Effects of wrong adjoints for RTM in TTI media.

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In order to obtain accurate images of the subsurface, anisotropic modeling and imaging is necessary. However, the twenty-one parameter complete wave-equation is too computationally expensive to be of use in this case. The transverse tilted isotropic wave-equation is then the next best feasible representation of the physics to use for imaging. The main complexity arising from transverse tilted isotropic imaging is to model the receiver wavefield (back propagation of the data or data residual) for the imaging condition. Unlike the isotropic or the full physics wave-equations, the transverse tilted isotropic wave-equation is not not self-adjoint. This difference means that time-reversal will not model the correct receiver wavefield and this can lead to incorrect subsurface images. In this work, we derive and implement the adjoint wave-equation to demonstrate the necessity of exact adjoint modeling for anisotropic modeling and compare our result with adjoint-free time-reversed imaging.

TTI modeling

The TTI wave equation with the conventional physical parametrization $m(\mathbf{x})$, $\epsilon(\mathbf{x})$,$\delta(\mathbf{x})$, $\theta(\mathbf{x})$,$\phi(\mathbf{x})$ is given by:

$$
\begin{aligned}
&m(x)\frac{d^2 p(\mathbf{x},t)}{dt^2} - (1+2\epsilon(\mathbf{x}))(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(\mathbf{x},t) \\
&- \sqrt{(1+2\delta(\mathbf{x}))}G_{\bar{z}\bar{z}}r(\mathbf{x},t) = q, \\
&m(x)\frac{d^2 r(\mathbf{x},t)}{dt^2} - \sqrt{(1+2\delta(\mathbf{x}))}(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(\mathbf{x},t) \\
&- G_{\bar{z}\bar{z}}r(\mathbf{x},t) = q,
\end{aligned}
$$

This is a coupled wave equation and this equation is not self adjoint

TTI adjoint vs time-reverse

The adjoint wave equation with the same Thomsen parameters is then given by:

$$
\begin{aligned}
&m(x)\frac{d^2 p_a(\mathbf{x},t)}{dt^2} - \\
&(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})((1+2\epsilon(\mathbf{x}))p_a(\mathbf{x},t) + \sqrt{1+2\delta(\mathbf{x})}r_a(\mathbf{x},t)) = q_a, \\
&m(x)\frac{d^2 r_a(\mathbf{x},t)}{dt^2} - G_{\bar{z}\bar{z}}(\sqrt{1+2\delta(\mathbf{x})}p_a(\mathbf{x},t) + r_a(\mathbf{x},t)) = q_a,
\end{aligned}
$$

and consists of a system of split veritcal/horizontal wave equations. The solution to this system is not the solution of the forward wave equation solved backward in time with the data as a source. We show the difference between the time reversed TTI equation and the adjoint TTI equation on Figure … where we compare the impulse response (single receiver) of the system of equation. We look at the two components of the wave as well as their sum and compare a vertical and horizontal trace for each of the snapshots.

