

Multiscale aspects of waveform tomography

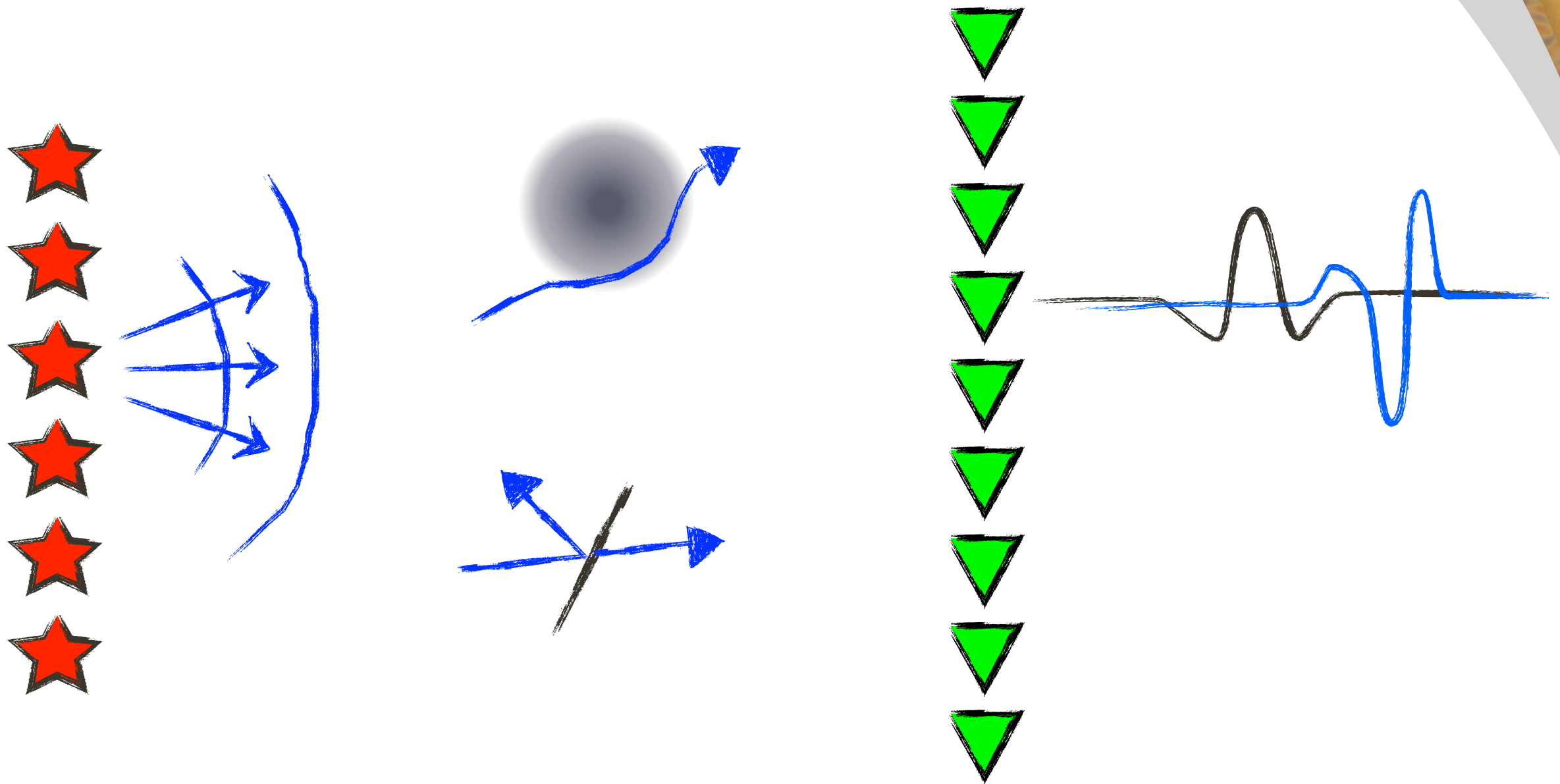
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SIAM GeoSciences 2011 – MS61

Waveform imaging



Overview

- **Waveform tomography**
- **Wavefrontset detection**
- **Misfit criteria**
- **Numerical example**
- **Future work & Conclusions**

Waveform tomography

Model the data as

$$\left[\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right] u = w(t) \delta(x - s),$$

$$d(t, s, r) = u(t, x, s)|_{x=r} \equiv F[c].$$

**Goal is to find the velocity given
the data and source signature**

Waveform tomography

Such inverse problems have been extensively studied. Major findings:

- recovery via LS is problematic for bandlimited data**
- some form of traveltime fitting needed for `complete' reconstruction**

Waveform Tomography

Sensitivity of the data to velocity perturbations can be studied in high-frequency asymptotic regime

$$\hat{d}(\omega, s, r) \simeq A(\omega, s, r) \exp[i\omega T(s, r)]$$

- amplitude

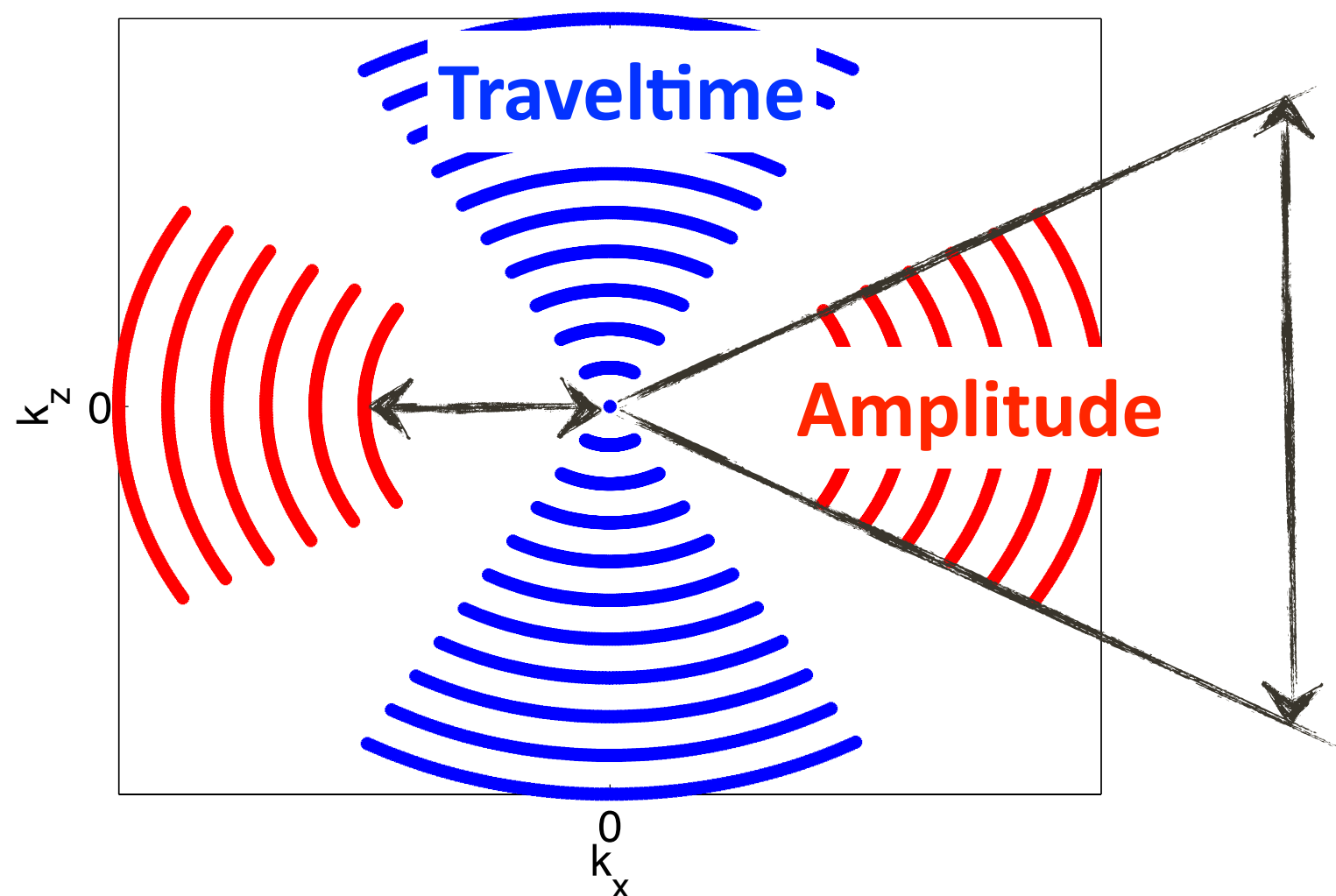
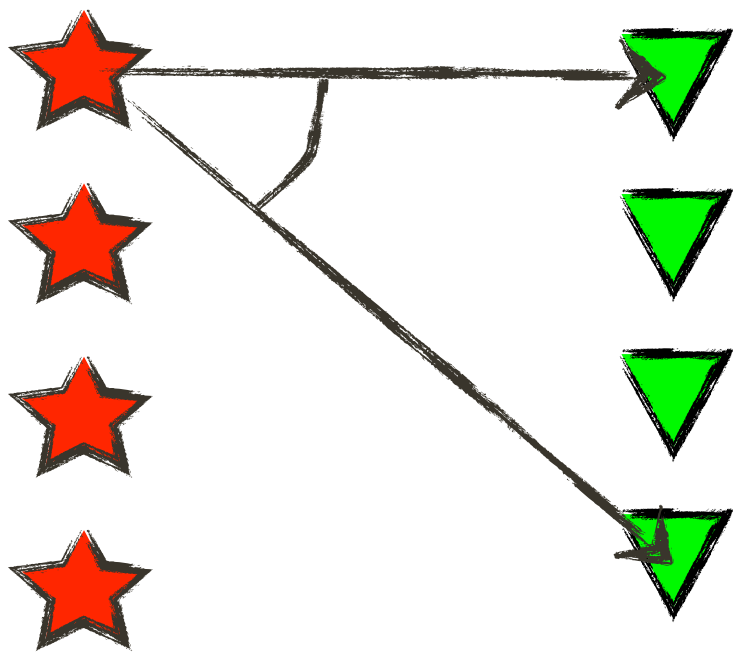
$$\widehat{\delta d} \simeq \int d\omega \int dx \, \hat{w}(\omega) \delta c(x) \exp[i\omega (T(s, x) - T(x, r))]$$

- travelttime

$$\nabla \delta T \cdot \nabla T = \delta c$$

Waveform tomography

Wavenumber coverage with limited aperture



[Stork; Bube; Natterer;]

