Velocity continuation with Fourier neural operators for accelerated uncertainty quantification

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challenges of seismic imaging

- noisy data and linearization errors
- bandwidth and aperture limitations
- presence of shadow zones

uncertainty quantification compliments imaging

- assessing variability amongst feasible seismic images
- translation of imaging uncertainty to downstream tasks, e.g, horizon tracking
- challenges: high-dimensionality of seismic images and forward operator costs

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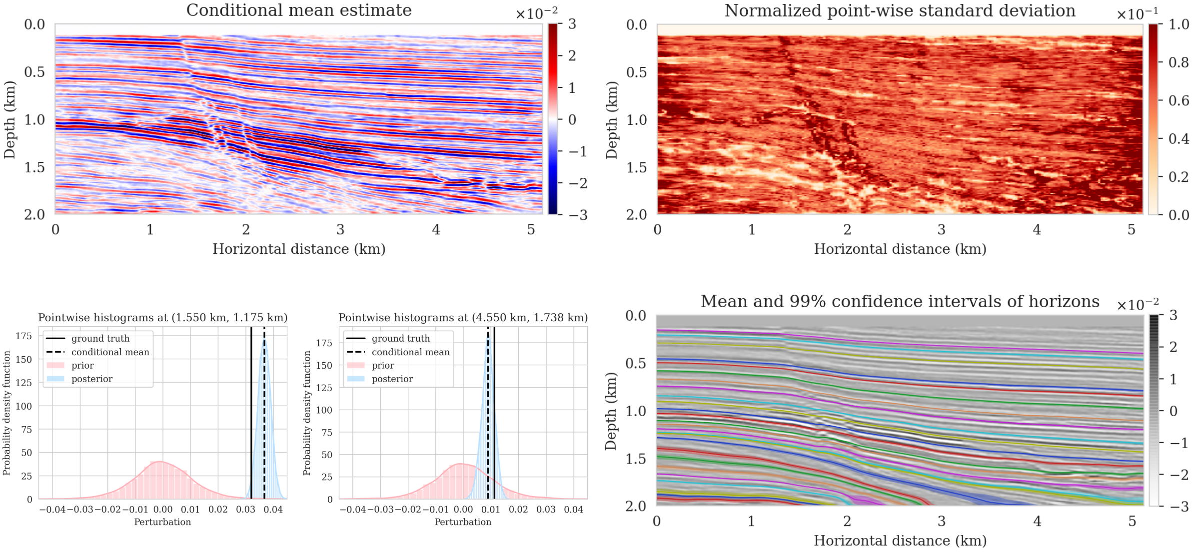
Gilles Lambaré, Jean Virieux, Raul Madariaga, and Side Jin. "Iterative asymptotic inversion in the acoustic approximation". In: *Geophysics* 57.9 (1992), pp. 1138–1154.

Gerard T Schuster. "Least-squares cross-well migration". In: *63rd Annual International Meeting, SEG*. Expanded Abstracts, 1993, pp. 110–113. DOI: 10.1190/1.1822308.



Ali Siahkoohi, Gabrio Rizzuti, and Felix J. Herrmann. "Deep Bayesian inference for seismic imaging with tasks". In: *Geophysics* 87.5 (June 2022). DOI: 10.1190/geo2021-0666.1. URL: https://arxiv.org/abs/2110.04825.

Two main source of uncertainty

errors in the data, e.g., measurement noise

forward modeling (background velocity model) errors

Pierre Thore, Arben Shtuka, Magali Lecour, Taoufik Ait-Ettajer, and Richard Cognot. "Structural uncertainties: Determination, management, and applications". In: *Geophysics* 67.3 (2002), pp. 840–852.

Konstantin Osypov et al. "Model-uncertainty quantification in seismic tomography: method and applications". In: *Geophysical Prospecting* 61.6-Challenges of Seismic Imaging and Inversion Devoted to Goldin (2013), pp. 1114–1134.

Gregory Ely, Alison Malcolm, and Oleg V. Poliannikov. "Assessing uncertainties in velocity models and images with a fast nonlinear uncertainty quantification method". In: *GEOPHYSICS* 83.2 (2018), R63–R75. DOI: 10.1190/geo2017-0321.1. eprint: https://doi.org/10.1190/geo2017-0321.1. URL: https://doi.org/10.1190/geo2017-0321.1.

