Neural Operators with Accurate Jacobian for High-Fidelity Image-Domain Seismic Inversion

Jeongjin (Jayjay) Park[1], Huseyin Tuna Erdinc[1], Grant Bruer[1], Richard Rex[2], Nisha Chandramoorthy[3], Felix J. Herrmann[1]

[1]Georgia Institute of Technology [2]Terra AI [3]University of Chicago

Motivation

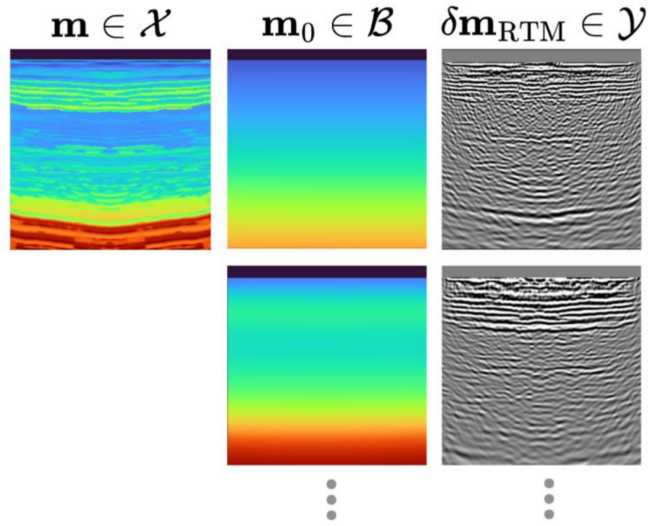
- **Problem 1**: Neural operators (NOs) are fast surrogates for wave-equation imaging, but accurate forward prediction alone *does not* *guarantee accurate sensitivity of NO*.
- **Problem 2** : Inversion *updates depend on the adjoint Jacobian*, a NO with incorrect sensitivity can introduce *unphysical artifacts and miss deep structure.*

GOAL: Supervise the NO's Jacobian — not just its forward map — along a set of well-illuminated directions, so the training cost stays computationally feasible while the surrogate learns accurate sensitivities alongside accurate outputs.

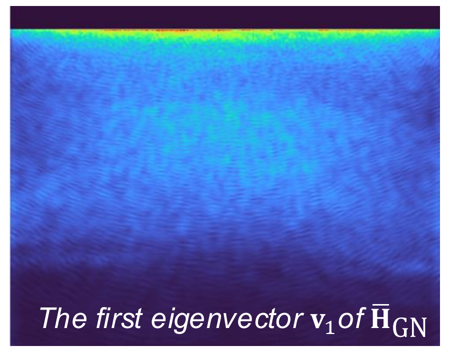
Dataset Construction (Offline)

Step 1: $\mathbf{m}, \mathbf{m}_0, \delta\mathbf{m}_{\mathrm{RTM}}$

- built from *randomized smoothing in depth and time*
- to expose the operator to diverse imaging scenarios.
- Simulated with JUDI.jl on 256 x 256 Compass model
- 202 training samples / 22 test samples



Step 2: Finding well-illuminated directions $\bar{\mathbf{V}}_r$ and Jacobian in illuminated directions, $\mathbf{J}\bar{\mathbf{V}}_r$



The first eigenvector $\mathbf{v}_1$ of $\bar{\mathbf{H}}_{\mathrm{GN}}$

Dominant *r* eigenvectors of the Gauss–Newton Hessian, $\bar{\mathbf{H}}_{\mathrm{GN}}$, averaged over the training distribution

