Weak deep priors for seismic imaging

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Introduction

Agenda

regularization w/ deep priors

weak deep priors—a computationally feasible alternative

examples—seismic imaging

Seismic imaging

estimate reflectivity $\delta\mathbf{m}$ given observed data $\{\mathbf{d}_i\}_{i=1}^{n_s}$

$$\min_{\delta\mathbf{m}} \quad \frac{1}{2\sigma^2}\sum_{i=1}^{n_s}\|\delta\mathbf{d}_i - \mathbf{J}(\mathbf{m}_0,\mathbf{q}_i)\delta\mathbf{m}\|_2^2$$

linearized Born operator, $\mathbf{J}(\mathbf{m}_0,\mathbf{q}_i)$

linearized data, $\delta\mathbf{d}_i$

noise variance, σ^2

Challenges

expensive forward operator

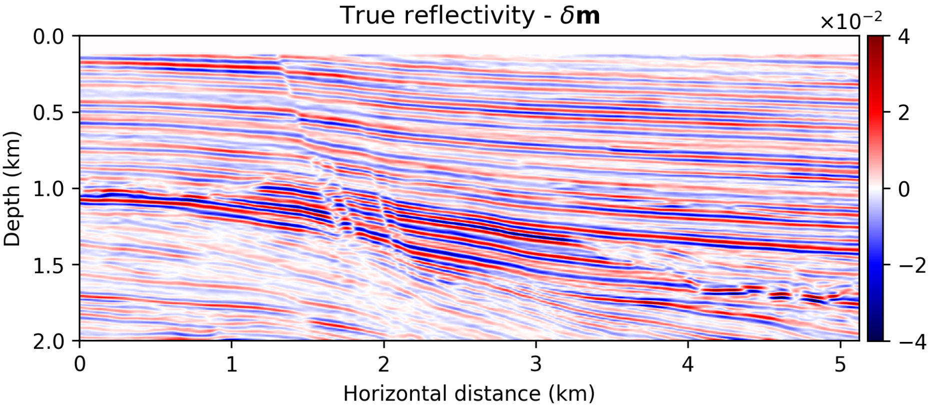
inconsistent, mildly ill-conditioned

2D slice from Parihaka dataset

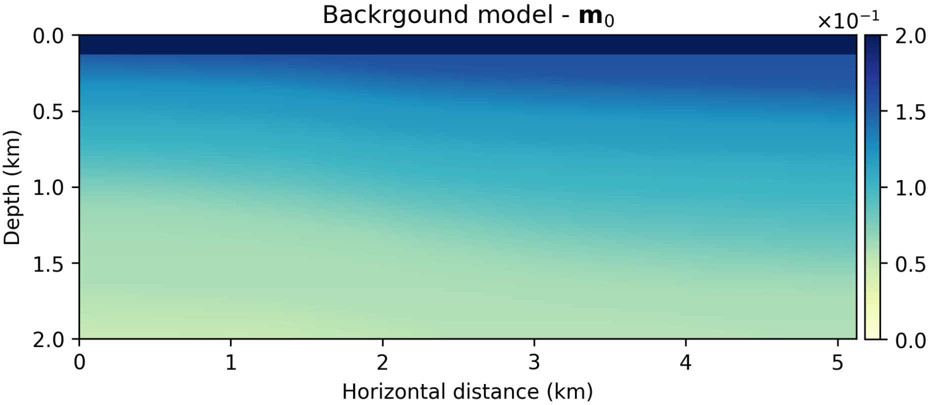
finite-difference simulations w/ Devito

WesternGeco. *Parihaka 3D PSTM Final Processing Report*. Tech. rep. New Zealand Petroleum Report 4582. New Zealand Petroleum & Minerals, Wellington, 2012. URL: `https://wiki.seg.org/wiki/Parihaka-3D`.

