3D seismic survey design by maximizing the spectral gap

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Introduction

The massive cost of 3D acquisition calls for methods to reduce the number of receivers by designing optimal receiver sampling masks. Enriching the current studies in 2D [1] and time-lapse [2], we propose a simulation-free method to generate optimal receiver locations in 3D seismic by maximizing the spectral gap of the subsampling mask via a simulated annealing algorithm. Within several hours, this method can produce near optimal receiver locations that are favorable to wavefield recovery algorithms based on low-rank matrix completion. Numerical experiments based on a frequency slice of the 3D Compass dataset confirm improvement of the proposed method over jitter sampling.

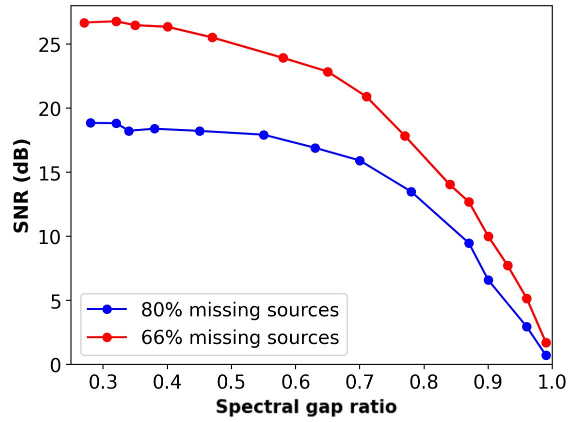
Methodology

3D wavefield reconstruction based on low-rank matrix completion relies on the non-canonical Source-X/Receiver-X (columns) Source-Y/Receiver-Y (rows) organization of the data into a matrix in order to leverage the inherent low-rank characteristics of seismic data [3]. Check the panel on the left for a visual illustration.

Spectral Gap Ratio (SGR)

The quality of wavefield reconstruction through low-rank matrix completion can be predicted by evaluating the ratio between the first two singular values of binary subsampling masks [4].

$$\mathrm{SGR}(\mathbf{M}) = \frac{\sigma_1(\mathbf{M})}{\sigma_2(\mathbf{M})}$$



- σ_1 first singular value
- σ_2 second singular value
- $\mathbf{M}$ binary subsampling mask
- measures connectivity of the graph spanned by the binary matrix
- lower SGR → better graph connectivity → improved wavefield recovery

Goal: ***find receiver positions suitable for wavefield reconstruction via low-rank matrix completion in non-canonical domain by minimizing the SGR of the binary mask***

Spectral gap ratio minimization

$$\underset{\mathbf{M}}{\text{minimize}} \quad \frac{\sigma_2(\mathbf{M})}{\sigma_1(\mathbf{M})} \quad \text{subject to} \quad \mathbf{M} \in \mathscr{C} = \bigcap_{i=1}^{4} \mathscr{C}_i$$

Here, $\mathscr{C}$ represents constraints, including

- cardinality $\mathscr{C}_1 = \{\mathbf{M} \mid \#(\mathbf{M}) = \lfloor N_{rx} \times N_{ry} \times \rho \rfloor \times N_{sx} \times N_{sy}\}$
- binary $\mathscr{C}_2 = \{\mathbf{M} \mid \mathbf{M} \in \{0,1\}^{(N_{sx}\times N_{rx})\times(N_{sy}\times N_{ry})}\}$
- $\mathscr{C}_3 = \{\mathbf{M} \mid \#(\mathbf{M}_i) \geq m \quad \text{for} \quad i = 1,\cdots,N_{sx}\times N_{rx}\}$
- $\mathscr{C}_4 = \{\mathbf{M} \mid \#(\mathbf{M}^j) \geq n \quad \text{for} \quad j = 1,\cdots,N_{sy}\times N_{ry}\}$

where

- ρ is receiver subsampling ratio
- $N_{sx}, N_{sy}, N_{rx}, N_{ry}$ are the number of sources/receivers in x/y direction
- $\mathbf{M}_i, \mathbf{M}^j$ denote the i-th row or j-th column
- $\mathscr{C}_3$ and $\mathscr{C}_4$ enforce lower bounds for the number of subsampling points for each row and column of the binary subsampling matrix

Simulated annealing algorithm

A stochastic local search algorithm to solve the optimization problem above. Please check the panel on the right.

