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High-frequency wavefield recovery with weighted matrix factorizations

Yijun Zhang, Shashin Sharan and Felix J. Herrmann
SEG 2019

SLIM
Georgia Institute of Technology

Motivation

Fully sampled data

- needed for multiple removal, migration & FWI

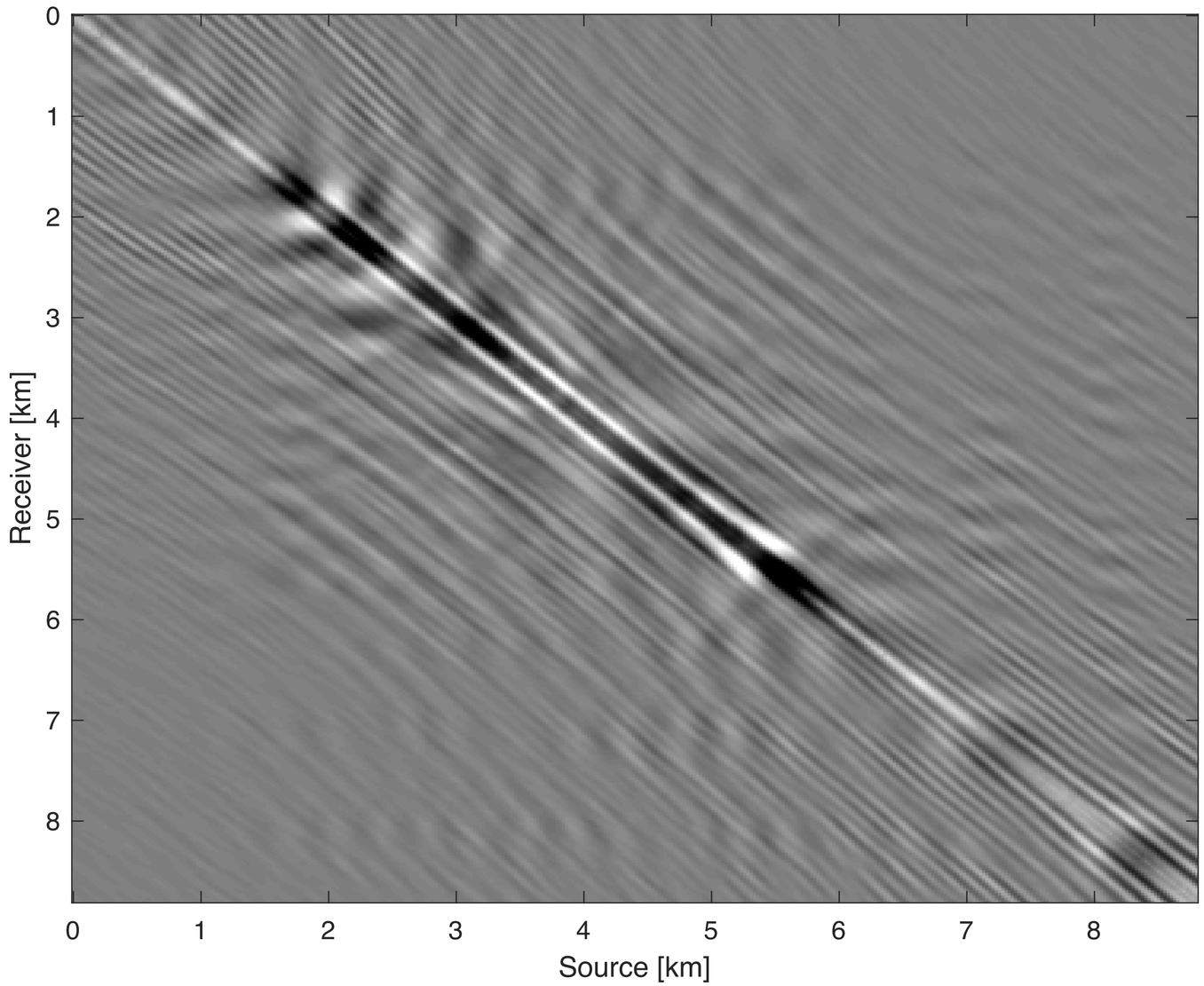
Seismic data

- expensive to acquire

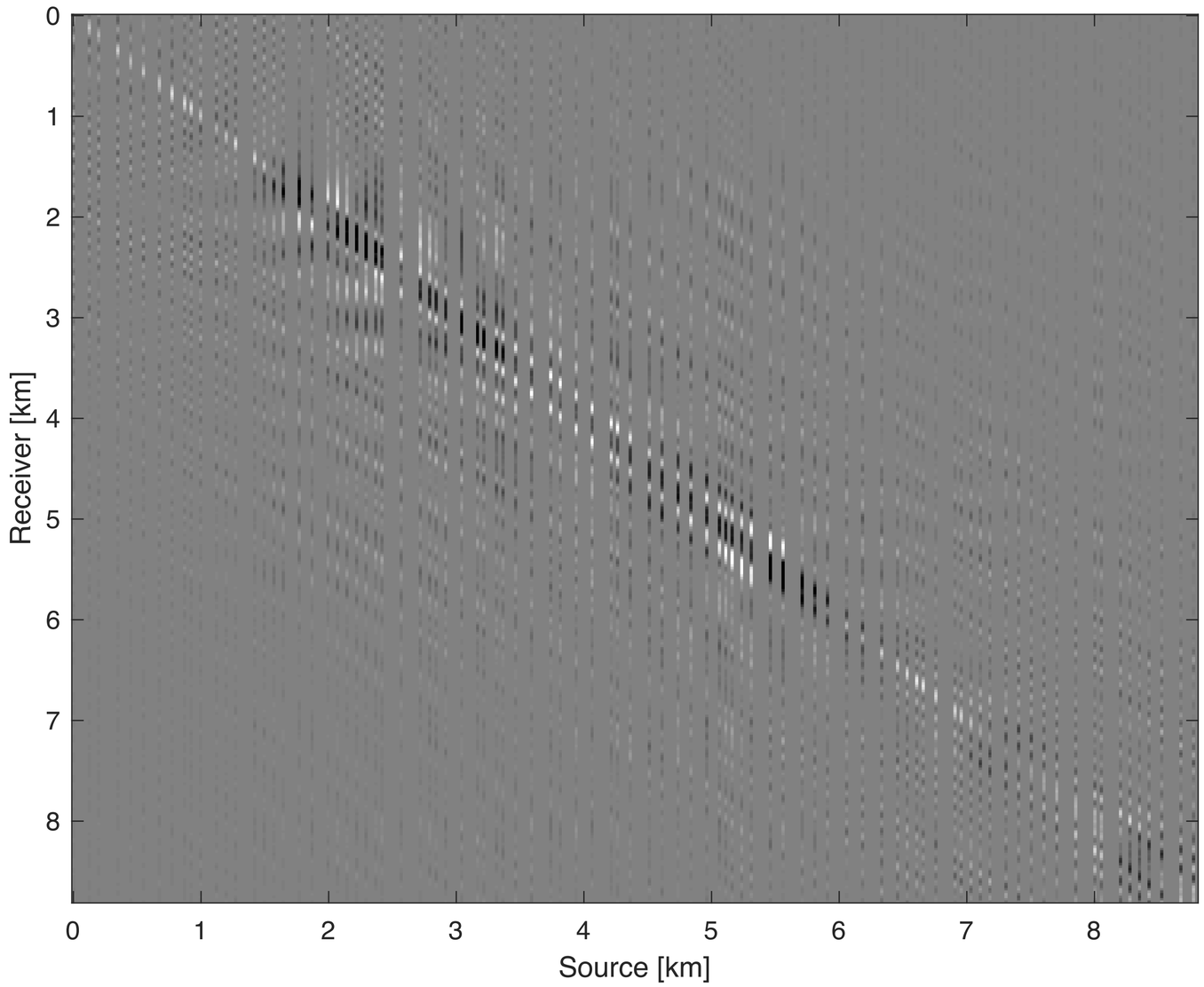
Conventional matrix completion

- exploits low-rank structure for recovery
- performance degrades w/ increasing frequency

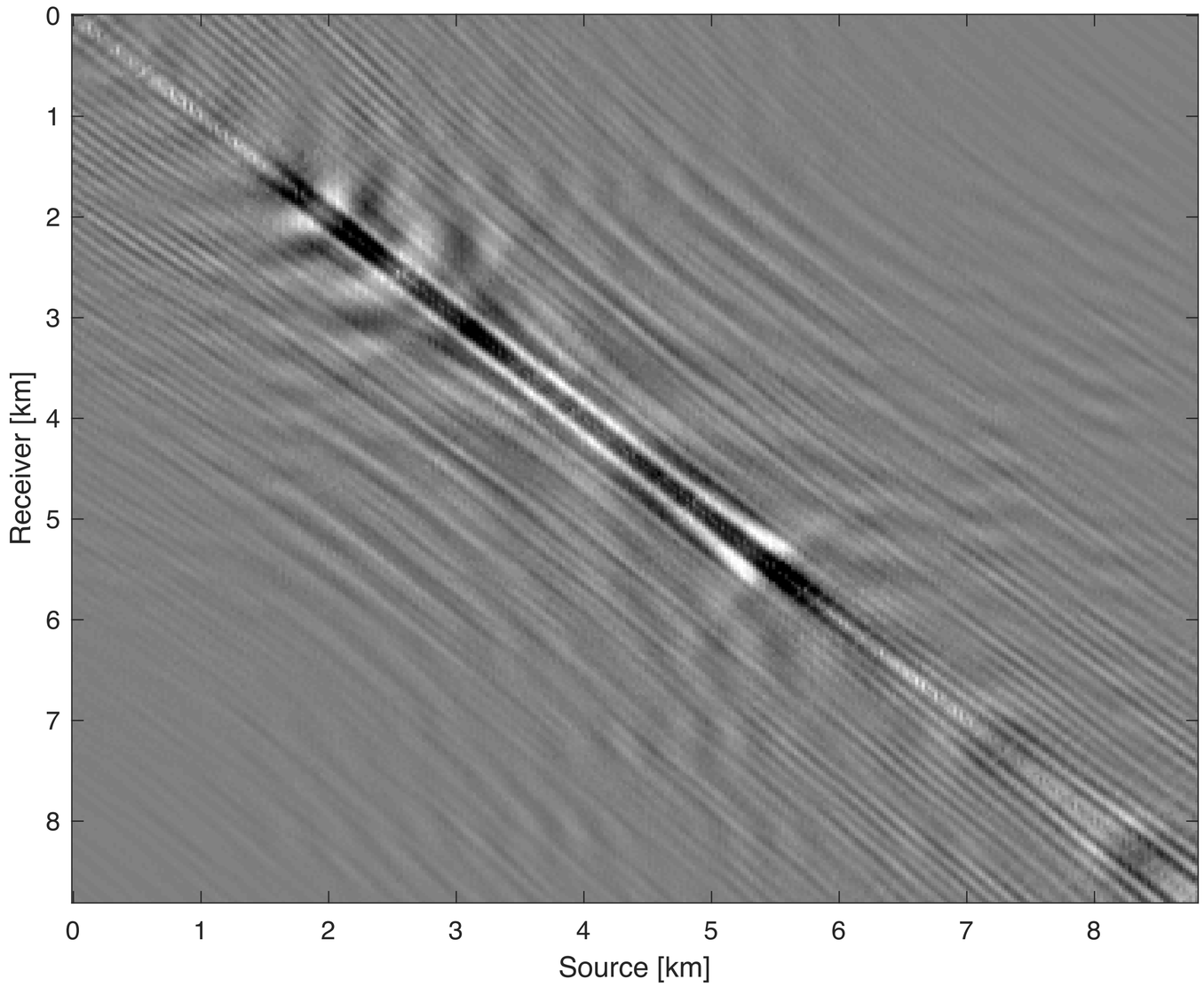
Fully sampled data: 30 Hz slice



Observed data (75% jittered subsampled)

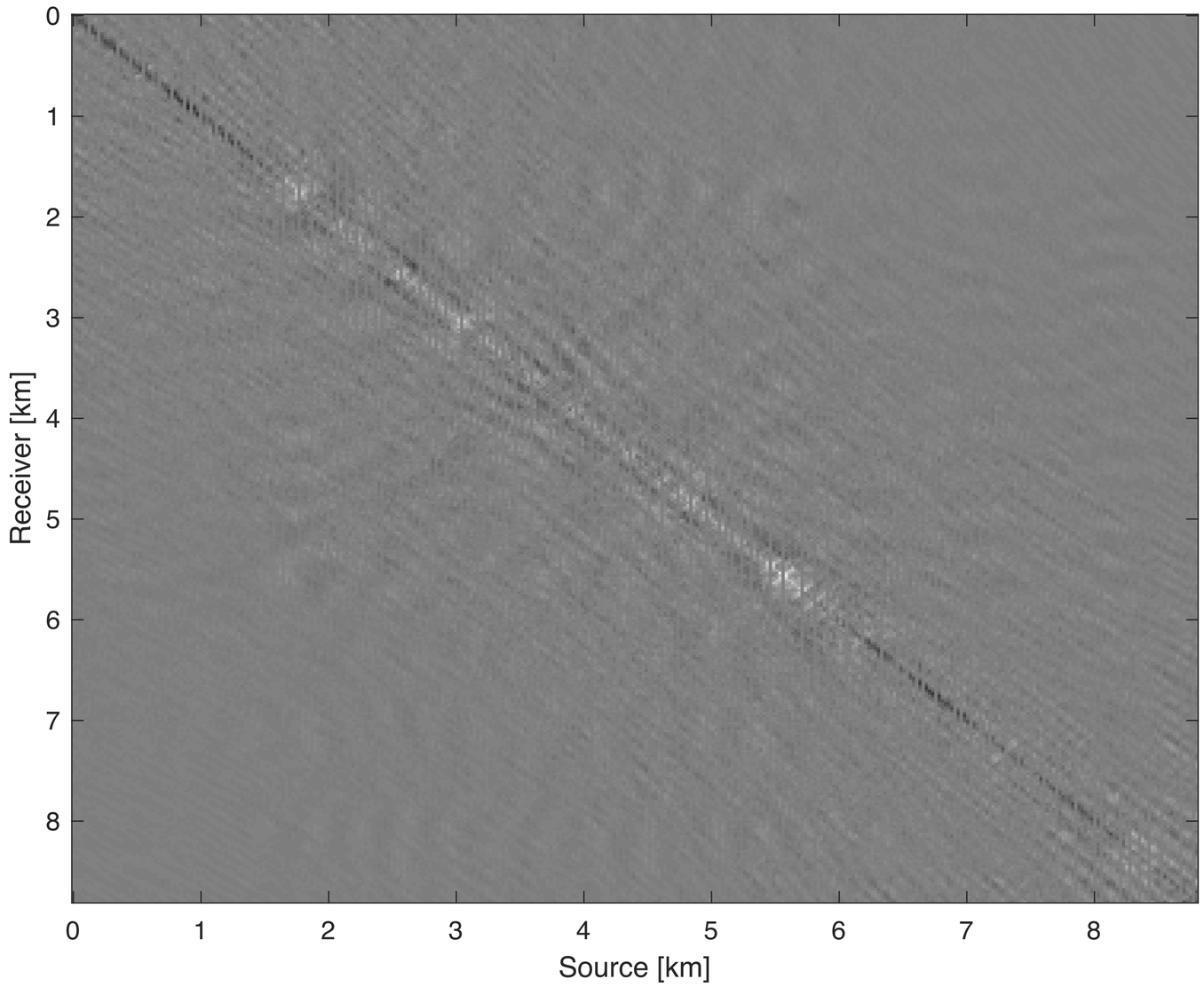


Recovery with conventional method (30Hz)

