# Removing density effects in LS-RTM with a low-rank matched filter

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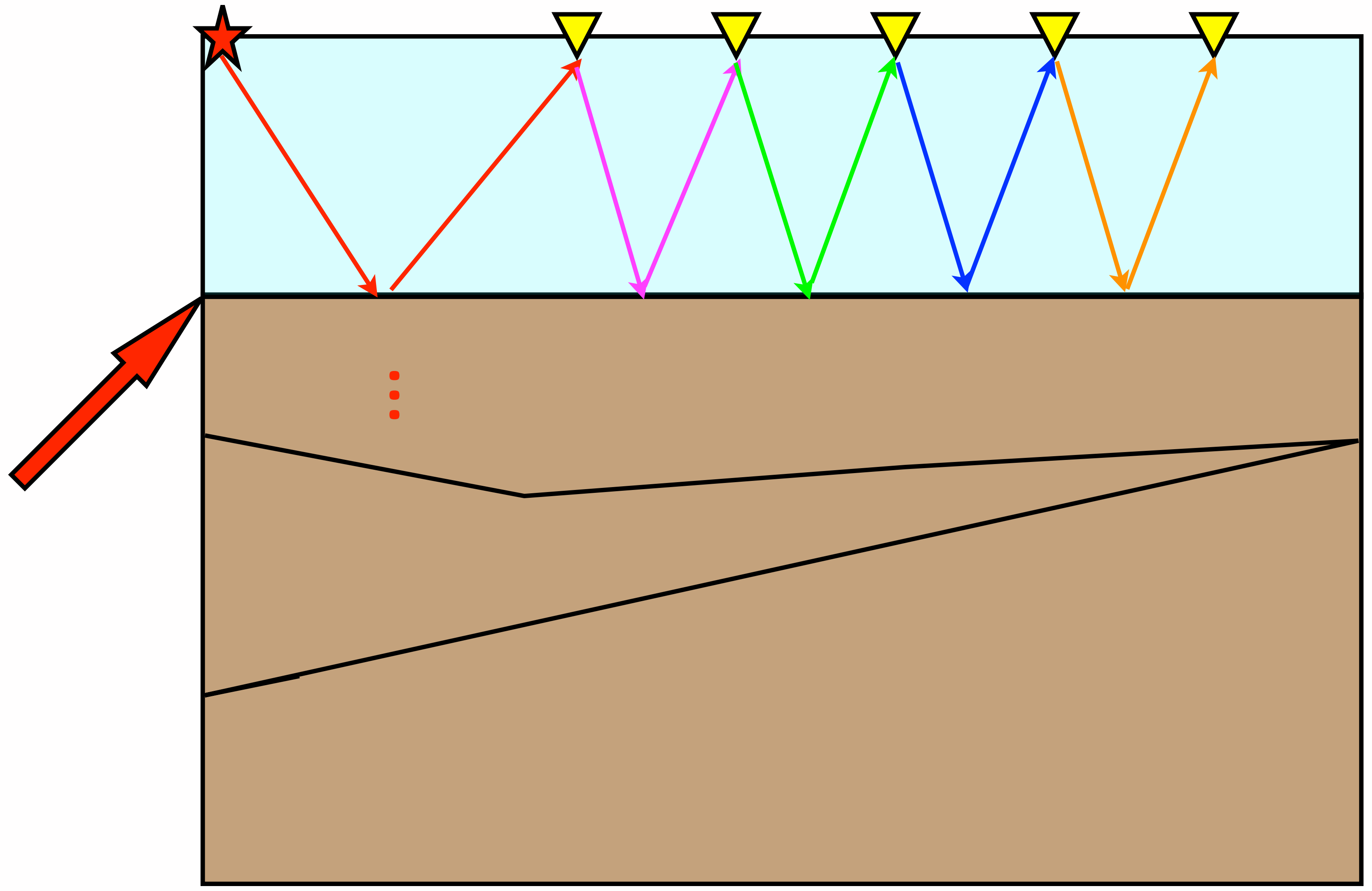
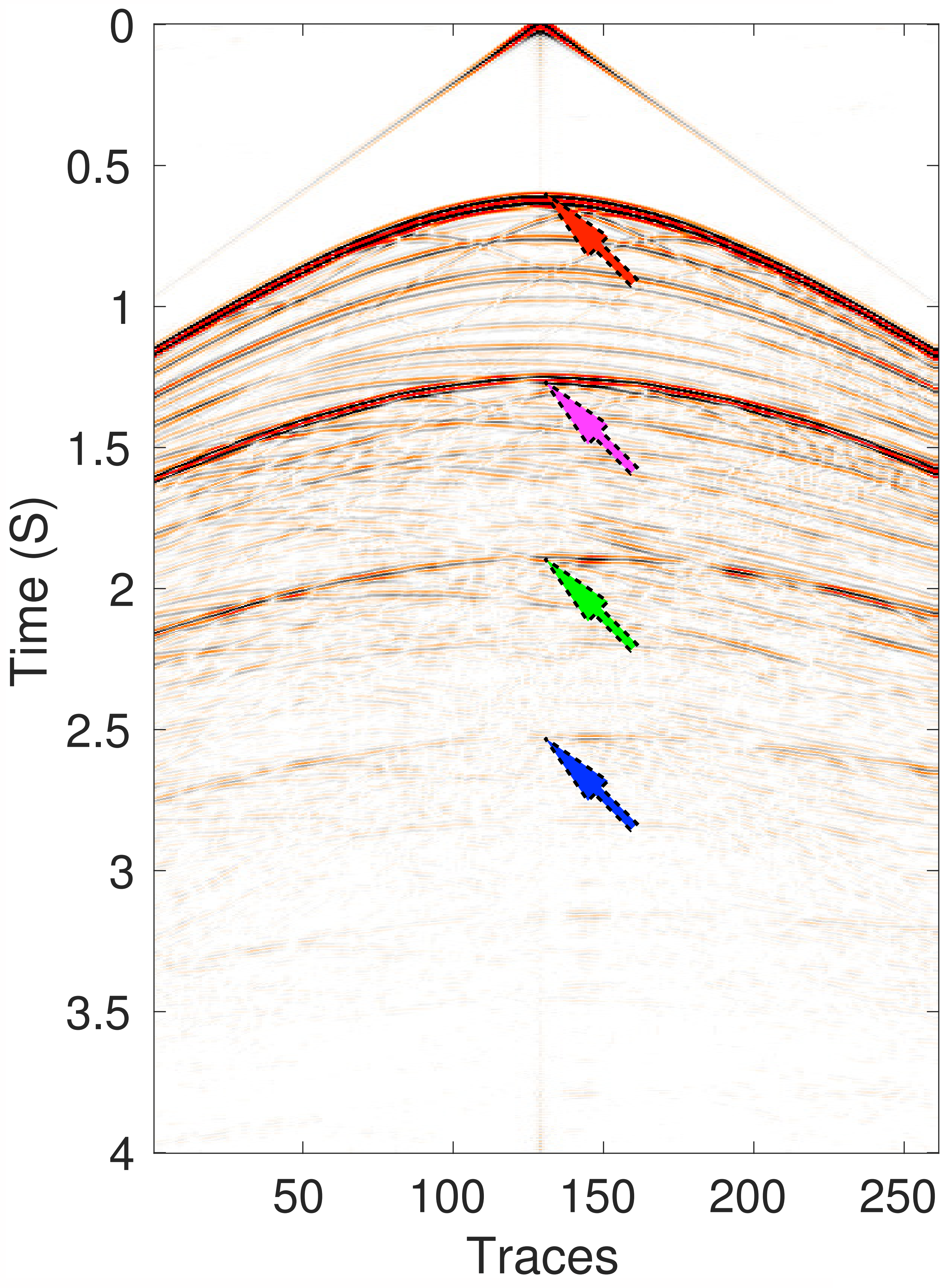
# Situation
# Multiples in marine data

- Strong density variations at ocean bottom and free surface play important roles in multiples generation
- The inverted velocity perturbation (w/ areal source injection) will have strong dummy components at ocean bottom





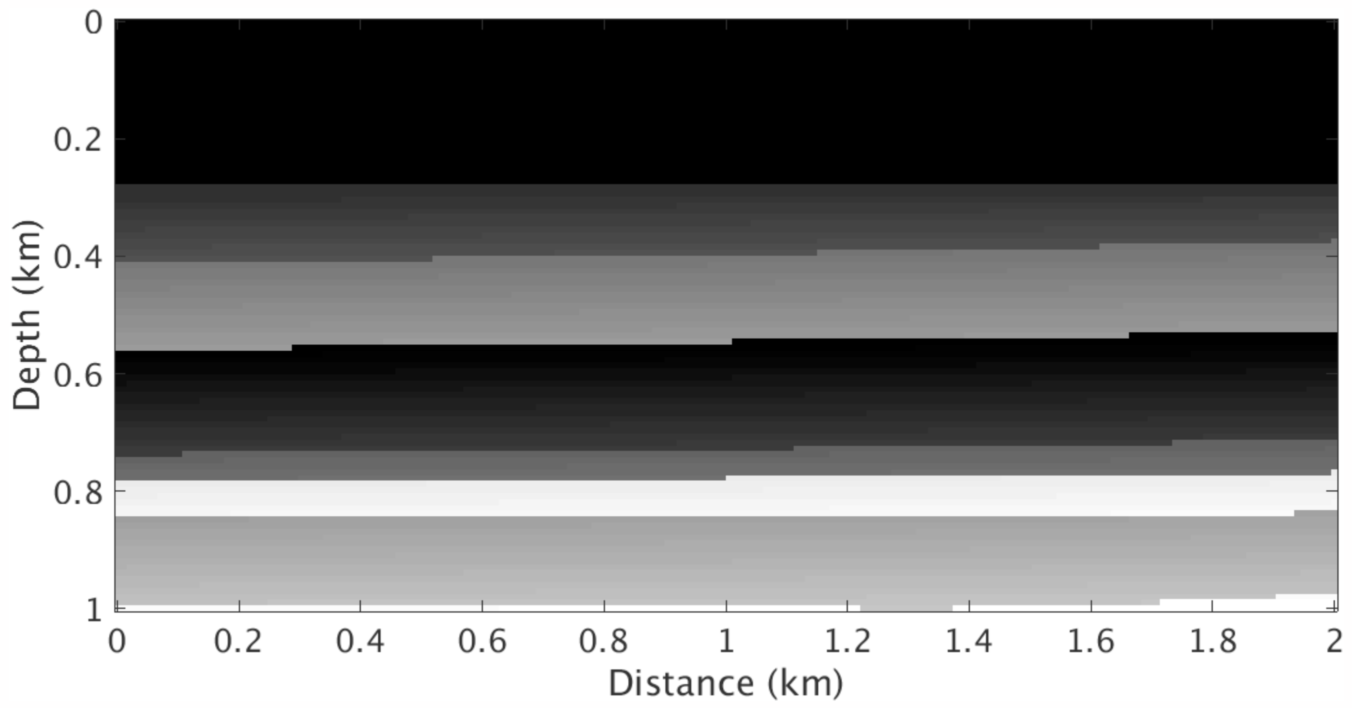
# Situation

Now simplify the problem into imaging only primaries, generated w/ absorbing surface and ocean model that contains strong density perturbation at the ocean bottom

