Released to public domain under Creative Commons license type BY (https://creativecommons.org/licenses/by/4.0).
Copyright (c) 2018 SLIM group @ Georgia Institute of Technology.

A debiasing approach to microseismic

Shashin Sharan, Rajiv Kumar, Rongrong Wang and Felix J. Herrmann

SEG 2018

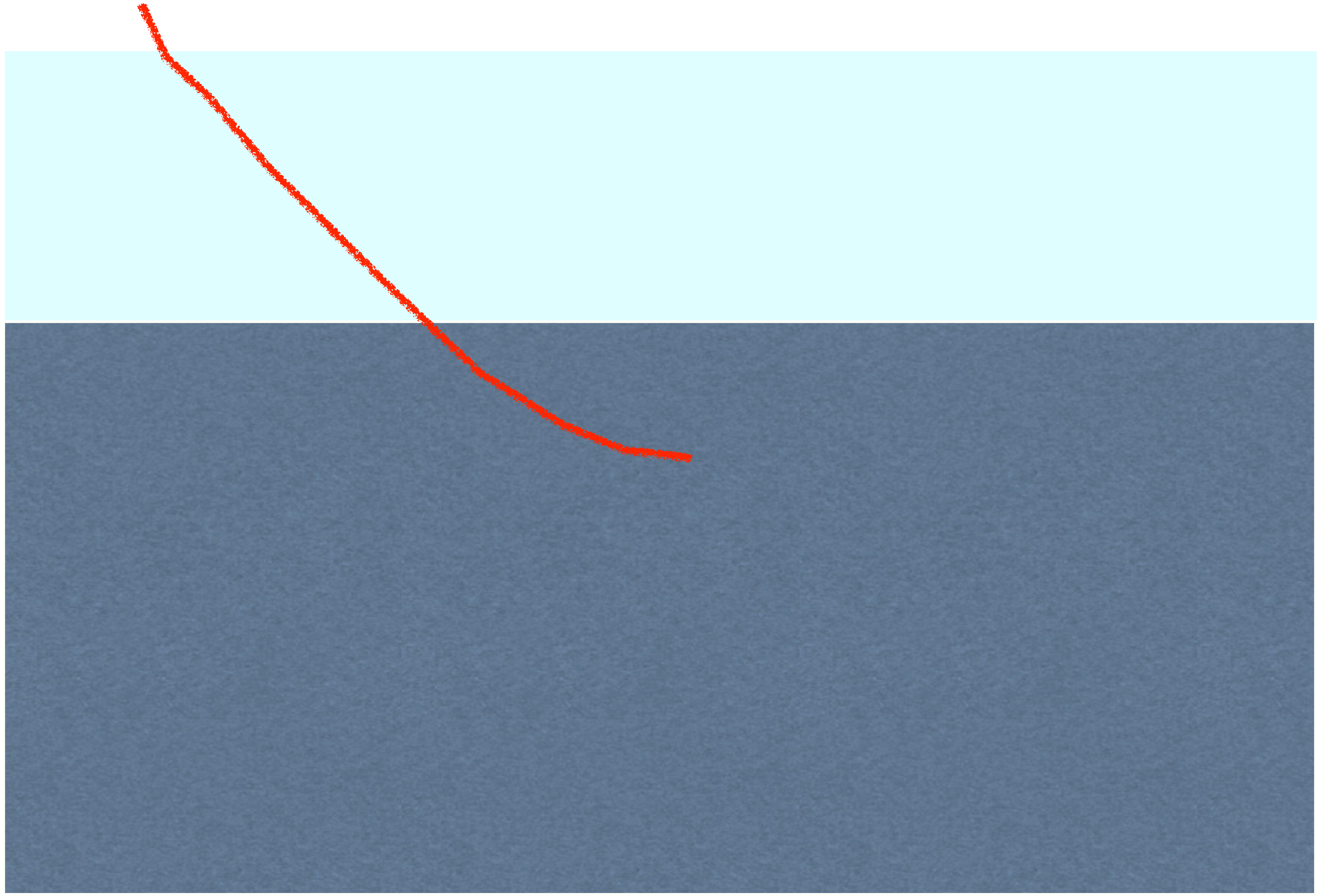



SLIM
Georgia Institute of Technology

Motivation

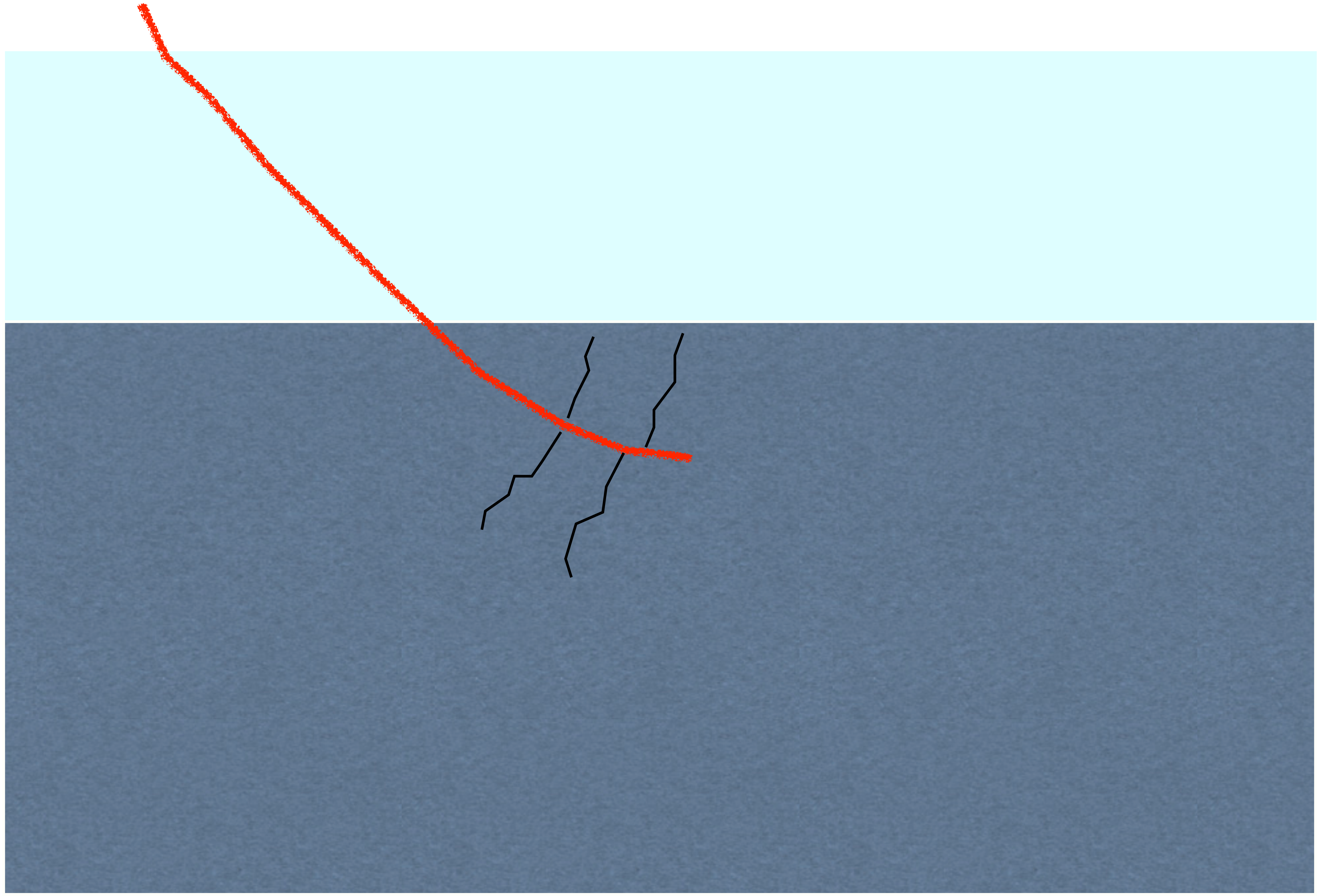
Unconventional
Reservoir Schematic

Motivation



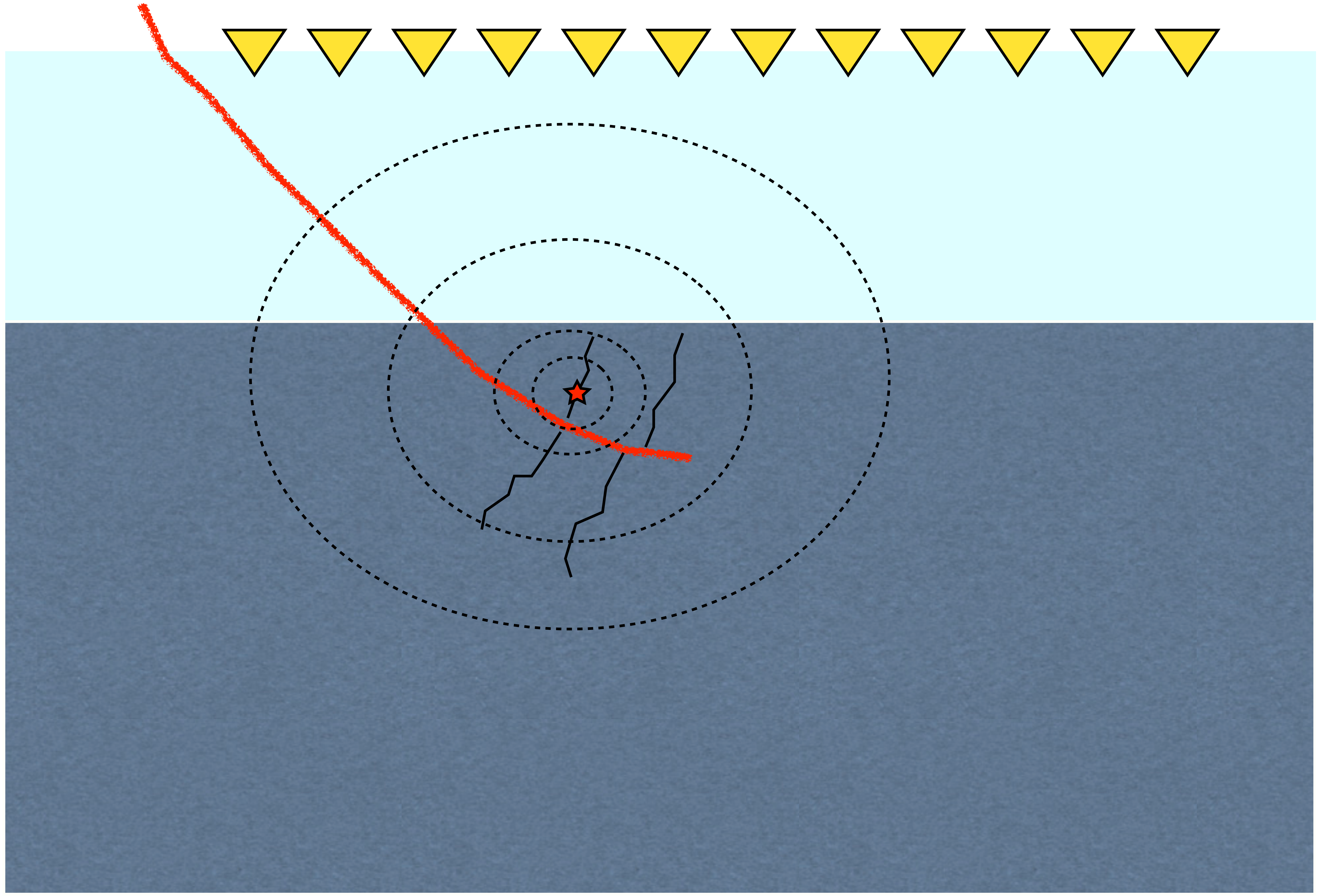
Unconventional Reservoir Schematic

Motivation



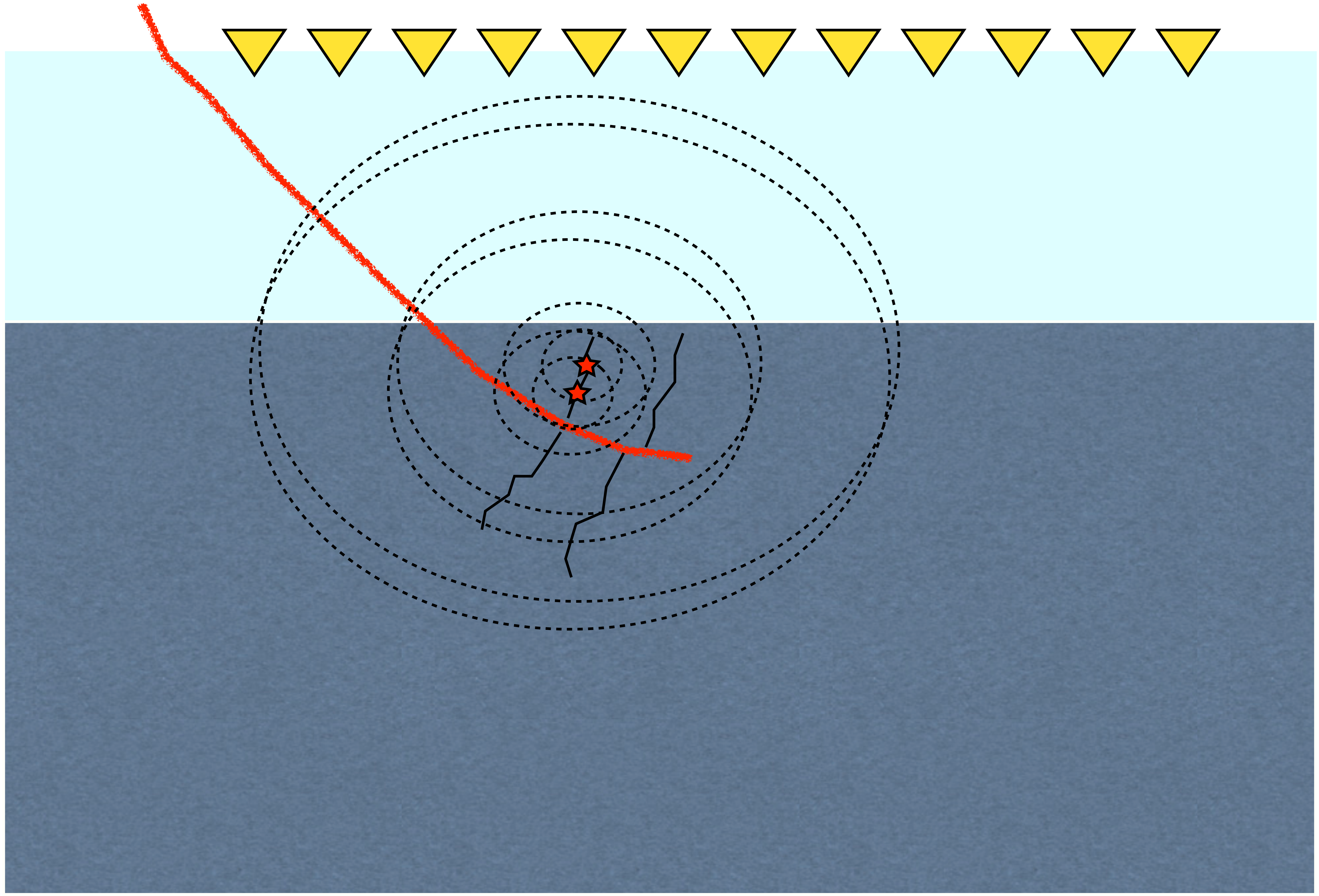
Unconventional
Reservoir Schematic

Motivation



Unconventional
Reservoir Schematic

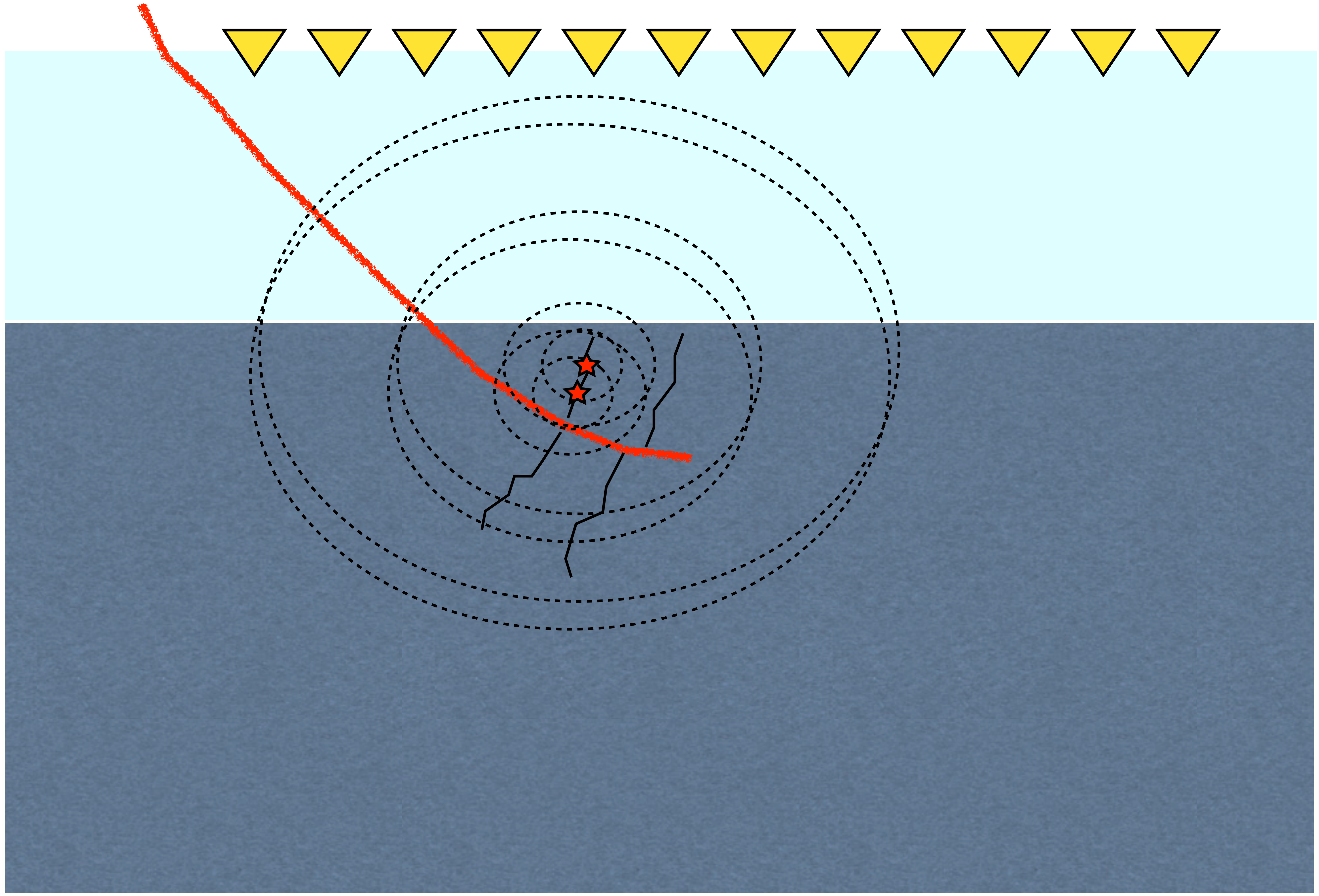
Motivation



Unconventional Reservoir Schematic

SLIM

Motivation

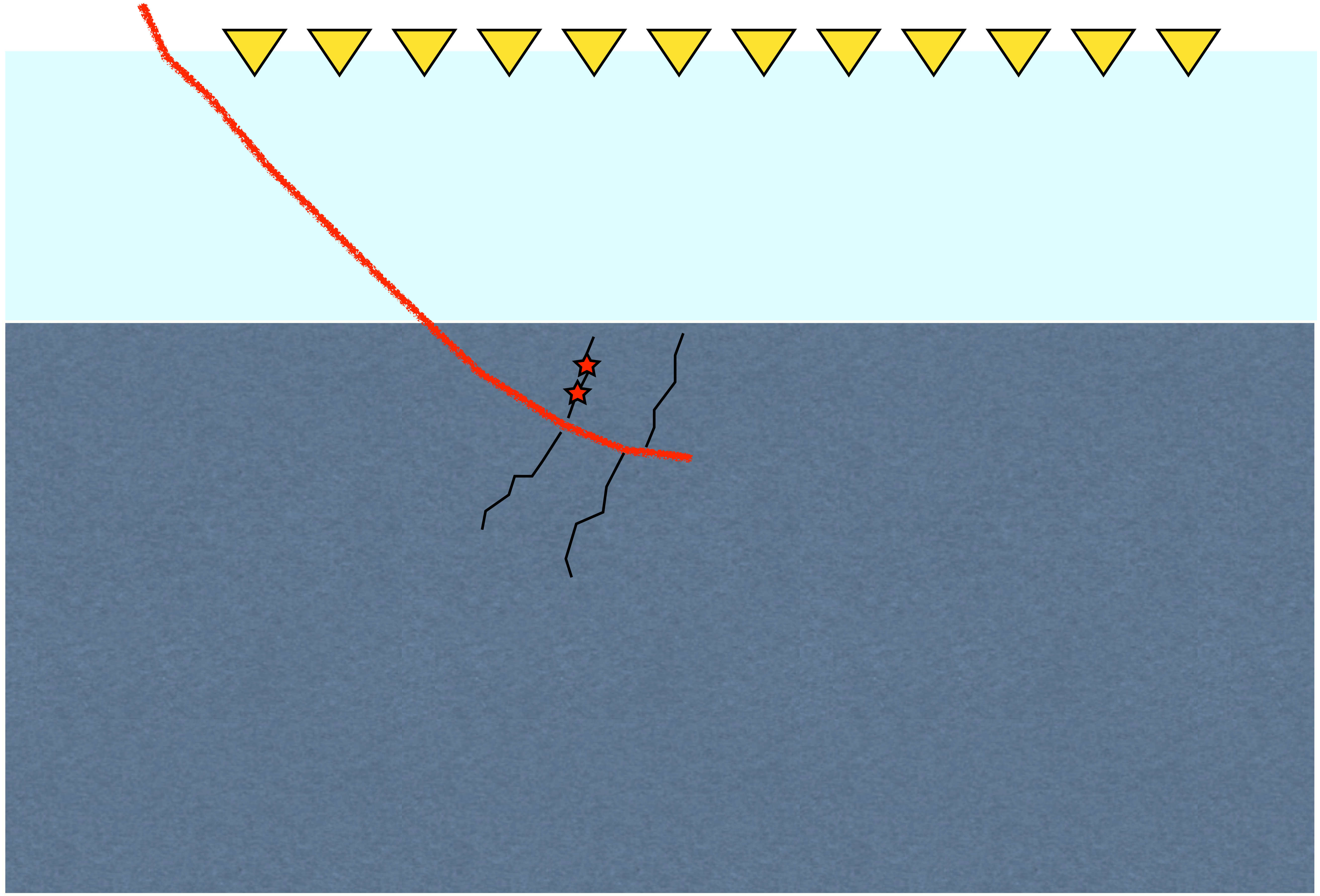


Unconventional Reservoir Schematic

Objectives

- detection of microseismic events in space and time
- estimation of source time function

Proposed method w/ sparsity promotion

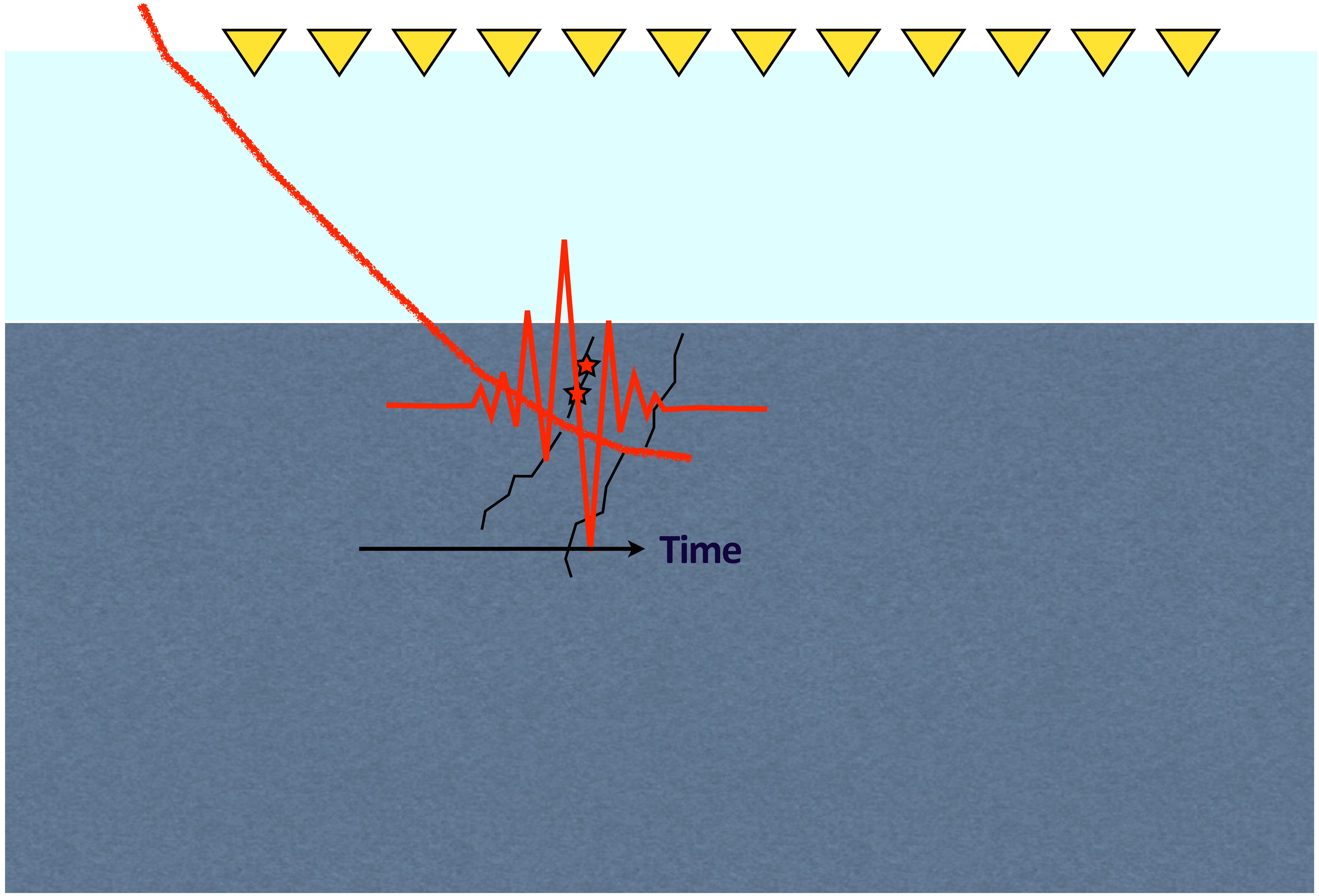


Unconventional Reservoir Schematic

Assumptions

- localized in space

Proposed method w/ sparsity promotion



Unconventional Reservoir Schematic

Assumptions

- localized in space
- finite energy along time

[Van Den Berg et al.,’08]

Proposed method w/ sparsity promotion

$$\underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1}$$
$$\text{subject to } \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2 \leq \epsilon$$

Source wavefield

Noise level

Forward modeling operator

Slowness square

Observed data

$\mathbf{Q} \in \mathbb{R}^{n_x \times n_t}$

n_x: number of grid points

n_t: number of time samples

[Van Den Berg et al.,'08]

SLIM

Proposed method w/ sparsity promotion

Source wavefield

Noise level

$$\underset{\mathbf{Q}}{\text{minimize}} \quad \|\mathbf{Q}\|_{2,1}$$
$$\text{subject to } \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2 \leq \epsilon$$

Forward modeling operator

Slowness square

Observed data

$\mathbf{Q} \in \mathbb{R}^{n_x \times n_t}$

n_x: number of grid points

n_t: number of time samples

Similar to classic Basis pursuit denoising (BPDN)

SLIM

Solving w/ Linearized Bregman

$$\underset{\mathbf{Q}}{\text{minimize}} \;\; \|\mathbf{Q}\|_{2,1} + \frac{1}{2\mu}\|\mathbf{Q}\|_F^2$$
$$\text{subject to } \|\mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}\|_2 \leq \epsilon$$

*where $\|.\|_F$ is the Frobenius norm

- Recent successful application to seismic imaging problem
- Three-step algorithm simple to implement
- Choice of μ controls the trade off between sparsity and the Frobenius norm
- $\mu \uparrow \infty$ corresponds to solving original BPDN problem

[Lorentz et al.,'14; Combettes et al.,'11; Sharan et al.,'16]

SLIM

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

[Lorentz et al.,'14; Combettes et al.,'11; Sharan et al.,'16]

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu \ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

*where $t_k = \frac{\|\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}\|^2}{\|\mathcal{F}^\top[\mathbf{m}](\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d})\|^2}$ is the dynamic step length

[Lorentz et al.,'14; Combettes et al.,'11; Sharan et al.,'16]

SLIM

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \text{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

*where $t_k = \frac{\|\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}\|^2}{\|\mathcal{F}^\top[\mathbf{m}](\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d})\|^2}$ is the dynamic step length

* $\text{Prox}_{\mu\ell_{2,1}}(\mathbf{C}) := \arg\min_{\mathbf{B}} \|\mathbf{B}\|_{2,1} + \frac{1}{2\mu}\|\mathbf{C} - \mathbf{B}\|_F^2$ is the proximal mapping of the $\ell_{2,1}$ norm

[Lorentz et al.,’14; Combettes et al.,’11; Sharan et al.,’16]

Linearized Bregman algorithm

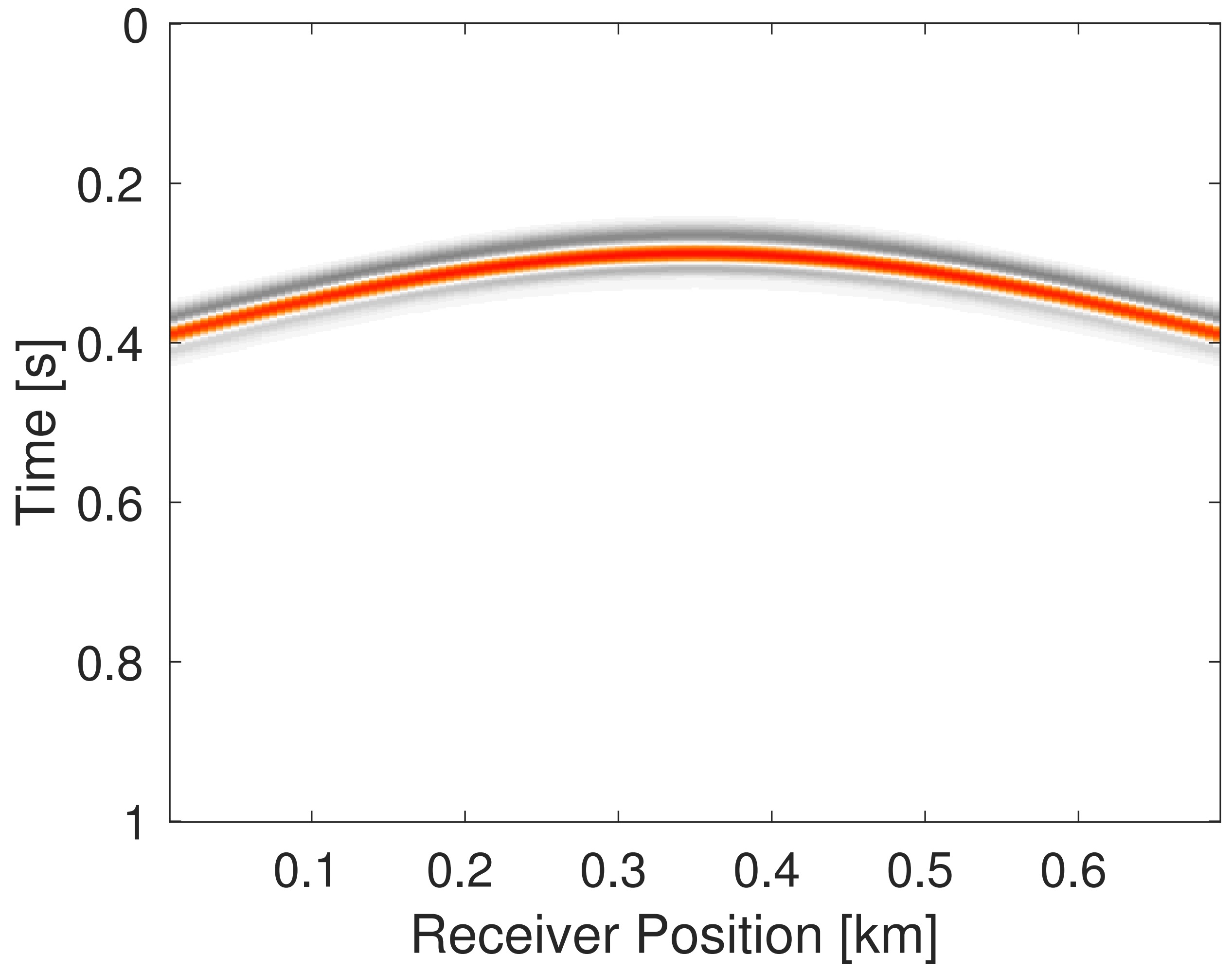
1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