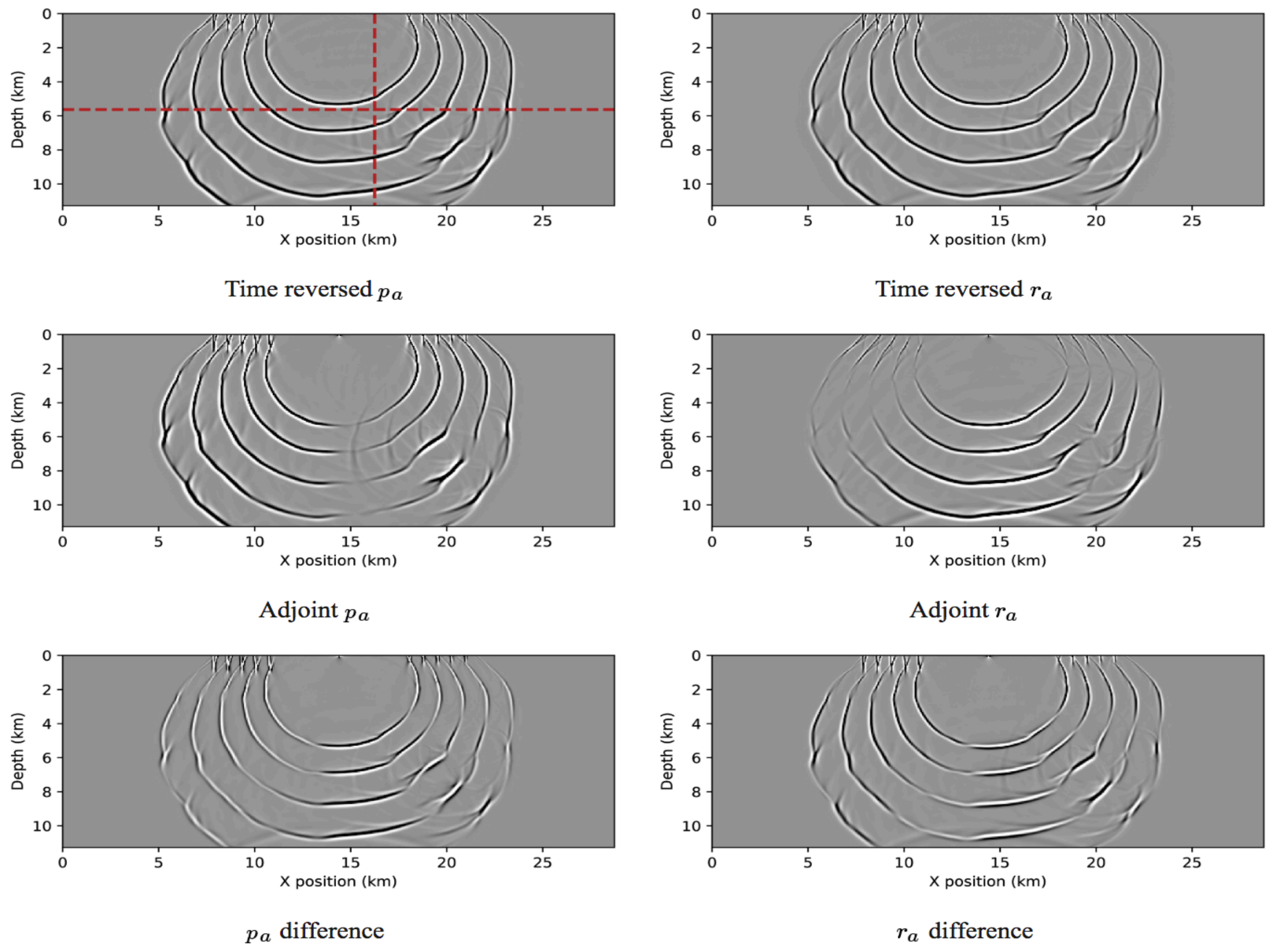
We will show the difference between considering the TTI system self adjoint and using the true adjoint for imaging in two steps:

- We compare the adjoint wakefield to the forward wakefield for a point source and highlight the discrepancies in amplitude and illumintaion.

