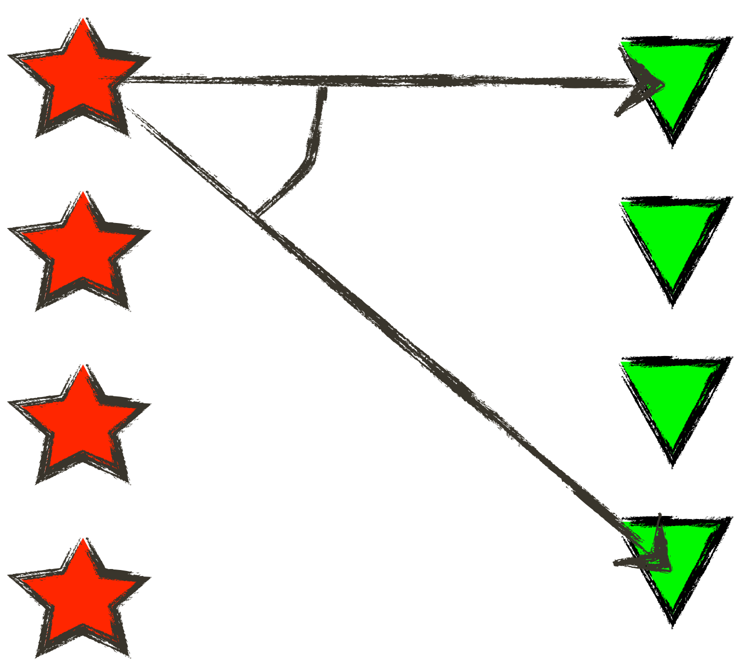
$$\widehat{\delta d} \simeq \int \mathrm{d}\omega \int \mathrm{d}x\, \hat{w}(\omega)\delta c(x) \exp[\imath\omega(T(s, x) - T(x, r))]$$

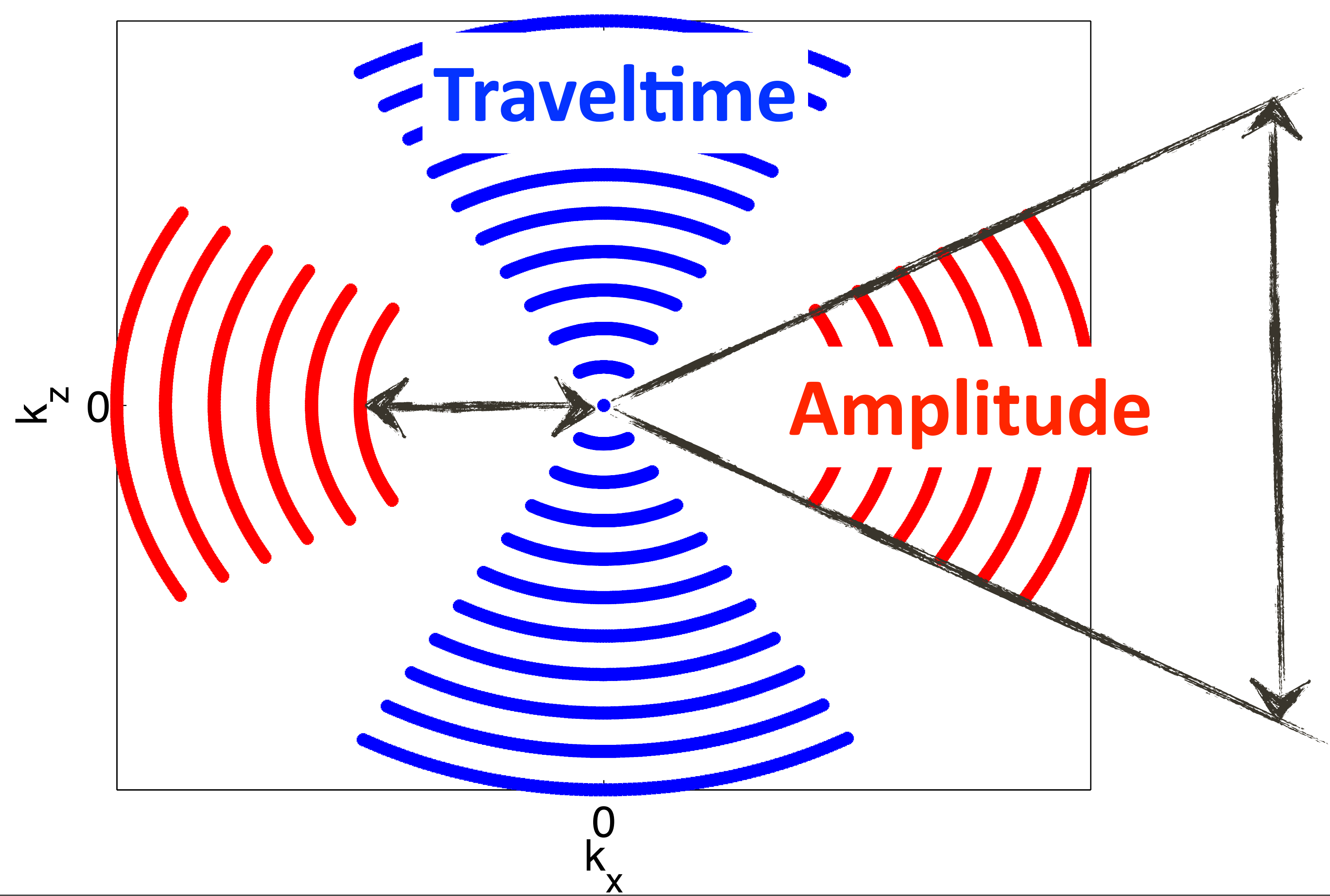
- traveltime

$$\nabla\delta T \cdot \nabla T = \delta c$$

Waveform tomography

Wavenumber coverage with limited aperture



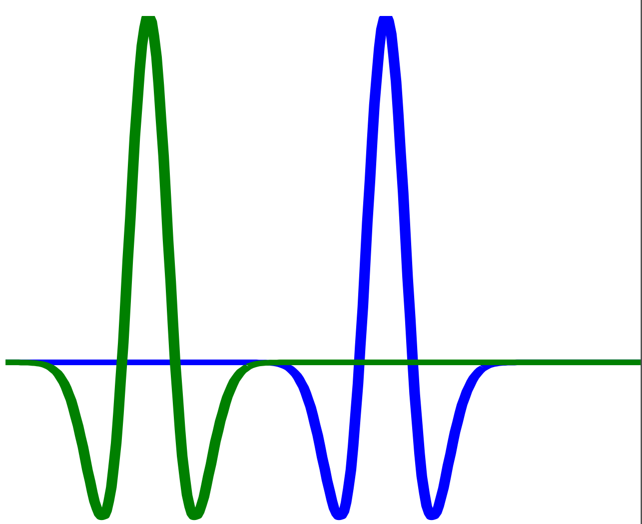


[Stork; Bube; Natterer;]

Waveform tomography

Full waveform inversion:

$$\min_c ||F[c] - \bar{d}||_2^2$$

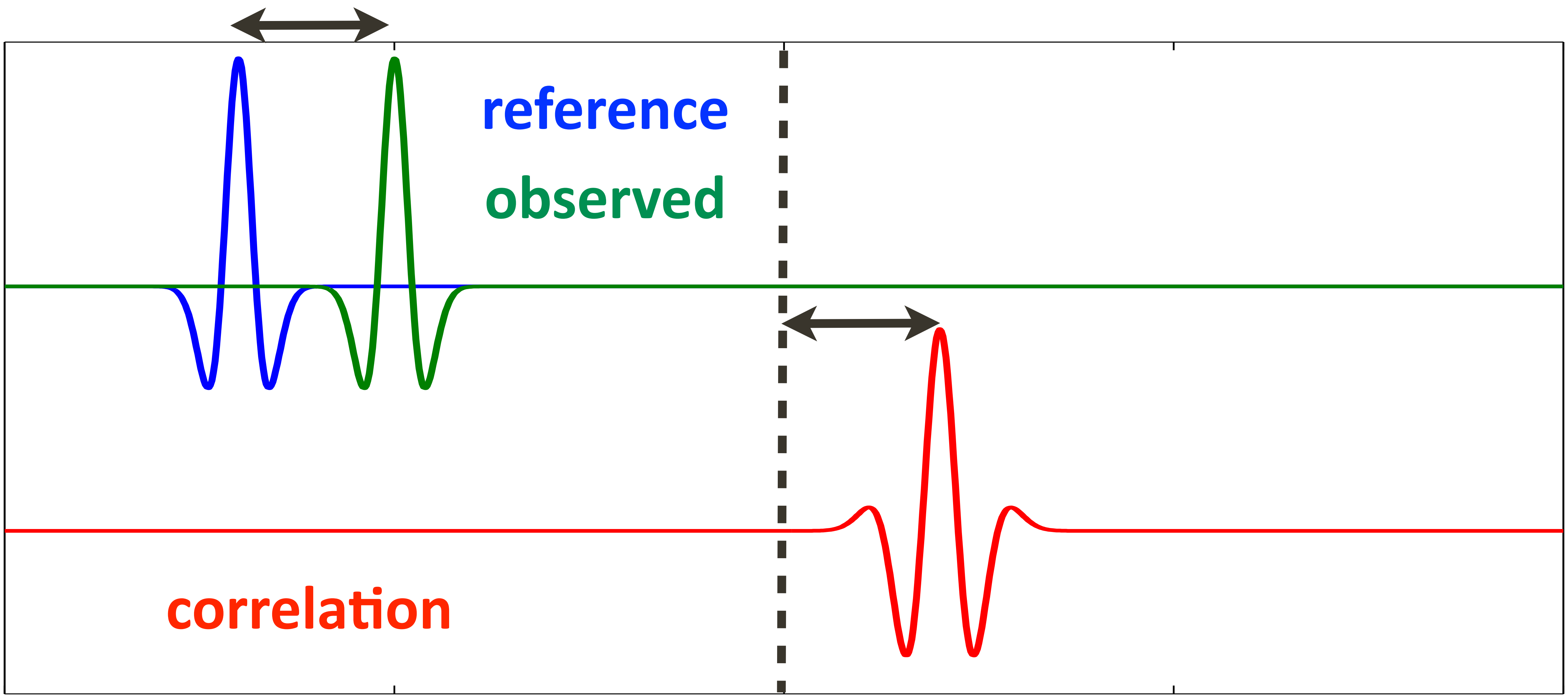


- **need large aperture and low frequencies**
- **loopskipping if velocity error is too big, can be mitigated partly by `multiscale' FWI**

