Dynamics-driven error reduction for extremely large problems in geophysics

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SLIM
University of British Columbia

Motivation: least-squares migration

Consider

$$\min_{\mathbf{x}} ||\mathbf{x}||_1$$

s.t.

$$\sum_{i=1}^{n_s} ||\mathbf{J}_i[\mathbf{m}_0, \mathbf{q}_i]\mathbf{C}^*\mathbf{x} - \mathbf{b}_i||_2 \leq \sigma$$

- $\mathbf{x}$ is the vector of Curvelet coefficients,
- $\mathbf{J}_i$ is the Born modelling operator,
- $\mathbf{m}_0$ is the background model for the velocity,
- $\mathbf{b}_i$ is the vectorized reflection of the i-th shot,
- $\mathbf{C}^*$ is the transpose of the curvelet transform,
- σ is the tolerance for noise.

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Solution w/ Linearized Bregman (LB)

$$\begin{aligned} z_{k+1} &= z_k - t_k A_k^T (A_k x_k - b_k) \\ x_{k+1} &= S_\lambda(z_{k+1}). \end{aligned}$$

where

$$S_\lambda(z_k) = \max(|z_k| - \lambda, 0)\mathrm{sign}(z_k)$$

and

$$t_k = \frac{||A_k x_k - b_k||_2^2}{||A_k^T (A_k x_k - b_k)||_2^2}$$

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NOTE: Subsampling is necessary for large data sets

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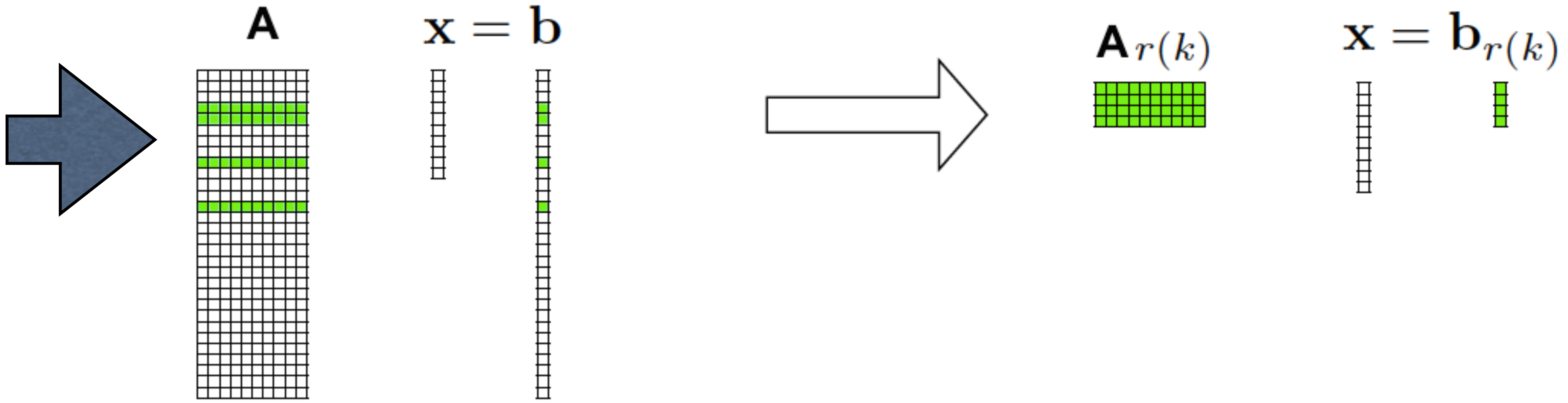

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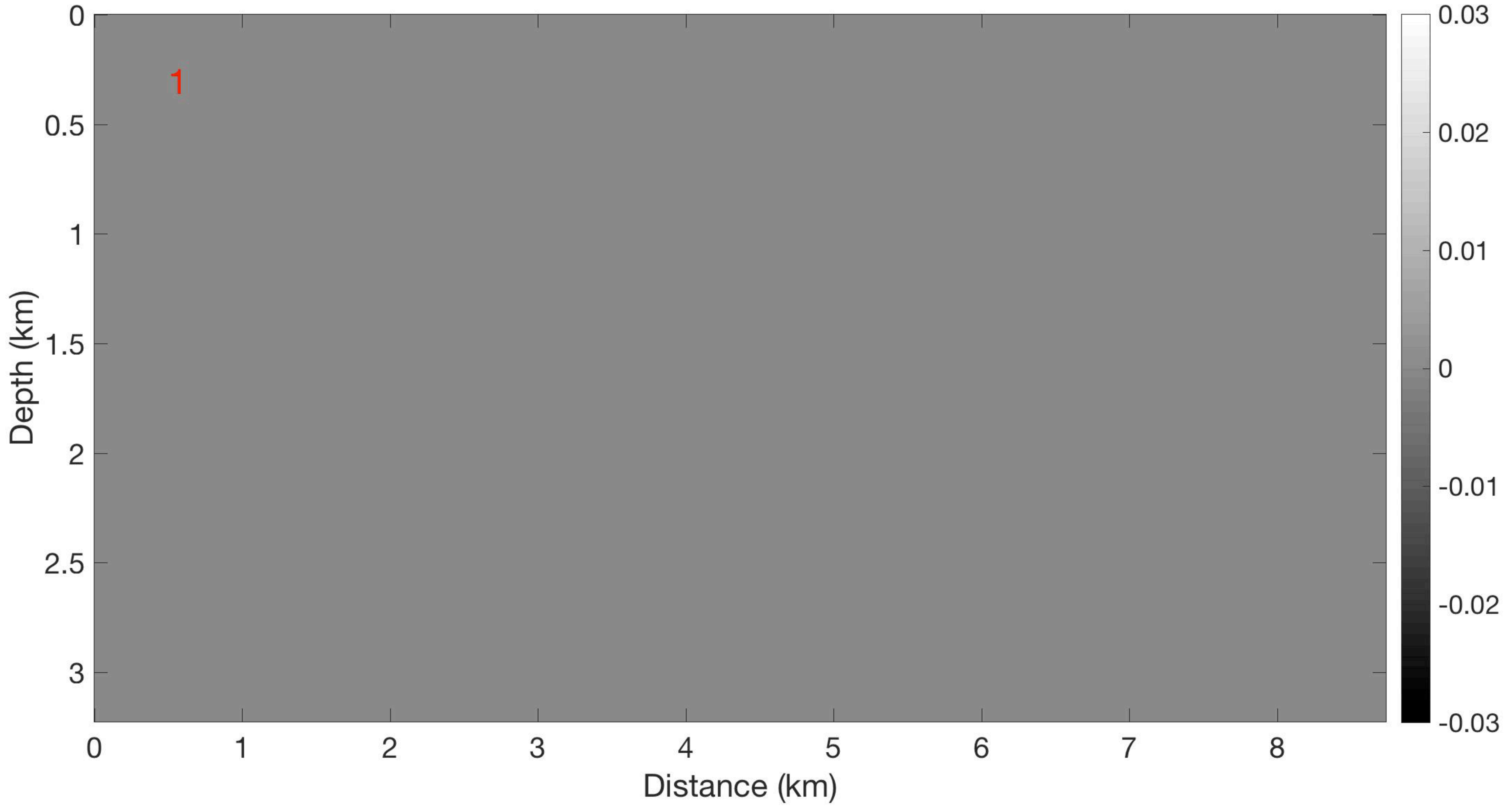