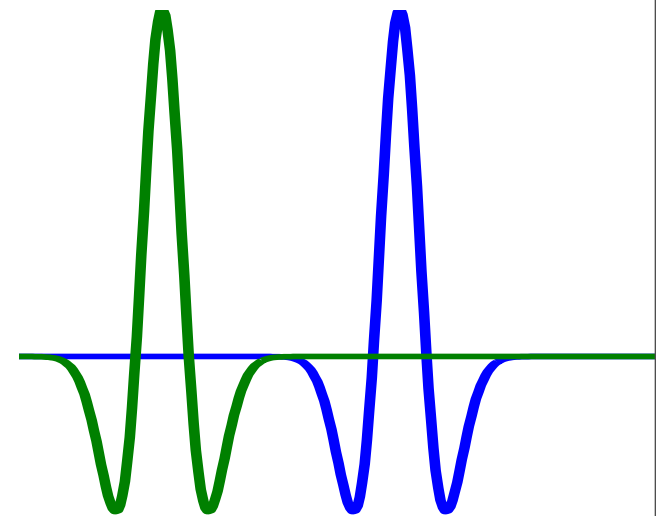
Waveform tomography

Full waveform inversion:

$$\min_c ||F[c] - \bar{d}||_2^2$$

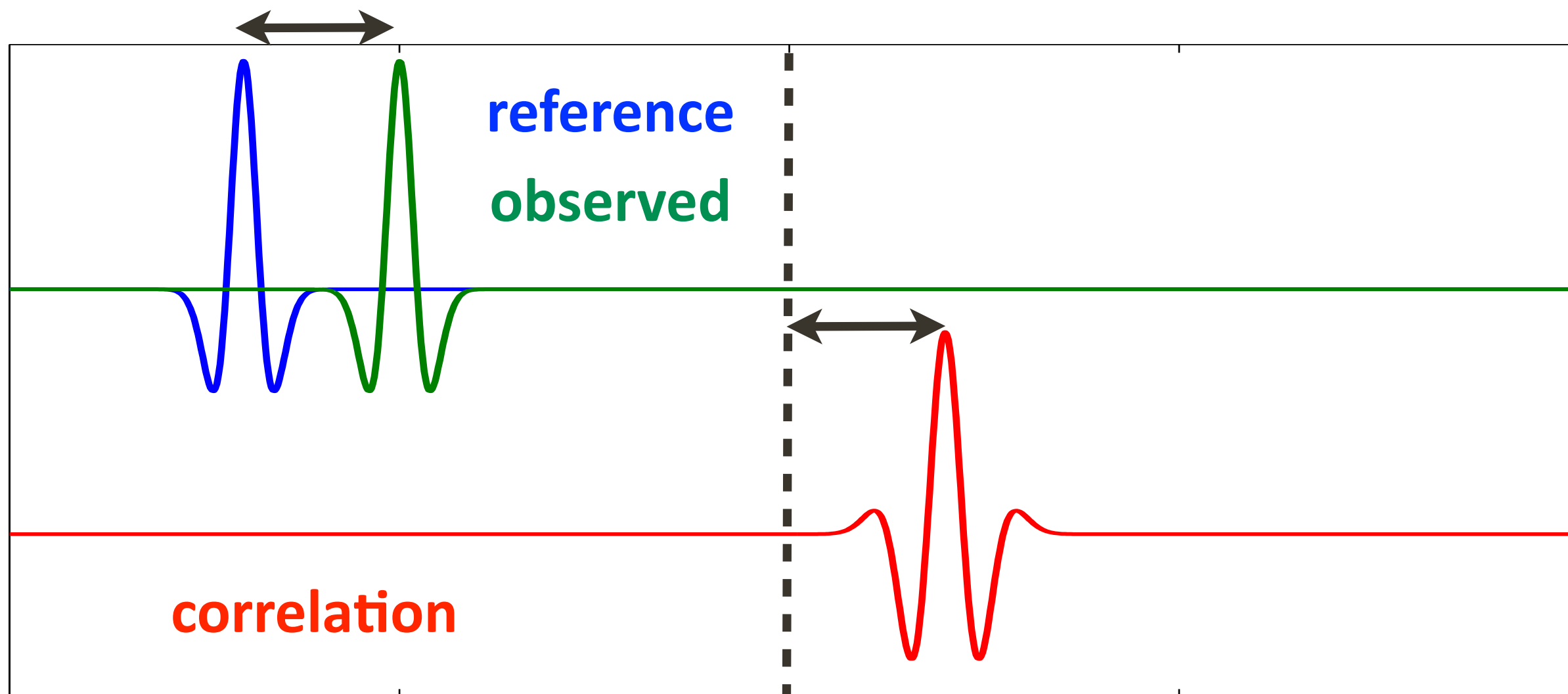
- need large aperture and low frequencies
- loopskipping if velocity error is too big, can be mitigated partly by `multiscale' FWI

[Tarantola 84; Bunks 98; Shin 09]



Waveform tomography

Wave-equation travelttime tomography



Waveform tomography

WE traveltime tomography:

- **relies on detecting shift of singular support**
- **widely used criterion: maximum of the correlation**

$$\min_c ||\tau[c]||_2^2, \quad \tau[c] = \operatorname{argmax}_t (d * \bar{d})(t)$$

[Cara 87; Luo 91; Dahlen 10; Hormann 02; de Hoop 05;Brytik 10]

Waveform tomography

LS may be re-formulated as
maximizing the normalized zero-
lag correlation

$$||d - \bar{d}||_2^2 = ||d||_2^2 + ||\bar{d}||_2^2 - 2 \underbrace{\langle d, \bar{d} \rangle}_{(\bar{d} * d)|_{t=0}}$$

`picking approach' is a clever
extension of this

Wavefrontset detection

Given a function of the form

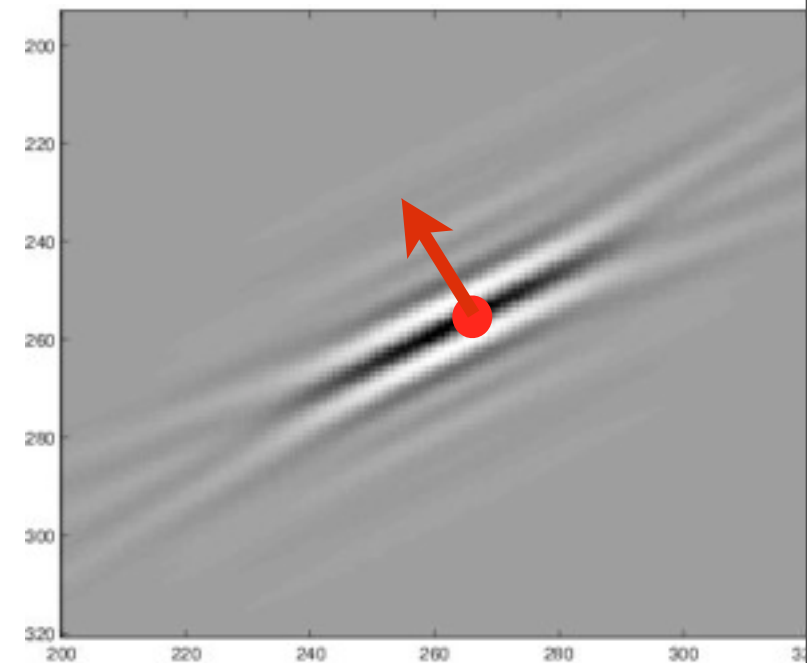
$$f(x, t) = \int d\omega a(\omega, x, t) \exp[i\phi(\omega, x, t)]$$

the wavefrontset is given by

$$\text{WF}(f) \subseteq \{x, t; \partial_x \phi, \partial_t \phi \mid \partial_\omega \phi = 0\}$$

In particular:

$$\text{WF}(\bar{d} * d) \subseteq \{s, r, \bar{T} - T; \nabla(\bar{T} - T), i\omega\}$$



Wavefrontset detection

- **Multiscale WF detection via the FBI transform:**

$$G[f](t, \omega, \sigma) = \frac{1}{\sqrt{\sigma}} \int dt' f(t') W[(t - t')/\sigma] \exp[i\omega t']$$