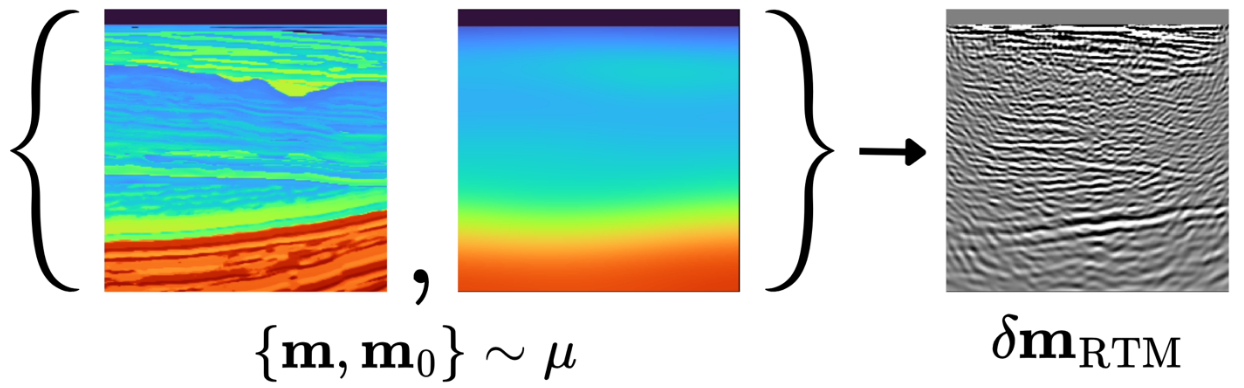
- give globally informative,
- well-illuminated directions

Step 3: Training Dataset

$$\mathcal{D} = \left\{ \left(\mathbf{m}^{(i)}, \mathbf{m}_0^{(i)}, \delta\mathbf{m}_{\mathrm{RTM}}^{(i)}, \bar{\mathbf{V}}_r, \left(\frac{\partial \mathcal{R}}{\partial \mathbf{m}} \Big|_{\mathbf{m}^{(i)}} \right) \bar{\mathbf{V}}_r \right) \right\}_{i=1}^{N}$$

Step 1: Jacobian-Informed Training (Offline)

$$\mathcal{R} : \mathbf{J}^\top(\mathcal{F}(\mathbf{m}) - \mathcal{F}(\mathbf{m}_0))$$