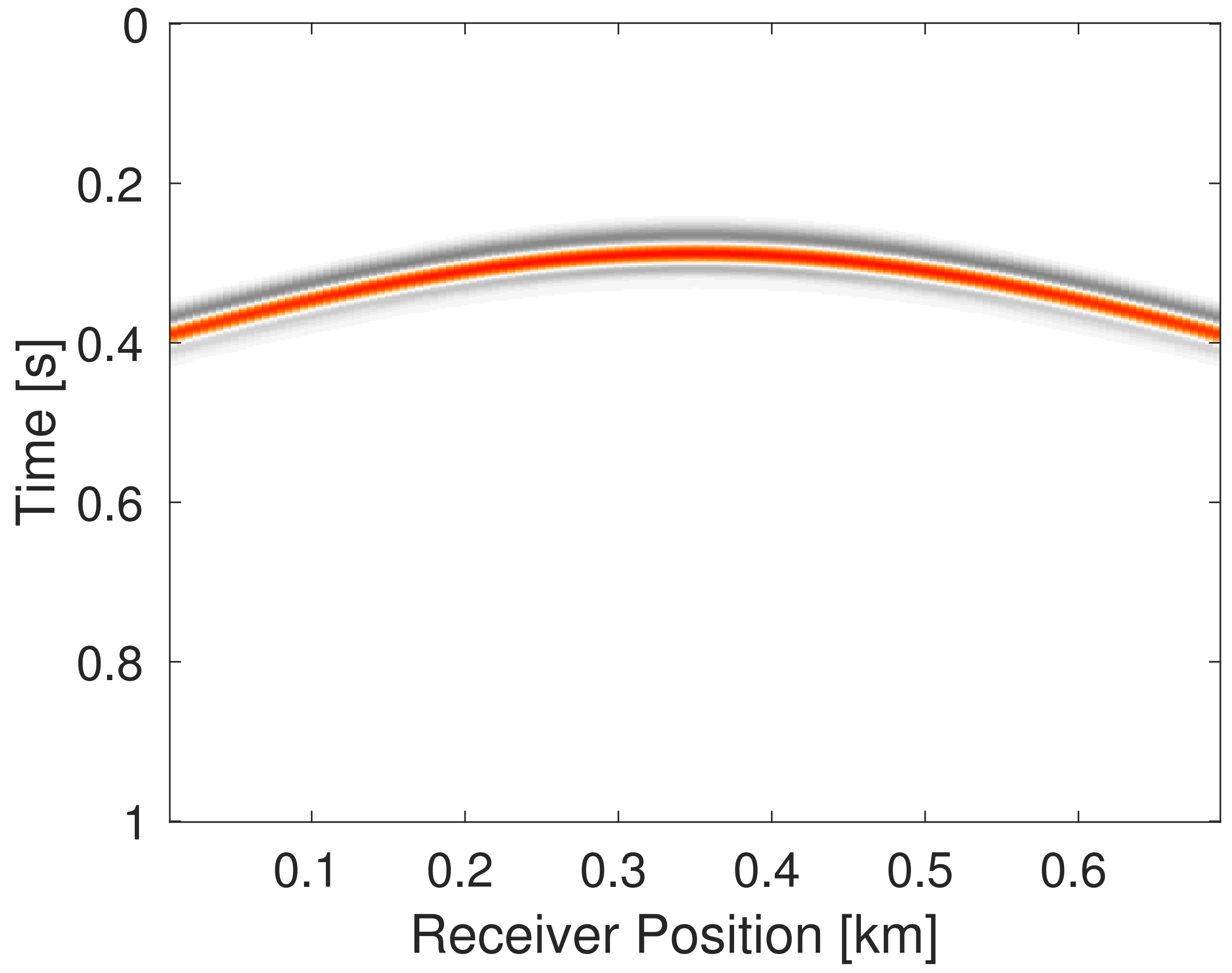
*where $t_k = \frac{\|\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}\|^2}{\|\mathcal{F}^\top[\mathbf{m}](\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d})\|^2}$ is the dynamic step length

* $\mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{C}) := \arg\min_{\mathbf{B}} \|\mathbf{B}\|_{2,1} + \frac{1}{2\mu}\|\mathbf{C} - \mathbf{B}\|_F^2$ is the proximal mapping of the $\ell_{2,1}$ norm

- **Source location**: estimated as outlier in intensity plot
- **Source-time function**: temporal variation of wavefield at estimated source location

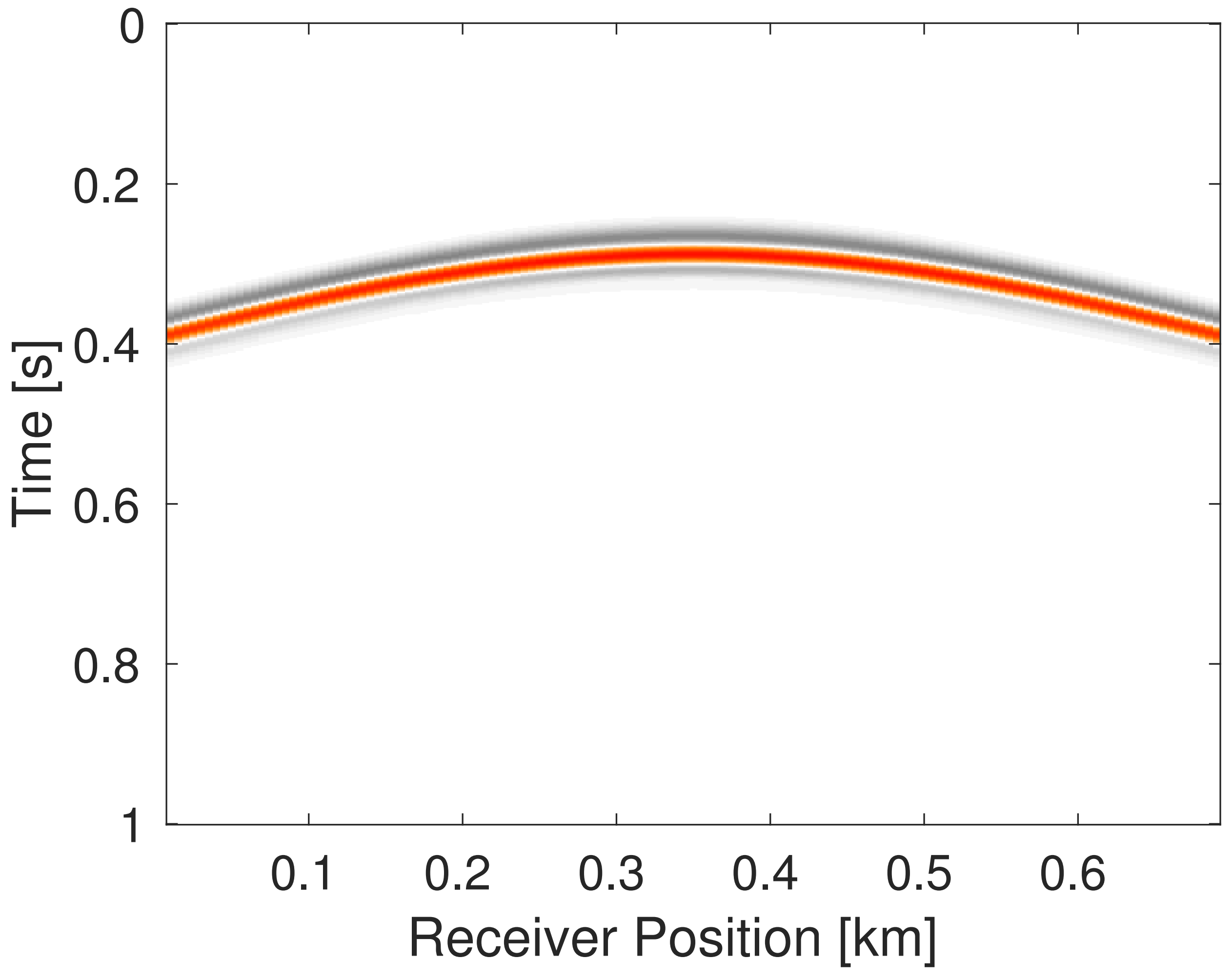


0
0.2
0.4
0.6
0.8
1
Time [s]
0.1
0.2
0.3
0.4
0.5
0.6
Receiver Position [km]



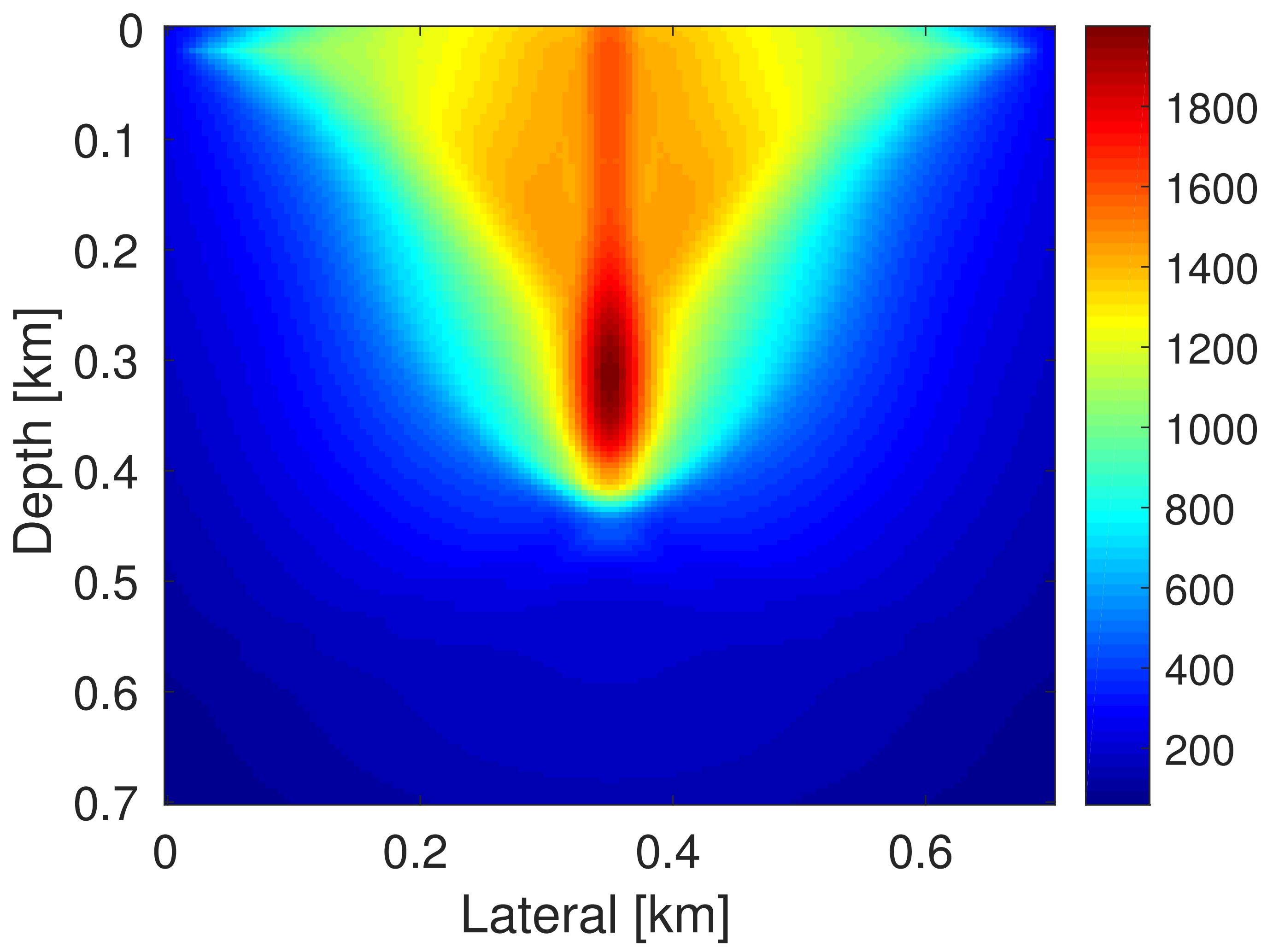
$$\mathbf{V}_1 = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_0) - \mathbf{d}))$$

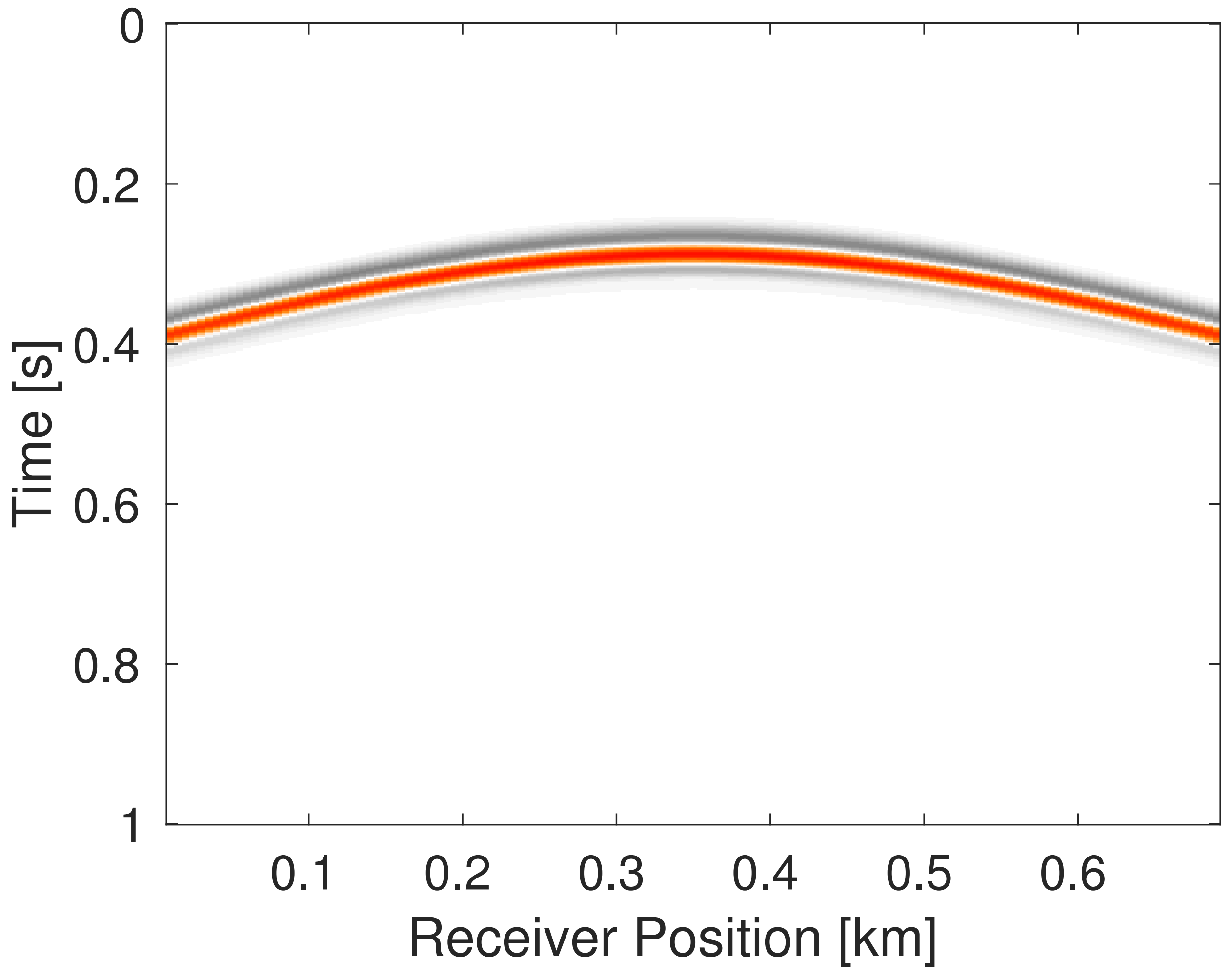
Adjoint solve



$$\mathbf{V}_1 = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_0) - \mathbf{d}))$$

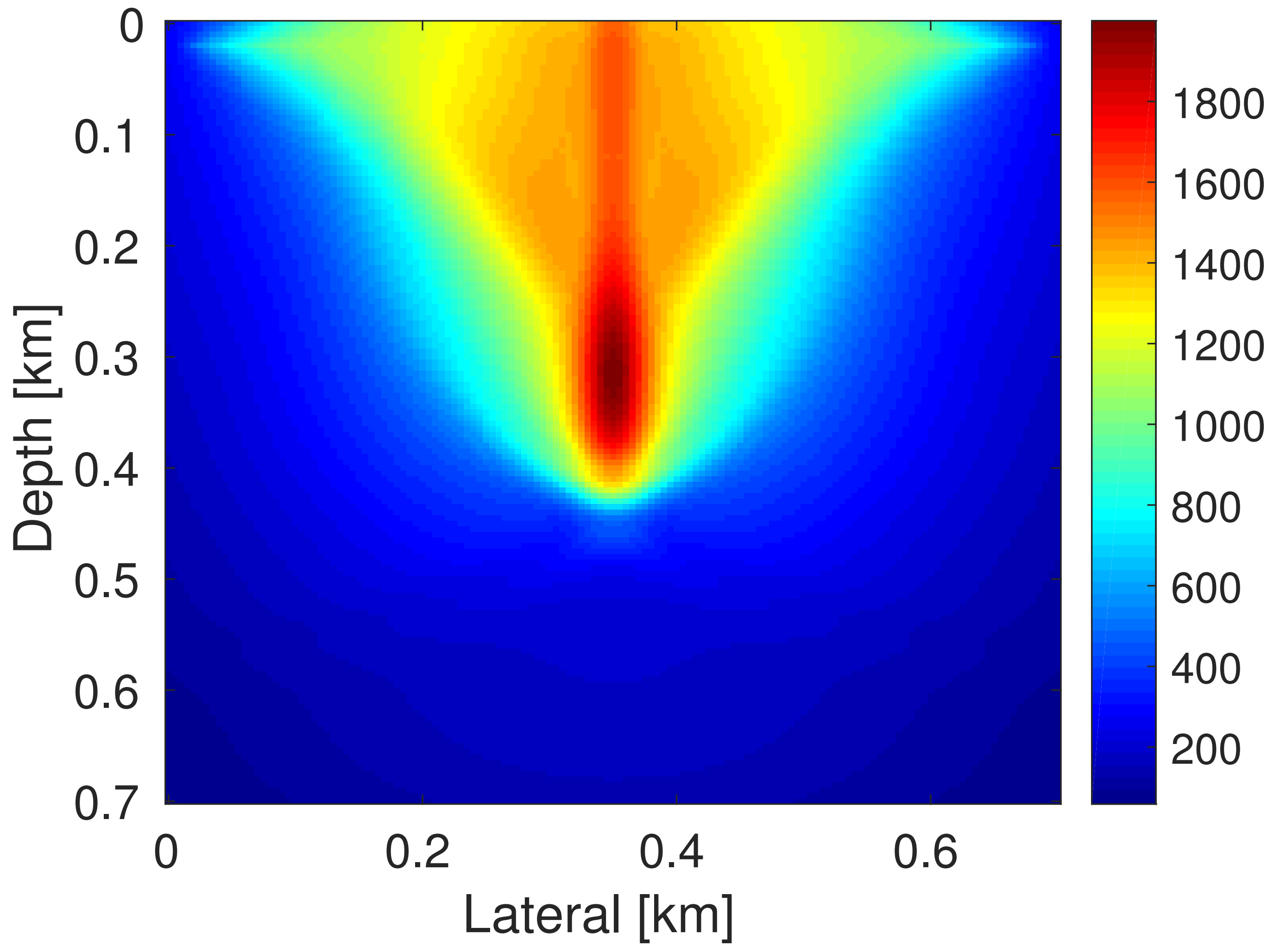
Adjoint solve





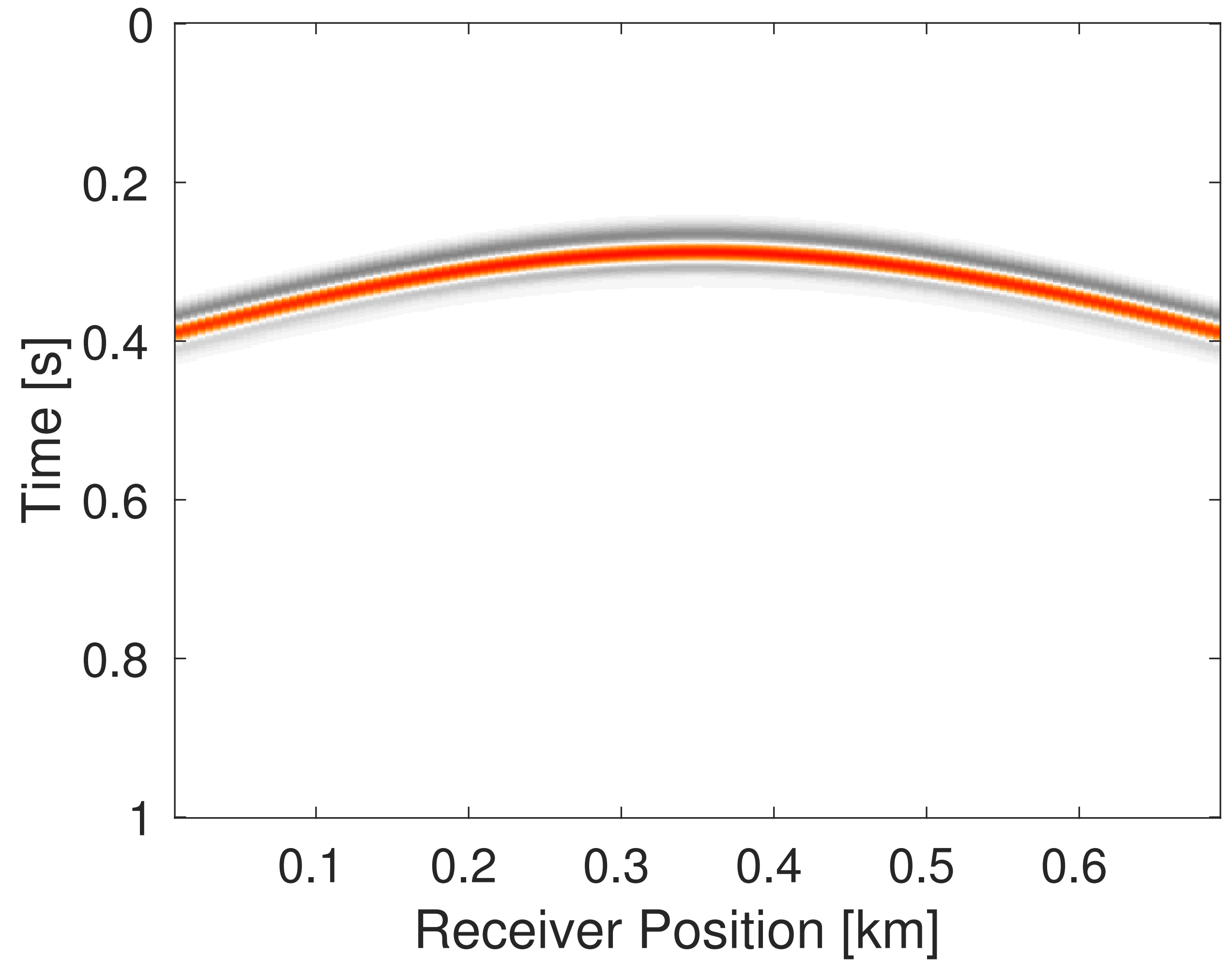
$$\mathbf{V}_1 = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_0) - \mathbf{d}))$$

Adjoint solve



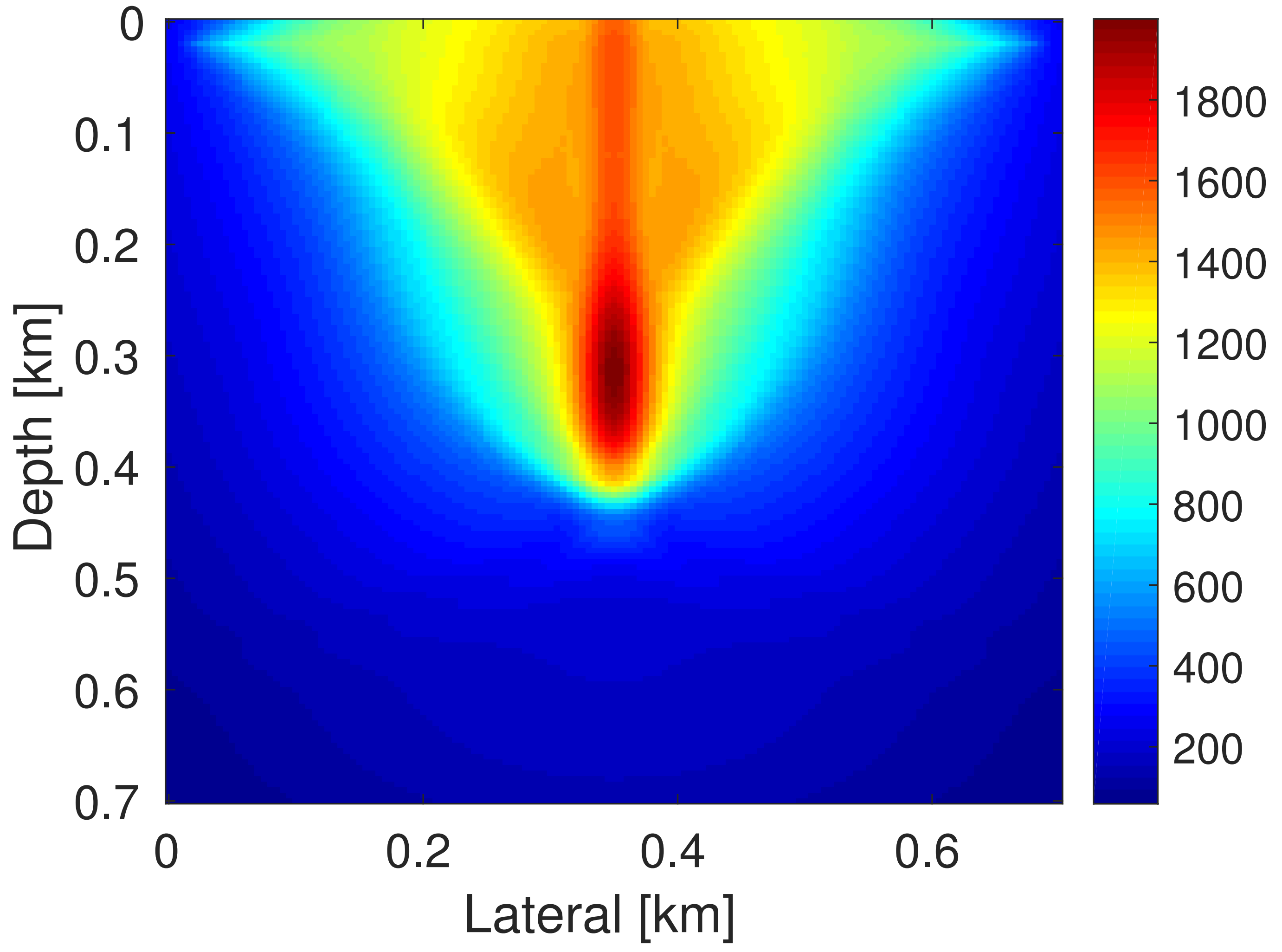
Auxiliary variable update

$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1\mathbf{V}_1$$



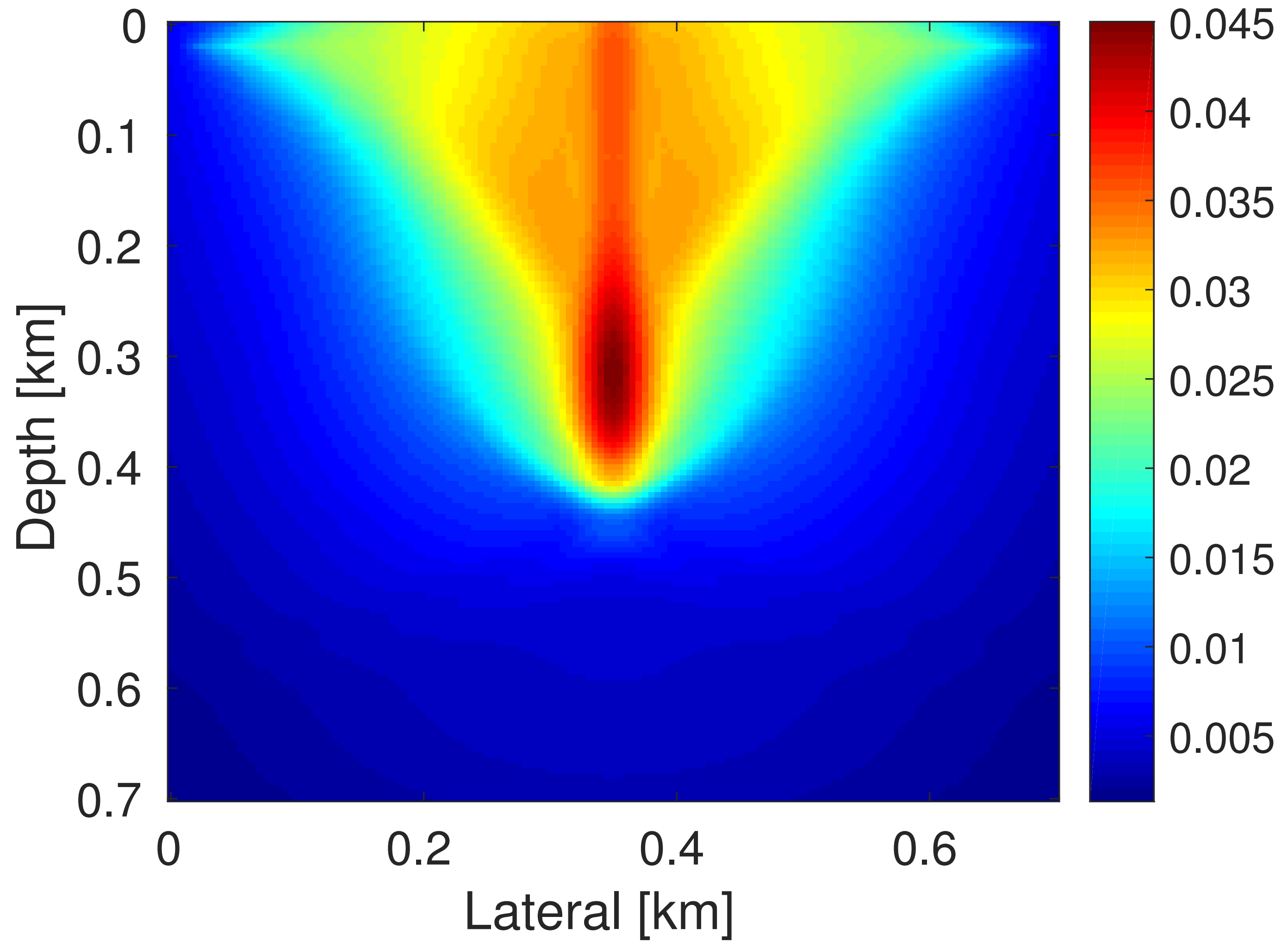
$$\mathbf{V}_1 = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_0) - \mathbf{d}))$$