- **if $t \notin \text{WF}(f)$ then for fixed ω and any $N \in \mathbb{N}$**

$$|G[f](t, \omega, \sigma)| \leq \sigma^N \quad \text{as } \sigma \downarrow 0$$

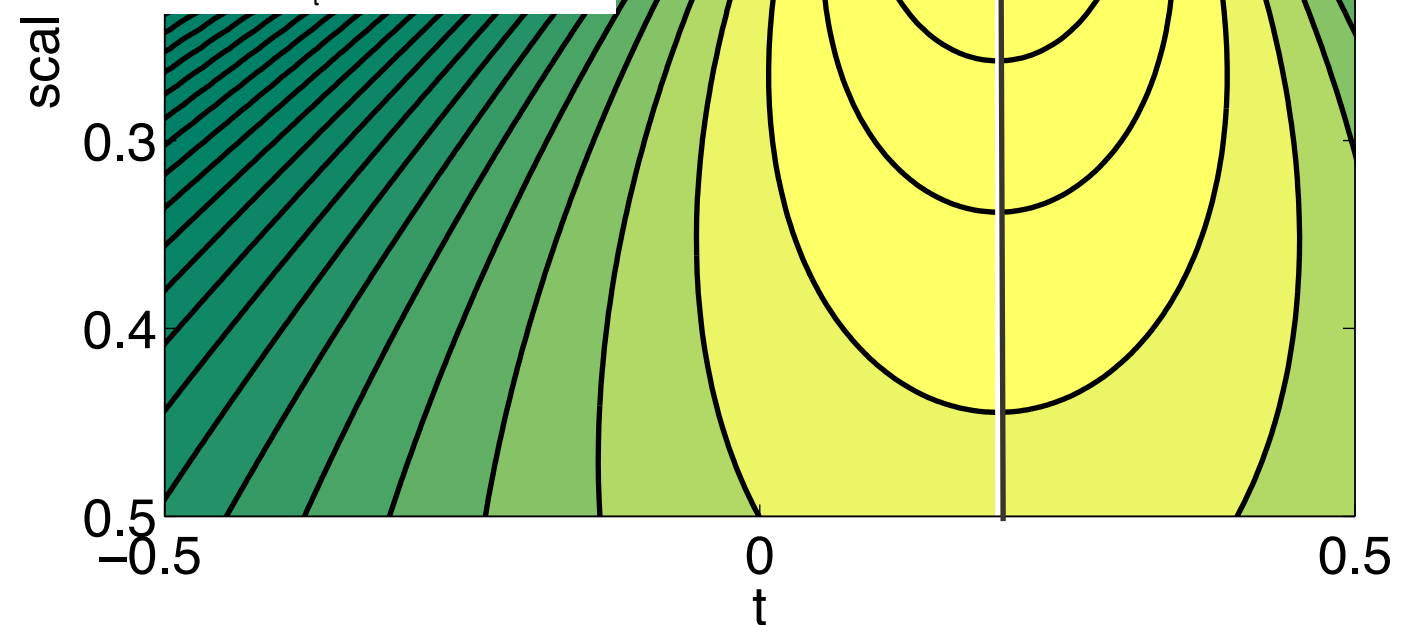
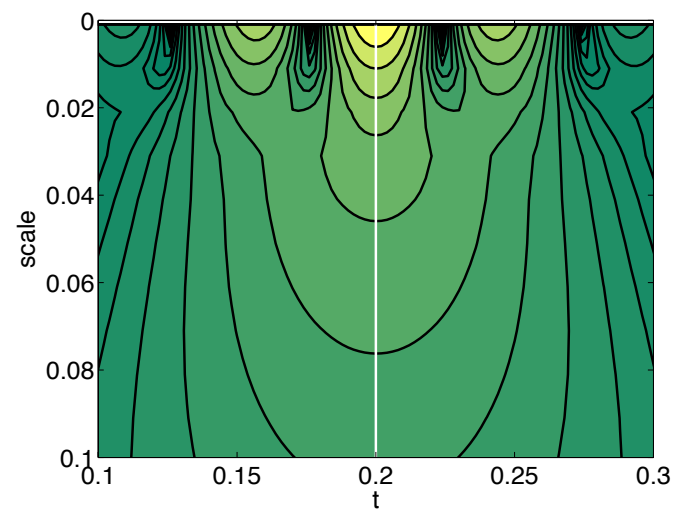
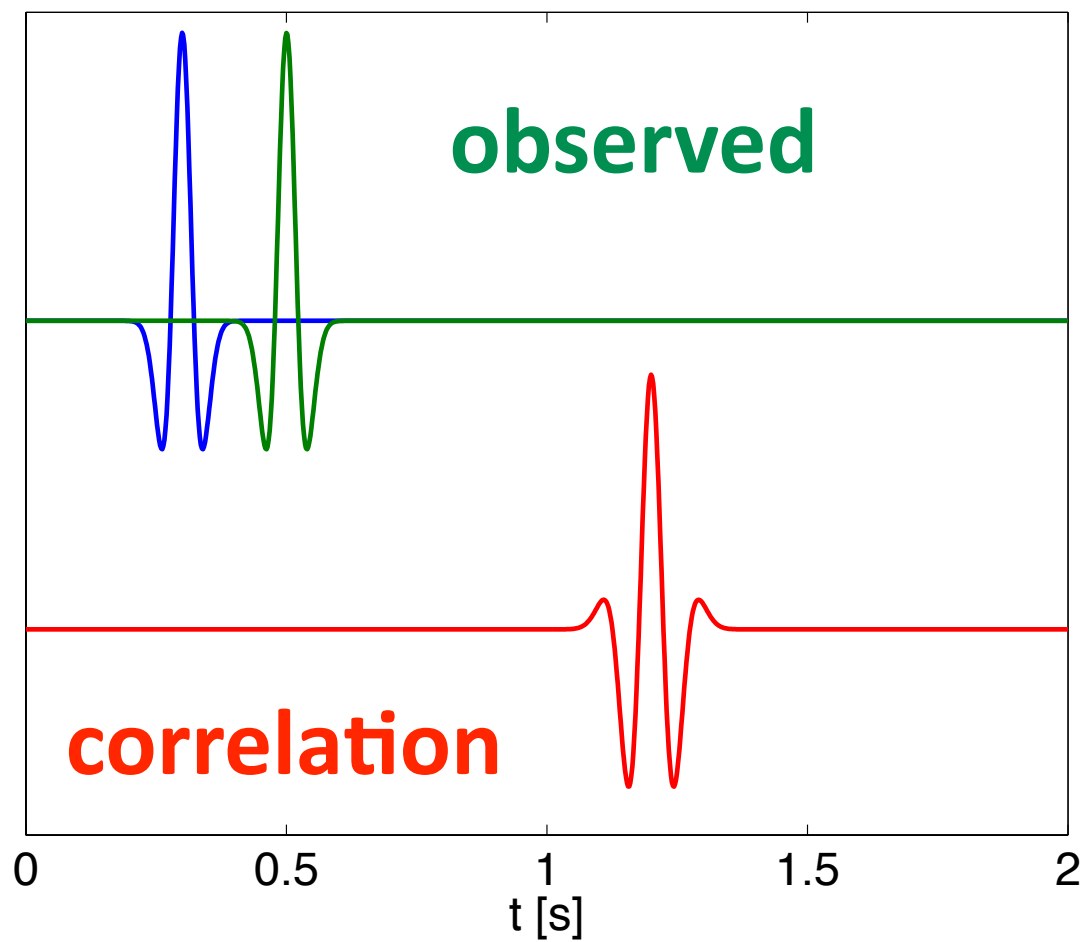
[Hormander 83; Hormann 02; de Hoop 05]

Wavefrontset detection

reference

observed

correlation

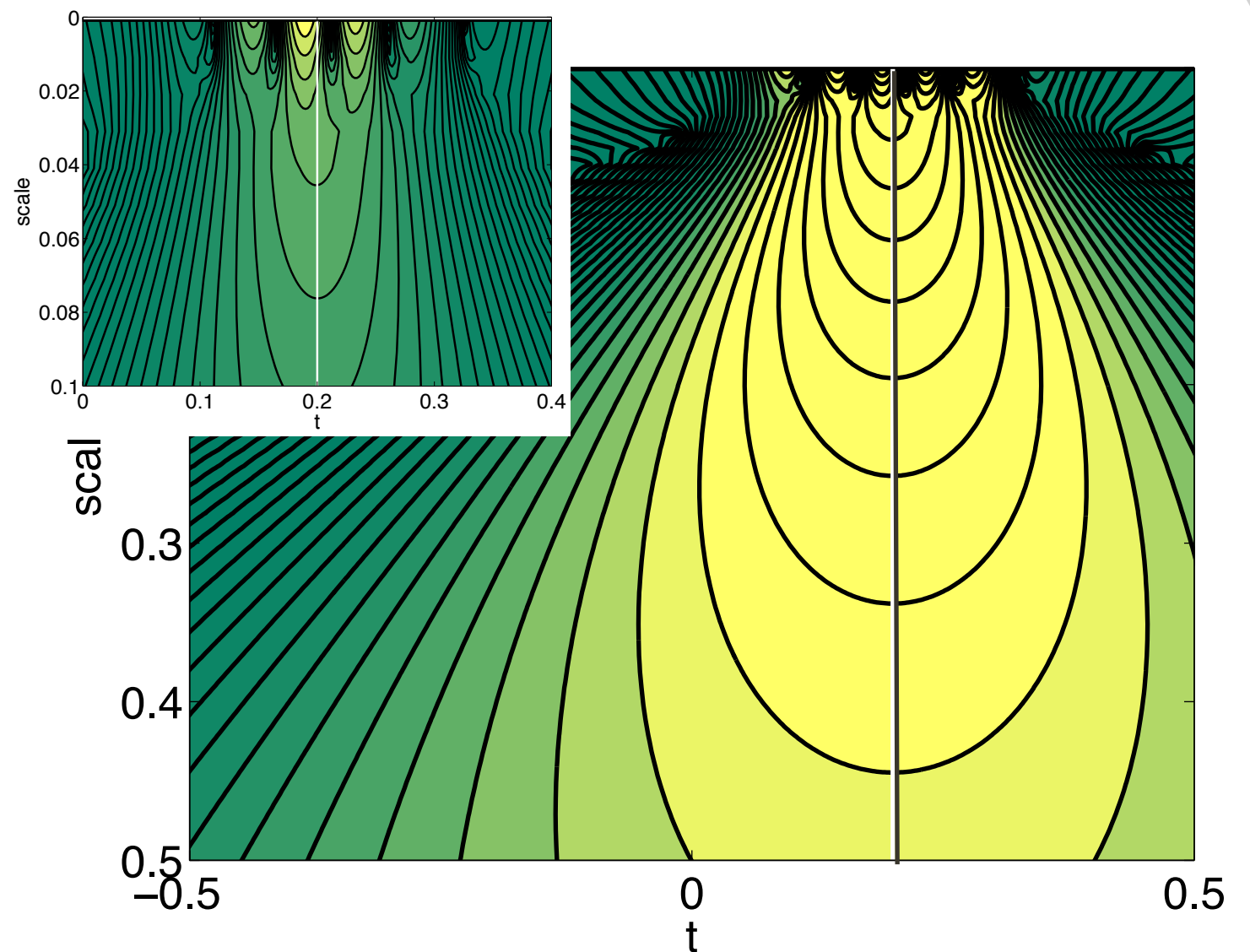
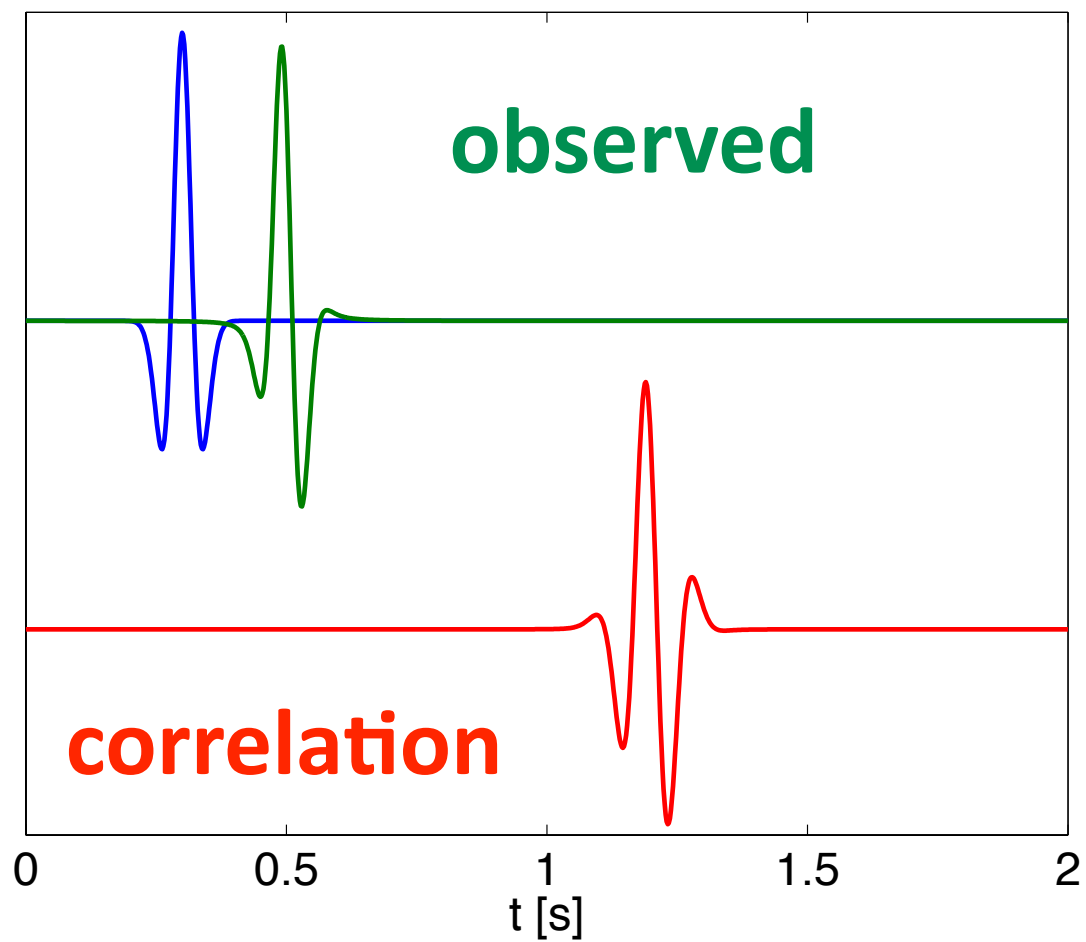