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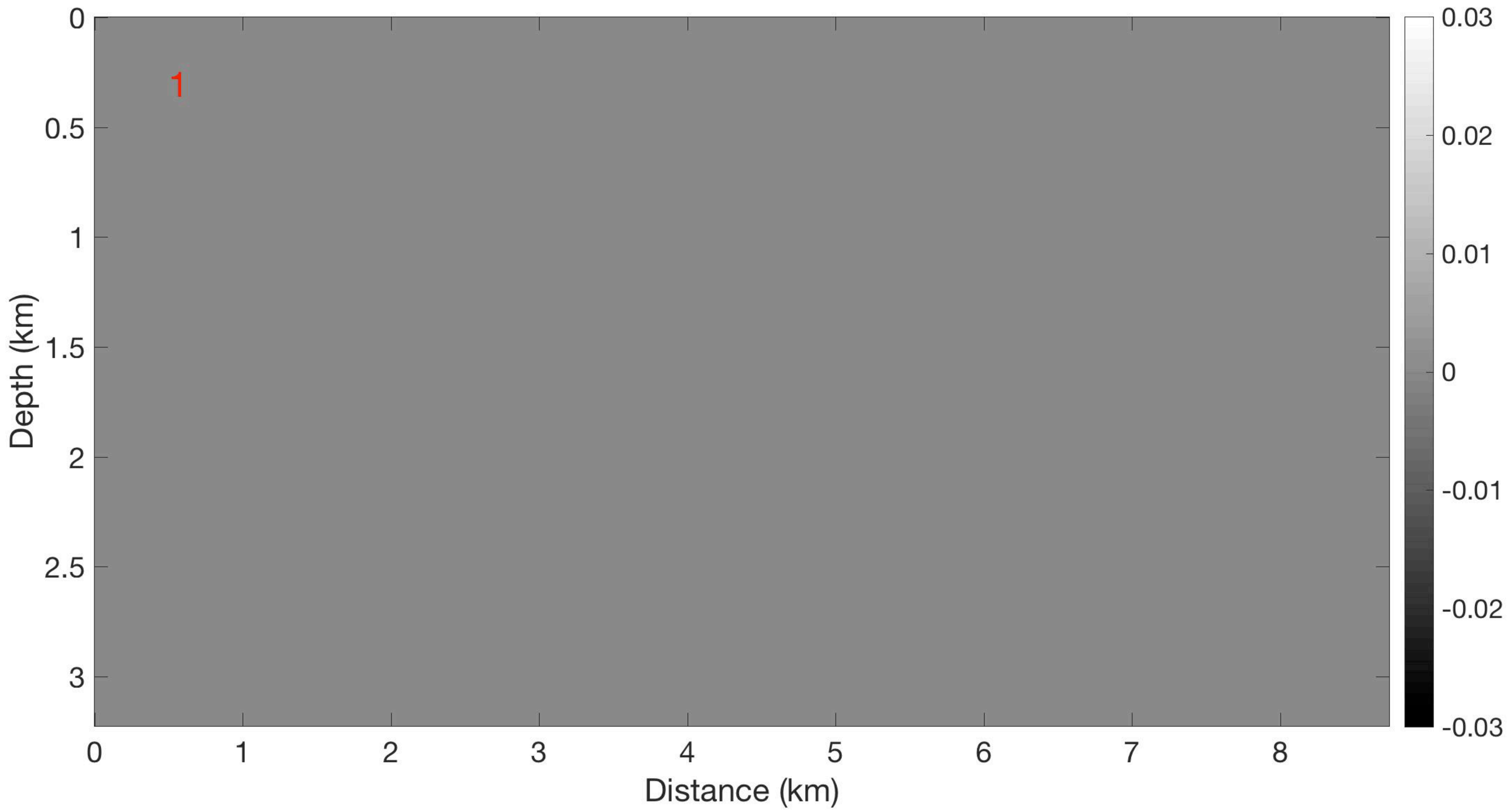

LB Method: w/ weighted increment

LB method

$$
\begin{aligned}
z_{k+1} &= z_k - t_k A_k^T (A_k x_k - b_k) \\
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\end{aligned}
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where

$$S_\lambda(z_k) = \max(|z_k| - \lambda, 0)\mathrm{sign}(z_k)$$

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$$t_k = \frac{||A_k x_k - b_k||_2^2}{||A_k^T (A_k x_k - b_k)||_2^2}$$

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LB method w/ weighted increment

$$\begin{aligned} z_{k+1} &= z_k - \tau_k \odot A_k^T (A_k x_k - b_k) \\ x_{k+1} &= S_\lambda(z_{k+1}), \end{aligned}$$

where

$$S_\lambda(z_k) = \max(|z_k| - \lambda, 0)\text{sign}(z_k)$$

and

$$\tau_k^i = t_k \frac{|\sum_{j=1}^{k} \text{sign}([A_j^T (A_j x_j - b_j)]_i)|}{k}$$

with

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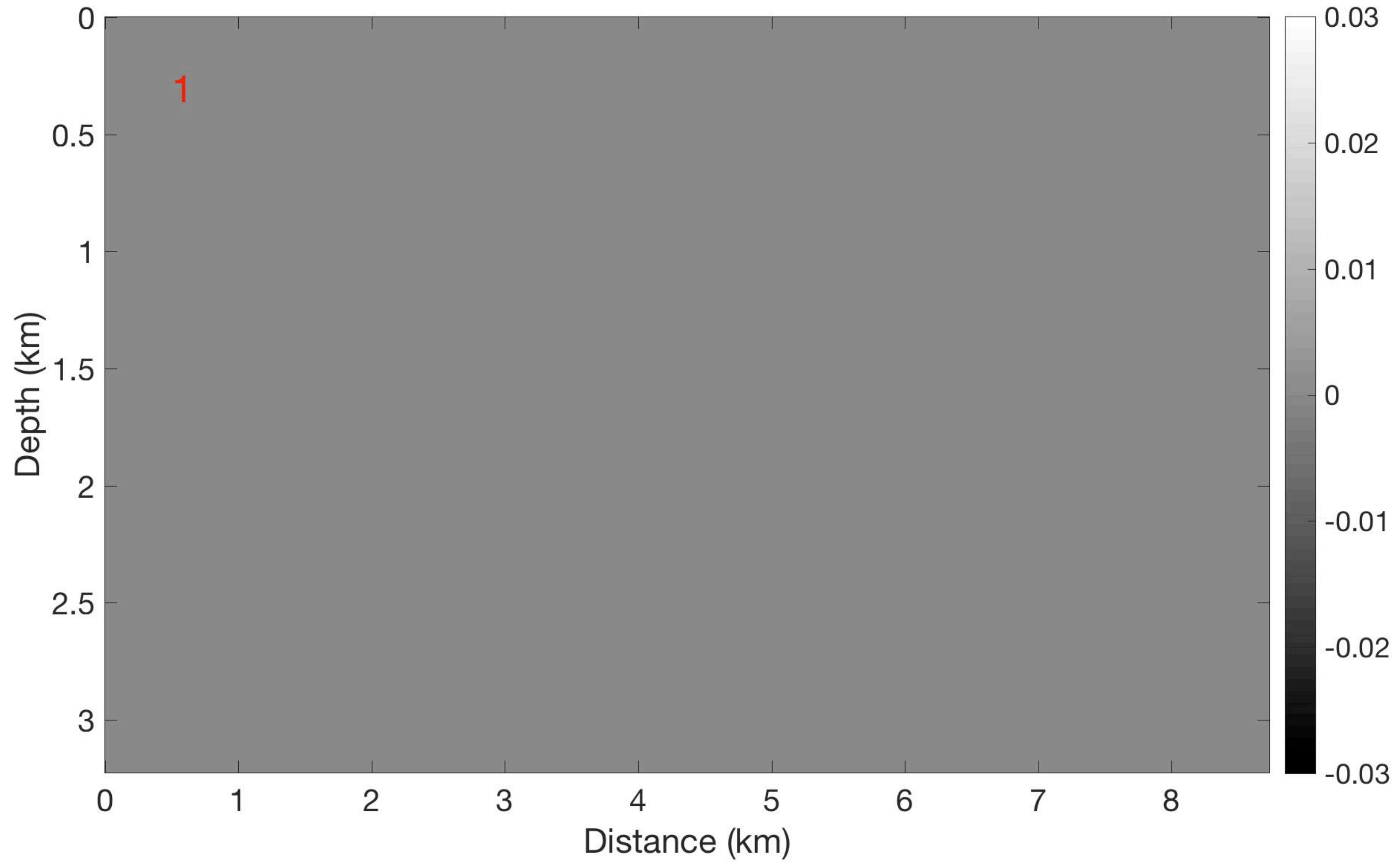
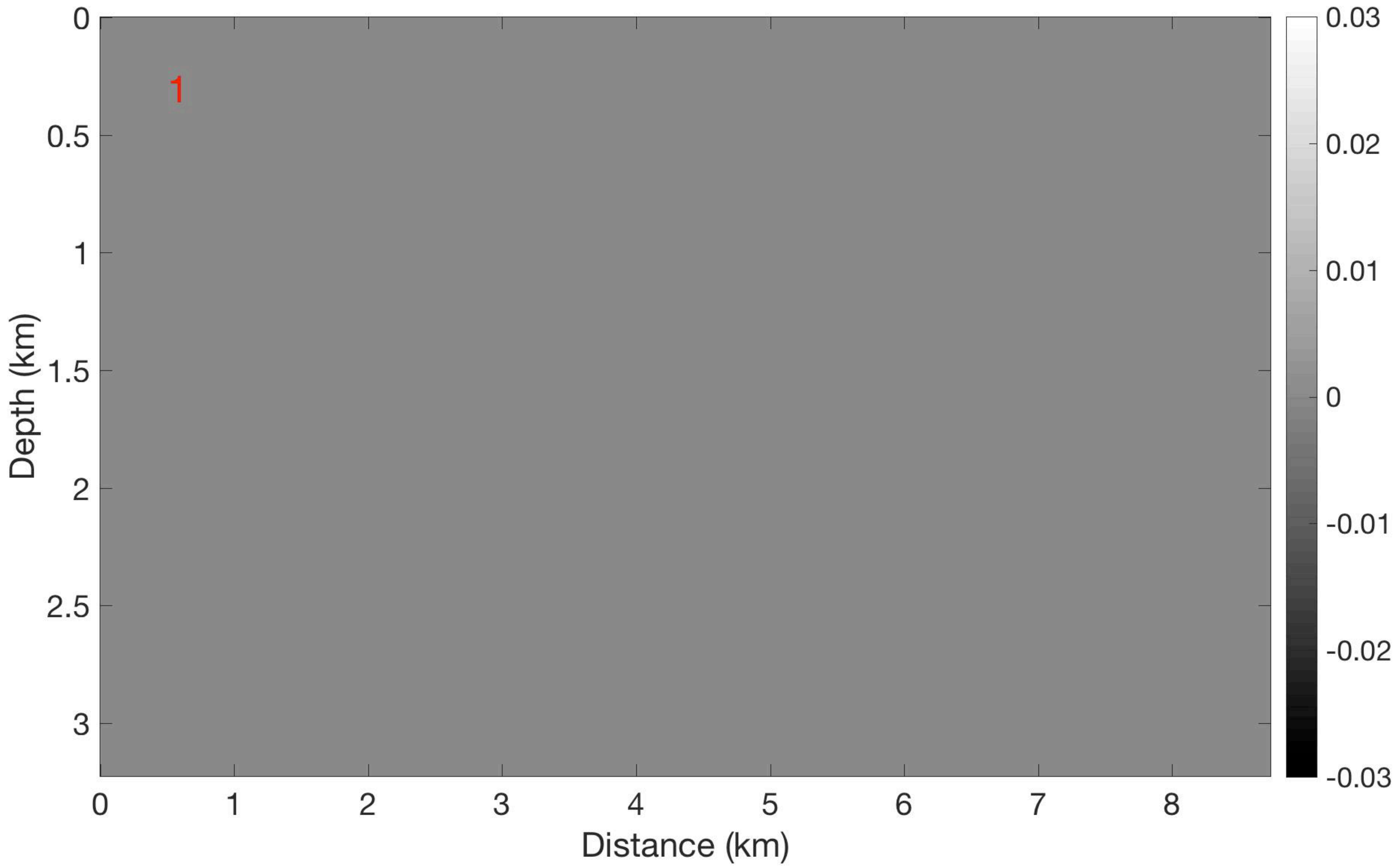
LB method