[Tarantola 84; Bunks 98; Shin 09]

Waveform tomography

Wave-equation traveltime tomography



Waveform tomography

WE traveltime tomography:

- **relies on detecting shift of singular support**
- **widely used criterion: maximum of the correlation**

$$\min_c ||\tau[c]||_2^2, \quad \tau[c] = \mathrm{argmax}_t (d * \bar{d})(t)$$

[Cara 87; Luo 91; Dahlen 10; Hormann 02; de Hoop 05;Brytik 10]

Waveform tomography

LS may be re-formulated as maximizing the normalized zero-lag correlation

$$||d - \bar{d}||_2^2 = ||d||_2^2 + ||\bar{d}||_2^2 - 2 \underbrace{\langle d, \bar{d} \rangle}_{(\bar{d} * d)|_{t=0}}$$

`picking approach' is a clever extension of this

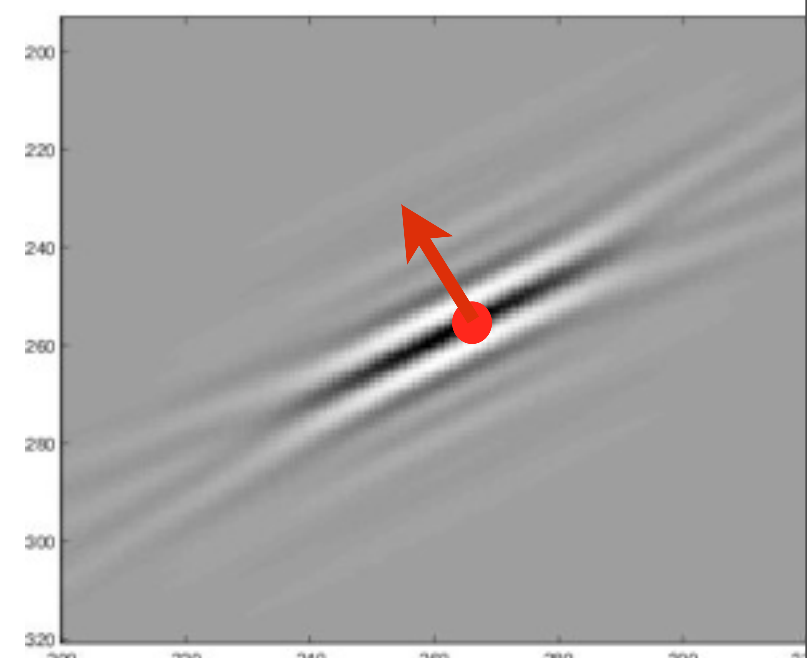
Wavefrontset detection

Given a function of the form

$$f(x,t) = \int \mathrm{d}\omega \, a(\omega, x, t) \exp[\imath \phi(\omega, x, t)]$$

the wavefrontset is given by

$$\mathrm{WF}(f) \subseteq \{x, t; \partial_x \phi, \partial_t \phi \,|\, \partial_\omega \phi = 0\}$$



In particular:

$$\mathrm{WF}(\bar{d} * d) \subseteq \{s, r, \bar{T} - T; \nabla(\bar{T} - T), \imath\omega\}$$

Wavefrontset detection

- **Multiscale WF detection via the FBI transform:**

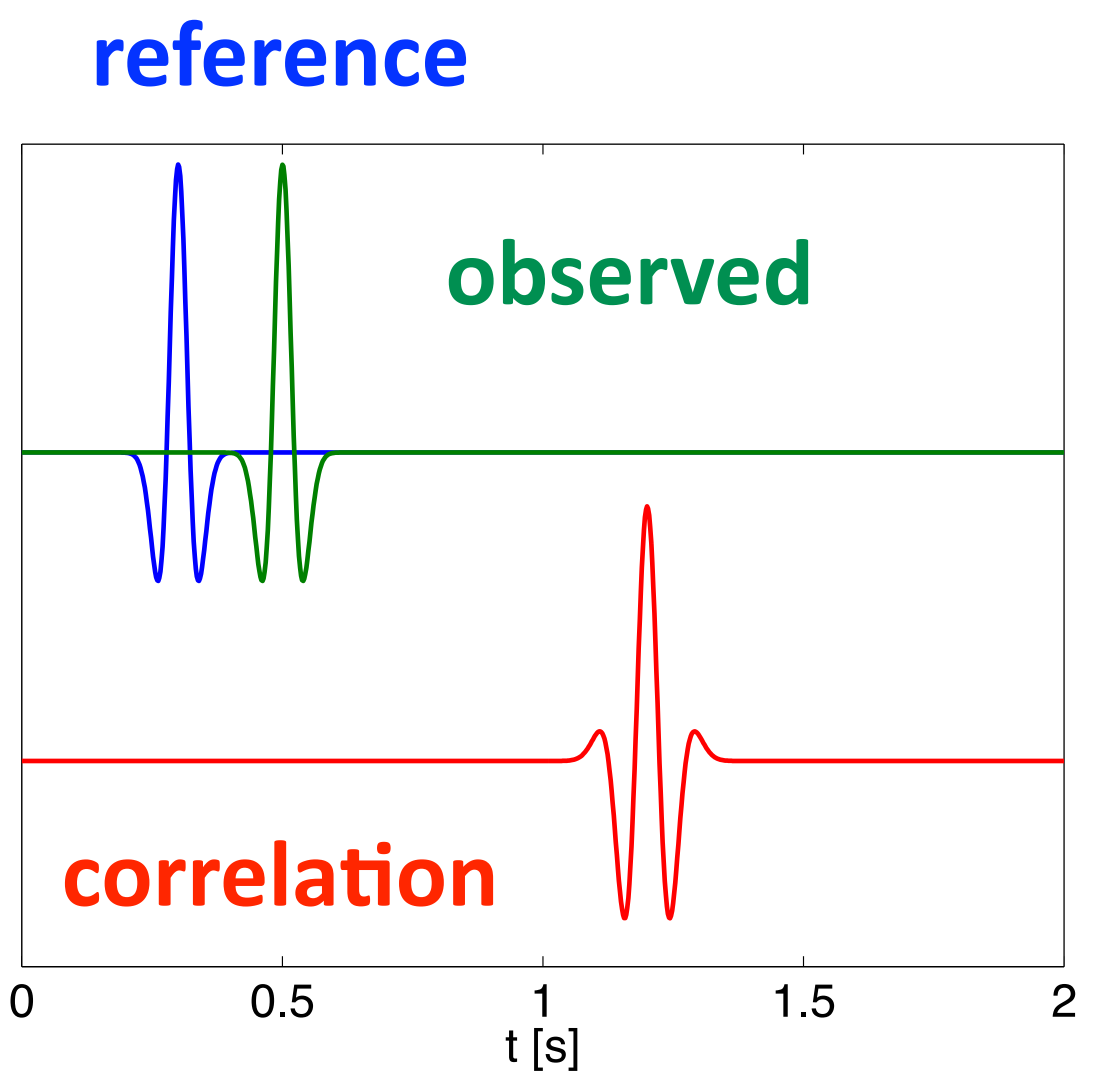
$$G[f](t,\omega,\sigma) = \frac{1}{\sqrt{\sigma}} \int \mathrm{d}t' \, f(t') W[(t-t')/\sigma] \exp[\imath\omega t']$$

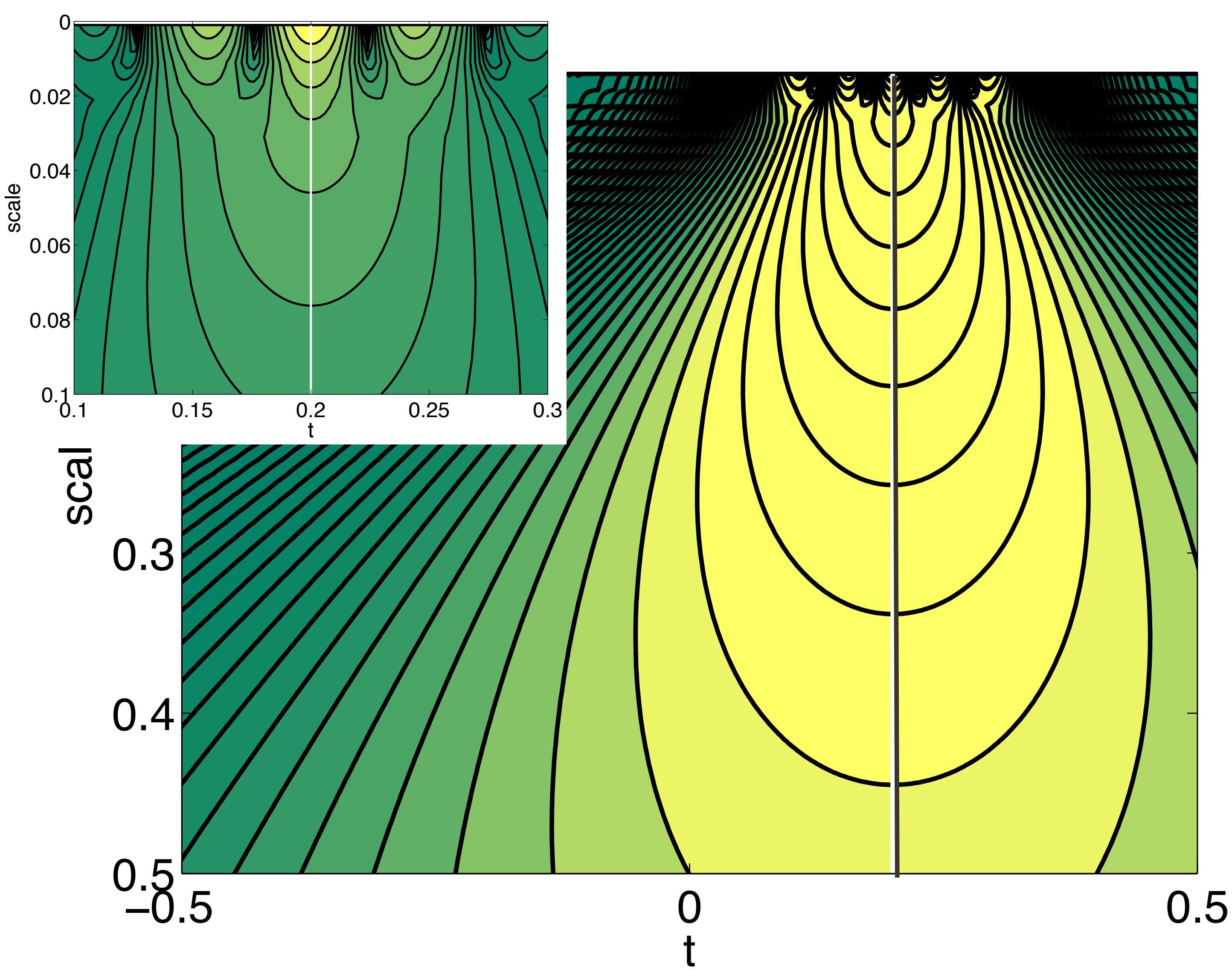
- **if** $t \not\subset \mathrm{WF}(f)$ **then for fixed** ω **and any** $N \in \mathbb{N}$