Wavefrontset detection

reference

observed

correlation

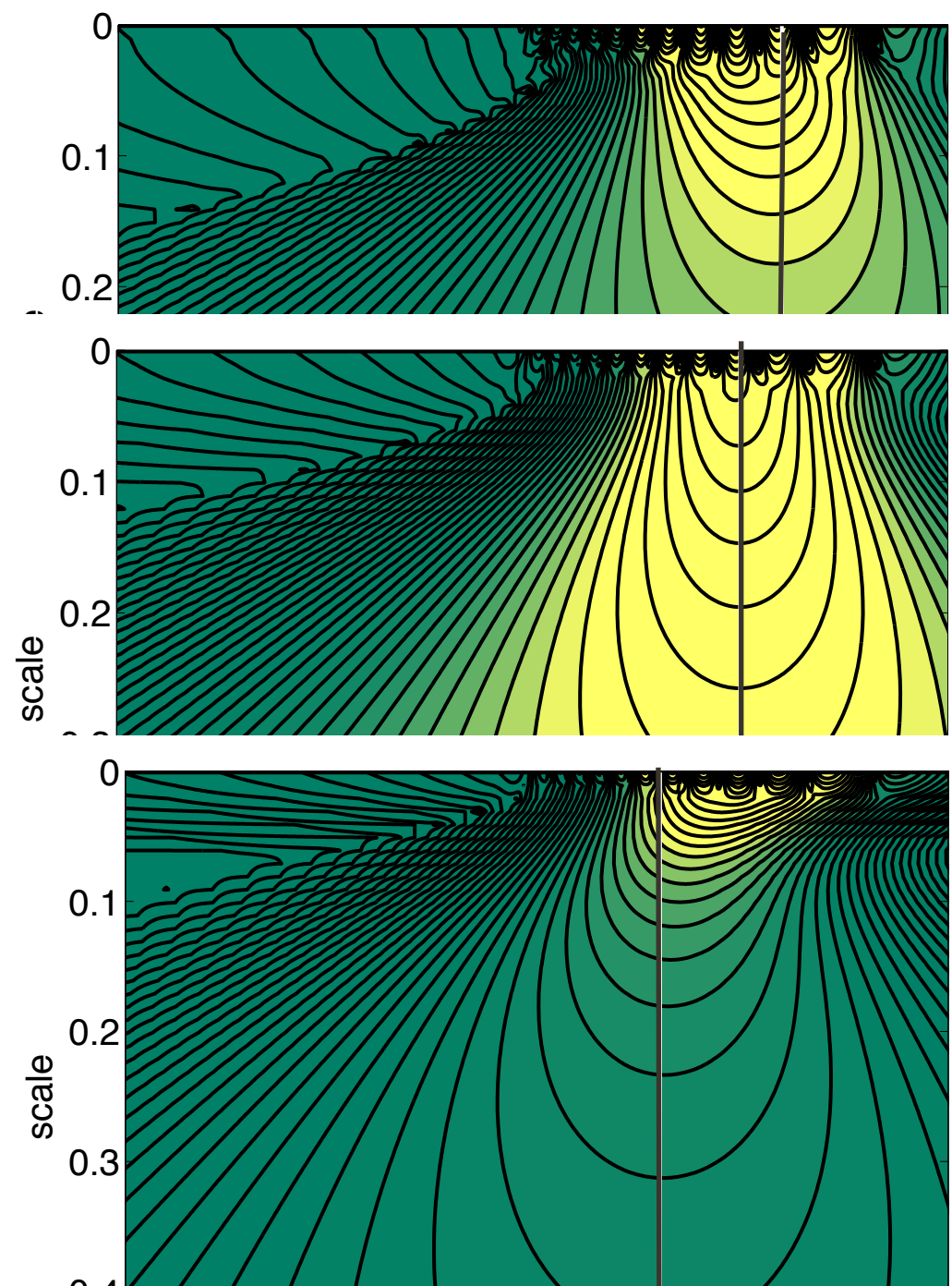
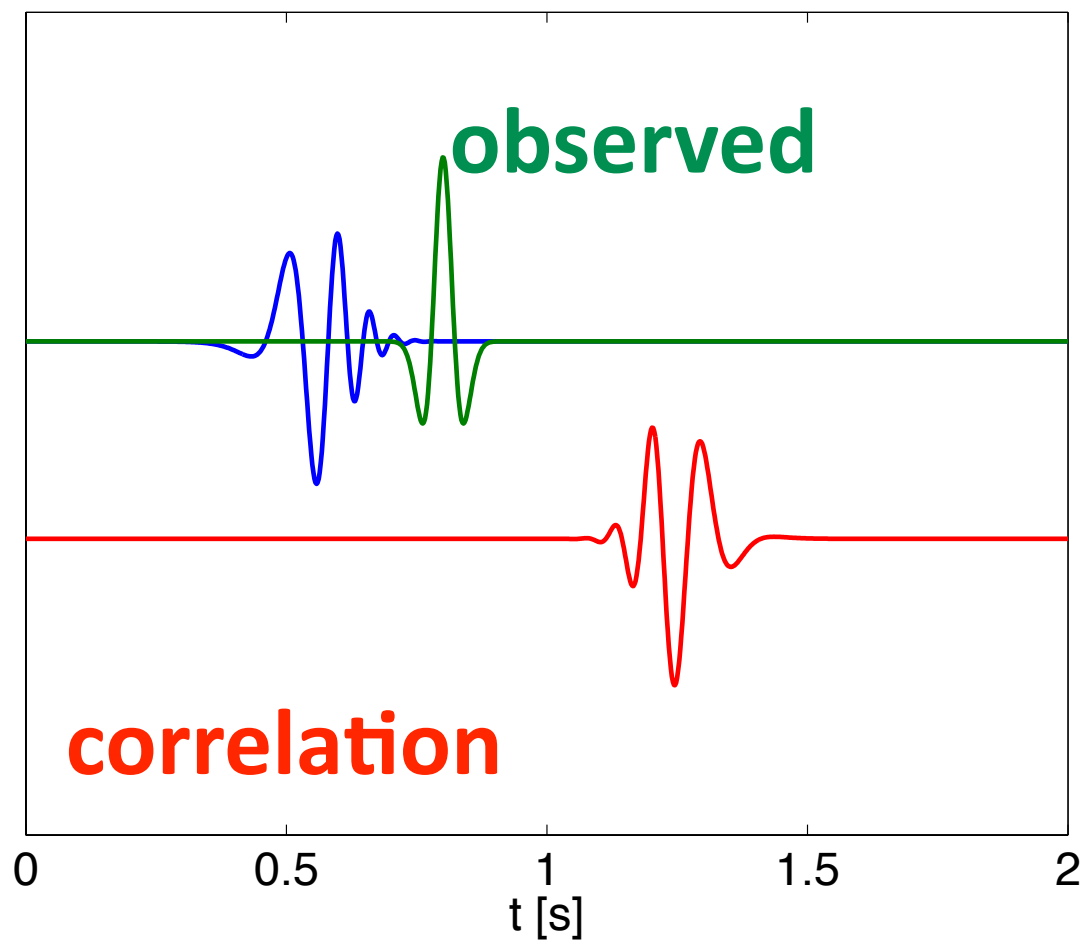