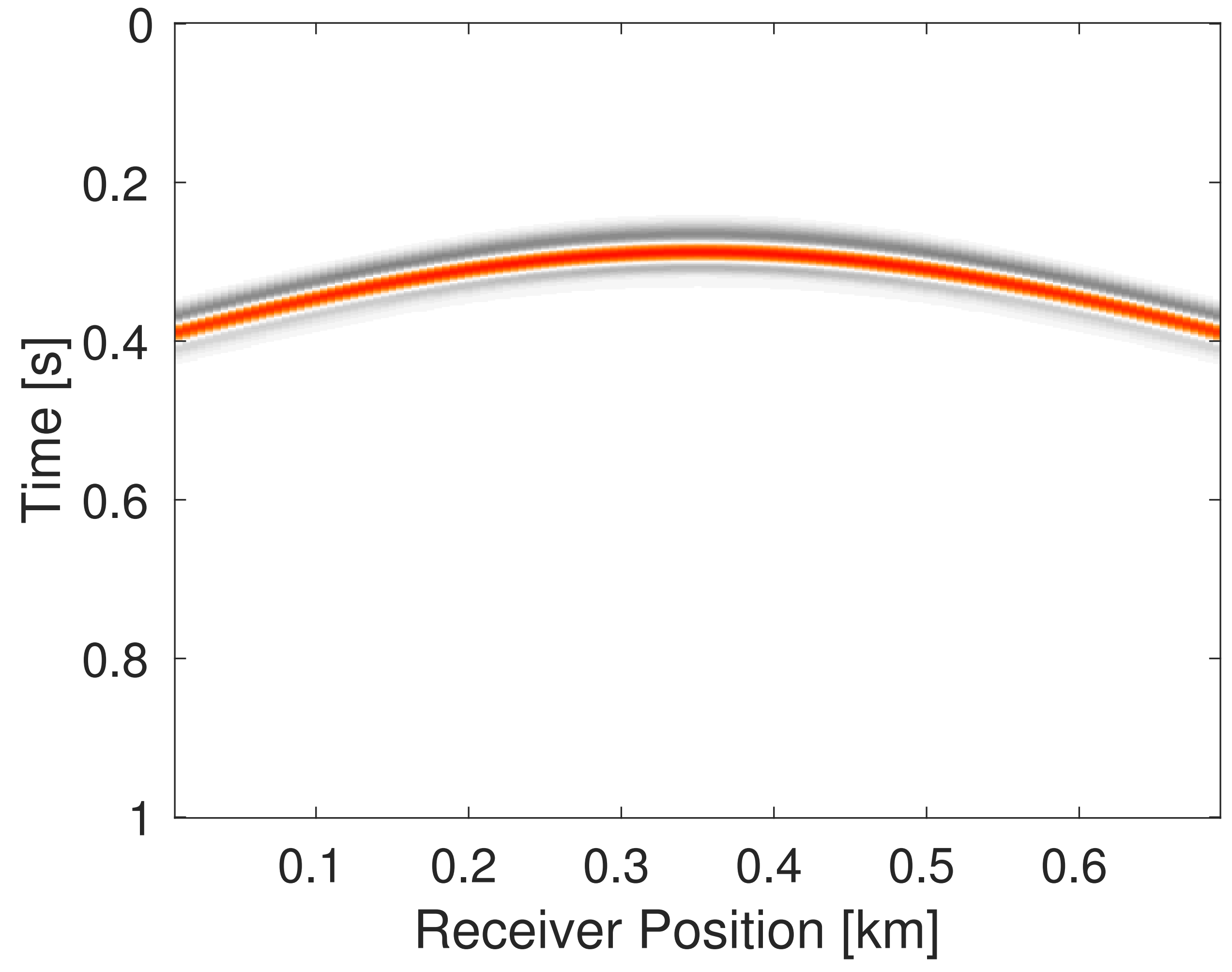
Adjoint solve



Auxiliary variable update

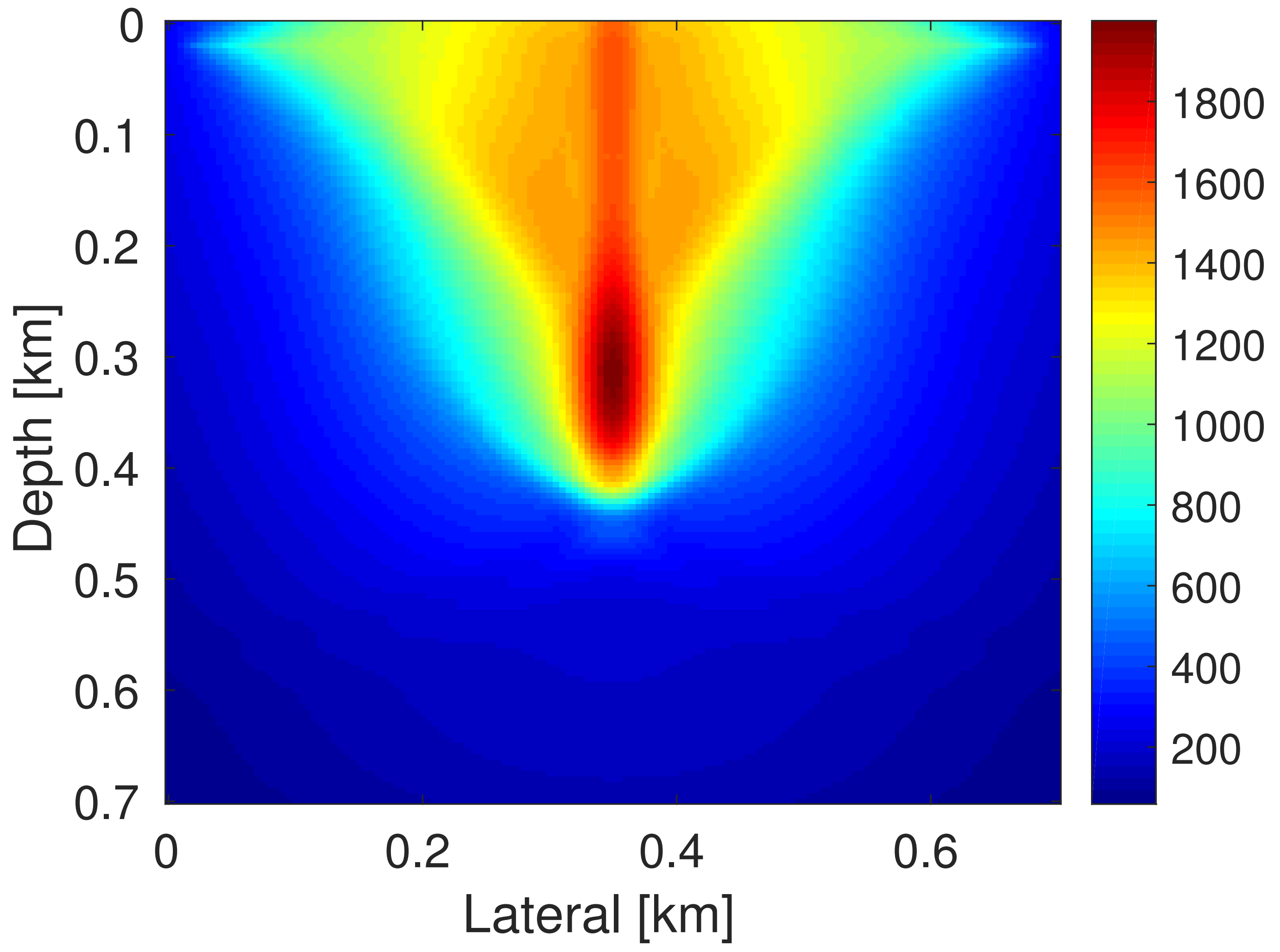
$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1$$





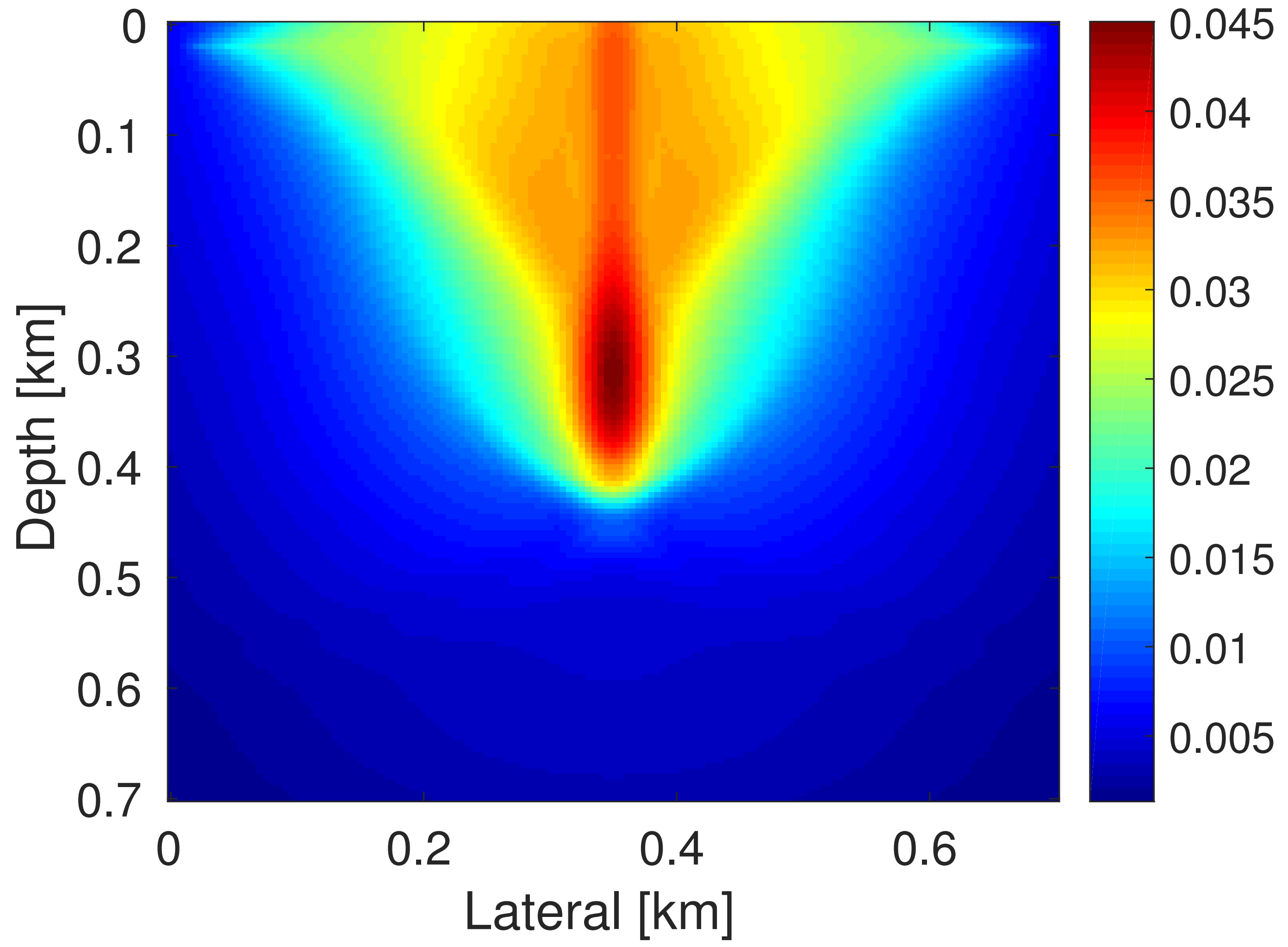
$$\mathbf{V}_1 = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_0) - \mathbf{d}))$$

Adjoint solve



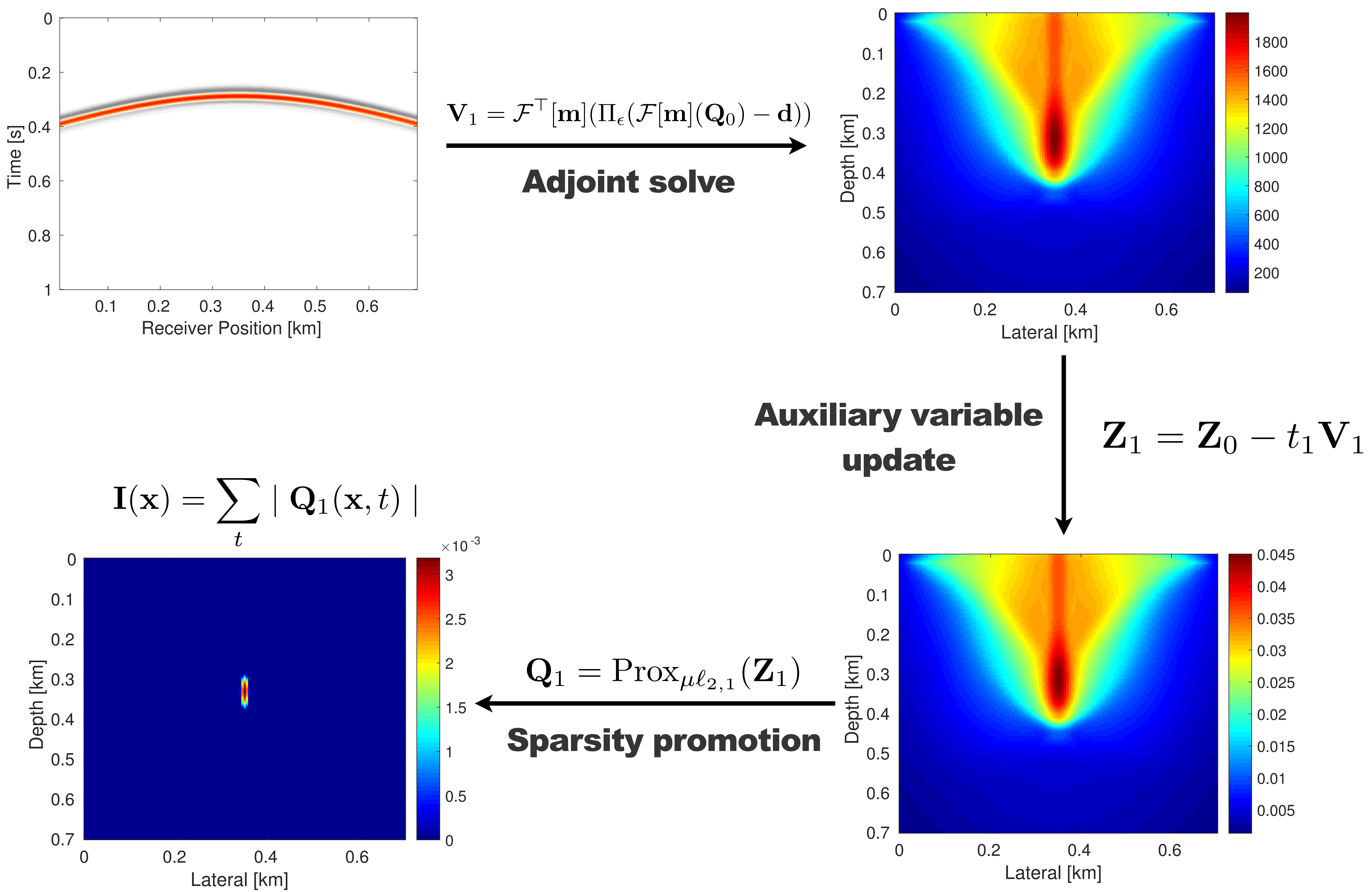
Auxiliary variable update

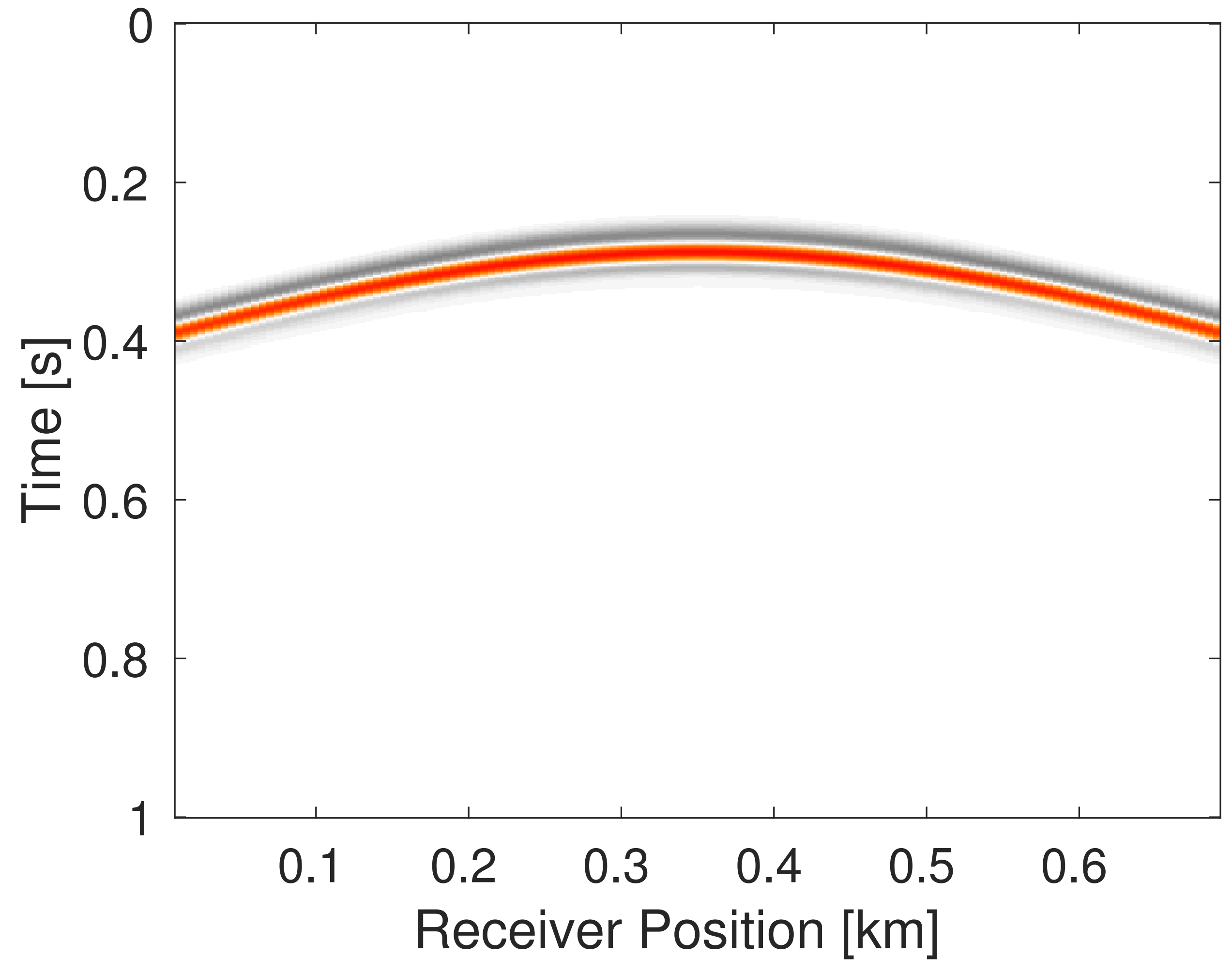
$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1\mathbf{V}_1$$



$$\mathbf{Q}_1 = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_1)$$

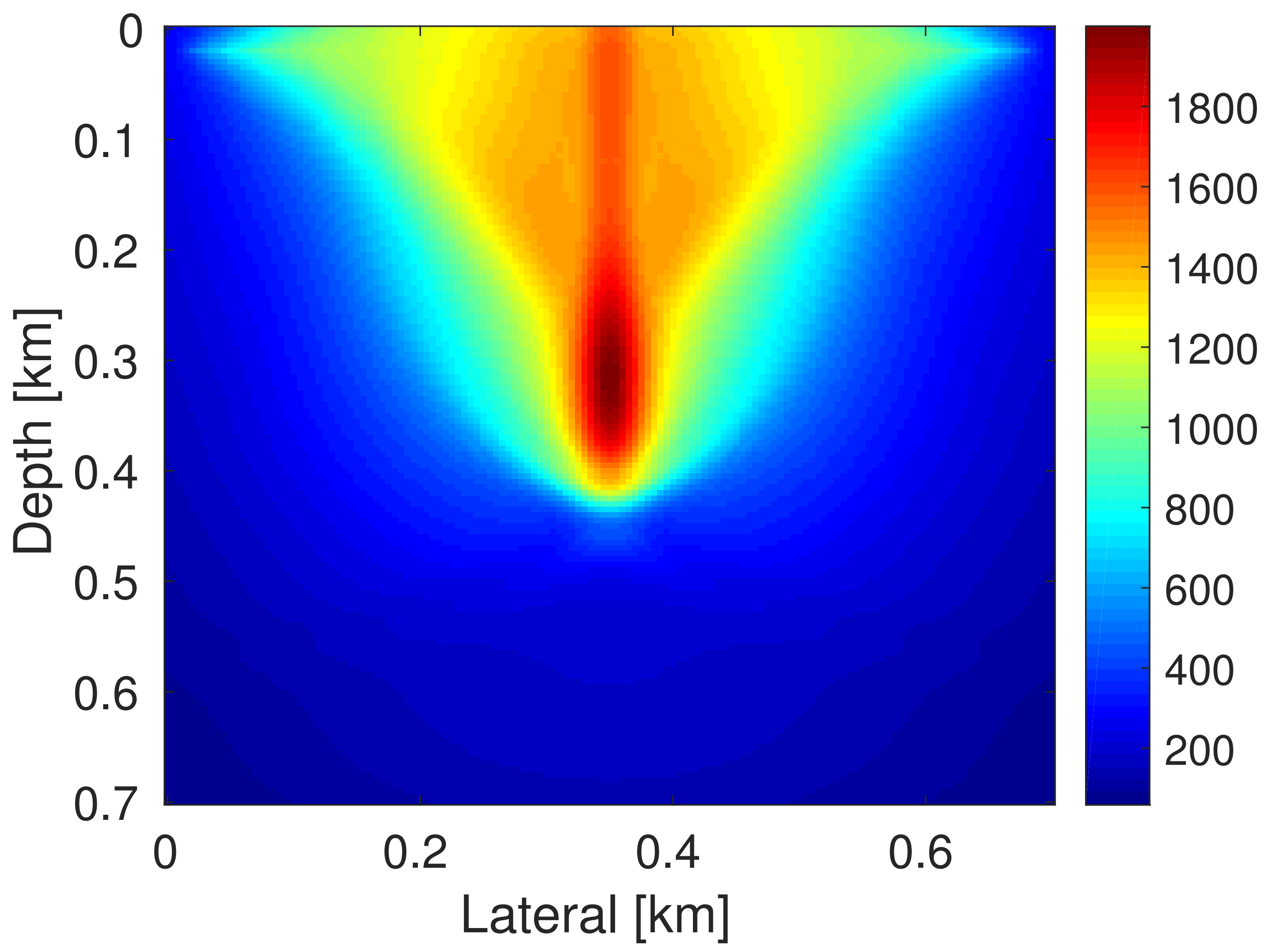
Sparsity promotion


Time [s]
Receiver Position [km]
$\mathbf{V}_1 = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_0) - \mathbf{d}))$
Adjoint solve
Depth [km]
Lateral [km]
Auxiliary variable update
$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1$
$\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}_1(\mathbf{x}, t) \mid$
$\mathbf{Q}_1 = \mathrm{Prox}_{\mu \ell_{2,1}}(\mathbf{Z}_1)$
Sparsity promotion
$\times 10^{-3}$



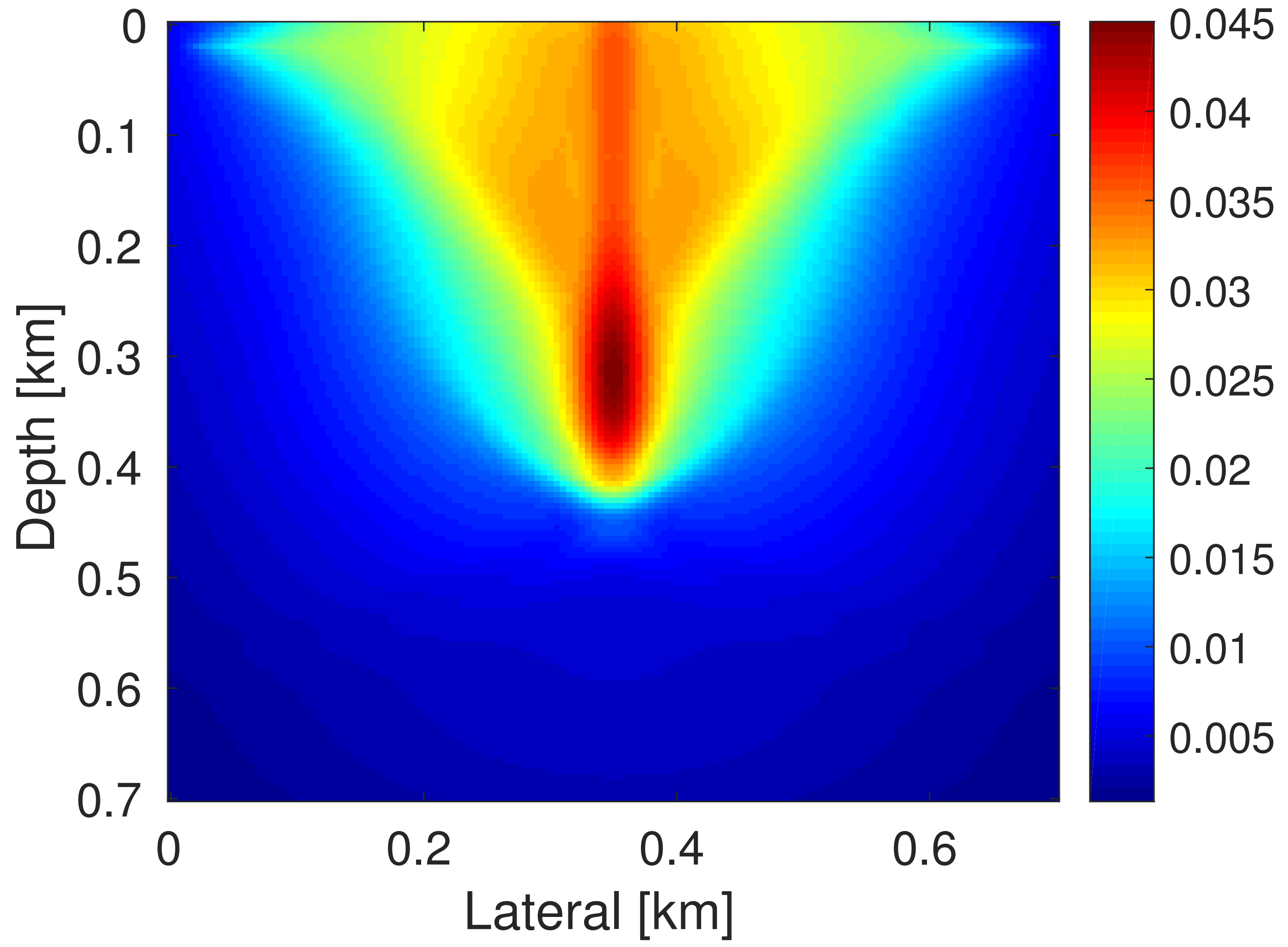
$$\mathbf{V}_1 = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_0) - \mathbf{d}))$$

Adjoint solve



Auxiliary variable update

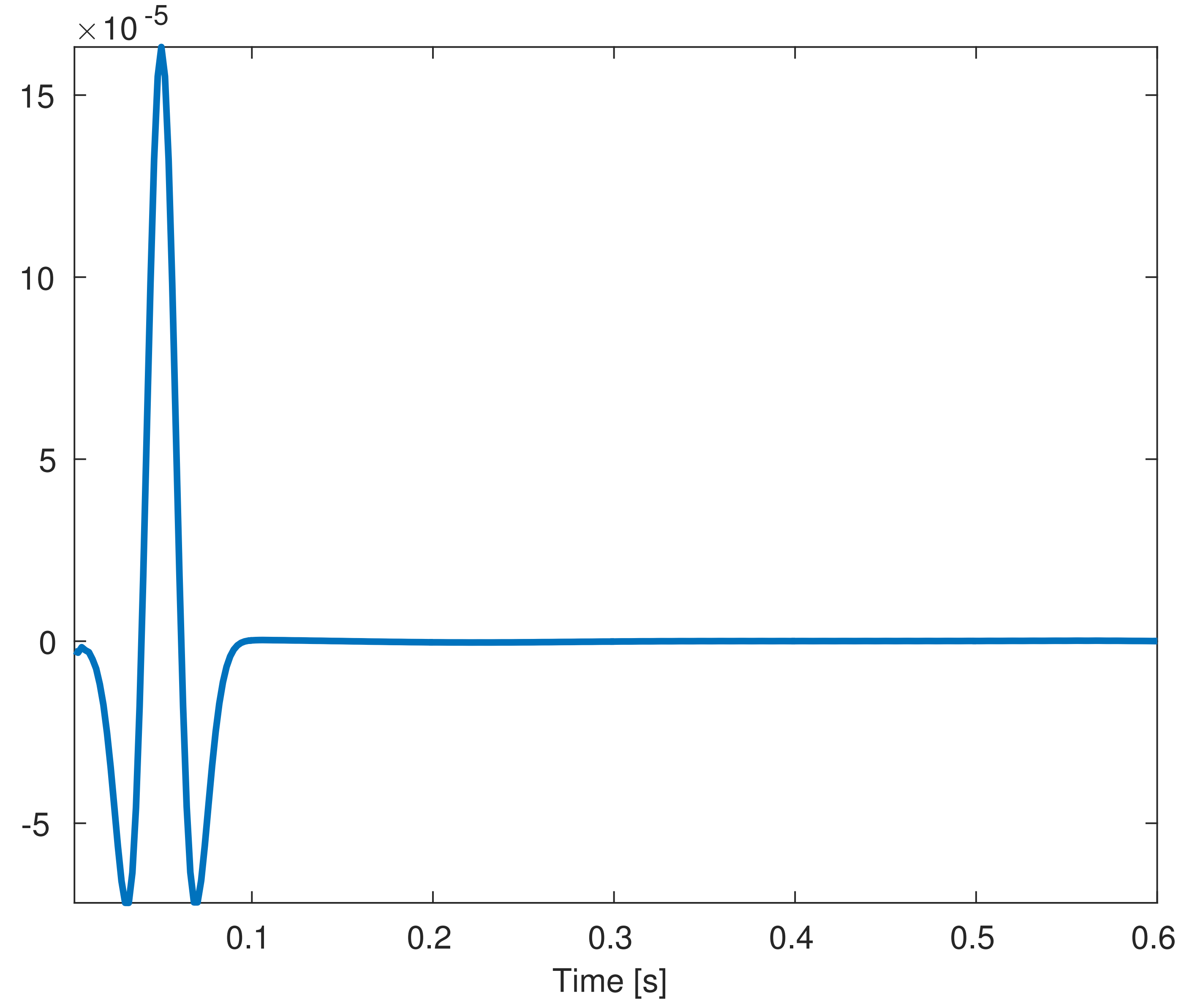
$$\mathbf{Z}_1 = \mathbf{Z}_0 - t_1\mathbf{V}_1$$



$$\mathbf{Q}_1 = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_1)$$

Sparsity promotion

Source-time function



[Lorentz et al.,'14; Combettes et al.,'11]

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \text{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

[Lorentz et al.,'14; Combettes et al.,'11]

SLIM

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k\mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

*where $t_k = \frac{\|\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}\|^2}{\|\mathcal{F}^\top[\mathbf{m}](\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d})\|^2}$ is the dynamic step length

[Lorentz et al.,'14; Combettes et al.,'11]

SLIM

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

*where $t_k = \frac{\|\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}\|^2}{\|\mathcal{F}^\top[\mathbf{m}](\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d})\|^2}$ is the dynamic step length

* $\mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{C}) := \arg\min_{\mathbf{B}} \|\mathbf{B}\|_{2,1} + \frac{1}{2\mu}\|\mathbf{C} - \mathbf{B}\|_F^2$ is the proximal mapping of the $\ell_{2,1}$ norm

[Lorentz et al.,'14; Combettes et al.,'11]

SLIM

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t \mid \mathbf{Q}(\mathbf{x}, t) \mid$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

*where $t_k = \frac{\|\mathcal{F}[\mathbf{m}](\mathbf{Q}_k)-\mathbf{d}\|^2}{\|\mathcal{F}^\top[\mathbf{m}](\mathcal{F}[\mathbf{m}](\mathbf{Q}_k)-\mathbf{d})\|^2}$ is the dynamic step length

* $\mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{C}) := \arg\min_{\mathbf{B}} \|\mathbf{B}\|_{2,1} + \frac{1}{2\mu}\|\mathbf{C} - \mathbf{B}\|_F^2$ is the proximal mapping of the $\ell_{2,1}$ norm

- **Source location**: estimated as outlier in intensity plot
- **Source-time function**: temporal variation of wavefield at estimated source location

[Lorentz et al.,'14; Combettes et al.,'11]

SLIM

Linearized Bregman algorithm

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t |\, \mathbf{Q}(\mathbf{x}, t)\, |$ //Intensity plot

* $\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$ the projection on to ℓ_2 norm ball

*where $t_k = \frac{\|\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}\|^2}{\|\mathcal{F}^\top[\mathbf{m}](\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d})\|^2}$ is the dynamic step length

* $\mathrm{Prox}_{\mu\ell_{2,1}}(\mathbf{C}) := \arg\min_{\mathbf{B}} \|\mathbf{B}\|_{2,1} + \frac{1}{2\mu}\|\mathbf{C} - \mathbf{B}\|_F^2$ is the proximal mapping of the $\ell_{2,1}$ norm

- **Source location**: estimated as outlier in intensity plot
- **Source-time function**: temporal variation of wavefield at estimated source location

High level of noise

$$\Pi_\epsilon(\mathbf{r}) = \max\{0, 1 - \frac{\epsilon}{\|\mathbf{r}\|}\}.(\mathbf{r})$$

* $\mathbf{r} = \mathcal{F}[\mathbf{m}](\mathbf{Q}) - \mathbf{d}$ is the data residual

$\epsilon \gg \|\mathbf{r}\|$ for very noisy data

$\downarrow$

$$\Pi_\epsilon(\mathbf{r}) = 0$$

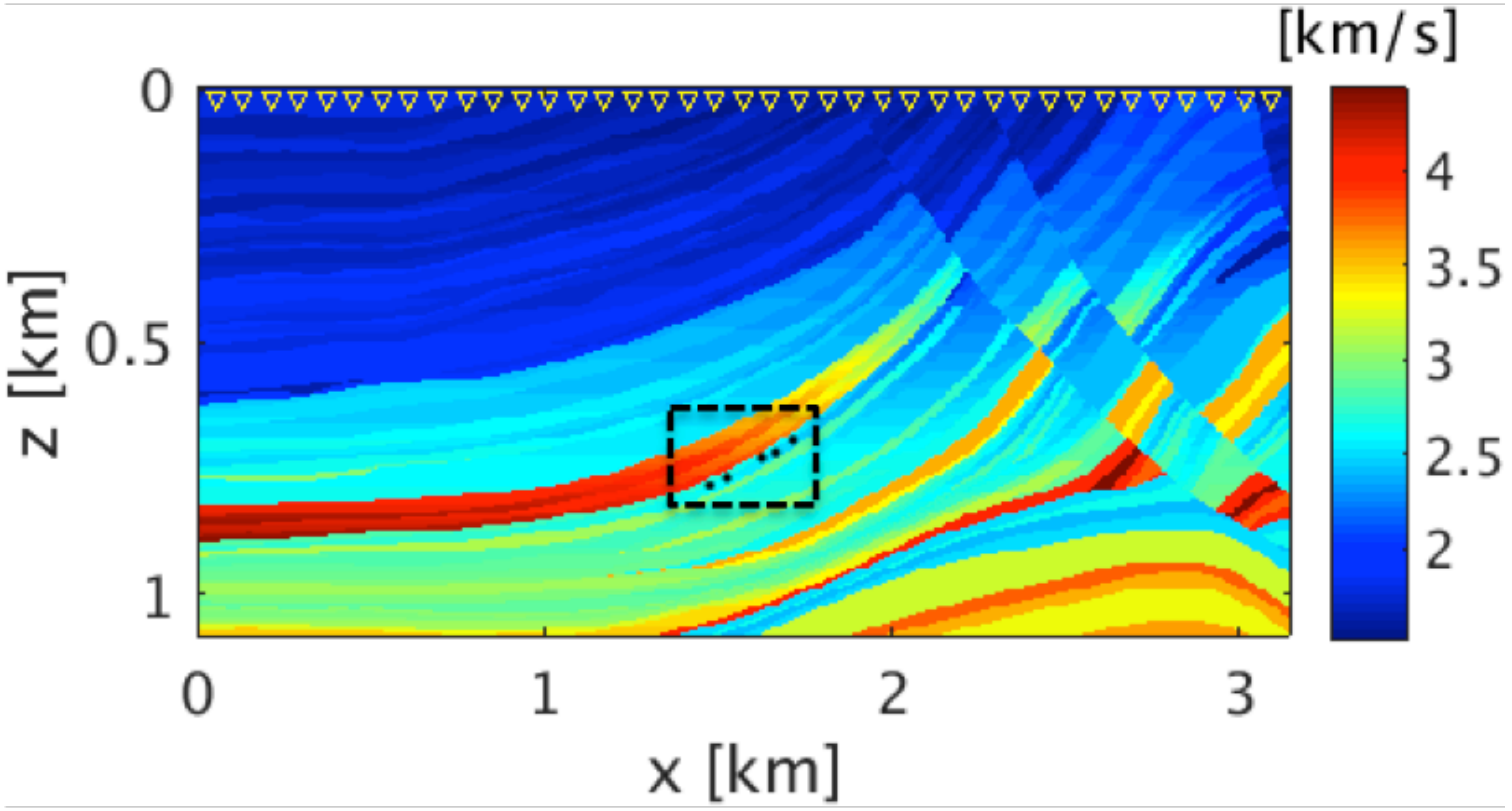
SLIM

Linearized Bregman algorithm with extreme noise

1. **Data d, slowness square m** //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\epsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$ //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$ //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \mathrm{Prox}_{\mu \ell_{2,1}}(\mathbf{Z}_{k+1})$ //sparsity promotion
6. **end**
7. $\mathbf{I}(\mathbf{x}) = \sum_t |\mathbf{Q}(\mathbf{x}, t)|$ //Intensity plot