M. Louboutin et al. “Devito (v3.1.0): an embedded domain-specific language for finite differences and geophysical exploration”. In: *Geoscientific Model Development* 12.3 (2019), pp. 1165–1187. DOI: `10.5194/gmd-12-1165-2019`. URL: `https://www.geosci-model-dev.net/12/1165/2019/`.



$\delta\mathbf{m}$—"true" reflectivity model obtain from Parihaka dataset



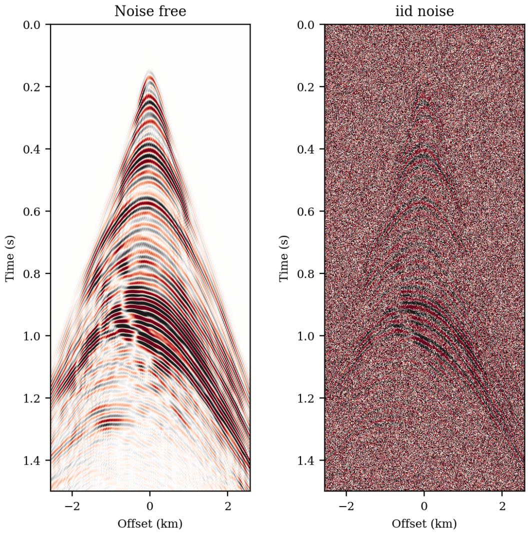
$\mathbf{m}_0$—made up background squared-slowness $\left(\frac{s^2}{km^2}\right)$

205 sources w/ 25 m sampling rate

410 receivers w/ 12.5 m sampling rate

1.5 s recording time

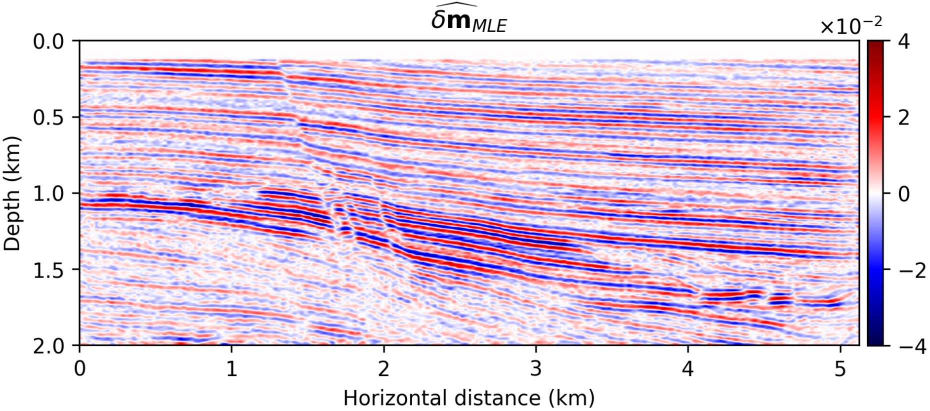
Ricker source wavelet w/ 30 Hz central frequency



Noise-free (left) and noisy (right) linearized data — SNR: -8.74 dB

Maximum likelihood estimate

$$\min_{\delta \mathbf{m}} \quad \frac{1}{2\sigma^2} \sum_{i=1}^{n_s} \|\delta \mathbf{d}_i - \mathbf{J}(\mathbf{m}_0, \mathbf{q}_i)\delta \mathbf{m}\|_2^2$$



MLE — no regularization (prior)

Deep priors

regularization w/ an untrained CNN

V. Lempitsky, A. Vedaldi, and D. Ulyanov. "Deep Image Prior". In: *2018 IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2018, pp. 9446–9454. DOI: 10.1109/CVPR.2018.00984.