$$|G[f](t,\omega,\sigma)| \leq \sigma^N \quad \text{as} \quad \sigma \downarrow 0$$

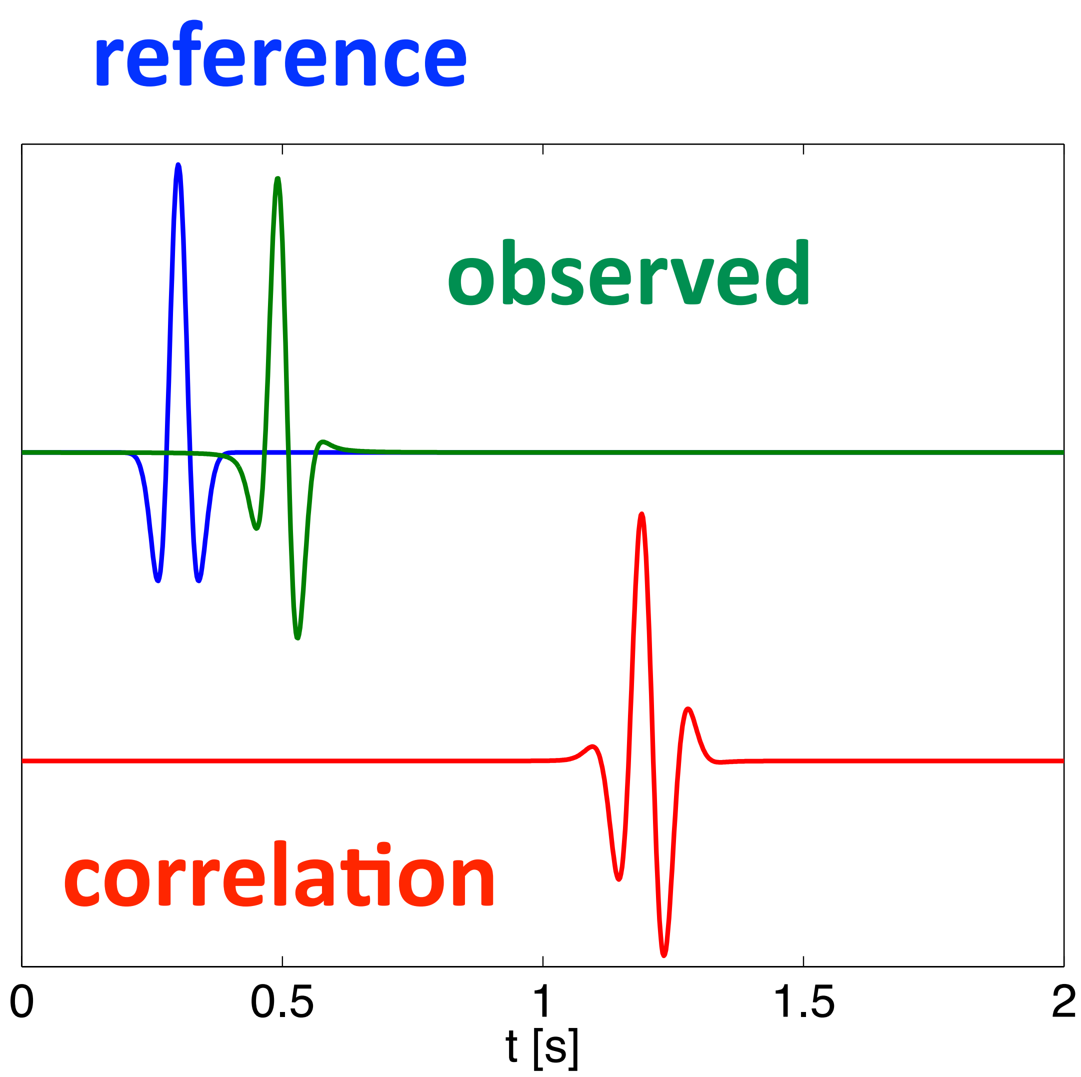
[Hormander 83; Hormann 02; de Hoop 05]