No updates

Numerical Experiment: Extreme noise

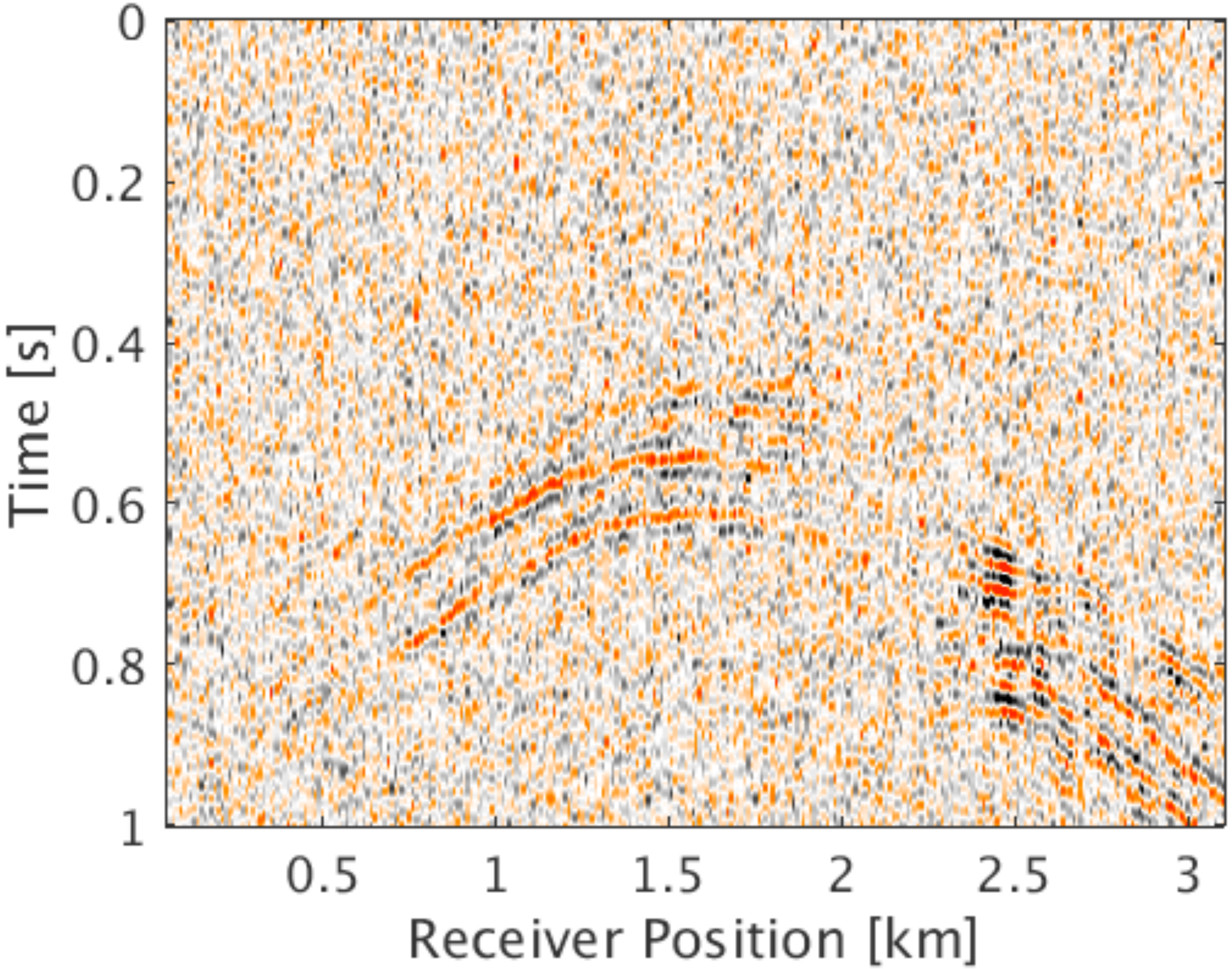


Marmousi model

Modeling information:

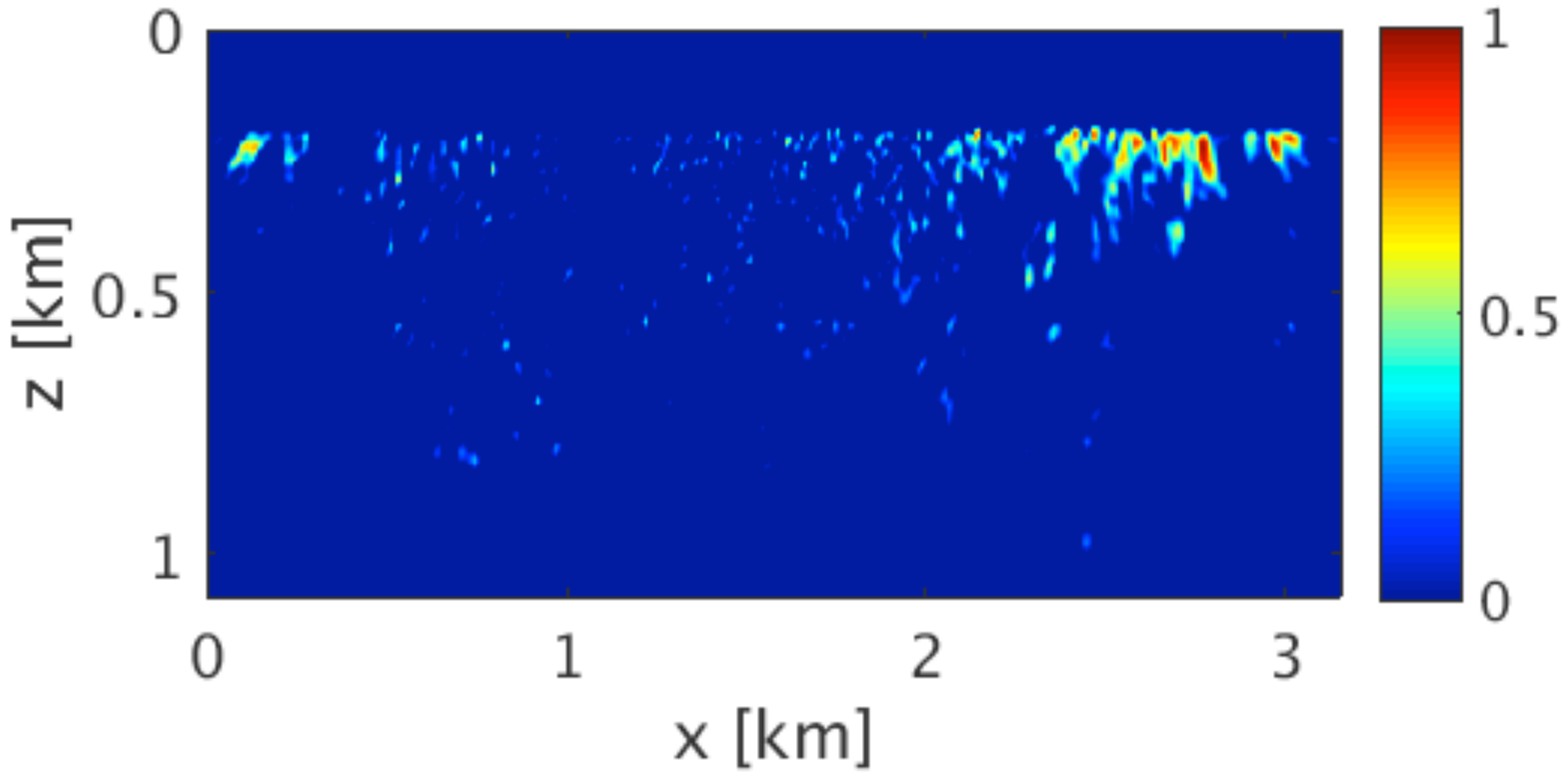
Model size: 3.15 km x 1.08 km
Grid spacing: 5 m
Total number of sources: 5
Peak frequency : 25 Hz & 30 Hz
Receiver spacing: 10m
Receiver depth: 20m
Sampling interval: 0.5 ms
Recording length: 1 s
Free surface: No
Amplitude ratio of sources: 2:1

Noisy Data



SNR: -7.3 dB

Estimated source location w/o denoising



[Forghani-Arani et al.,’12; St-Onge et al.,’11]

SLIM

Properties of noise in microseismic data

Amplitude of ambient noise is similar or higher than the amplitude of signal

Frequency range of noise is similar to the frequency range of signal

This makes signal and noise separation difficult

Eventually causes problem in detecting microseismic sources

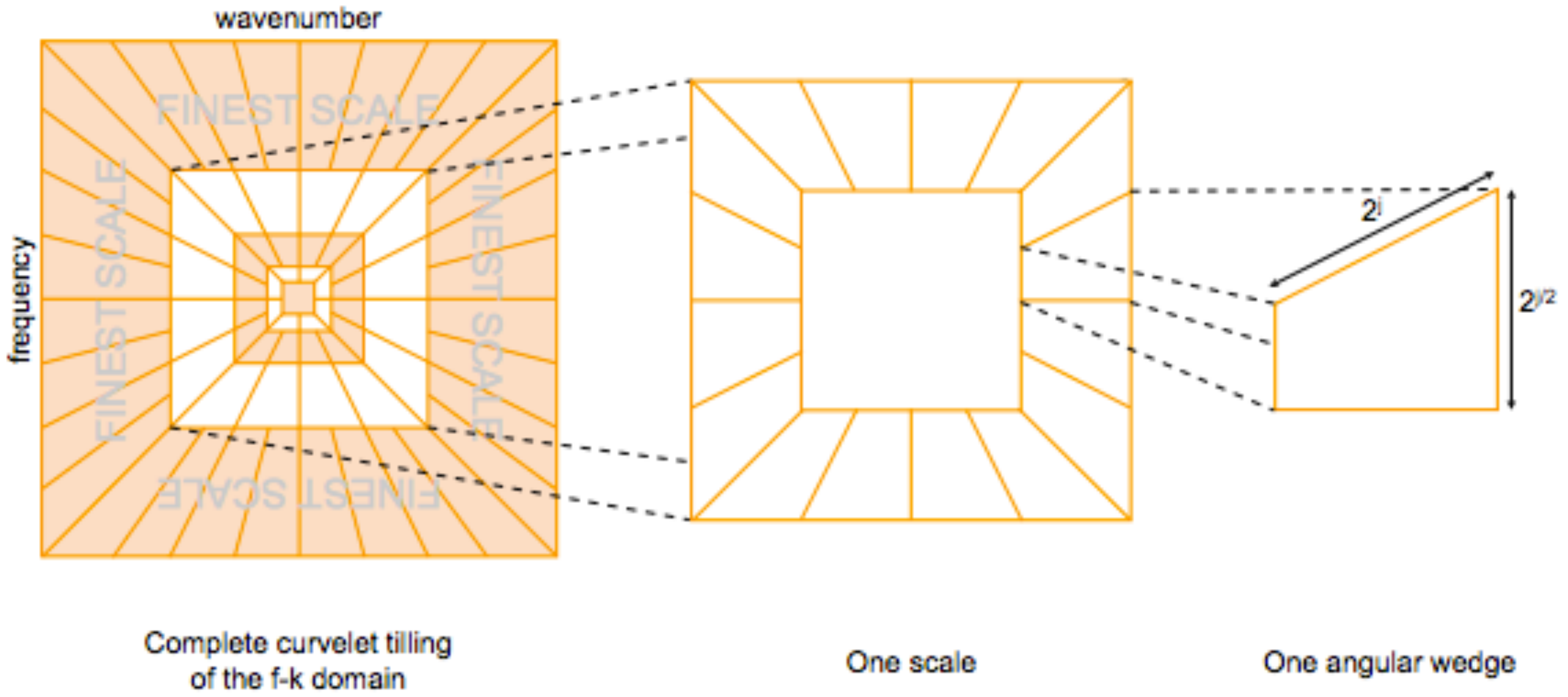
Road ahead

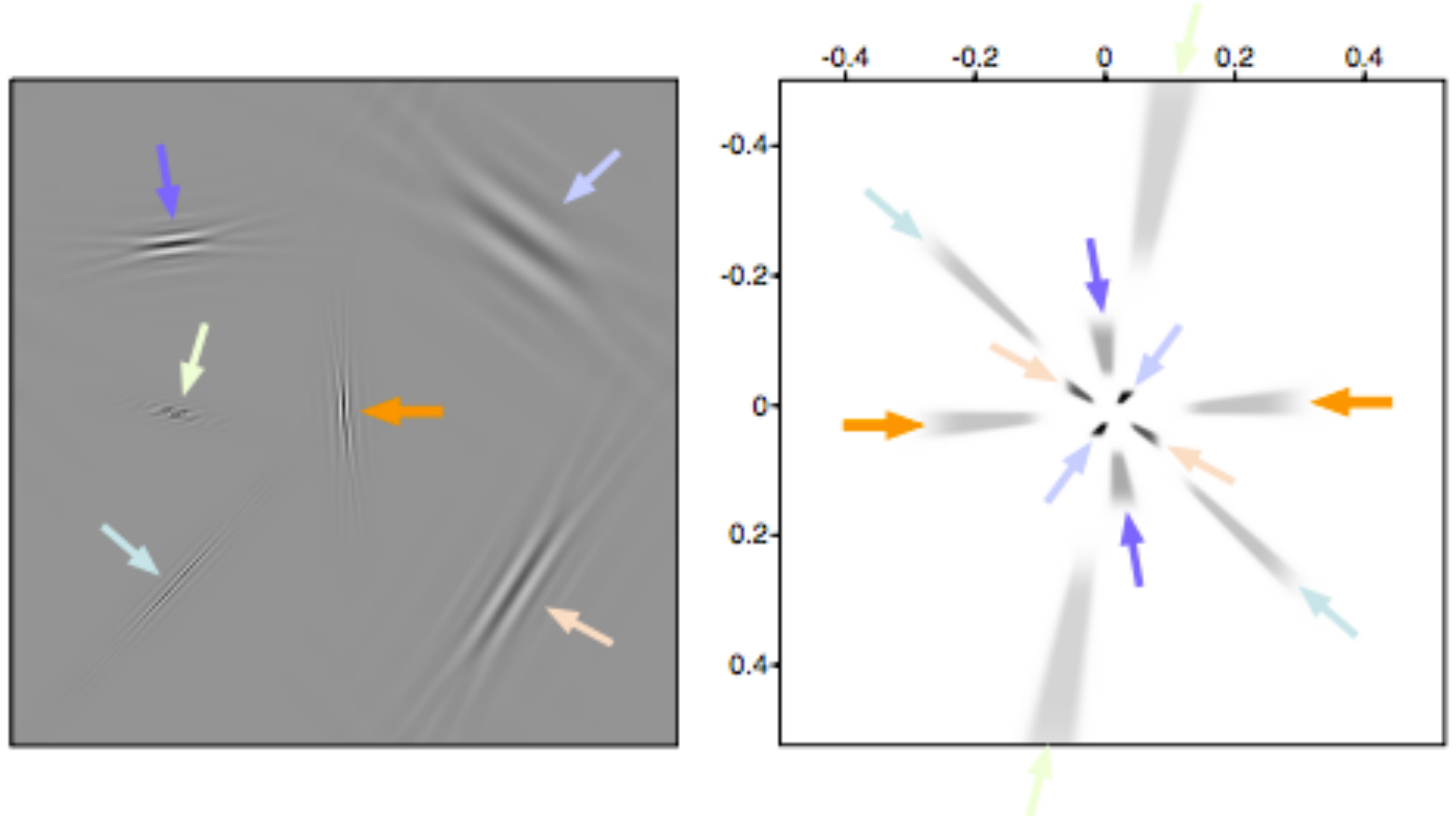
Noise is a big problem in microseismic data

Transform based methods which discriminate signal components based on directionality and scale can be useful in separating microseismic signal and noise

[Candes et al.,'00; Herrmann et al.,'08]

Curvelet transform

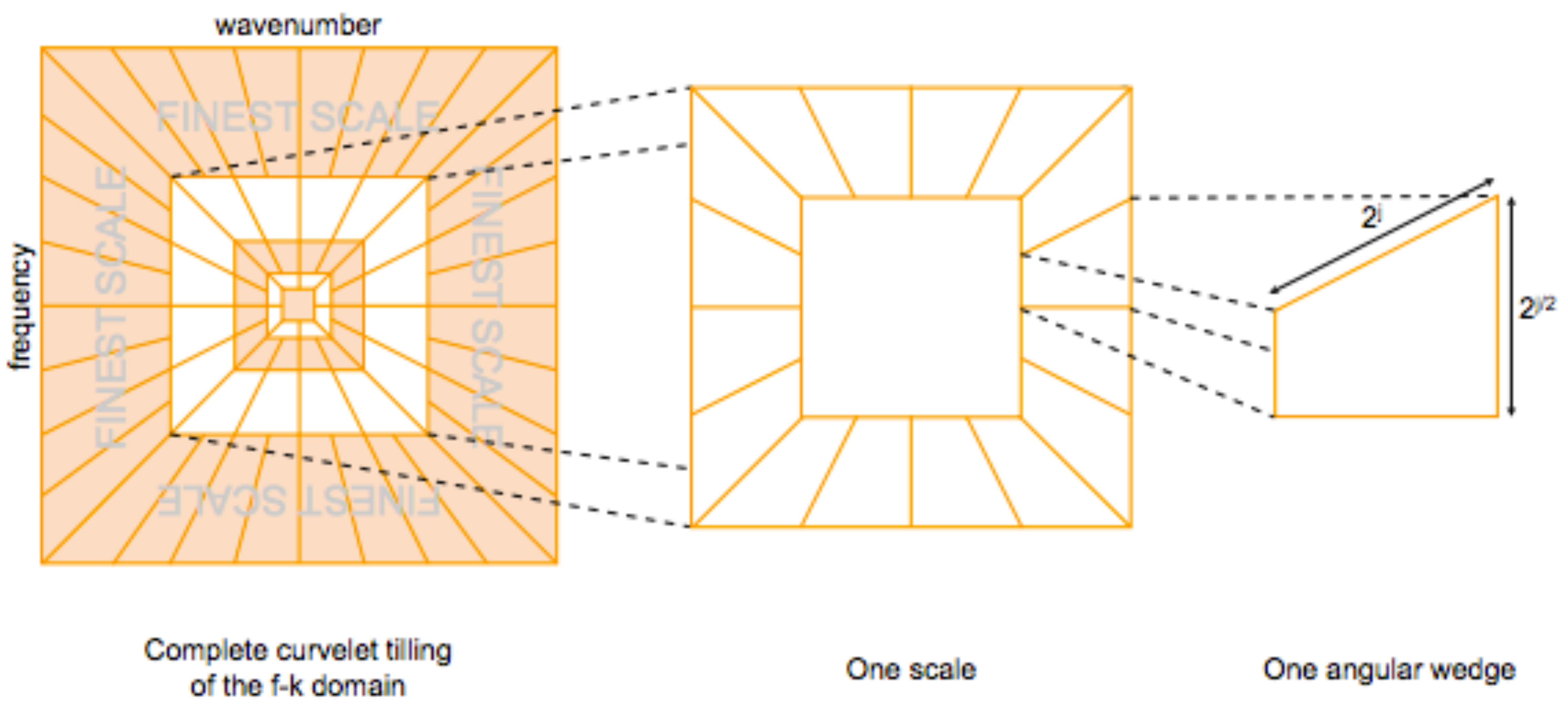


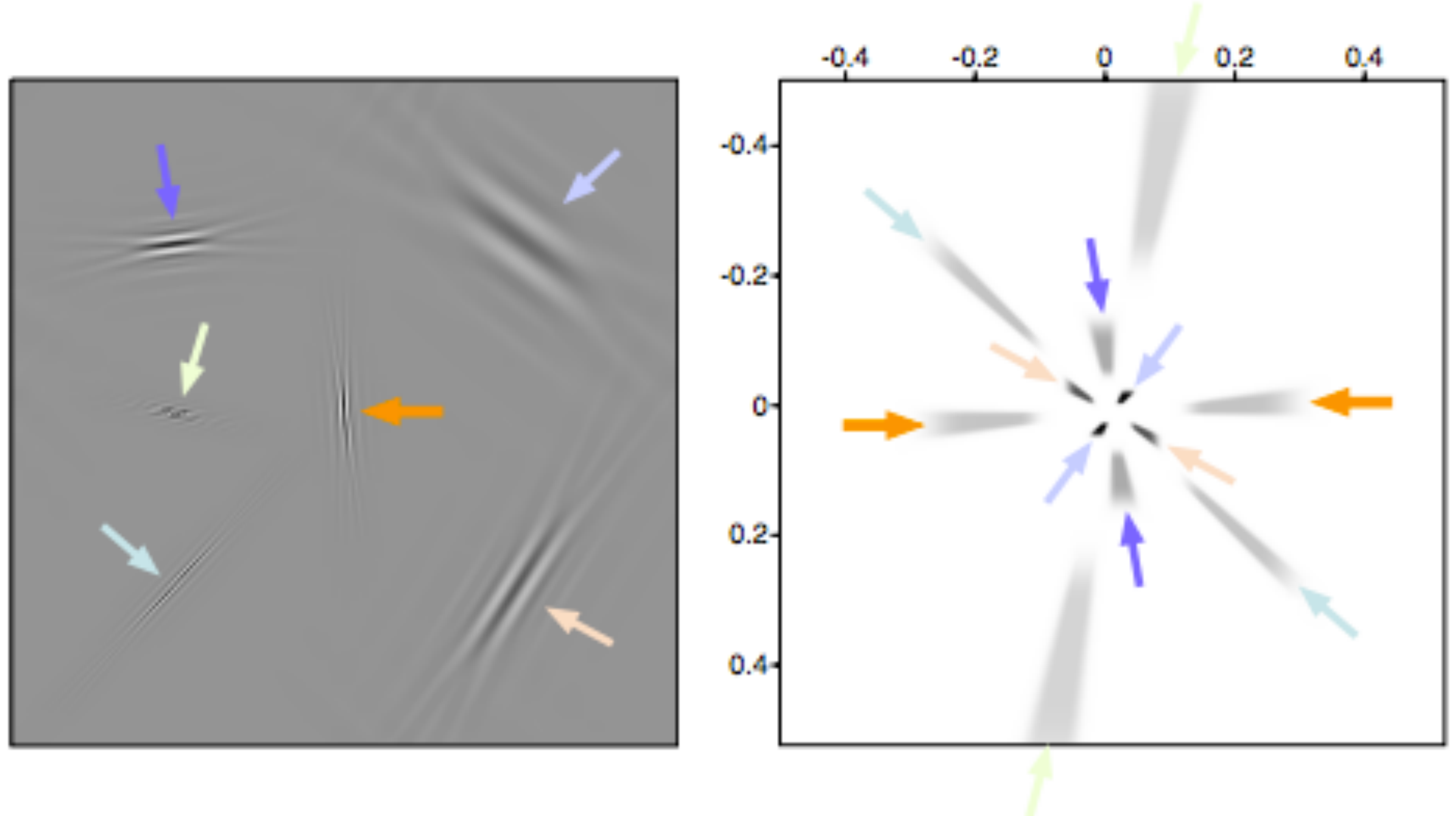


[Candes et al.,'00; Herrmann et al.,'08]

SLIM

Curvelet transform



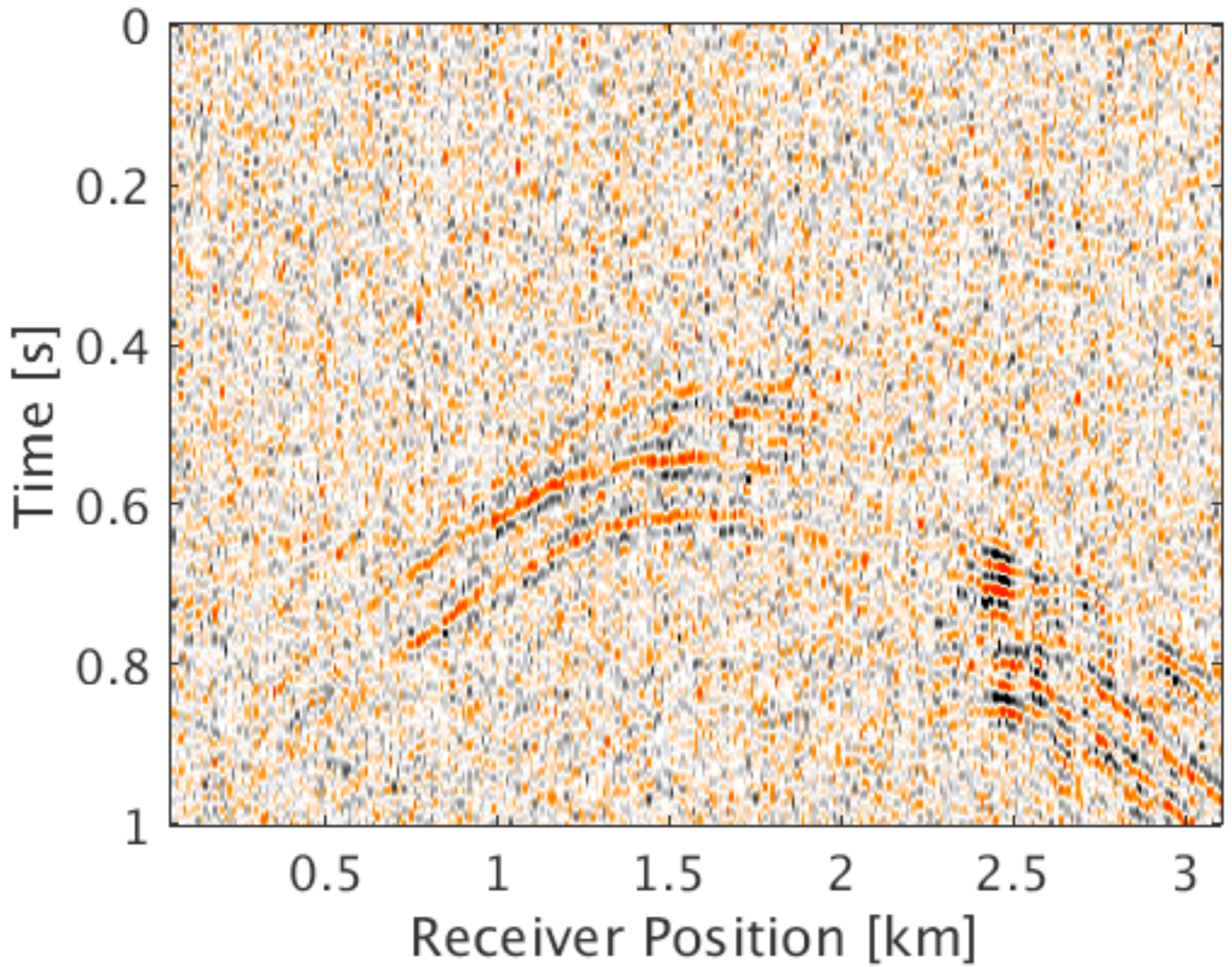


- **Curvelet transform:** multi-scale and multi-directional transform
- Maps seismic data into angular wedges of different scales in f-k domain
- Better separation of signal and noise in transform domain

Curvelet based denoising steps

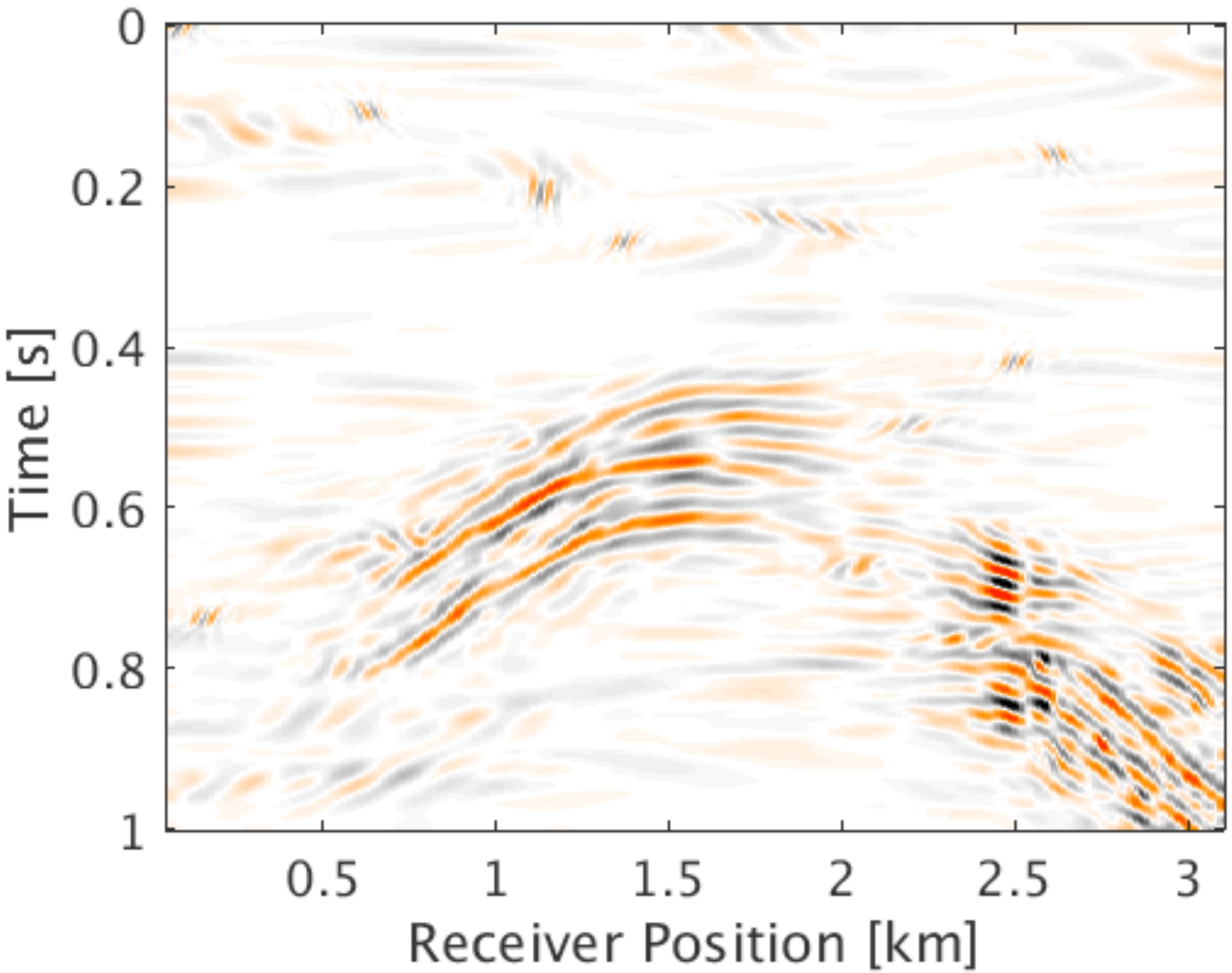
1. **Noisy Data d, forward curvelet transform operator C, Threshold parameter** λ //Input
2. $\mathbf{b} = \mathbf{C}\mathbf{d}$ //Forward curvelet transform
3. $[\mathbf{sb}, \mathbf{idx}] = \mathrm{Sort}(|\ \mathbf{b}\ |)$ //Sorting in descending order where **idx** stores the indices of sorted curvelet coefficients
4. $\mathbf{e}_h = \sqrt{\frac{\sum_{i=1}^{h} \mathbf{sb}_i^2}{\sum_{i=1}^{l} \mathbf{sb}_i^2}}$ //normalized cumulative energy
5. **Find the smallest index** p **such that** $\mathbf{e}_p \geq \lambda$
6. $\mathbf{S} = \mathbf{C}^H(\mathbf{idx}(1:p), :)$ //New inverse curvelet transform operator
7. $\mathbf{b}_{dn} = (\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H\mathbf{d}$ //Solving the normal equation
8. $\mathbf{d}_{dn} = \Re(\mathbf{S}\mathbf{b}_{dn})$ //denoising

