High-resolution fast microseismic source collocation and source-time function estimation

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Motivation

Unconventional Reservoir Schematic
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Objectives

- detection of microseismic events in space and time
- estimation of source time function
Motivation

Accurate fracture detection
- by locating closely spaced point microseismic sources along a fracture within half a dominant wavelength

Estimation of fracture evolution in time
- by estimating the source-time function of point microseismic sources without any assumptions on the shape of source-time function source-time functions can be used for estimating the source mechanism
Pre-existing methods

Arrival time picking based methods:
- estimate the location and origin time
- can be challenging in the presence of noise

Imaging based methods:
- do not require arrival time picking
- based on back propagation
- estimate the location and origin time
- require scanning of complete 4D volume (3D in space and 1D in time)

[Rentsch et al., ’07; McMechan, ’82; Gajewski et al., ’05; Nakata et al.,’16; Bazargani et al.,’16]
[Thurber et al., ’00; Waldhauser et al.,’00]
Pre-existing methods

Dictionary learning based methods:

- simultaneously estimate location, origin time and source mechanism
- require forming large dictionaries based on number of sources, number of receivers and number of time samples
- require prior knowledge of source-time function

Full-waveform inversion (FWI) based methods:

- invert for source parameters
- some of these methods assume prior knowledge of source-time function
  - source-time function to be a gaussian function

[Sjögreen et al.,'14; Wu et al.,’96; Kim et al.,’11; Michel et al.,’14; Kaderli et al.,’15]
[Rodriguez et al.,’12; Ely et al., ’13]
Proposed method w/ sparsity promotion

Estimates complete source field in:

- space and
- time
Proposed method w/ sparsity promotion

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No assumptions on:
- shape of source-time function
- prior knowledge of source-time function
Proposed method w/ sparsity promotion

Estimates complete source field in:
- space and
- time

No assumptions on:
- shape of source-time function
- prior knowledge of source-time function

Needs:
- sufficiently accurate medium velocity model
- position of receivers
Proposed method w/ sparsity promotion

Unconventional Reservoir Schematic

Assumptions

- localized in space
Proposed method w/ sparsity promotion

Unconventional Reservoir Schematic

Assumptions

- localized in space
- finite energy along time
Proposed method w/ sparsity promotion

\[ \text{minimize} \quad \|Q\|_{2,1} \]

subject to \[ \|\mathcal{F}[m](Q) - d\|_2 \leq \epsilon \]

\[ Q \in \mathbb{R}^{n_x \times n_t} \]

\( n_x \): number of grid points

\( n_t \): number of time samples

Similar to classic Basis pursuit denoising (BPDN)

Source field

Forward modeling operator

Slowness square

Observed data

Noise level
Proposed method w/ sparsity promotion

\[ \text{minimize} \quad \|Q\|_{2,1} \]

subject to \( \| \mathcal{F}[m](Q) - d \|_2 \leq \epsilon \)

\( Q \in \mathbb{R}^{n_x \times n_t} \)

\( n_x \): number of grid points

\( n_t \): number of time samples
Solving w/ Linearized Bregman

\[
\begin{align*}
\text{minimize} & \quad \|Q\|_{2,1} + \frac{1}{2\mu} \|Q\|_F^2 \\
\text{subject to} & \quad \|\mathcal{F}[m](Q) - d\|_2 \leq \epsilon \\
& \quad \text{where } \|\cdot\|_F \text{ is the Frobenius norm}
\end{align*}
\]

- Recent successful application to seismic imaging problem
- Three-step algorithm simple to implement
- Choice of $\mu$ controls the trade off between sparsity and the Frobenius norm
- $\mu \uparrow \infty$ corresponds to solving original BPDN problem
1. **Data** $\mathbf{d}$, **slowness square** $\mathbf{m}$    //Input
2. **for** $k = 0, 1, \cdots$
3. $\mathbf{V}_k = \mathcal{F}^\top[\mathbf{m}](\Pi_\varepsilon(\mathcal{F}[\mathbf{m}](\mathbf{Q}_k) - \mathbf{d}))$    //adjoint solve
4. $\mathbf{Z}_{k+1} = \mathbf{Z}_k - t_k \mathbf{V}_k$    //auxiliary variable update
5. $\mathbf{Q}_{k+1} = \text{Prox}_{\mu \ell_2,1}(\mathbf{Z}_{k+1})$    //sparsity promotion
6. **end**
7. $I(\mathbf{x}) = \sum_t |Q(\mathbf{x}, t)|$    //Intensity plot