Focus of this talk

quantifying uncertainty w.r.t errors in the background model

main contributing factor to imaging uncertainty due to its effect on reflector positioning

requires solving numerous imaging problems, e.g., via reverser-time migration

Sergey Fomel and Evgeny Landa. "Structural uncertainty of time-migrated seismic images". In: *Journal of Applied Geophysics* 101 (2014), pp. 27–30.

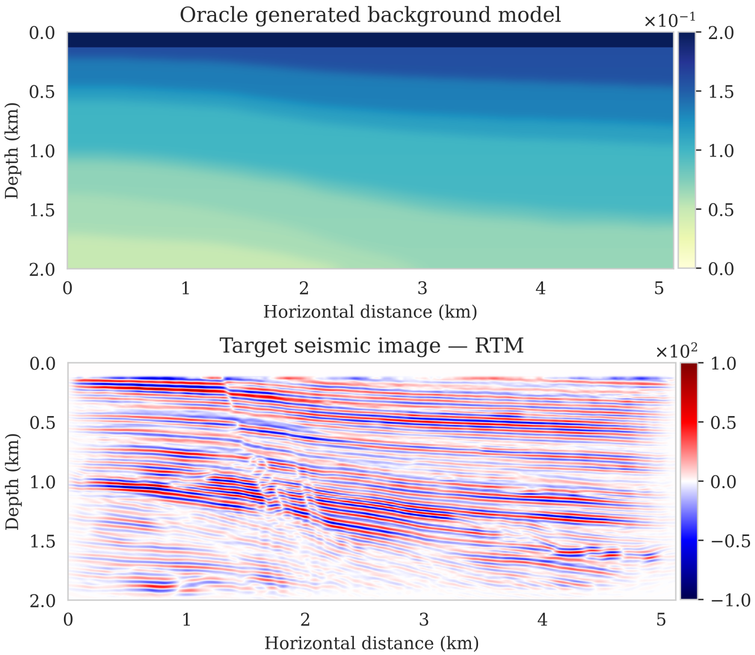
Oleg V Poliannikov and Alison E Malcolm. "The effect of velocity uncertainty on migrated reflectors: Improvements from relative-depth imaging". In: *Geophysics* 81.1 (2016), S21–S29.

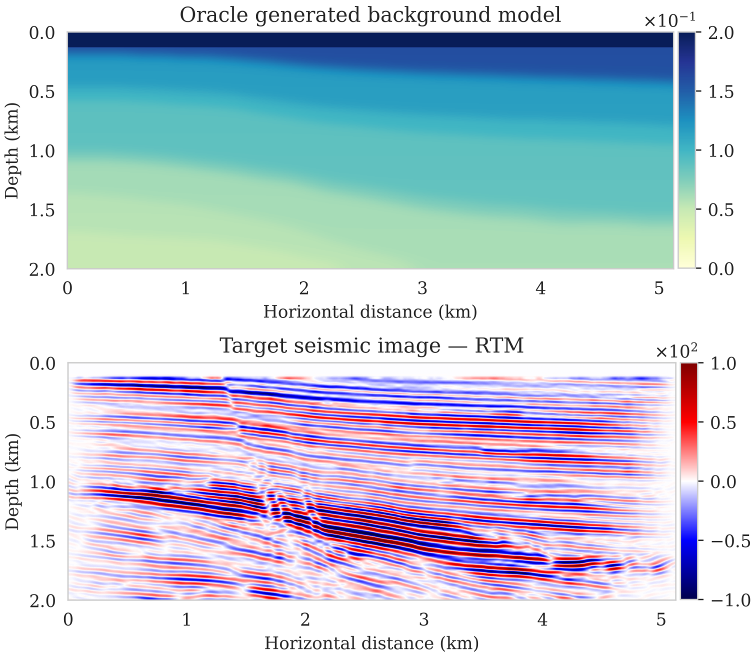
Parihaka dataset

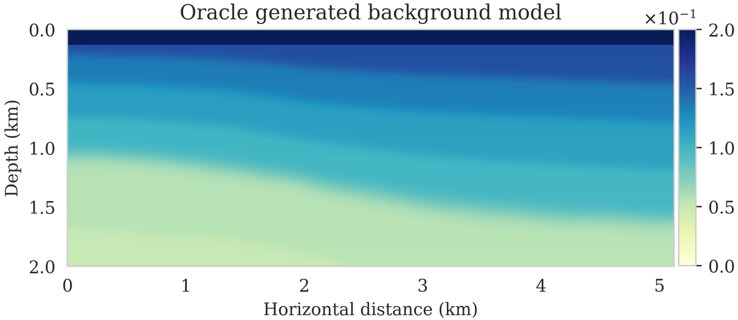
real prestack Kirchhoff time migrated dataset

Veritas. *Parihaka 3D Marine Seismic Survey - Acquisition and Processing Report*. Tech. rep. New Zealand Petroleum Report 3460. New Zealand Petroleum & Minerals, Wellington, 2005.