LB method w/ weighted increment



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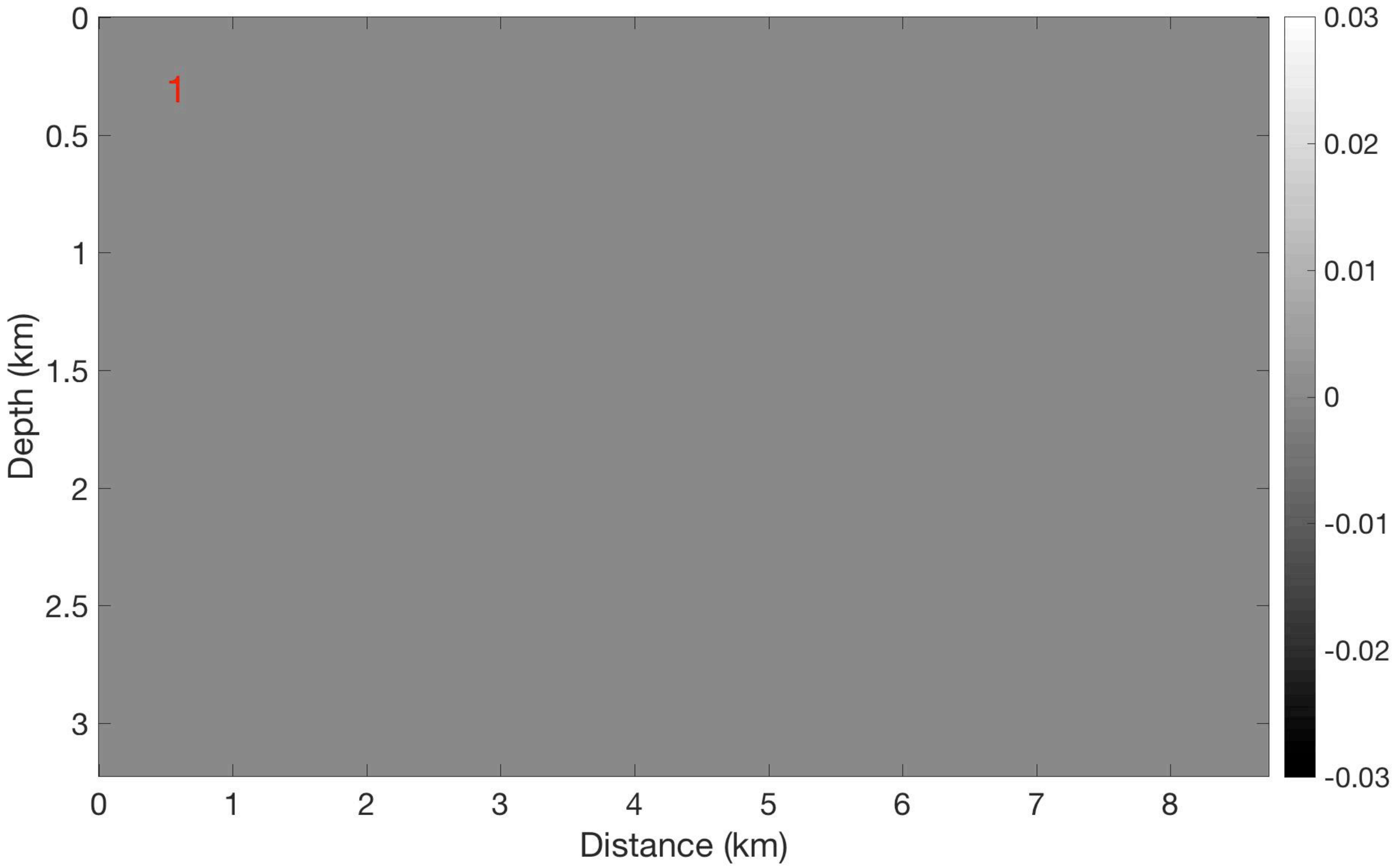

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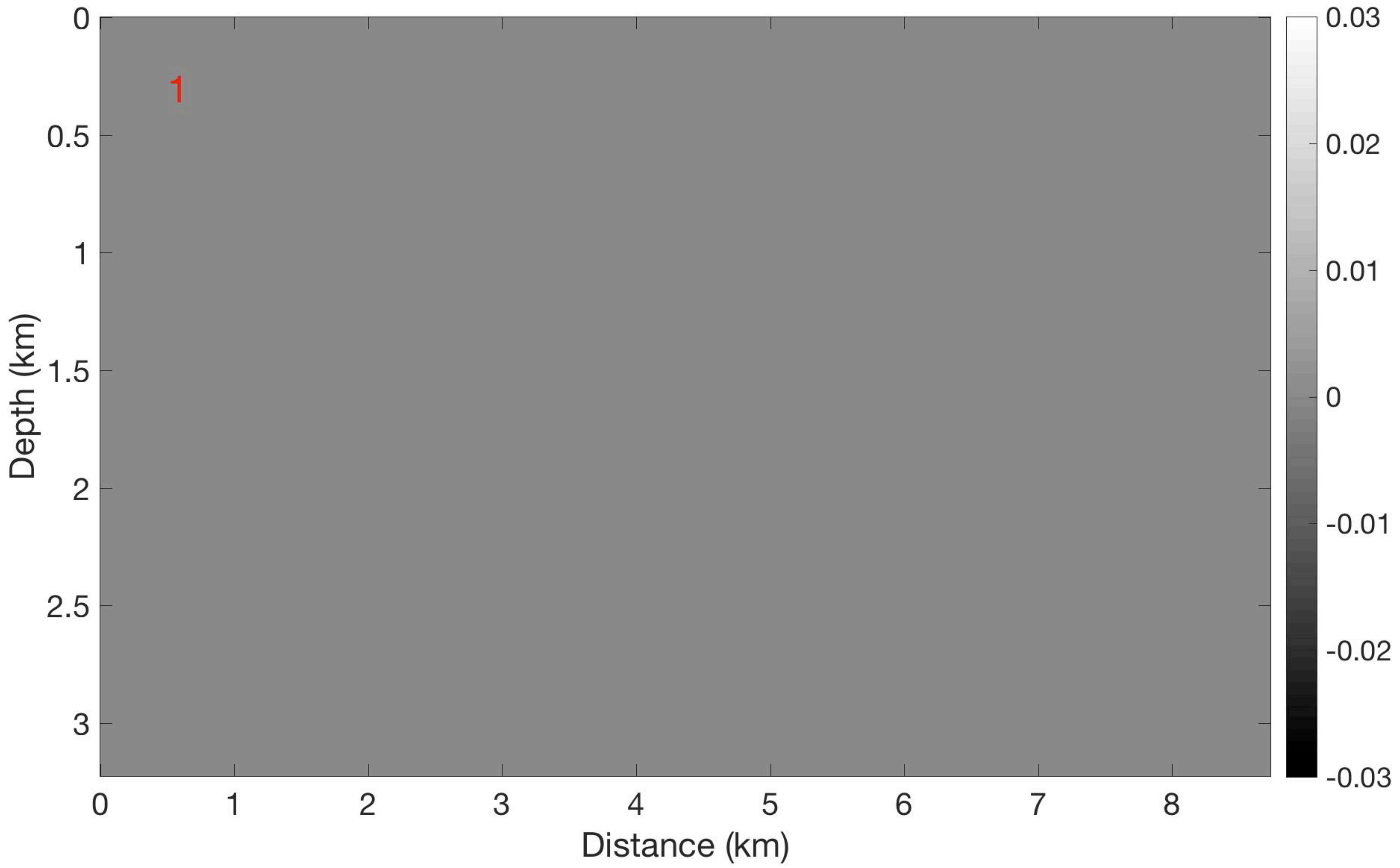
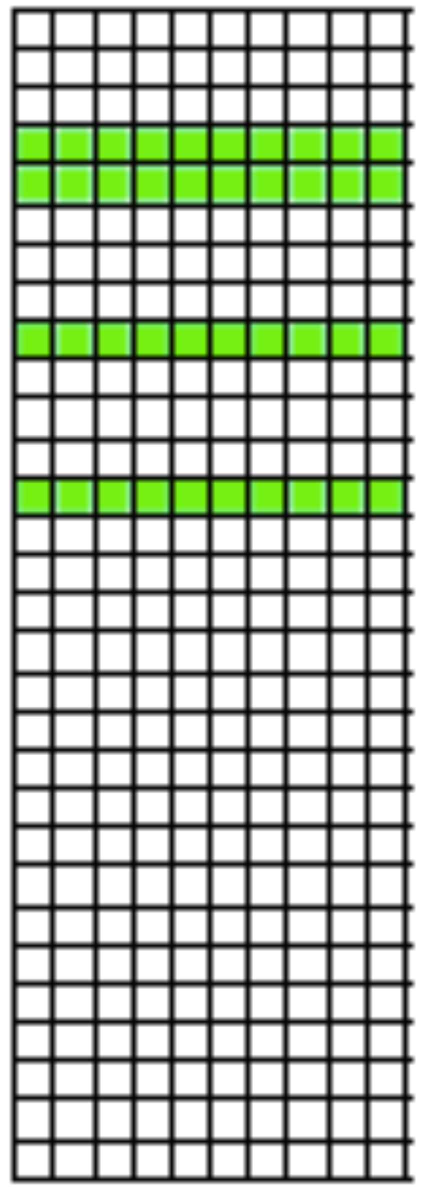