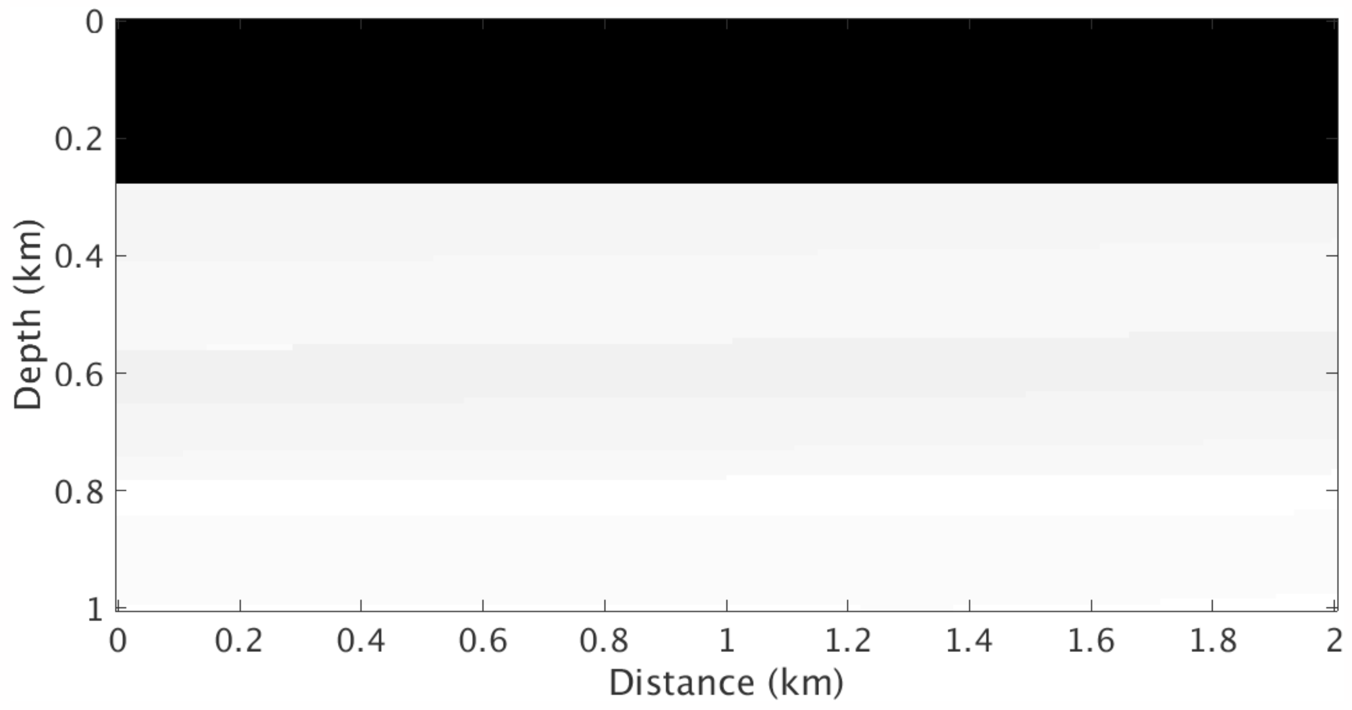
## Situation

- In subsurface, density variations often co-occur w/ velocity variations
- Density & velocity have different radiation patterns for certain scattering angles
- LS-RTM will have errors for Born modeling w.r.t velocity variations only
- Simultaneous inversion for velocity & density can lead to crosstalk