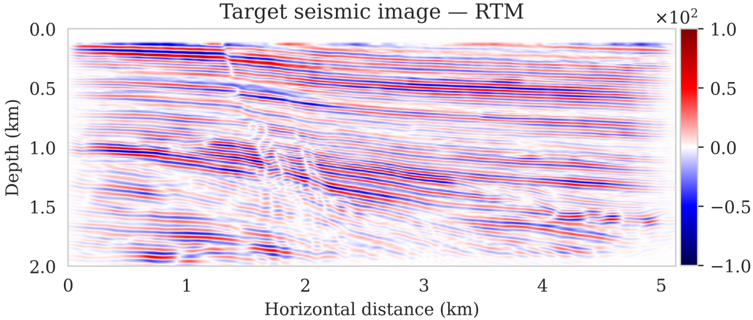
WesternGeco. *Parihaka 3D PSTM Final Processing Report*. Tech. rep. New Zealand Petroleum Report 4582. New Zealand Petroleum & Minerals, Wellington, 2012.


Oracle generated background model
×10⁻¹
Depth (km)
Horizontal distance (km)
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4
5
Target seismic image — RTM
×10²
Depth (km)
Horizontal distance (km)
1.0
0.5
0.0
−0.5
−1.0

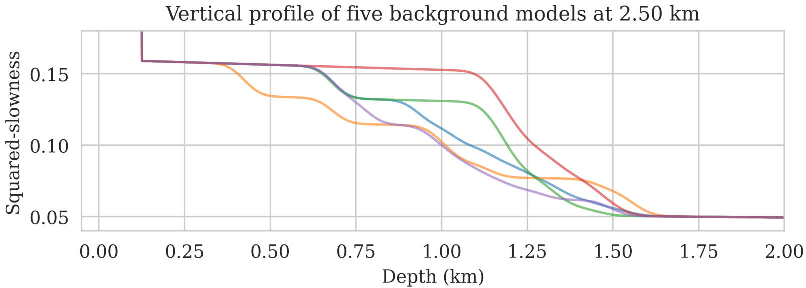

Oracle generated background model
×10⁻¹
Depth (km)
Horizontal distance (km)
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4
5
Target seismic image — RTM
×10²
−0.5
−1.0



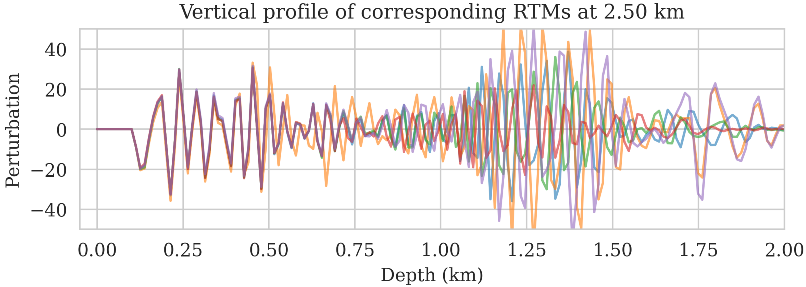
Oracle generated background model
×10⁻¹
Depth (km)
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4
5
Horizontal distance (km)
2.0
1.5
1.0
0.5
0.0



Target seismic image — RTM
×10²
Depth (km)
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4
5
Horizontal distance (km)
1.0
0.5
0.0
−0.5
−1.0



Vertical profile of five background models at 2.50 km
Squared-slowness
0.15
0.10
0.05
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
Depth (km)



Vertical profile of corresponding RTMs at 2.50 km
Perturbation
40
20
0
−20
−40
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
Depth (km)