Toy problem

$A \in \mathbb{R}^{20000 \times 1000}$

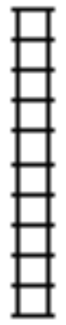
Gaussian matrix



*

$x \in \mathbb{R}^{1000}$

Sparse vector



=

$b \in \mathbb{R}^{20000}$

Noisy data vector

Optimization problems

1. l_1-minimization problem (consistent)

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad Ax = b$$

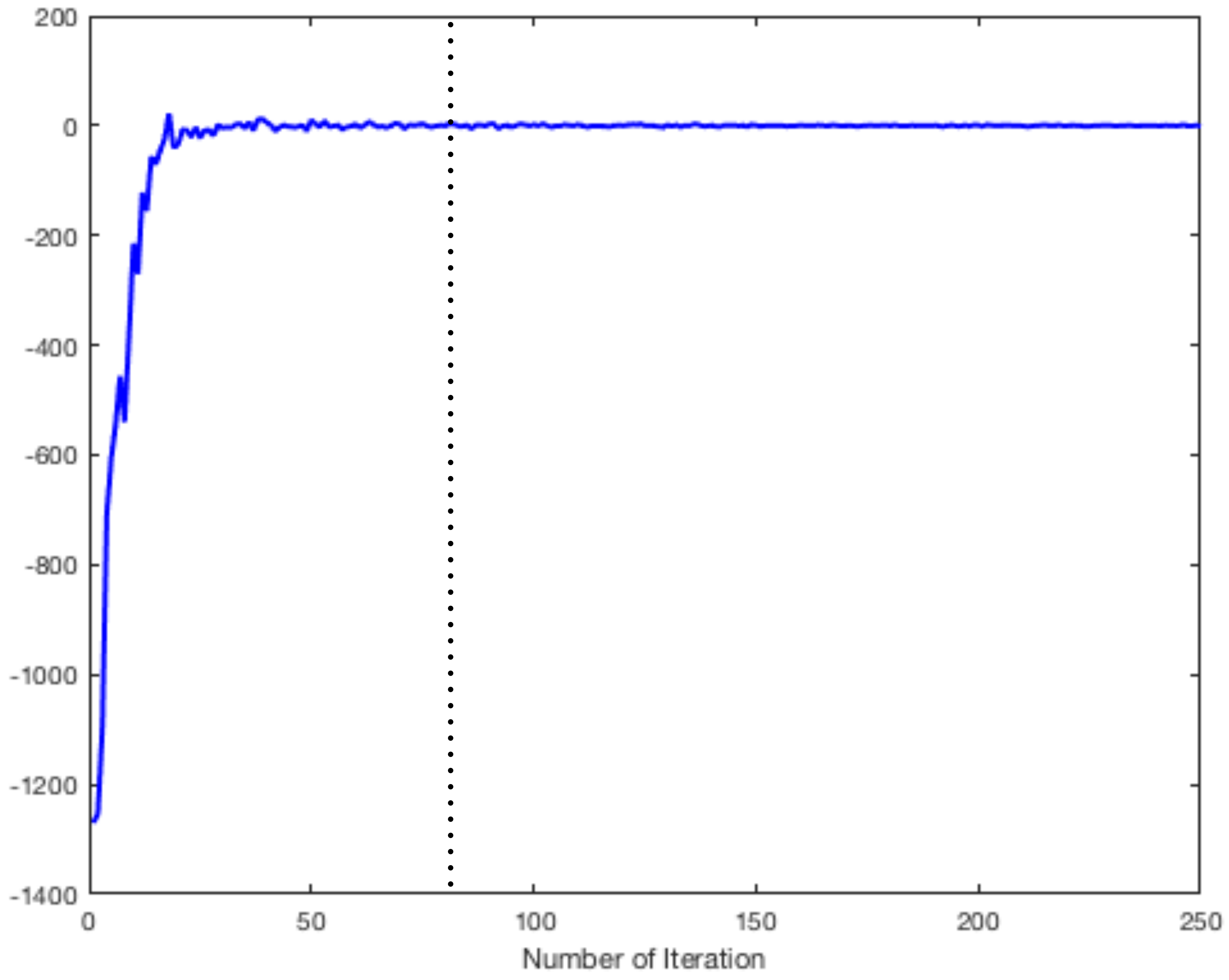
2. BPDN problem (inconsistent)

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad ||Ax - b||_2 \leq \sigma$$

Intuition: gradient entry for weighted increments

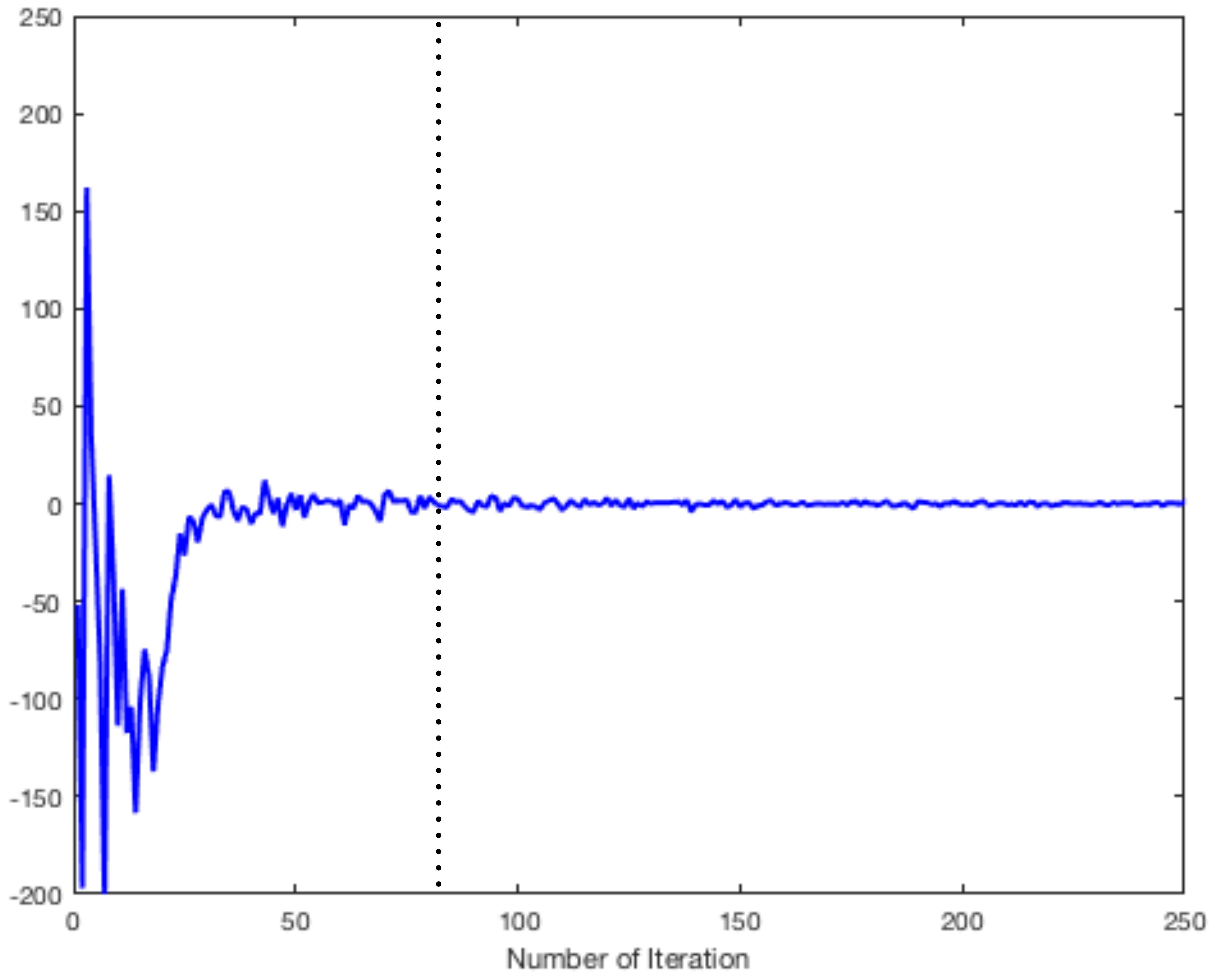
(for consistent problems)

