Uncertainty quantification in imaging and automatic horizon tracking—a Bayesian deep-prior based approach

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Seismic imaging

estimate reflectivity $\delta m$ given observed data $\{d_i\}_{i=1}^{n_s}$
\[
\min_{\delta m} \frac{1}{2\sigma^2} \sum_{i=1}^{n_s} \| \delta d_i - J(m_0, q_i) \delta m \|^2_2
\]

linearized Born operator, \( J(m_0, q_i) \)

linearized data, \( \delta d_i \)

noise variance, \( \sigma^2 \)
Challenges

expensive forward operator

inconsistent, mildly ill-conditioned
2D slice from Parihaka dataset

finite-difference simulations w/ Devito

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$\delta m$—"true" reflectivity model obtain from Parihaka dataset
\( \mathbf{m}_0 \) — made up background squared-slowness model \( \left( \frac{s^2}{\text{km}^2} \right) \)
205 sources w/ 25 m sampling rate

410 receivers w/ 12.5 m sampling rate

1.5 s recording time

Ricker source wavelet w/ 30 Hz central frequency
Noise-free (left) and noisy (right) linearized data — SNR: $-8.7466 \text{ dB}$
Maximum likelihood estimate

$$\min_{\delta m} \frac{1}{2\sigma^2} \sum_{i=1}^{n_s} \| \delta d_i - J(m_0, q_i)\delta m \|^2_2$$
MLE—no regularization (prior)
Deep priors

regularization w/ an untrained CNN
Deep priors in seismic data reconstruction

denoising


Deep priors in seismic imaging

full-waveform inversion


Deep prior

\[ \delta m = g(z, w), \quad w \sim N(0, \lambda^{-2}I) \]

untrained CNN, \( g(z, w) \)

CNN weights, \( w \)

fixed input, \( z \)

Negative-log posterior

\[- \log \pi_{w|\delta d}(w | \{\delta d_i\}_{i=1}^{n_s})\]

\[= \frac{1}{2\sigma^2} \sum_{i=1}^{n_s} \|\delta d_i - J(m_0, q_i)g(z, w)\|_2^2 + \frac{\lambda^2}{2} \|w\|_2^2 + \text{const} \]

negative-log likelihood

negative-log prior

Ind. of \(w\)
Posterior distribution, $\pi_{w|\delta d}$

MAP, conditional mean, pointwise standard deviation (UQ), ...
MAP estimate, $\hat{\delta m}_{MAP} = g(z, \hat{w}_{MAP})$

$\hat{w}_{MAP} := \arg \max_w \pi_{w|d} (w | \{\delta d_i\}_{i=1}^{n_s})$
conditional (posterior) mean, \( \hat{\delta m}_{CM} \)

\[
\hat{\delta m}_{CM} := \mathbb{E}_{w \sim \pi_{w|\delta d}} [g(z, w)] \approx \frac{1}{n_W} \sum_{j=1}^{n_W} g(z, \hat{w}_j),
\]

samples from posterior, \( \{\hat{w}_j\}_{j=1}^{n_W} \sim \pi_{w|\delta d}(w | \{\delta d_i\}_{i=1}^{n_s}) \)
pointwise standard deviation (UQ)

\[ \hat{\sigma}^2 := \frac{1}{n_w - 1} \sum_{j=1}^{n_w} \left( g(z, \hat{w}_j) - \hat{\delta m}_{CM} \right) \odot \left( g(z, \hat{w}_j) - \hat{\delta m}_{CM} \right), \]

samples from posterior, \( \{ \hat{w}_j \}_{j=1}^{n_w} \sim \pi_{w|\delta d}(w | \{ \delta d_i \}_{i=1}^{n_s}) \)
Comparison