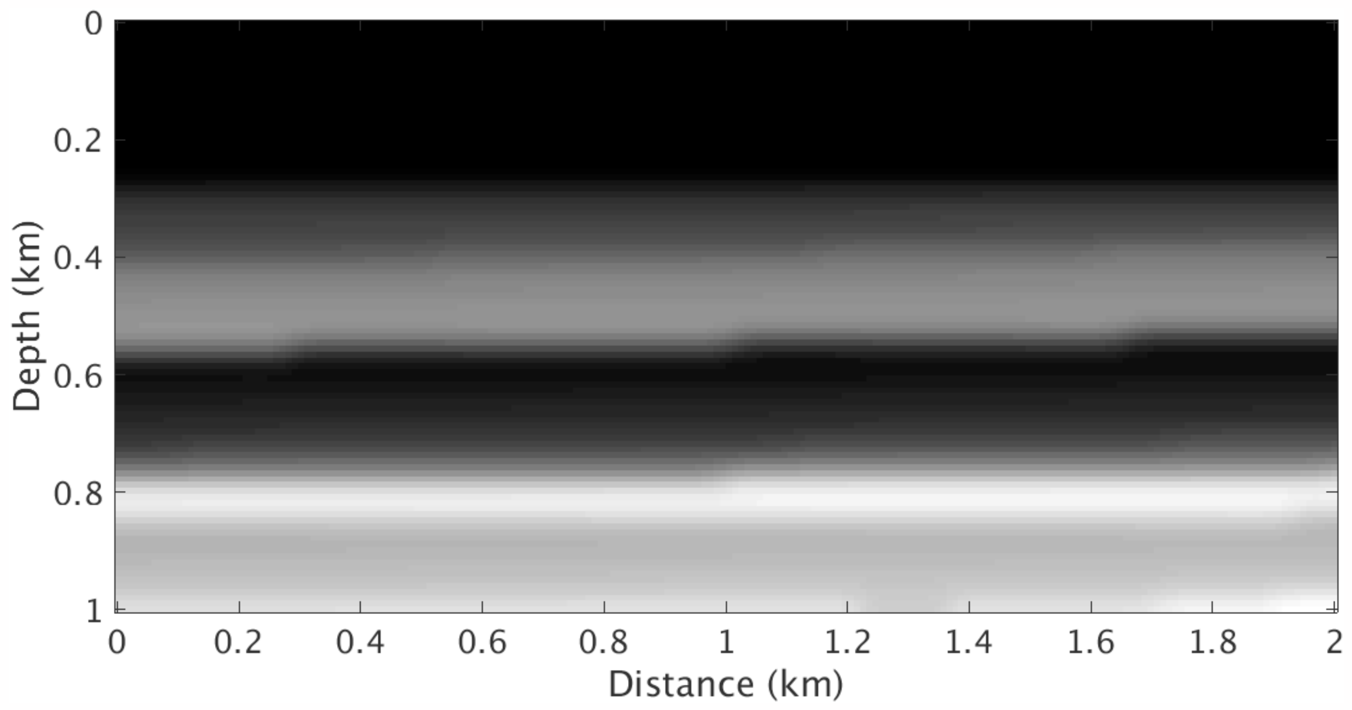
# Ocean bottom example



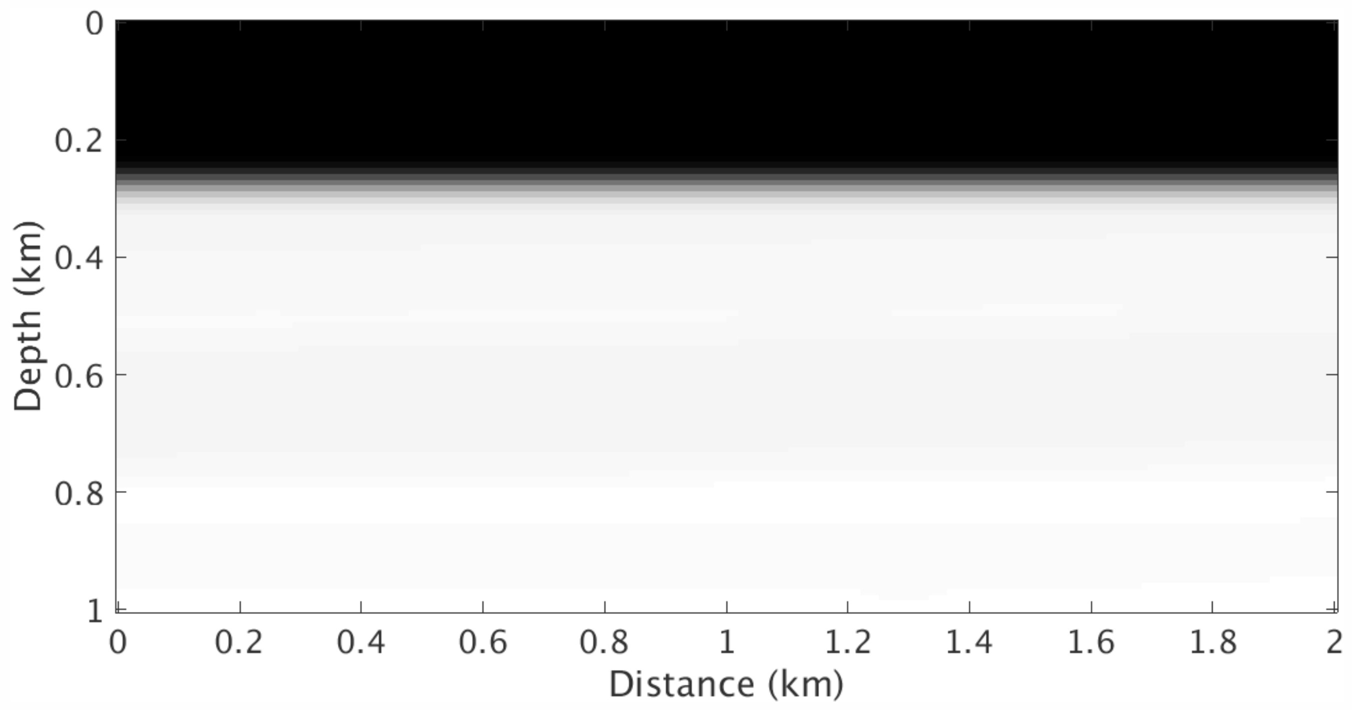

velocity model




density model

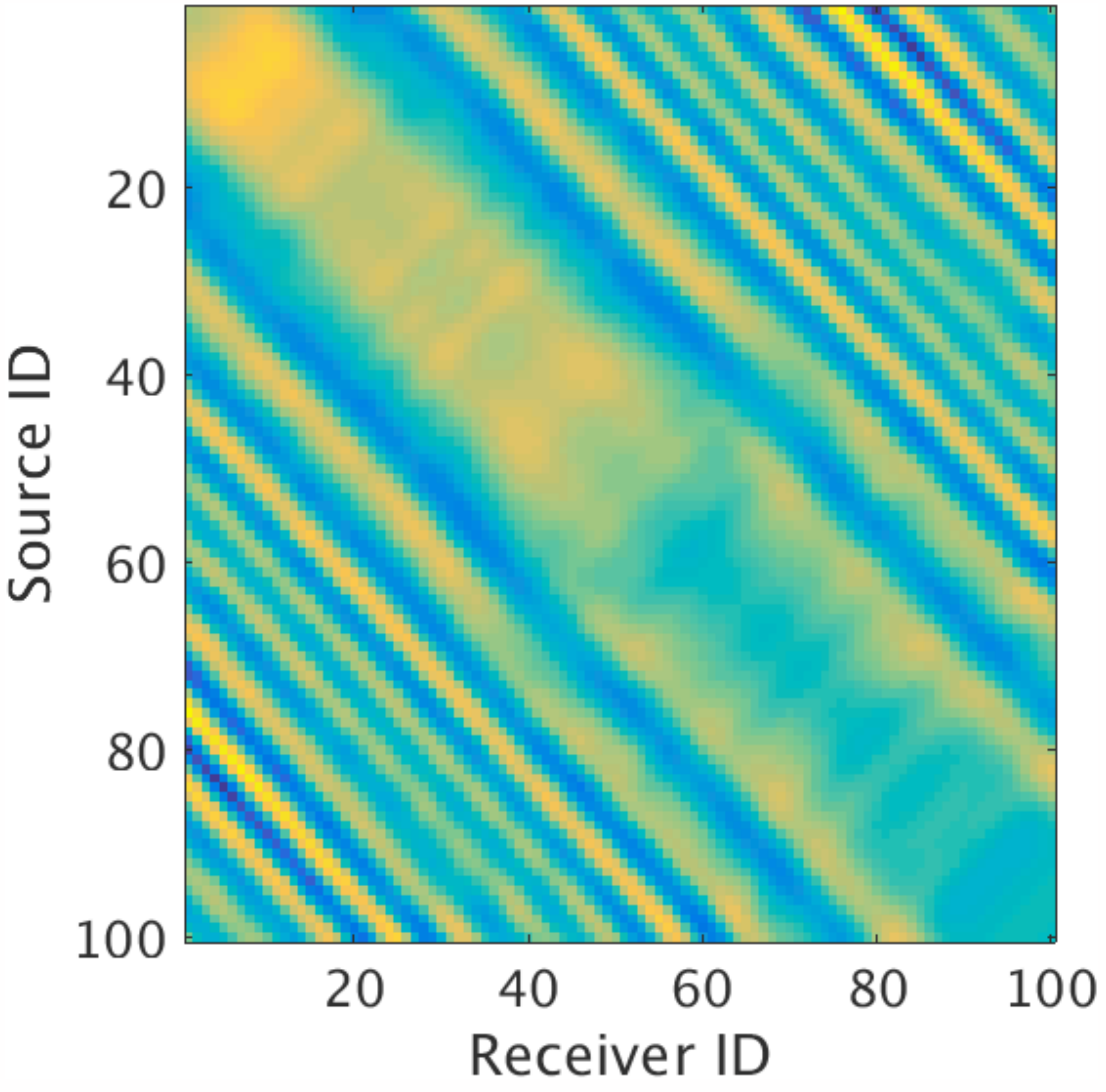



back ground velocity model

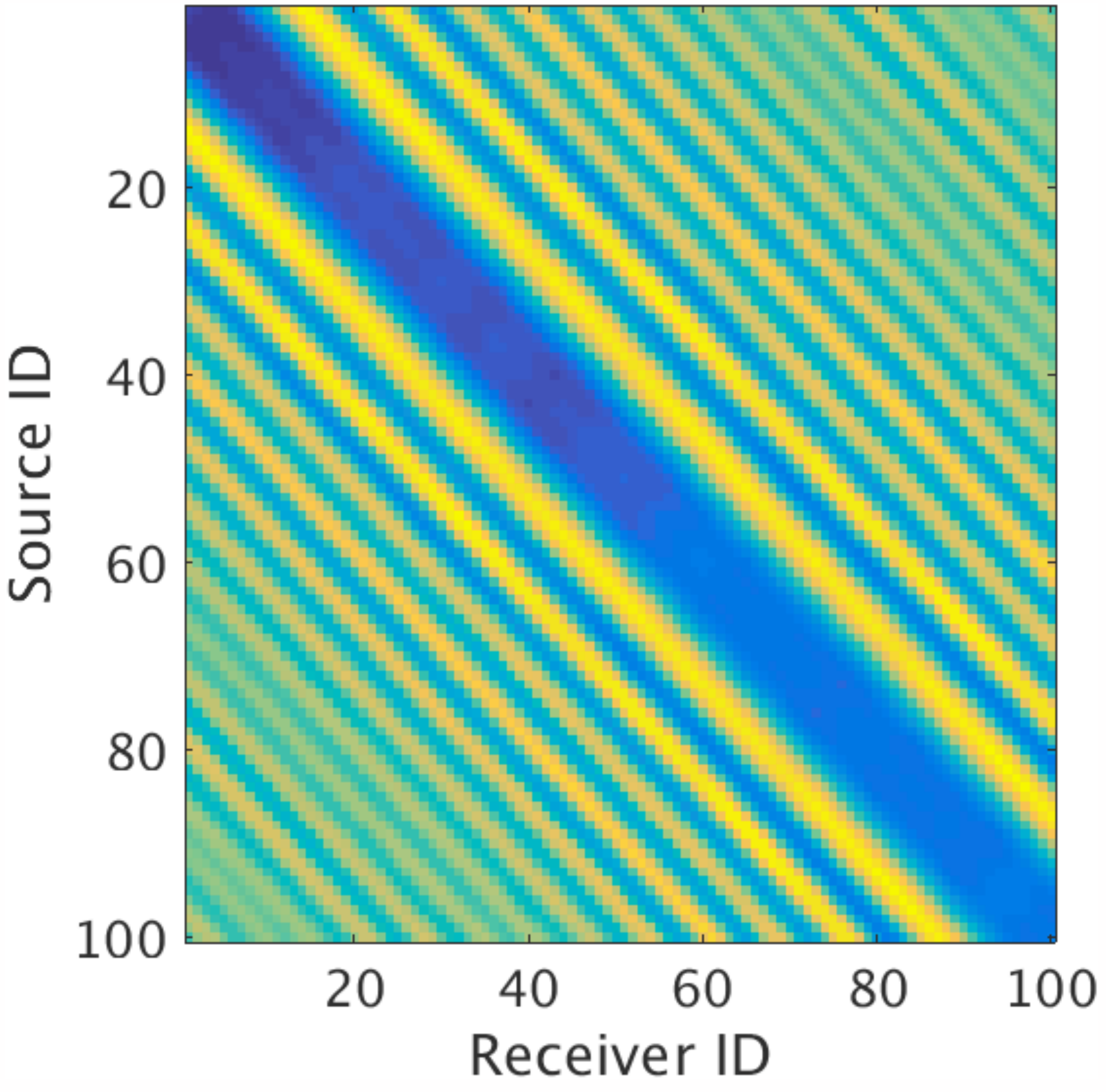



back ground density model

# Linearized vs nonlinear data – velocity only

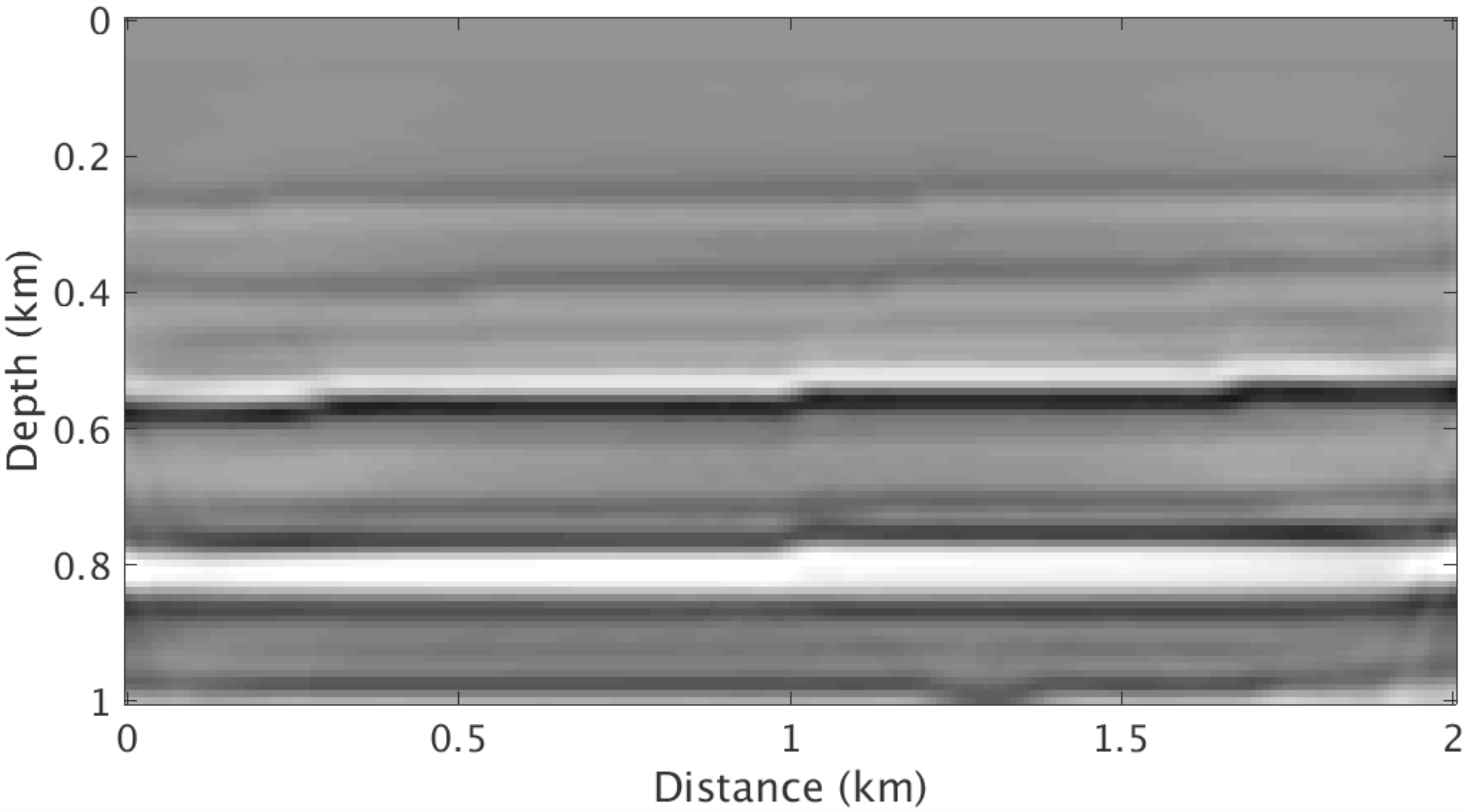



Linear Data 12Hz

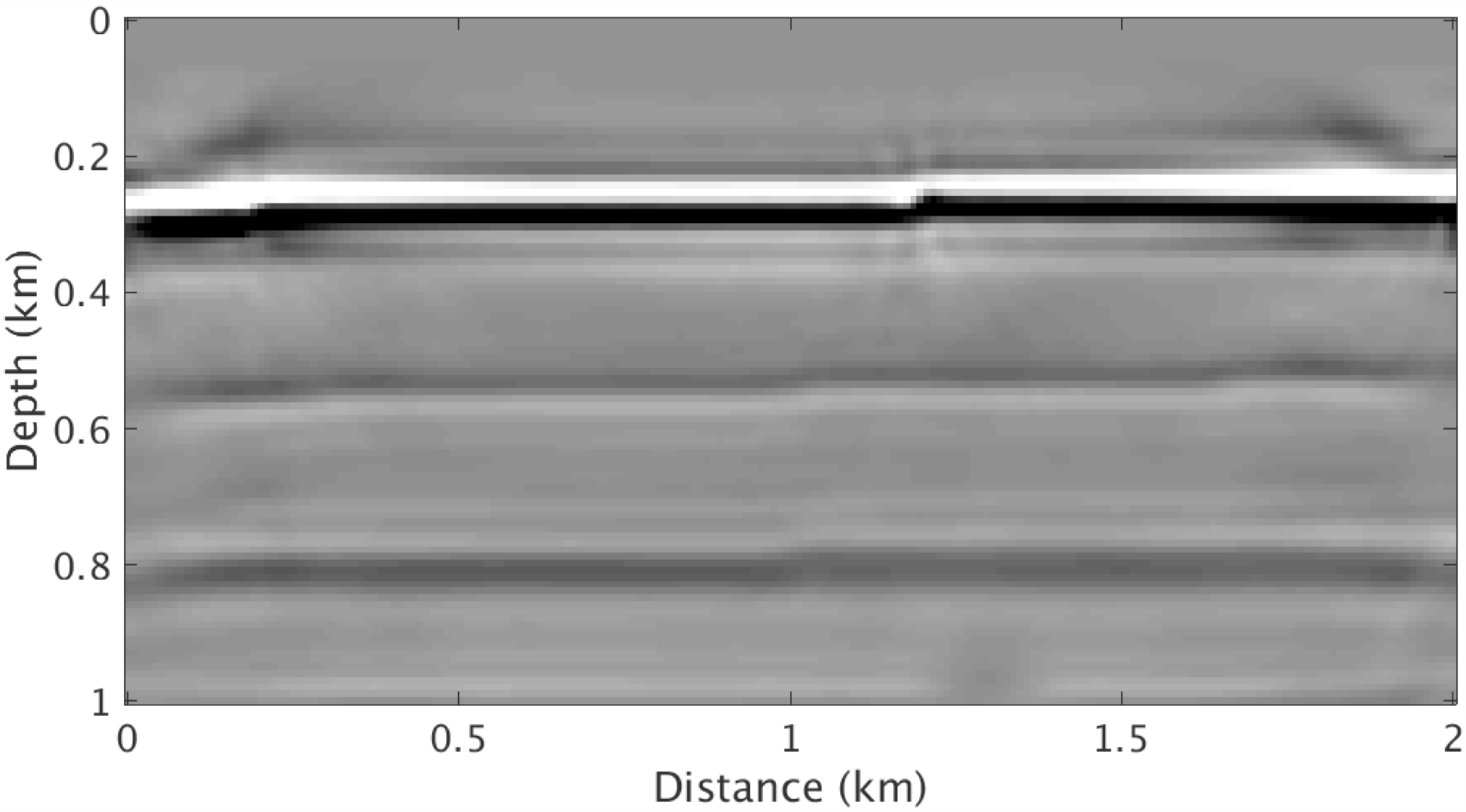



Nonlinear Data 12Hz

# LS-RTM – linearized data for velocity only



# LS-RTM – nonlinear data for velocity & density



# Motivation

- Remove artifacts related to strong density perturbations
- Avoid calculation of Born modeling w.r.t density