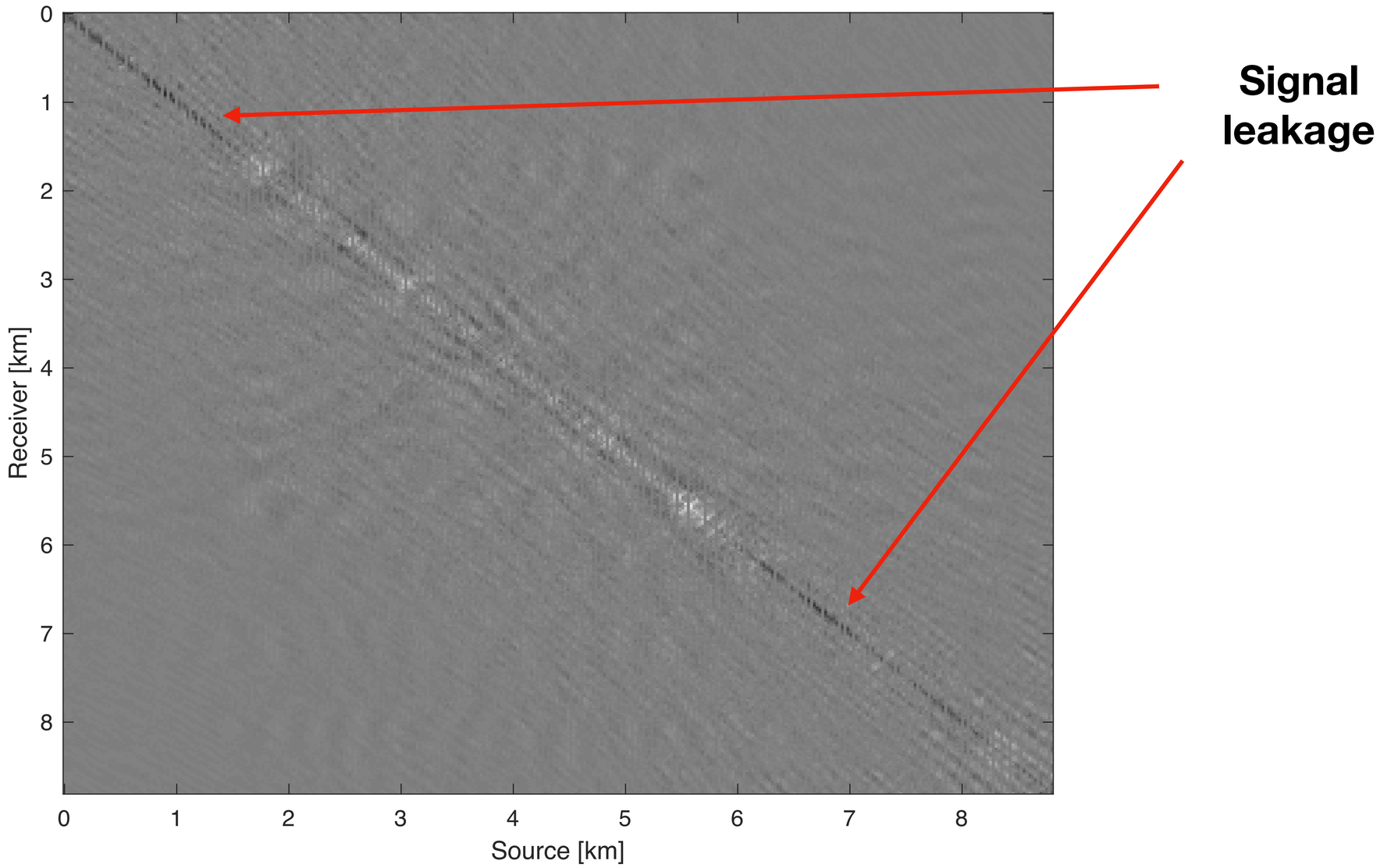


SNR = 11.48 dB
Rank = 30

Difference: True - Recovery w/ conventional



Difference: True - Recovery w/ conventional



Eftekhari, A. et al. “Weighted matrix completion and recovery with prior subspace information.” IEEE Transactions on Information Theory, 2018.
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Why weighted matrix completion?

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Why weighted matrix completion?

Reliable prior information

- useful for data reconstruction quality improvement

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In seismic data

- similarity between adjacent frequency slices can be used as prior

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Why weighted matrix completion?

Reliable prior information

- useful for data reconstruction quality improvement

In seismic data

- similarity between adjacent frequency slices can be used as prior

Similarity depends

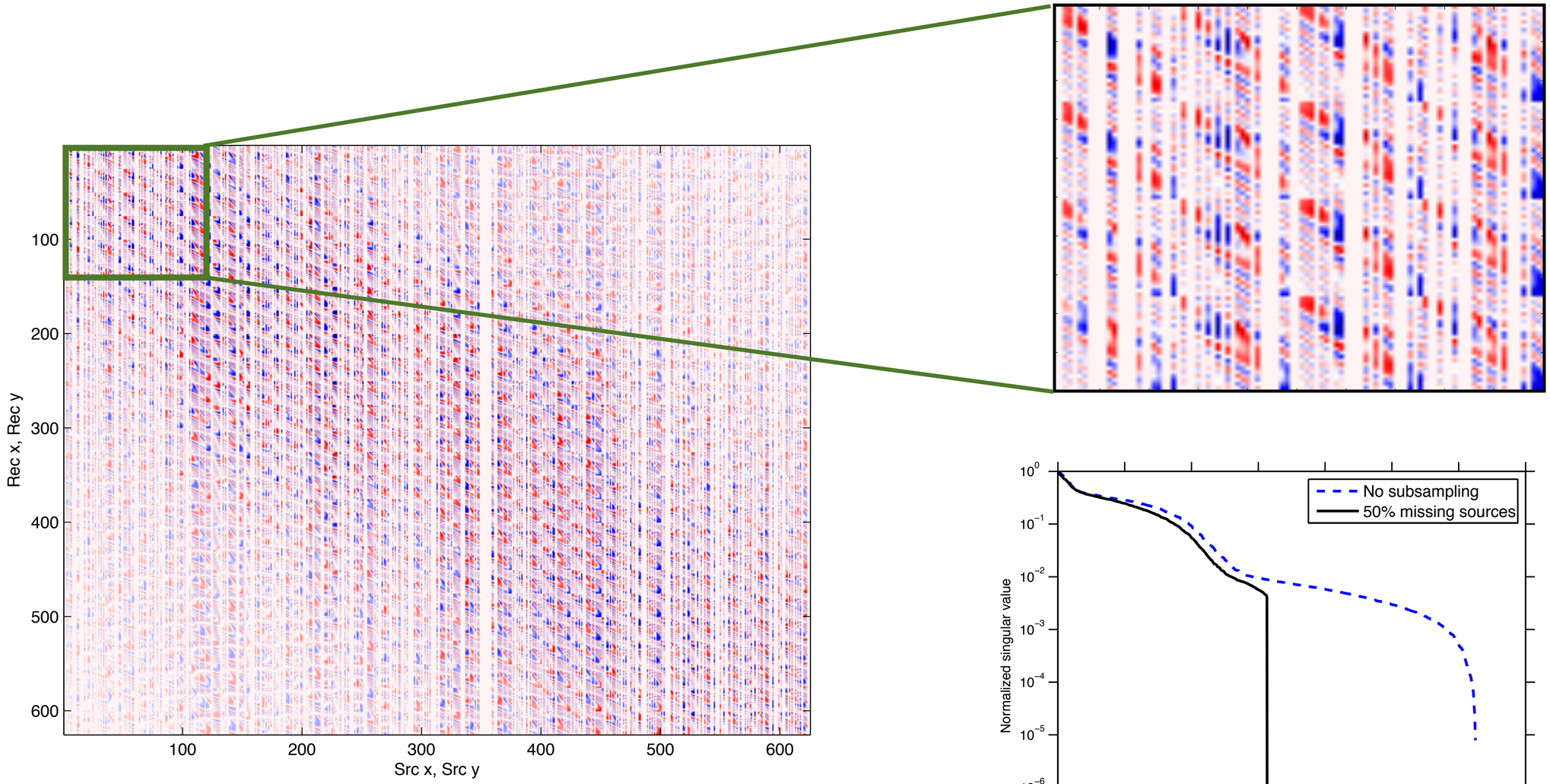
- on largest angle between adjacent frequency slices

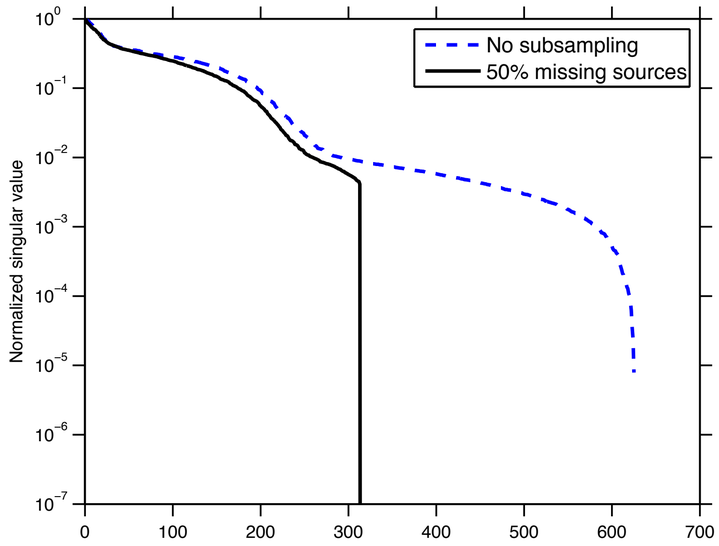
Matrix completion

Successful reconstruction schemes

- exploit structure
  - low rank/fast decay of singular values in “transform domain”
- sample randomly
  - increases rank in “transform domain”

Low-rank structure
Monochromatic slice, Sx-Sy matricization





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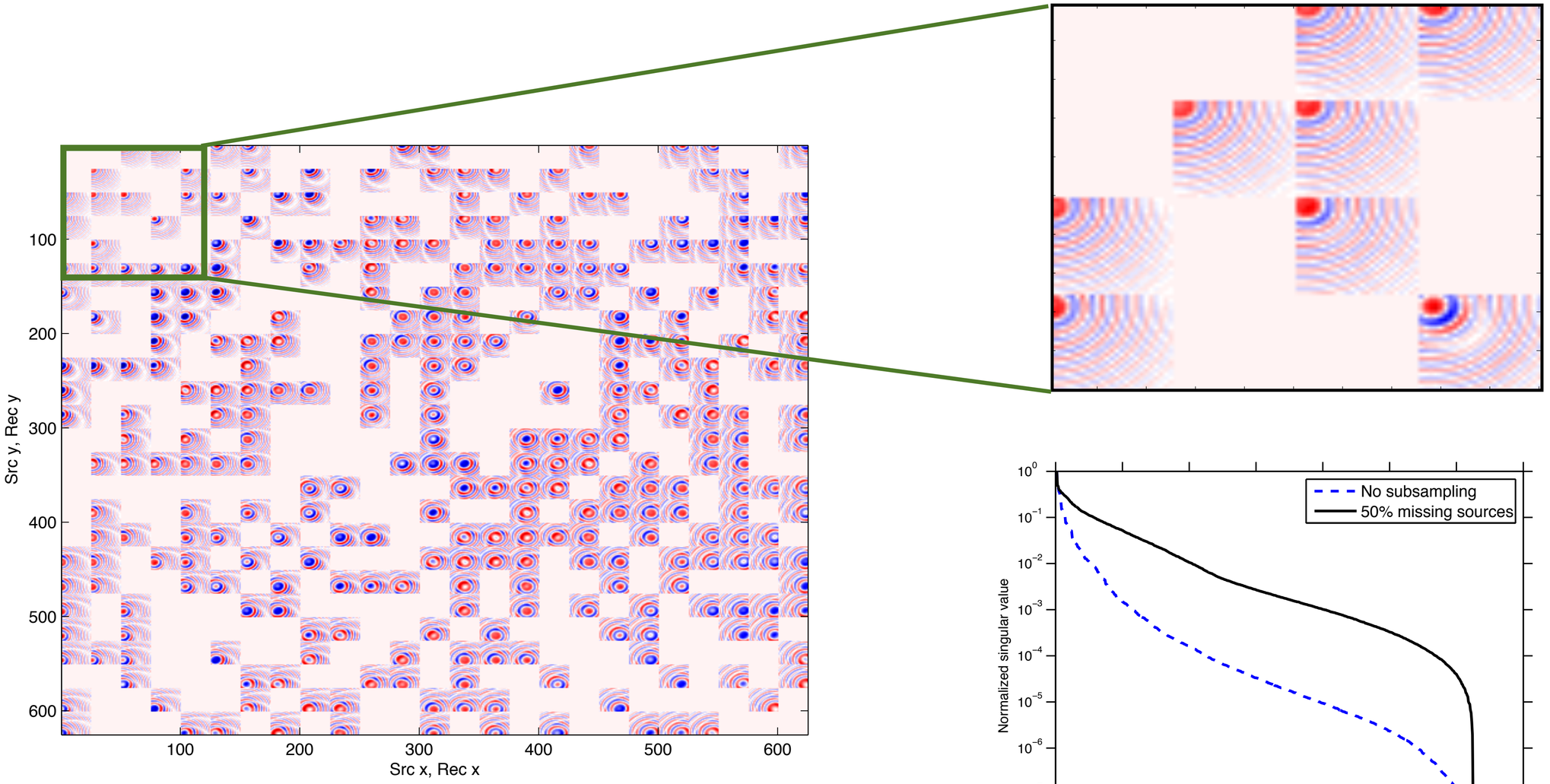
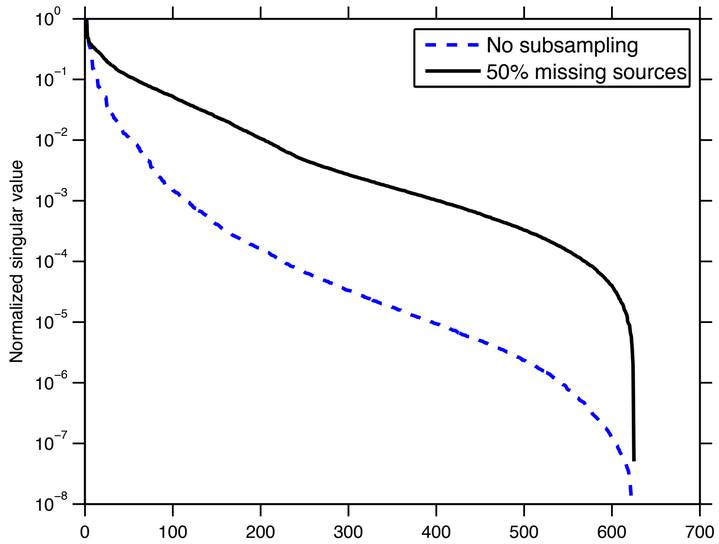
Low-rank structure
Monochromatic slice, Sx-Rx matricization





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Matrix completion

Successful reconstruction schemes

- exploit structure
  - low rank/fast decay of singular values in “transform domain”
- sample randomly
  - increases rank in “transform domain”
- optimization
  - via rank-minimization (nuclear norm-minimization)

Conventional method

Rank minimization

$$\underset{\mathbf{X}\in\mathbb{C}^{m\times n}}{\text{minimize}} \quad \underbrace{\text{rank}(\mathbf{X})}_{\textbf{number of singular values of } \mathbf{X}} \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F \leq \epsilon$$

where

$\mathcal{A}$	acquisition mask
$\mathbf{B} \in \mathbb{C}^{m\times n}$	observed data
$\|.\|_F$	Frobenius norm

Conventional method

Rank minimization

Hard to solve

$$\underset{\mathbf{X}\in\mathbb{C}^{m\times n}}{\text{minimize}} \quad \underbrace{\text{rank}(\mathbf{X})}_{\textbf{number of singular values of } \mathbf{X}} \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{B}\|_F \leq \epsilon$$

where

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Conventional method

Low-rank matrix completion

$$\underset{\mathbf{X}\in\mathbb{C}^{m\times n}}{\text{minimize}} \quad \underbrace{\|\mathbf{X}\|_*}_{\textbf{sum of singular values of } \mathbf{X}} \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{X})-\mathbf{B}\|_F \leq \epsilon$$