An intriguing property of spectral gap ratio in 3D seismic

A property to reduce the computational cost of SGR calculation, which further accelerates the turnaround time for solving the optimization problem above. Please check the panel on the left.

Numerical Experiments - 3D Compass dataset

Data dimension: $100 \times 100 \times 41 \times 41 \times 501$ $(N_{rx} \times N_{ry} \times N_{sx} \times N_{sy} \times N_t)$
Frequency slice: 16.8 Hz
Source sampling interval: 150 m
Receiver sampling interval: 25 m
Time sampling interval: 10 ms

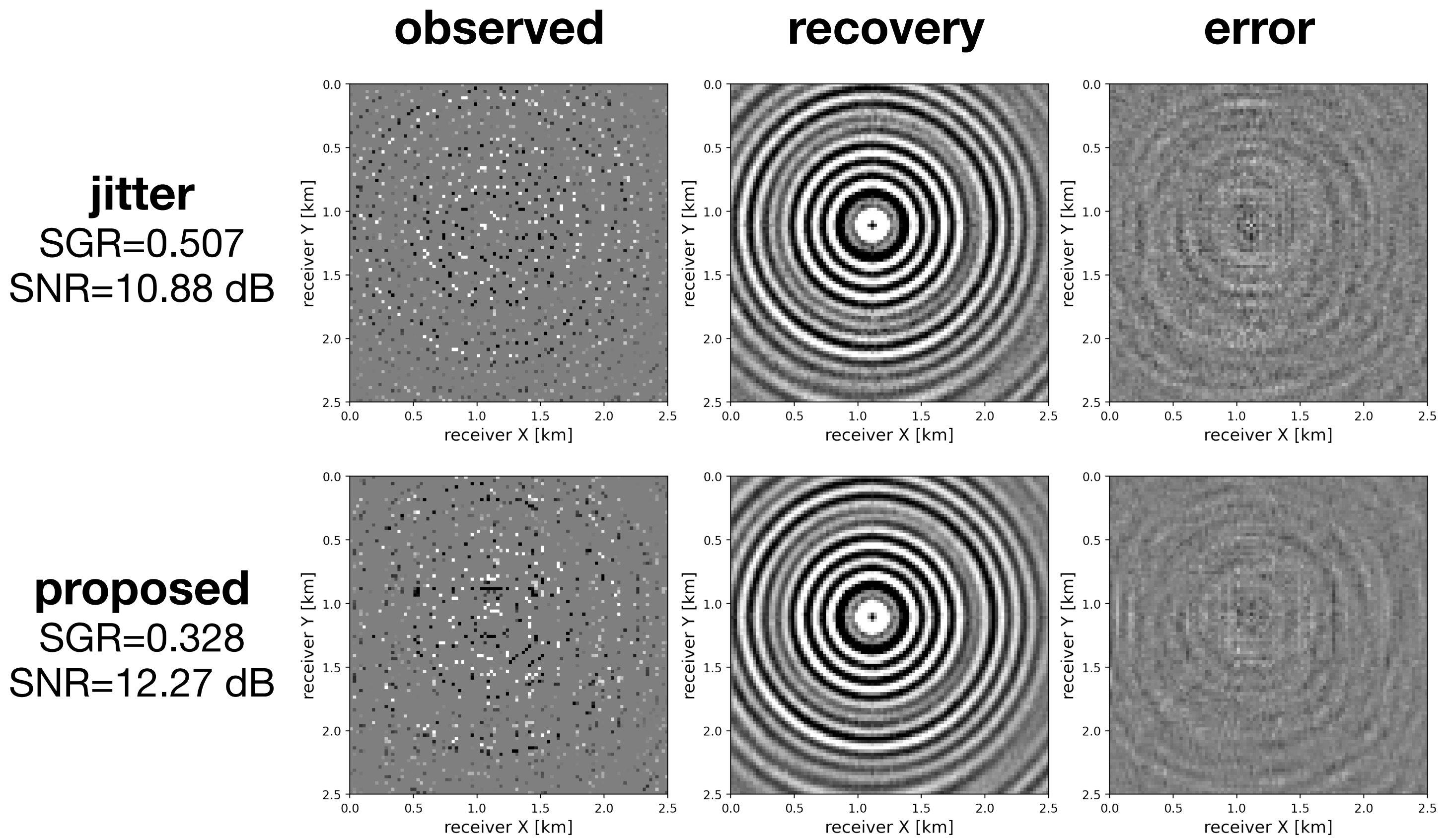


Figure: Comparison of data reconstruction performance for receiver locations sampled by the jittered method and the proposed method. 90% of receivers

Conclusions

Proposed method for seismic survey design is
- *simulation-free*
- best suitable for wavefield reconstruction via low-rank matrix completion
- adaptable to 2D [1] & time-lapse [2] & *3D (this poster)*
- based on the arguments from graph theory

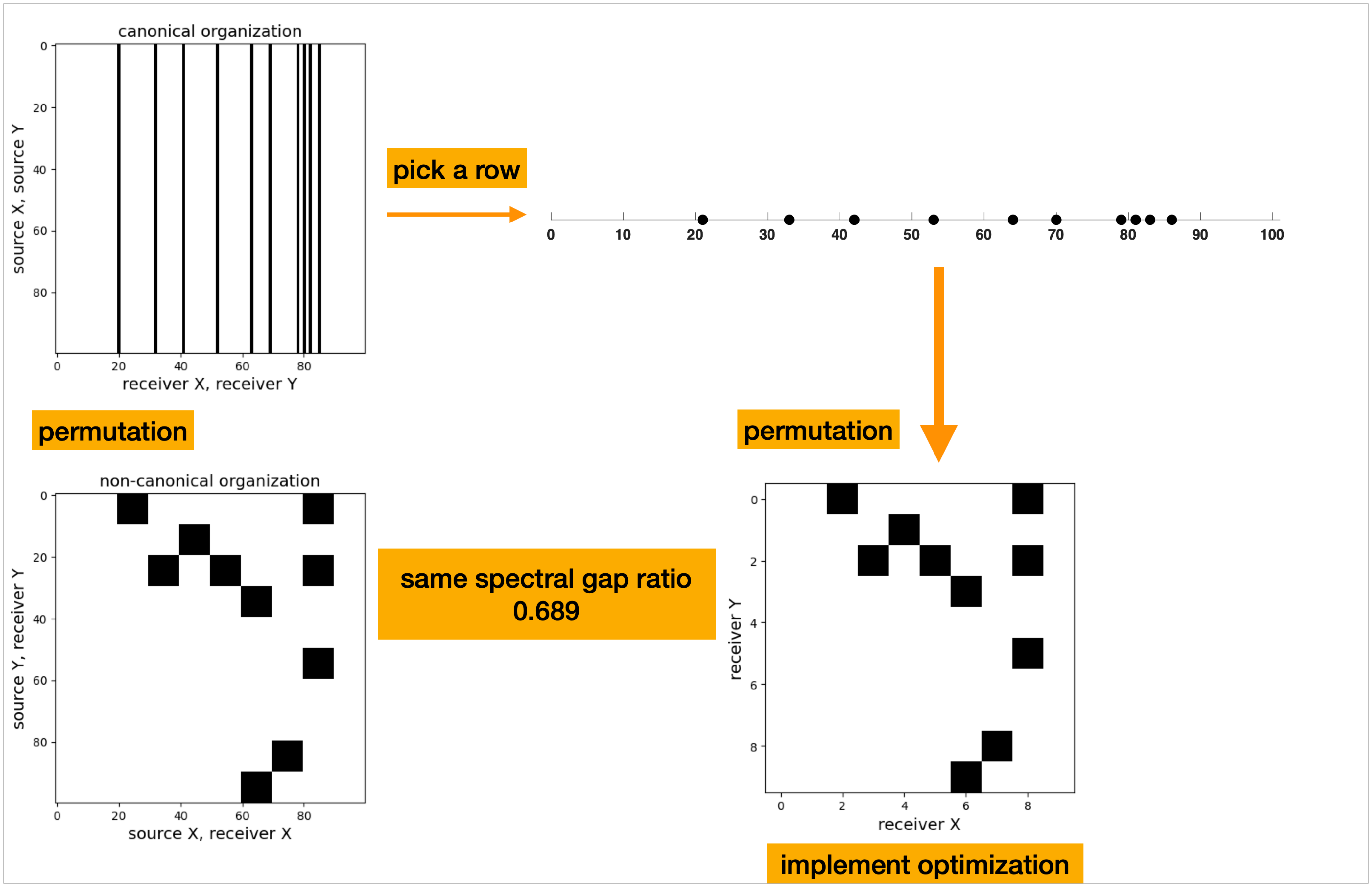
Future work
- off-the-grid
- entire field dataset
- tensor representation of seismic data