$[A_k^T(A_k x_k - b_k)]_{136}$



Largest entry of the exact solution x^*

$[A_k^T(A_k x_k - b_k)]_{147}$



One of the small entries of the exact solution x^*

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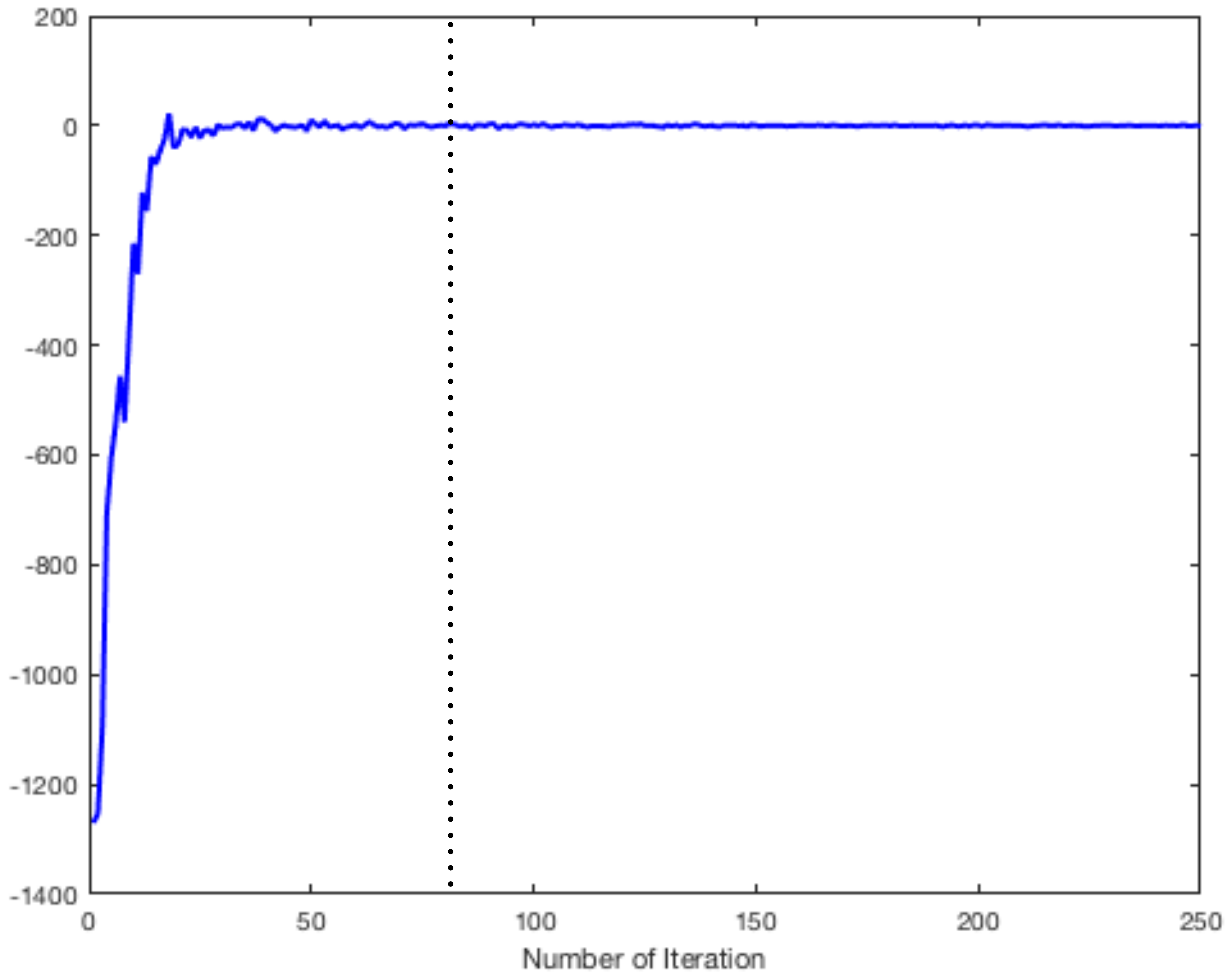

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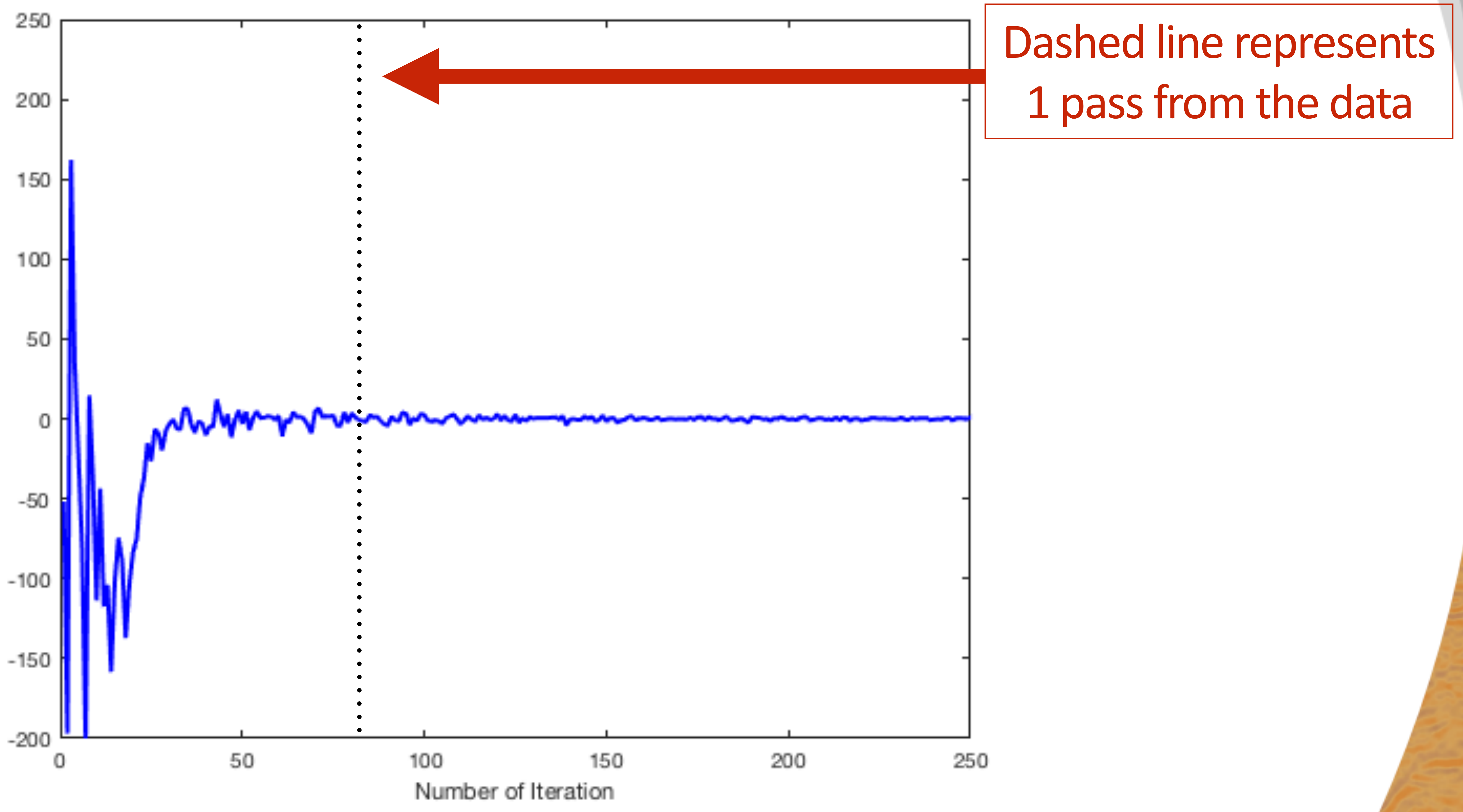