Proposed method

a novel velocity continuation method

mapping images associated with one background model to another virtually for free

based on the recent advances in deep learning for operator learning

Linearized wave-equation based imaging

$$\mathbf{d}_i = \mathcal{F}(\mathbf{m}, \mathbf{q}_i) + \boldsymbol{\epsilon}_i$$

$$\approx \mathcal{F}\left(\mathbf{m}_0, \mathbf{q}_i\right) + \underbrace{\nabla_{\mathbf{m}} \mathcal{F}(\mathbf{m}, \mathbf{q}_i)\Big|_{\mathbf{m}=\mathbf{m}_0}}_{\mathbf{J}(\mathbf{m}_0, \mathbf{q}_i):\ \text{Born modeling operator}} \delta\mathbf{m} + \underbrace{\mathcal{O}\left(\delta\mathbf{m}^\top \delta\mathbf{m}\right)}_{\text{linearization error}} + \boldsymbol{\epsilon}_i$$

observed seismic data $\mathbf{d} = \{\mathbf{d}_i\}_{i=1}^{n_s}$

discretized acoustic wave equation $\mathcal{F}(\mathbf{m}, \mathbf{q}_i)$

squared-slowness model of the subsurface $\mathbf{m}$

source signature $\mathbf{q}_i$

measurement noise $\boldsymbol{\epsilon}_i$

smooth squared-slowness model $\mathbf{m}_0$

Velocity continuation

$$\mathcal{T}_{(\mathbf{m}_{\text{init}}, \mathbf{m}_{\text{target}})} : \delta\mathcal{M} \rightarrow \delta\mathcal{M}$$

space of seismic images $\delta\mathcal{M}$

initial and target background models $\mathbf{m}_{\text{init}}, \mathbf{m}_{\text{target}}$

maps images associated with one background model to another without explicitly imaging

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Sergey Fomel. "Time-migration velocity analysis by velocity continuation". In: *Geophysics* 68.5 (2003), pp. 1662–1672.

Mengmeng Yang, Marie Graff, Rajiv Kumar, and Felix J Herrmann. "Low-rank representation of omnidirectional subsurface extended image volumes". In: *Geophysics* 86.3 (2021), S165–S183.

Fourier neural operator surrogate

$$\mathcal{G}_{\mathbf{w}^*}(\mathbf{m}_{\mathsf{target}}, \delta\mathbf{m}_{\mathsf{init}}) \approx \mathcal{T}_{(\mathbf{m}_{\mathsf{init}}, \mathbf{m}_{\mathsf{target}})}(\delta\mathbf{m}_{\mathsf{init}}) = \delta\mathbf{m}_{\mathsf{target}},$$

Fourier neural operator $\mathcal{G}_{\mathbf{w}}$

initial and target seismic images $\delta\mathbf{m}_{\mathsf{init}}, \delta\mathbf{m}_{\mathsf{target}}$

Zongyi Li et al. "Fourier Neural Operator for Parametric Partial Differential Equations". In: *9th International Conference on Learning Representations*. OpenReview.net, 2021. URL: https://openreview.net/forum?id=c8P9NQVtmnO.

Training

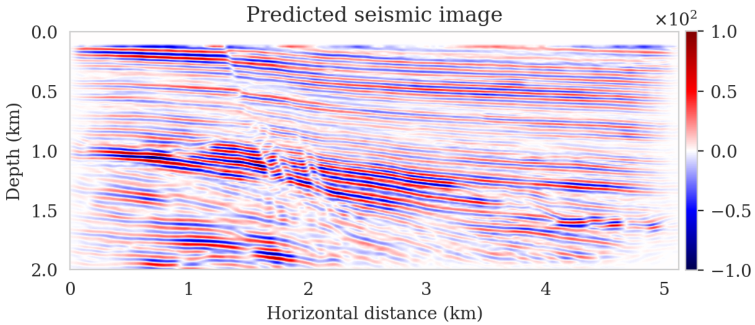
$$\mathbf{w}^* = \arg\min_{\mathbf{w}} \frac{1}{n_{\mathsf{train}}} \sum_{i=1} \|\mathcal{G}_{\mathbf{w}}(\mathbf{m}_0^{(i)}, \delta\mathbf{m}_{\mathsf{init}}) - \delta\mathbf{m}_{\mathsf{RTM}}^{(i)}\|_2^2$$

training tuples $\left\{\left((\mathbf{m}_0^{(i)}, \delta\mathbf{m}_{\mathsf{init}}), \delta\mathbf{m}_{\mathsf{RTM}}^{(i)}\right) \,\middle|\, i = 1, \ldots, n_{\mathsf{train}}\right\}$