References

[1] Zhang, Yijun, Mathias Louboutin, Ali Siahkoohi, Ziyi Yin, Rajiv Kumar, and Felix J. Herrmann. "A simulation-free seismic survey design by maximizing the spectral gap." In *Second International Meeting for Applied Geoscience & Energy*, pp. 15-20, 2022.
[2] Zhang, Yijun, Ziyi Yin, Oscar López, Ali Siahkoohi, Mathias Louboutin, Rajiv Kumar, and Felix J. Herrmann."Optimized time-lapse acquisition design via spectral gap ratio minimization." *Geophysics* 88, no. 4 (2023): A19-A23.
[3] Kumar, Rajiv, Curt Da Silva, Okan Akalin, Aleksandr Y. Aravkin, Hassan Mansour, Benjamin Recht, and Felix J. Herrmann. "Efficient matrix completion for seismic data reconstruction." *Geophysics* 80, no. 5 (2015): V97-V114.
[4] López, Oscar, Rajiv Kumar, Nick Moldoveanu, and Felix J. Herrmann. "Spectral gap-based seismic survey design." *IEEE Transactions on Geoscience and Remote Sensing* 61 (2023): 1-9.

An intriguing property of spectral gap ratio in 3D seismic

When sources are fully sampled, each single-receiver block of the global sampling matrix is either fully sampled or empty depending on whether that specific receiver is sampled. Consequently, the block structure of the global matrix leads to the exact same singular values as a single-source receiver sampling mask. We can therefore optimize a single-source mask to obtain the global optimized mask. The main computational cost therein is computing the first two singular values of the receiver sampling mask, which is negligible compared to approaches that require wave simulations.