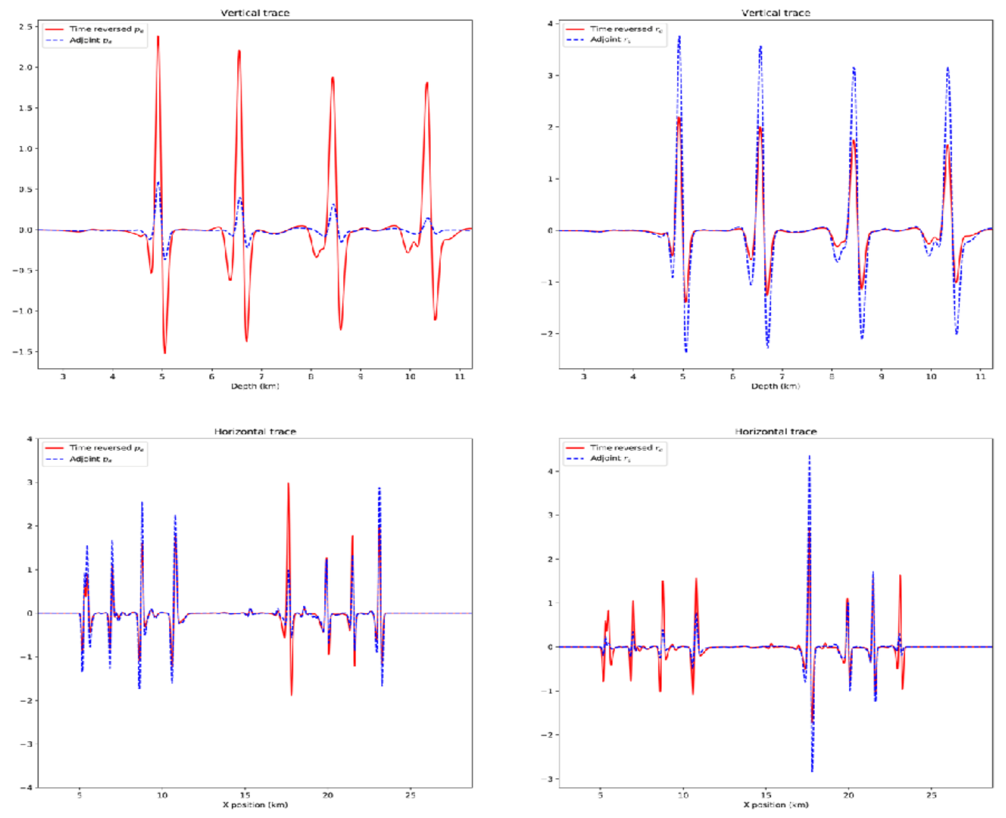
- We show an RTM result on the 2007 BP TTI model imaging the raw data. In this case we compare the image obtained with the correlation of the forward and adjoint wavefield, and the correlation of the forward and backward in time forward wavefield

Adjoint versus forward wakefields

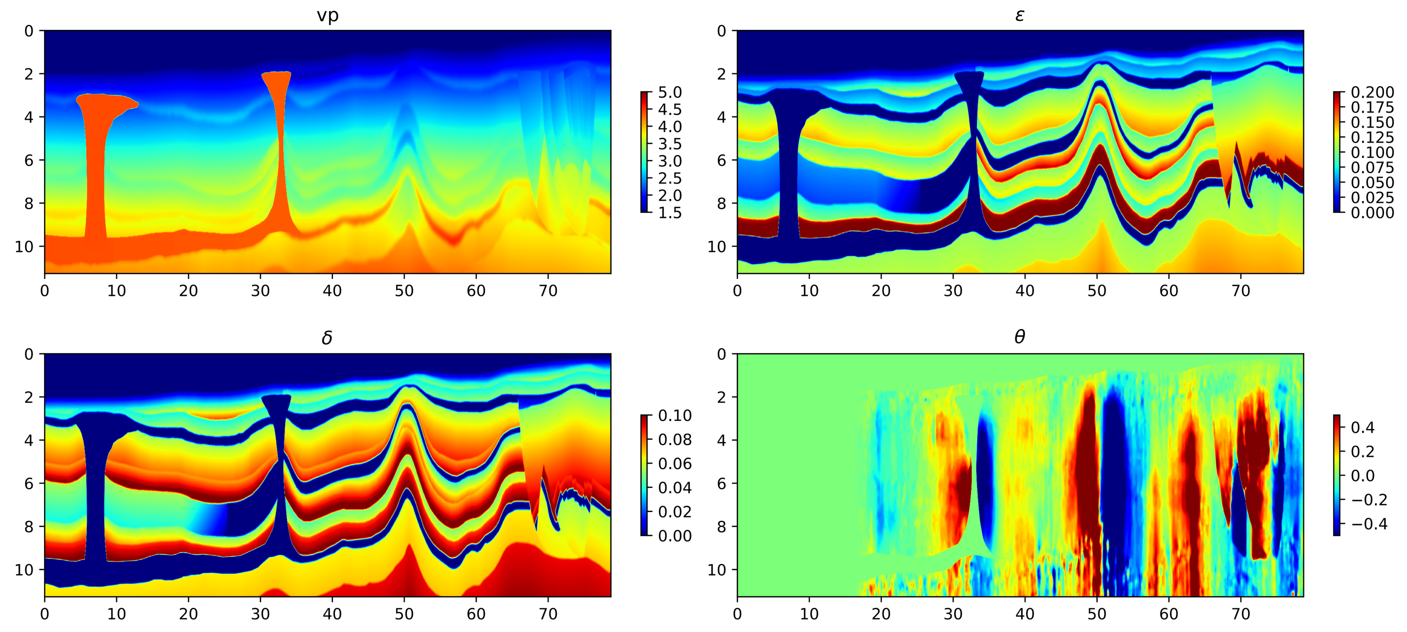
We compare the impulse response of the forward and adjoint TTI wave equations to demonstrate that the TTI system is not self adjoint. We will show in the imaging example the impact on the final RTM image.