Wavefrontset detection I

reference

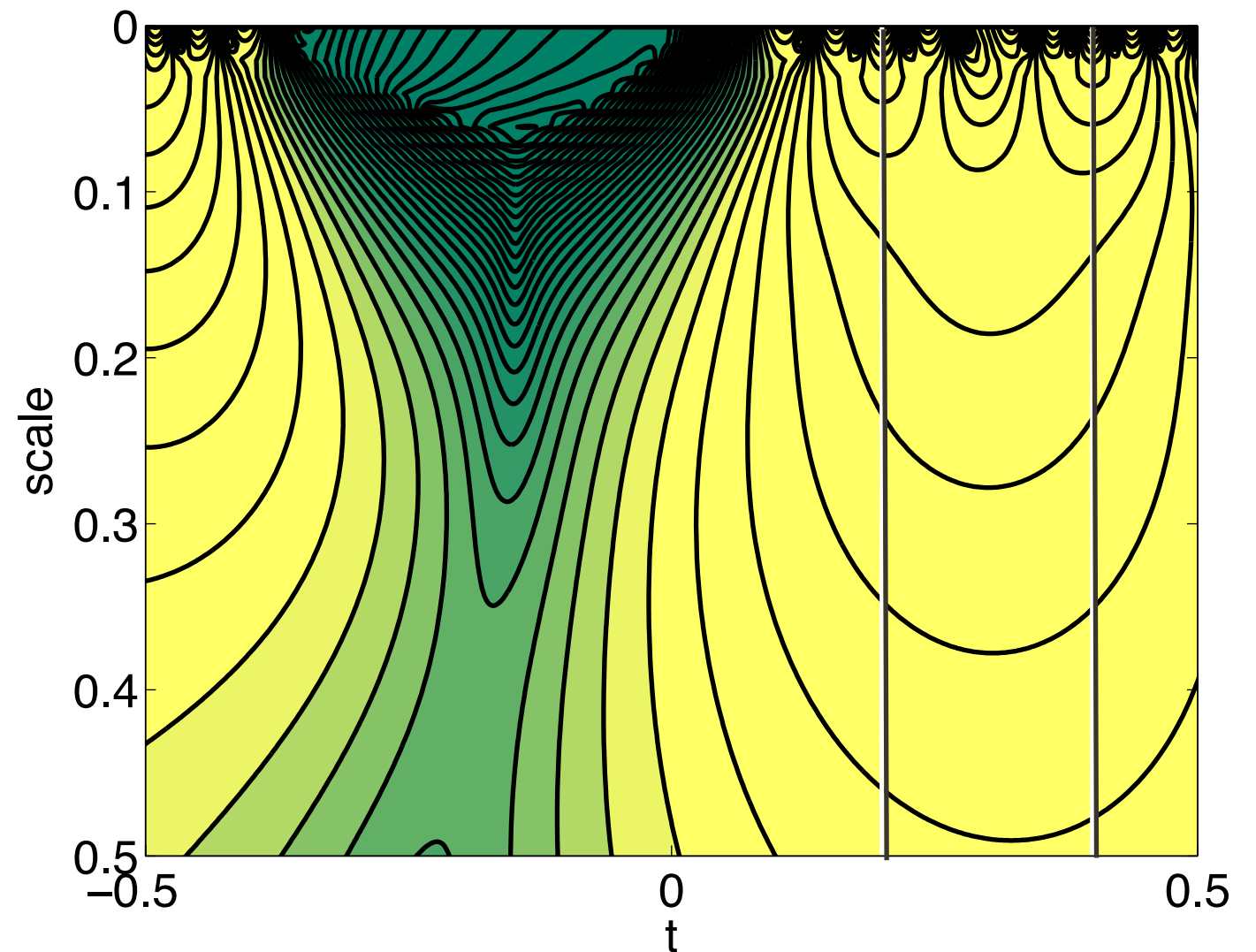
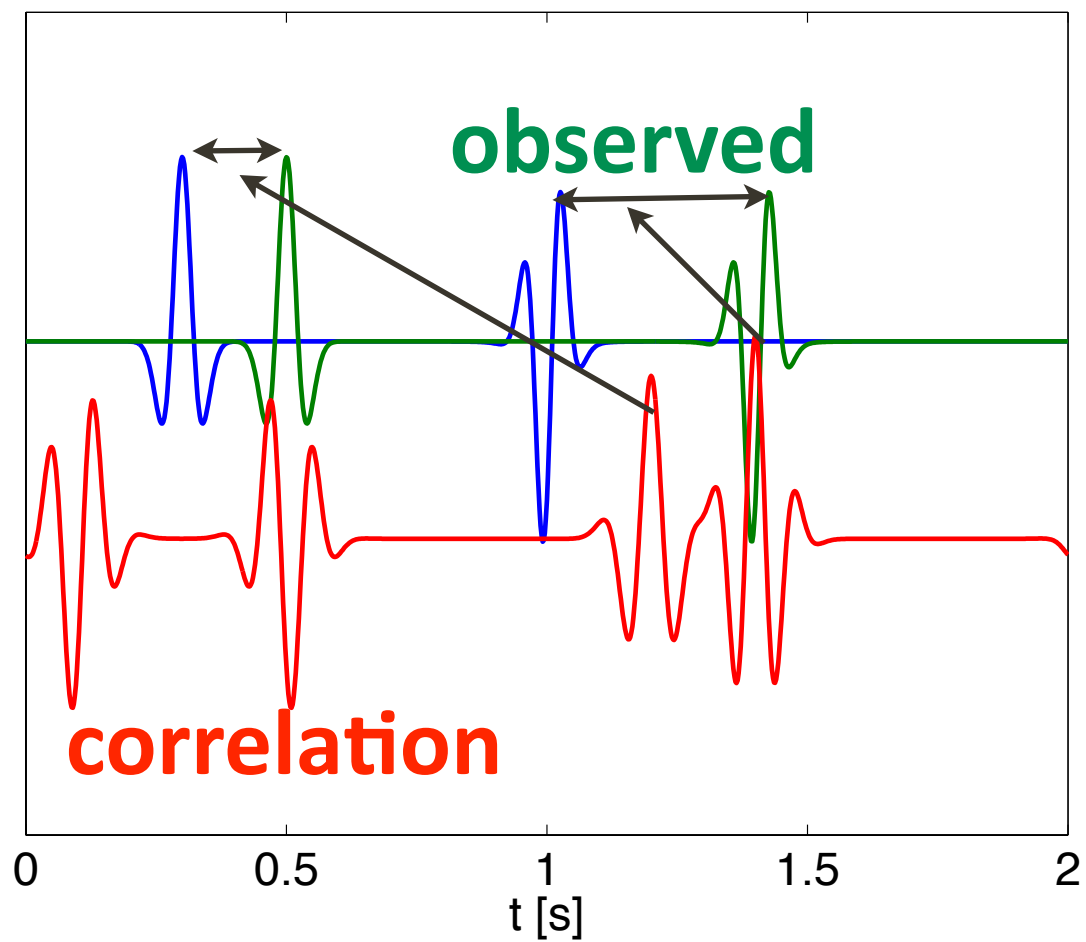
observed

correlation



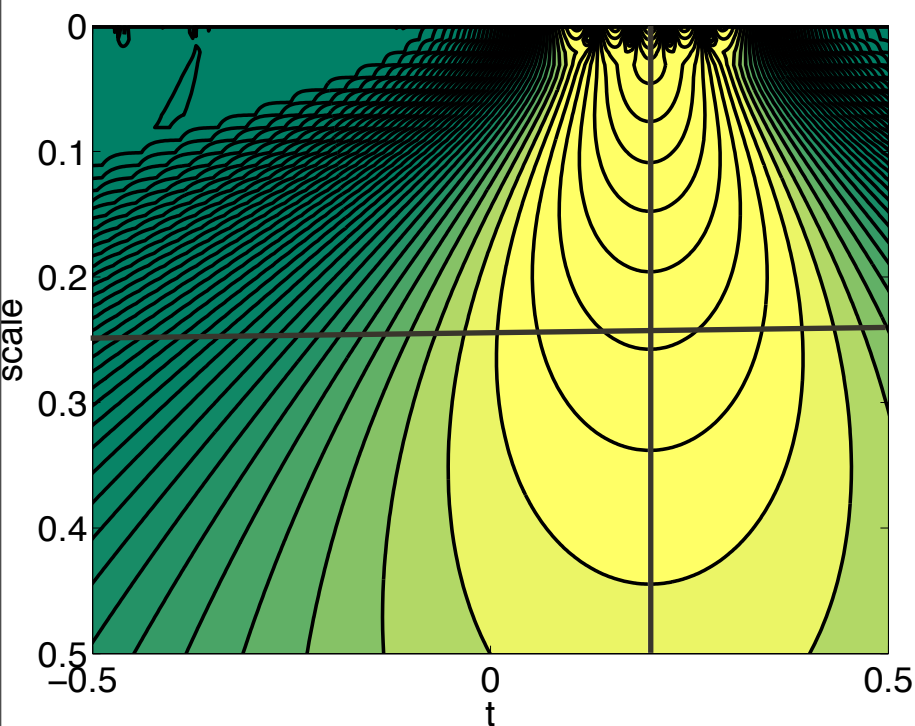
Wavefrontset detection

reference



Misfit criteria

- $\tau[\omega, \sigma] = \operatorname{argmax}_t G[\bar{d} * d](t, \omega, \sigma)$
converges to picking approach
as $\sigma \downarrow 0$ and $\omega = 0$



- **Maximize** $\|G[\bar{d} * d](0, \cdot, \sigma)\|_2^2$
- **Minimize** $\|\partial_t G[\bar{d} * d](0, \cdot, \sigma)\|_2^2$

Misfit criteria

Rewrite:

$$G[f](0, \omega, \sigma) = (\widehat{W_\sigma \cdot f})(\omega)$$

$$\partial_t G[f](0, \omega, \sigma) = (\widehat{W'_\sigma \cdot f})(\omega)$$

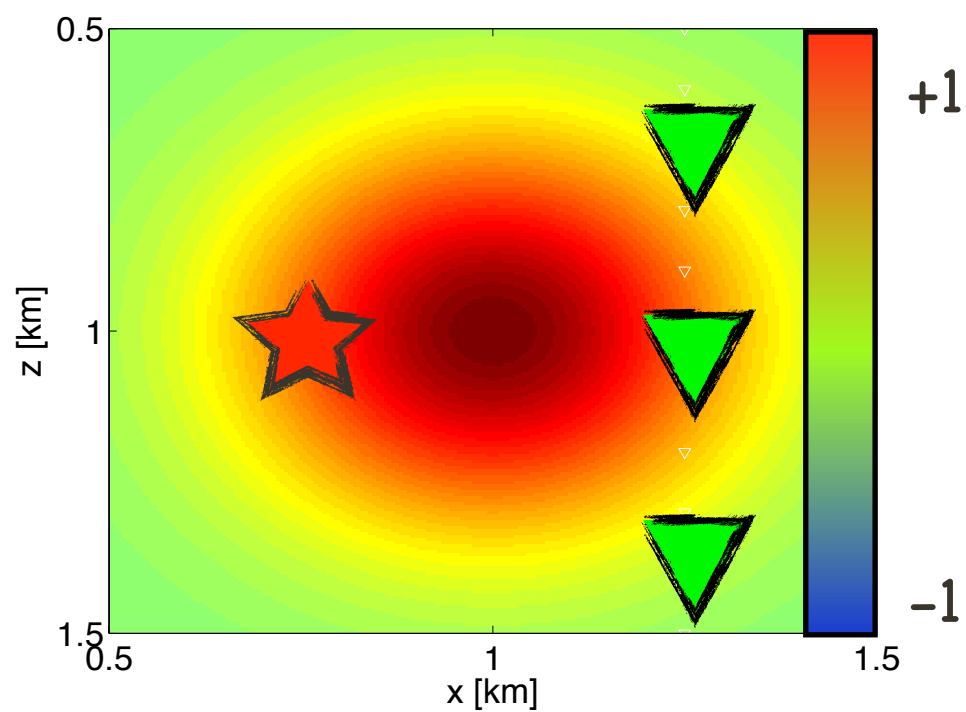
where $W_\sigma(t) = \frac{1}{\sqrt{\sigma}} \exp[-(t/\sigma)^2]$