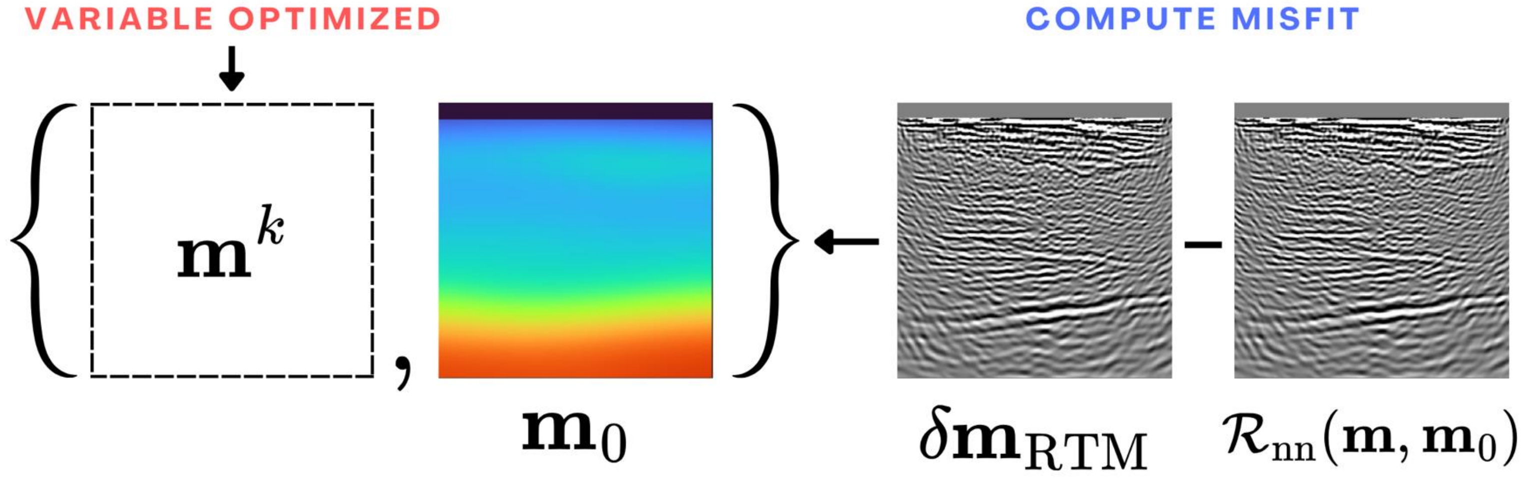


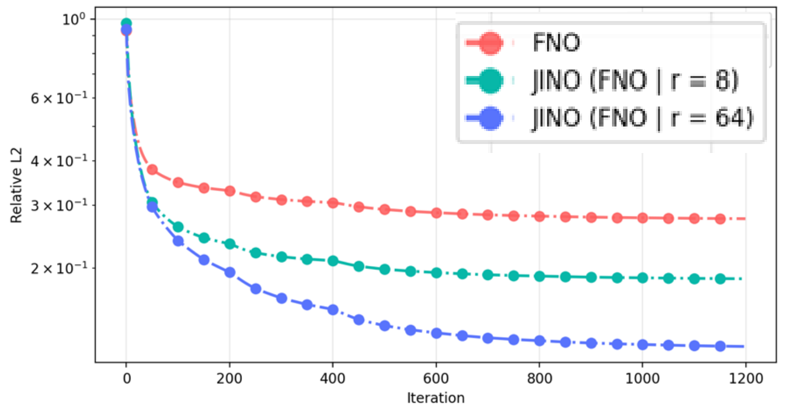
The NO, $\mathcal{R}_{\mathrm{nn}}$, is trained to match both the RTM image and the Jacobian action along *r* directions of $\bar{\mathbf{H}}_{\mathrm{GN}}$:

$$\mathcal{L}_{\mathrm{train}} = \|\mathcal{R}_{\mathrm{nn}}(\mathbf{m}, \mathbf{m}_0) - \delta\mathbf{m}_{\mathrm{RTM}}\|_2^2 + \lambda \left\| \frac{\partial \mathcal{R}_{\mathrm{nn}}}{\partial \mathbf{m}} \bar{\mathbf{V}}_r - \frac{\partial \mathcal{R}}{\partial \mathbf{m}} \bar{\mathbf{V}}_r \right\|_2^2$$

- **Model Architecture**: Fourier Neural Operator
- **Jacobian-informed Fourier Neural Operator (JI-FNO)** with *r* = 8 and *r* = 64 Jacobian directions (0.01% and 0.1% of the 256×256 model size)