MLE (no deep prior), MAP, and conditional mean
MLE—no (deep) prior
MAP estimate—SNR: 8.77 dB
\( \delta m \) — "true" reflectivity model obtained from Parihaka dataset
Conditional mean estimate—SNR: 9.66 dB
Pointwise marginals at (0.73 km, 0.32 km)
Pointwise marginals at (1.55 km, 1.18 km)
Pointwise marginals at (4.55 km, 1.74 km)
Sampling from the posterior, $\pi_{\mathbf{w}|\delta \mathbf{d}}$

stochastic gradient Langevin dynamics (SGLD)
SGLD—an stochastic-approximation based MCMC sampling approach

\[
\mathbf{w}_{k+1} = \mathbf{w}_k + \frac{\epsilon}{2} \mathbf{M} \nabla_{\mathbf{w}} \left[ n_s \log \pi_{\text{like}} (\delta \mathbf{d}_i | \mathbf{w}_k) \right. \\
+ \log \pi_{\text{prior}} (\mathbf{w}_k) \left. \right] + \mathbf{\eta}_k, \quad \mathbf{\eta}_k \sim \mathcal{N}(0, \epsilon \mathbf{M}),
\]


Devito4PyTorch

integrating Devito’s PDE solvers into PyTorch

Horizon tracking and uncertainty analysis
Contribution

propagate uncertainties from imaging into horizon tracking

\[ \pi_{h|\delta d}^* \]

* \( h \) horizons (random variable)
Assumption

horizon tracker does not directly use observed data
Horizon tracking inference

\[ \mathbb{E}_{\mathbf{h} \sim \pi_{\mathbf{h} | \delta d}} [ f(\mathbf{h}) ] = \mathbb{E}_{\delta \mathbf{m} \sim \pi_{\delta \mathbf{m} | \delta d}} \mathbb{E}_{\mathbf{h} \sim \pi_{\mathbf{h} | \delta \mathbf{m}}} [ f(\mathbf{h}) ] \]

nonuniqueness in horizon tracking

nonuniqueness in seismic imaging

arbitrary function, \( f \)—e.g.,

\[ f(\mathbf{h}) = \mathbf{h} \]

\[ f(\mathbf{h}) = (\mathbf{h} - \mathbb{E}[\mathbf{h}]) \odot (\mathbf{h} - \mathbb{E}[\mathbf{h}]) \]
Automatic horizon tracker, $\mathcal{H}$

uses local slopes of the image

needs control points

open-source software

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**deterministic** horizon tracker, $\mathcal{H}$

\[
\mathbb{E}_{h \sim \pi_h|\delta_d} \left[ f(h) \right] \approx \frac{1}{n_w} \sum_{j=1}^{n_w} f \left( \mathcal{H} \left( g \left( z, \hat{w}_j \right) \right) \right)
\]

samples from posterior, $\{\hat{w}_j\}_{j=1}^{n_w} \sim \pi_{w|\delta_d}(w \mid \{\delta_d_i\}_{i=1}^{n_s})$
Five sets of control points identifying 25 horizons of interest
Uncertainty in horizon tracking due to uncertainties in imaging
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Uncertainty in horizon tracking due to uncertainties in imaging
nondeterministic horizon tracker, $\mathcal{H}$

\[
\mathbb{E}_{h \sim \pi_{h|\delta}} \left[ f(h) \right] \approx \frac{1}{n_c n_W} \sum_{j=1}^{n_W} \sum_{k=1}^{n_c} f\left( \mathcal{H}(c_k, g(z, \hat{w}_j)) \right)
\]

sets of control points, $\{c_k\}_{k=1}^{n_c}$
Uncertainty in horizon tracking due uncertainties in imaging and control points
Conclusions

imaging w/ deep prior—expensive but circumvents artifacts

SGLD—reasonable first and second moments of posterior

uncertainties from imaging affect deep, close to boundary, and complex horizons
Contributions

regularization w/ deep priors

imaging uncertainty analysis via SGLD

propagate uncertainties from imaging into horizon tracking

github.com/slimgroup/Software.SEG2020