Conventional method

Low-rank matrix completion

Expensive for large scale

$$\underset{\mathbf{X}\in\mathbb{C}^{m\times n}}{\text{minimize}} \quad \underbrace{\|\mathbf{X}\|_*}_{\textbf{sum of singular values of } \mathbf{X}} \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{X})-\mathbf{B}\|_F \leq \epsilon$$

Low-rank matrix factorization

$$\underset{\mathbf{L},\mathbf{R}}{\text{minimize}} \quad \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}\right\|_F^2 \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{L}\mathbf{R}^H)-\mathbf{B}\|_F \leq \epsilon$$

where

$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$
$$\mathbf{L} \in \mathbb{C}^{m\times r}$$
$$\mathbf{R} \in \mathbb{C}^{n\times r}$$
$$\mathbf{B} \in \mathbb{C}^{m\times n}$$

Weighted method

Weighted matrix completion

$$\underset{\mathbf{X}\in\mathbb{C}^{m\times n}}{\text{minimize}} \quad \|\mathbf{QXW}\|_* \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{X})-\mathbf{B}\|_F \leq \epsilon$$

where

$$\mathbf{X}_{\text{prior}} \approx \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$$

$$\mathbf{Q} = w_1\mathbf{U}\mathbf{U}^H + \mathbf{U}^{\perp}\mathbf{U}^{\perp H}$$

$$\mathbf{W} = w_2\mathbf{V}\mathbf{V}^H + \mathbf{V}^{\perp}\mathbf{V}^{\perp H}$$

Scalars $w_1, w_2 \in (0,1]$ are weights

Weighted method

Weighted matrix completion

$$\underset{\mathbf{X}\in\mathbb{C}^{m\times n}}{\text{minimize}} \quad \|\mathbf{QXW}\|_* \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{X})-\mathbf{B}\|_F \leq \epsilon$$

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Scalars $w_1, w_2 \in (0,1]$ are weights

Weighted matrix factorization

$$\underset{\mathbf{L},\mathbf{R}}{\text{minimize}} \quad \frac{1}{2}\left\|\begin{bmatrix}\mathbf{QL}\\ \mathbf{WR}\end{bmatrix}\right\|_F^2 \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{LR}^H)-\mathbf{B}\|_F \leq \epsilon$$

Weighted method

Weighted matrix completion

$$\underset{\mathbf{X}\in\mathbb{C}^{m\times n}}{\text{minimize}} \quad \|\mathbf{QXW}\|_* \quad \text{subject to} \quad \|\mathcal{A}(\mathbf{X})-\mathbf{B}\|_F \leq \epsilon$$

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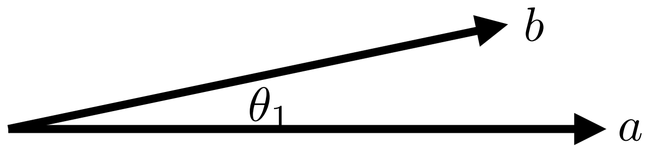
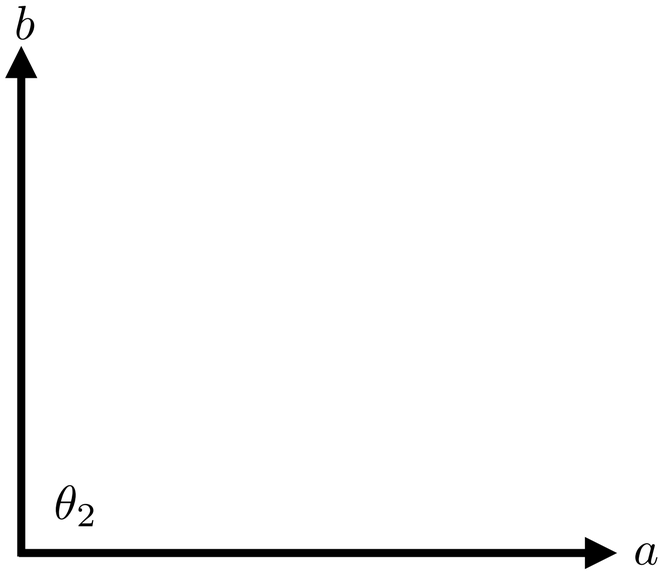
Smaller weights correspond to more confidence on prior information

Understanding similarity



Small angle ⟶ more similarity



Large angle ⟶ less similarity

2D Field data example: Gulf of Suez

Data acquisition area: Gulf of Suez

Data dimension: 354 x 354 x 1024 (nr x ns x nt)

Dimension of each frequency slice: 354 x 354

Source sampling interval: 25 m

Receiver sampling interval: 25 m

Field data example: Gulf of Suez

Observed data: 75 % missing sources

Result shown for Frequency slice: 30 Hz

Optimization information

Rank: 30

Number of iterations per frequency: 150

Scenarios compared

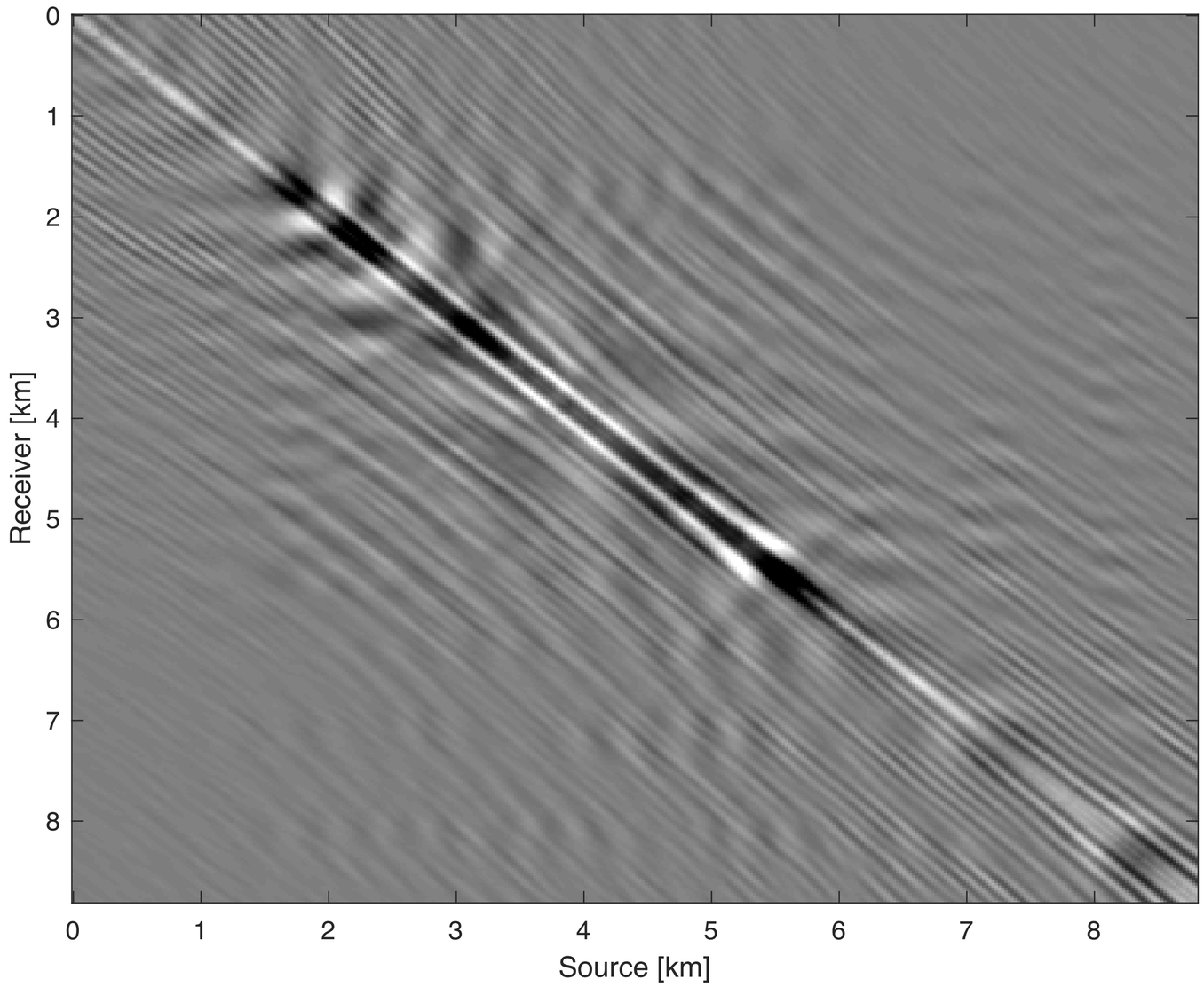
Scenario 1: without using any prior information

Scenario 2: using single pair prior information

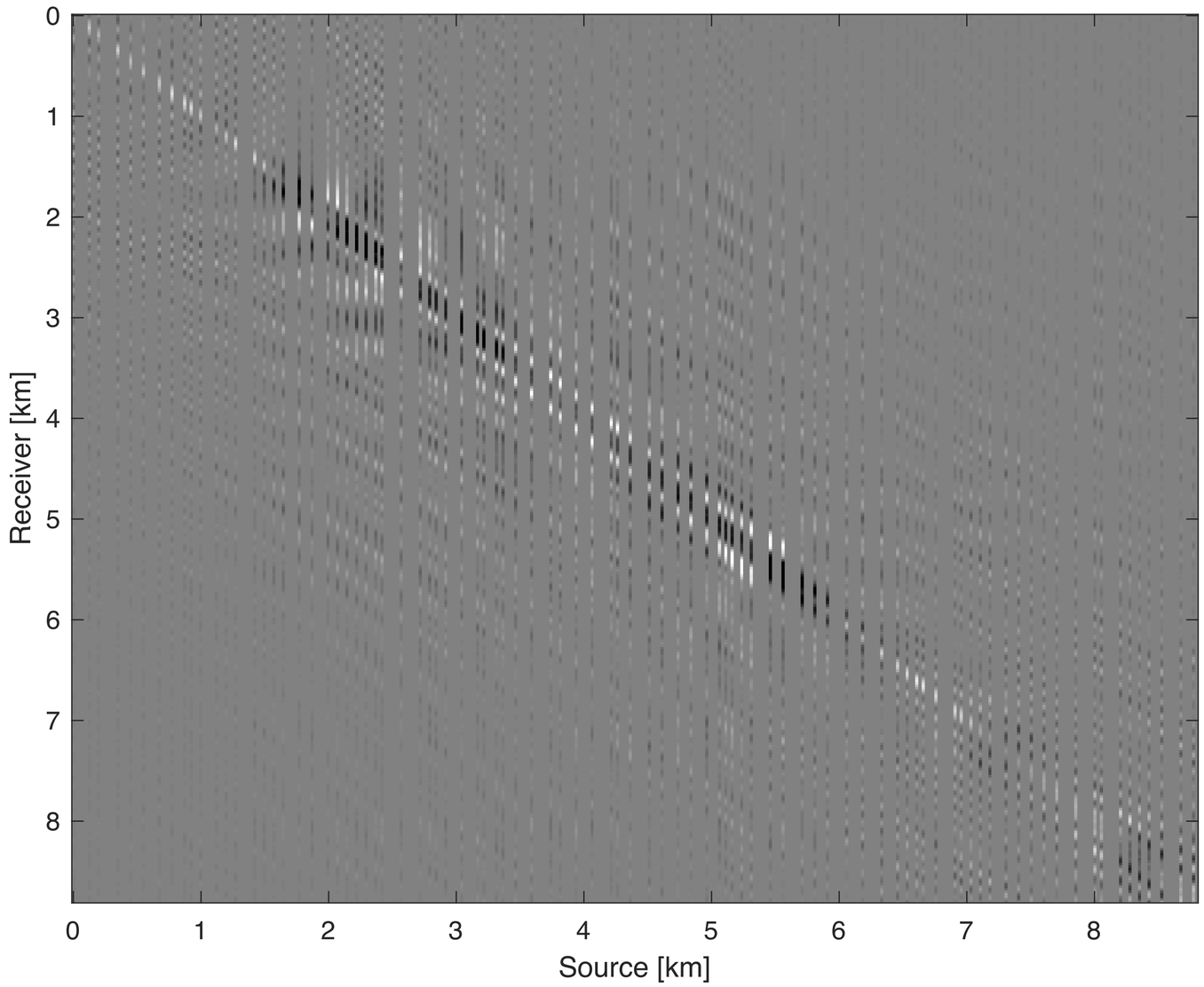
Scenario 3: using recursive prior information

Frequency slice: 30 Hz

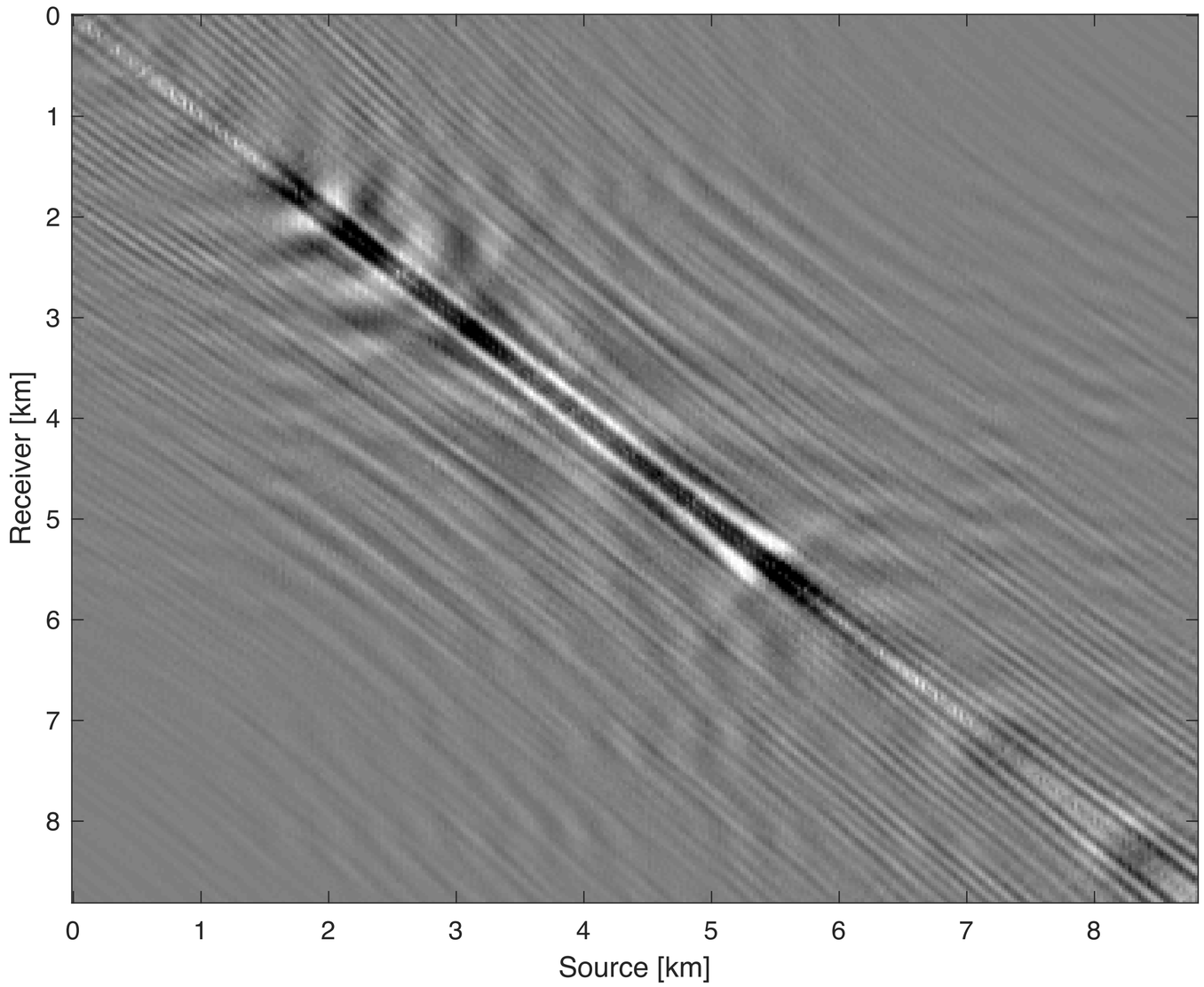
Fully sampled data



Observed data (75% jittered subsampled)