[Lorentz et al.,’14; Combettes et al.,’11]
Linearized Bregman algorithm

1. **Data** \( d, \text{ slowness square } m \)  //Input
2. **for** \( k = 0, 1, \cdots \)
3. \( \mathbf{V}_k = \mathcal{F}^\top [m](\Pi_\varepsilon(\mathcal{F}[m](\mathbf{Q}_k) - d)) \)  //adjoint solve
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6. **end**
7. \( \mathbf{I}(\mathbf{x}) = \sum_t | \mathbf{Q}(\mathbf{x}, t) | \)  //Intensity plot

* \( \Pi_\varepsilon(x) = \max\{0, 1 - \frac{\varepsilon}{\|x\|}\}\), \( x \) the projection on to \( \ell_2 \) norm ball
Linearized Bregman algorithm

1. Data $d$, slowness square $m$  //Input
2. for $k = 0, 1, \cdots$
3. $V_k = \mathcal{F}^\top[m](\Pi(\mathcal{F}[m](Q_k) - d))$  //adjoint solve
4. $Z_{k+1} = Z_k - t_k V_k$  //auxiliary variable update
5. $Q_{k+1} = \text{Prox}_{\mu\ell_2,1}(Z_{k+1})$  //sparsity promotion
6. end
7. $I(x) = \sum_t |Q(x, t)|$  //Intensity plot

* $\Pi(x) = \max\{0, 1 - \frac{\epsilon}{\|x\|}\}x$ the projection on to $\ell_2$ norm ball

*where $t_k = \frac{\|\mathcal{F}[m](Q_k) - d\|^2}{\|\mathcal{F}[m](\mathcal{F}[m](Q_k) - d)\|^2}$ is the dynamic step length
Linearized Bregman algorithm

1. Data d, slowness square m  //Input
2. for k = 0, 1, ⋯
3. \[ V_k = \mathcal{F}^\top[m](\Pi_\varepsilon(\mathcal{F}[m](Q_k) - d)) \] //adjoint solve
4. \[ Z_{k+1} = Z_k - t_k V_k \] //auxiliary variable update
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* \( \Pi_\varepsilon(x) = \max\{0, 1 - \frac{\varepsilon}{\|x\|}\} \), \( \varepsilon \) the projection on to \( \ell_2 \) norm ball

*where \( t_k = \frac{\|\mathcal{F}[m](Q_k) - d\|^2}{\|\mathcal{F}[m](\mathcal{F}[m](Q_k) - d)\|^2} \) is the dynamic step length

* \( \text{Prox}_{\mu \ell_2,1}(C) := \arg \min_B \|B\|_{2,1} + \frac{1}{2\mu}\|C - B\|^2_F \) is the proximal mapping of the \( \ell_{2,1} \) norm
Linearized Bregman algorithm

1. Data $d$, slowness square $m$  //Input
2. for $k = 0, 1, \cdots$
3. $V_k = F^\top[m](\Pi_\varepsilon(F[m](Q_k) - d))$  //adjoint solve
4. $Z_{k+1} = Z_k - t_k V_k$  //auxiliary variable update
5. $Q_{k+1} = \text{Prox}_{\mu\ell_2,1}(Z_{k+1})$  //sparsity promotion
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*where $t_k = \frac{\|F[m](Q_k) - d\|^2}{\|F[m][F[m](Q_k) - d]\|^2}$ is the dynamic step length

*Prox$_{\mu\ell_2,1}(C) := \arg\min_B \|B\|_{2,1} + \frac{\mu}{2\pi}\|C - B\|^2_F$ is the proximal mapping of the $\ell_{2,1}$ norm

- **Source location**: estimated as outlier in intensity plot
- **Source-time function**: temporal variation of wavefield at estimated source location
$V_1 = F^\dagger [m] (\Pi_{\epsilon} (F[m](Q_0) - d))$
V_1 = \mathcal{F}^\dagger [m] (\Pi \epsilon (\mathcal{F}[m](Q_0) - d))

Adjoint solve
\[ V_1 = \mathcal{F}^\dagger [m] (\Pi_{\epsilon}(\mathcal{F}[m](Q_0) - d)) \]

Adjoint solve

\[ Z_1 = Z_0 - t_1 V_1 \]
\[ V_1 = F^\dagger [m] (H \epsilon (F[m](Q_0) - d)) \]

**Adjoint solve**

\[ Z_1 = Z_0 - t_1 V_1 \]

**Auxiliary variable update**
\[
V_1 = \mathcal{F}^\dagger [m] (\Pi_{\epsilon} \mathcal{F}[m](Q_0 - d))
\]

**Adjoint solve**

\[
Z_1 = Z_0 - t_1 V_1
\]

**Auxiliary variable update**

\[
Q_1 = \text{Prox}_{\mu\ell_{2,1}}(Z_1)
\]

**Sparsity promotion**
0.1 0.2 0.3 0.4 0.5 0.6
Receiver Position [km]

0 0.2 0.4 0.6
Time [s]