Misfit:

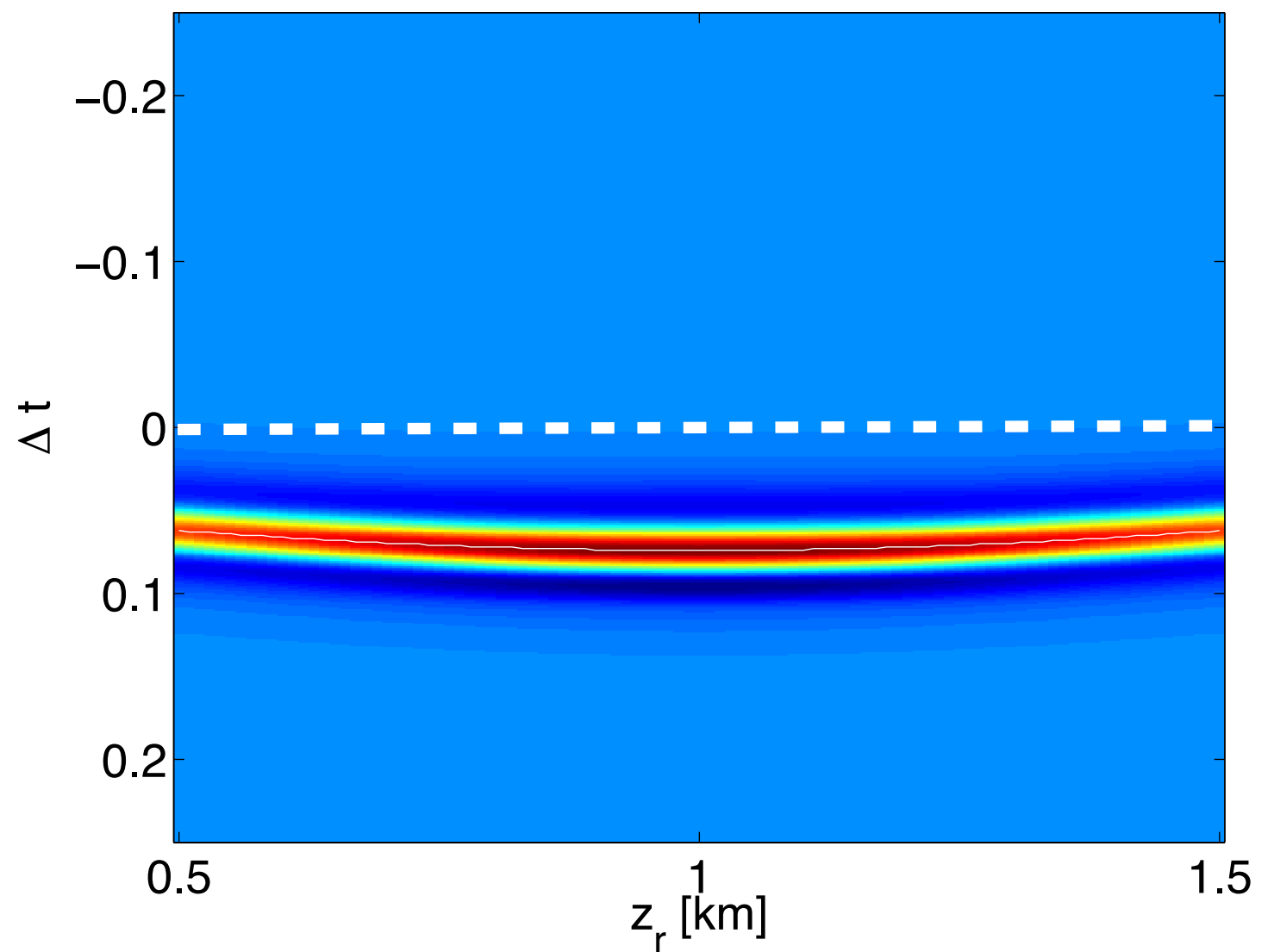
$$\phi = \frac{||W_\sigma \cdot (\bar{d} * d)||_2^2}{||d||_2^2}$$

[TvL 08; TvL 10]