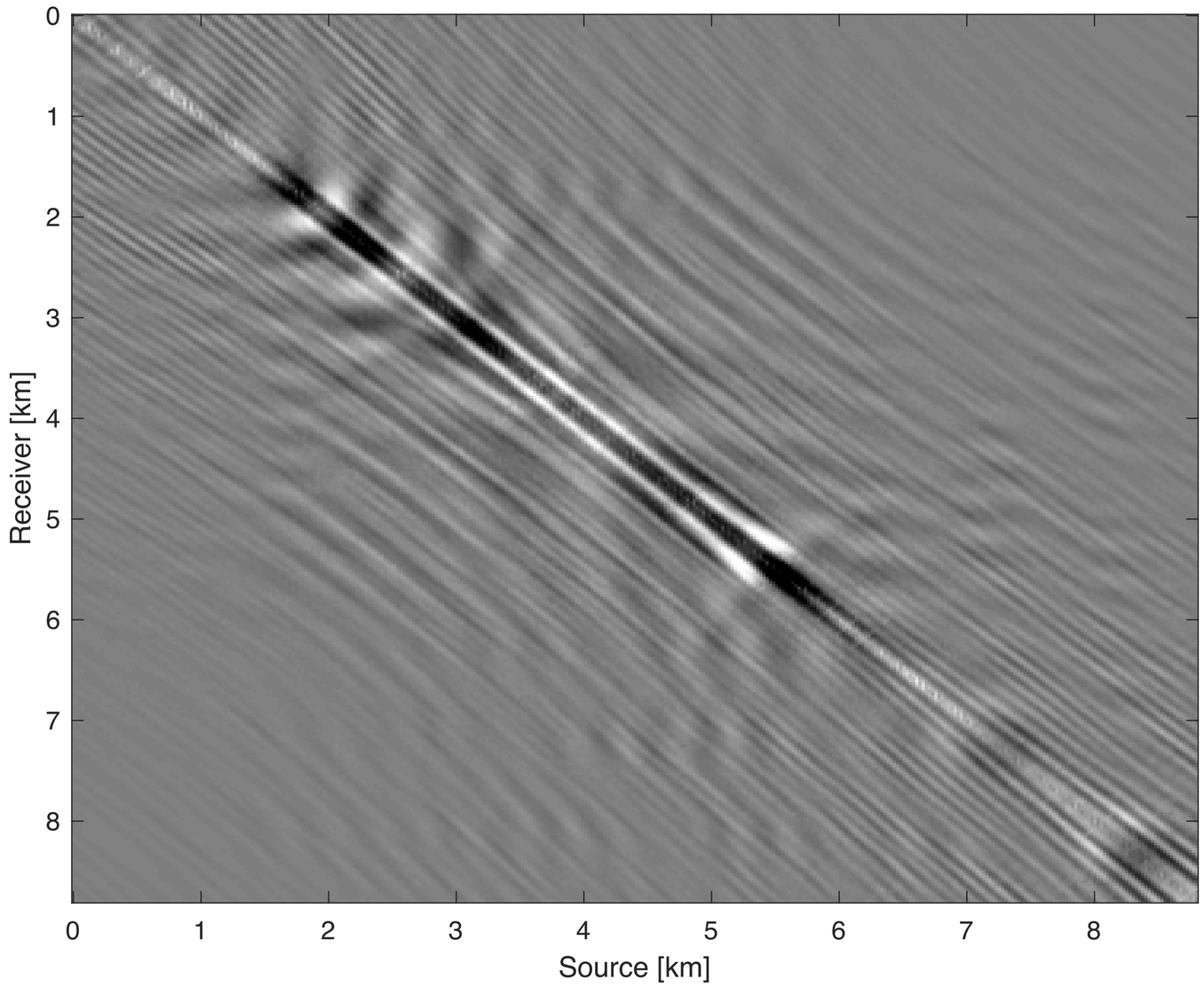


Recovery w/ conventional method



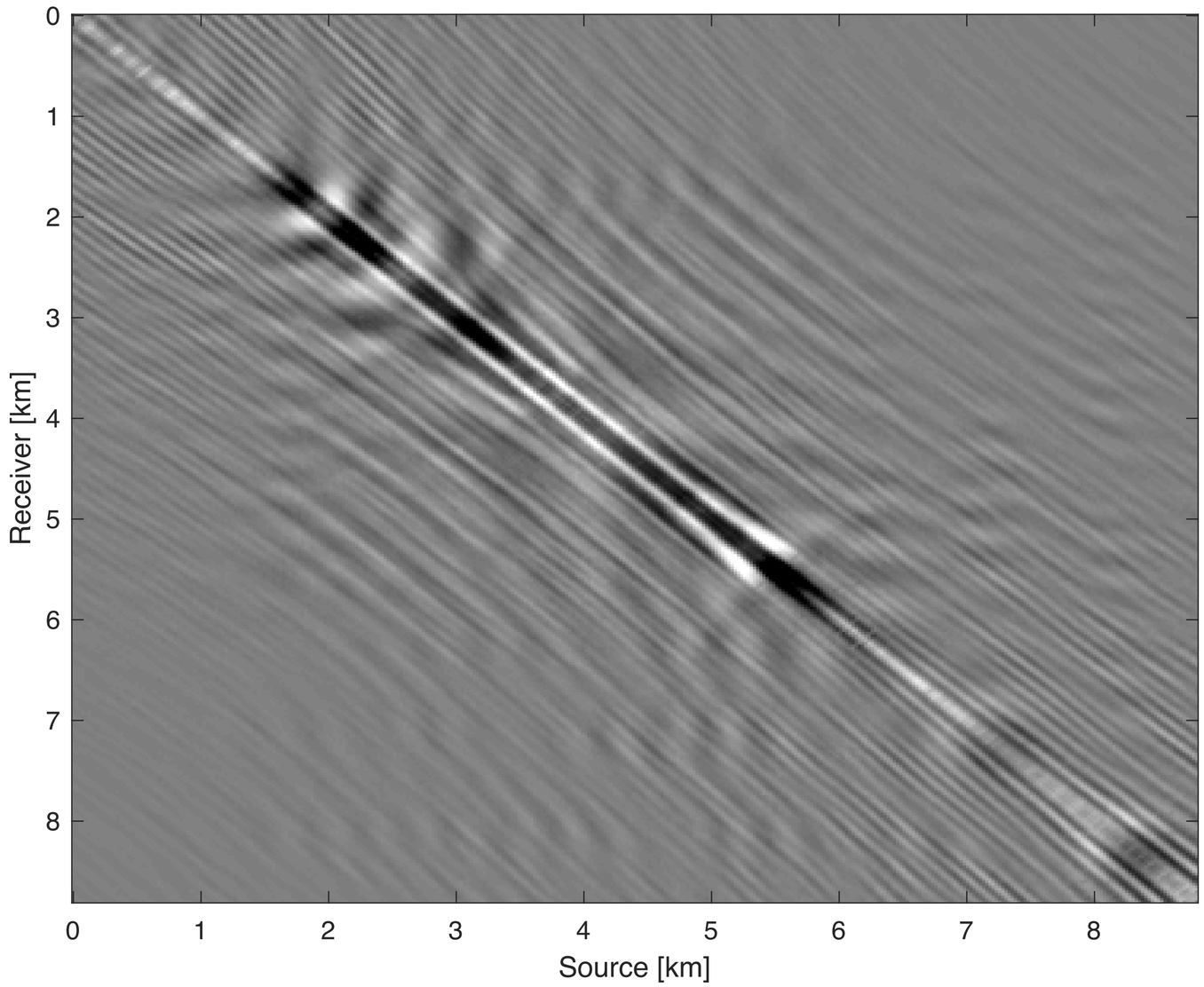
SNR = 11.48 dB
Rank = 30

Recovery w/ weighted pair



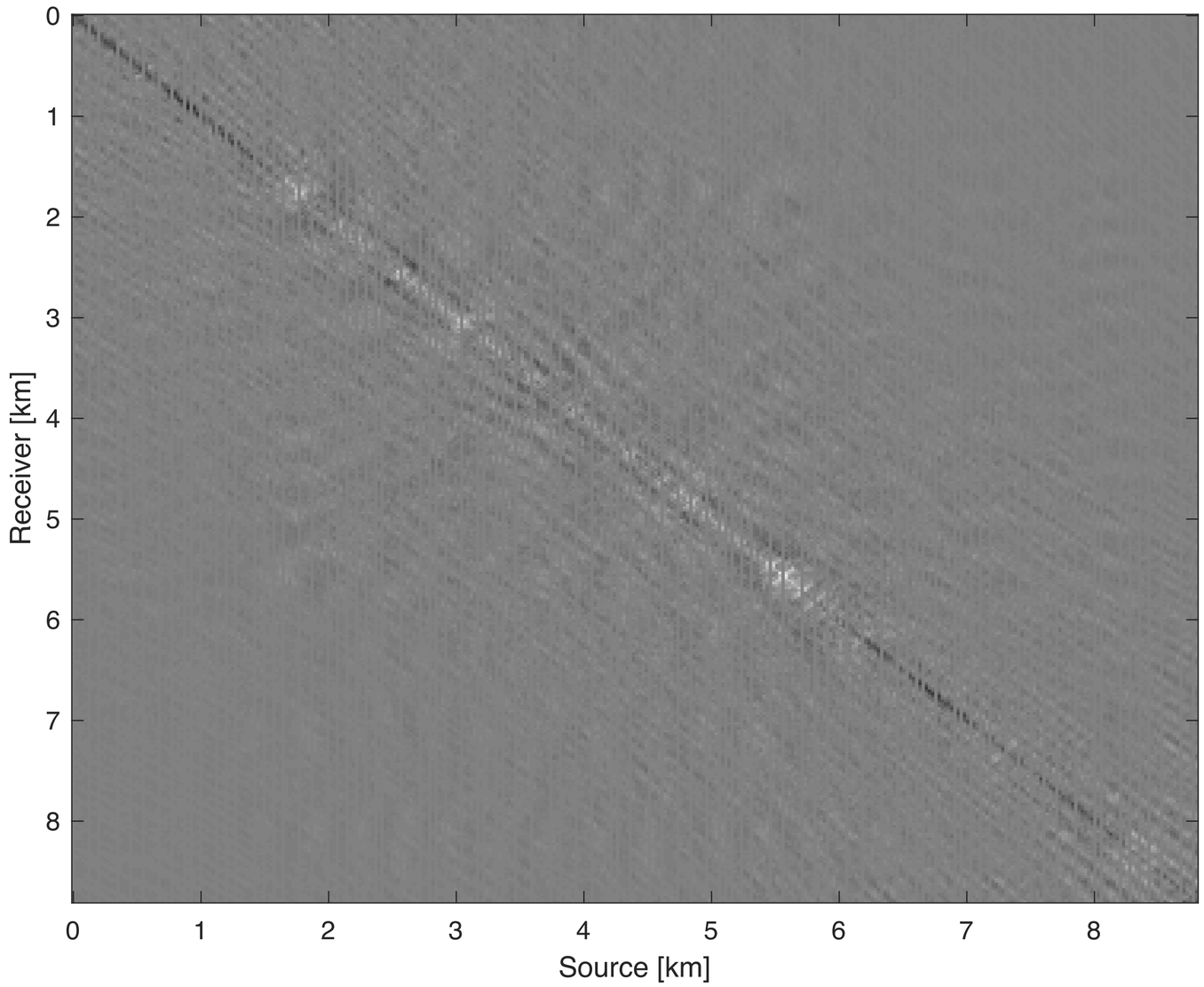
SNR = 15.36 dB
Rank = 30
Weight = 0.75

Recovery w/ weighted recursive



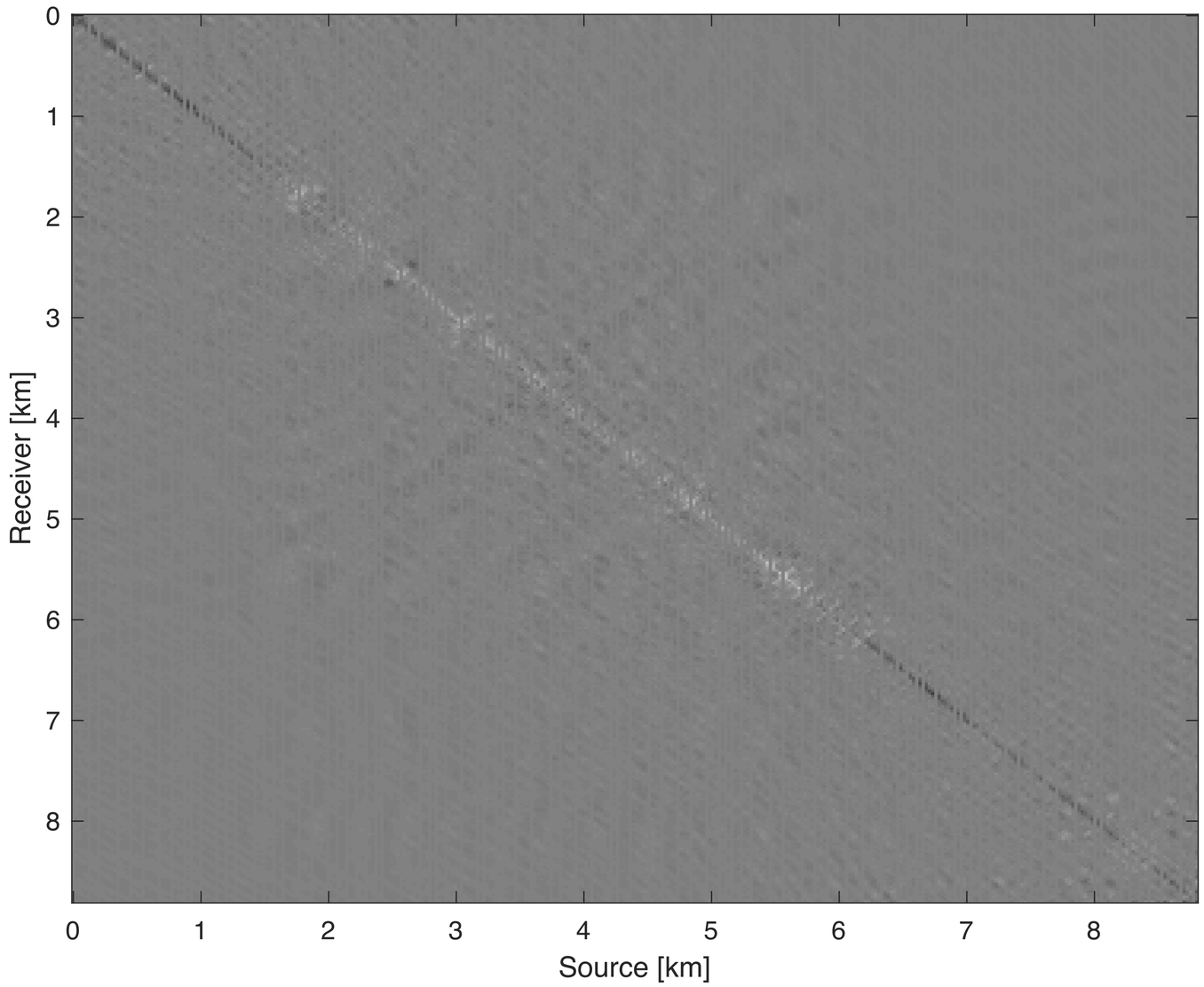
SNR = 18.63 dB
Rank = 30
Weight = 0.75

Difference: True - Recovery w/ conventional



SNR = 11.48 dB
Rank = 30