Deep prior

$$\delta\mathbf{m} = g(\mathbf{z}, \mathbf{w}), \quad \mathbf{w} \sim \mathrm{N}(\mathbf{0}, \lambda^{-2}\mathbf{I})$$

untrained CNN, $g(\mathbf{z}, \mathbf{w})$

CNN weights, $\mathbf{w}$

fixed input, $\mathbf{z}$

V. Lempitsky, A. Vedaldi, and D. Ulyanov. "Deep Image Prior". In: *2018 IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2018, pp. 9446–9454. DOI: 10.1109/CVPR.2018.00984.

data misfit—negative-log likelihood

$$
-\log \pi_{\text{like}} \left(\{\delta \mathbf{d}_i\}_{i=1}^{n_s} \mid \mathbf{w} \right)
$$

$$
= \frac{1}{2\sigma^2} \sum_{i=1}^{n_s} \|\delta \mathbf{d}_i - \mathbf{J}(\mathbf{m}_0, \mathbf{q}_i) g(\mathbf{z}, \mathbf{w})\|_2^2 + \underbrace{\text{const}}_{\text{Ind. of } \mathbf{w}}
$$

regularization—negative-log prior

$$-\log \pi_{\text{prior}}\left(\mathbf{w}\right) = \frac{\lambda^2}{2}\|\mathbf{w}\|_2^2 + \underbrace{\text{const}}_{\text{Ind. of } \mathbf{w}}$$

Deep-prior based imaging—objective

$$\min_{\mathbf{w}} \quad \underbrace{\frac{1}{2\sigma^2}\sum_{i=1}^{n_s}\|\delta\mathbf{d}_i - \mathbf{J}(\mathbf{m}_0, \mathbf{q}_i)g(\mathbf{z}, \mathbf{w})\|_2^2}_{\text{negative-log likelihood}} + \underbrace{\frac{\lambda^2}{2}\|\mathbf{w}\|_2^2}_{\substack{\text{negative-log}\\ \text{prior}}}$$

MAP estimate, $\widehat{\delta\mathbf{m}}_{\text{MAP}} = g(\mathbf{z}, \widehat{\mathbf{w}}_{\text{MAP}})$

Deep priors in seismic

data reconstruction

denoising

Qun Liu, Lihua Fu, and Meng Zhang. "Deep-seismic-prior-based reconstruction of seismic data using convolutional neural networks". In: *arXiv preprint arXiv:1911.08784* (2019).

Yunzhi Shi, Xinming Wu, and Sergey Fomel. "Deep learning parameterization for geophysical inverse problems". In: *SEG 2019 Workshop: Mathematical Geophysics: Traditional vs Learning, Beijing, China, 5-7 November 2019*. Society of Exploration Geophysicists. 2020, pp. 36–40. DOI: 10.1190/iwmg2019_09.1.

Deep priors in seismic

imaging

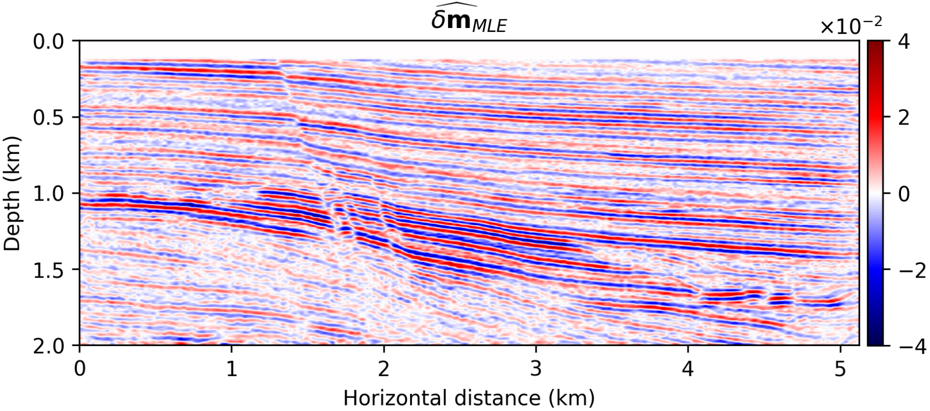
full-waveform inversion

Ali Siahkoohi, Gabrio Rizzuti, and Felix J. Herrmann. "A Deep-Learning Based Bayesian Approach to Seismic Imaging and Uncertainty Quantification". In: 2020.1 (2020), pp. 1–5. DOI: https://doi.org/10.3997/2214-4609.202010770. URL: https://arxiv.org/pdf/2001.04567.pdf.