Misfit criteria

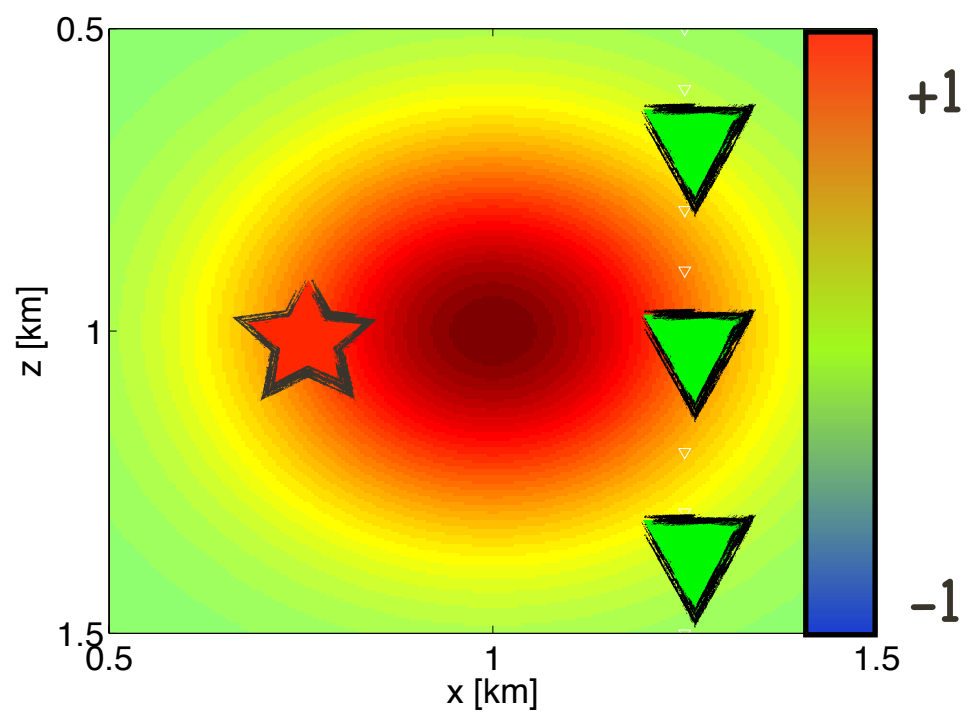


velocity perturbation

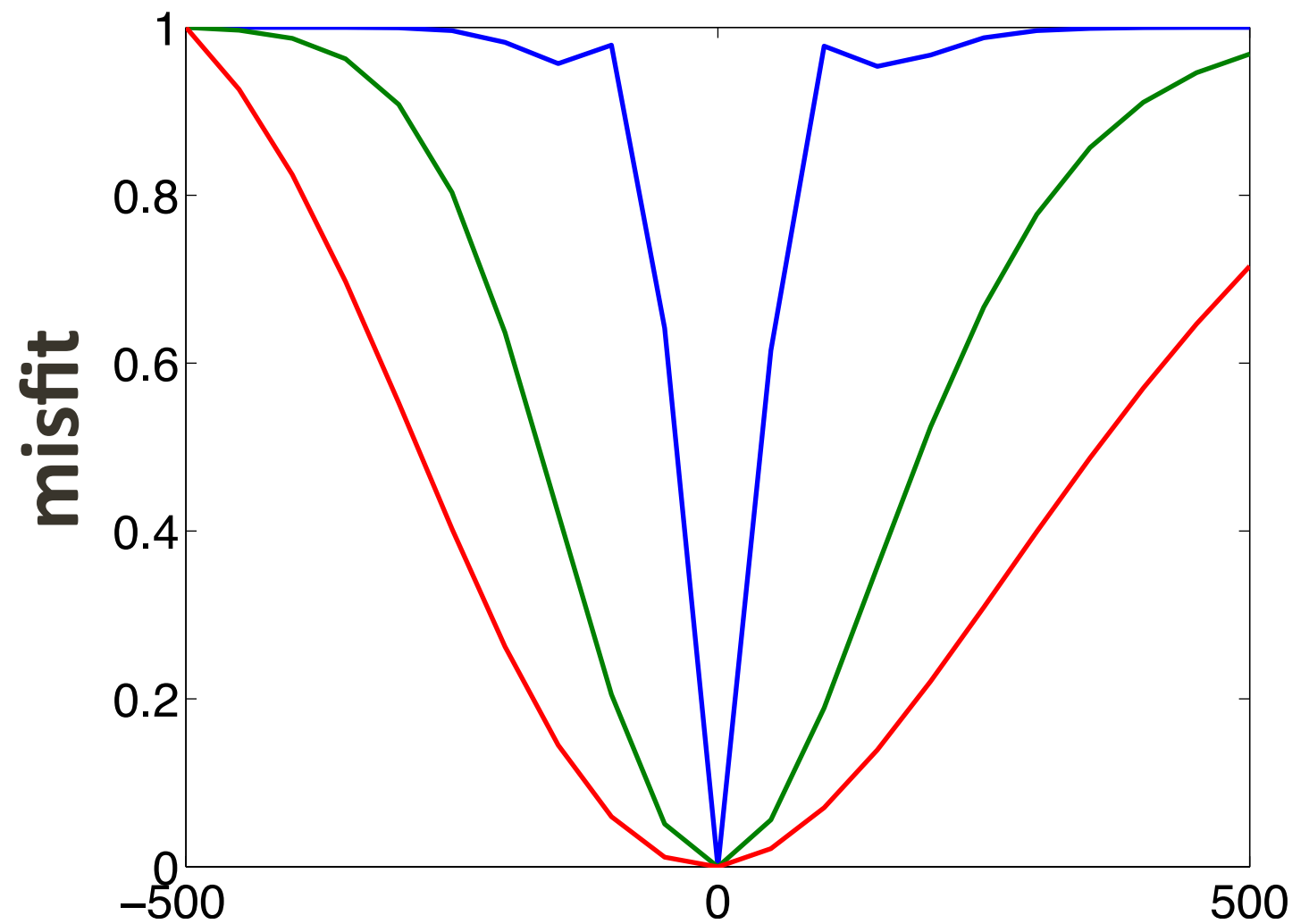


Misfit criteria

small, medium, large

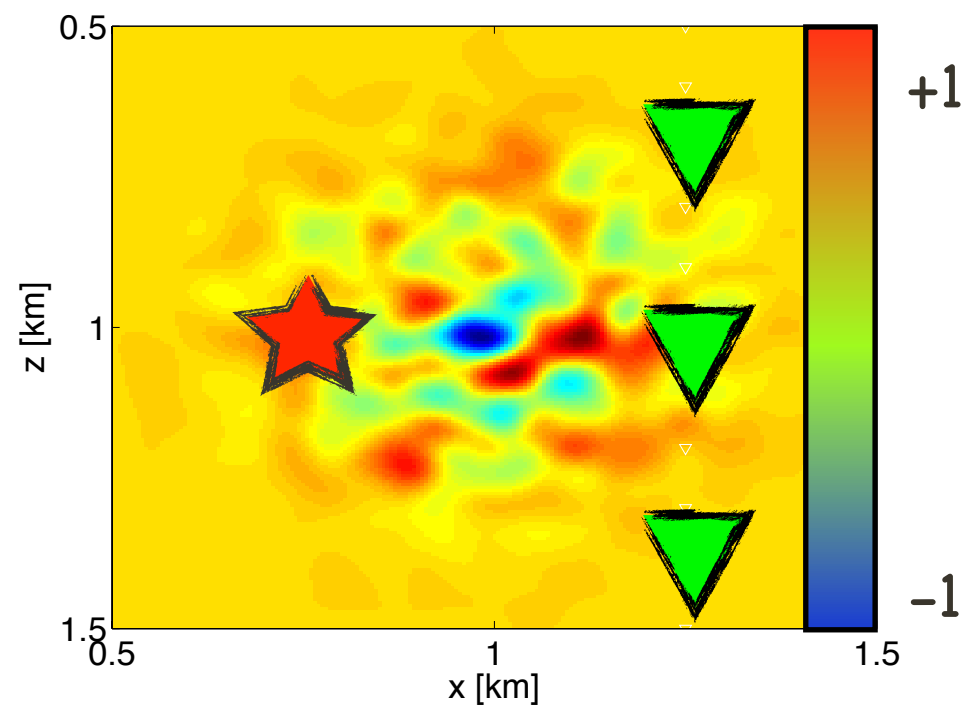


velocity perturbation

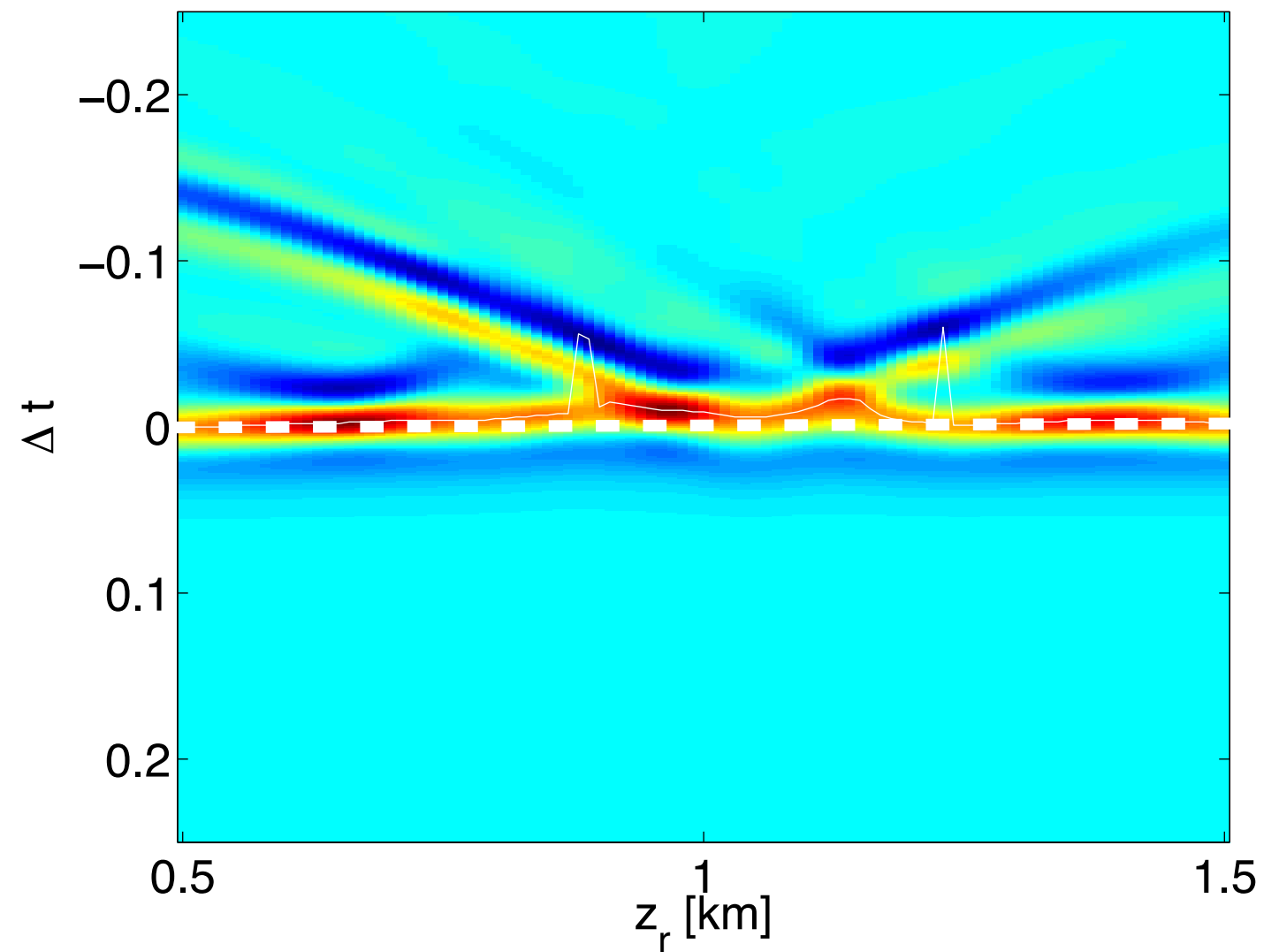


velocity [m/s]