Difference: True - Recovery w/ weighted pair



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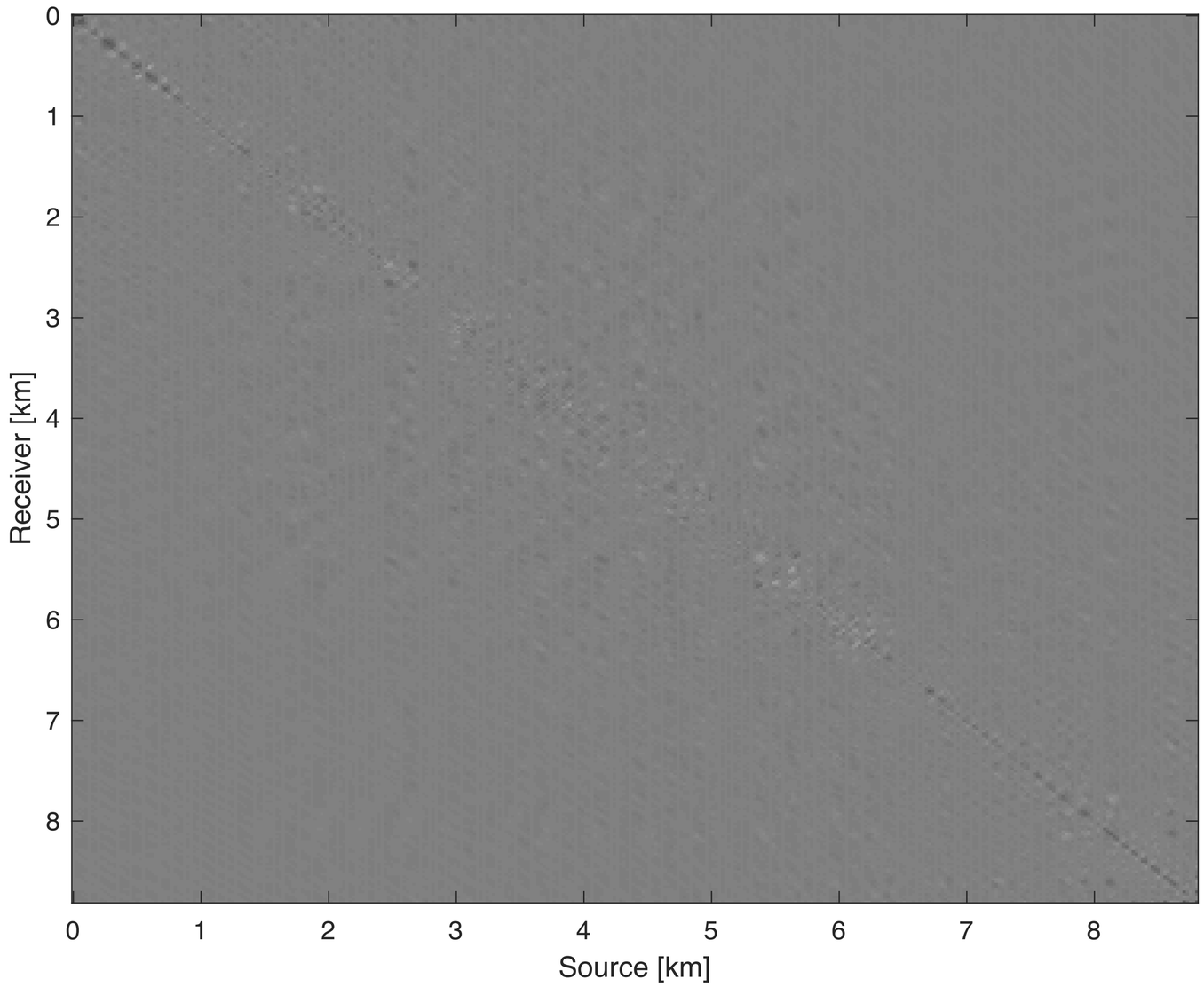

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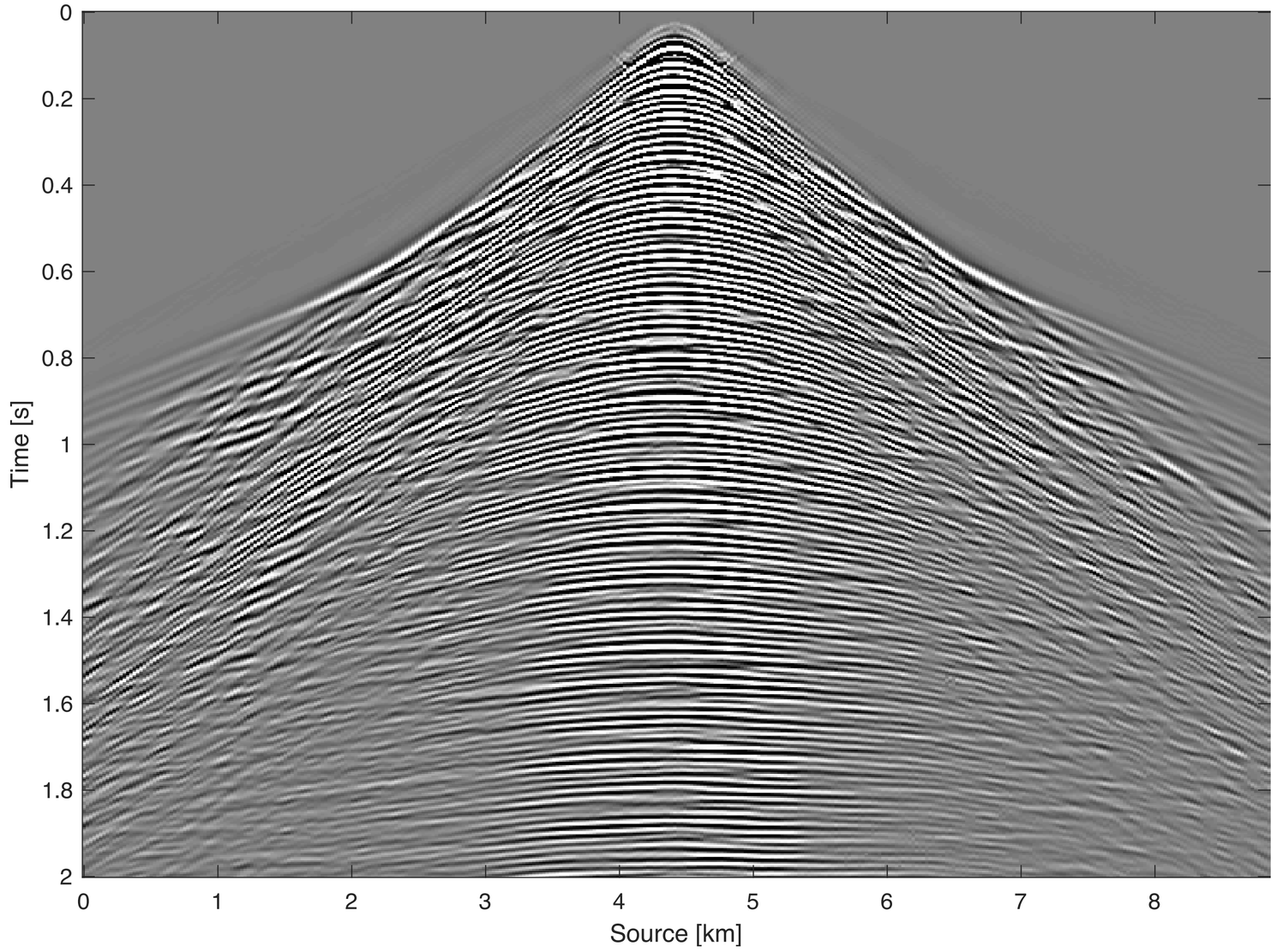
Common Receiver Gather

Fully sampled data



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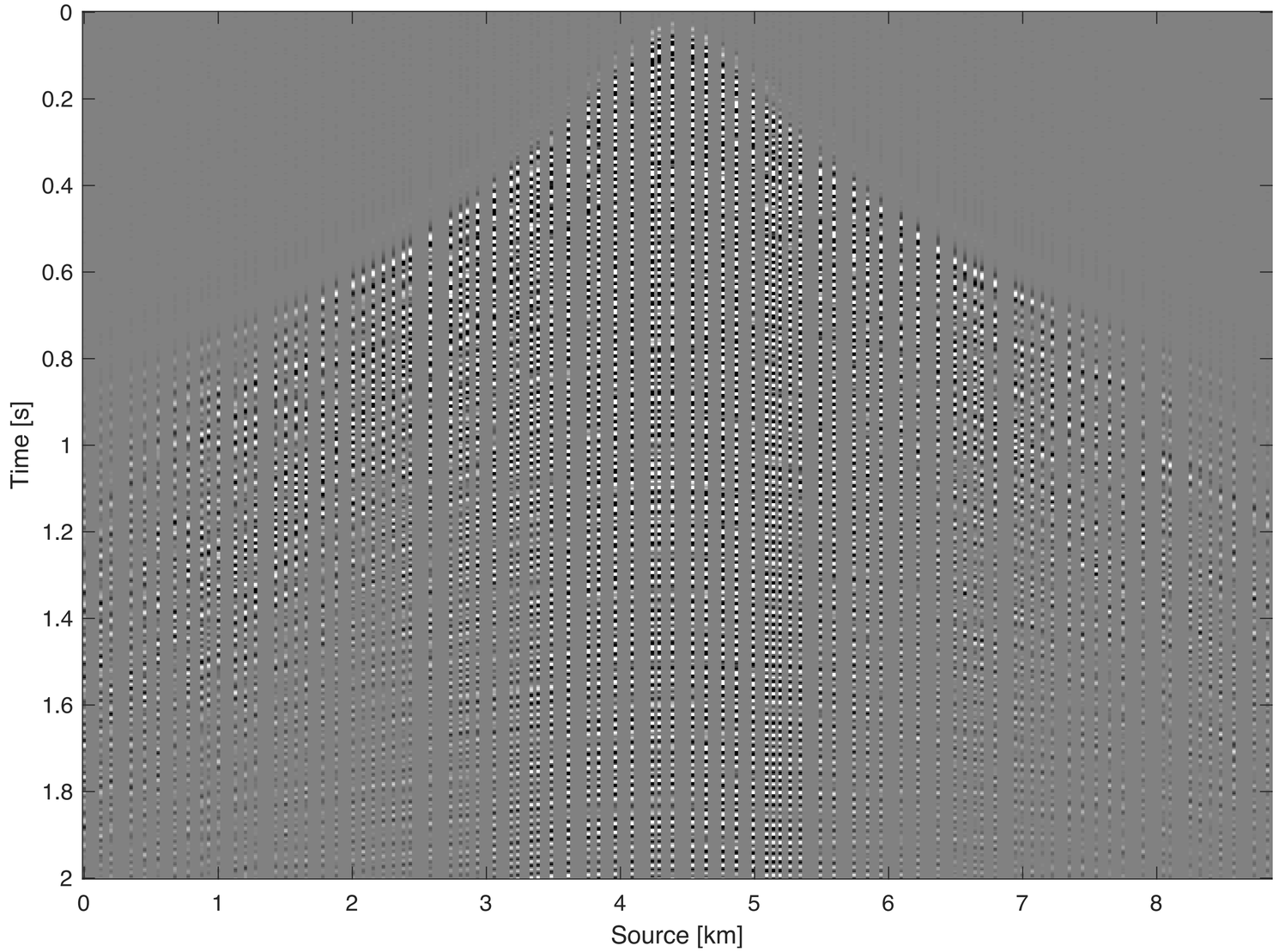
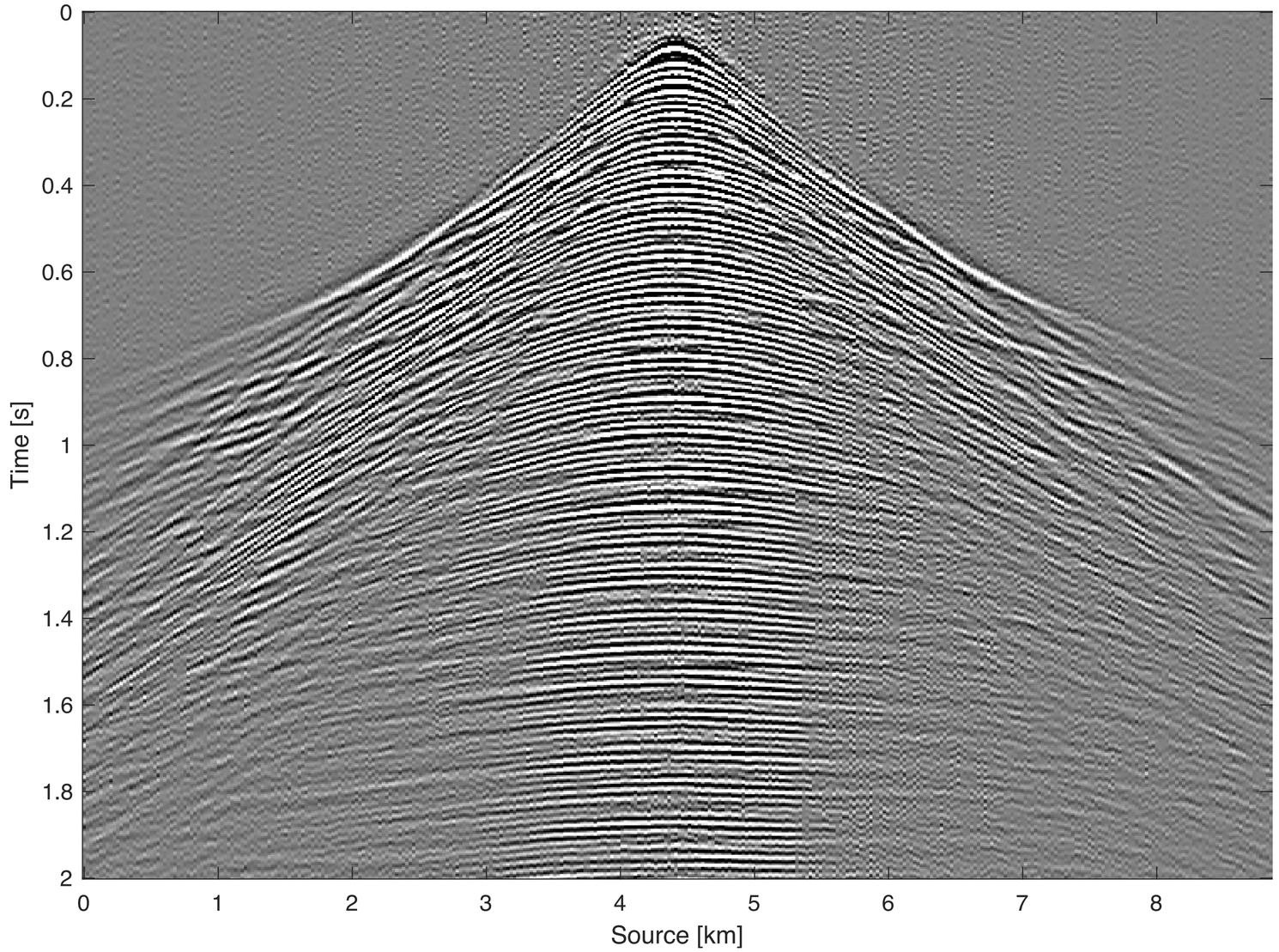
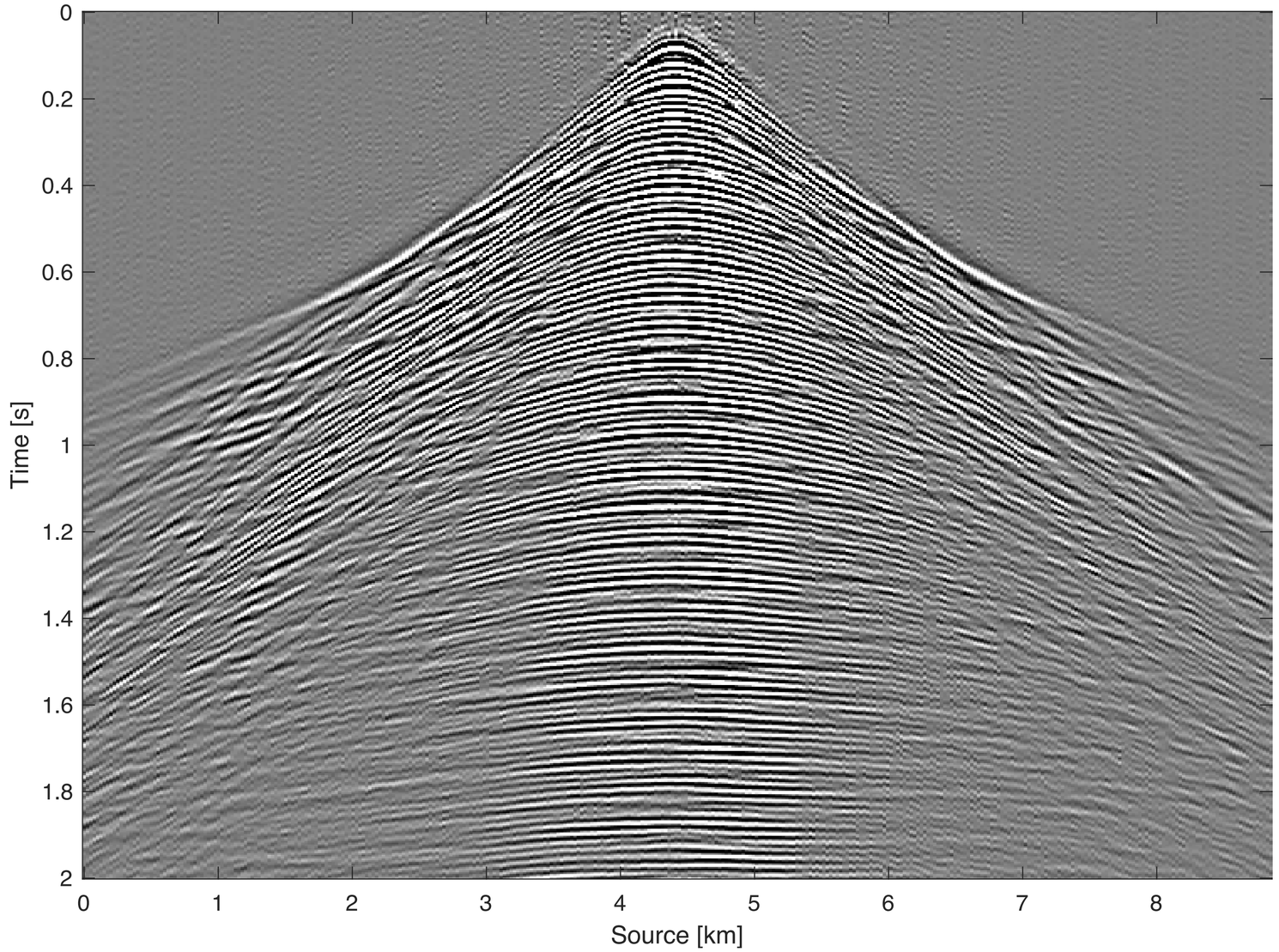