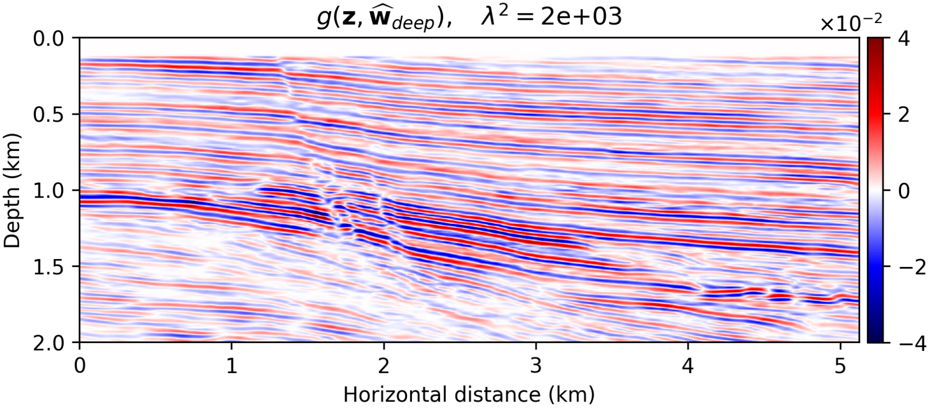
Yulang Wu and George A McMechan. "Parametric convolutional neural network-domain full-waveform inversion". In: *GEOPHYSICS* 84.6 (2019), R881–R896. DOI: 10.1190/geo2018-0224.1.

Comparison

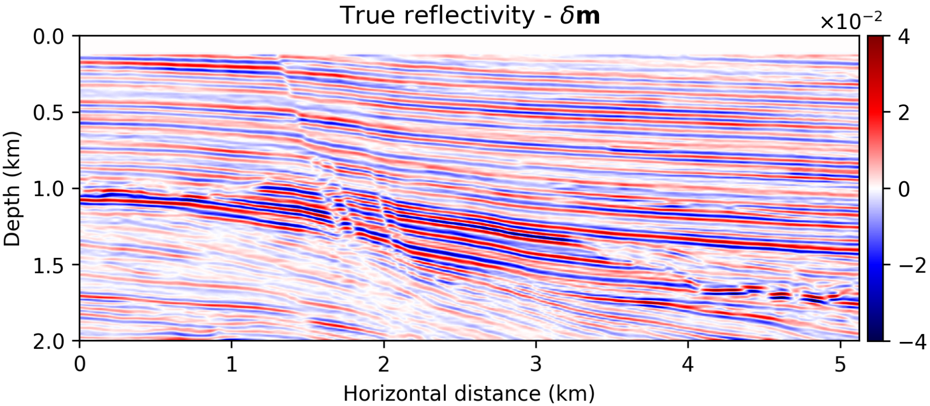
MLE (no deep prior) and MAP (deep prior)



MLE — no regularization (prior)



Imaging w/ deep prior, $\lambda = 2 \times 10^3$



$\delta\mathbf{m}$—"true" reflectivity model obtain from Parihaka dataset

Limitations

needs stopping criteria

expensive—many wave-equation solves

Contribution

weak deep priors—a computationally practical formulation

relaxing the deep prior—i.e., $\delta\mathbf{m} \sim \mathrm{N}(g(\mathbf{z}, \mathbf{w}), \gamma^{-2}\mathbf{I})$

$$\min_{\delta \mathbf{m}, \mathbf{w}} \left[\frac{1}{2\sigma^2} \sum_{i=1}^{N} \|\delta \mathbf{d}_i - \mathbf{J}(\mathbf{m}_0, \mathbf{q}_i)\delta \mathbf{m}\|_2^2 + \frac{\lambda^2}{2} \|\mathbf{w}\|_2^2 \right]$$

subject to $\delta \mathbf{m} = g(\mathbf{z}, \mathbf{w})$

(deep prior)

subject to $\delta \mathbf{m} = g(\mathbf{z}, \mathbf{w}) + \mathrm{N}(\mathbf{0}, \gamma^{-2}\mathbf{I})$

