Velocity continuation with Fourier neural operators for accelerated uncertainty quantification

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challenges of seismic imaging

- noisy data and linearization errors
- bandwidth and aperture limitations
- presence of shadow zones

uncertainty quantification compliments imaging

- assessing variability amongst feasible seismic images
- translation of imaging uncertainty to downstream tasks, e.g., horizon tracking
- challenges: high-dimensionality of seismic images and forward operator costs


Two main sources of uncertainty

errors in the data, e.g., measurement noise

forward modeling (background velocity model) errors
Focus of this talk

quantifying uncertainty w.r.t errors in the background model

main contributing factor to imaging uncertainty due to its effect on reflector positioning

requires solving numerous imaging problems, e.g., via reverse-time migration

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Parihaka dataset

real prestack Kirchhoff time migrated dataset

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Proposed method

a novel velocity continuation method

mapping images associated with one background model to another virtually for free

based on the recent advances in deep learning for operator learning
Linearized wave-equation based imaging

\[ d_i = \mathcal{F}(m, q_i) + \epsilon_i \]

\[
\approx \mathcal{F}(m_0, q_i) + \nabla_m \mathcal{F}(m, q_i) \bigg|_{m=m_0} \delta m + \mathcal{O}(\delta m^\top \delta m) + \epsilon_i
\]

- observed seismic data \( d = \{d_i\}_{i=1}^{n_s} \)
- discretized acoustic wave equation \( \mathcal{F}(m, q_i) \)
- squared-slowness model of the subsurface \( m \)
- source signature \( q_i \)
- measurement noise \( \epsilon_i \)
- smooth squared-slowness model \( m_0 \)
Velocity continuation

\[ T(m_{\text{init}}, m_{\text{target}}) : \delta \mathcal{M} \rightarrow \delta \mathcal{M} \]

space of seismic images \( \delta \mathcal{M} \)

initial and target background models \( m_{\text{init}}, m_{\text{target}} \)

maps images associated with one background model to another without explicitly imaging


Mengmeng Yang, Marie Graff, Rajiv Kumar, and Felix J Herrmann. “Low-rank representation of omnidirectional subsurface extended image volumes”. In: Geophysics 86.3 (2021), S165–S183.
Fourier neural operator surrogate

\[ G_w^* (m_{\text{target}}, \delta m_{\text{init}}) \approx \mathcal{T}(m_{\text{init}}, m_{\text{target}})(\delta m_{\text{init}}) = \delta m_{\text{target}}, \]

Fourier neural operator \( G_w \)

initial and target seismic images \( \delta m_{\text{init}}, \delta m_{\text{target}} \)

Training

$$w^* = \arg \min_w \frac{1}{n_{\text{train}}} \sum_{i=1}^{n_{\text{train}}} \| G_w(m_0^{(i)}, \delta m_{\text{init}}) - \delta m_{\text{RTM}}^{(i)} \|_2^2$$

training tuples \[ \left\{ (\mathbf{m}_0^{(i)}, \delta m_{\text{init}}^{(i)}, \delta m_{\text{RTM}}^{(i)}) \mid i = 1, \ldots, n_{\text{train}} \right\} \]

$$\delta m_{\text{RTM}} = \sum_{i=1}^{n_s} J(m_0, \mathbf{q}_i) \top \mathbf{d}_i.$$ 

trained with only 200 background and seismic image pairs

Vertical profile at 2.50 km

- Prediction
- RTM

Vertical profile at 4.12 km

- Prediction
- RTM
Uncertainty in the tracked horizons

Conclusions

Uncertainty quantification is rendered impractical when
the forward operators are expensive to evaluate
the problem is high dimensional

Fourier neural operator surrogate for velocity continuation

can lead to computational improvements

can be pretrained on existing data followed by a cheaper finetuning stage to the
pertaining survey

can act as “summary functions” for scalable amortized variational inference, i.e., full
posterior learning
Acknowledgment

This research was carried out with the support of Georgia Research Alliance and partners of the ML4Seismic Center
Contributions

a survey-specific Fourier neural operator surrogate to velocity continuation
mapping images associated with one background model to another virtually for free
trained with only 200 background and seismic image pairs
enables accelerated uncertainty quantification w.r.t background model

Code:  https://github.com/slimgroup/fno4vc