## Solution

Include a matrix-valued low-rank matched filter

# LS-RTM W/ low-rank matched filter

## Original LS-RTM

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) = \sum_{i=1}^{n_f} \|\mathbf{B}_i - \mathbb{J}_i(\mathbf{m}_0)\mathbf{x}\|_F$$

$\mathbf{x}$ : velocity perturbation
$i$ : frequency index
$n_f$ : total frequencies
$\mathbf{B}_i$ : the $i_{th}$ frequency slice of the observed data
$\mathbf{m}_0$ : the smooth background velocity model
$\mathbb{J}_i$ : the $i_{th}$ frequency's monochromatic Born modelling wrt velocity
$F$ : Frobenius norm

# LS-RTM with a low-rank matched filter

LS-RTM w/ matched filter

$$\min_{\mathbf{x},\mathbf{M}_i} \quad f(\mathbf{x},\mathbf{M}_i) = \sum_{i=1}^{nf} \|\mathbf{B}_i\mathbf{M}_i - \mathbb{J}_i(\mathbf{m}_0)\mathbf{x}\|_F$$

$\mathbf{M}_i$ : low-rank matrix that matches the ideal linear data and observed data

# Modification

Apply low-rank constraints

$$\min_{\mathbf{x},\mathbf{M}_i} \quad f(\mathbf{x},\mathbf{M}_i) = \sum_{i=1}^{n_f} \|\mathbf{B}_i\mathbf{M}_i - \mathbb{J}_i(\mathbf{m}_0)\mathbf{x}\|_F$$

$$\text{s.t.} \quad \text{rank}(\mathbf{M}_i) = k_i$$

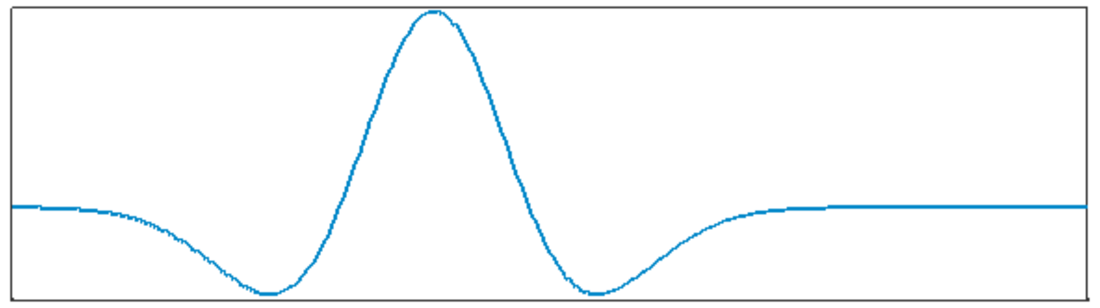
To avoid relying tightly on the provided rank

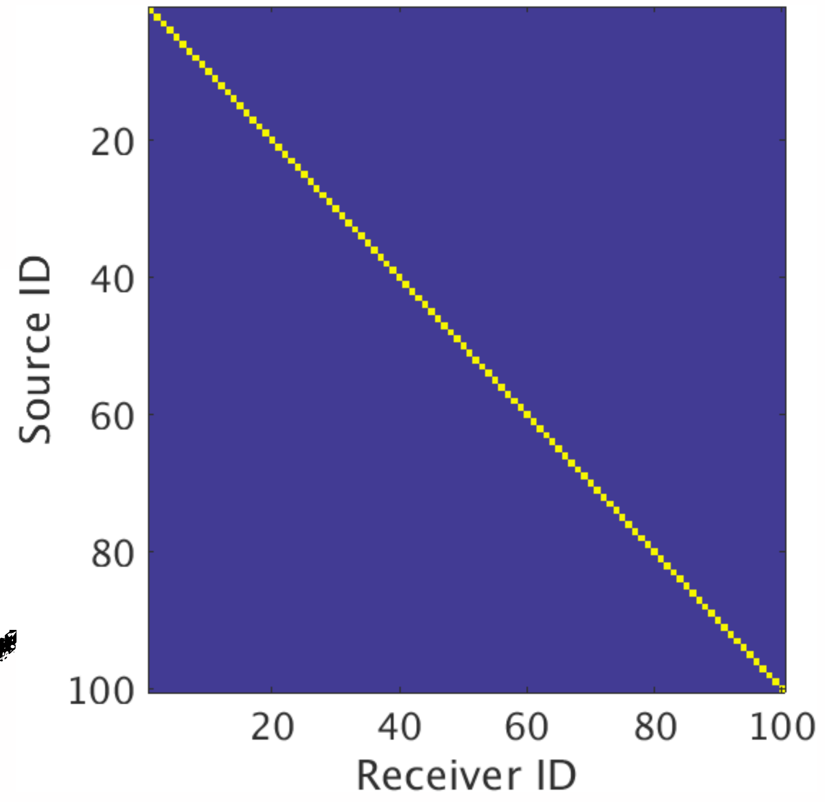
$$\min_{\mathbf{x},\mathbf{M}_i} \quad f(\mathbf{x},\mathbf{M}_i) = \sum_{i=1}^{n_f} \|\mathbf{B}_i\mathbf{M}_i - \mathbb{J}_i(\mathbf{m}_0)\mathbf{x}\|_F + \tau\|\mathbf{M}_i\|_*$$

$\tau$ : parameter that balances the Frobenius norm and nuclear norm

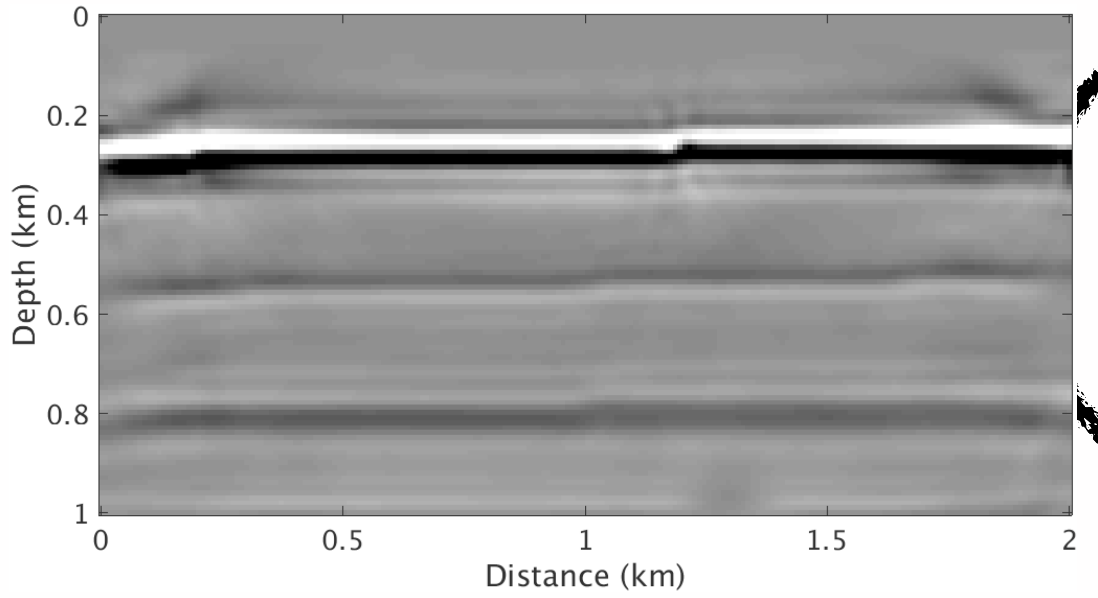
# Workflow

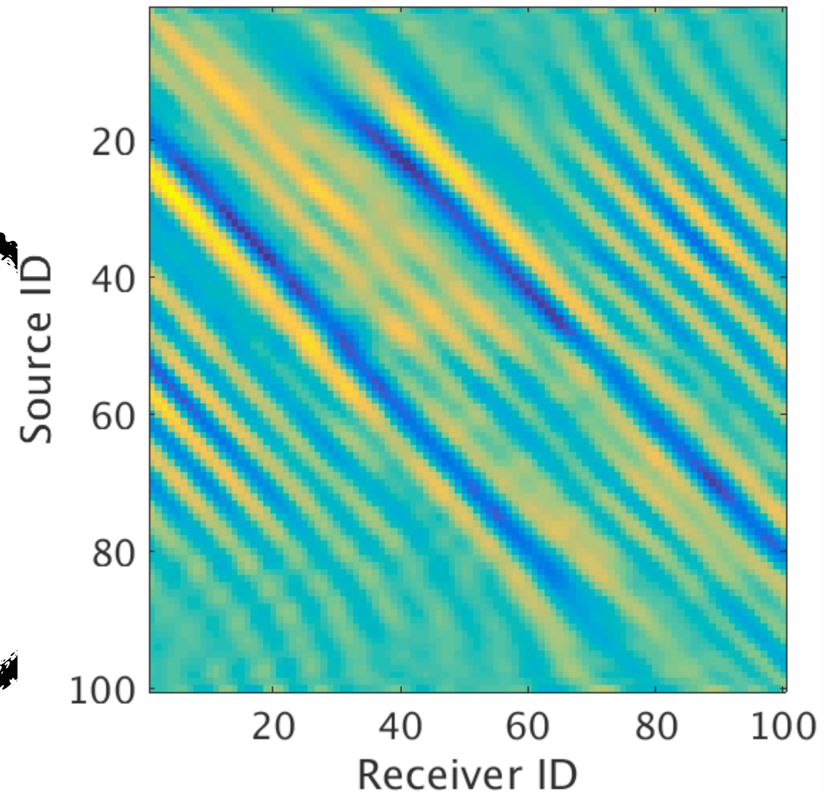
start with full-rank matrix






update $\mathbf{x}$ w/ GD




Invert for matrix filter
w/ Singular Value Thresholding (SVT)

## Workflow

$$
\begin{array}{l}
\text{Initialize } \mathbf{x}_0 = \mathbf{0}, \tau \\
\text{for } k = 1 \cdots N \\
\quad \text{for } i = 1 \cdots n_f \\
\quad\quad \mathbf{D}_{ki} = \mathbb{J}_i(\mathbf{m}_0)\mathbf{x}_k \\
\quad\quad \text{Initialize } \mathbf{Y}_0 = \mathbf{0}, \mathbf{M}_{ki,0} \\
\quad\quad \text{for } j = 1 \cdots J \\
\quad\quad\quad \mathbf{Y}_j = \mathbf{Y}_{j-1} + \delta_j(\mathbf{D}_{ki} - \mathbf{B}_i\mathbf{M}_{ki,j-1}) \\
\quad\quad\quad \mathbf{M}_{ki,j} = \mathbb{D}_\tau(\mathbf{B}_i^T\mathbf{Y}_j) \\
\quad\quad \text{end} \\
\quad \text{end} \\
\quad \mathbf{g}_k = \sum_{i=1}^{nf} \mathbb{J}_i^T \text{Vec}(\mathbf{B}_i\mathbf{M}_{ki,J} - \mathbf{D}_{ki}) \\
\quad \mathbf{x}_k = \mathbf{x}_{k-1} + s_k\mathbf{g}_k \\
\text{end}
\end{array}
$$