Noisy Data



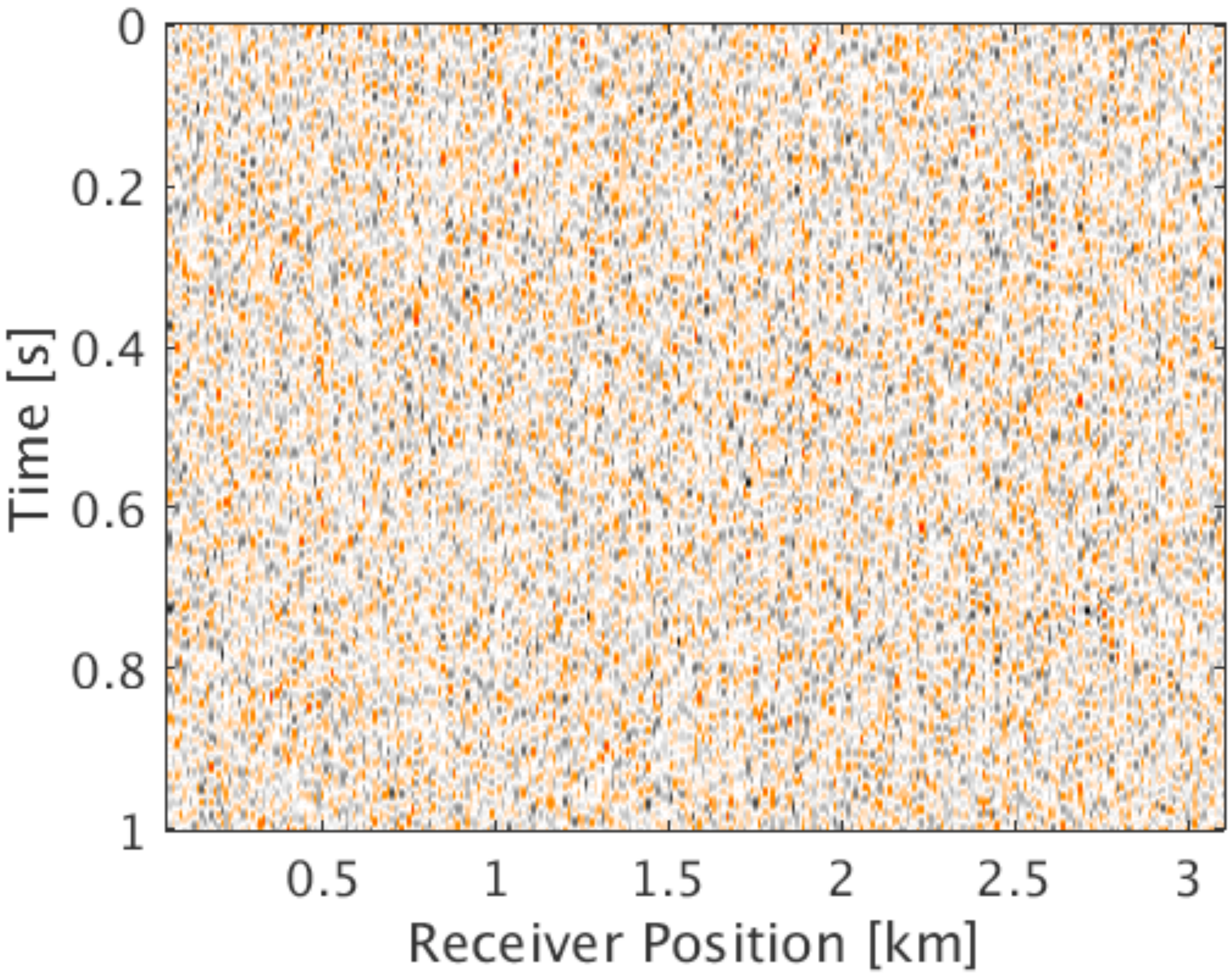
SNR: -7.3 dB

Denoised data w/ curvelet based denoising

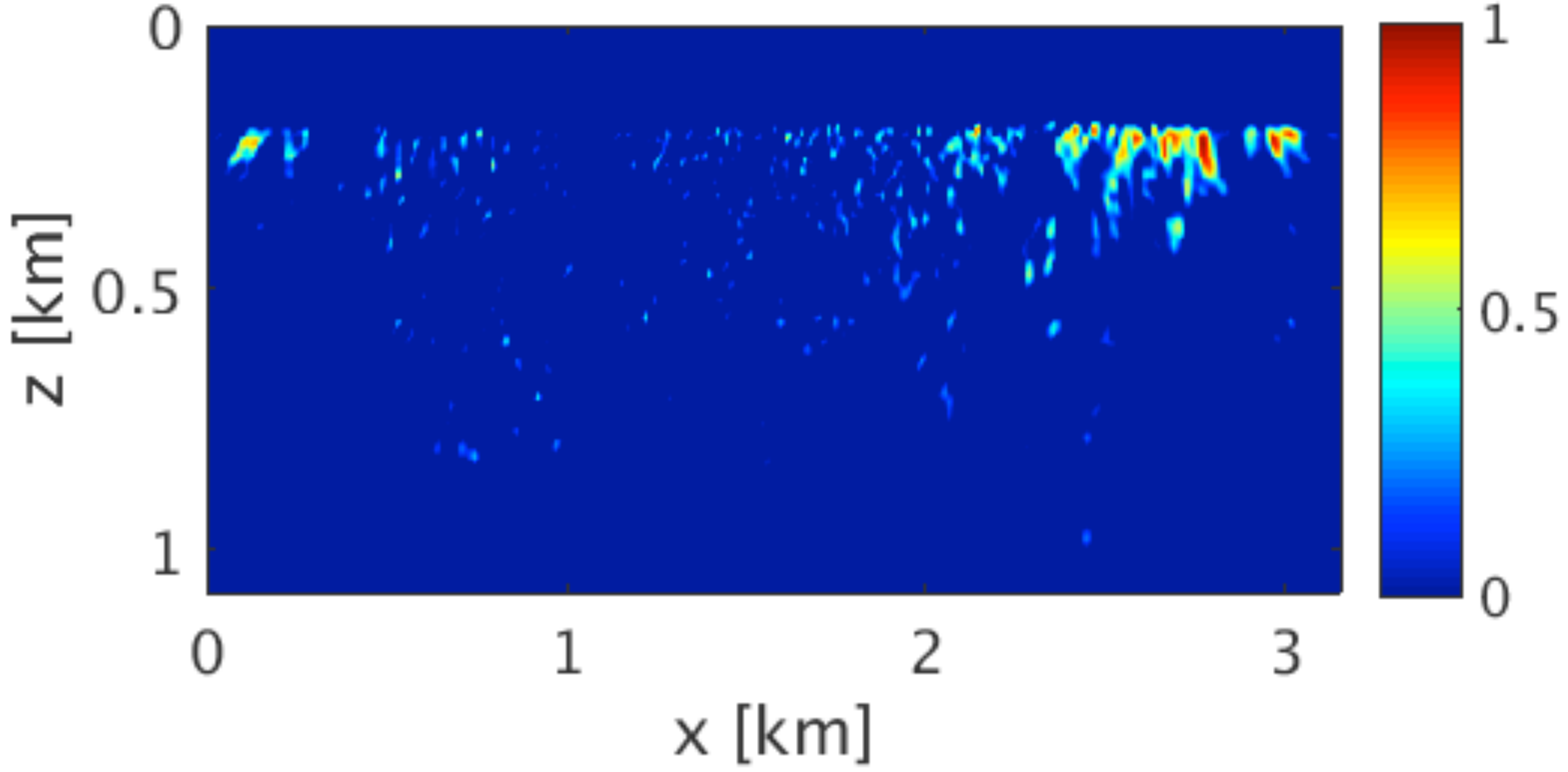


SNR: 3.5 dB

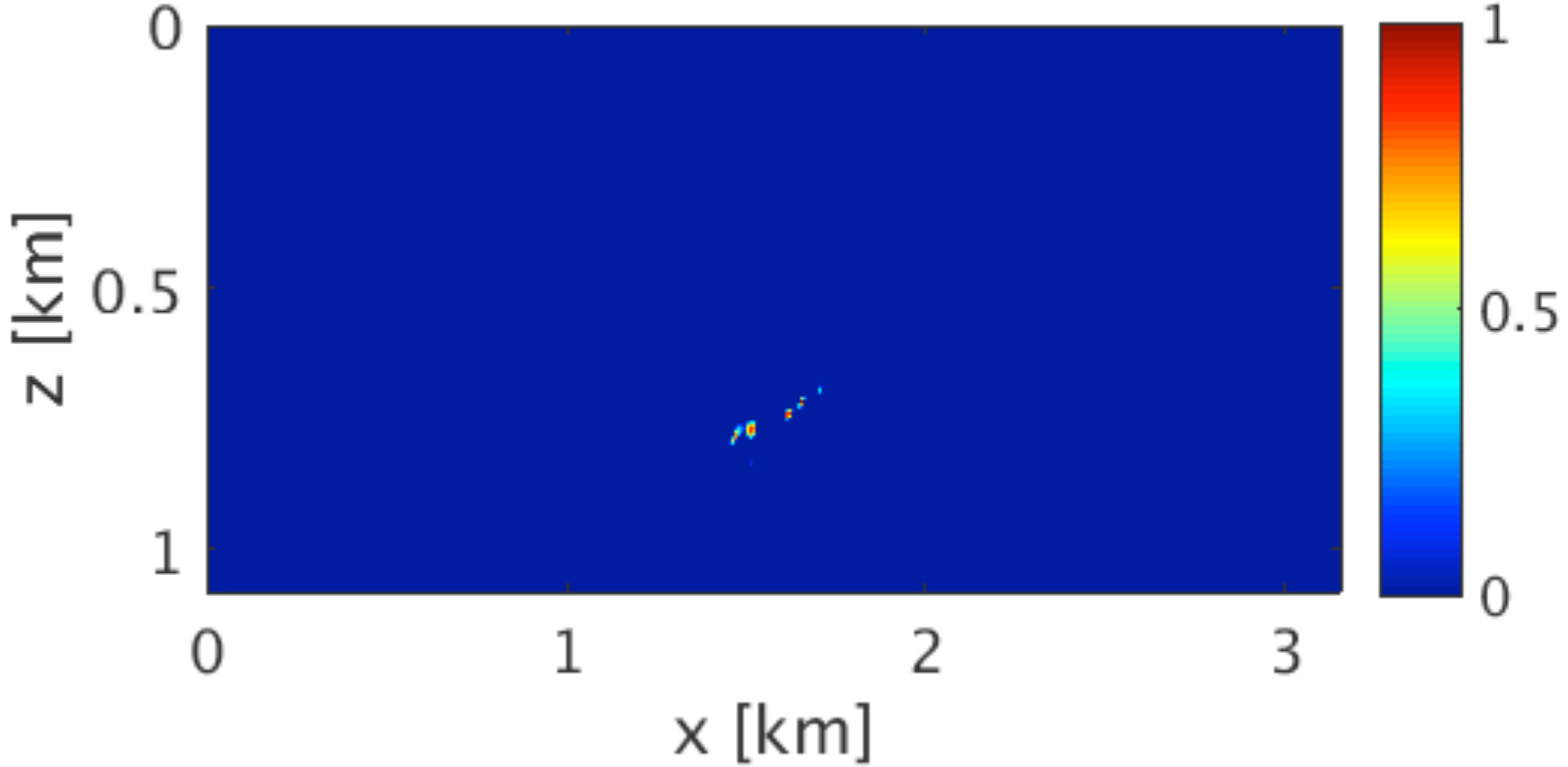
Difference



Estimated source location w/o denoising

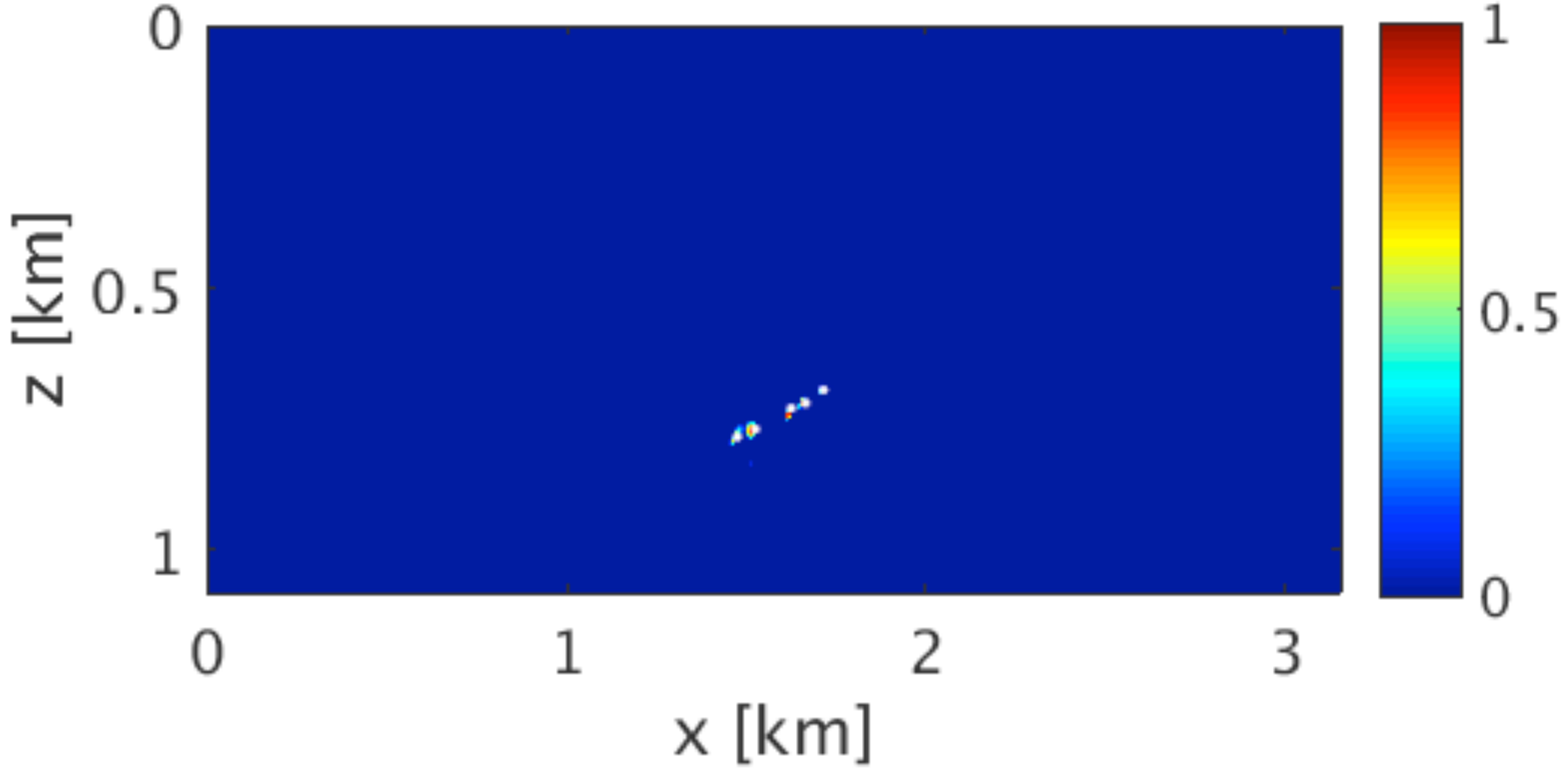


Estimated source location after denoising

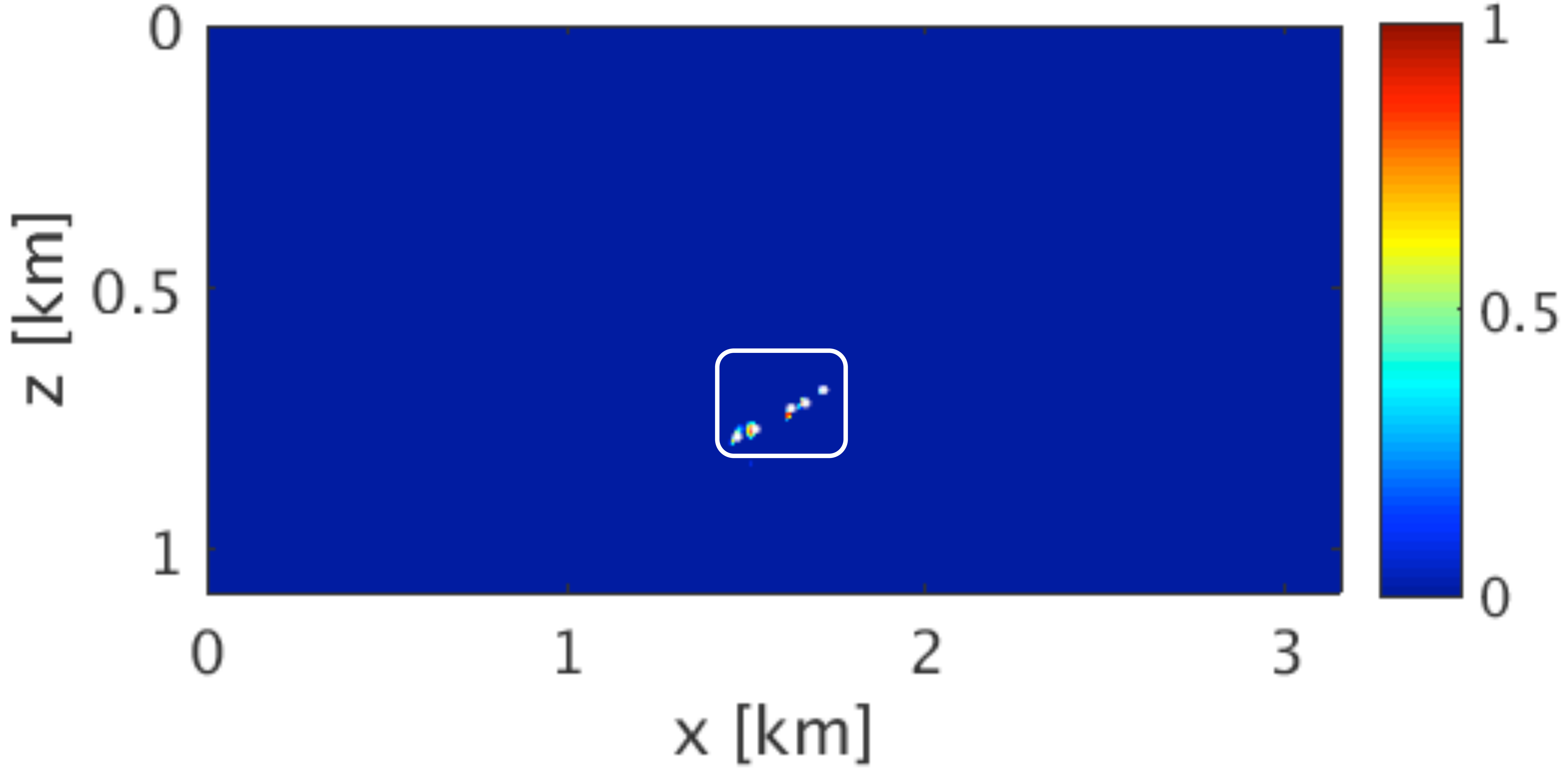


SLIM

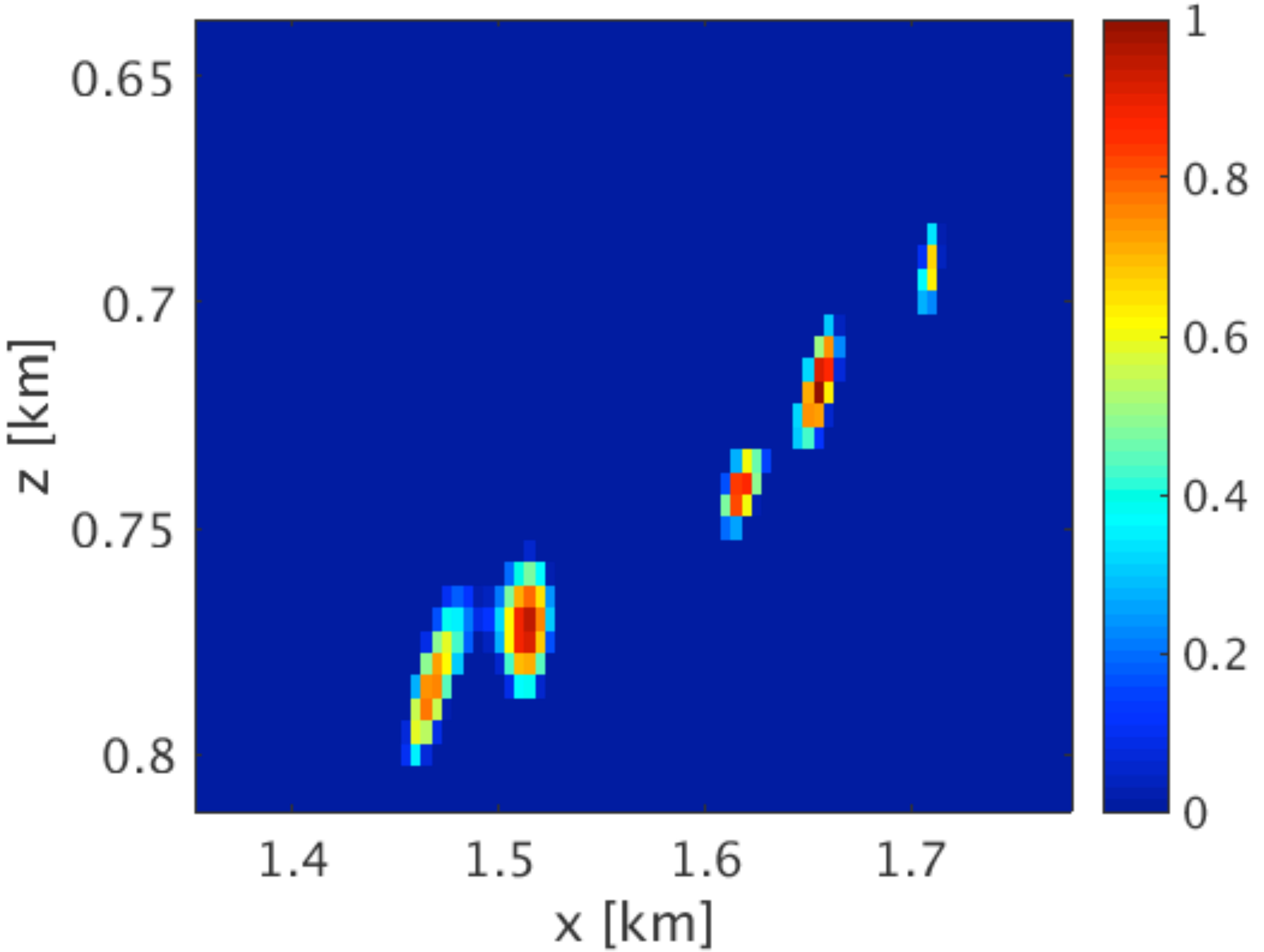
Estimated source location after denoising



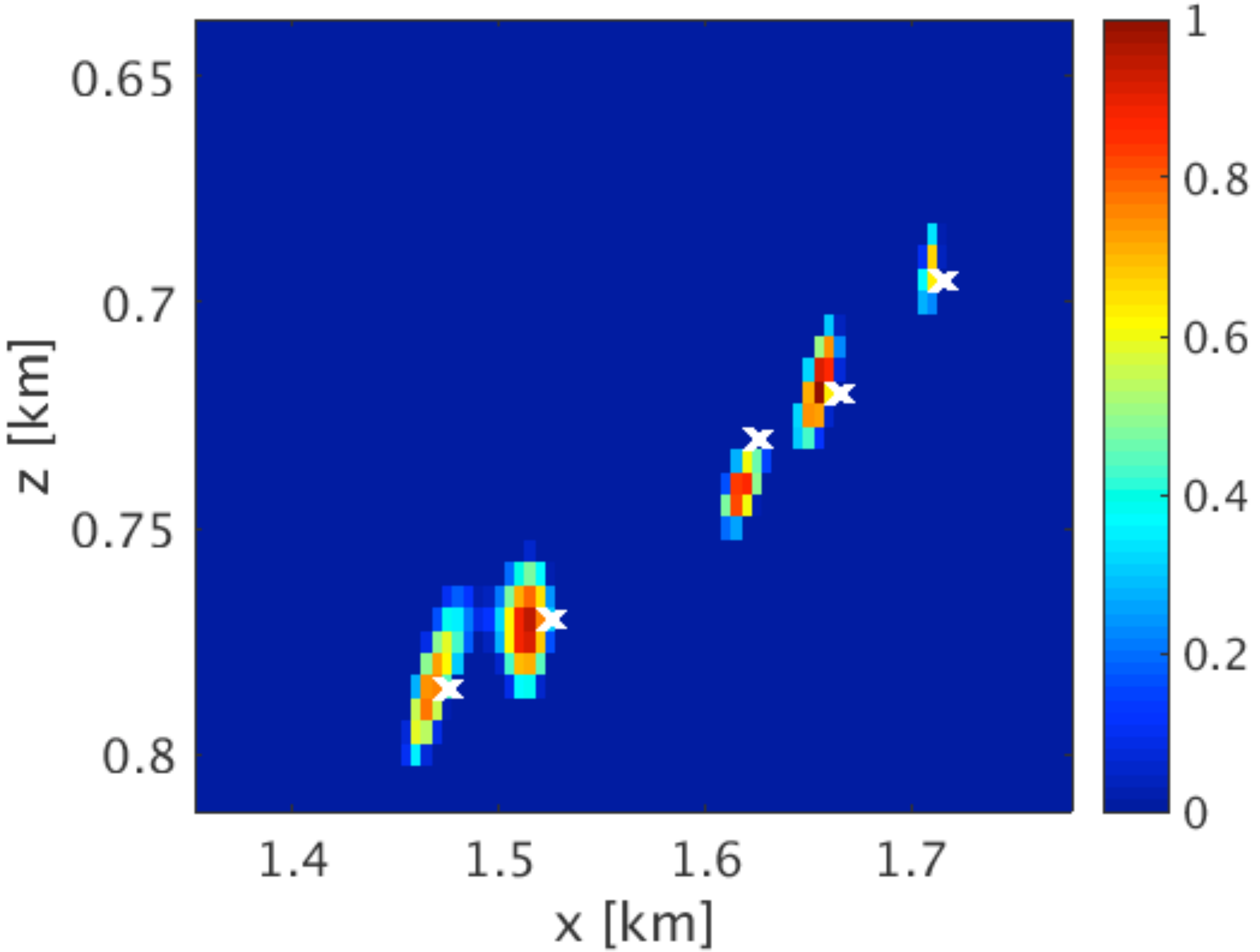
Estimated source location after denoising



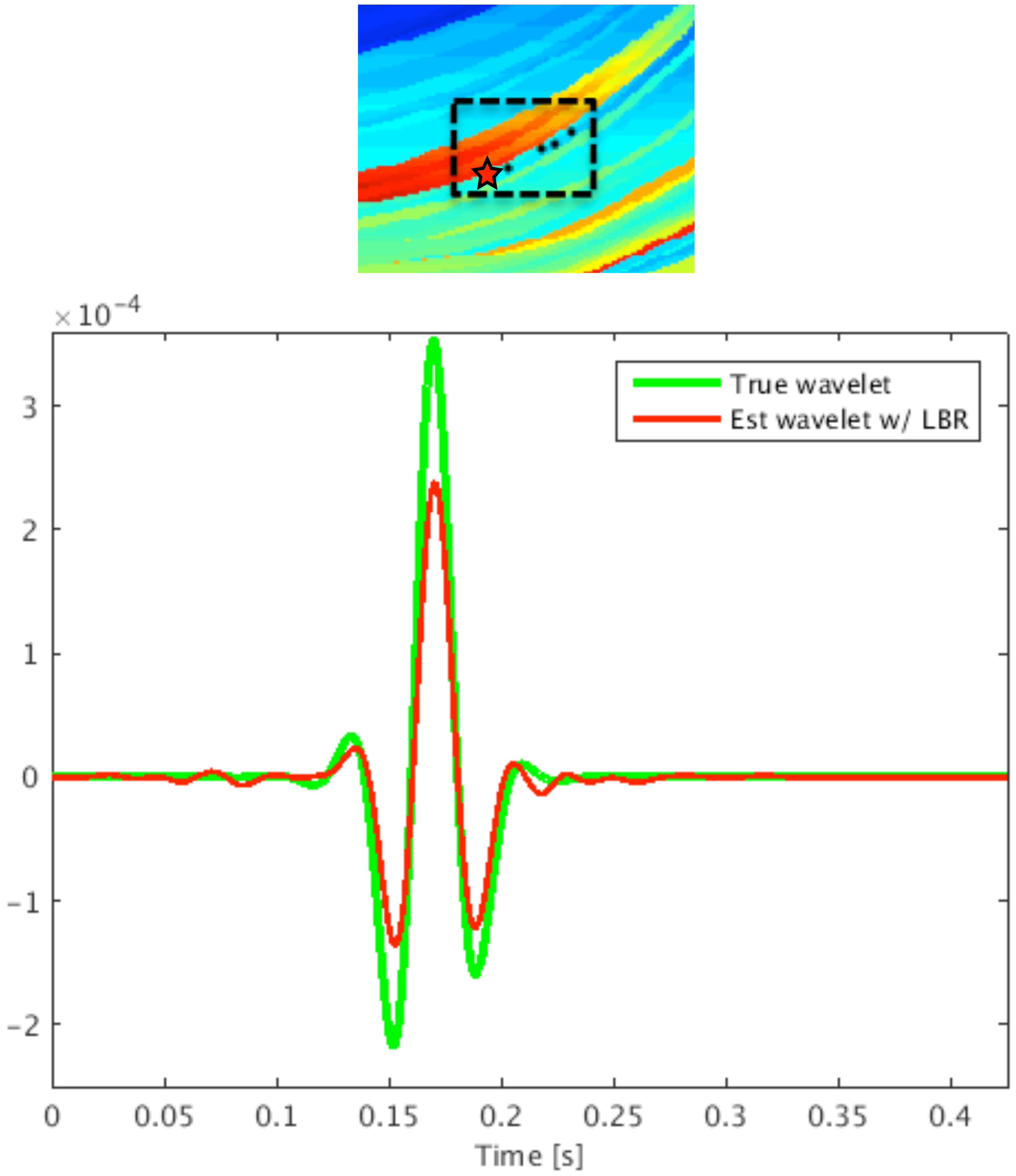
Estimated source location after denoising (zoomed)

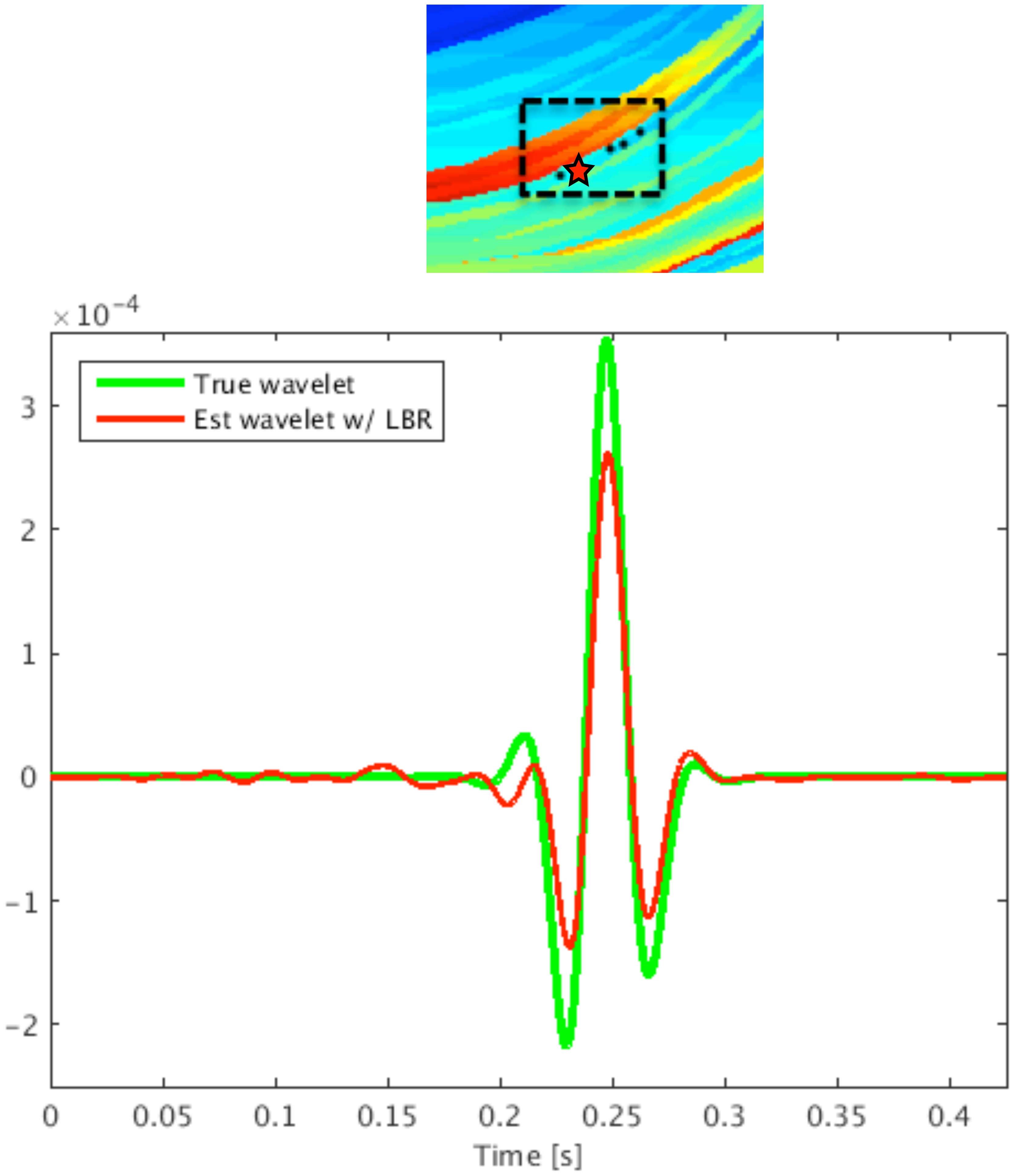


Estimated source location after denoising (zoomed)