0 0.1 0.2 0.3 0.4 0.5 0.6
Lateral [km]

0.005 0.01 0.015 0.02 0.025 0.03 0.035 0.04 0.045
Depth [km]

0 0.5 1 1.5 2 2.5 3
× 10^{-3}

0 0.1 0.2 0.3 0.4 0.5 0.6
Lateral [km]

0 0.5 1 1.5 2 2.5 3
× 10^{-3}

0 0.1 0.2 0.3 0.4 0.5 0.6
Lateral [km]

0 0.5 1 1.5 2 2.5 3
× 10^{-3}

\[ \mathbf{V}_1 = \mathcal{F}^\dagger [\mathbf{m}] (\mathcal{I}_\varepsilon (\mathcal{F}[\mathbf{m}] (Q_0) - d)) \]

**Adjoint solve**

**Auxiliary variable update**

\[ Z_1 = Z_0 - t_1 \mathbf{V}_1 \]

**Sparsity promotion**

\[ \mathbf{Q}_1 = \text{Prox}_{\mu \ell_{2,1}} (Z_1) \]

\[ I(x) = \sum_t | Q_1(x, t) | \]
\[ \mathbf{V}_1 = \mathcal{F}^\dagger \left[ \mathbf{m} \left( \Pi_{\epsilon} (\mathcal{F} \mathbf{m} (\mathbf{Q}_0) - \mathbf{d}) \right) \right] \]

**Adjoint solve**

**Source-time function**

\[ \mathbf{Q}_1 = \text{Prox}_{\mu \ell_{2,1}} (\mathbf{Z}_1) \]

**Sparsity promotion**

**Auxiliary variable update**

\[ \mathbf{Z}_1 = \mathbf{Z}_0 - t_1 \mathbf{V}_1 \]
Case study: two far sources

Modeling information:

**Model size:** 0.7 km x 0.7 km  
**Grid spacing:** 5m  
**Receiver spacing:** 10m  
**Receiver depth:** 20m  
**Fixed spread:** 0.69km  
**Sampling interval:** 2 ms  
**Recording length:** 1s  
**Peak frequency:** 30 Hz  
**Dominant wavelength:** 46 m
Data and estimated location

**Microseismic data**

**Estimated location**
Data and estimated location

**Microseismic data**

**Estimated location**
Estimated wavelet
What happens when sources are very close?

Acquisition setup

Microseismic data

1400 m/s
Estimated location after 150 iterations

Estimated location, $\mu = 8e-4$
Estimated location after 150 iterations

Estimated location, $\mu = 8 \times 10^{-4}$
Estimated location after 150 iterations

Estimated location, $\mu = 8e-4$

Increased $\mu$

For high resolution

Estimated location, $\mu = 8e-3$
Estimated location after 150 iterations

Fails to resolve due to slow convergence

Estimated location, $\mu = 8 \times 10^{-3}$

For high resolution

Estimated location, $\mu = 8 \times 10^{-4}$
Convergence comparison
Convergence comparison

Motivation for locating closely spaced sources: for accurate fracture mapping
Convergence comparison

Motivation for locating closely spaced sources: for accurate fracture mapping

Challenges with Linearized Bregman algorithm:
- need higher value of $\mu$ to resolve closely spaced sources
- higher values of $\mu$ needs more iterations
Motivation for locating closely spaced sources: for accurate fracture mapping

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- higher values of $\mu$ needs more iterations

Acceleration with quasi-Newton:
- Linearized Bregman algorithm is equivalent to solving dual problem by gradient descent
- we accelerate the dual problem using quasi-Newton
Acceleration with quasi-Newton: Algorithm

1. **Data** $d$, slowness square $m$, number of iterations $k$  //Input
2. **Initialize** dual variable $y = 10^{-3}d$
3. $\hat{y} = \text{L-BFGS}(f(y), g(y), y, k)$  //Dual solution
   where $f(y) = \Psi(y) - \epsilon \|y\|_2$  //L-BFGS objective
   and $g(y) = \Psi'(y) - \epsilon y / \|y\|_2$  //L-BFGS gradient
4. $\hat{Q} = \text{Prox}_{\mu \ell_{2,1}}(\mu \mathcal{F}[m]^\top(\hat{y}))$  //Primal solution
5. $I(x) = \sum_t |\hat{Q}(x, t)|$  //Intensity plot

*where $\Psi(y) = \min_Q \|Q\|_{2,1} + \frac{1}{2\mu} \|Q\|_F - y^\top (\mathcal{F}[m](Q) - d)$

* $\Psi'(y) = d - \mathcal{F}[m](\text{Prox}_{\mu \ell_{2,1}}(\mu \mathcal{F}[m]^\top(y)))$ is the gradient of $\Psi(y)$
Acceleration with quasi-Newton: Algorithm