Recovery w/ conventional method



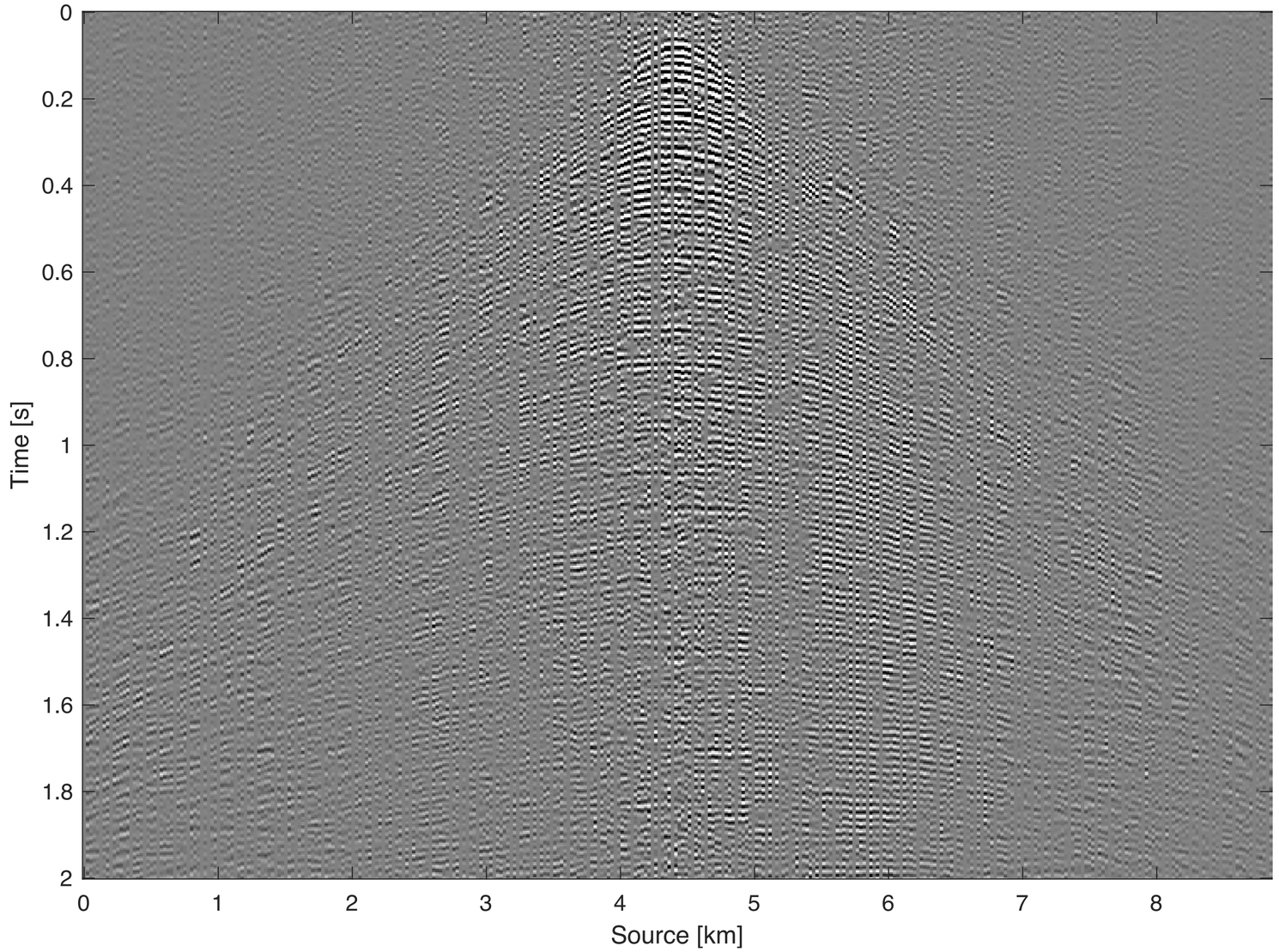
SNR = 6.90 dB
Rank = 30

Recovery w/ weighted recursive



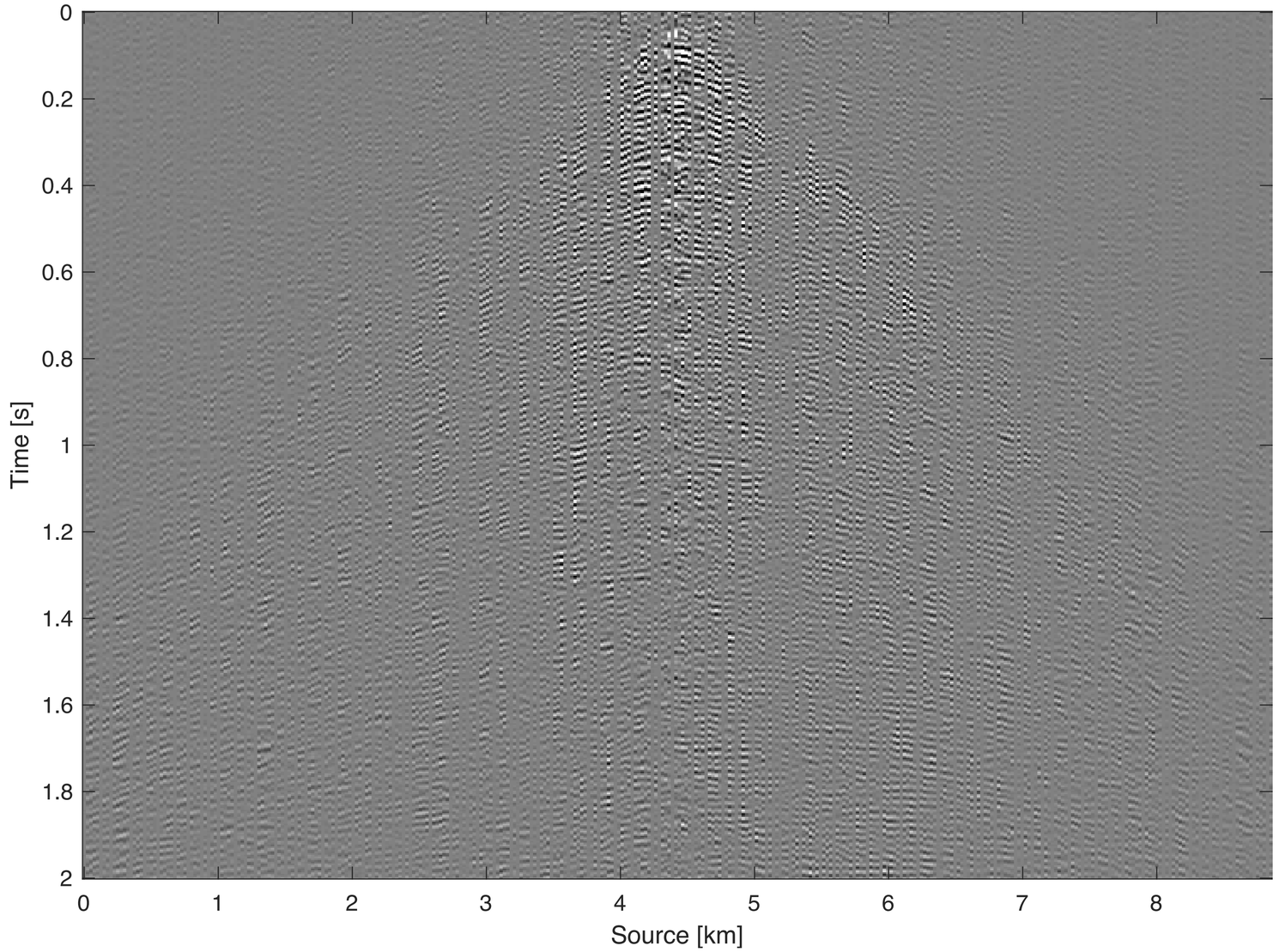
SNR = 11.75 dB
Rank = 30
Weight = 0.75

Difference: True - Recovery w/ conventional



SNR = 6.90 dB
Rank = 30

Difference: True - Recovery w/ weighted recursive



SNR = 11.75 dB
Rank = 30
Weight = 0.75

3D synthetic data example

Velocity model: Overthrust model

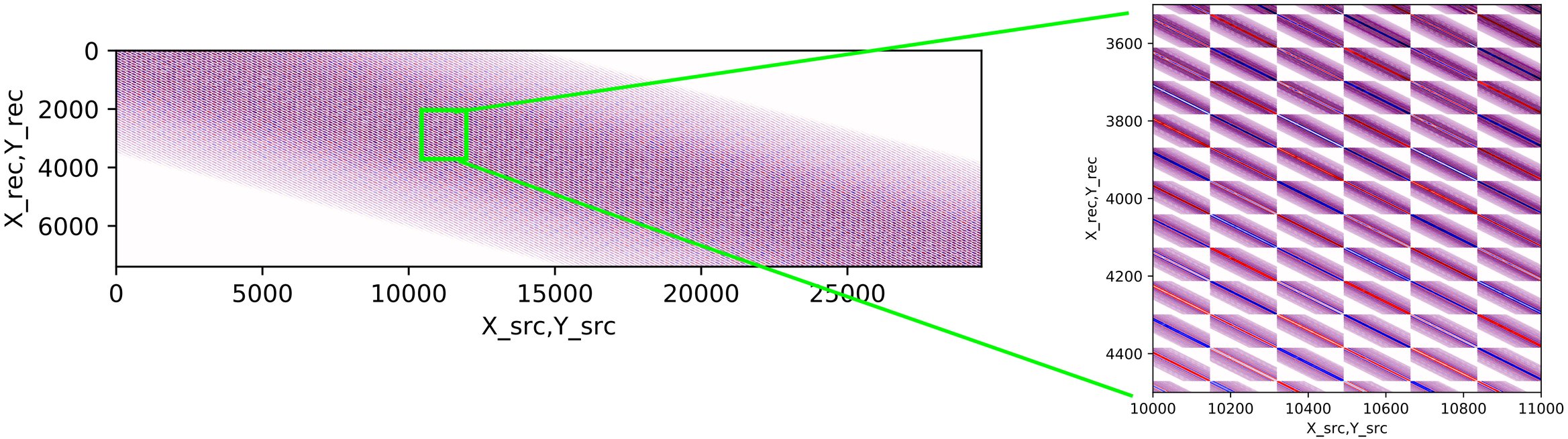
Dimension of each frequency slice: 86 x 86 x 172 x 172 (nrx x nry x nsx x nsy)

Source sampling interval: 100 m (in both directions)

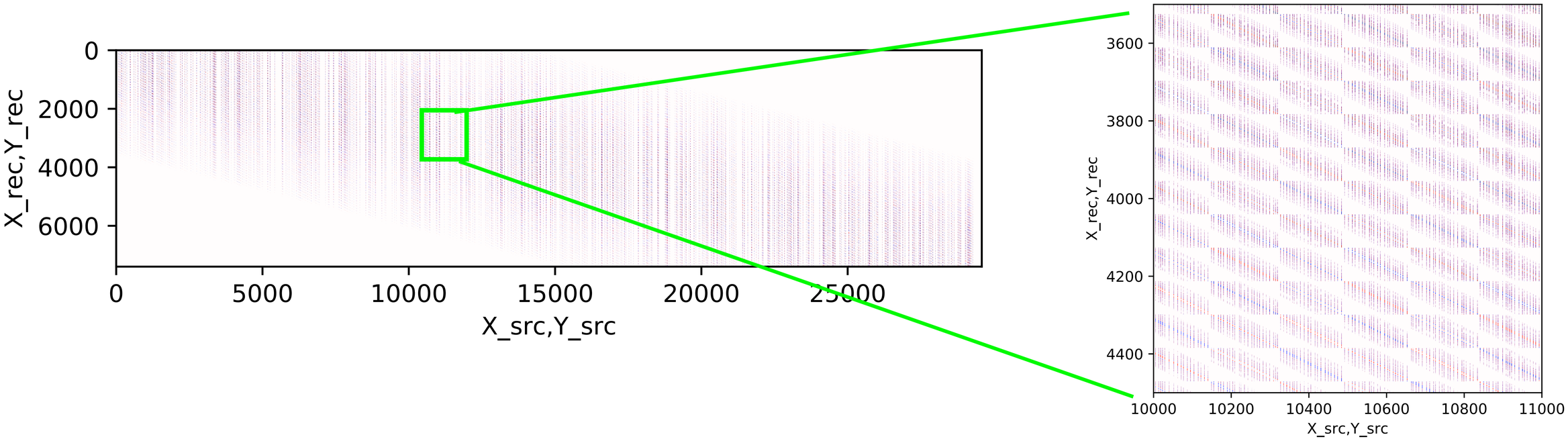
Receiver sampling interval: 50 m (in both directions)