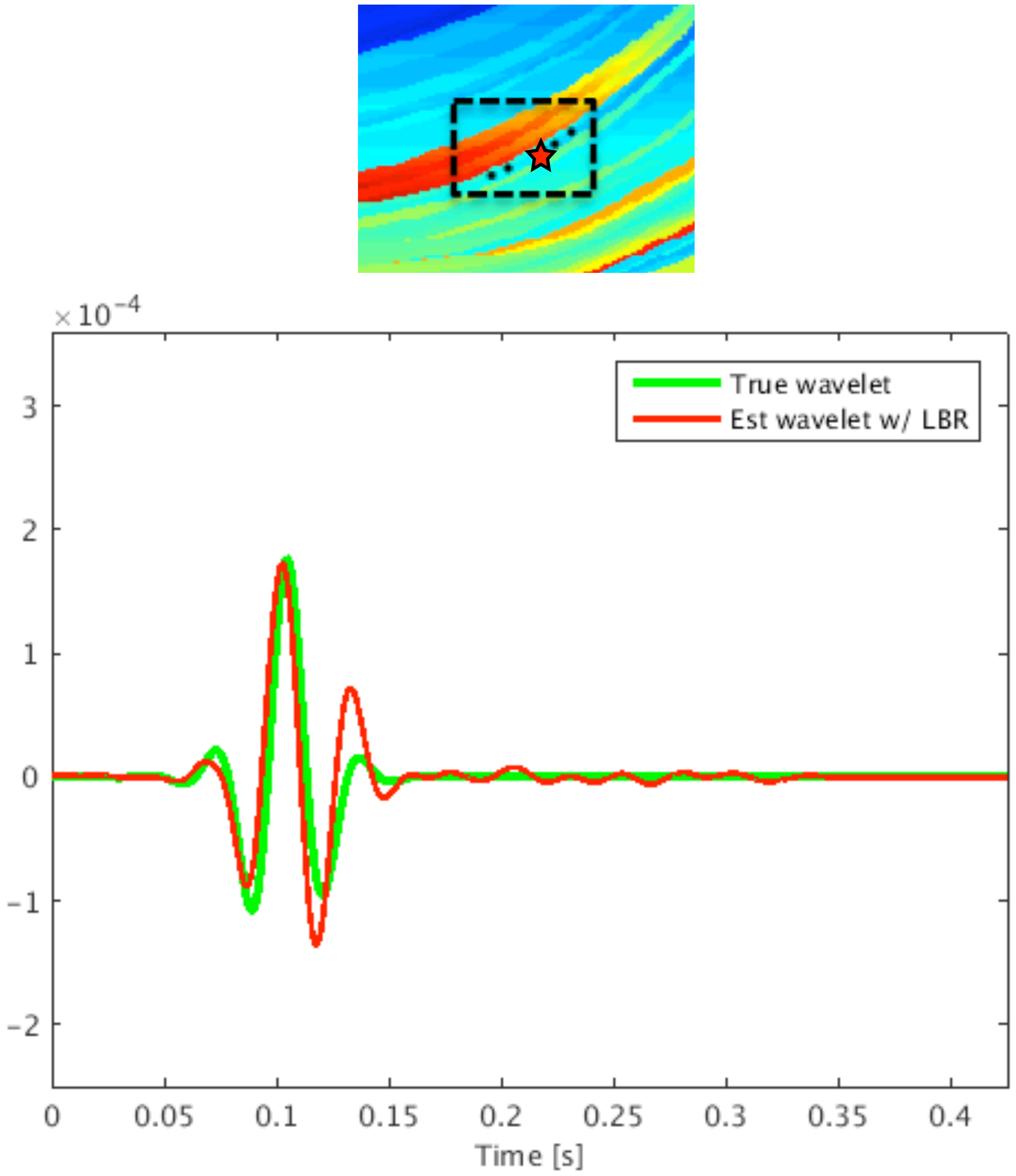


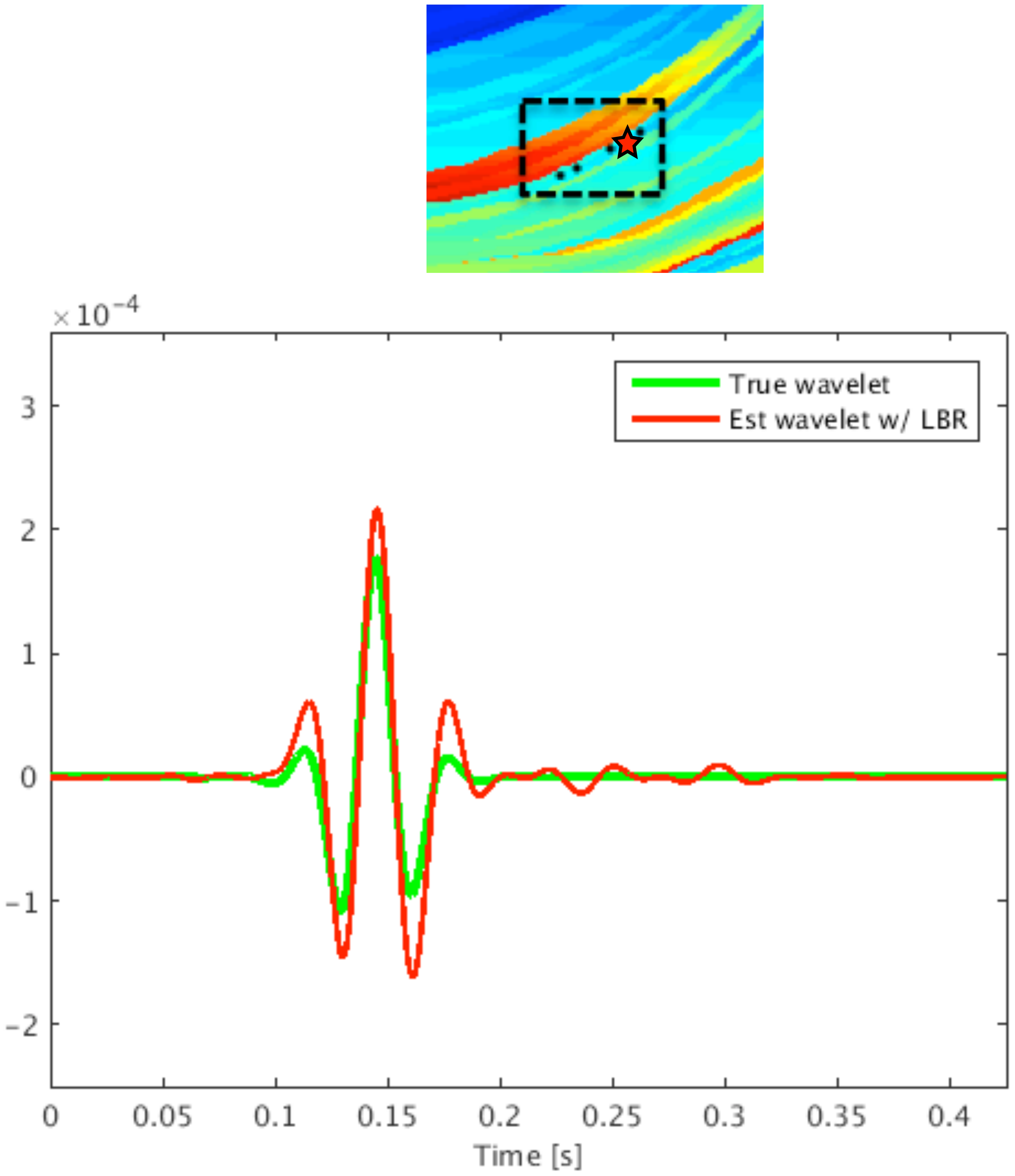
Source-time function comparison



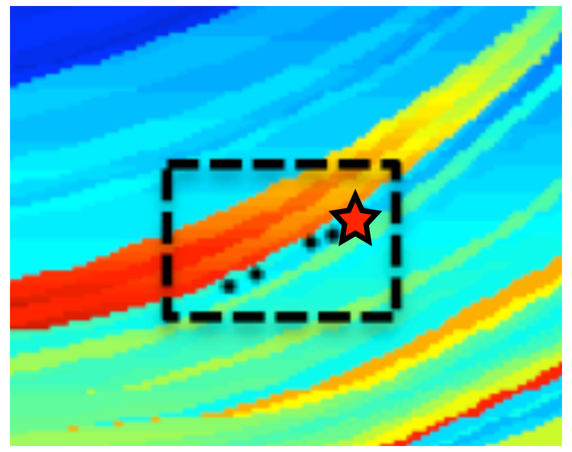


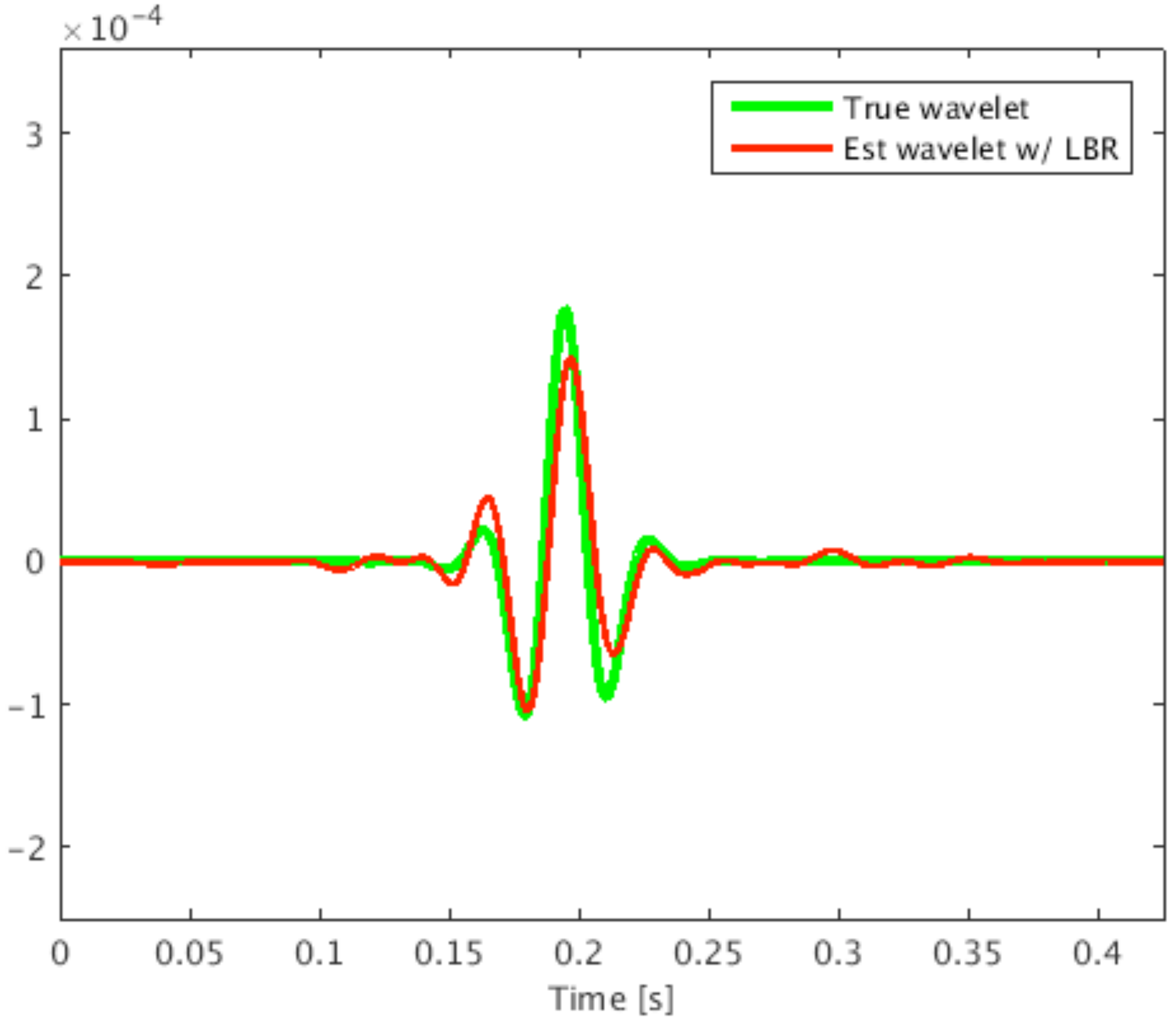
Source-time function comparison





Source-time function comparison

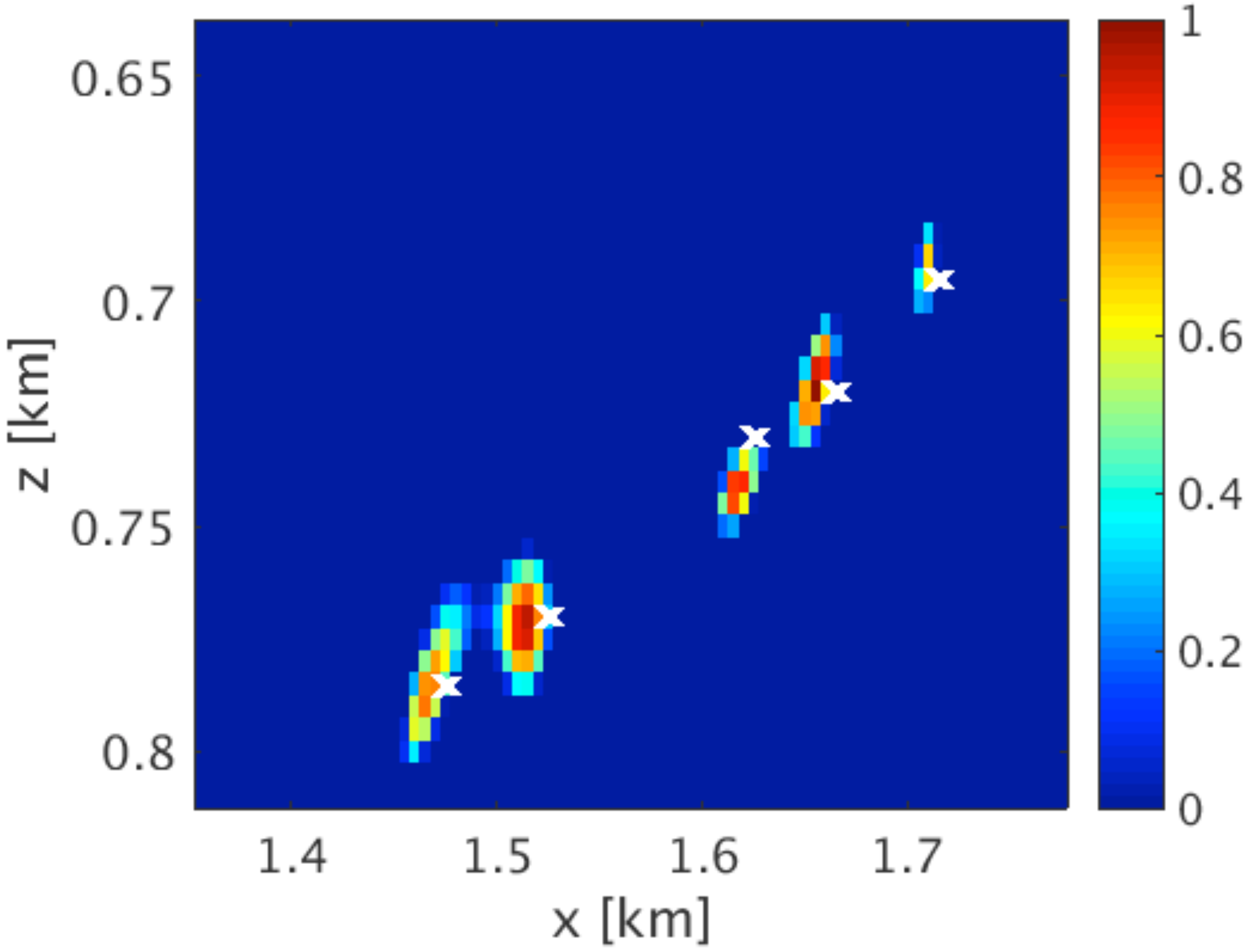




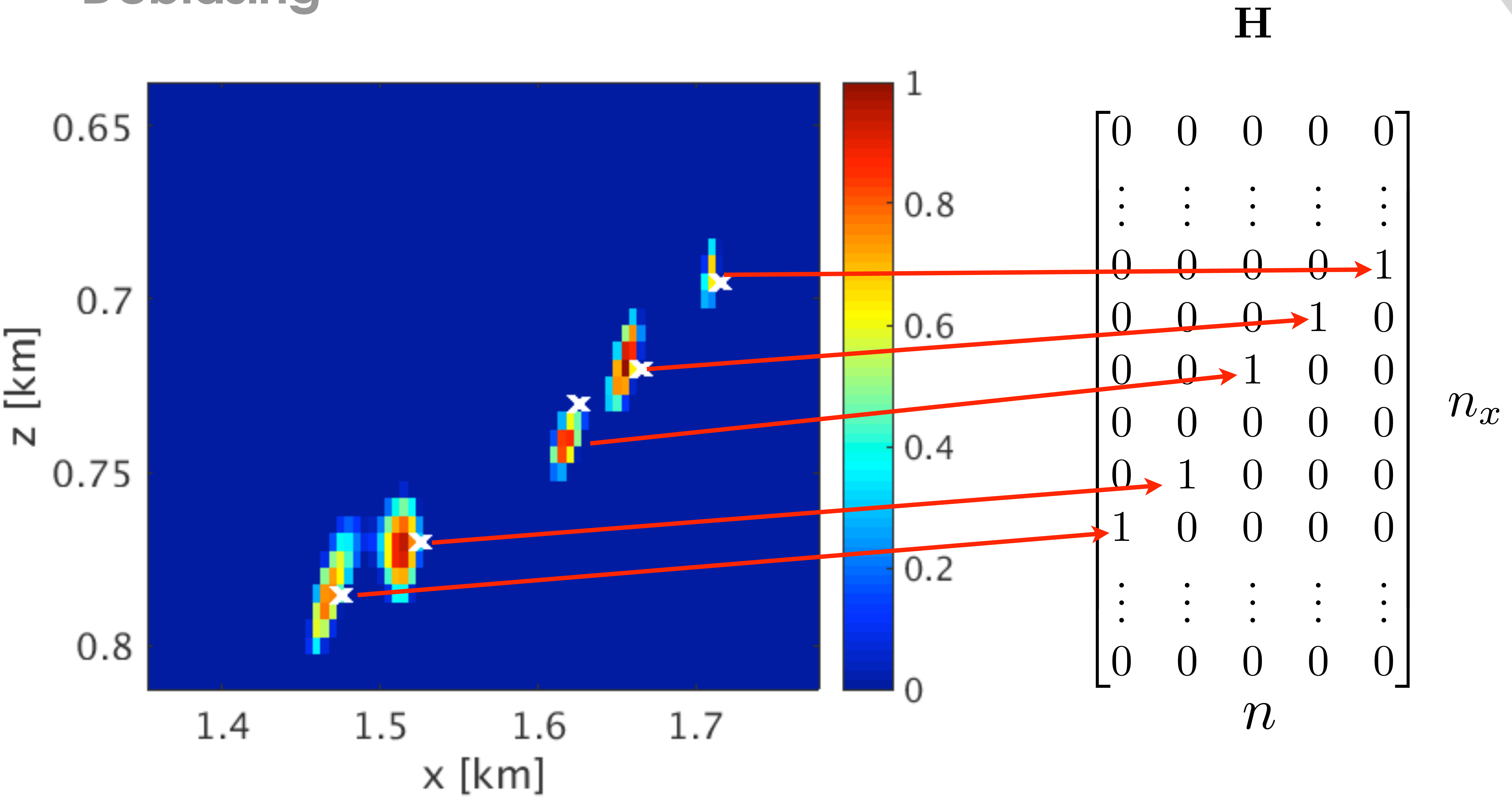
Source-time function

- Correct shape
- Incorrect amplitude
- 40 times amplified

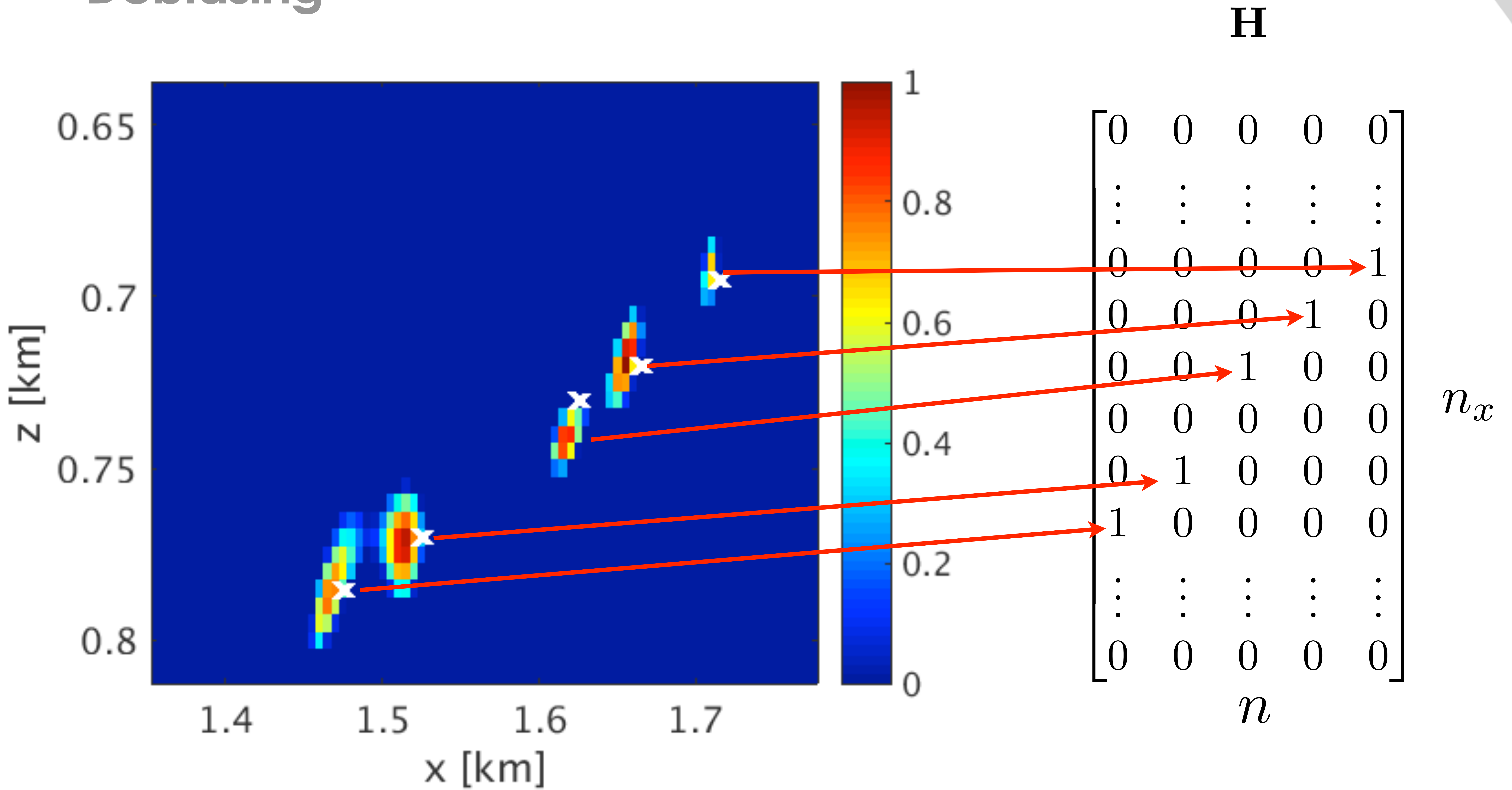
Debiasing



Debiasing



Debiasing



*where n is the number of detected microseismic sources

SLIM

Debiasing

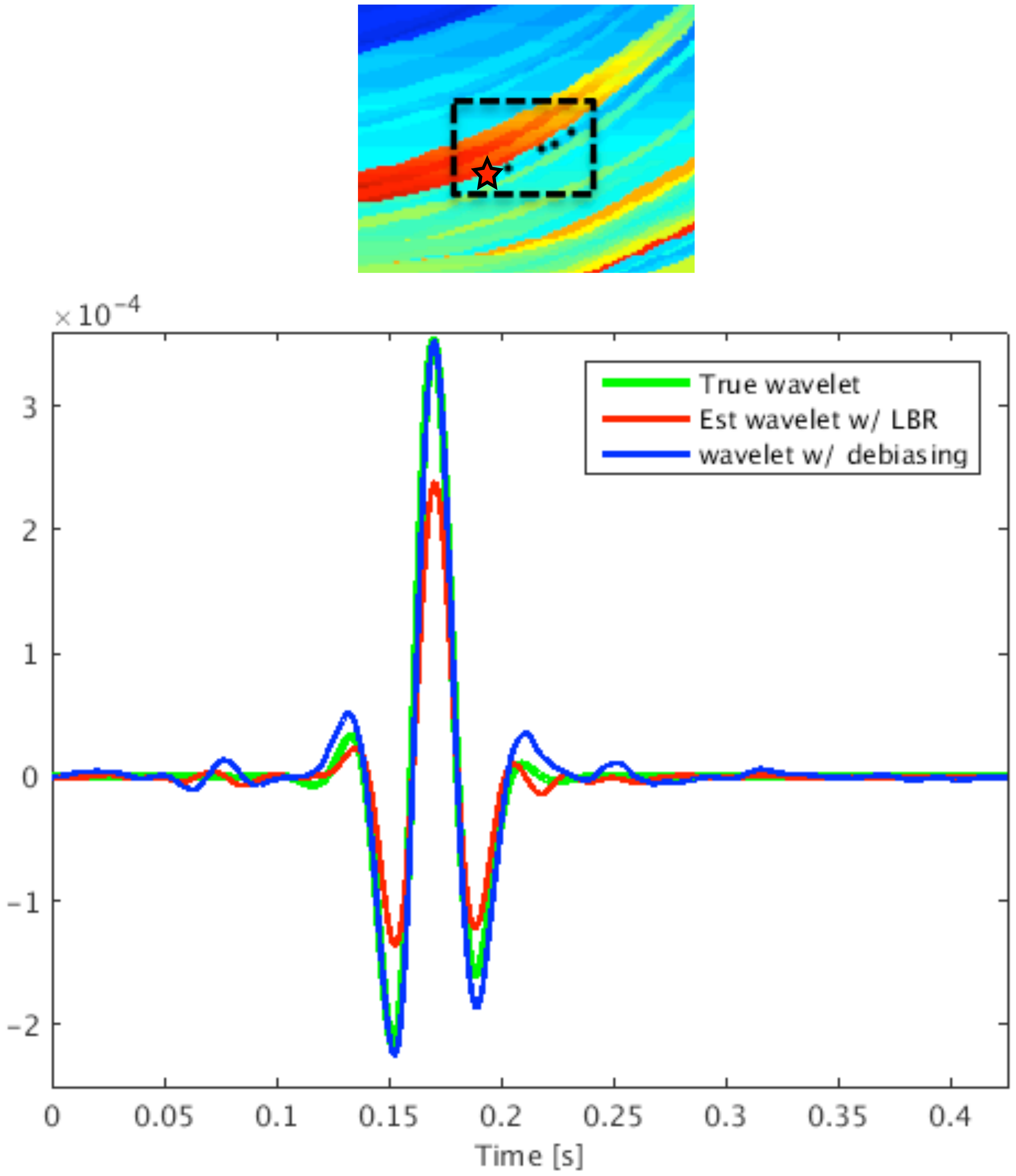
$$\tilde{\mathbf{W}} = \underset{\mathbf{W} \in \mathbb{R}^{n_t \times n}}{\arg\min} \quad \|\mathcal{F}(\mathbf{H}\mathbf{W}^\top) - \mathbf{d}\|$$

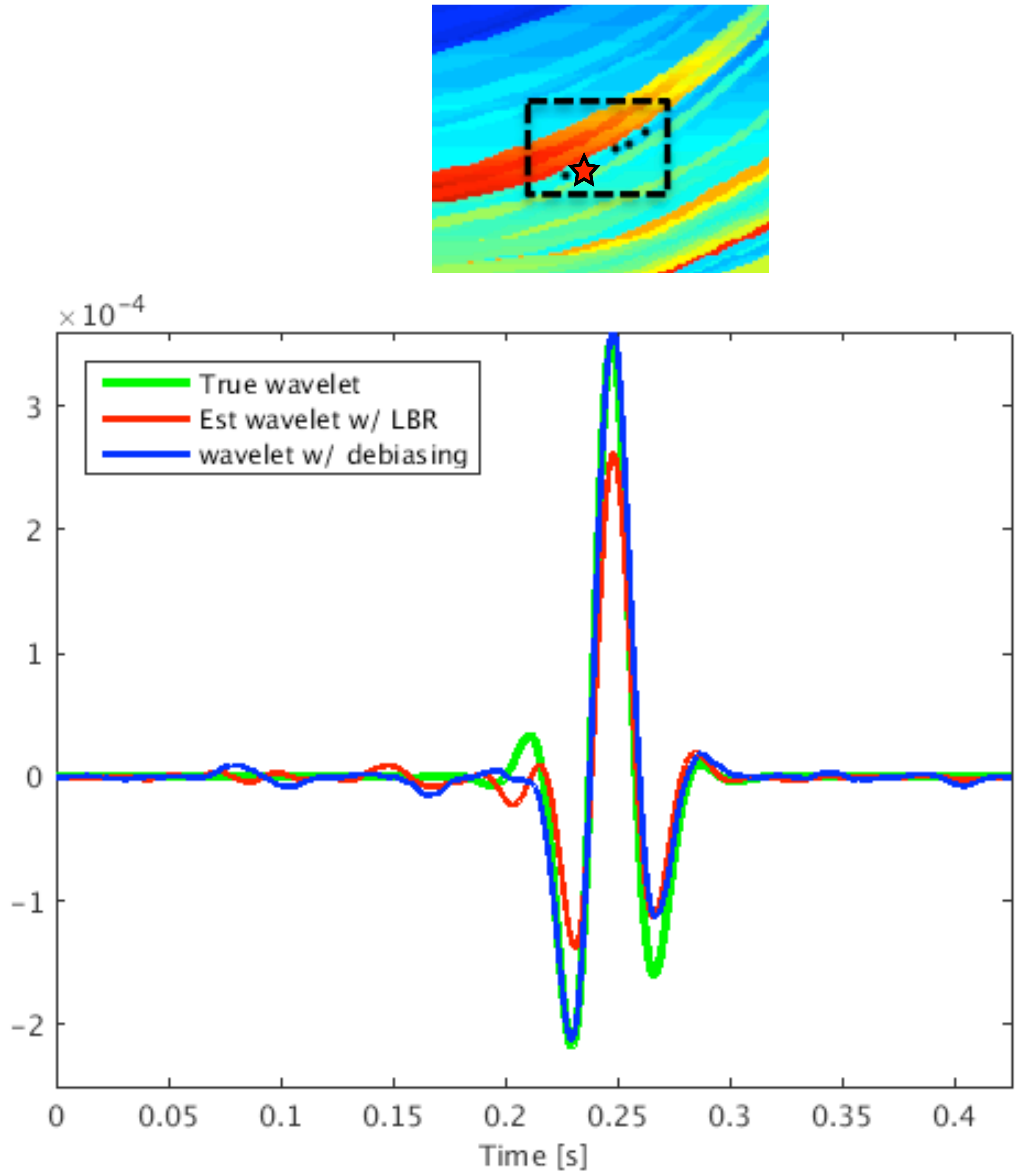
*where $\mathbf{H} \in n_x \times n$

*we use noisy data $\mathbf{d}$ to avoid any amplitude error

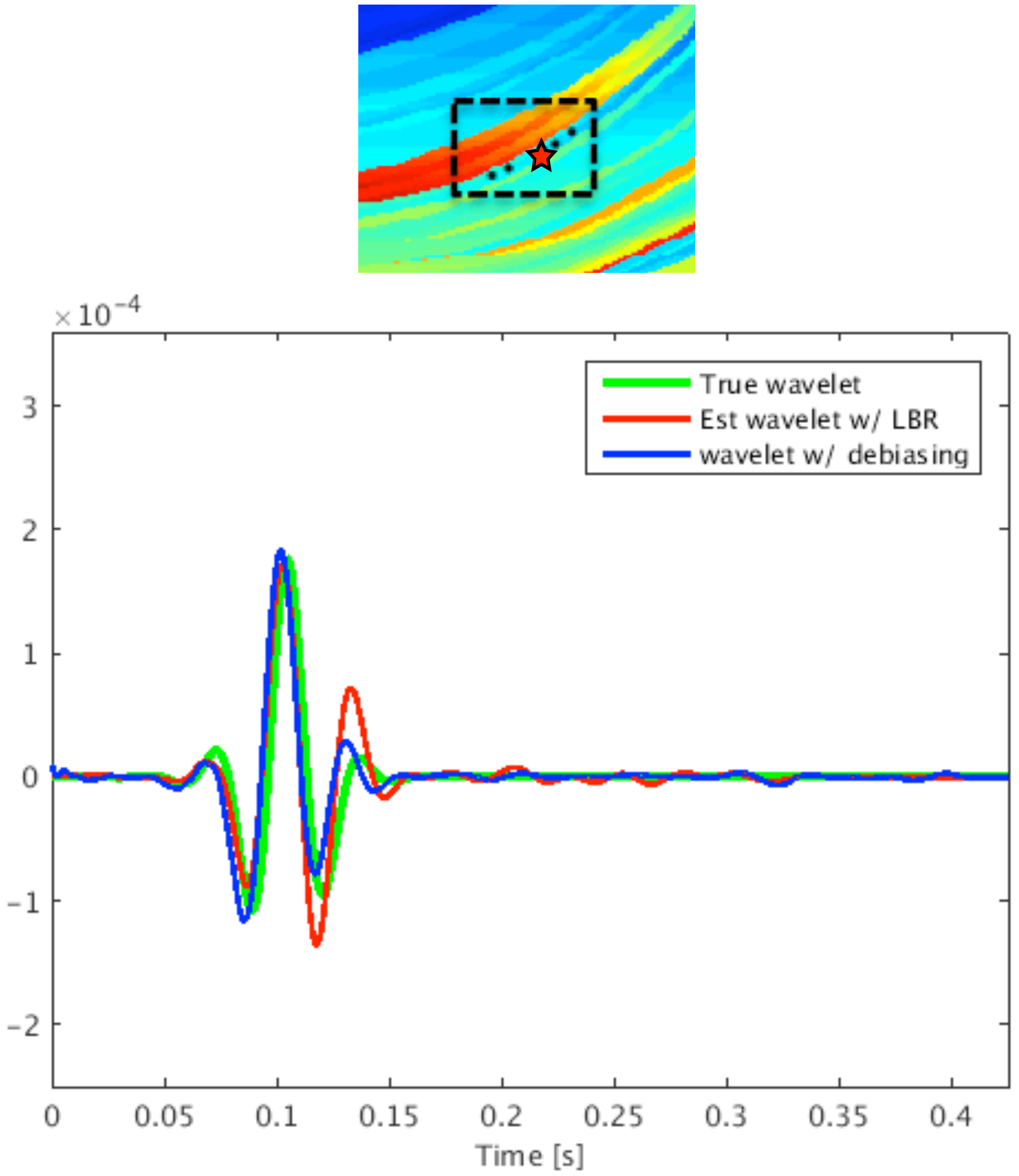
- Debiasing is possible because
- we are able to detect microseismic sources
- Even from noisy data with very low SNR