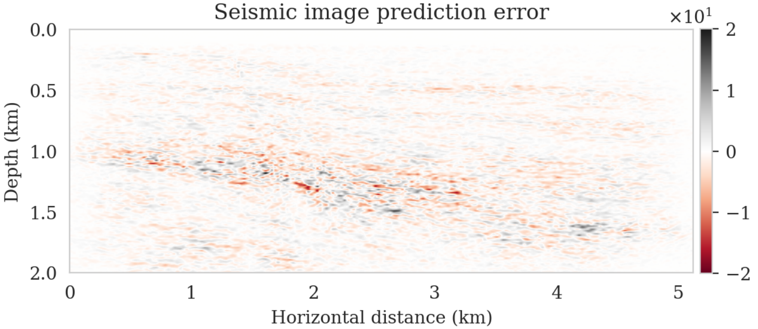
$\delta\mathbf{m}_{\mathsf{RTM}} = \sum_{i=1}^{n_s} \mathbf{J}(\mathbf{m}_0, \mathbf{q}_i)^\top \mathbf{d}_i.$

trained with only 200 background and seismic image pairs

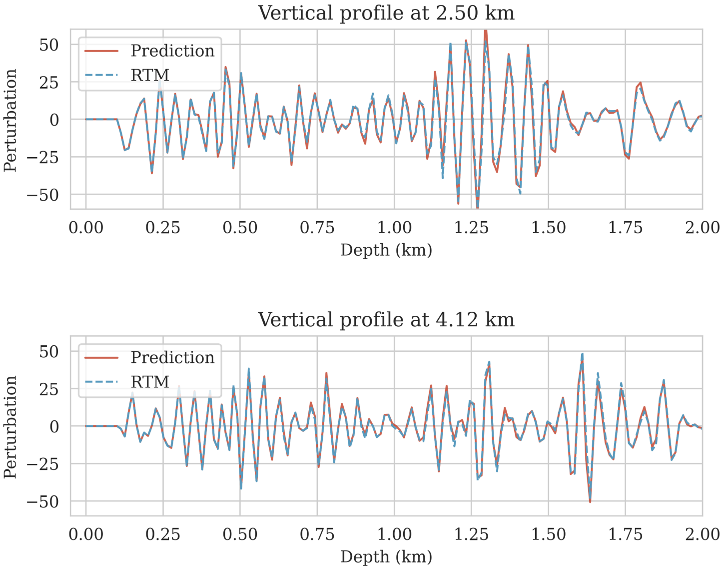
Maarten De Hoop, Daniel Zhengyu Huang, Elizabeth Qian, and Andrew M Stuart. "The Cost-Accuracy Trade-Off In Operator Learning With Neural Networks". In: *arXiv preprint arXiv:2203.13181* (2022).



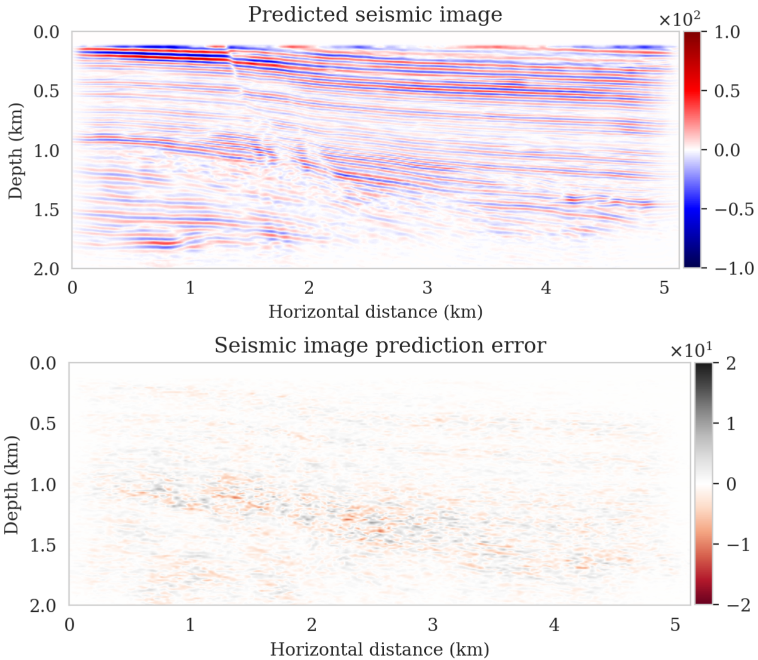
Predicted seismic image
×10²
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
Depth (km)
0
1
2
3
4
5
Horizontal distance (km)