Frequency slice: 10 Hz

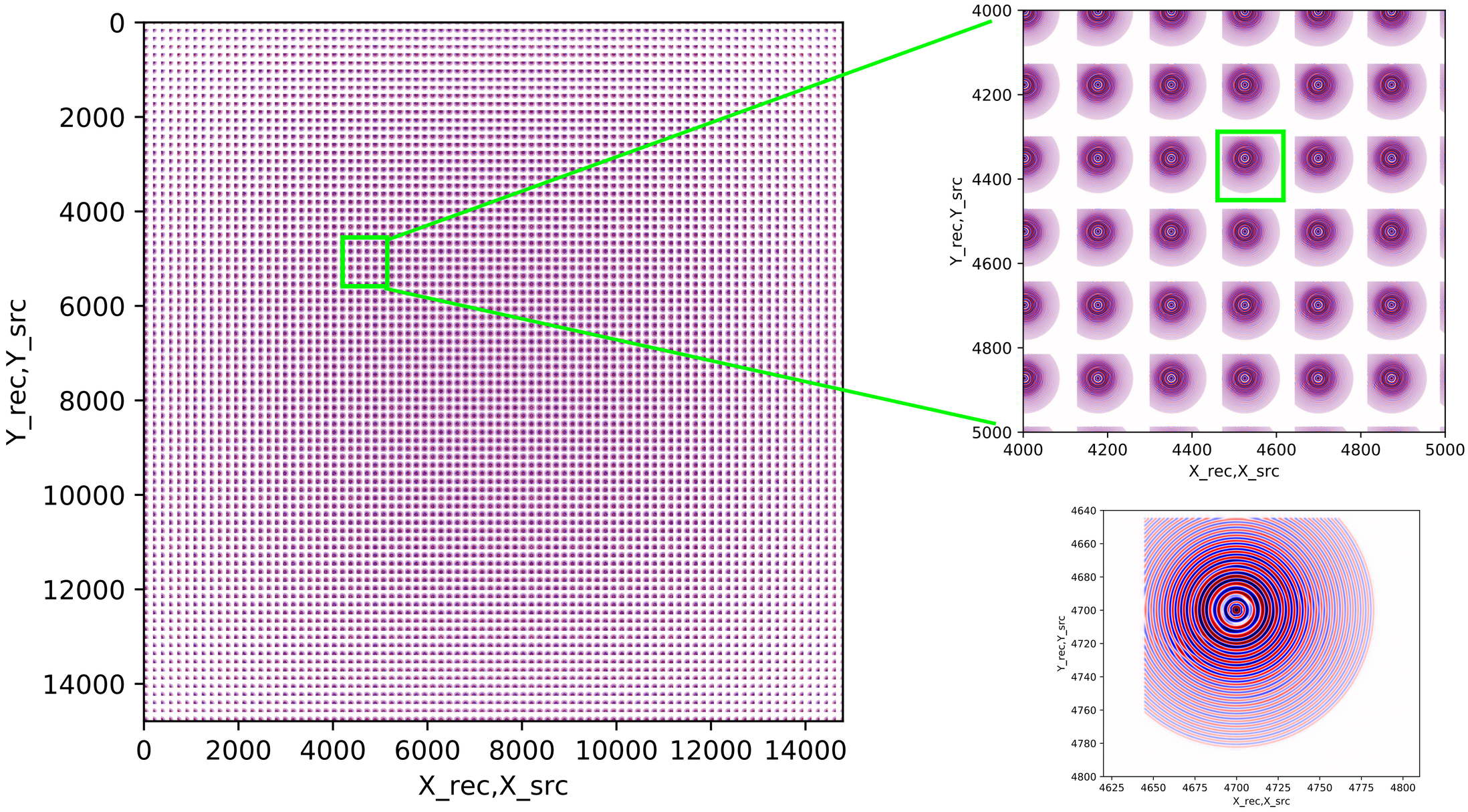
Fully sampled data: Sx-Sy matricization



Sub-sampled data: Sx-Sy matricization
90% missing sources

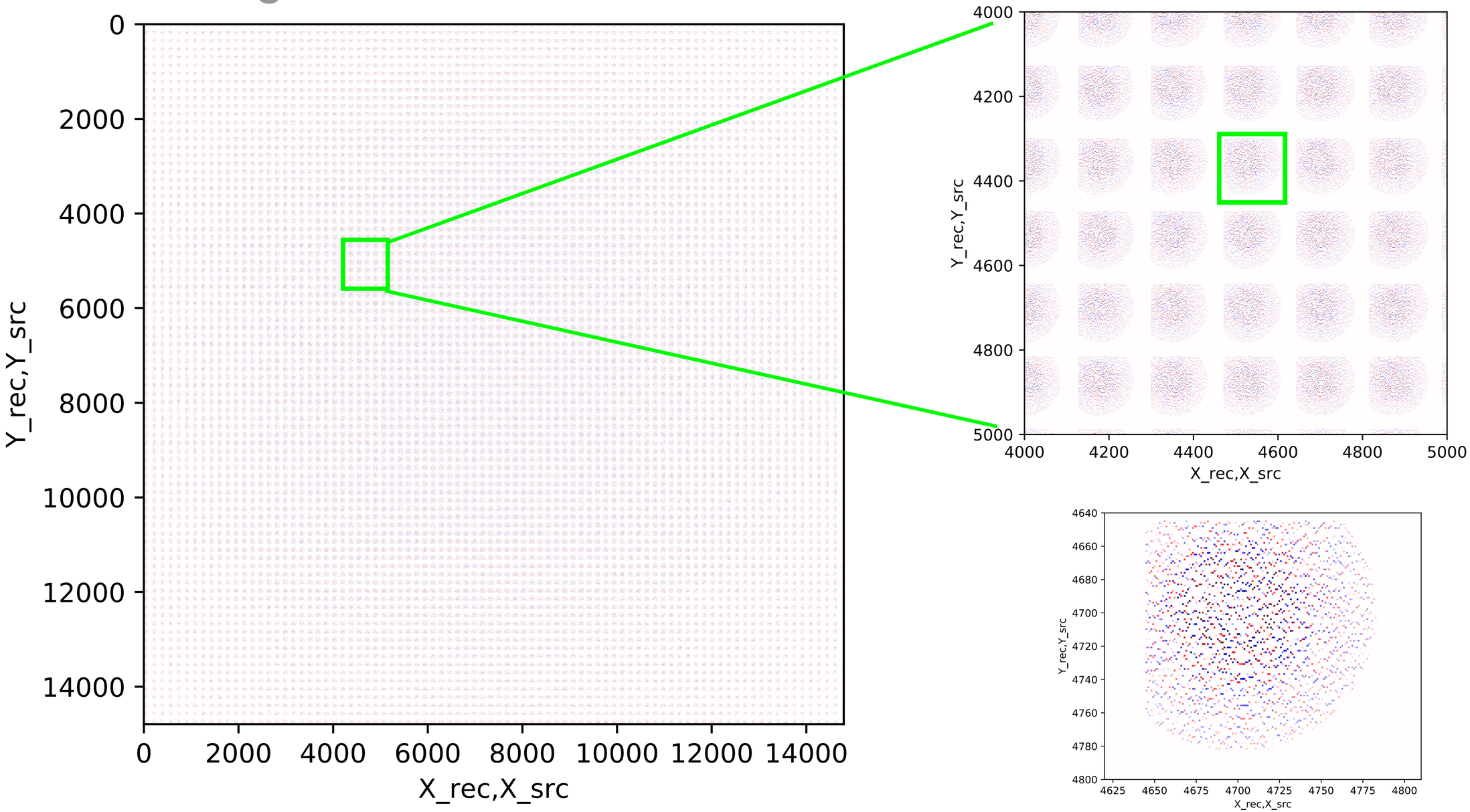


Fully sampled data: Sx-Rx matricization



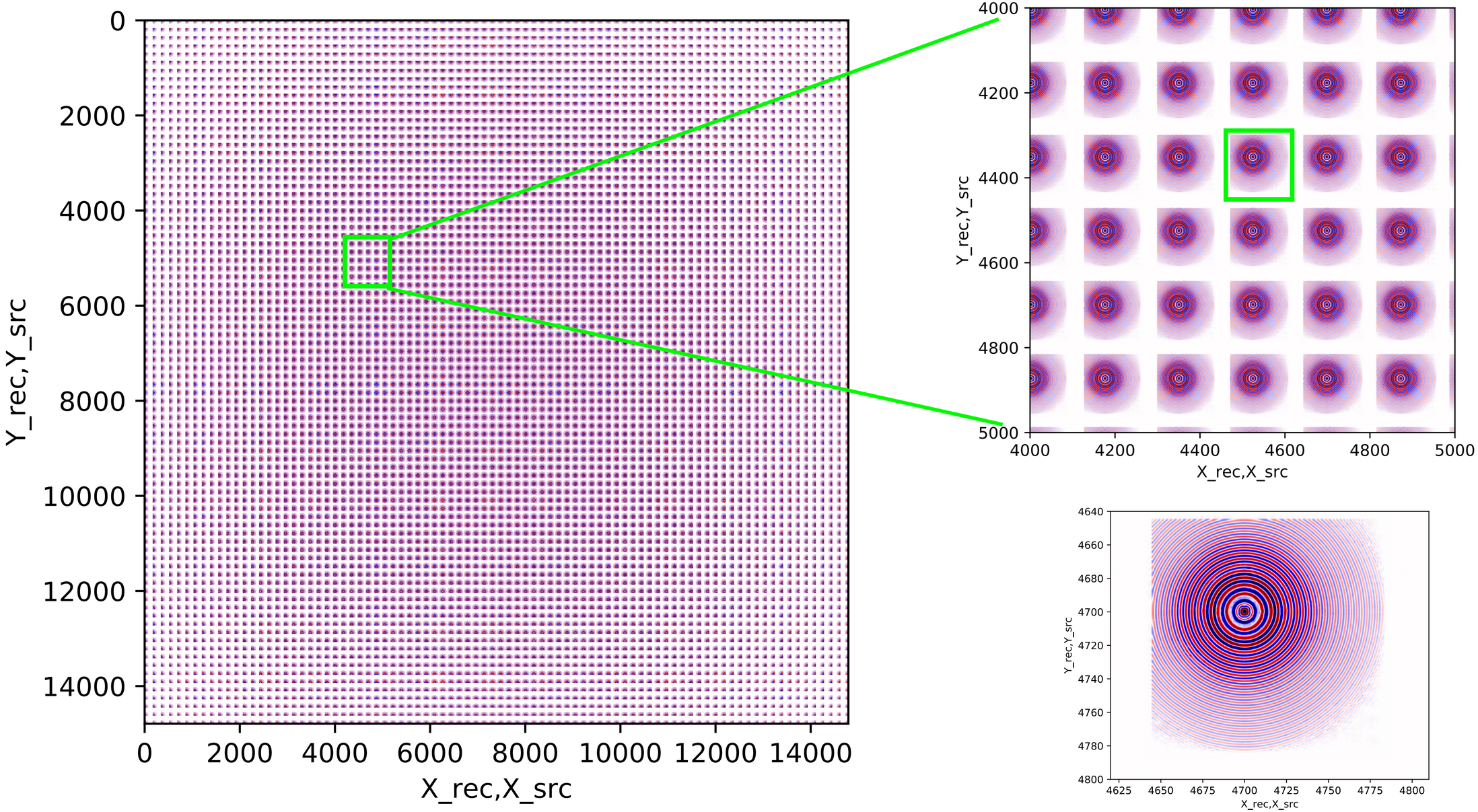
Sub-sampled data: Sx-Rx matricization

90% missing sources



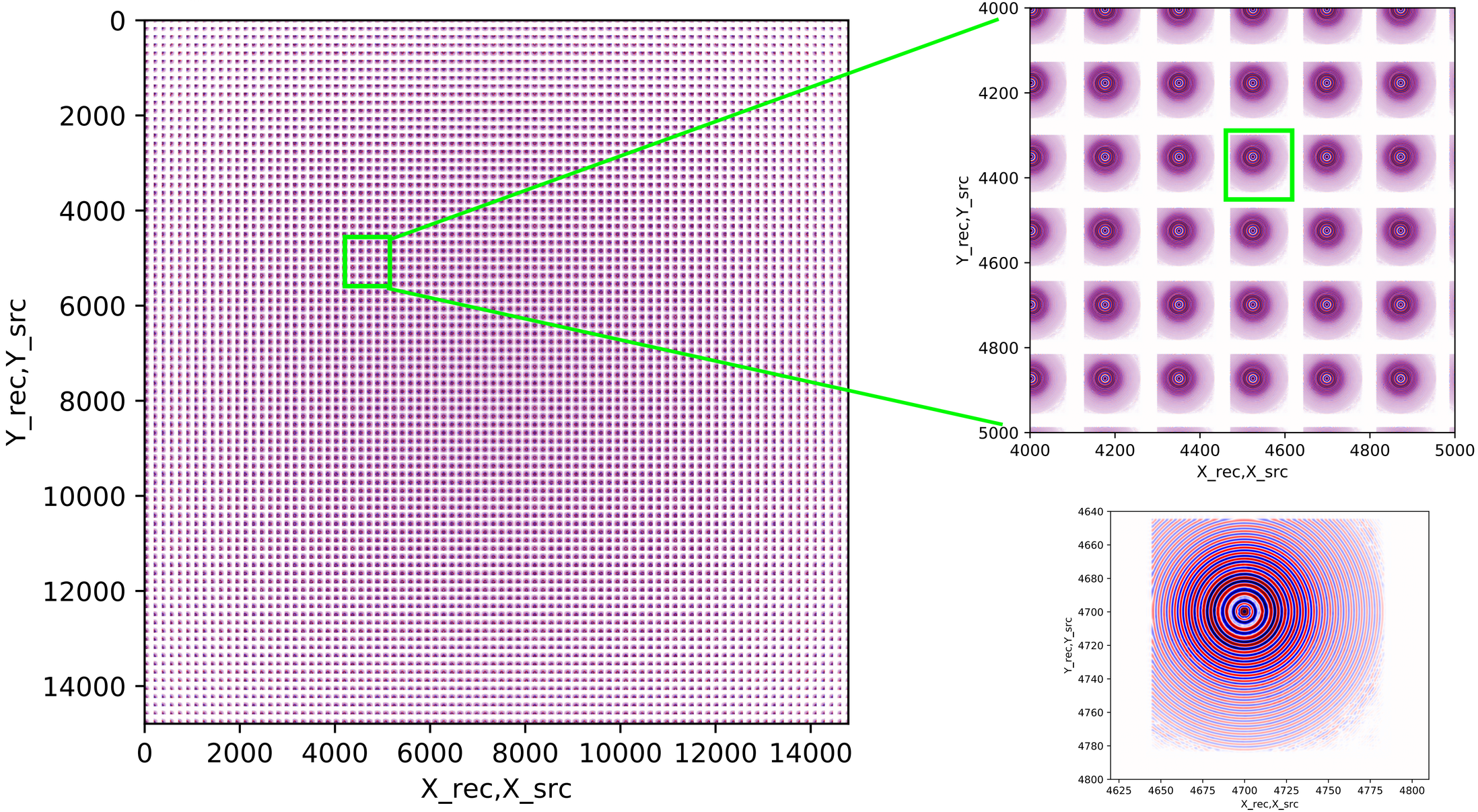
Interpolated data: Sx-Rx matricization

w/o weights SNR: 17.41dB



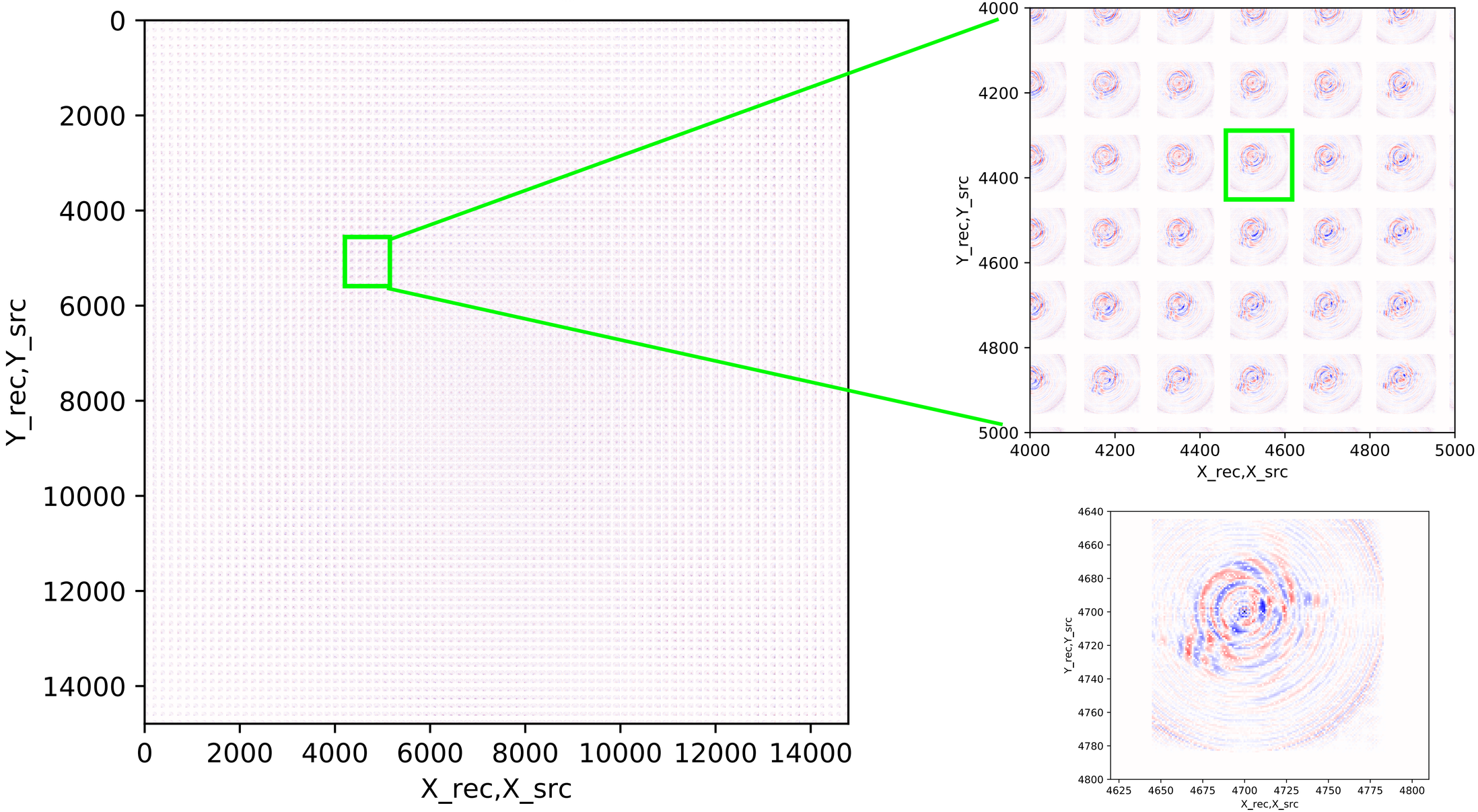
Interpolated data: Sx-Rx matricization

w/ weights SNR: 19.20dB



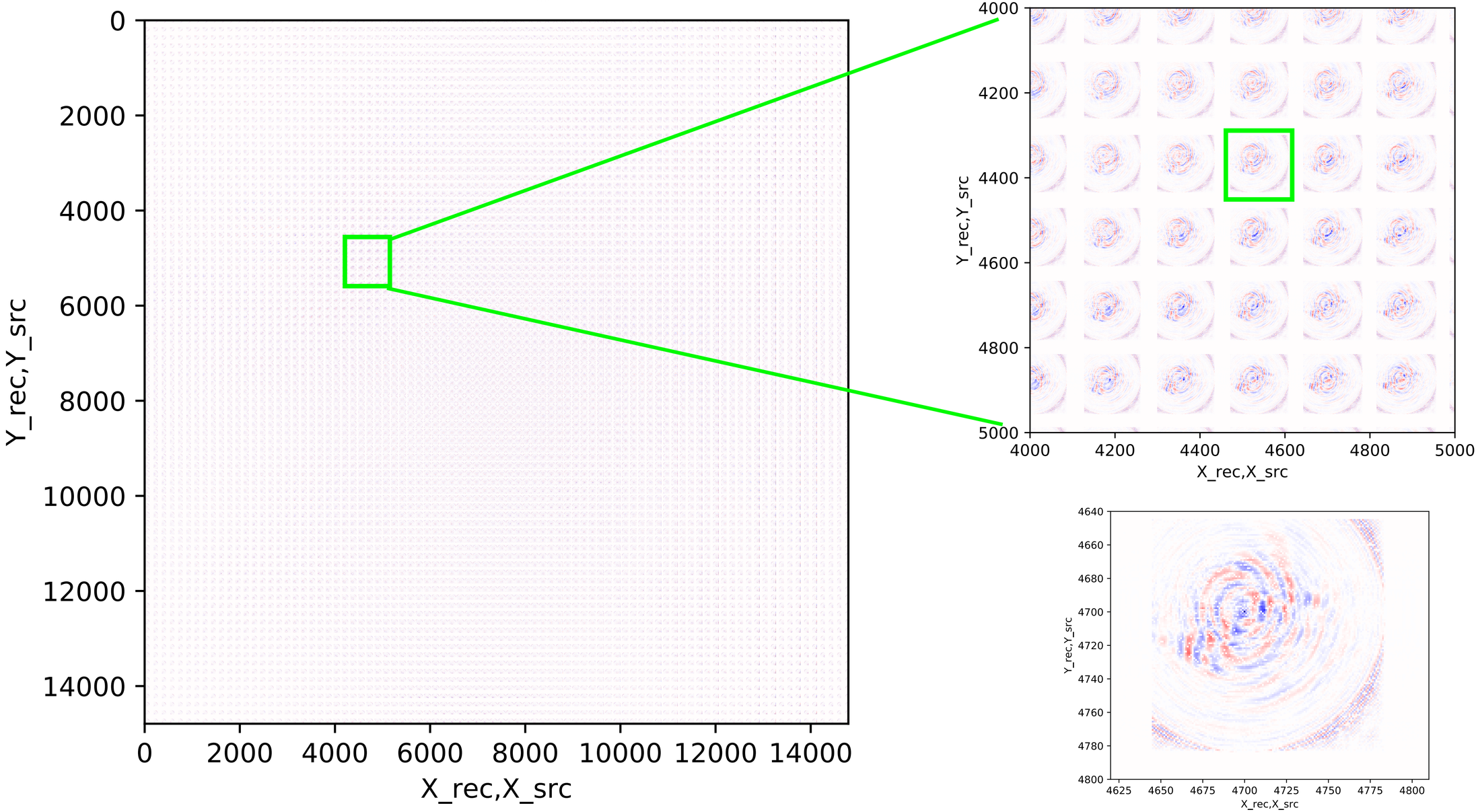
Difference: Sx-Rx matricization

True - interpolated w/o weights SNR: 17.41dB



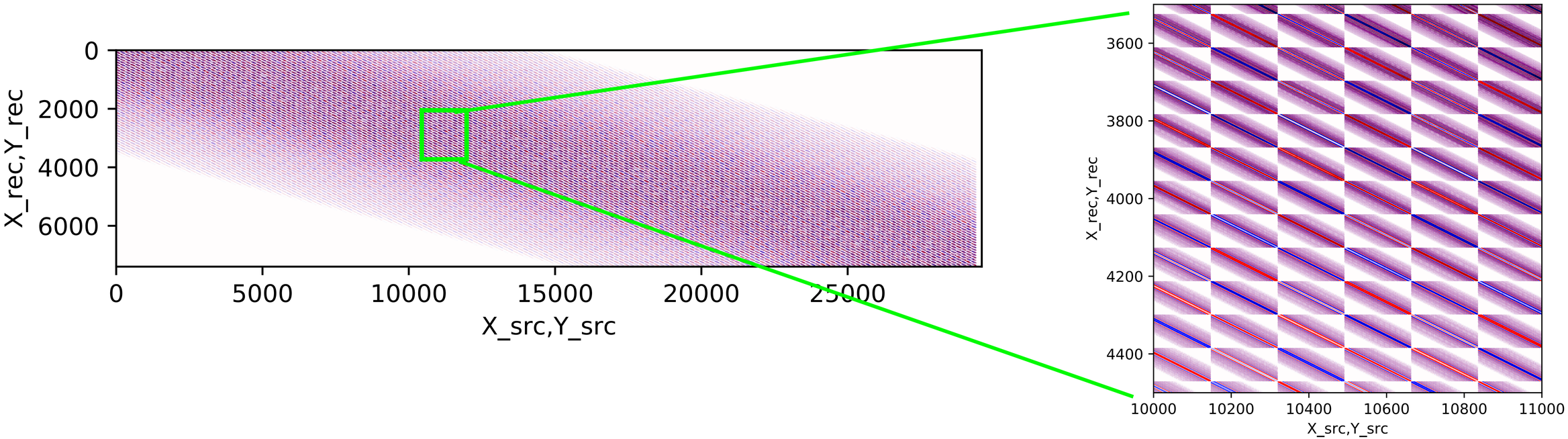