SVT

update $\mathbf{x}$

Note: $\mathbb{D}_\tau$ is the singular value shrinkage operator

$\delta_j$ and $s_k$ are stepsizes needed in update

# Experiment

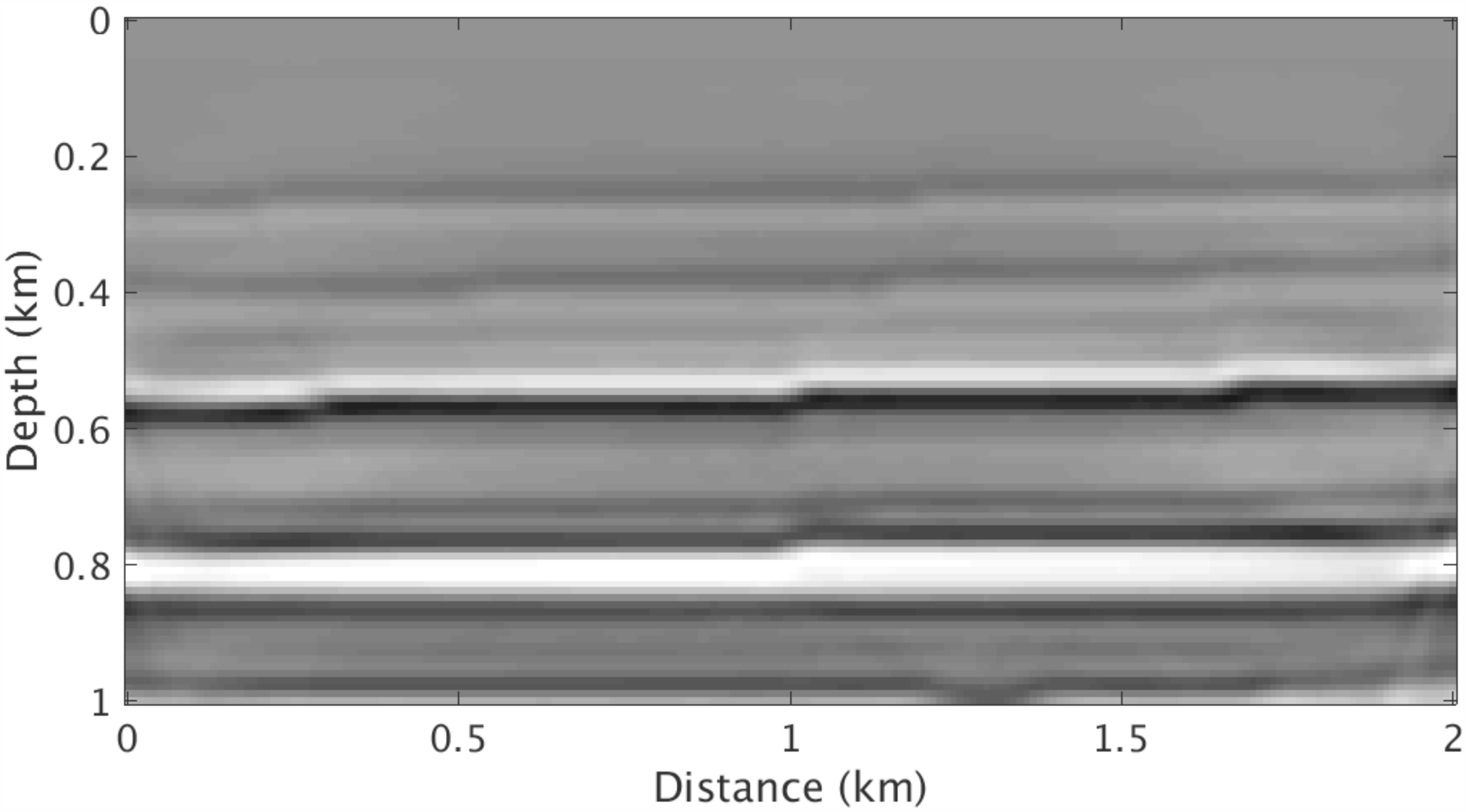
**Data:**

- 100 sources and receivers
- Ricker wavelet centered at 8 Hz
- $\mathbb{F}(\mathbf{m}, \rho) - \mathbb{F}(\mathbf{m}_0, \rho_0)$

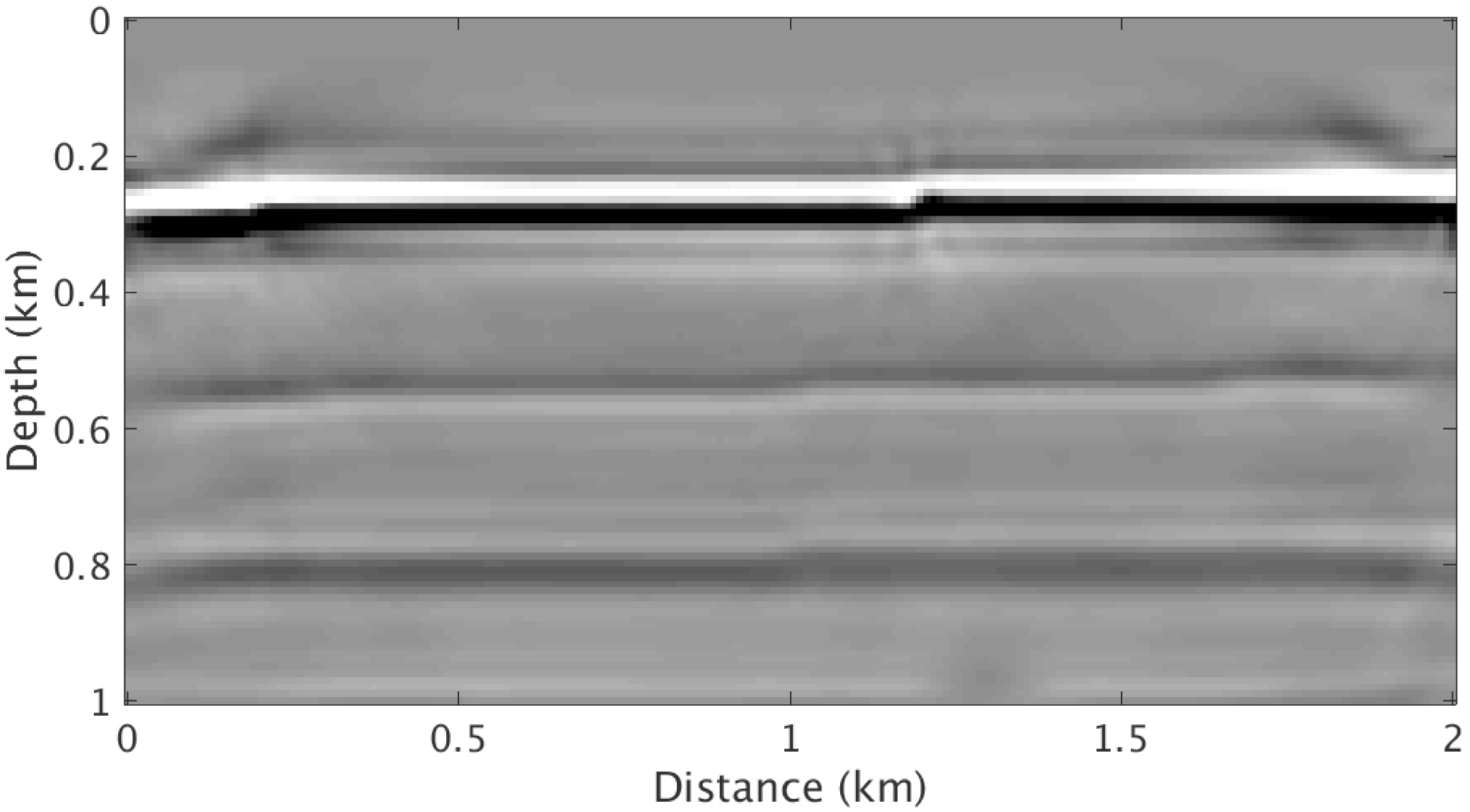
**Experiments:**

- true source wavelet
- Born modeling wrt only velocity
- 20 iterations for linear data inversion
- 100 iterations for our method