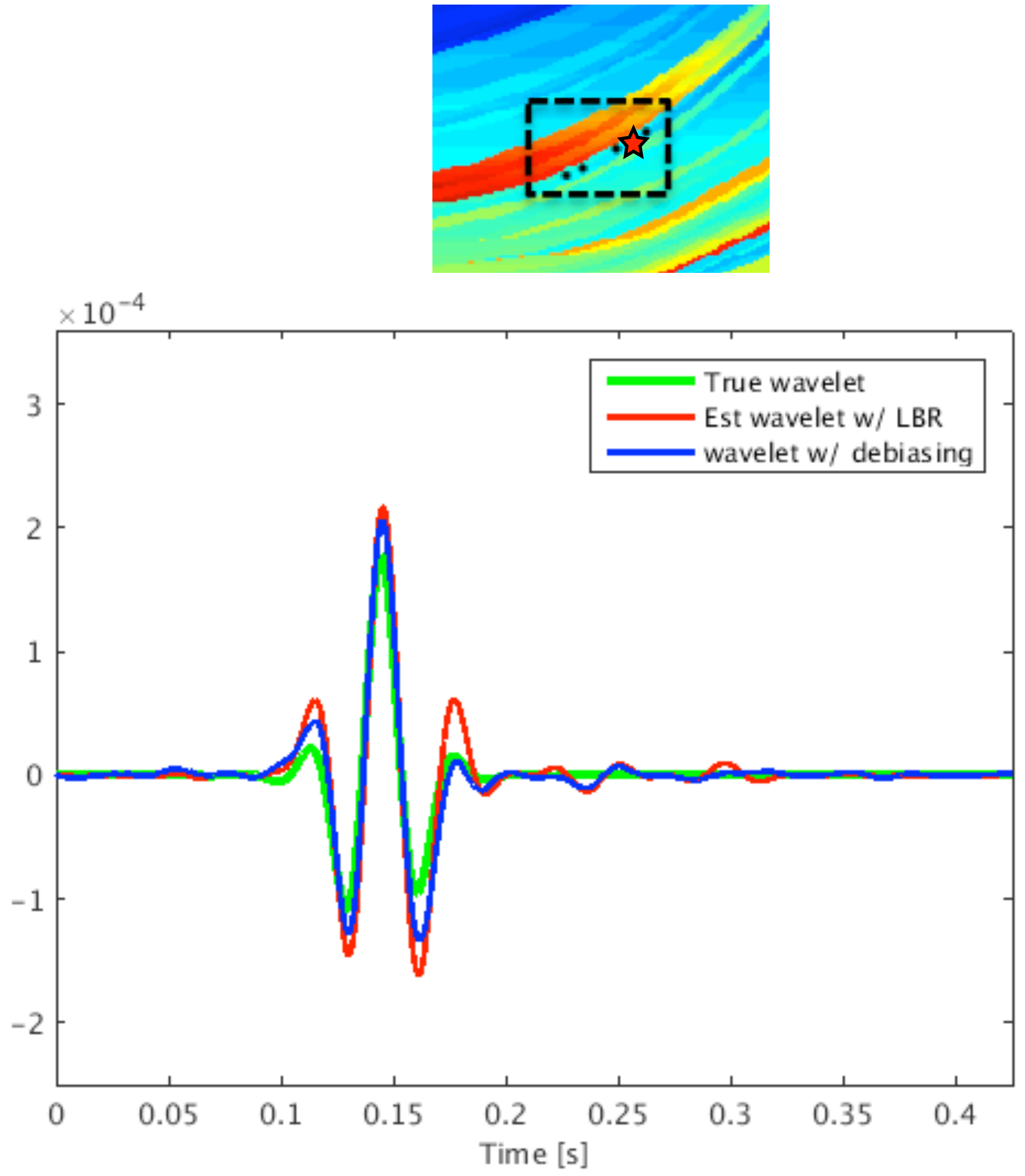
Source-time function comparison after debiasing



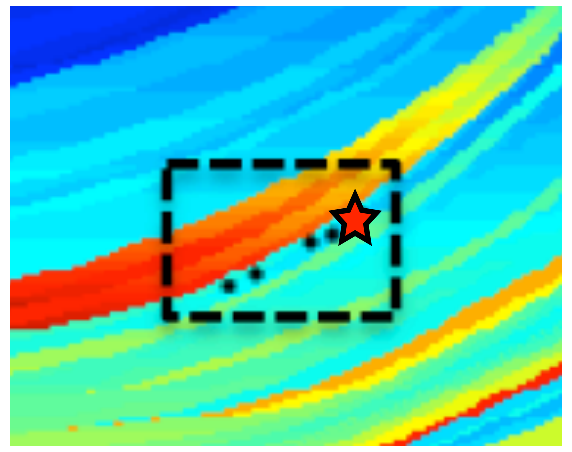


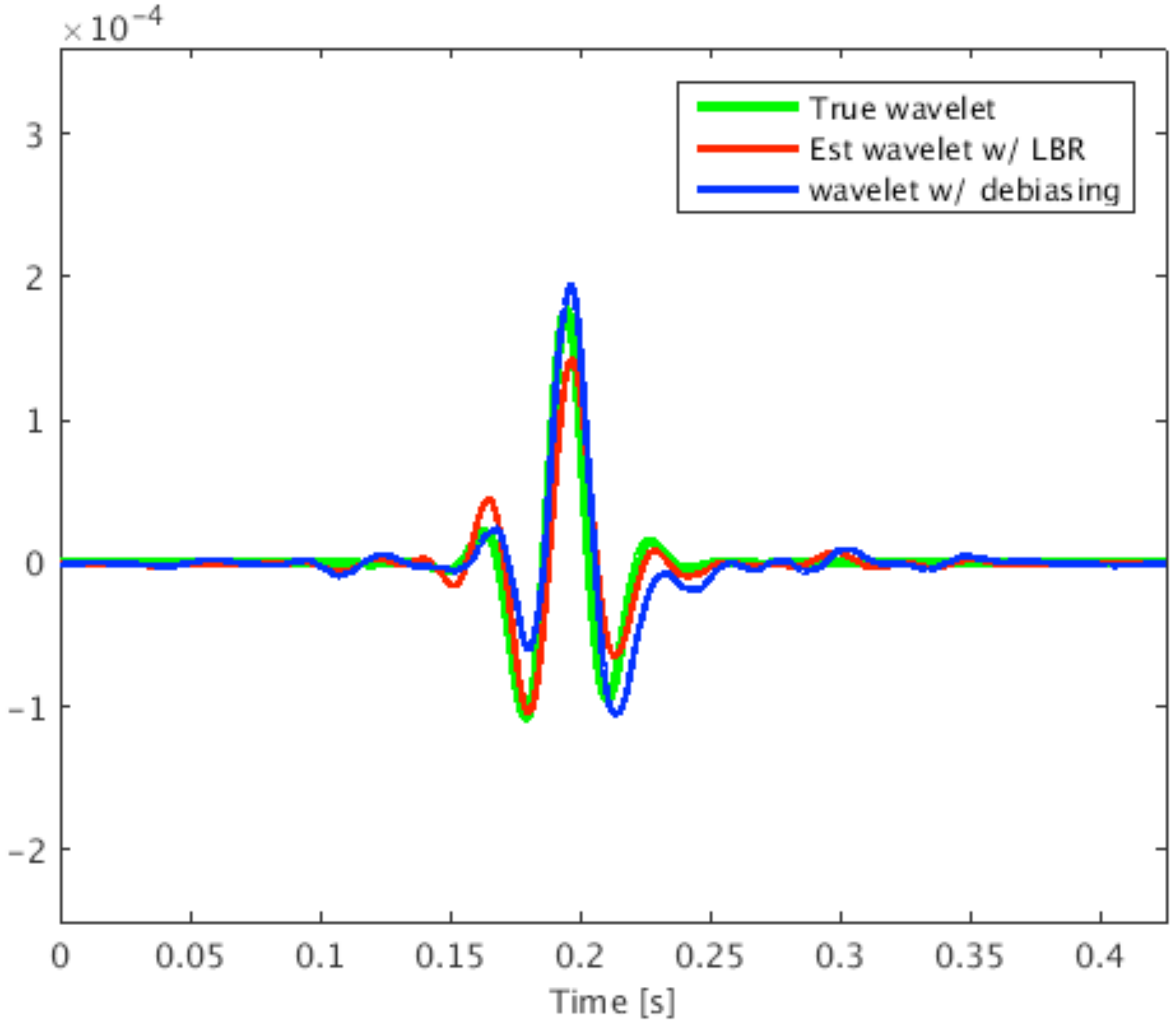
Source-time function comparison after debiasing





Source-time function comparison after debiasing

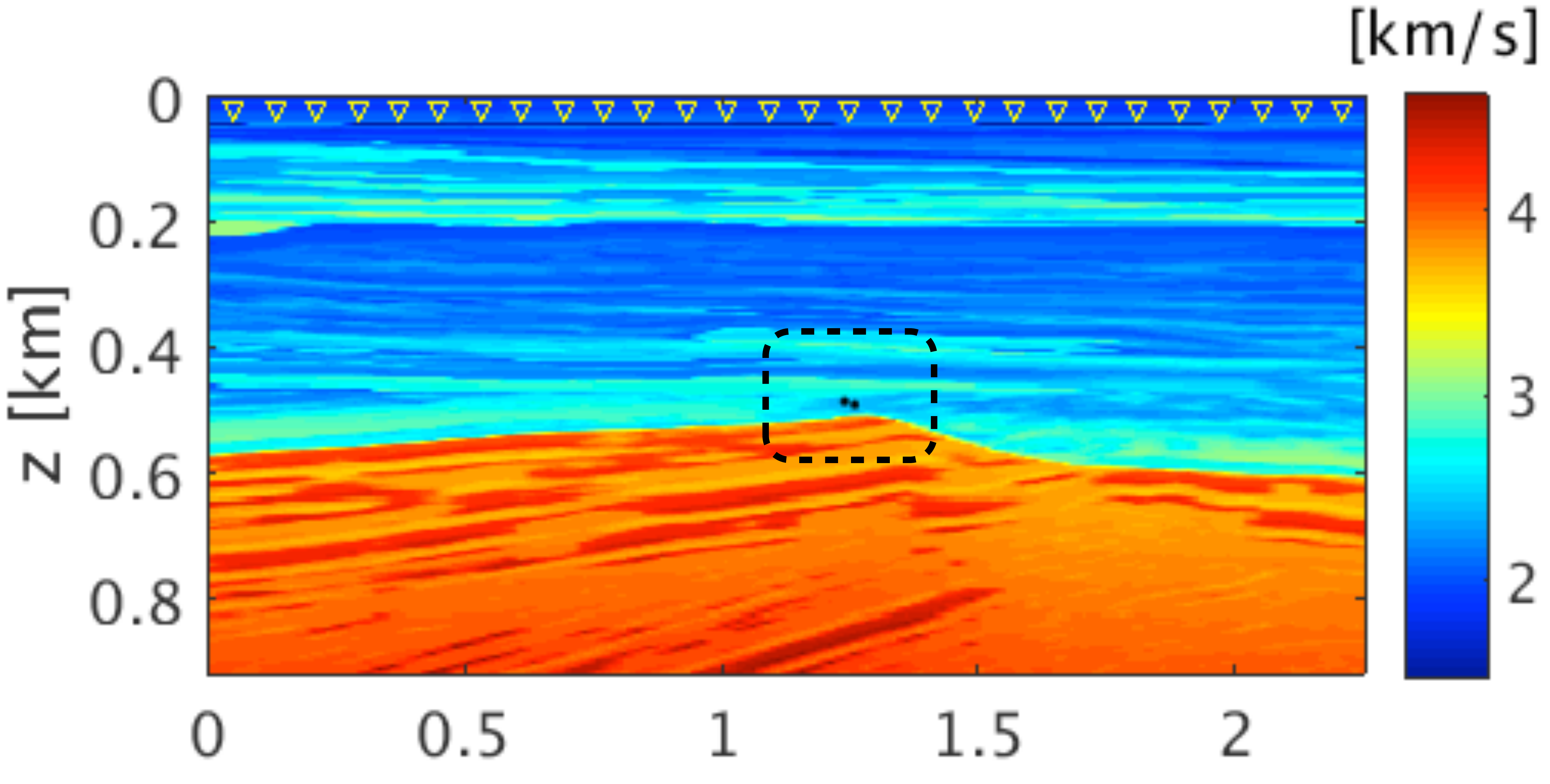




Source-time function

- Correct shape
- Correct amplitude

Numerical Experiment: BG Compass model

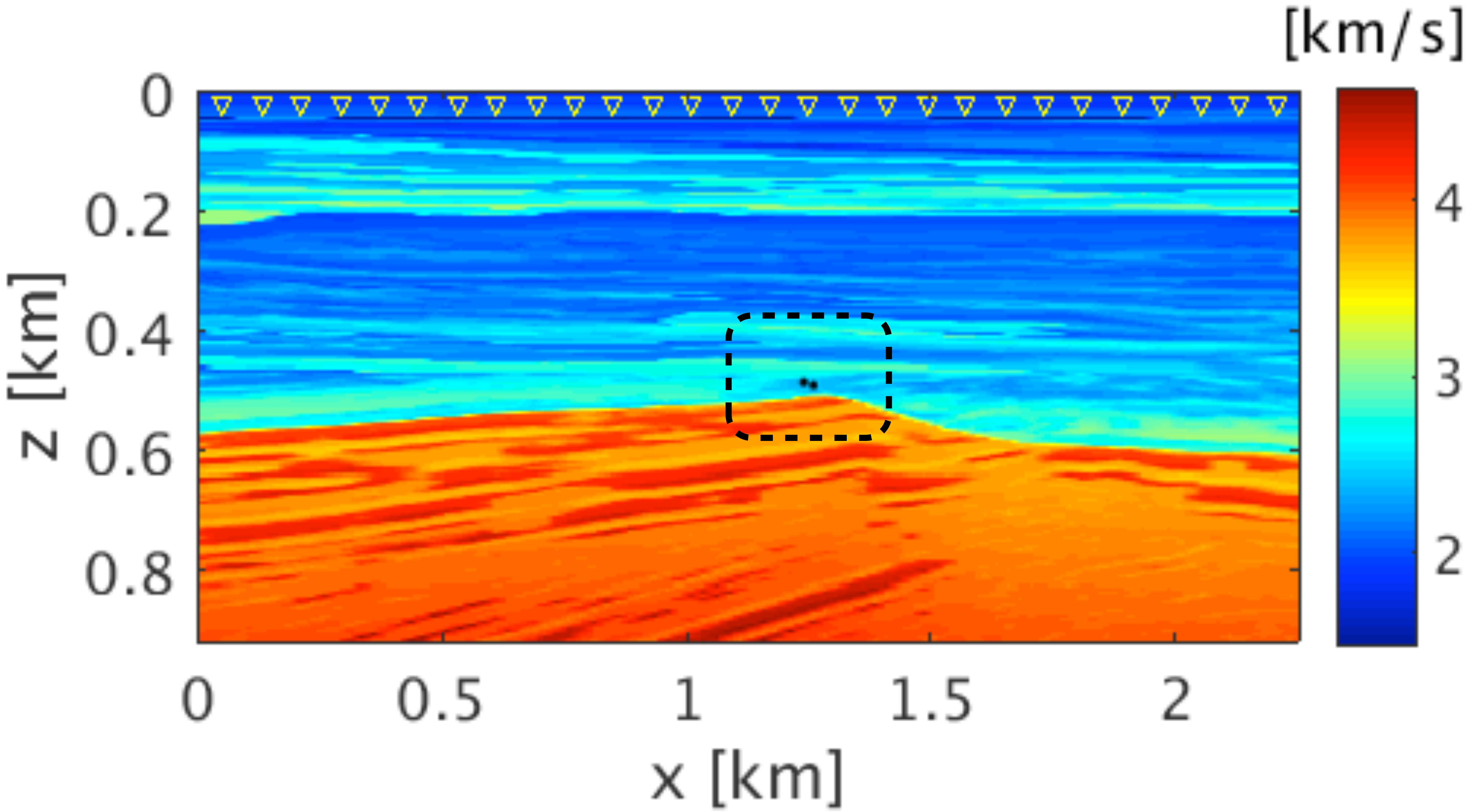


BG Compass model

Modeling information:

Model size: 2.20 km x 0.90 km
Grid spacing: 5 m
Total number of sources: 2
Peak frequency : 30 Hz
Receiver spacing: 10m
Receiver depth: 20m
Sampling interval: 0.5 ms
Recording length: 1 s
Free surface: No

Numerical Experiment: BG Compass model



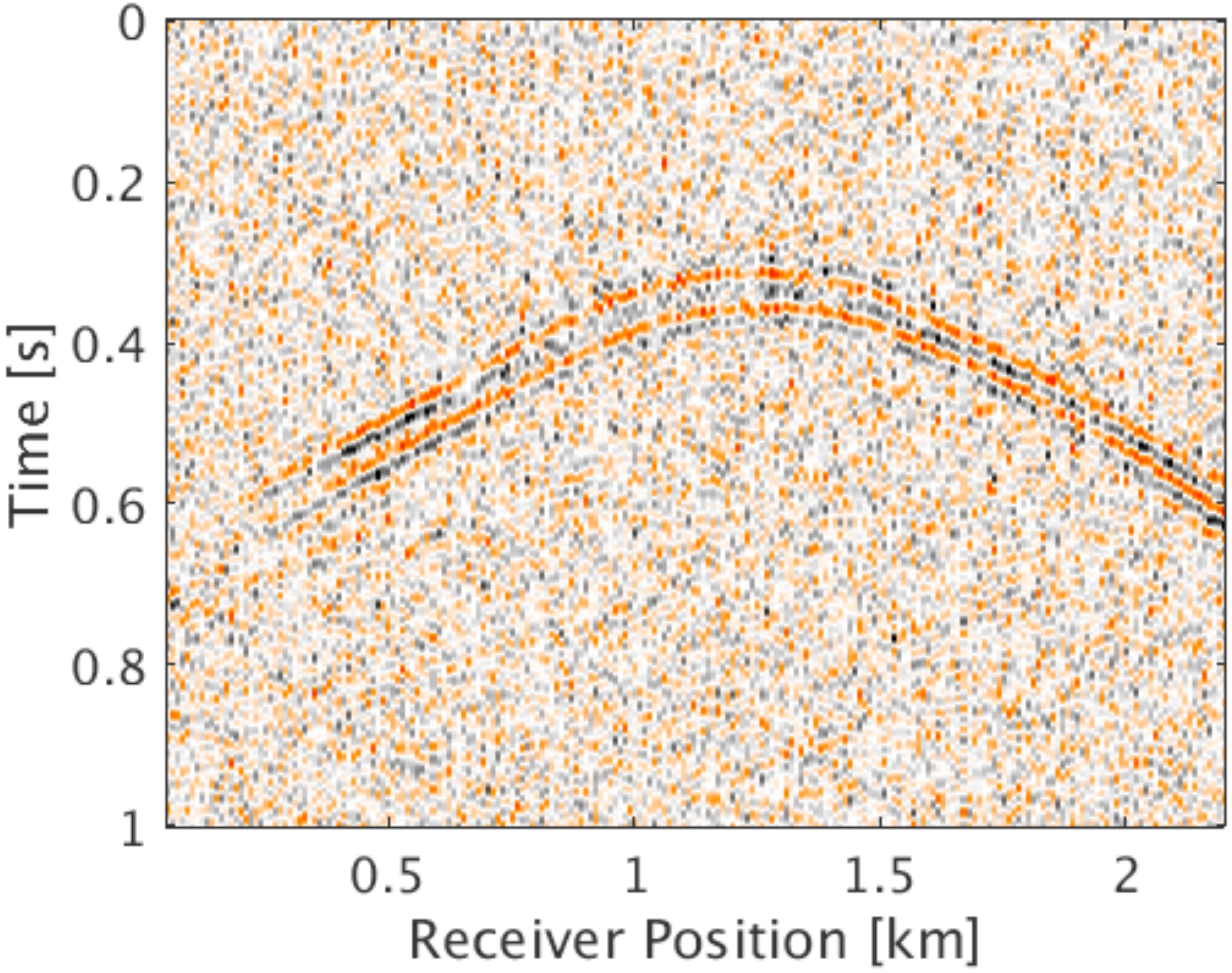
BG Compass model

Modeling information:

Model size: 2.20 km x 0.90 km
Grid spacing: 5 m
Total number of sources: 2
Peak frequency : 30 Hz
Receiver spacing: 10m
Receiver depth: 20m
Sampling interval: 0.5 ms
Recording length: 1 s
Free surface: No

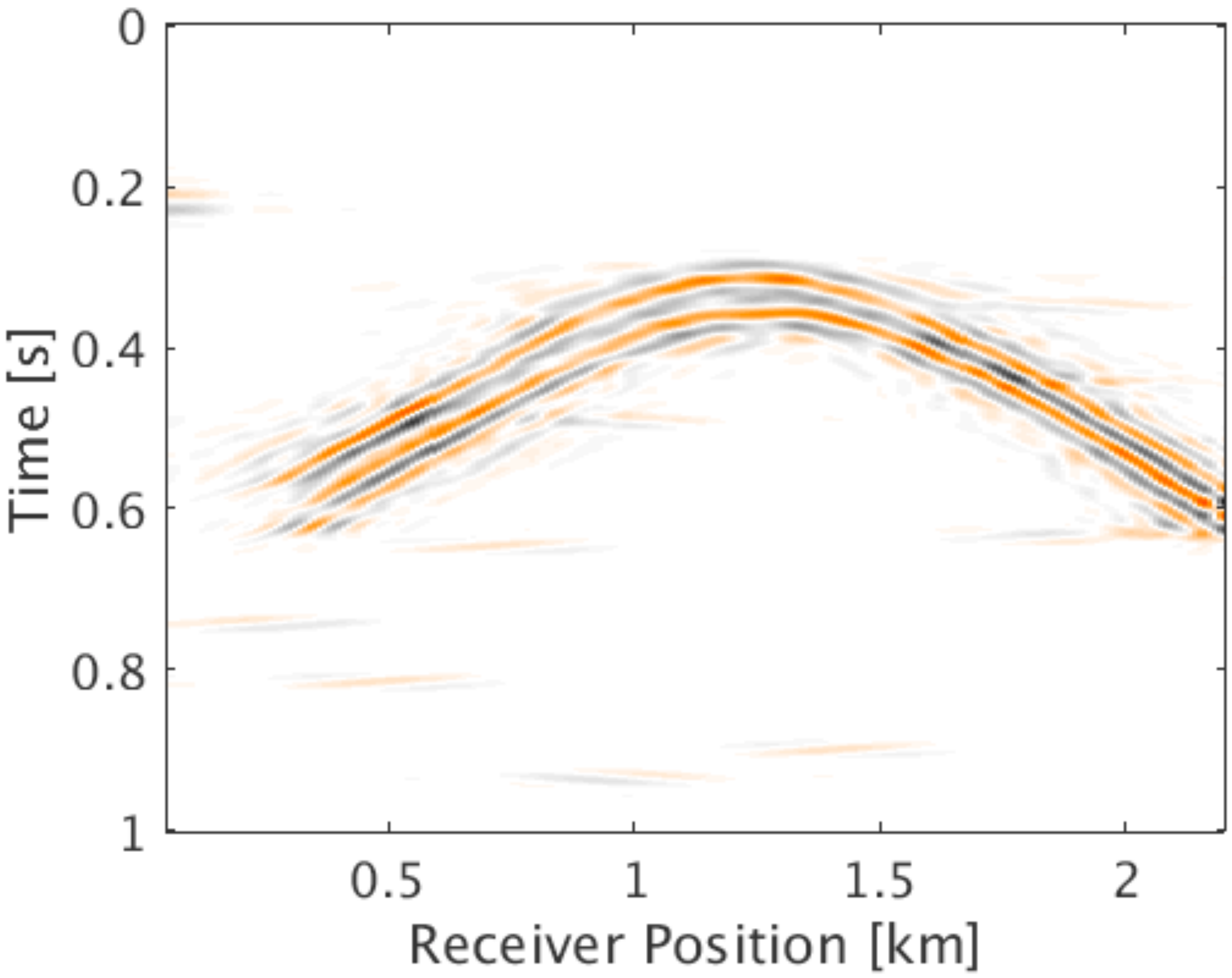
Sources are located within half a wavelength with overlapping source-time functions