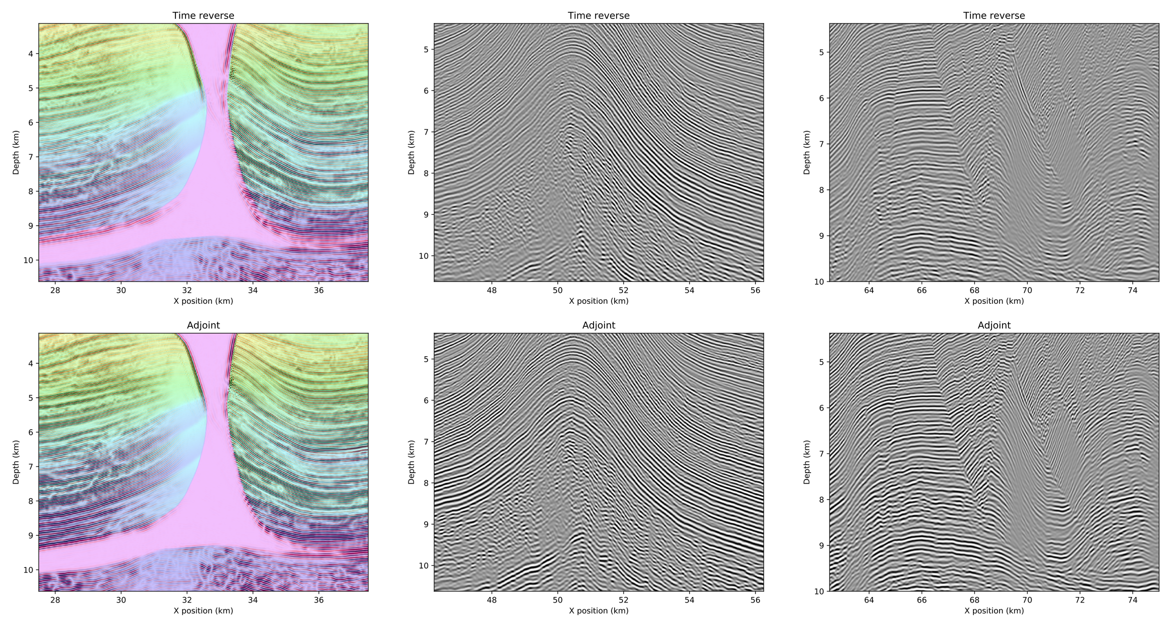
Snapshots of the two components of the forward wavefield and adjoint wavefield for a single source in the right part of the 2007 BP TTI model.