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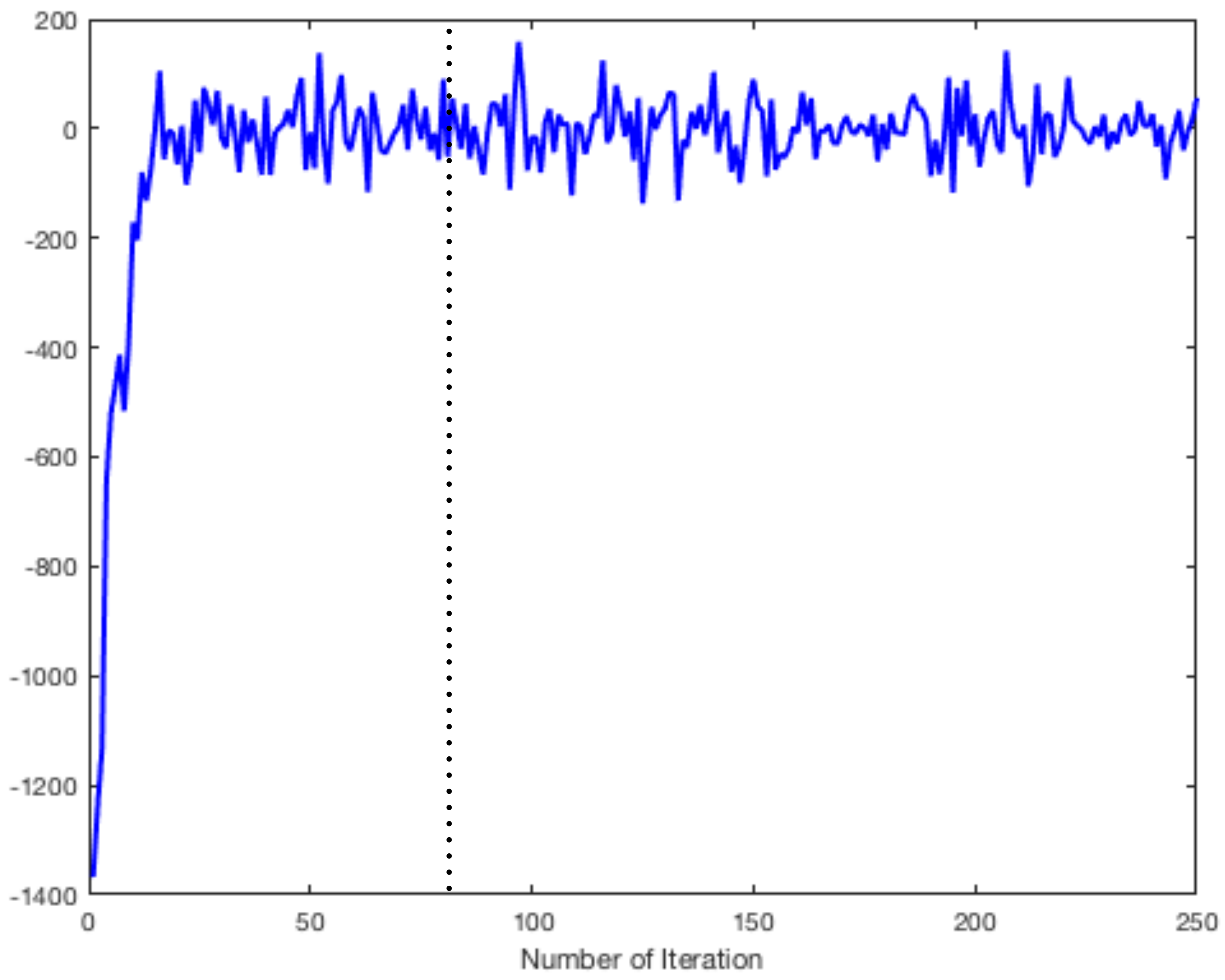

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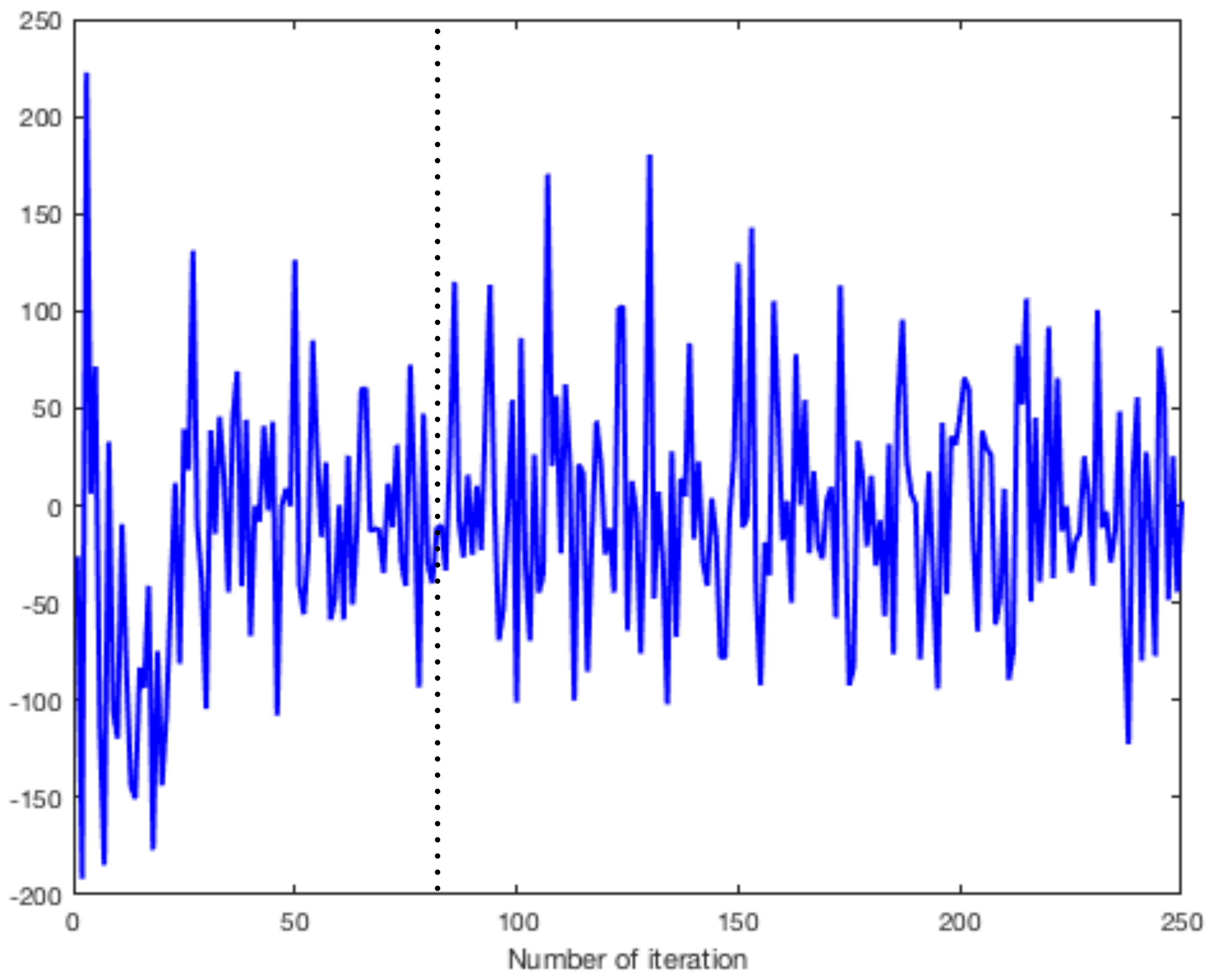
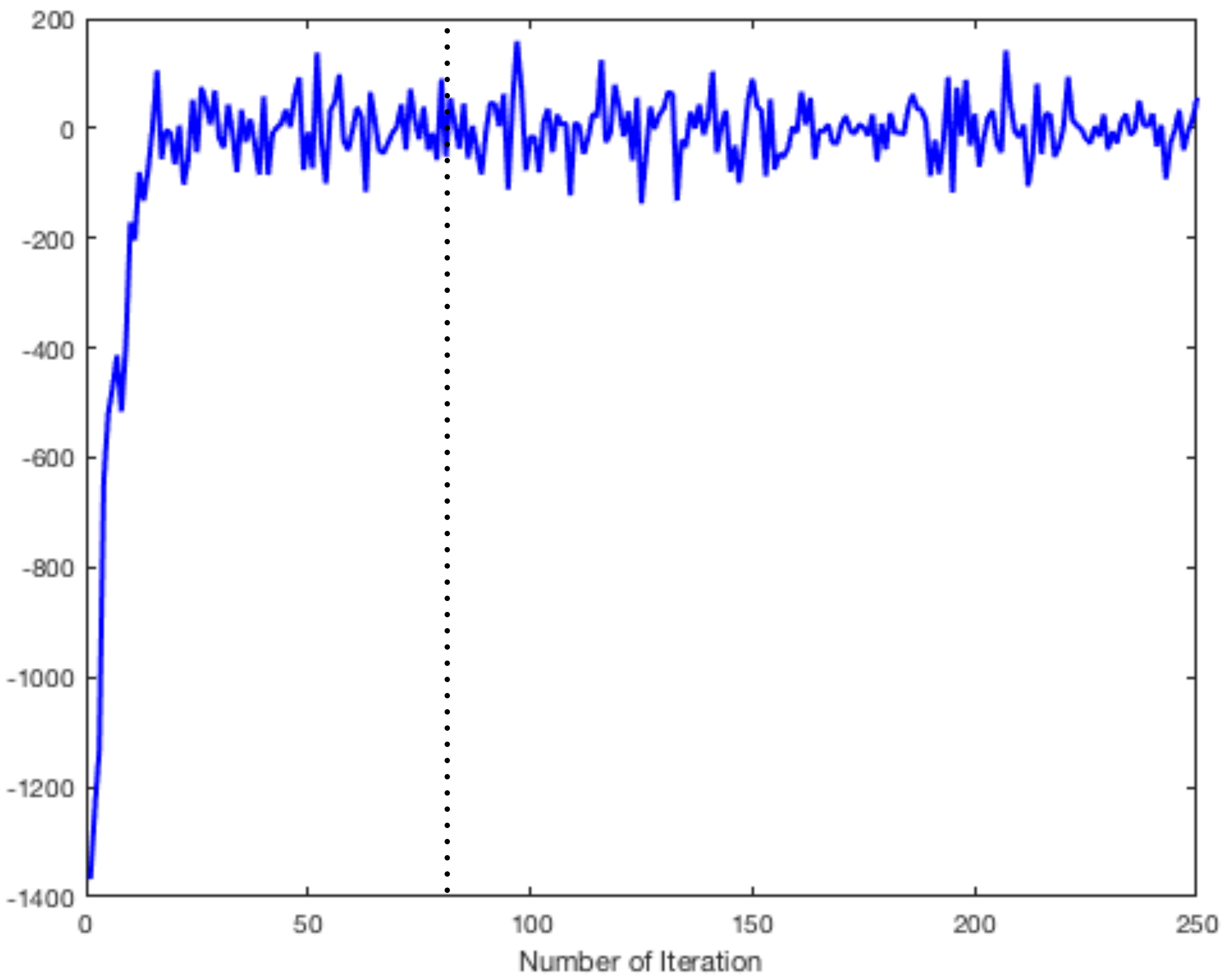