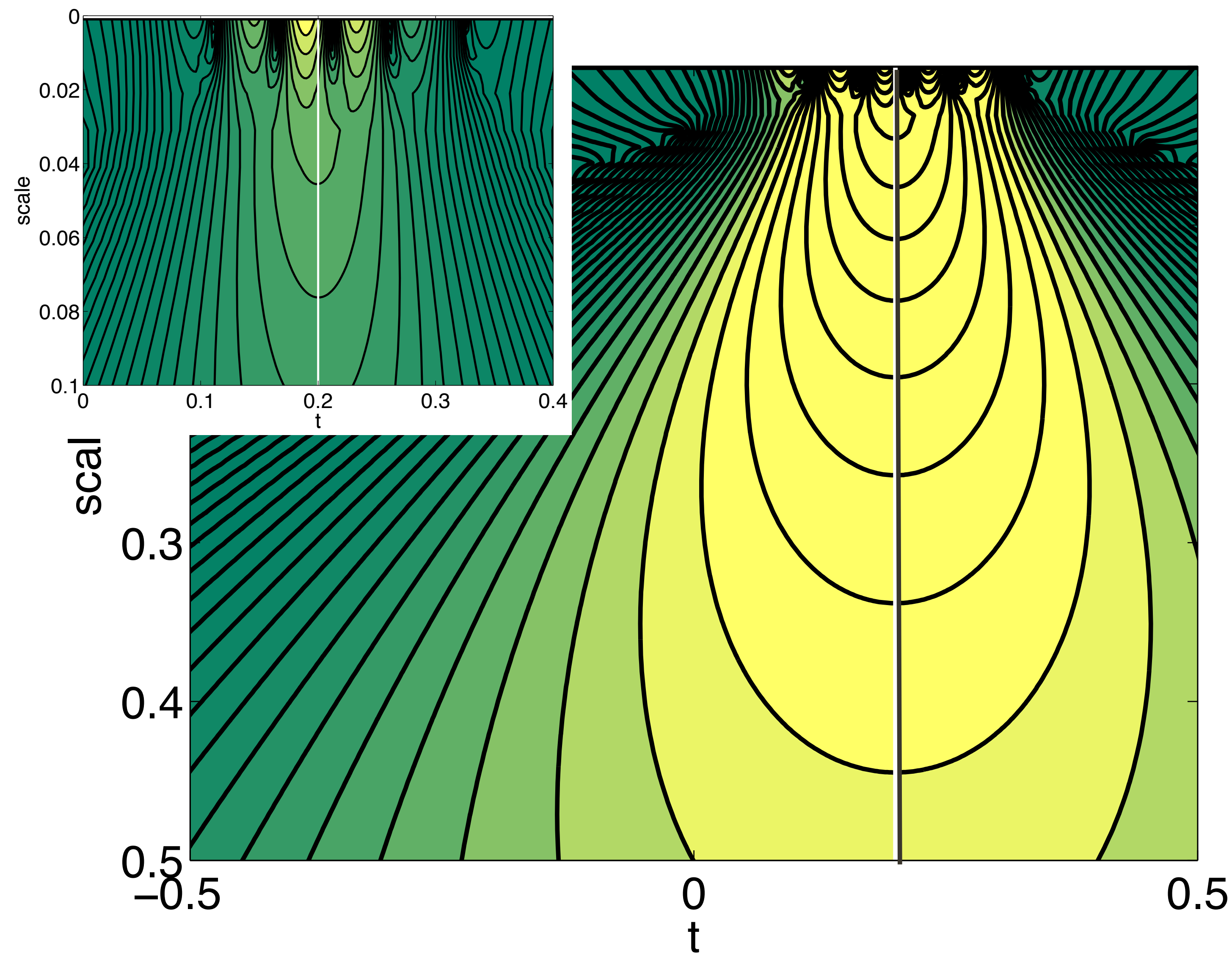
Wavefrontset detection



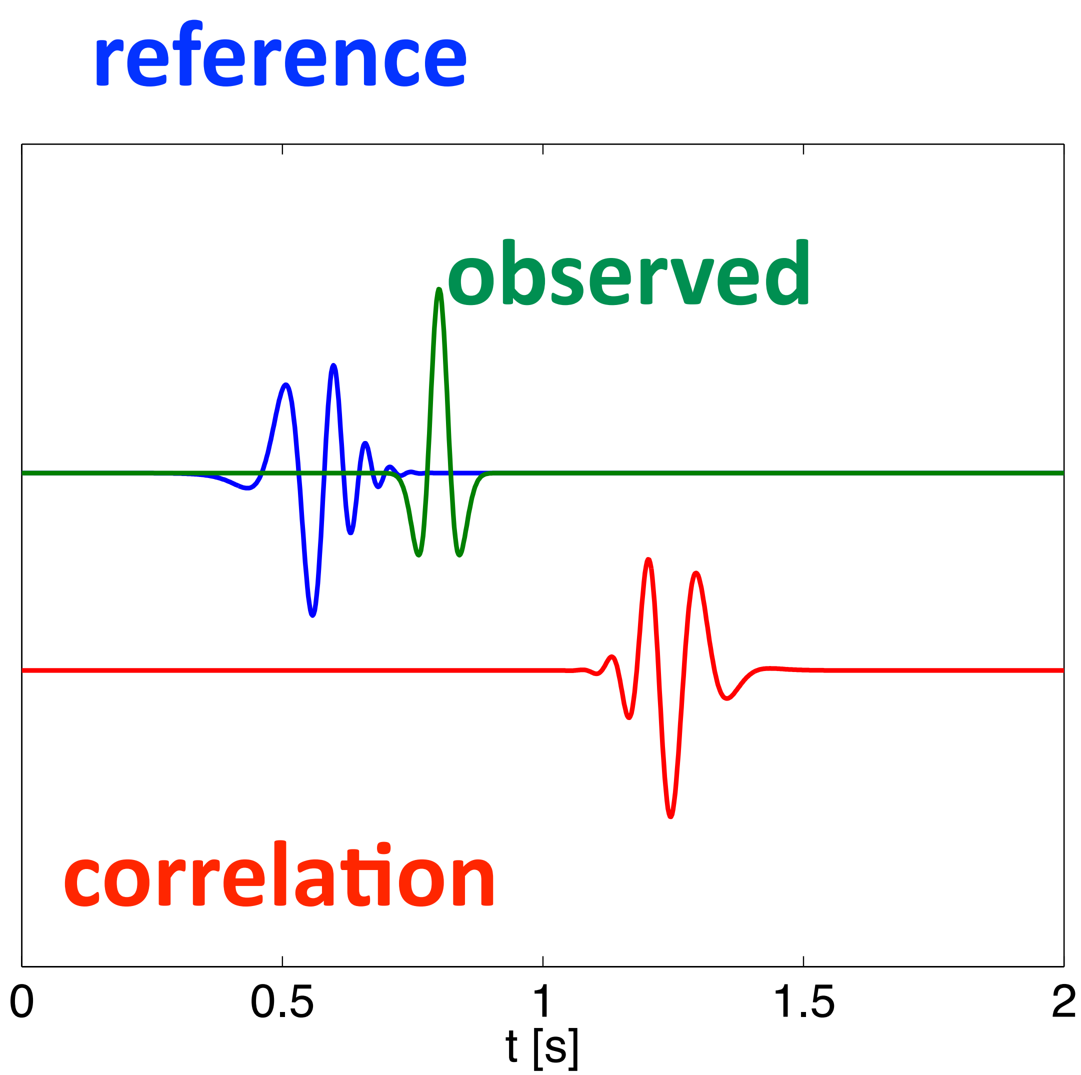


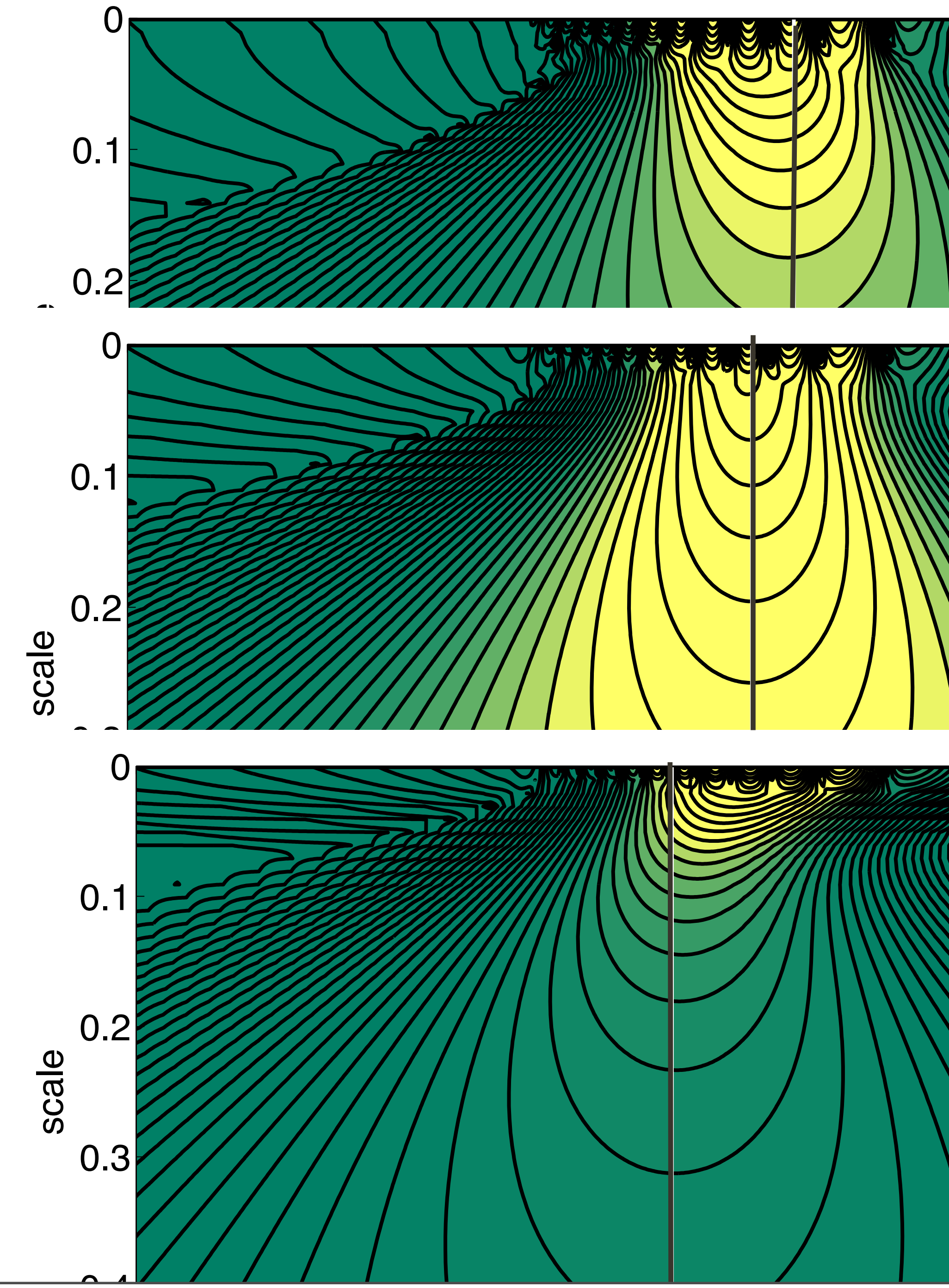
Wavefrontset detection





Wavefrontset detection I





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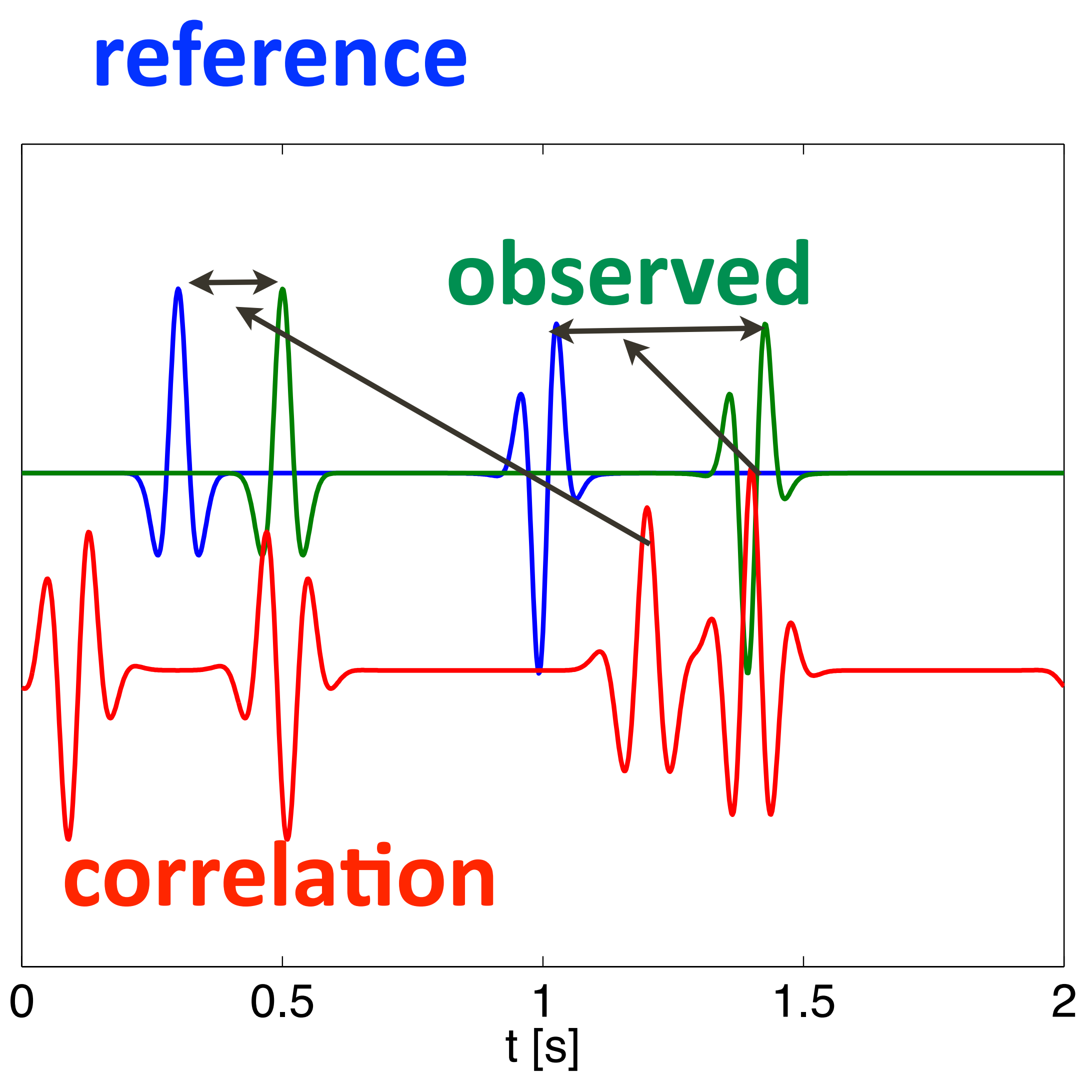
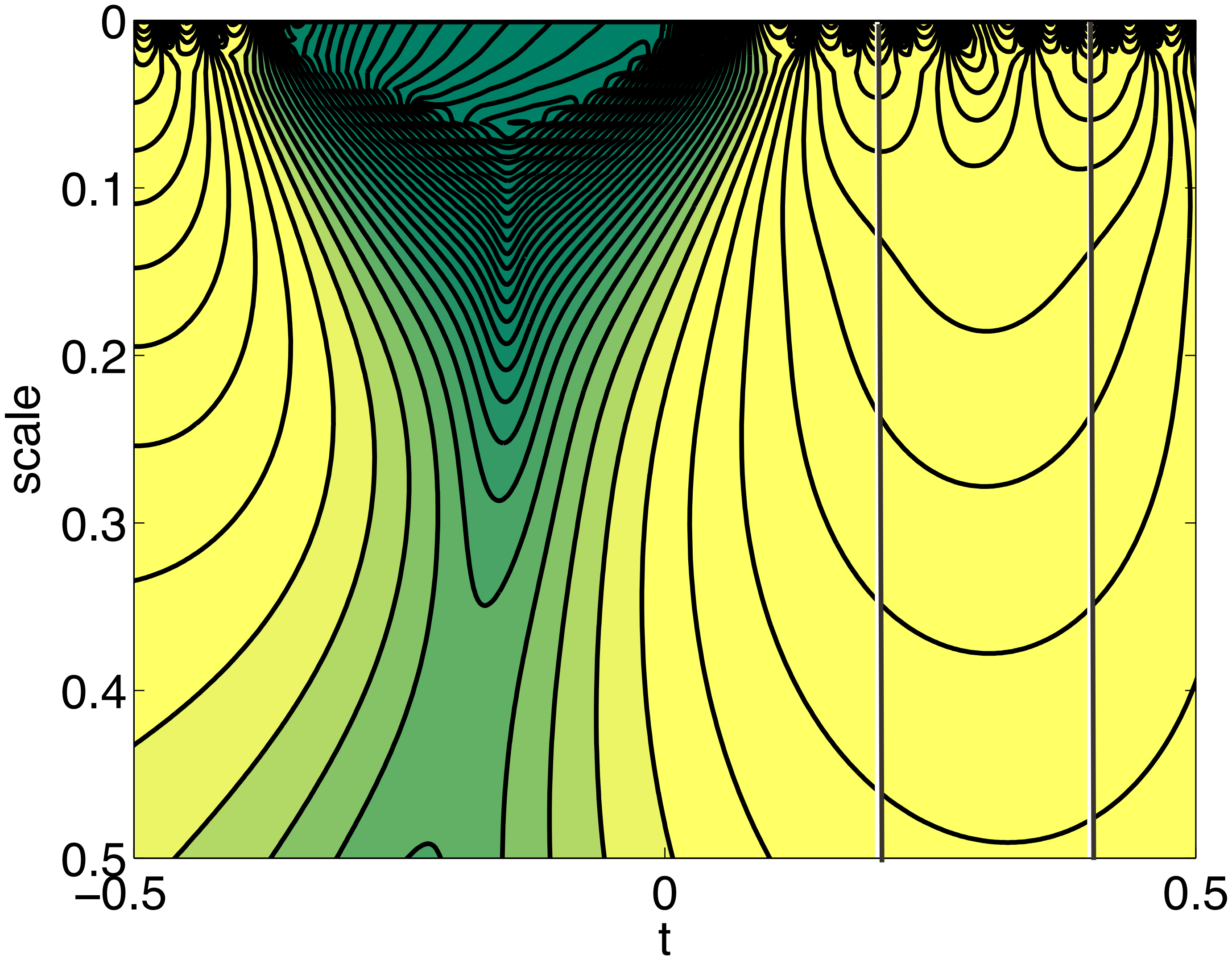
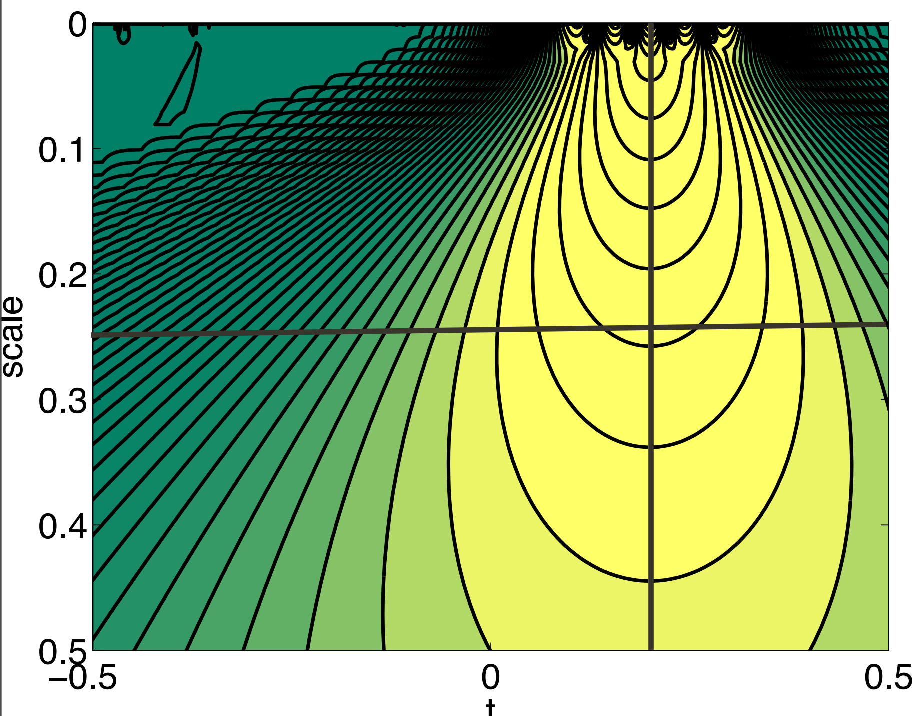
Wavefrontset detection





Misfit criteria