1. Data \( d \), slowness square \( m \), number of iterations \( k \) //Input
2. Initialize dual variable \( y = 10^{-3}d \)
3. \( \hat{y} = \text{L-BFGS}(f(y), g(y), y, k) \) //Dual solution
   where \( f(y) = \Psi(y) - \epsilon \| y \|_2 \) //L-BFGS objective
   and \( g(y) = \Psi'(y) - \epsilon y / \| y \|_2 \) //L-BFGS gradient
4. \( \hat{Q} = \text{Prox}_{\mu \ell_2,1}(\mu \mathcal{F}[m]^{\top}(\hat{y})) \) //Primal solution
5. \( I(x) = \sum_t |\hat{Q}(x, t)| \) //Intensity plot

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*where \( \Psi(y) = \min_Q \| Q \|_{2,1} + \frac{1}{2\mu} \| Q \|_F - y^{\top} (\mathcal{F}[m](Q) - d) \)*

* \( \Psi'(y) = d - \mathcal{F}[m](\text{Prox}_{\mu \ell_2,1}(\mu \mathcal{F}[m]^{\top}(y))) \) is the gradient of \( \Psi(y) \)
Further acceleration w/ 2D Preconditioning

Each iteration of L-BFGS requires solving at least one
- wave equation and
- its adjoint
Further acceleration w/ 2D Preconditioning

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Requires further reduction in the total number of iterations due to:
- the problem size and
- computational costs
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Left preconditioner:
- reduces the condition number of 2D forward modeling operator $F$
- accelerates the convergence
Further acceleration w/ 2D Preconditioning

In 2D, a point source implicitly assumes
- a line source
- extending infinitely in the out of plain direction

This causes wavefields to have:
- amplitude and
- phase differ from the wavefields of a true point source

We introduce:
- a symmetric half differentiation correction along time
- corrects for the amplitude and phase of 2D wavefield
- which acts as a left preconditioner
Modified problem w/ 2D Preconditioning

\[
\begin{align*}
\text{minimize} & \quad \|Q\|_{2,1} + \frac{1}{2\mu} \|Q\|_F^2 \\
\text{subject to} & \quad \|M_L \mathcal{F}[m](Q) - M_L d\|_2 \leq \gamma
\end{align*}
\]

*with $M_L := \frac{1}{|t|}^{1/2}$ is the half differentiation correction

*where $\frac{1}{|t|}^{1/2} = F^{-1}|\omega|^{1/2}F$

*F is the Fourier transform and $\omega$ is the frequency

*\(\gamma\) is the noise level
Result for two close sources

With L-BFGS and 2D preconditioner

10 iterations
Result for two close sources

With L-BFGS and 2D preconditioner

10 iterations
Convergence comparison: LBR vs L-BFGS

Convergence comparison
- Using same value of $\mu$

Improvement in convergence with
- Dual formulation and
- 2D Preconditioning
Numerical Experiment: Marmousi model

Modeling information:

- **Model size:** 3.15 km x 1.08 km
- **Grid spacing:** 5 m
- **Total number of sources:** 7
- **Peak frequency:** 22 Hz, 25 Hz & 30 Hz
- **Receiver spacing:** 10 m
- **Receiver depth:** 20 m
- **Sampling interval:** 0.5 ms
- **Recording length:** 1 s
- **Free surface:** No
Numerical Experiment: Marmousi model

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- **Model size:** 3.15 km x 1.08 km
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- **Recording length:** 1 s
- **Free surface:** No

Adjacent sources are located within half a wavelength with overlapping source-time functions
Data

Noise free microseismic data

Noisy Microseismic data, SNR = 2.9
Estimated source location in 10 iterations

w/noise free data and true velocity model

w/noisy data and smooth velocity model
Estimated source location in 10 iterations

w/noise free data and true velocity model

w/noisy data and smooth velocity model
Wavelet comparison

Peak frequency: 25 Hz
Wavelet comparison

Peak frequency: 30 Hz
Wavelet comparison

Peak frequency: 30 Hz

Peak frequency: 22 Hz
Wavelet comparison

- True Wavelet
- Wavelet w/ noise free data and true velocity
- Wavelet w/ noisy data and smooth velocity

Peak frequency: 22 Hz
Conclusions

Sparsity promotion based method:

- can simultaneously estimate multiple source locations & source-time functions
- can provide locations of fractures by resolving microseismic sources within half a wavelength
- works w/ sources of different frequencies & origin times
Conclusions

Sparsity promotion based method:
- can simultaneously estimate multiple source locations & source-time functions
- can provide locations of fractures by resolving microseismic sources within half a wavelength
- works w/ sources of different frequencies & origin times

Dual formulation provides a computationally efficient scheme.
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Thank you !!