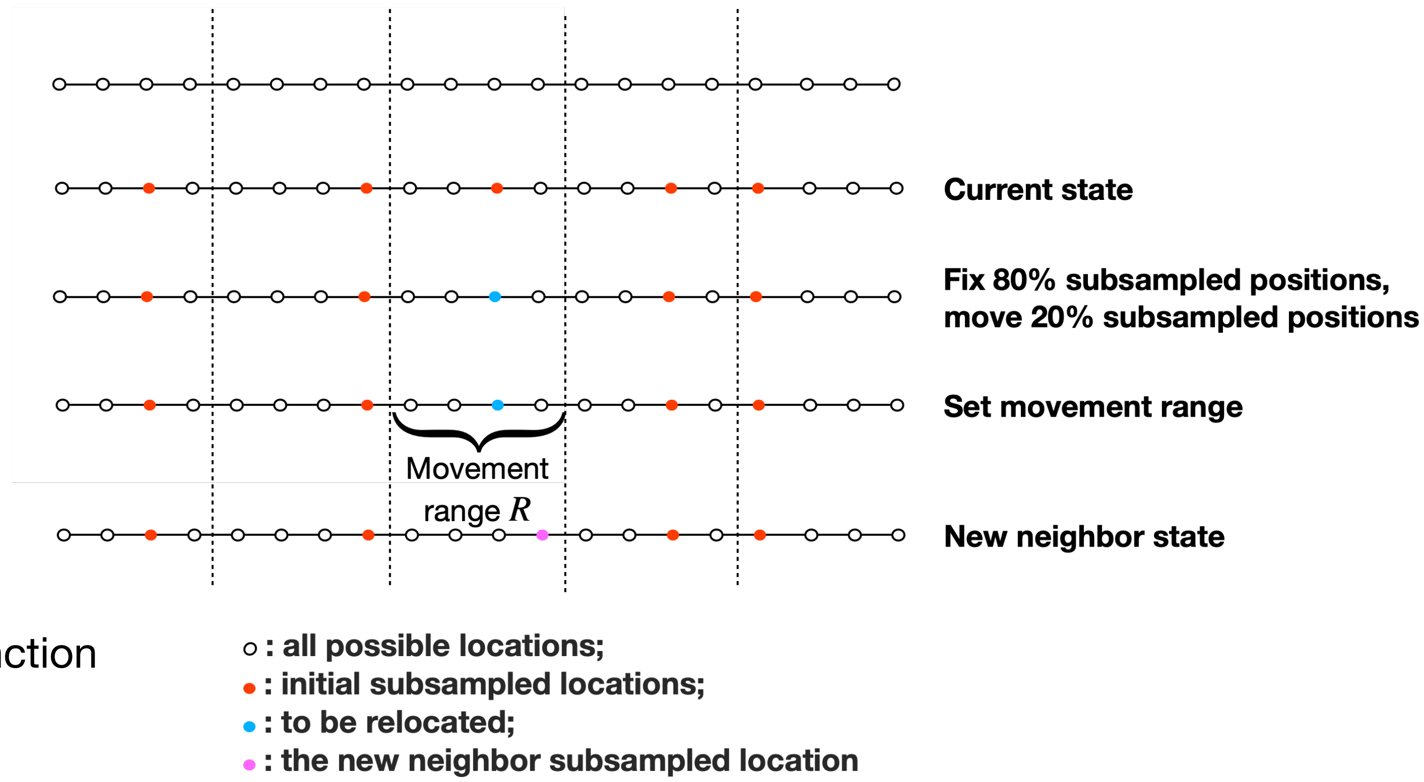


A black square is a matrix of all 1s
A white square is a matrix of all 0s

A black square is 1
A white square is 0

Simulated annealing algorithm for spectral gap ratio minimization

The design of acquisition masks that minimize the SGR is an NP-hard problem whose solution requires a brute-force search through all combinatorial possibilities. For this reason, we propose to obtain a near optimal solution using simulated annealing, a stochastic local search optimization technique that is straightforward to implement, apply, and computationally feasible.



Simulated annealing algorithm

Inputs:

$\mathbf{M}_0$: initial receiver sampling mask

maxiter: maximum number of iterations

$p(\delta L, k) = \exp\left(\frac{-\delta L}{T_0 \times \alpha^k}\right)$ transition probability function

for $k = 0, \cdots, \mathrm{maxiter}$

$\widetilde{\mathbf{M}_k} \leftarrow$ randomly select a neighboring state

$\delta L = \mathrm{SGR}(\widetilde{\mathbf{M}_k}) - \mathrm{SGR}(\mathbf{M}_k)$ # compute the change of SGR

if $\delta L \leq 0$

$\mathbf{M}_{k+1} = \widetilde{\mathbf{M}_k}$ # always take improving move

else # might take a worsening move

$$\mathbf{M}_{k+1} = \begin{cases} \widetilde{\mathbf{M}_k} & \text{with probability } p(\delta L, k) \\ \mathbf{M}_k & \end{cases}$$

endif

endfor

Output: $\mathbf{M}_{\mathrm{maxiter}}$