Difference: Sx-Rx matricization

True - interpolated w/ weights SNR: 19.20dB

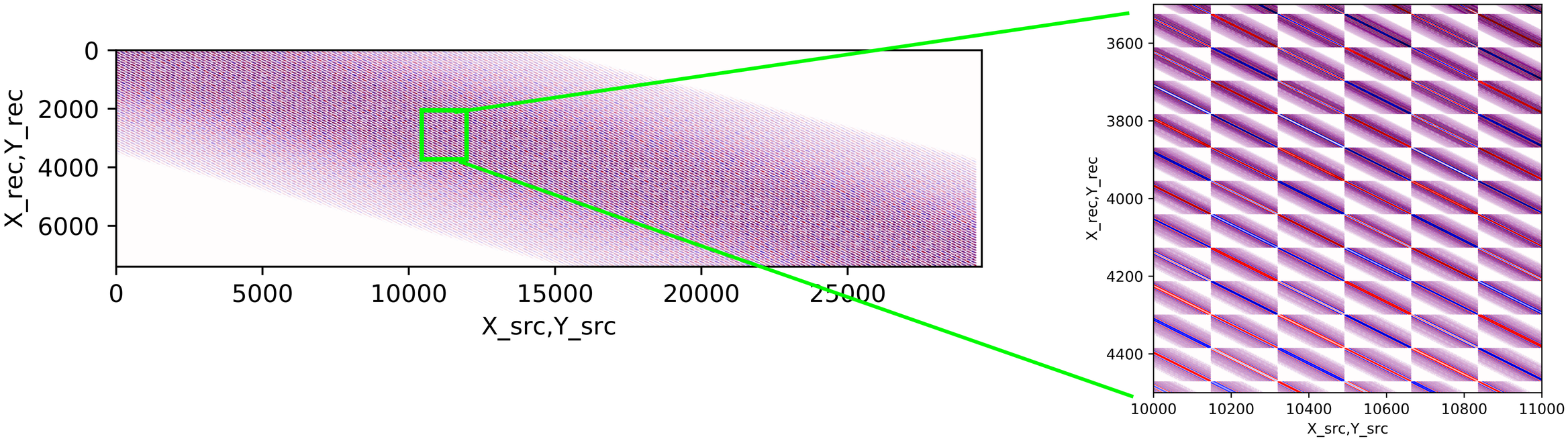


Interpolated data: Sx-Sy matricization

w/o weights SNR: 17.41dB

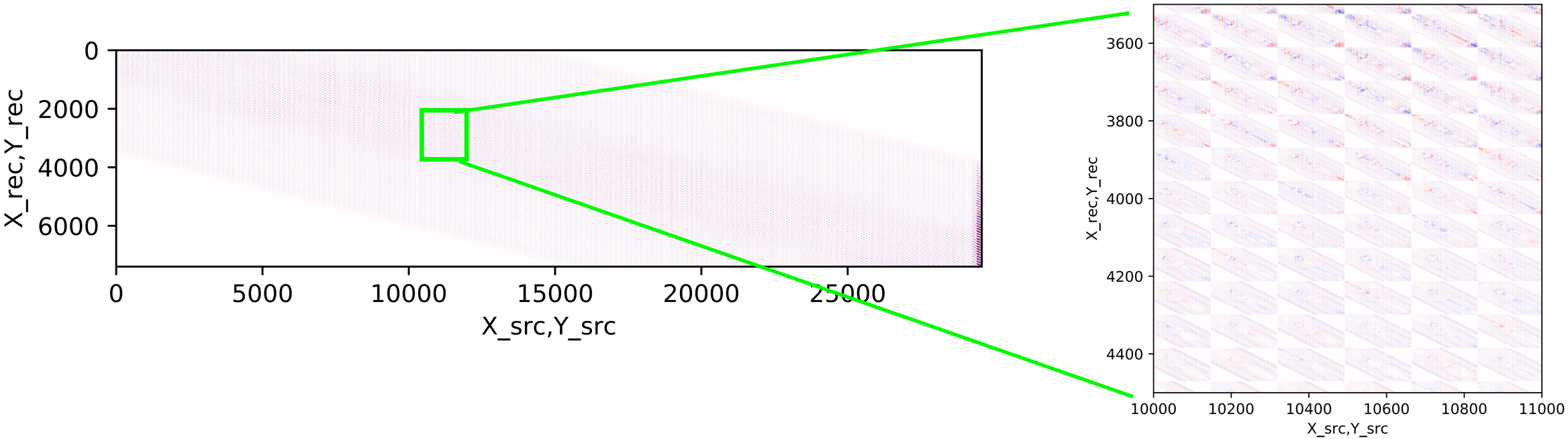


Interpolated data: Sx-Sy matricization
w/ weights SNR: 19.20dB



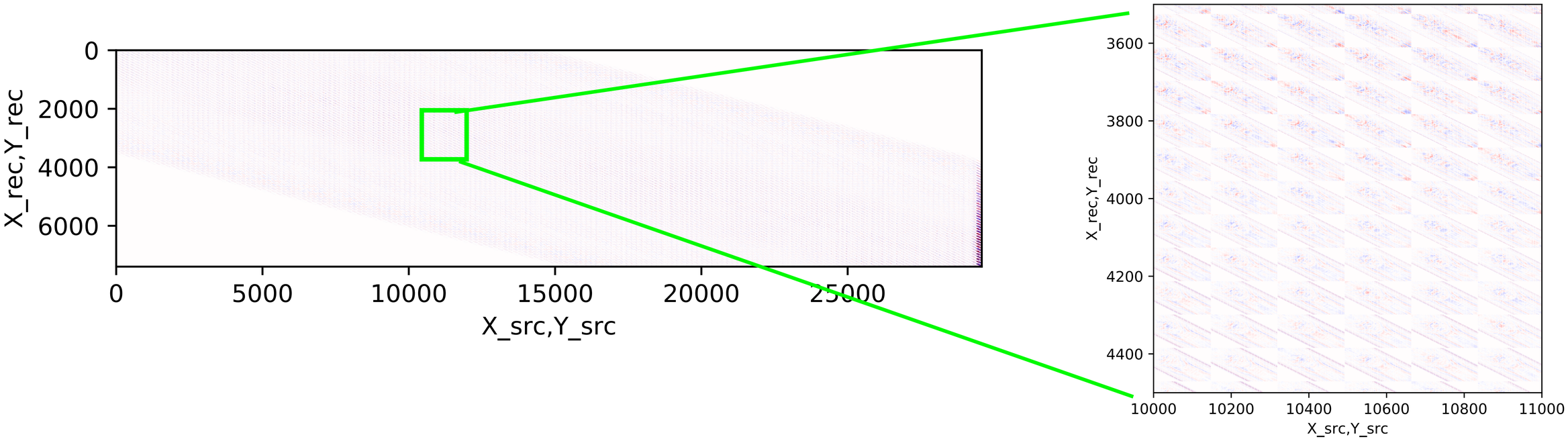
Difference: Sx-Sy matricization

True - interpolated w/o weights SNR: 17.41dB



Difference: Sx-Sy matricization

True - interpolated w/ weights SNR: 19.20dB



Conclusion & Future work

Recursive weighted strategy

- improves SNR at higher frequencies

High frequency 3D data interpolation

- needs parallel weighted framework

Simultaneous source separation

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