One of the small entries of the exact solution x^*

Intuition: Behaviour of the new weighted increment

$$[A_k^T(A_k x_k - b_k)]_{136}$$



Weighted increment

$$\tau_k^i = t_k \frac{|\sum_{j=1}^{k} \mathrm{sign}([A_j^T(A_j x_j - b_j)]_i)|}{k}$$

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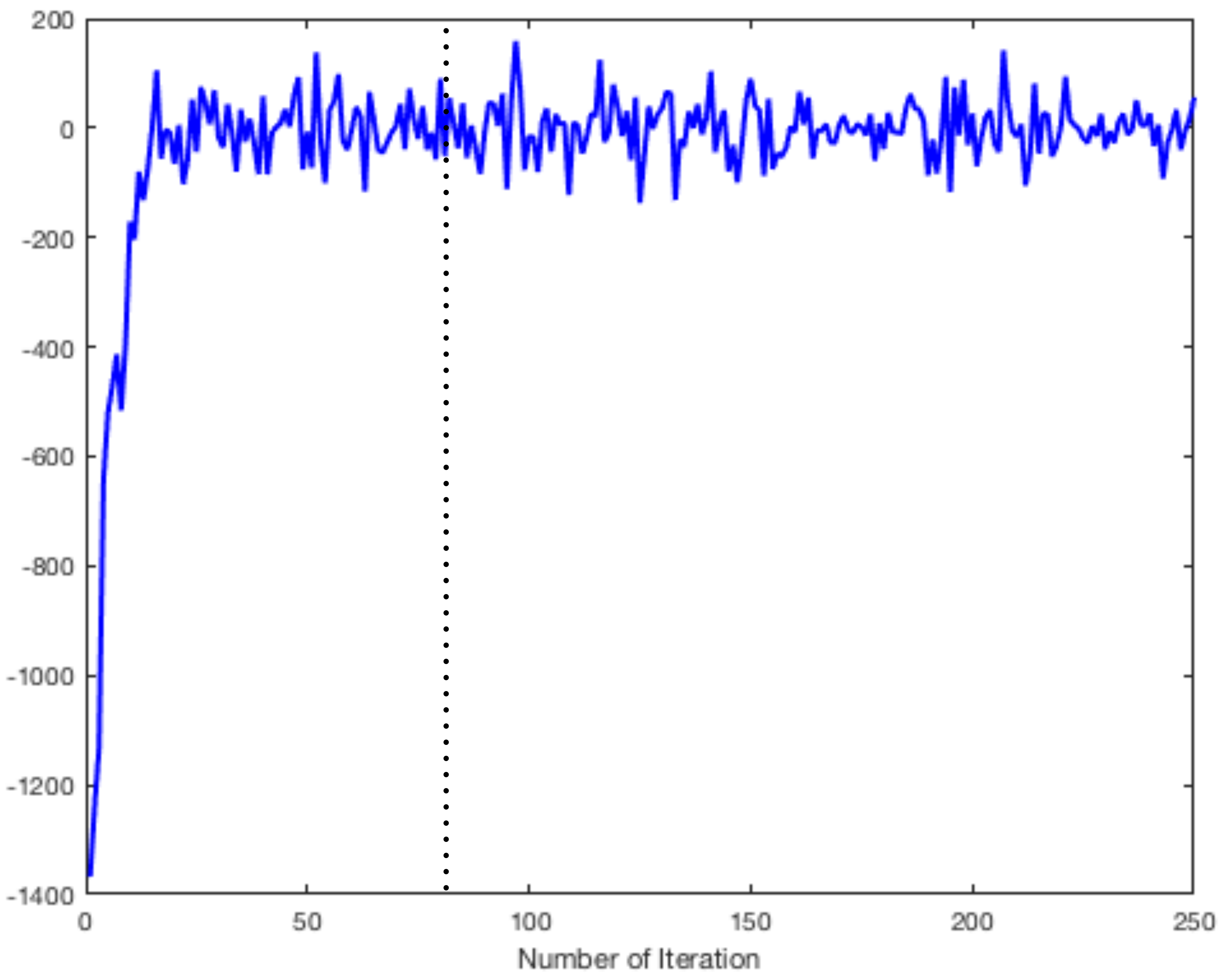
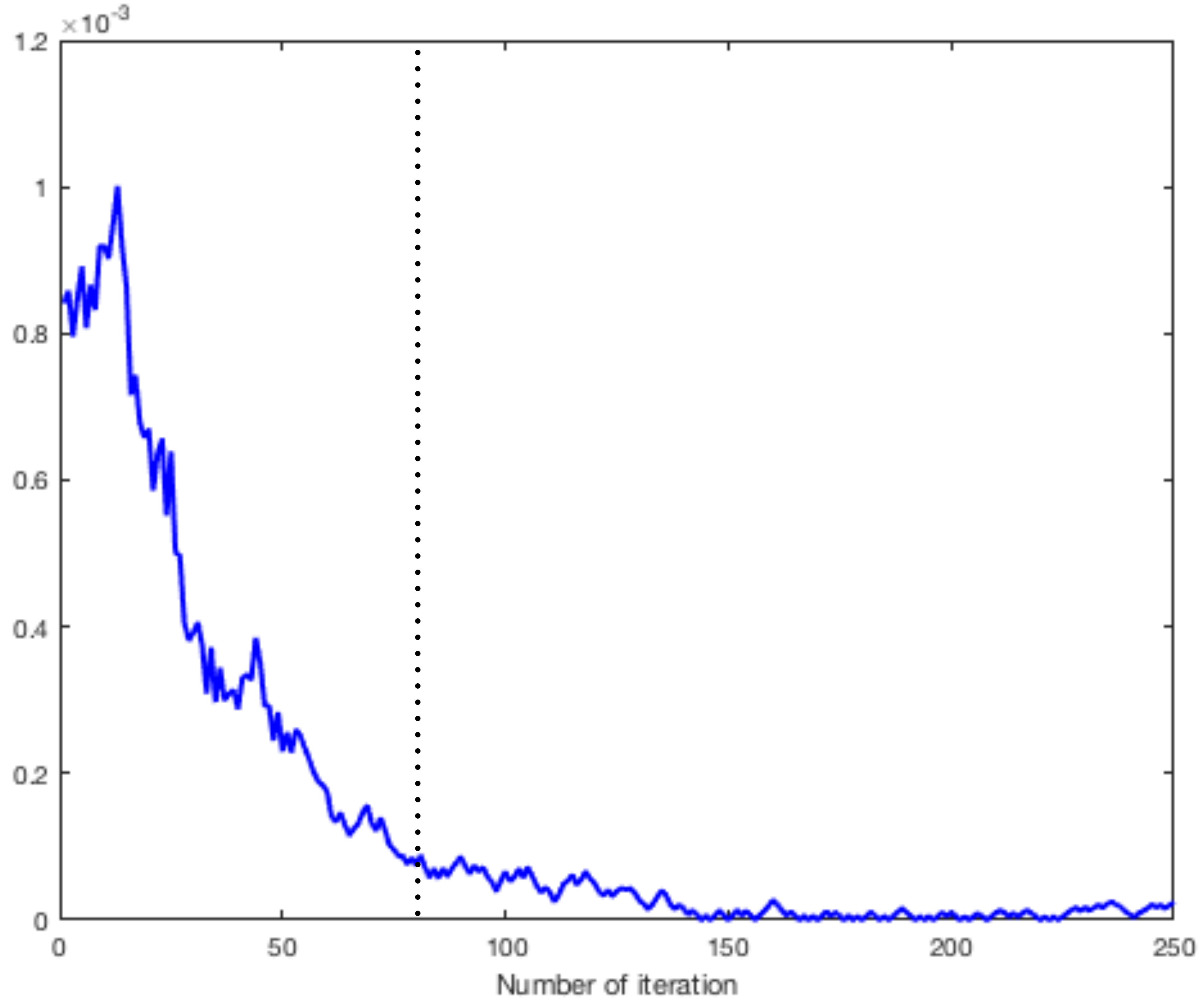