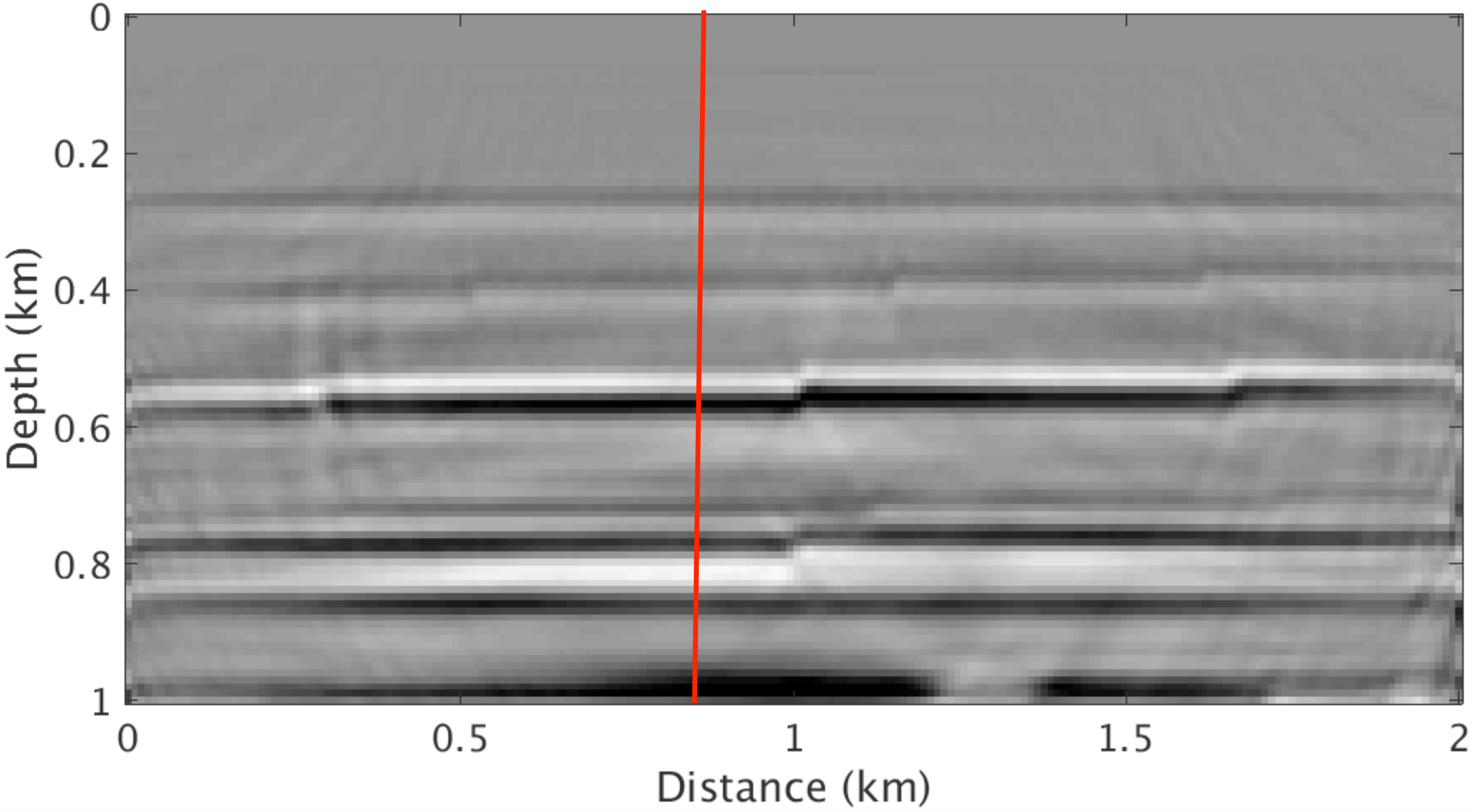
# LS-RTM – linearized data



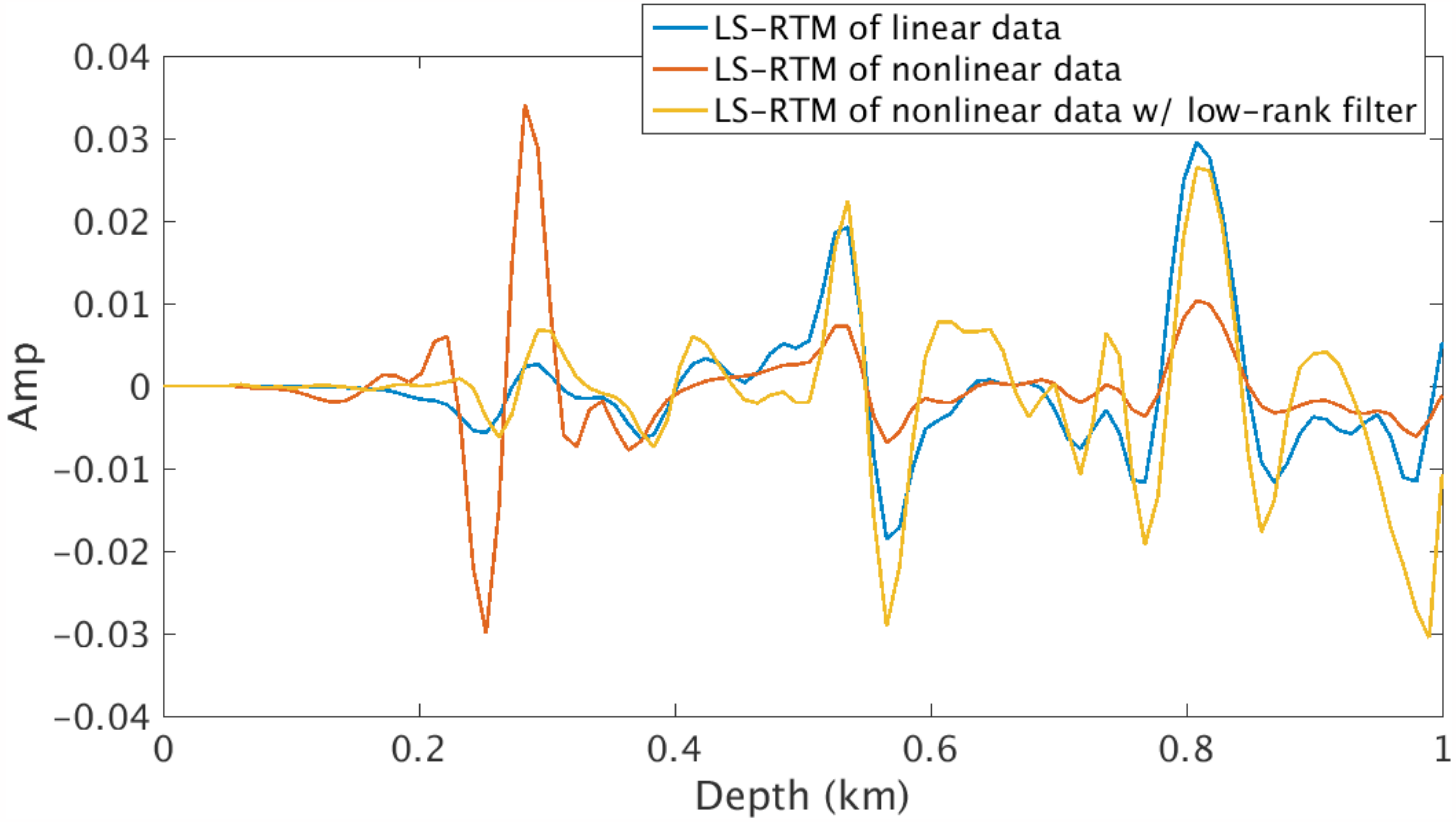
# LS-RTM – nonlinear data



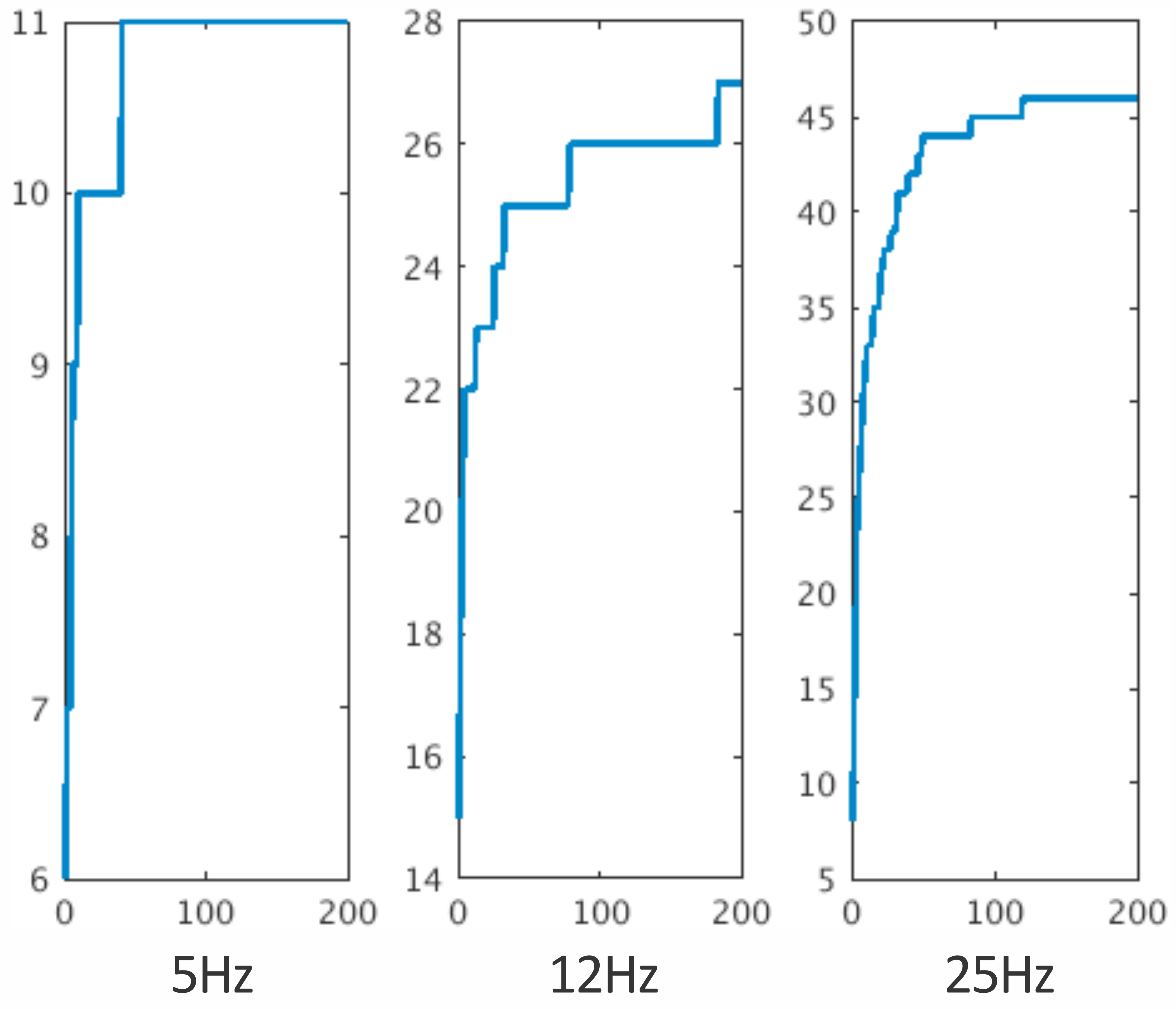
# LS-RTM – nonlinear data w/ low-rank filter