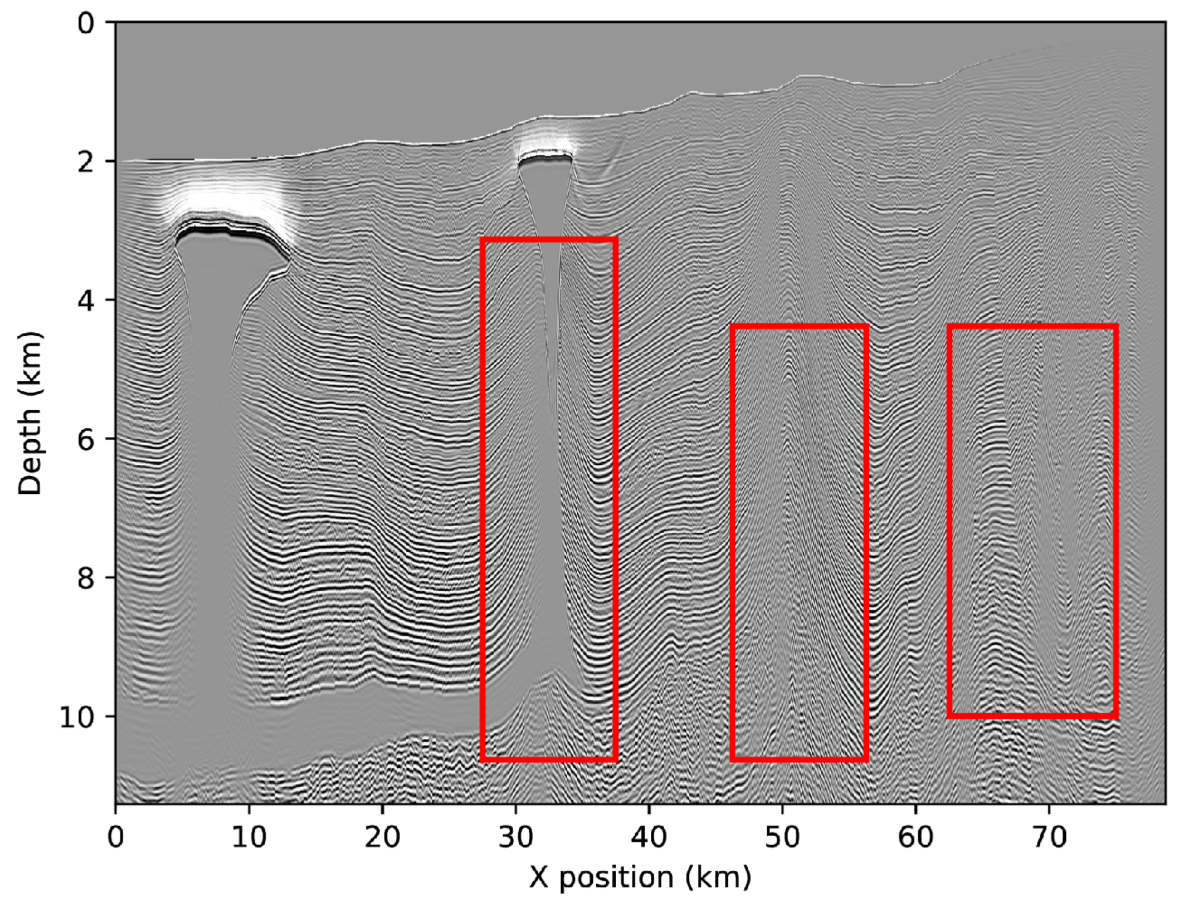
Traces highlighted in red in the snapshots that show the strongest difference in amplitude repartition in the subsurface.