Noisy data



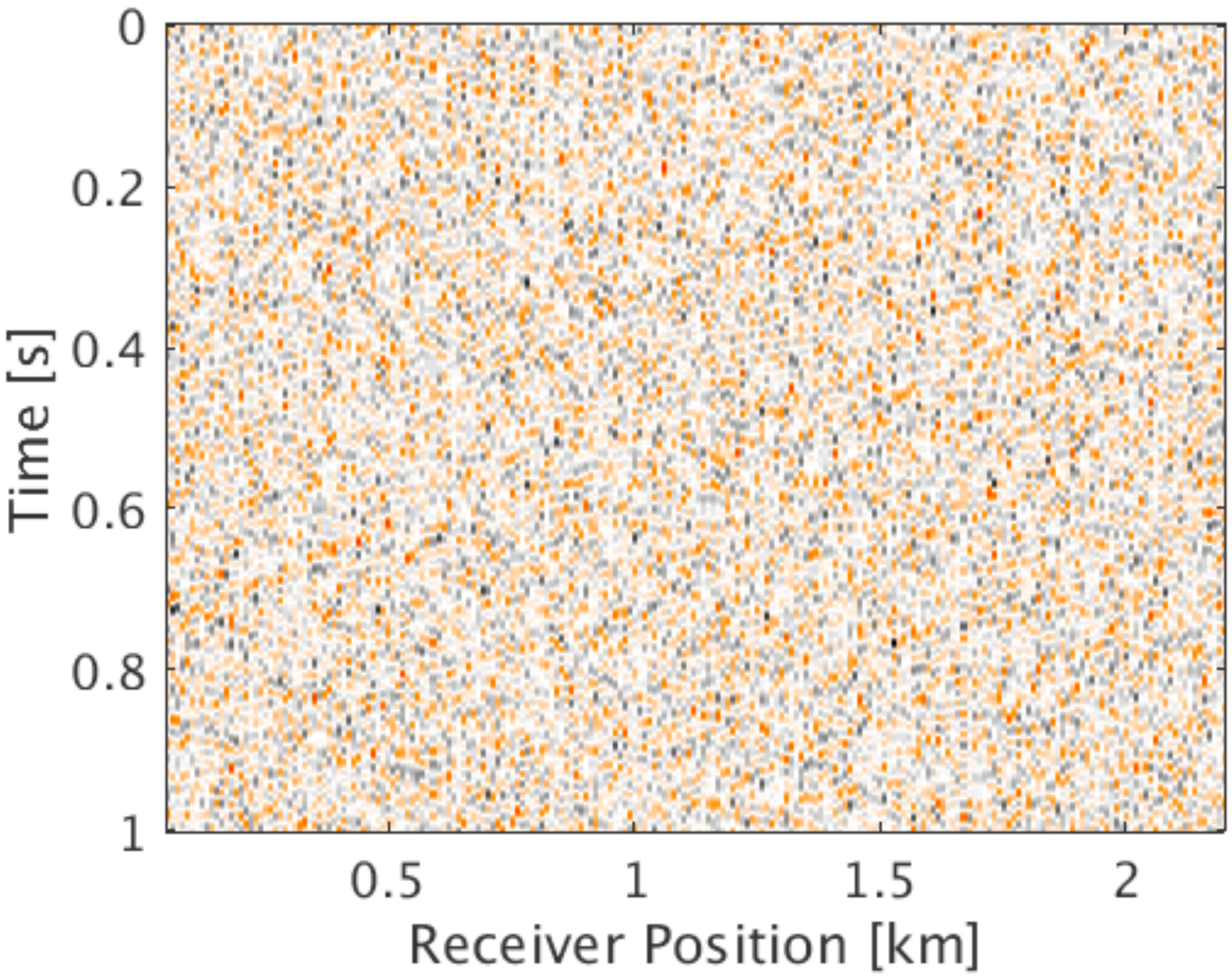
SNR: -8.2 dB

Denoised data w/ curvelet based denoising

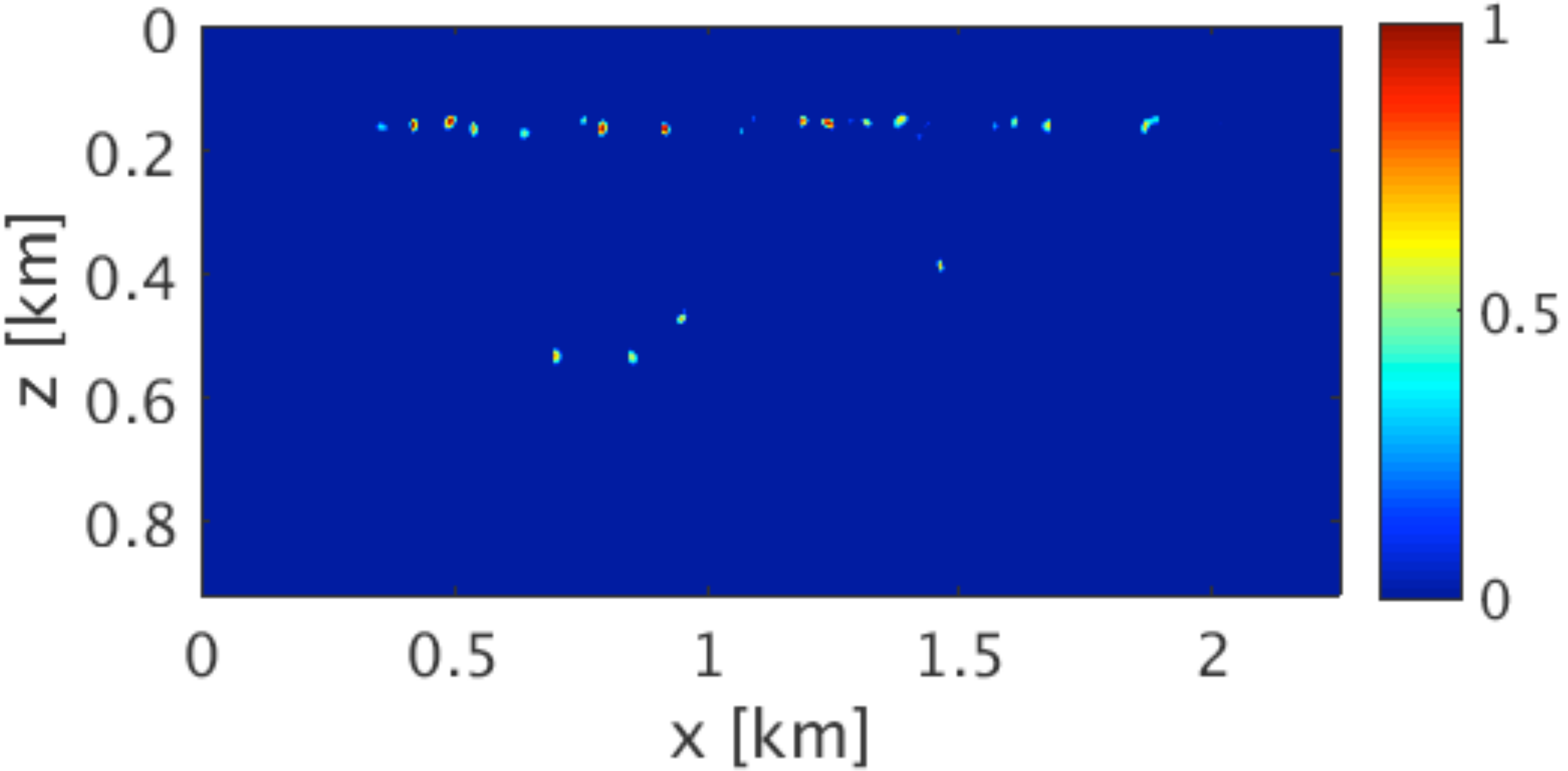


SNR: 5.9 dB

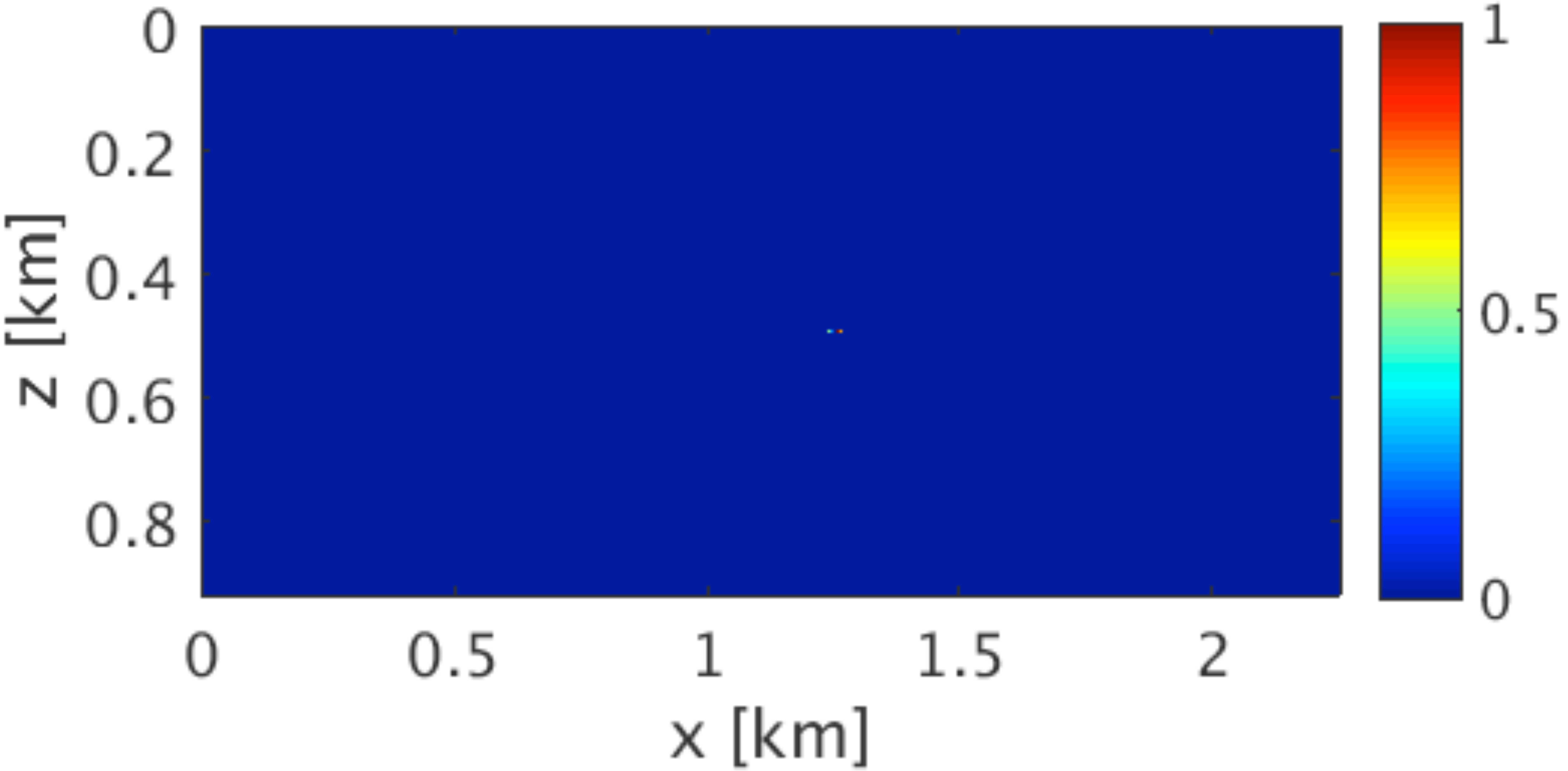
Difference



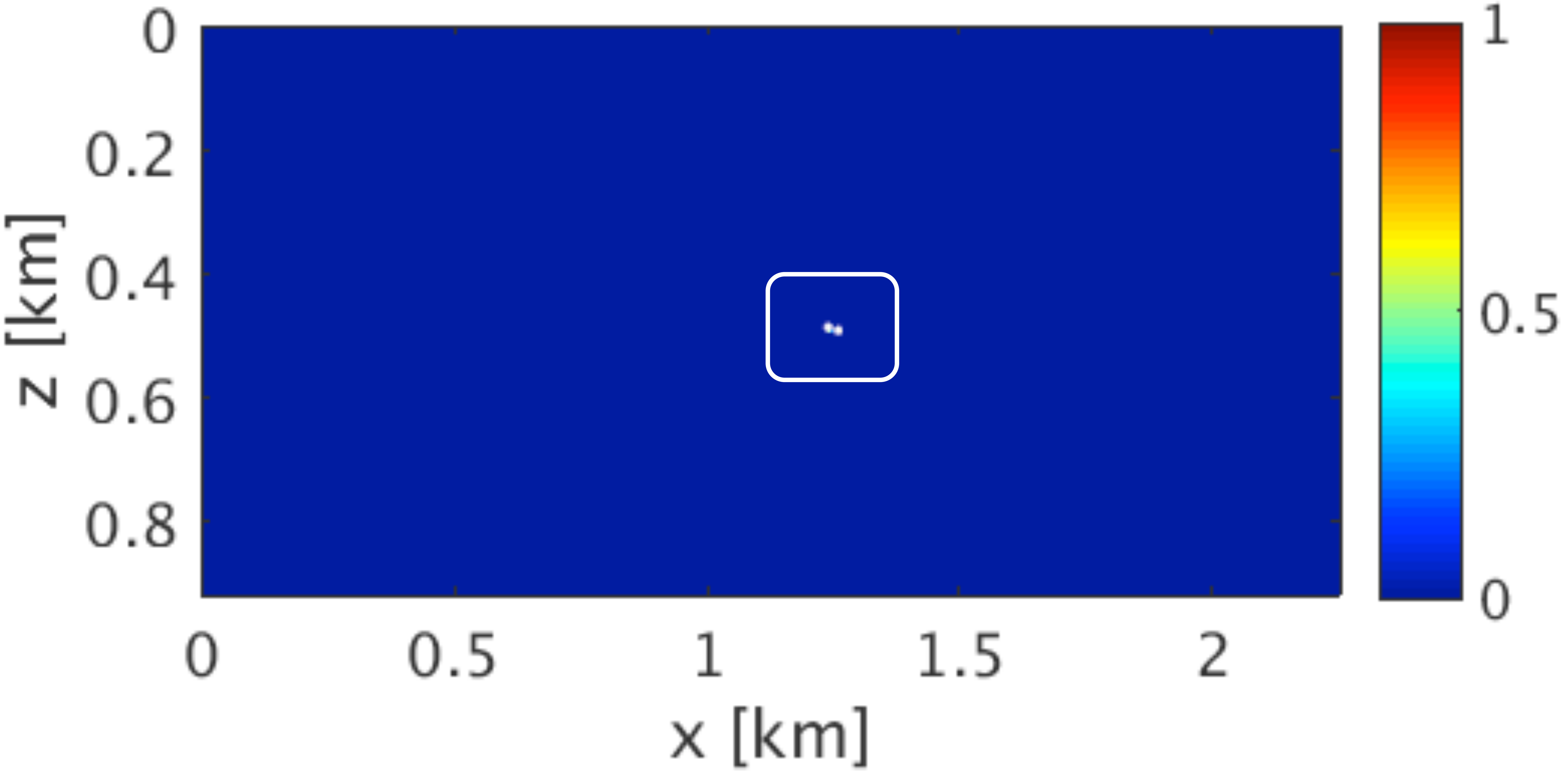
Estimated source location w/o denoising



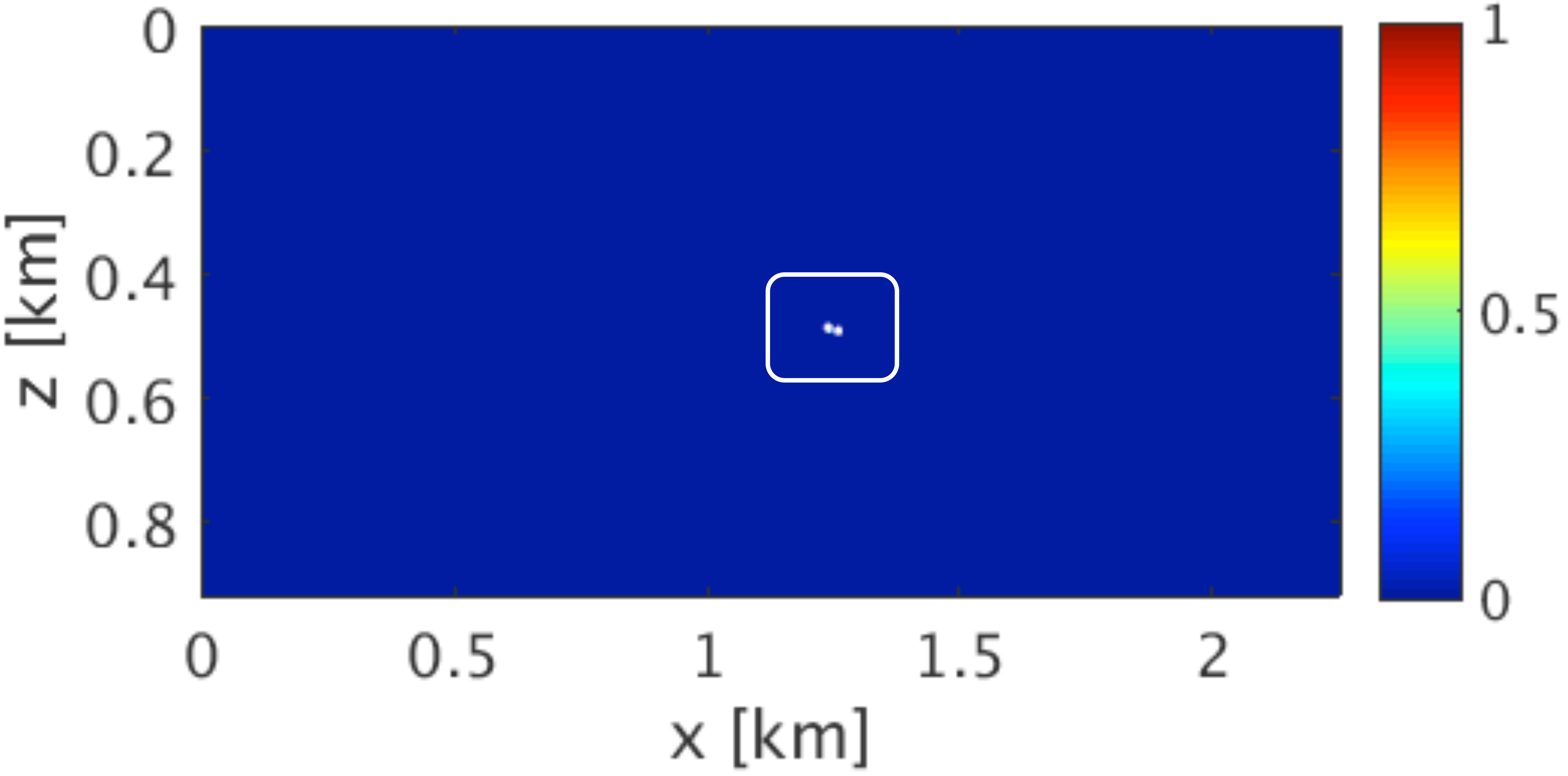
Estimated source location after denoising



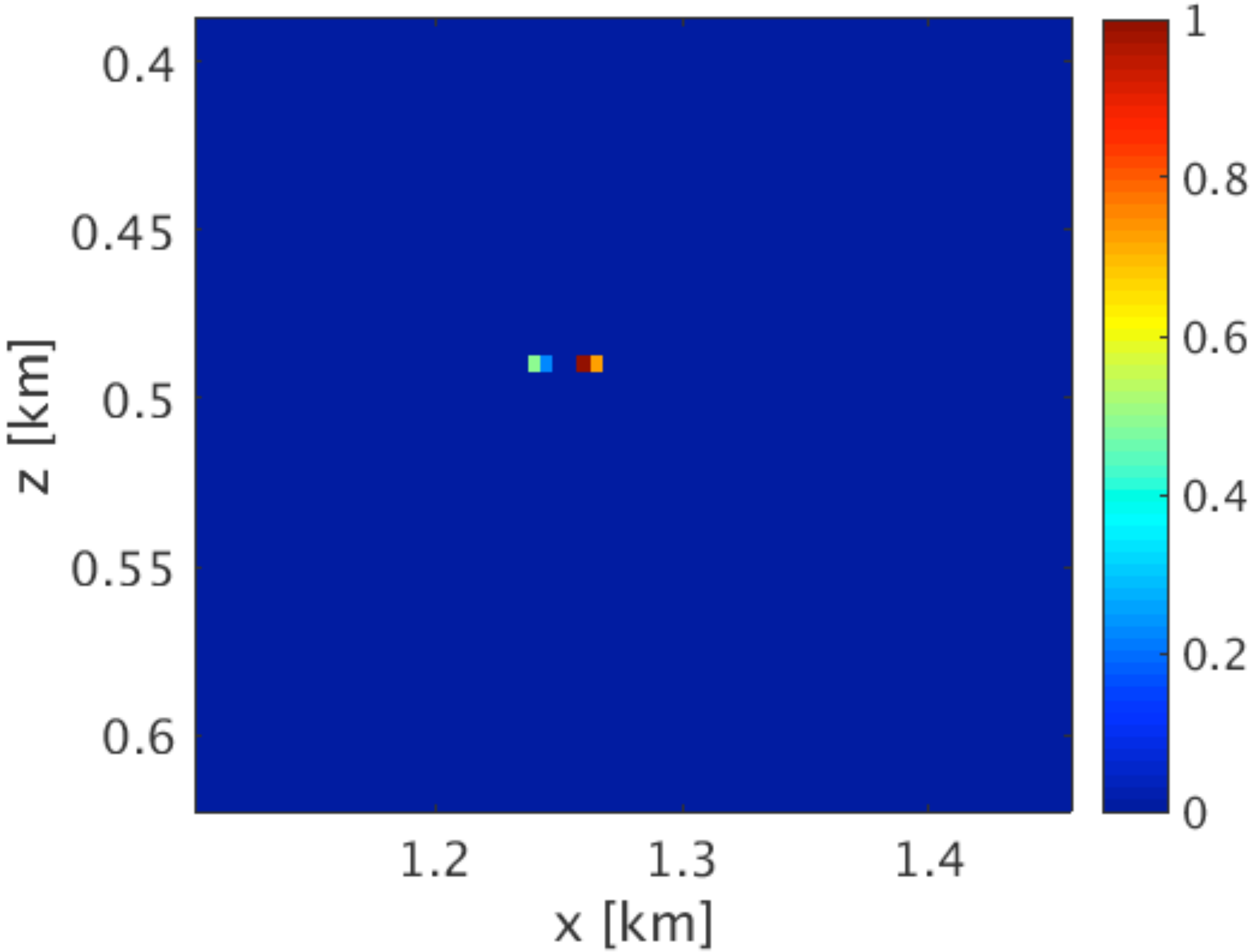
Estimated source location after denoising



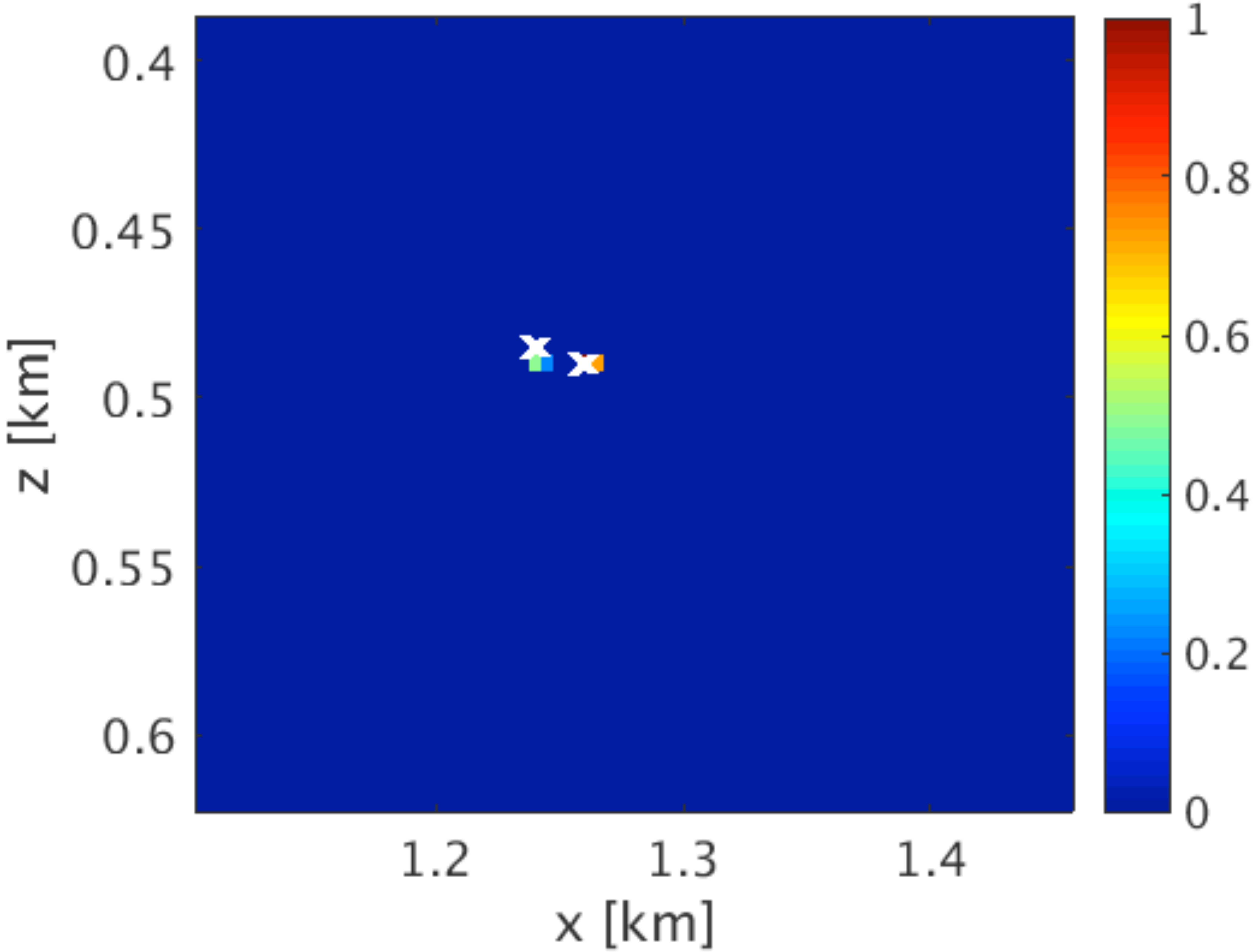
Estimated source location after denoising



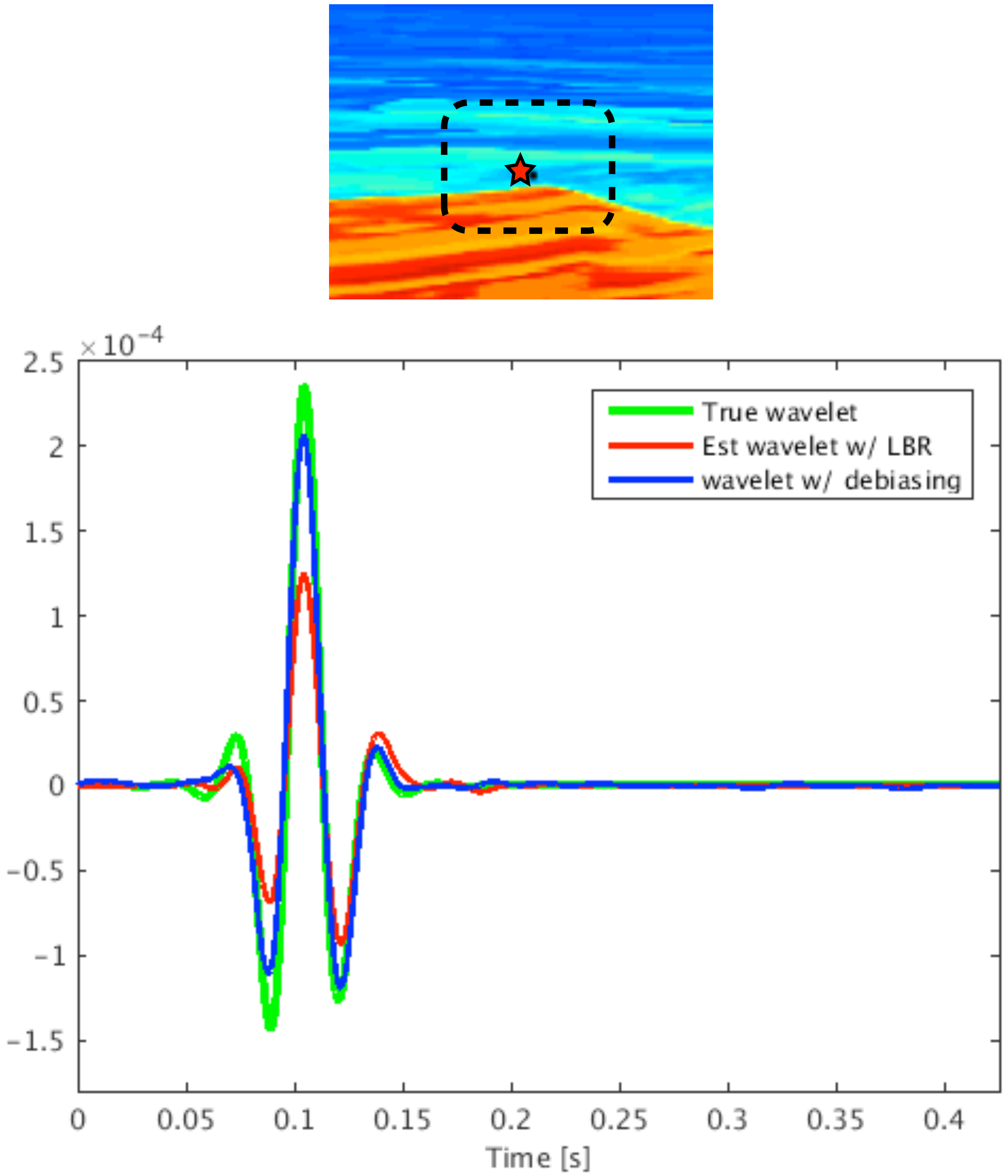
Estimated source location after denoising (zoomed)

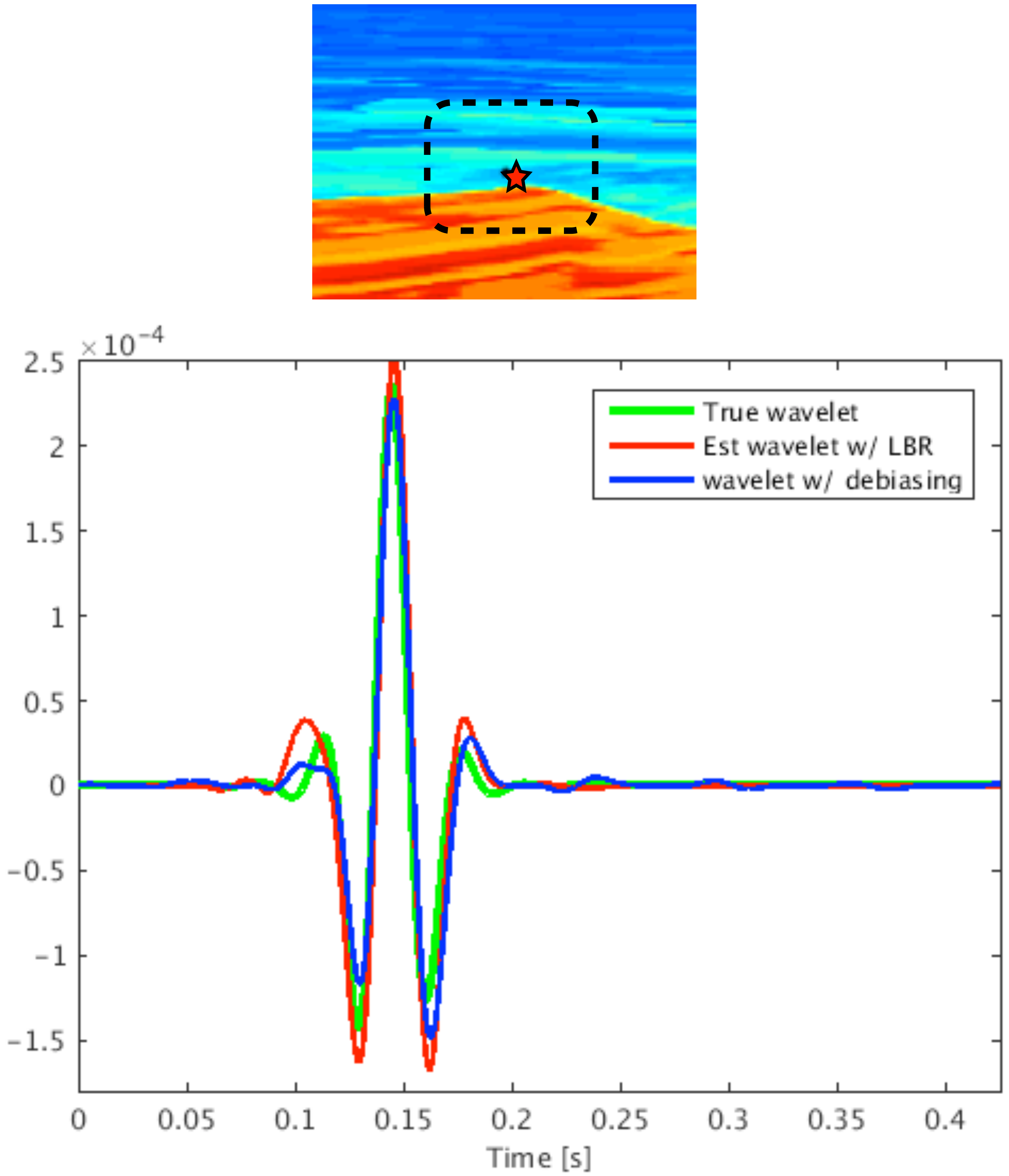


Estimated source location after denoising (zoomed)

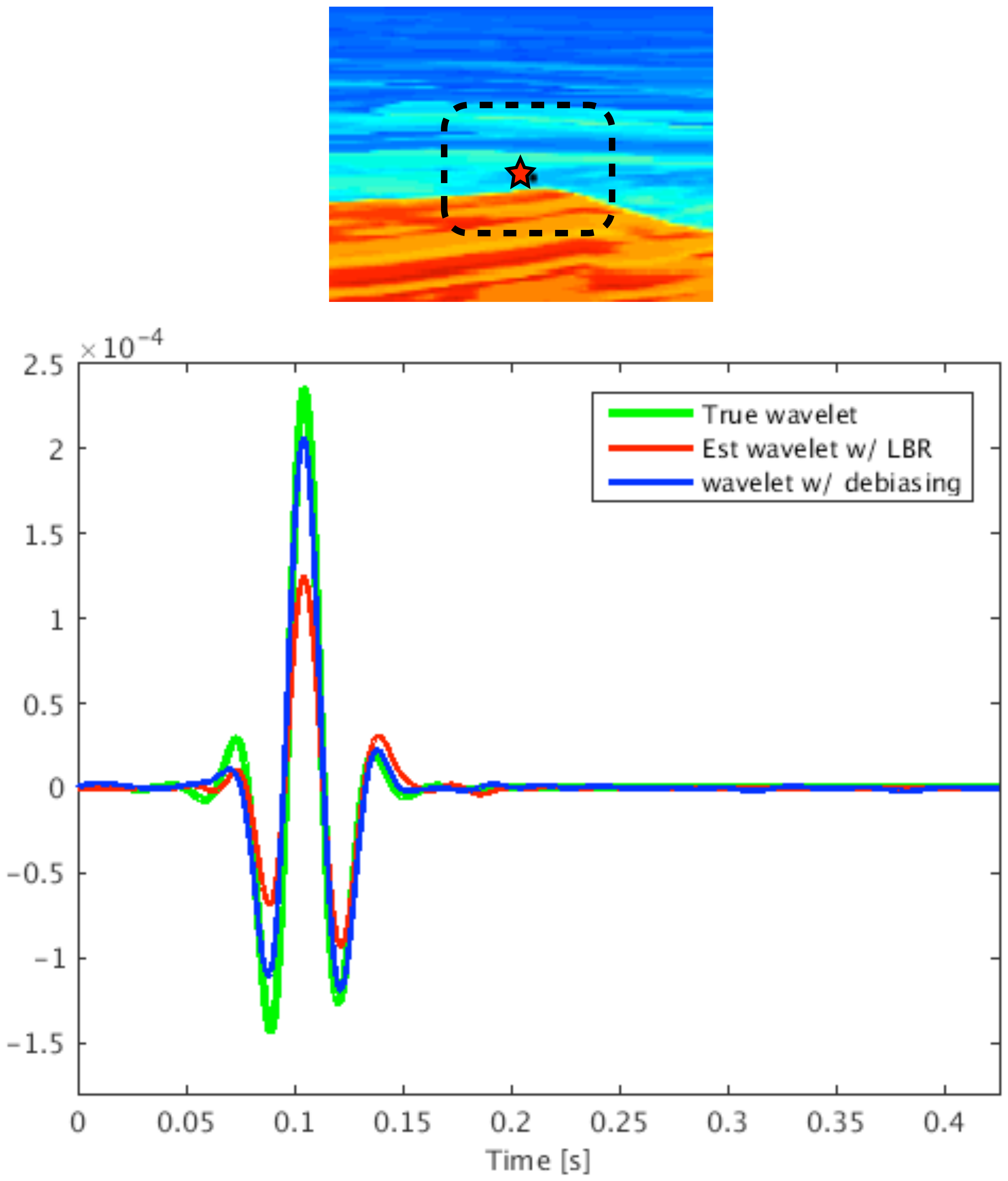


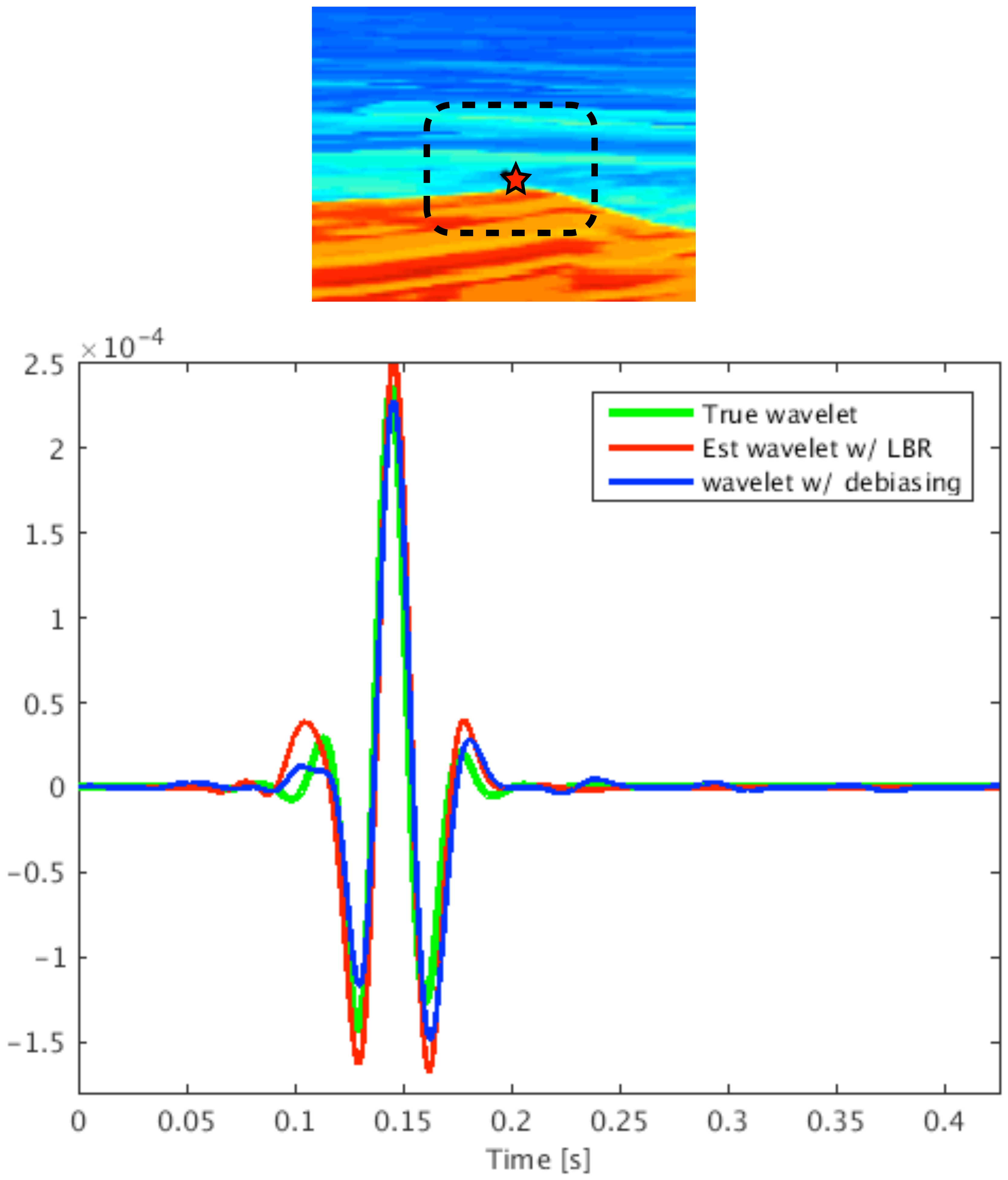
Source-time function comparison after debiasing





Source-time function comparison after debiasing





Estimated source-time function after denoising (without debiasing) is scaled by a factor of 4

Conclusions and future work

With debiasing based approach we are able to:

- locate closely spaced microseismic sources and
- estimate the associated source-time function with correct amplitude from data with very low SNR

Conclusions and future work

With debiasing based approach we are able to:

- locate closely spaced microseismic sources and
- estimate the associated source-time function with correct amplitude from data with very low SNR

Proposed approach is computationally cheap as it requires:

- very few forward and backward Curvelet transforms and
- few iterations of accelerated version of linearized Bregman algorithm

Conclusions and future work

With debiasing based approach we are able to:

- locate closely spaced microseismic sources and
- estimate the associated source-time function with correct amplitude from data with very low SNR

Proposed approach is computationally cheap as it requires:

- very few forward and backward Curvelet transforms and
- few iterations of accelerated version of linearized Bregman algorithm

Use PCA based techniques to deal with different kinds of noise

Acknowledgement

This research was carried out as part of the SINBAD project with the support of the member organizations of the SINBAD Consortium.

Thank you !!