Time domain sparsity promoting LSRTM with surface-related multiples in shallow-water case

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SUMMARY:

In the SRME relation, multiples are expressed as the multi-dimensional convolution between the the vertical derivative of the surface-free Green's function and the down-going receiver wavefield. This relation leads to the methods that separate the surface-related multiples and primaries. Instead of imaging separated multiples trivially, we introduced the SRME relation into wave equation by areal source injection, which costs nothing extra to involve multiples into the forward wavefields. The related image contains the phantoms components from cross-correlation between different-orders of events. Especially in shallow water, due to the strong magnitudes of multiples and their overlap with primaries, the artifacts contain strong phantoms not only of ocean bottom but also subsurface layer interfaces. We use sparsity-promoting inversions where the nicely curved structure of the subsurface models are detected to help cleaning up the phantoms. We reduce the costs by combining randomized source subsampling with linearized Bregman method and get cleaned image with one data pass.

METHOD

The SRME relation (Verschuur et al., 1992) forms the foundation of most multiples separation methods since it relates the vertical derivative of the surface-free Green's function and the downgoing wavefield to the total upgoing wavefield. By combining the SRME relation and the linearized forward Born modelling together, we get the following equation in time domain (Tu et al., 2013)

$$\mathbf{P} \approx \nabla \mathbf{F}[\mathbf{m}_0; \mathbf{Q} - \mathbf{P}]\delta\mathbf{m}, \tag{1}$$

where the matrix $\mathbf{P}$ stands for the total up-going wavefield at the surface. The matrix $\mathbf{Q}$ represents the down-going point source wavefield, acts as an areal source.$\nabla\mathbf{F}$ represents the vertical deriavtive of linearized Born modelling, which is linear in the model perturbation $\delta\mathbf{m}$ and the source term. So, in effect we replaced the expensive multi-dimensional convolutions in SRME by an areal source injection into Born modelling.

To generate realistic surface-related multiples in the water column, rather than introducing strong velocity perturbation at ocean bottom (Tu and Herrmann, 2015), we introduce density variations$\boldsymbol{\rho}$ at the ocean bottom. Equation 1 now becomes

$$\mathbf{P} \approx \nabla \mathbf{F}_{\mathbf{m}}[\mathbf{m}_0, \boldsymbol{\rho}_0; \mathbf{Q} - \mathbf{P}]\delta\mathbf{m}'. \tag{2}$$

In Equation 2, $\delta\mathbf{m}'$ contains the velocity perturbation $\delta\mathbf{m}$ and the strong component introduced by the density perturbation at ocean bottom. By including the density term, we are able to model realistic surface-related multiples in the water column without relying on unrealistic velocity perturbations which may cause numerical problems.

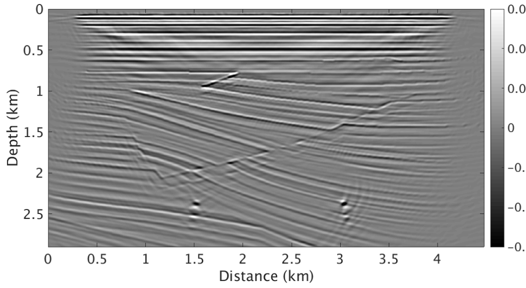
Tu (2015) demonstrated that sparsity-promoting LSRTM leads to artifact-free high-resolution images. Considering of the large scale of this ill-posed problem in time domain, and the goal of pursuing sparse solution, we use the linearized Bregman method with source subsampling. The images are obtained by solving

$$\begin{aligned}&\min_{\mathbf{x}} \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{x}\|_2^2 \\ &\qquad \text{subject to } \sum_j \|\nabla\mathbf{F}_j(\mathbf{m}_0, \rho_0, \mathbf{Q}_j - \mathbf{P}_j)\mathbf{C}^T\mathbf{x} - \mathbf{P}_j\|_2 \le \sigma,\end{aligned} \tag{3}$$

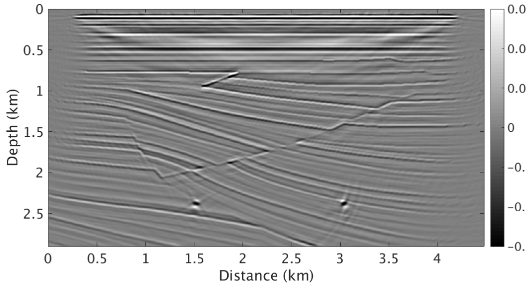
where $\mathbf{x}$ represents the curvelet coefficient vectors for the velocity perturbation $\delta\mathbf{m}'$. $\|\cdot\|_1$ and $\|\cdot\|_2$ stand for ℓ_1 and ℓ_2 norms, respectively. The sum runs over all shots, σ is the two-norm of the noise. We refer to Herrmann et al. (2015) and Yang et al. (2016) for discussions on the role of the thresholding parameter λ and the strategy to choose λ.

RESULTS

Here we show some synthetic examples using a part of the Sigsbee2A model. The observed data is generated by iWave with free-surface boundary condition, and inverted according to our method based on Devito with absorbing-surface. Data preprocessing like receiver-side deghost and data extrapolation from receivers to surface have already been applied. The results are shown as below.



(a) Inverted image with all the reflection data without areal source injection



(b) Inverted image with all the reflection data with areal source injection

Figure 1: Images by all the reflection data with and without areal source injection

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