- $\tau[\omega,\sigma] = \text{argmax}_t\, G[\bar{d} * d](t,\omega,\sigma)$ **converges to picking approach as** $\sigma \downarrow 0$ **and** $\omega = 0$



- **Maximize** $||G[\bar{d} * d](0,.,\sigma)||_2^2$

- **Minimize** $||\partial_t G[\bar{d} * d](0,.,\sigma)||_2^2$

Misfit criteria

Rewrite:

$$G[f](0,\omega,\sigma) = (\widehat{W_\sigma \cdot f})(\omega)$$

$$\partial_t G[f](0,\omega,\sigma) = (\widehat{W'_\sigma \cdot f})(\omega)$$

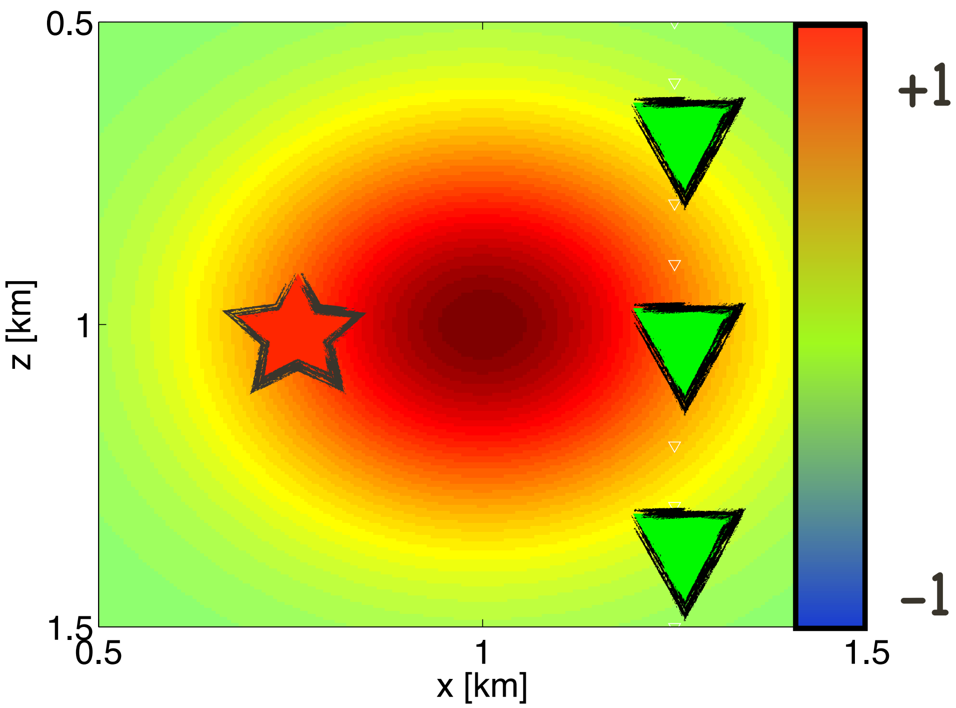
where $W_\sigma(t) = \frac{1}{\sqrt{\sigma}} \exp[-(t/\sigma)^2]$

Misfit:

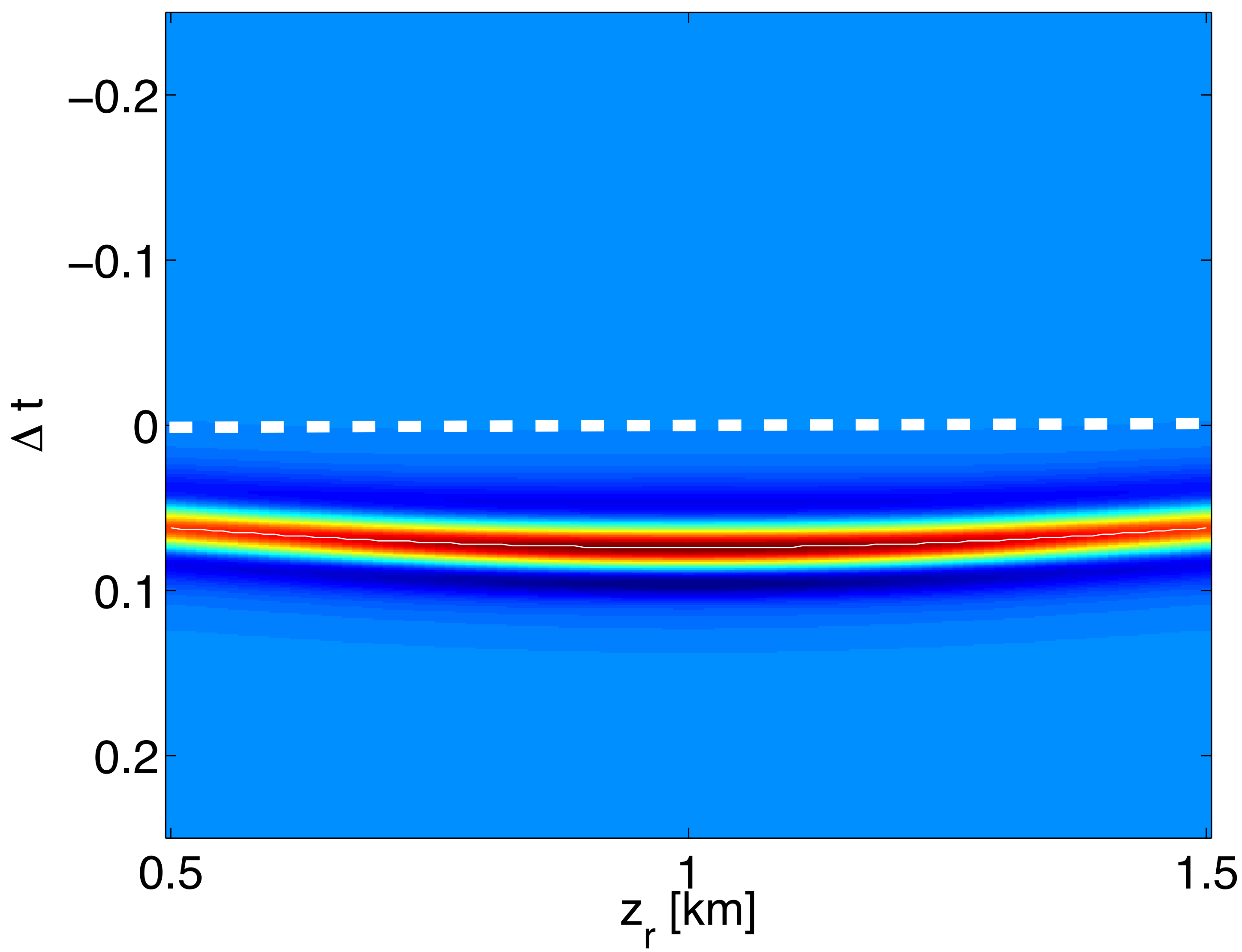
$$\phi = \frac{||W_\sigma \cdot (\bar{d} * d)||_2^2}{||d||_2^2}$$

[TvL 08; TvL 10]