(week deep prior)

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Weak peep-prior based imaging—objective

$$\min_{\delta\mathbf{m},\,\mathbf{w}} \quad \underbrace{\frac{1}{2\sigma^2}\sum_{i=1}^{n_s}\|\delta\mathbf{d}_i - \mathbf{J}(\mathbf{m}_0,\mathbf{q}_i)\delta\mathbf{m}\|_2^2}_{\text{negative-log likelihood}} + \underbrace{\frac{\gamma^2}{2}\|\delta\mathbf{m} - g(\mathbf{z},\mathbf{w})\|_2^2 + \frac{\lambda^2}{2}\|\mathbf{w}\|_2^2}_{\text{negative-log (weak) prior}}$$

Felix J. Herrmann, Ali Siahkoohi, and Gabrio Rizzuti. "Learned imaging with constraints and uncertainty quantification". In: *Neural Information Processing Systems (NeurIPS) 2019 Deep Inverse Workshop*. Dec. 2019. URL: https://arxiv.org/pdf/1909.06473.pdf.

Computational considerations

preconditioned stochastic optimization

unbiased estimation of posterior via one source experiment

Algorithm: Imaging w/ weak deep priors

Output : $\delta\mathbf{m}$

Initialization: $\mathbf{z} \sim \mathrm{N}(\mathbf{0}, \mathbf{I})$
$\mathbf{w} \sim \mathrm{N}(\mathbf{0}, \lambda^{-2}\mathbf{I})$
$\delta\mathbf{m} = \mathbf{0}$

repeat
  sample $(\delta\mathbf{d}, \mathbf{q})$ from $\{\delta\mathbf{d}_i, \mathbf{q}_i\}_{i=1}^{n_s}$ `// randomly select a shot`
  $\mathcal{L}(\delta\mathbf{m}) = \frac{n_s}{2\sigma^2}\|\delta\mathbf{d} - \mathbf{J}(\mathbf{m}_0, \mathbf{q})\delta\mathbf{m}\|_2^2 + \frac{\gamma^2}{2}\|\delta\mathbf{m} - g(\mathbf{z}, \mathbf{w})\|_2^2$ `//` $\delta\mathbf{m}$ `objective`
  $\delta\mathbf{m} \leftarrow \mathtt{Adagrad}(\mathcal{L}(\delta\mathbf{m}), \eta)$ `// update` $\delta\mathbf{m}$
  for $k \leftarrow 1$ **to** K **do** `// PDE-free updates to` $\mathbf{w}$
    $\mathcal{L}(\mathbf{w}) = \frac{\gamma^2}{2}\|\delta\mathbf{m} - g(\mathbf{z}, \mathbf{w})\|_2^2 + \frac{\lambda^2}{2}\|\mathbf{w}\|_2^2$ `//` $\mathbf{w}$ `objective`
    $\mathbf{w} \leftarrow \mathtt{RMSprop}(\mathcal{L}(\mathbf{w}), \tau)$ `// update` $\mathbf{w}$
  end
until *starting to overfit* `// e.g., 2 passes over shots`