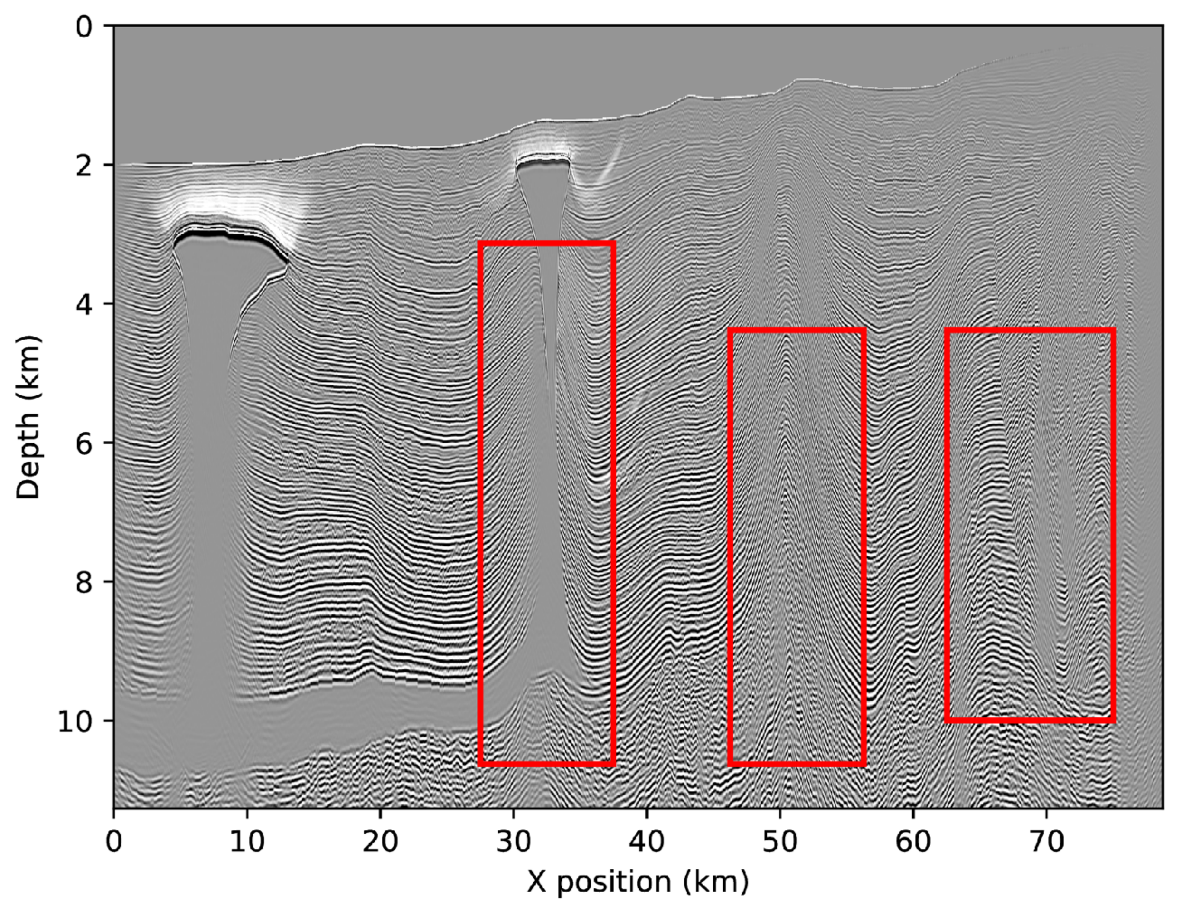
BP 2007 TTI model parameters



Closeup of the RTM images



RTM with time-reverse method



RTM with adjoint method

Conclusion

- **TTI is not self adjoint**
- **Using it as self adjoint leads to incorrect images at depth and large offset**
- **Correct adjoint of the TTI system leads to better images and focusing**
- **The error is most important in high anisotropy areas such as the right part of the model and the salt flanks**
- **More advanced methods, such as LSRTM, will require exact adjoint in order tom converge.**

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