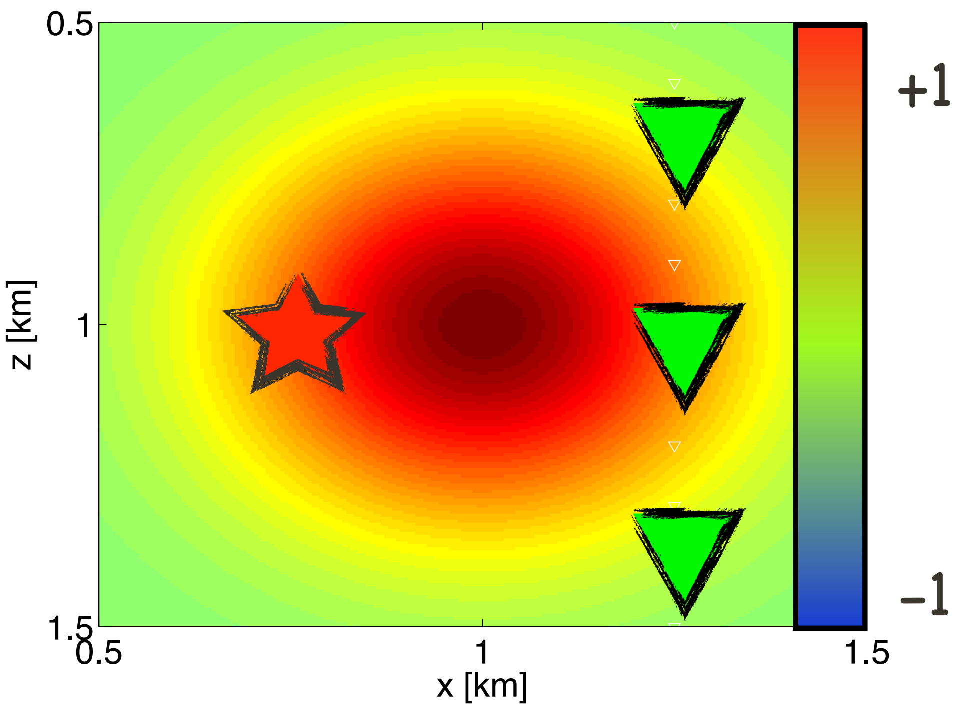
Misfit criteria



velocity perturbation

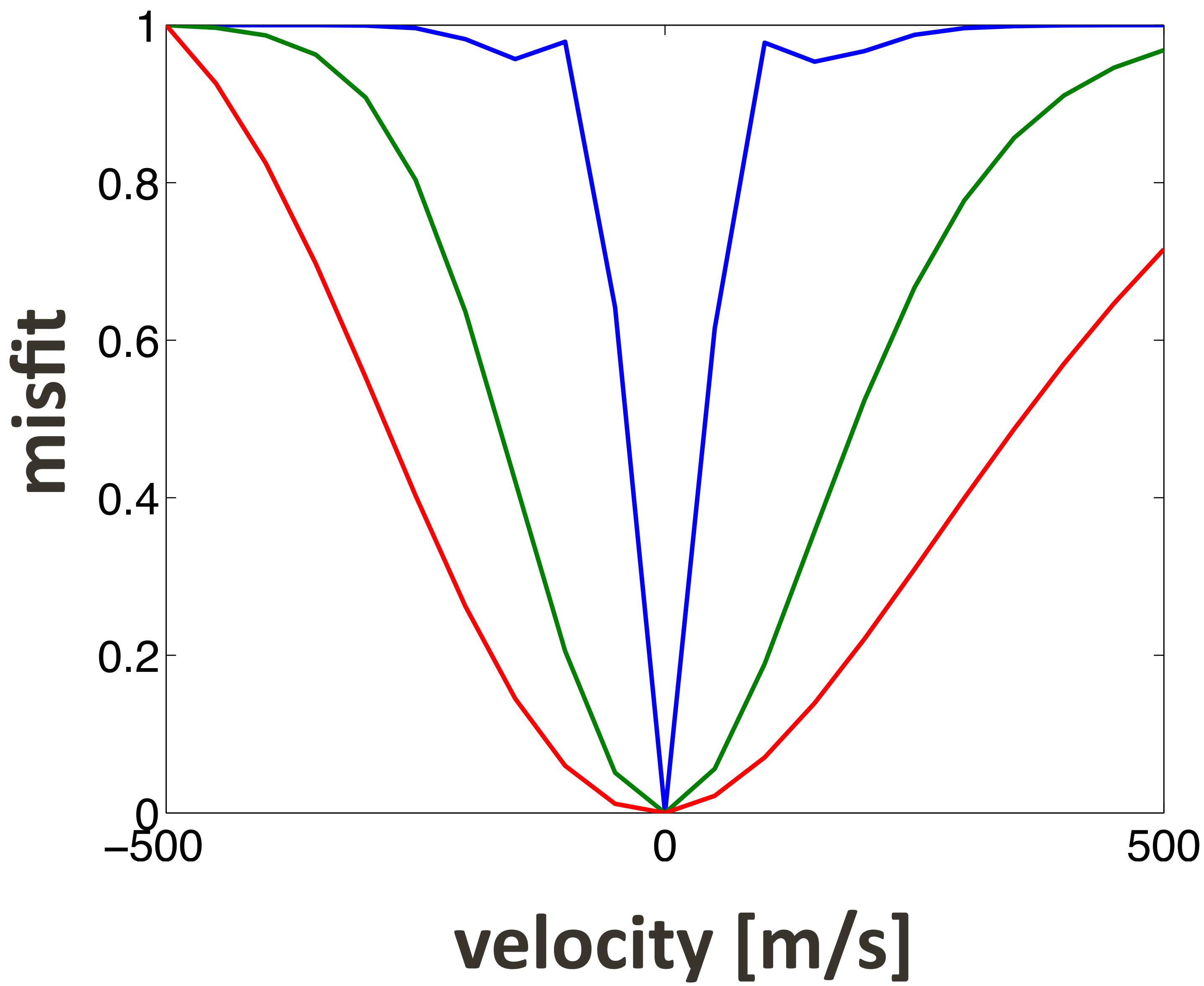


Misfit criteria

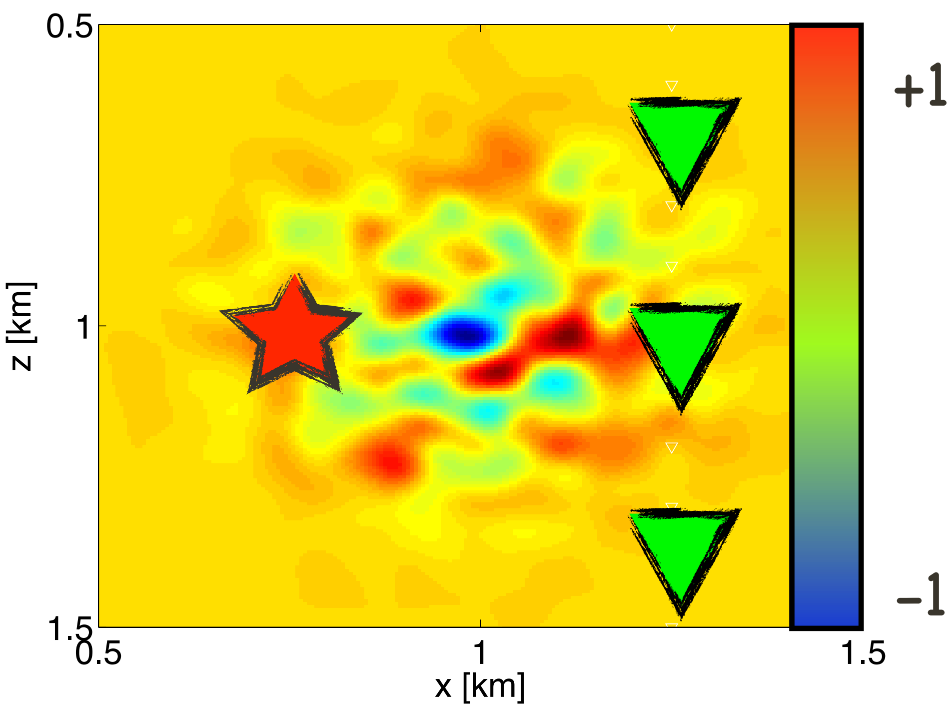


velocity perturbation

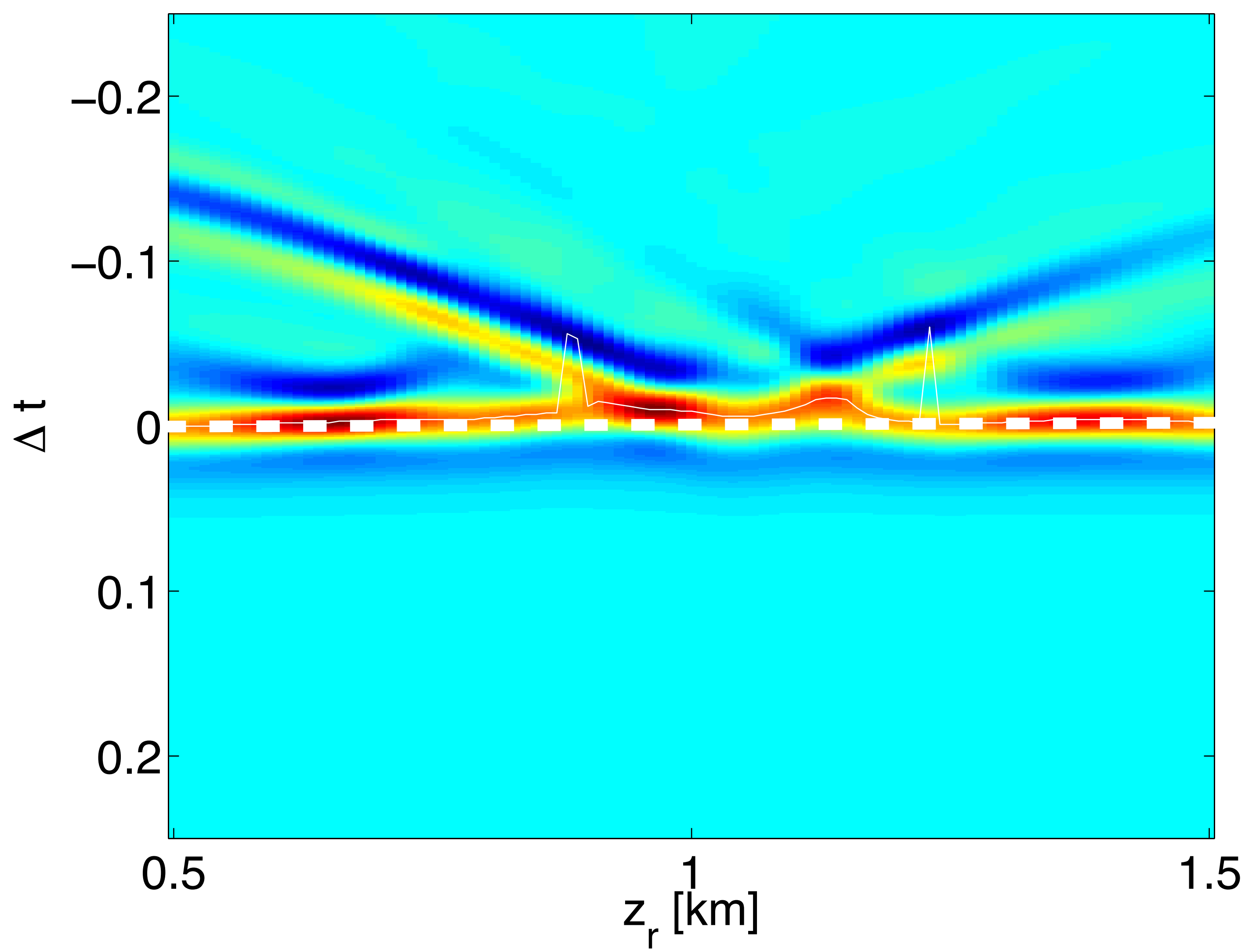
small, medium, large



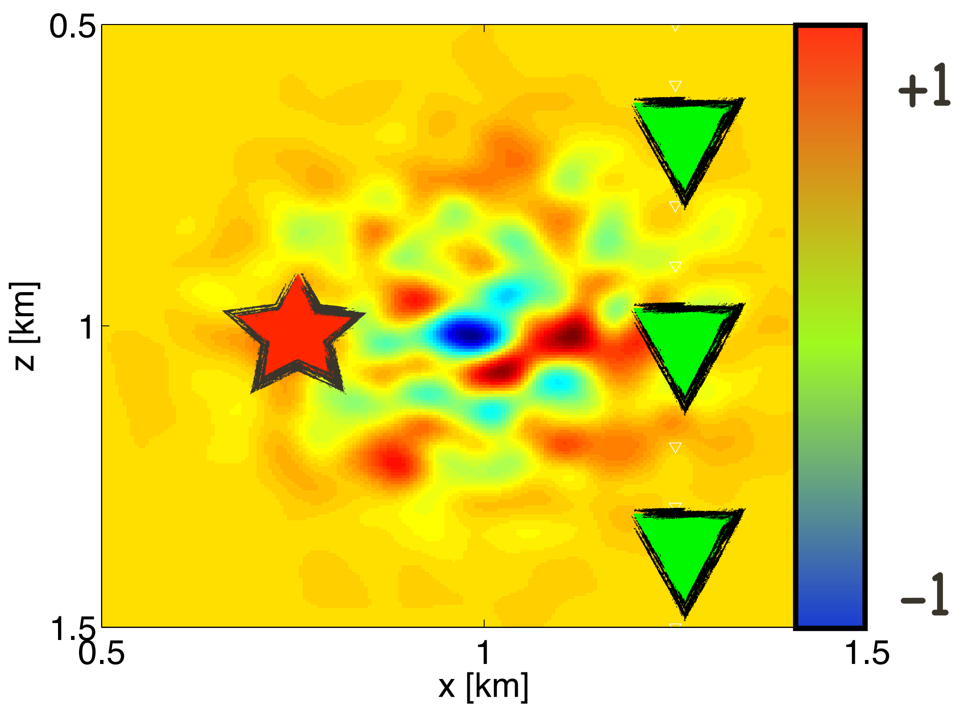
Misfit criteria



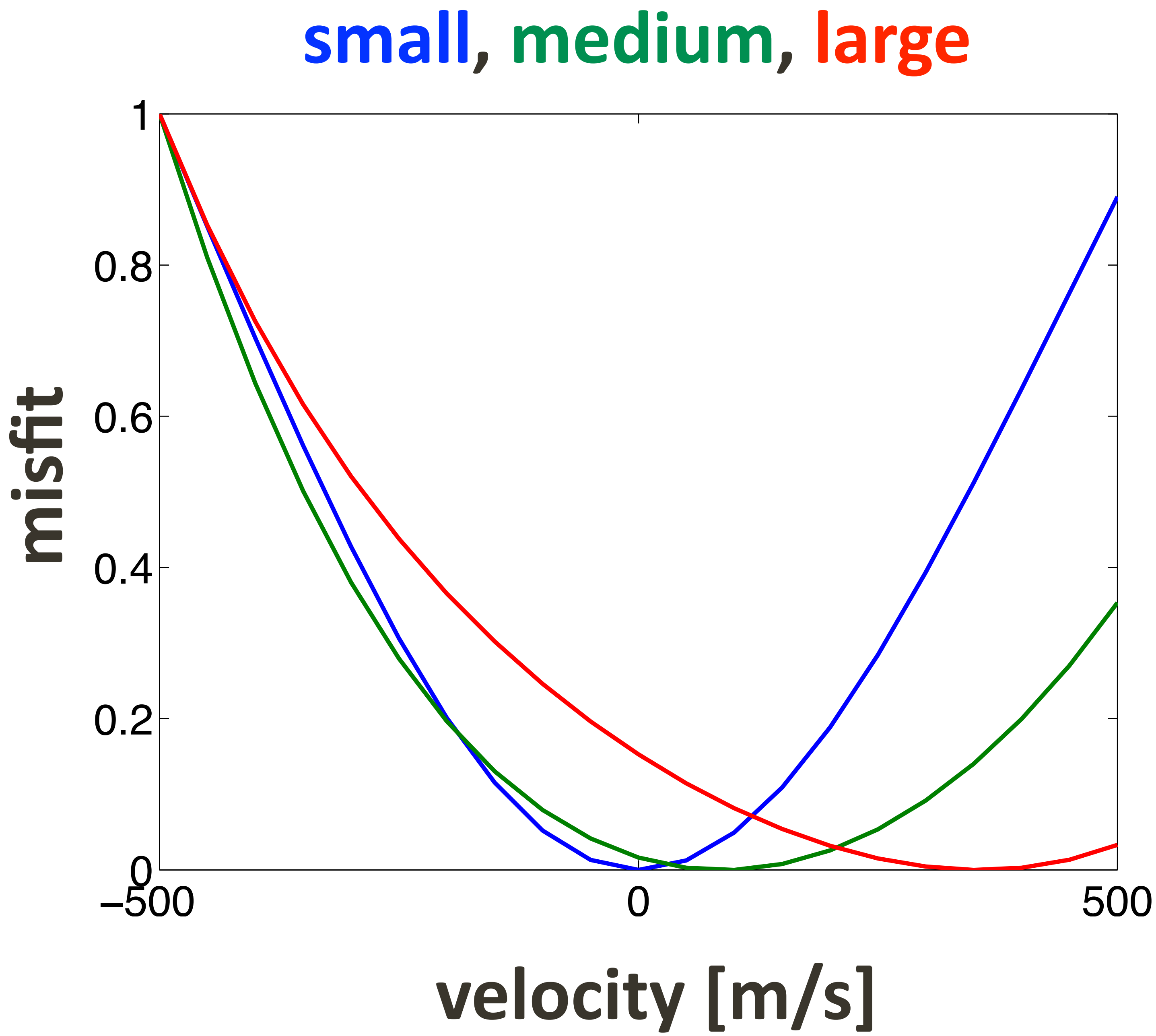
velocity perturbation



Misfit criteria



velocity perturbation



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Misfit criteria

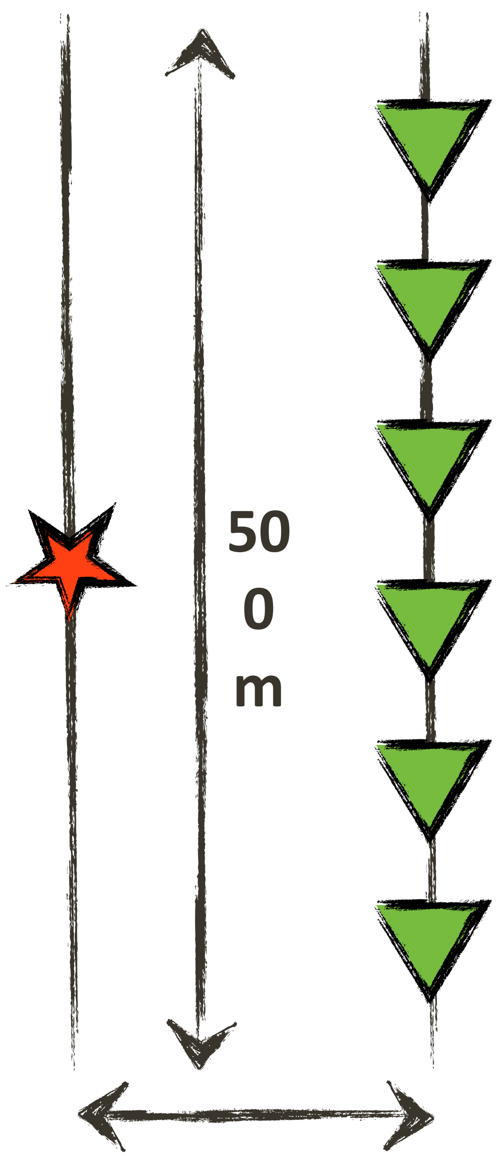
Multiscale WF detection allows us to move from

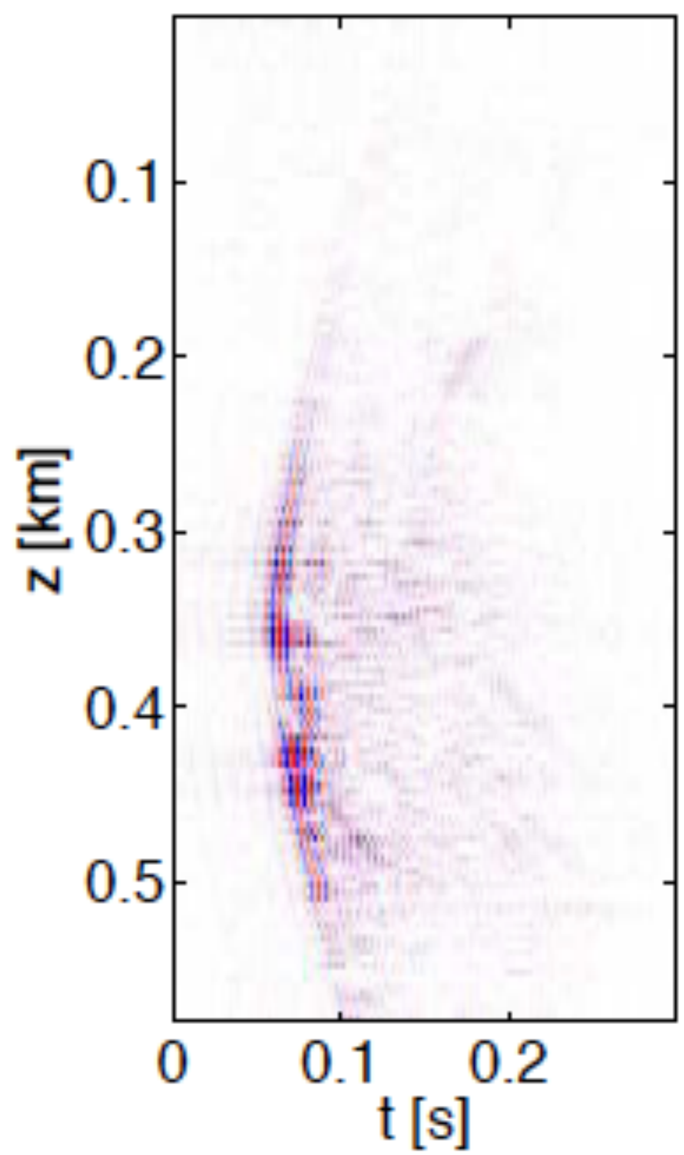
- **Traveltime fitting at large scale**

to