Comparison

deep prior and weak deep prior

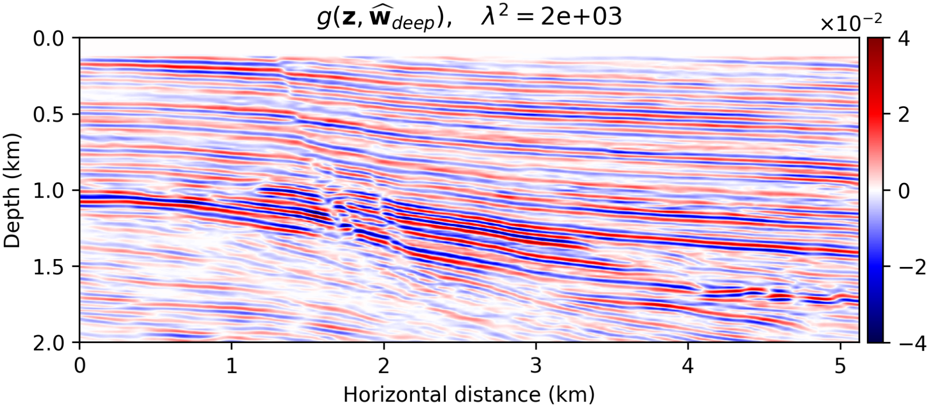

Image w/ deep prior, cost ≈ 15 reverse-time migrations

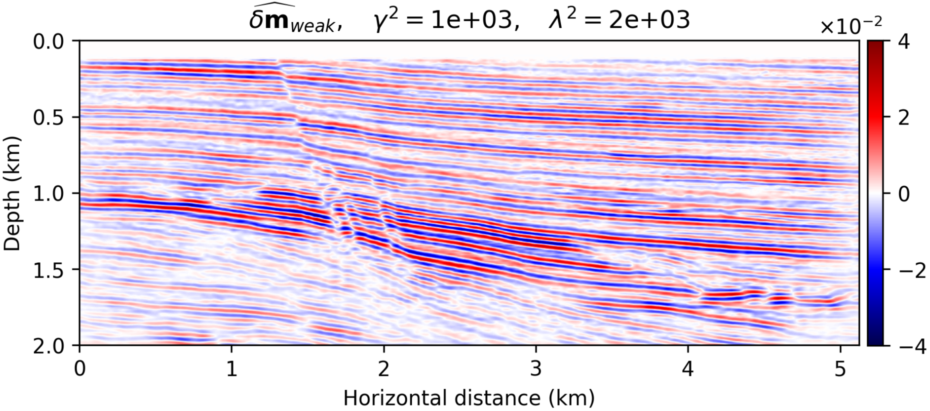

Image w/ weak deep prior, cost ≈ 2 reverse-time migrations

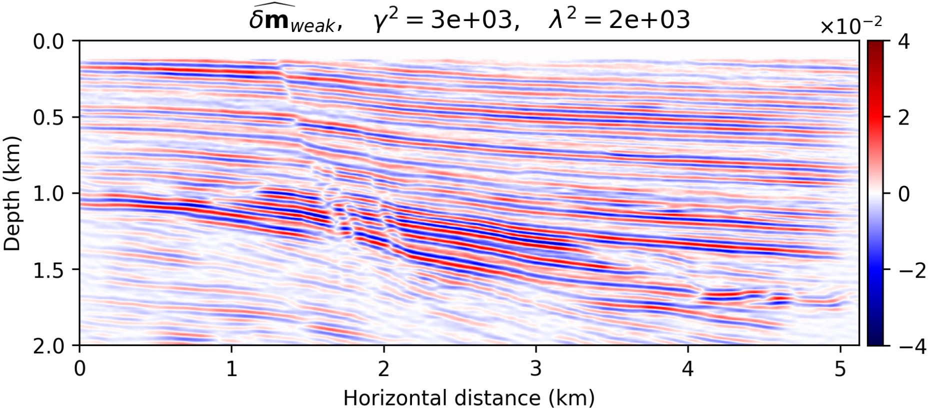

Image w/ weak deep prior, cost ≈ 2 reverse-time migrations

Conclusions

deep prior—circumvents artifacts but infeasible in large scale imaging

weak deep prior—decouple model and network updates

still able to resolve the imaging artifacts in presence of strong noise

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Contributions

regularization w/ deep priors

weak deep priors—a computationally feasible alternative

github.com/slimgroup/Software.SEG2020