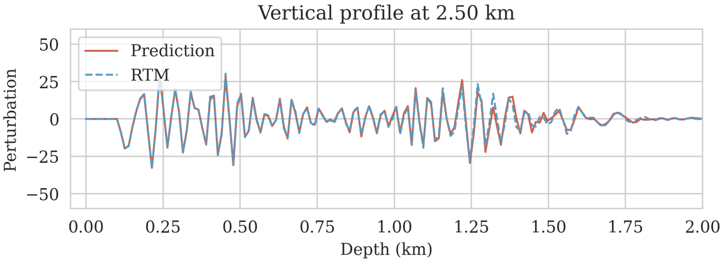


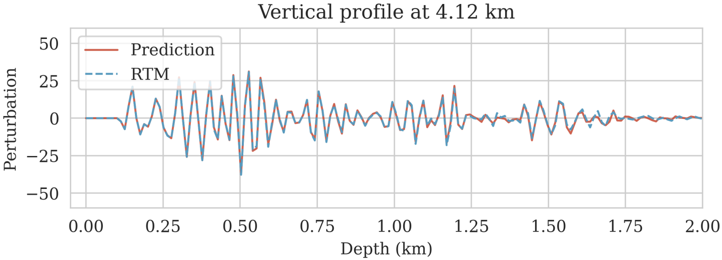
Seismic image prediction error
×10¹
2
1
0
−1
−2
0.0
0.5
1.0
1.5
2.0
Depth (km)
0
1
2
3
4
5
Horizontal distance (km)



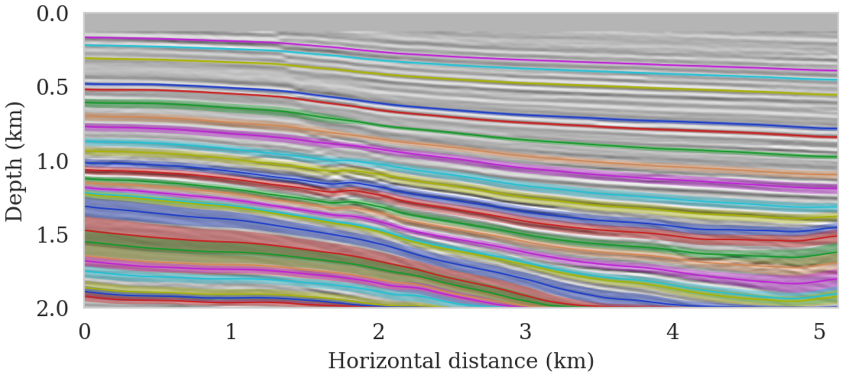
Vertical profile at 2.50 km
Prediction
RTM
Perturbation
50
25
0
−25
−50
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Depth (km)
Vertical profile at 4.12 km
Prediction
RTM
Perturbation
50
25
0
−25
−50
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Depth (km)


Predicted seismic image
Depth (km)
Horizontal distance (km)
×10²
Seismic image prediction error
Depth (km)
Horizontal distance (km)
×10¹
SLIM


Vertical profile at 2.50 km
Prediction
RTM
Perturbation
50
25
0
−25
−50
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Depth (km)


Vertical profile at 4.12 km
Prediction
RTM
Perturbation
50
25
0
−25
−50
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Depth (km)

SLIM



Uncertainty in the tracked horizons

Xinming Wu and Sergey Fomel. "Least-squares horizons with local slopes and multi-grid correlations". In: *GEOPHYSICS* 83 (4 2018), pp. IM29–IM40. DOI: 10.1190/geo2017-0830.1. URL: https://doi.org/10.1190/geo2017-0830.1.

Conclusions

Uncertainty quantification is rendered impractical when

- the forward operators are expensive to evaluate
- the problem is high dimensional

Fourier neural operator surrogate for velocity continuation

- can lead to computational improvements
- can be pretrained on existing data followed by a cheaper finetuning stage to the pertaining survey
- can act as "summary functions" for scalable amortized variational inference, i.e., full posterior learning

Acknowledgment

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Contributions

a survey-specific Fourier neural operator surrogate to velocity continuation

mapping images associated with one background model to another virtually for free

trained with only 200 background and seismic image pairs

enables accelerated uncertainty quantification w.r.t background model

Code: `https://github.com/slimgroup/fno4vc`