Misfit criteria

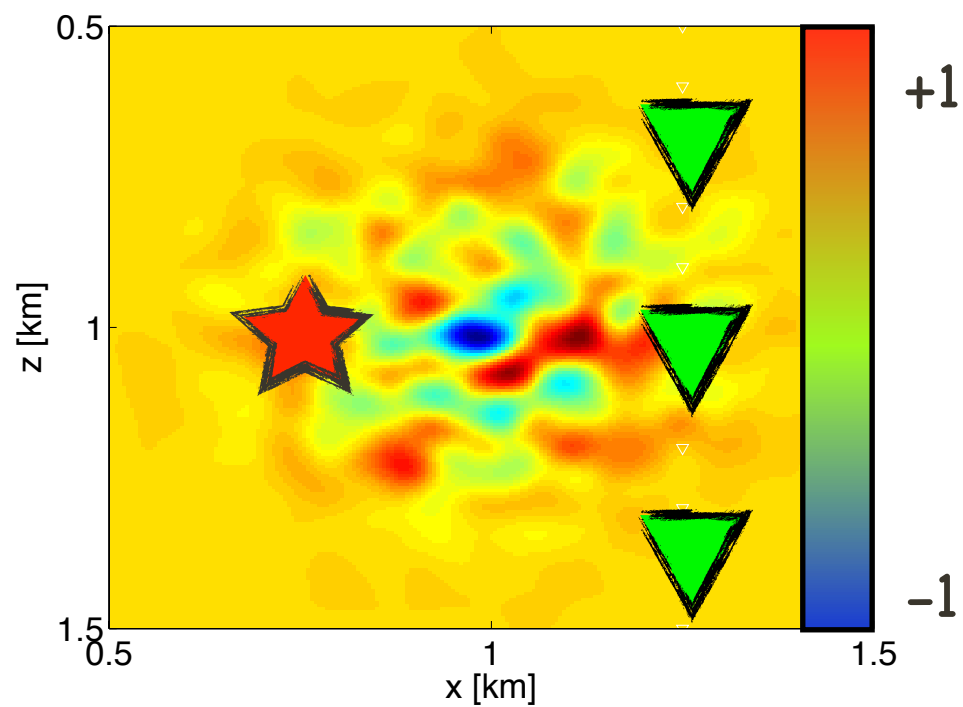


velocity perturbation

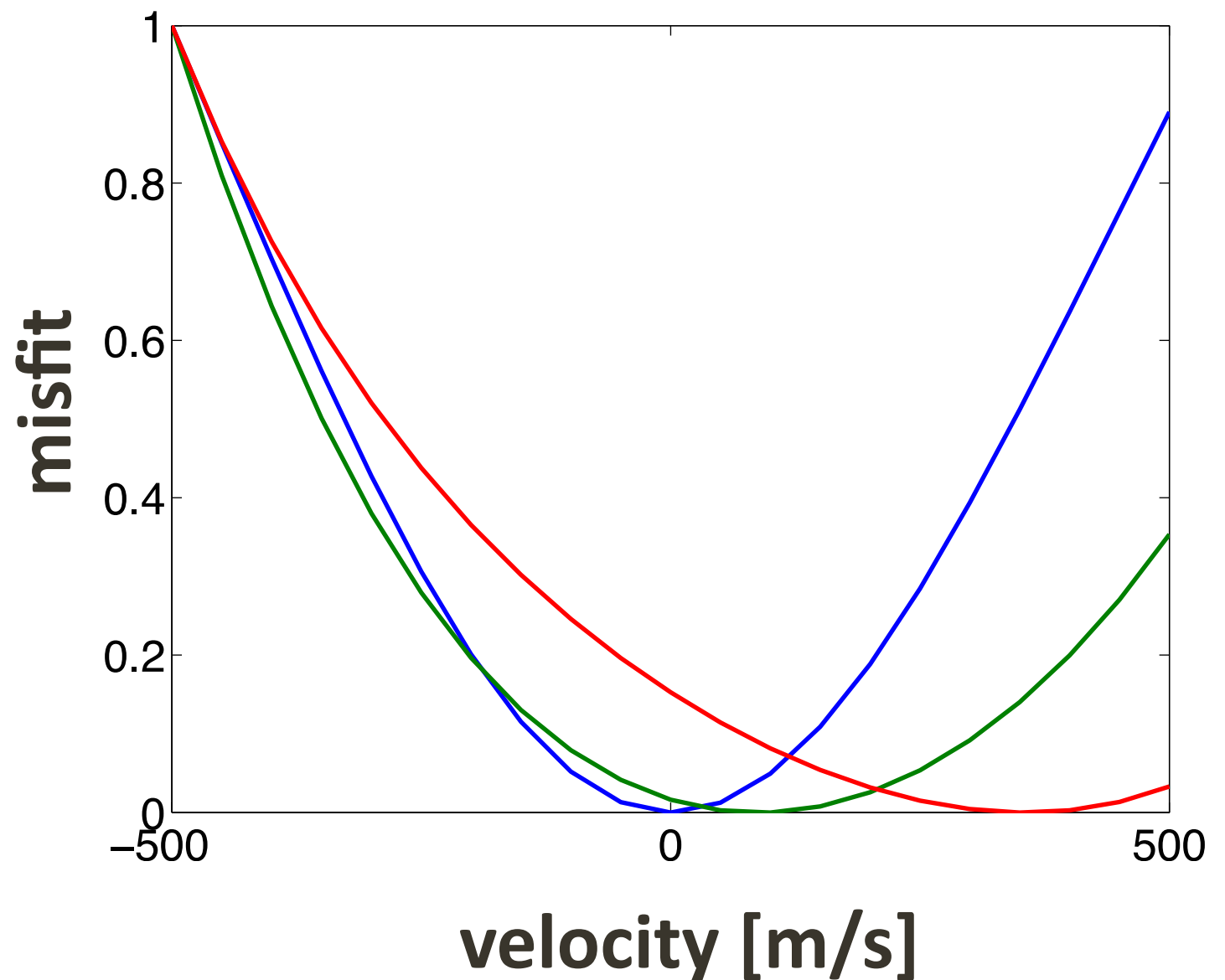


Misfit criteria

small, medium, large



velocity perturbation



Misfit criteria

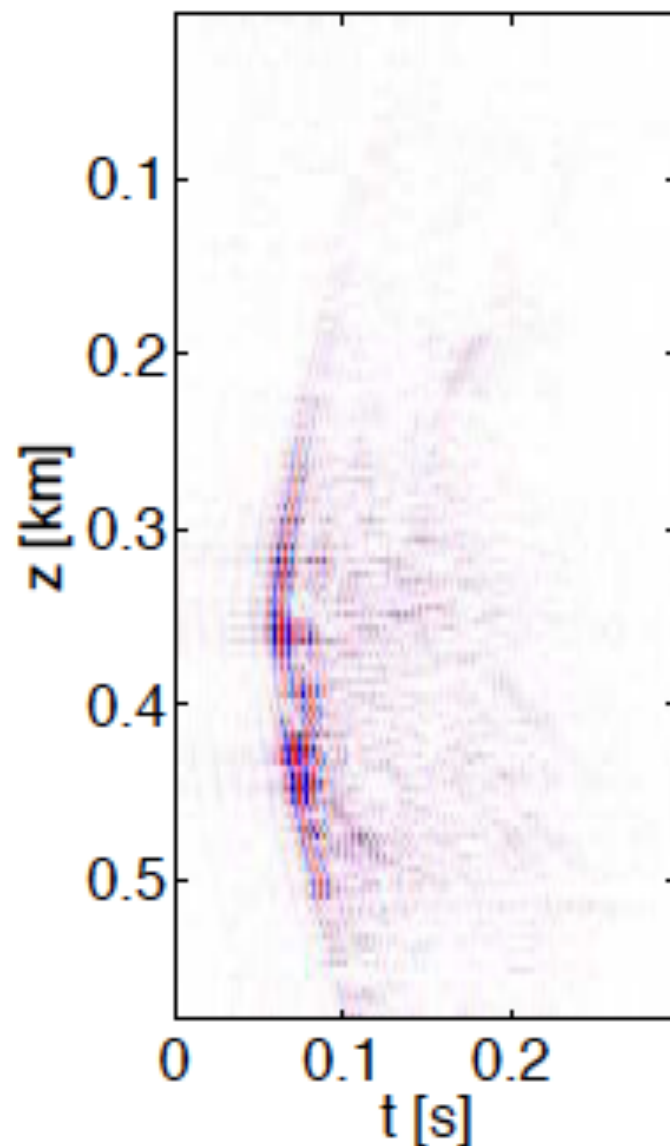
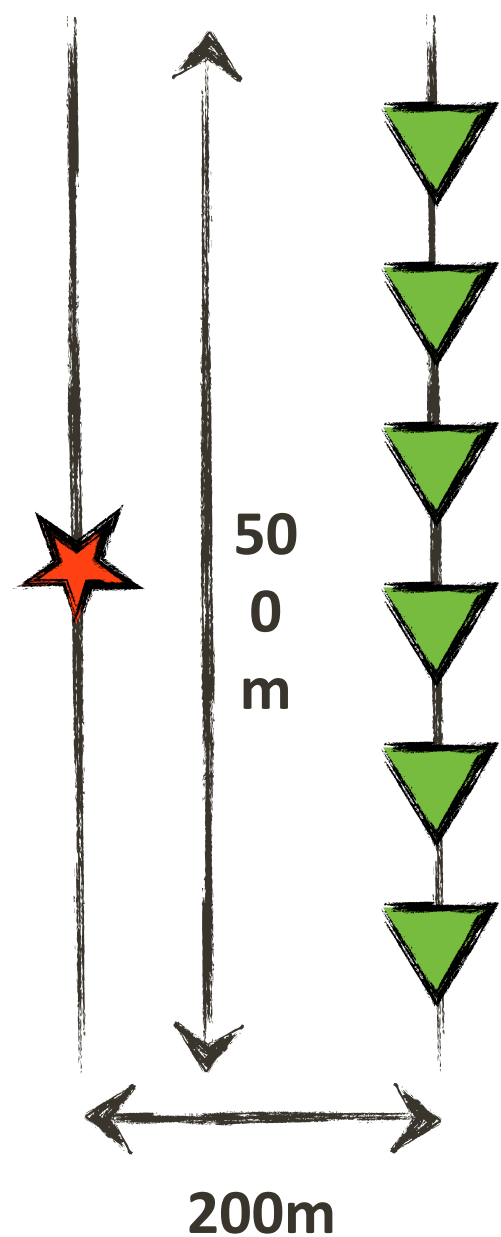
Multiscale WF detection allows us to move from

- **Traveltime fitting at large scale**
- to
- **`Stack power' at small scale**

Numerical example

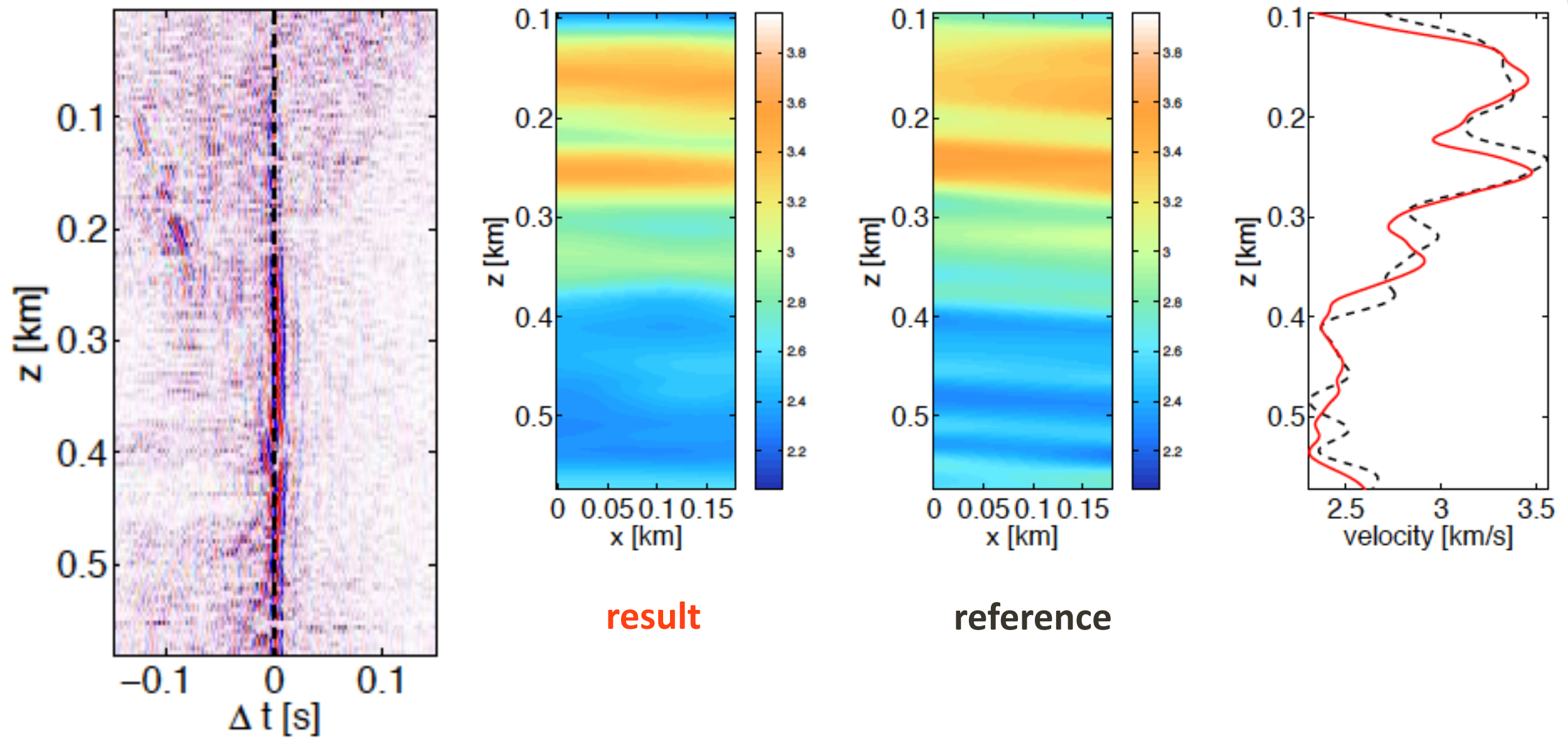
Real cross-well data set

- Frequency domain FD
- Adjoint-state for gradient
- L-BFGS for optimization
- different stages using different basis functions



[TvL 10;]

Numerical example



Future work

- **Curvelet-based WF detection**
- **Reflection tomography**
- **Scale dependent regularization
& study of sensitivity kernels**

Conclusions

- **Natural way to move from traveltimes to amplitude fitting, and overcome loopskipping**
- **Multiscale WF detection might be extended to dispersion and stereo tomography**

Acknowledgements

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