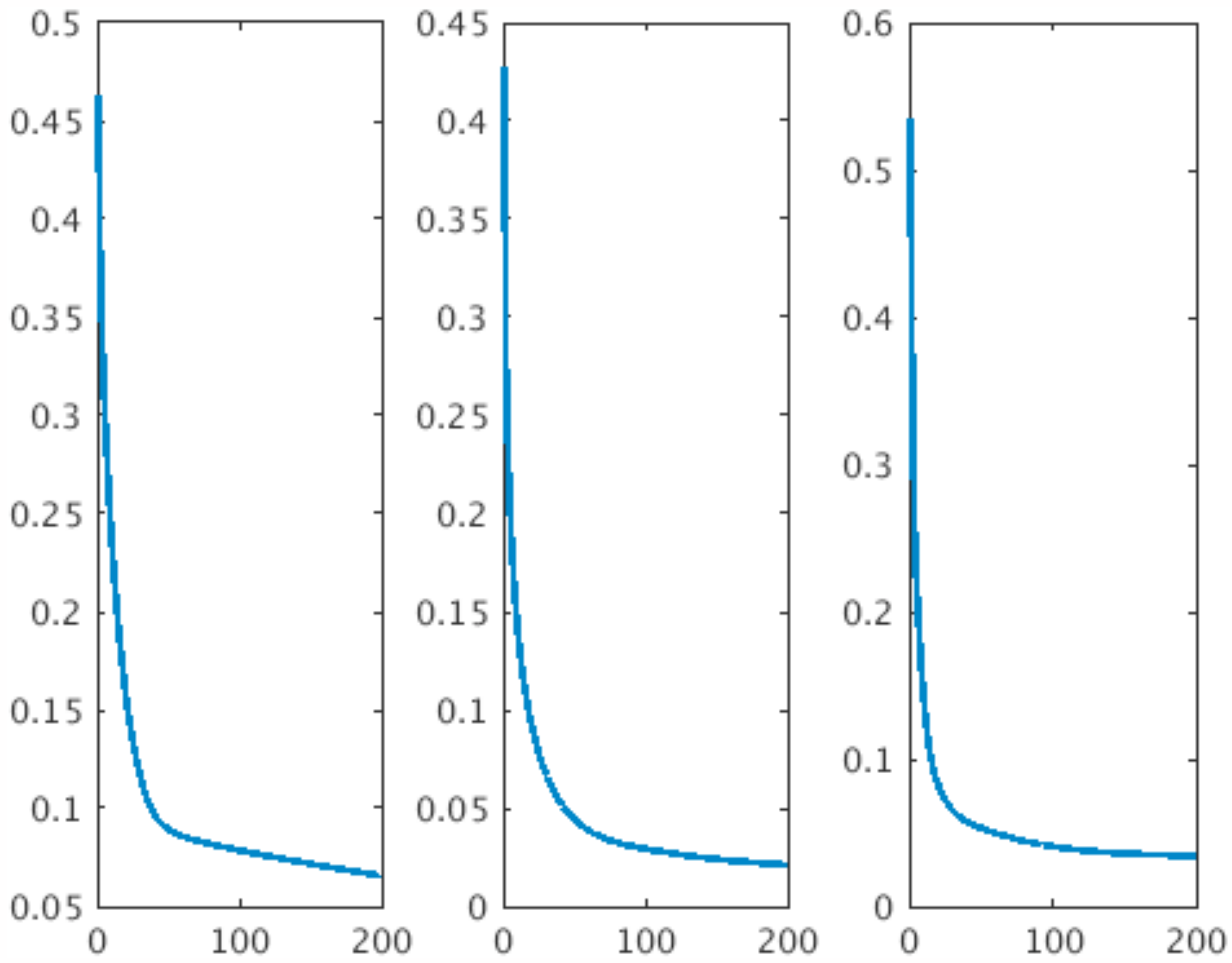
# Trace comparison



# Rank of filter during SVT iterations

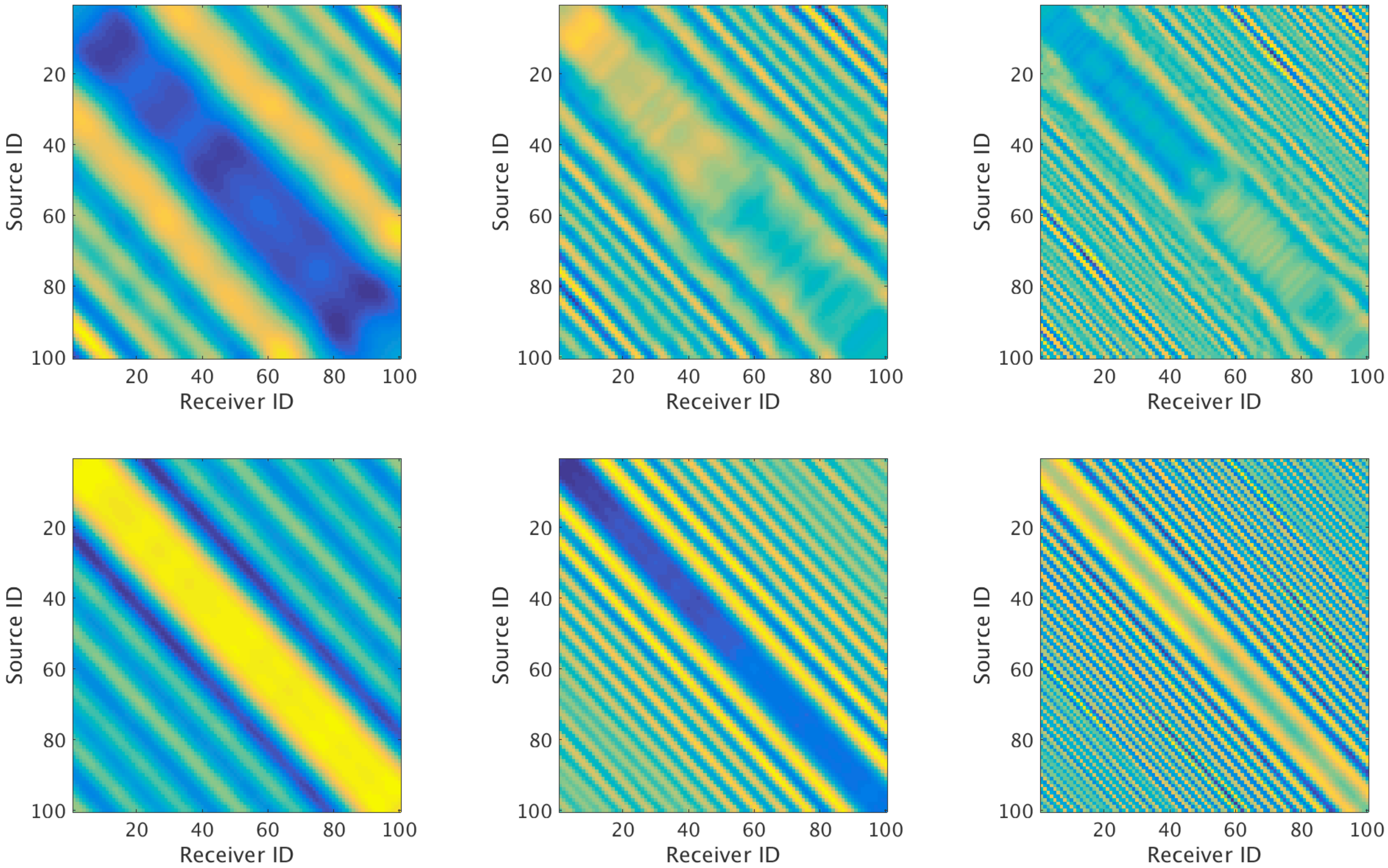


# Frobenius norm during SVT iterations




5Hz 12Hz 25Hz

# Data comparison




Linear data

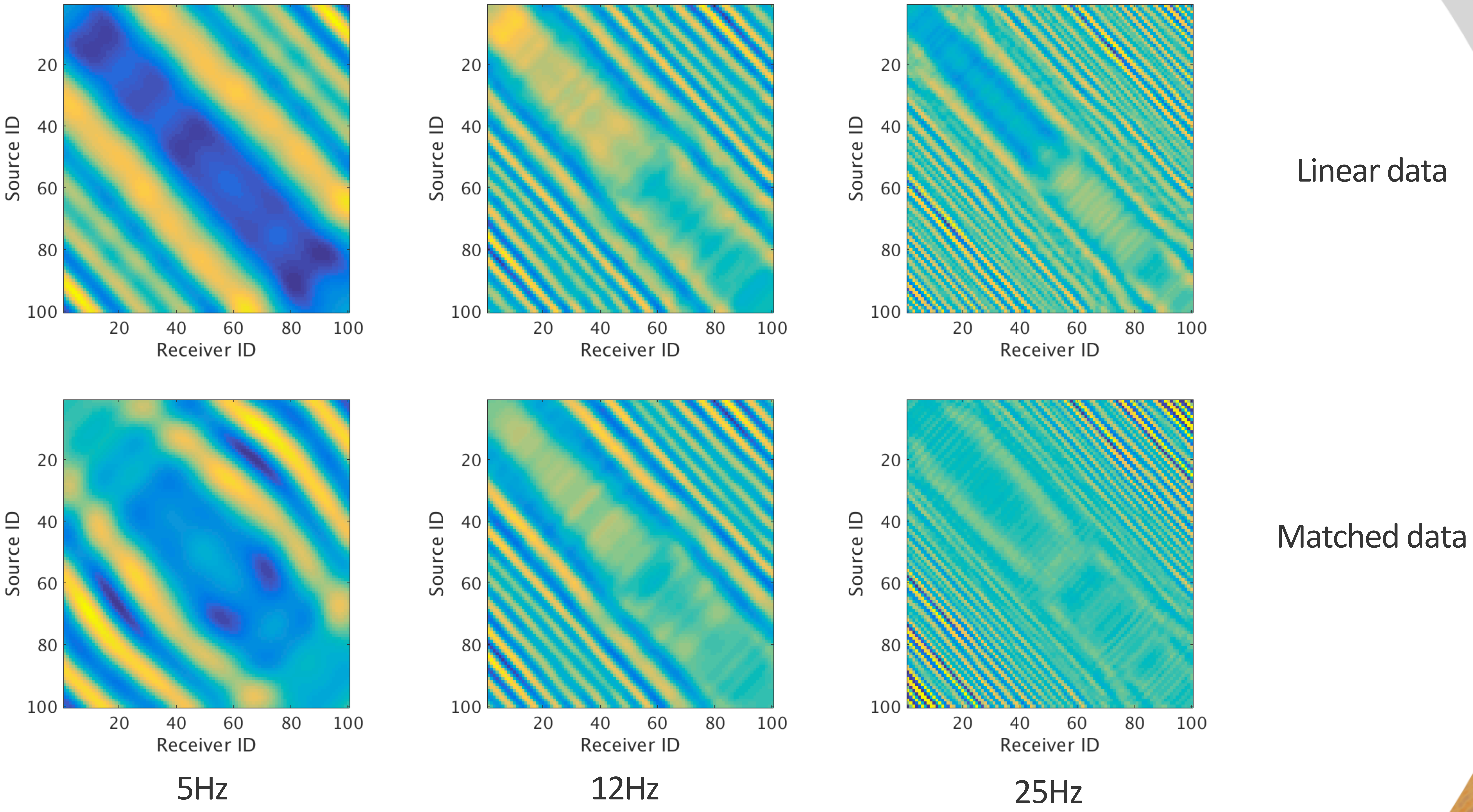
Nonlinear data

5Hz

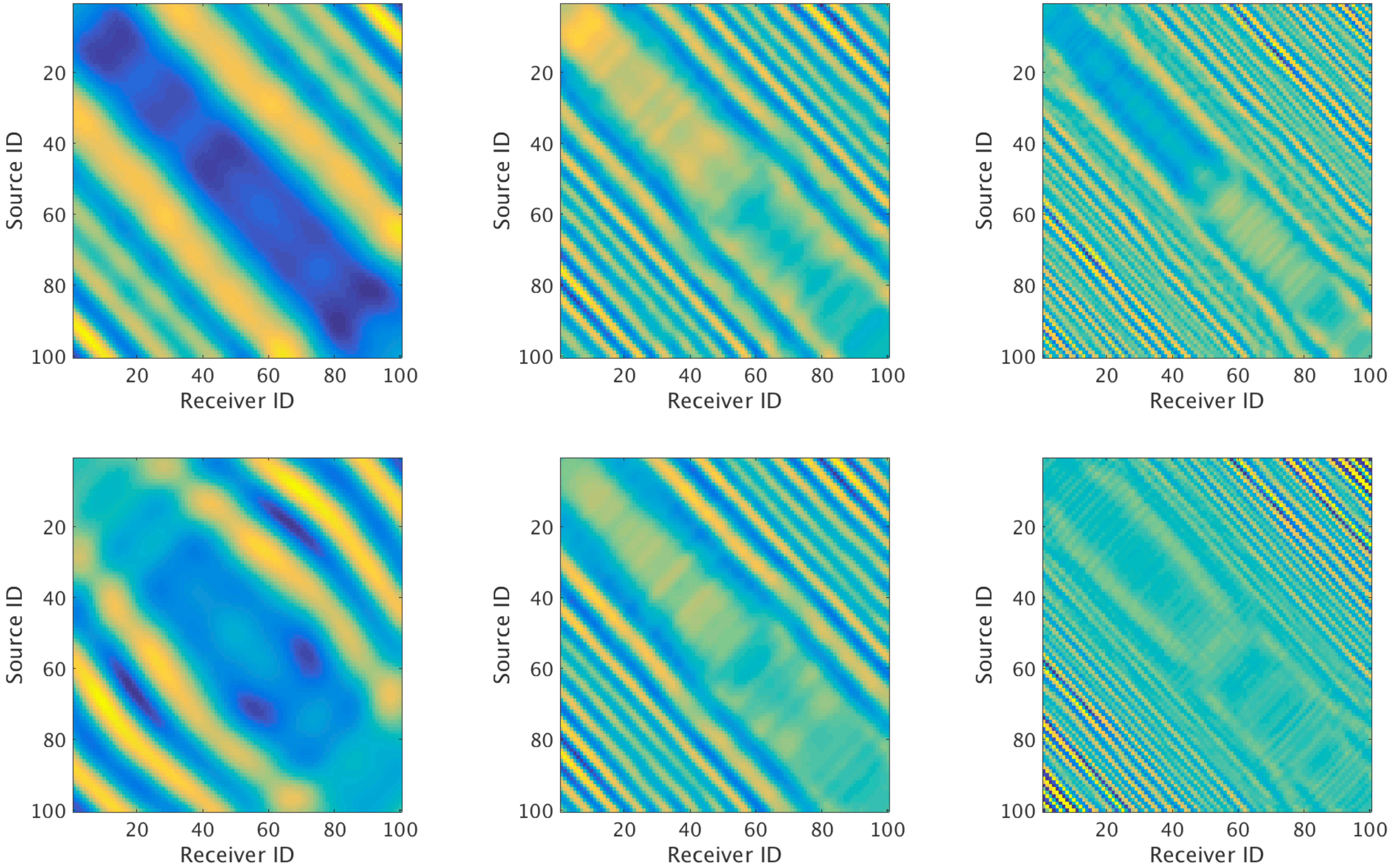
12Hz

25Hz

# Data comparison



# Data comparison




Linear data

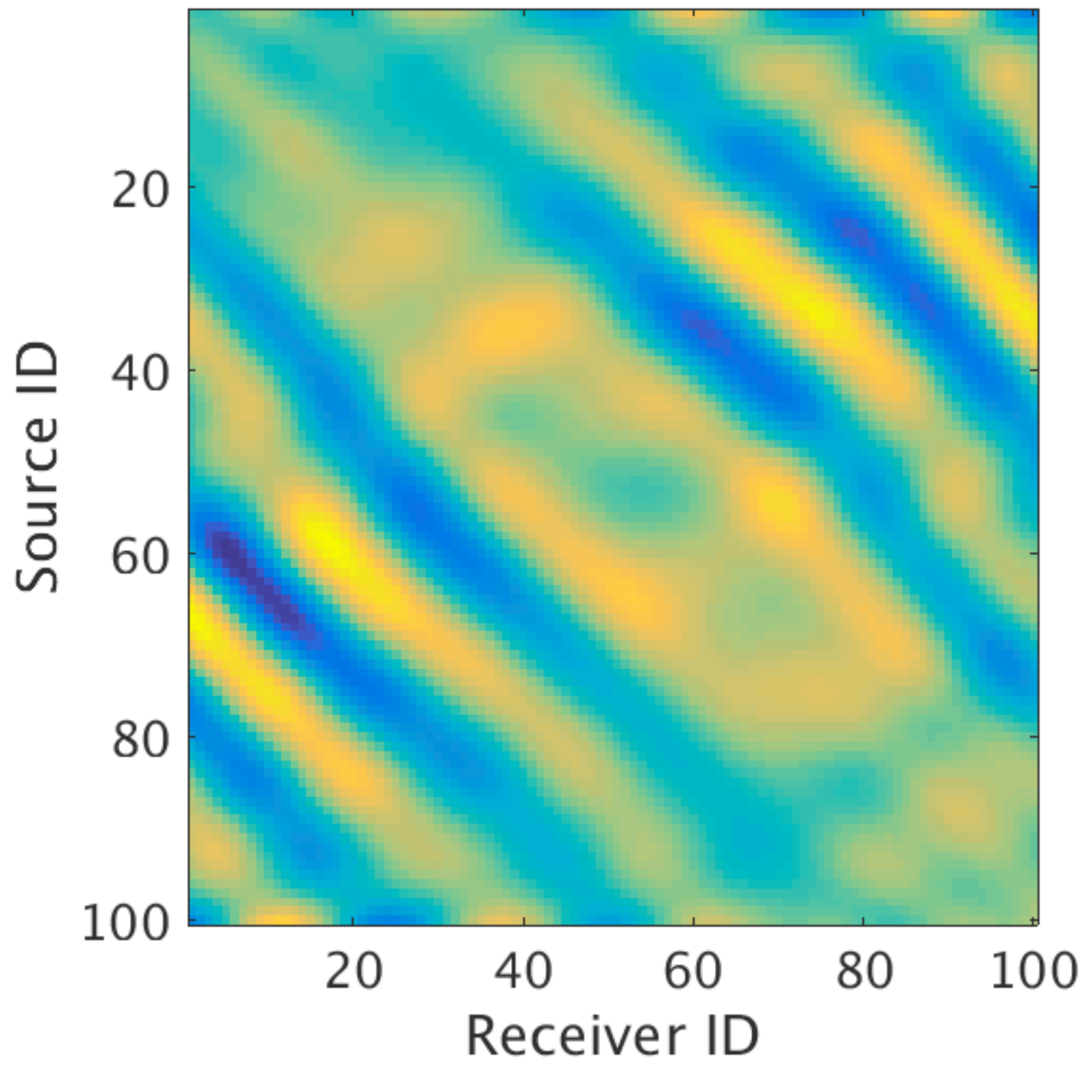
Nonlinear data w/ matched filter

5Hz

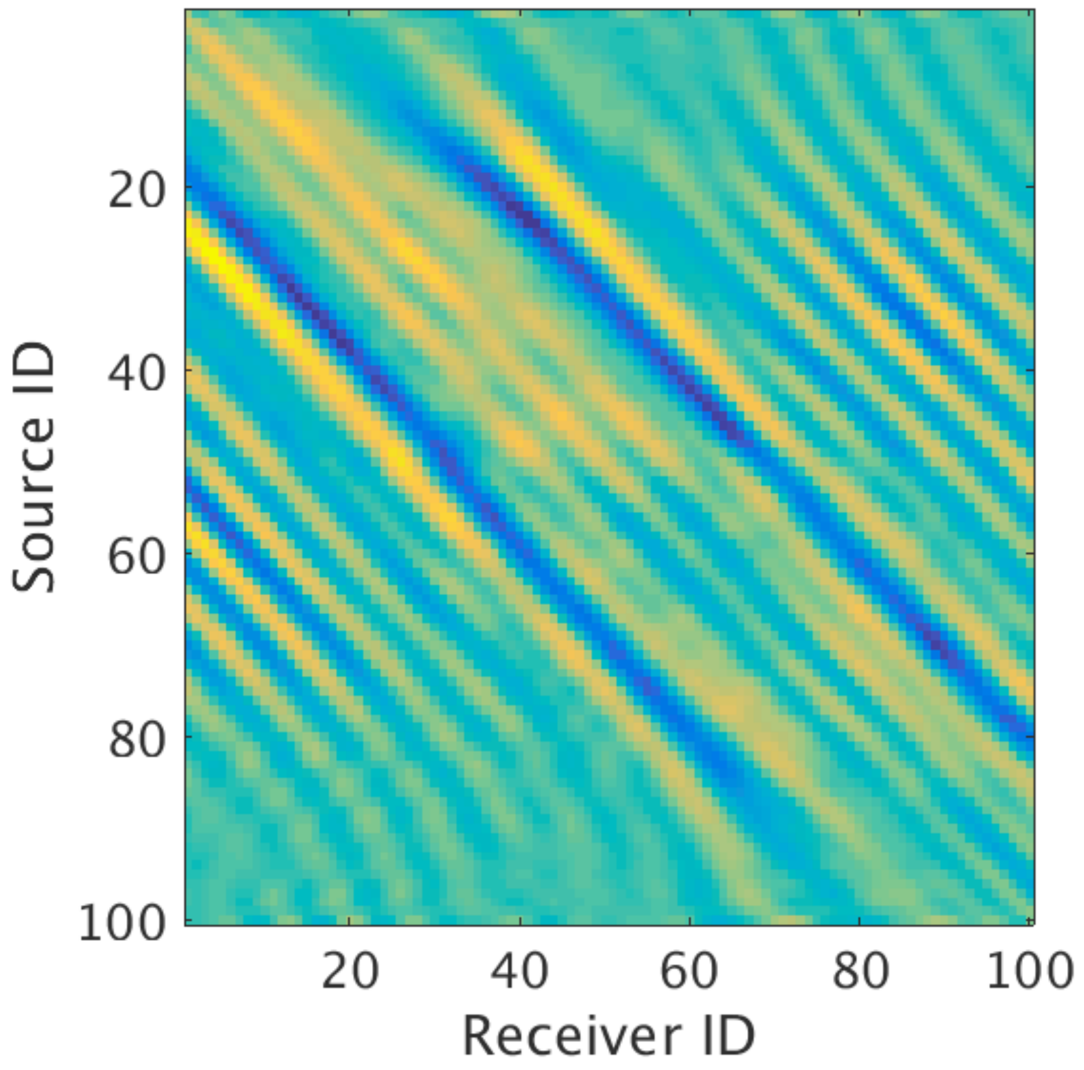
12Hz

25Hz

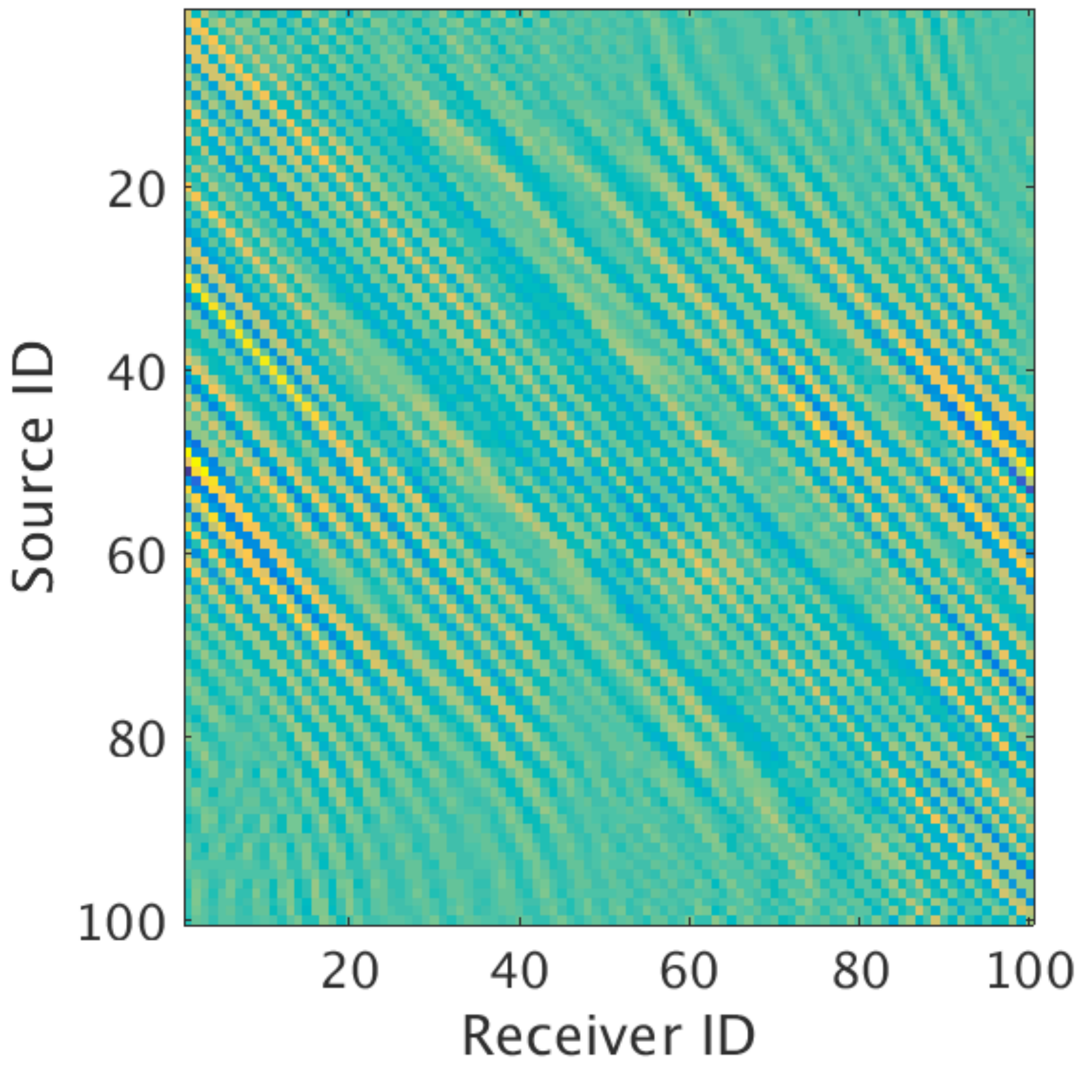
# Estimated matrix filter




5Hz




12Hz




25Hz

## Summary

- We propose a matched-filter LS-RTM formulation to remove events related to strong density variations
- This matched-filter method does not require additional PDE solves
- It maps nonlinear data to linear data for velocity variations only
- Artifacts from strong density perturbations at ocean bottom can be removed
- Calls for expensive iterations, every iteration costs 1 RTM and lots of SVD. Need to improve the convergence rate and the cost for single iteration, eg, source and frequency subsampling

Thanks for your attention