- **`Stack power' at small scale**

Numerical example



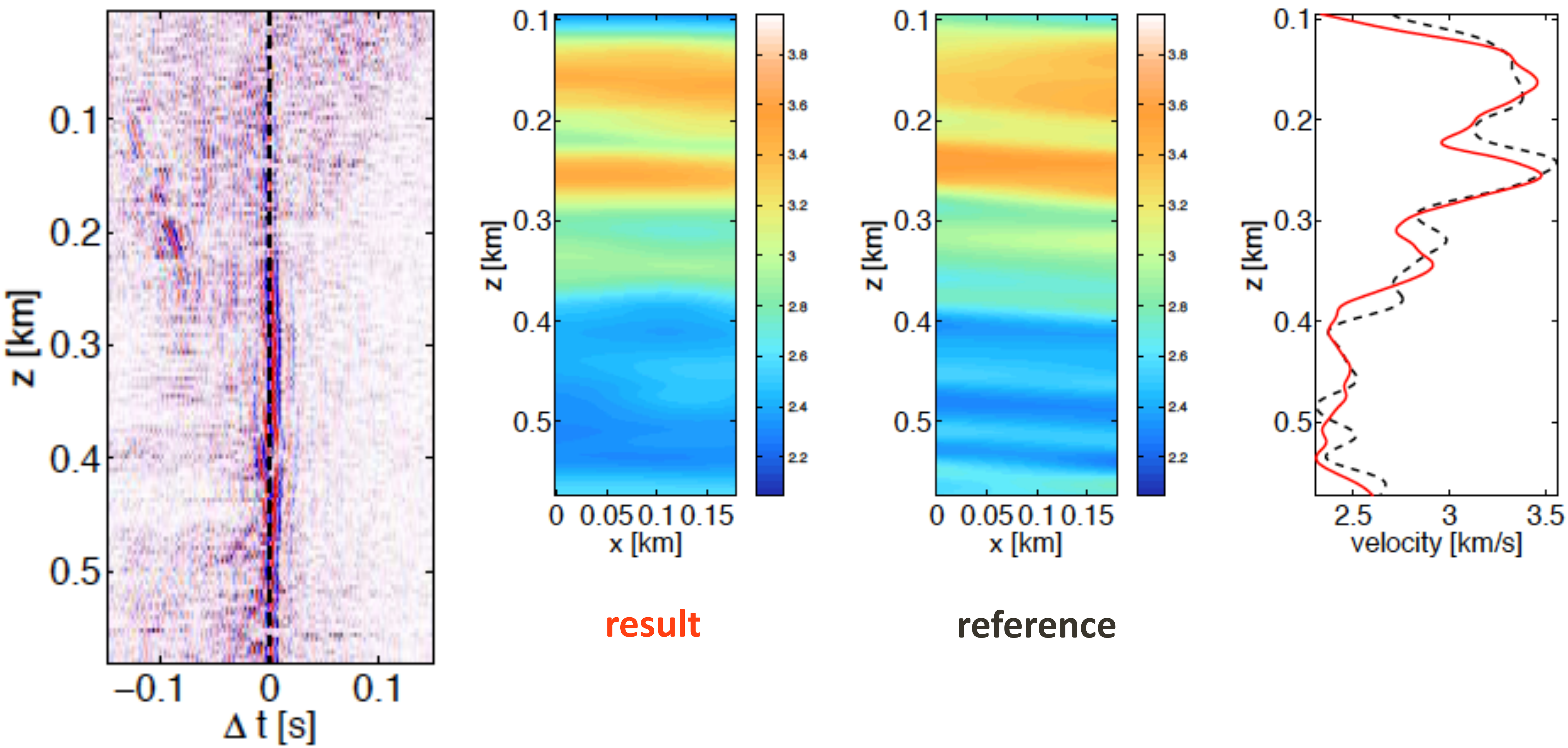


[TvL 10;]

Real cross-well data set

- **Frequency domain FD**
- **Adjoint-state for gradient**
- **L-BFGS for optimization**
- **different stages using different basis functions**

Numerical example



result

reference

Future work

- Curvelet-based WF detection
- Reflection tomography
- Scale dependent regularization & study of sensitivity kernels

Conclusions

- **Natural way to move from traveltime to amplitude fitting, and overcome loopskipping**
- **Multiscale WF detection might be extended to dispersion and stereo tomography**

Acknowledgements

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