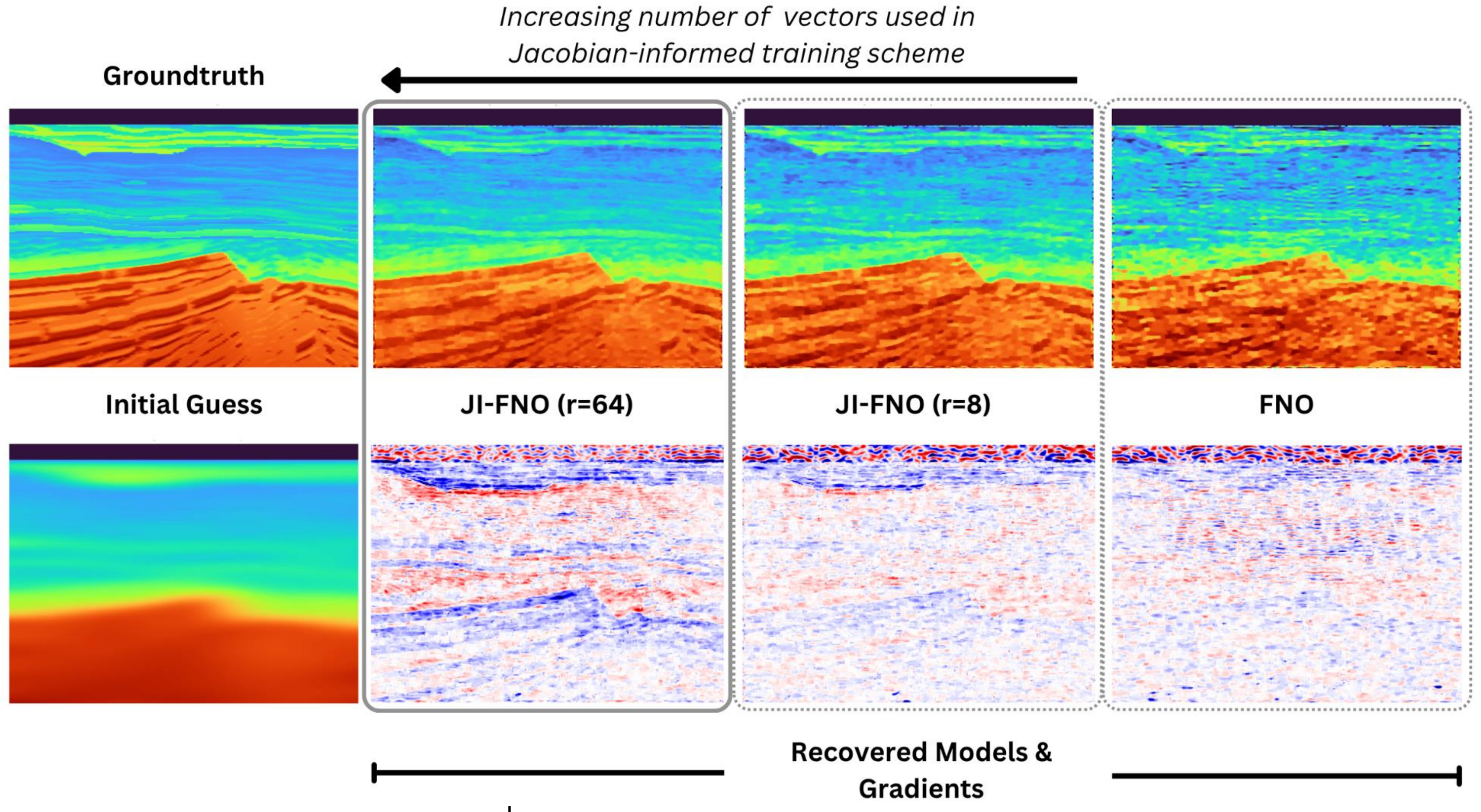
Step 2: Surrogate-Assisted Inversion (Online)





- **Inversion**: 1200 Adam iterations with a total-variation (TV) constraint ($\|\nabla \mathbf{m}\|_1 \leq \tau$)
- **Efficiency**: gradients via automatic differentiation → far cheaper per iteration than wave-equation-based inversion

Results



Metric (Rel. L2)	FNO	JI-FNO (r=8)	JI-FNO (r=64)
Model error (⬇)	0.29	**0.21**	**0.21** ✅
Data residual (⬇)	0.28	0.19	**0.12** ✅
Model SSIM (⬆)	0.50	**0.68**	**0.76** ✅

Key Observations & Conclusion

- **Gradient quality:** JI-FNO's gradients are structured and aligned with reflector geometry
  - reducing artifacts and improving deep-structure recovery,
  - unlike the noisy, high-frequency gradients from standard FNO.
- **Scaling with directions:** even r = 8 already gives substantial gains; r = 64 further sharpens continuity and depth accuracy.
- More directions → a more accurate surrogate Jacobian → larger expected gains in harder settings (poor background models, limited data).
- **Outlook:** more accurate Jacobians point toward data-driven nonlinear preconditioners for Gauss–Newton FWI, cutting GN iterations while preserving accuracy.

Ma, Xiao, and Tariq Alkhalifah. "Velocity model building from seismic images using a Convolutional Neural Operator." *EAGE Workshop on Enhancing Subsurface Practices using AI/ML*. Vol. 2025. No. 1. European Association of Geoscientists & Engineers, 2025.
Peters, B., and F. J. Herrmann, 2017, Constraints versus penalties for edge-preserving full-waveform inversion: The Leading Edge, 36, 94–100.
Park, J., N. T. Yang, and N. Chandramoorthy, 2024, When are dynamical systems learned from time series data statistically accurate?: Advances in Neural Information Processing Systems, 37, 43975–44008.