Weighted increment



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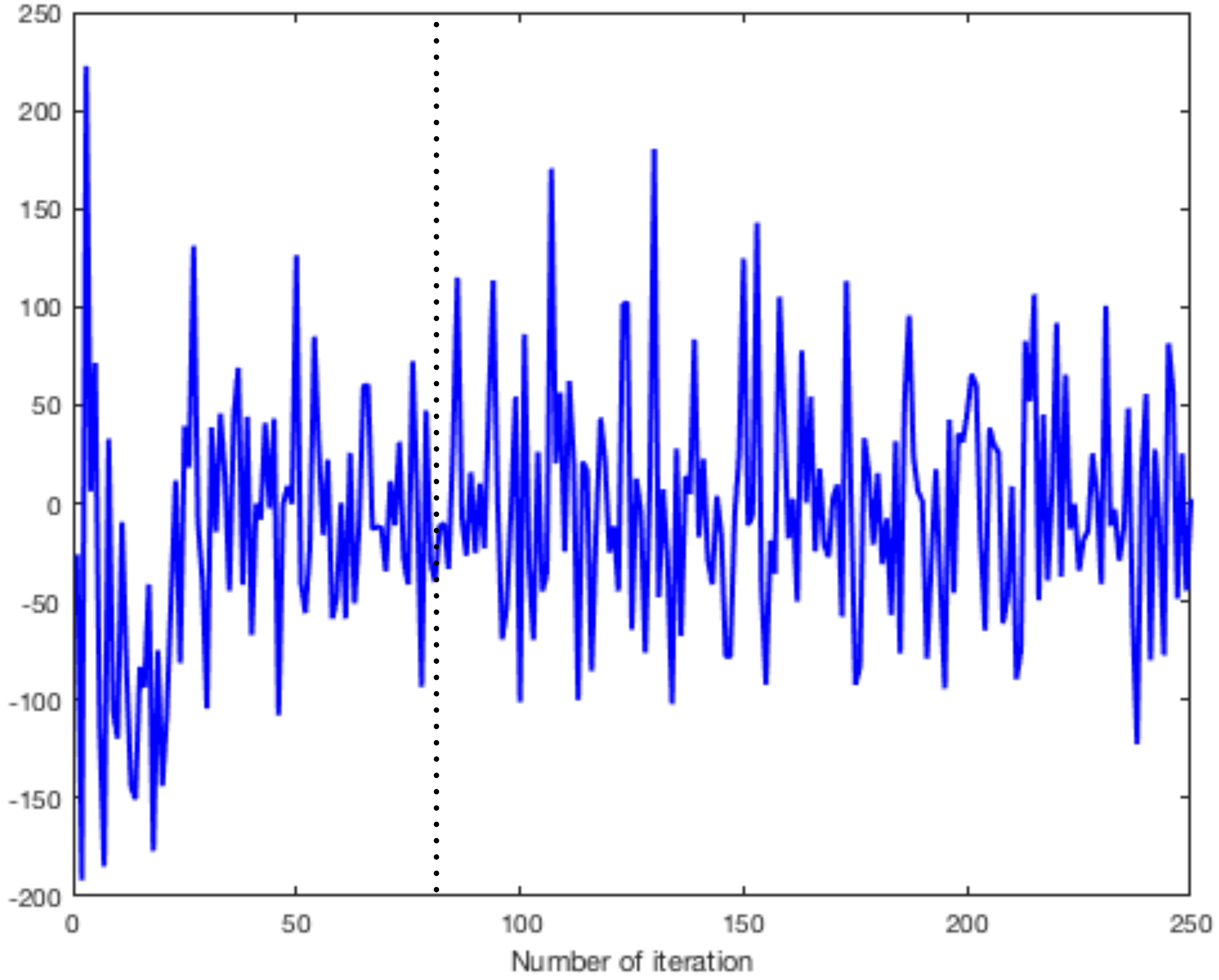
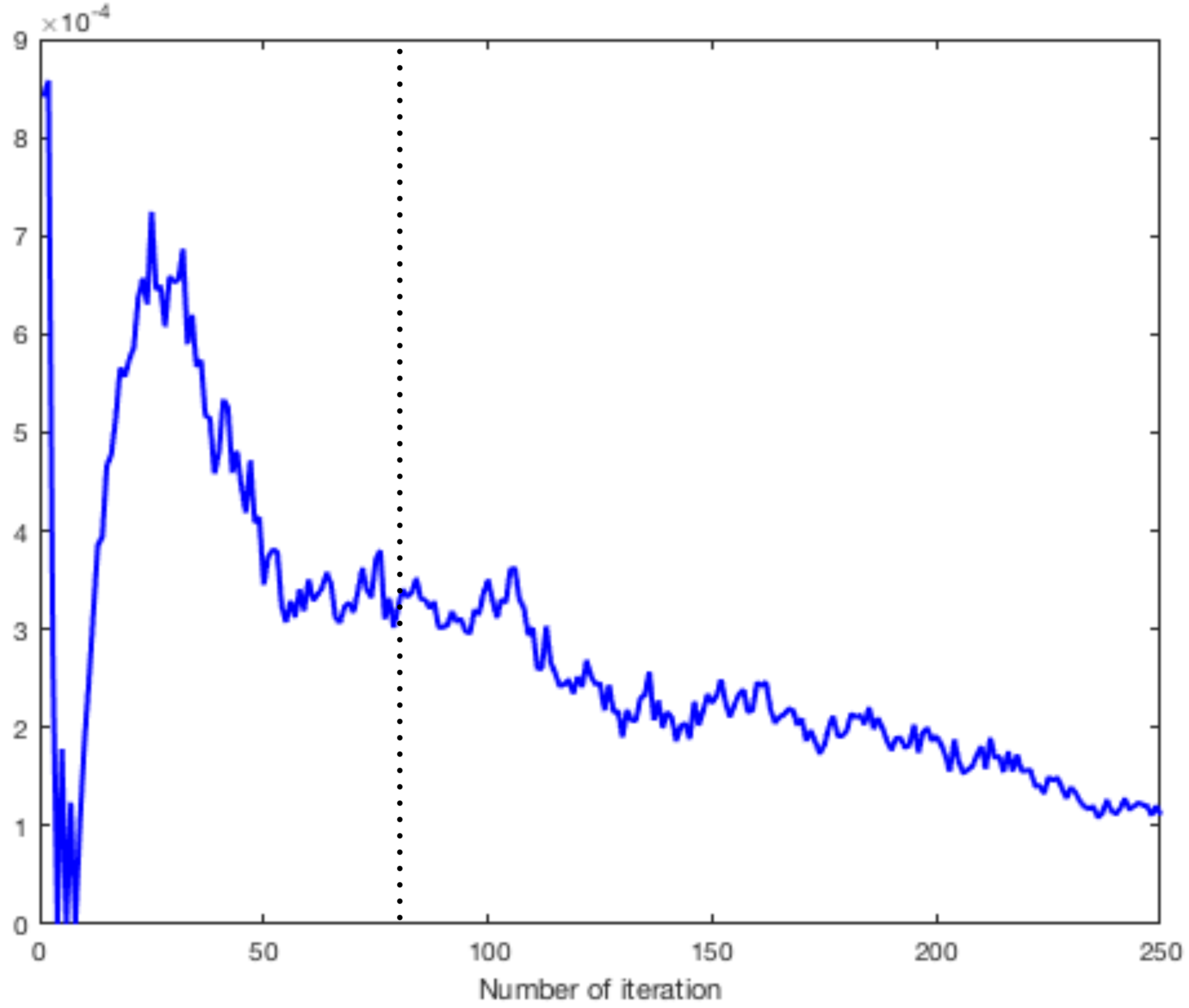
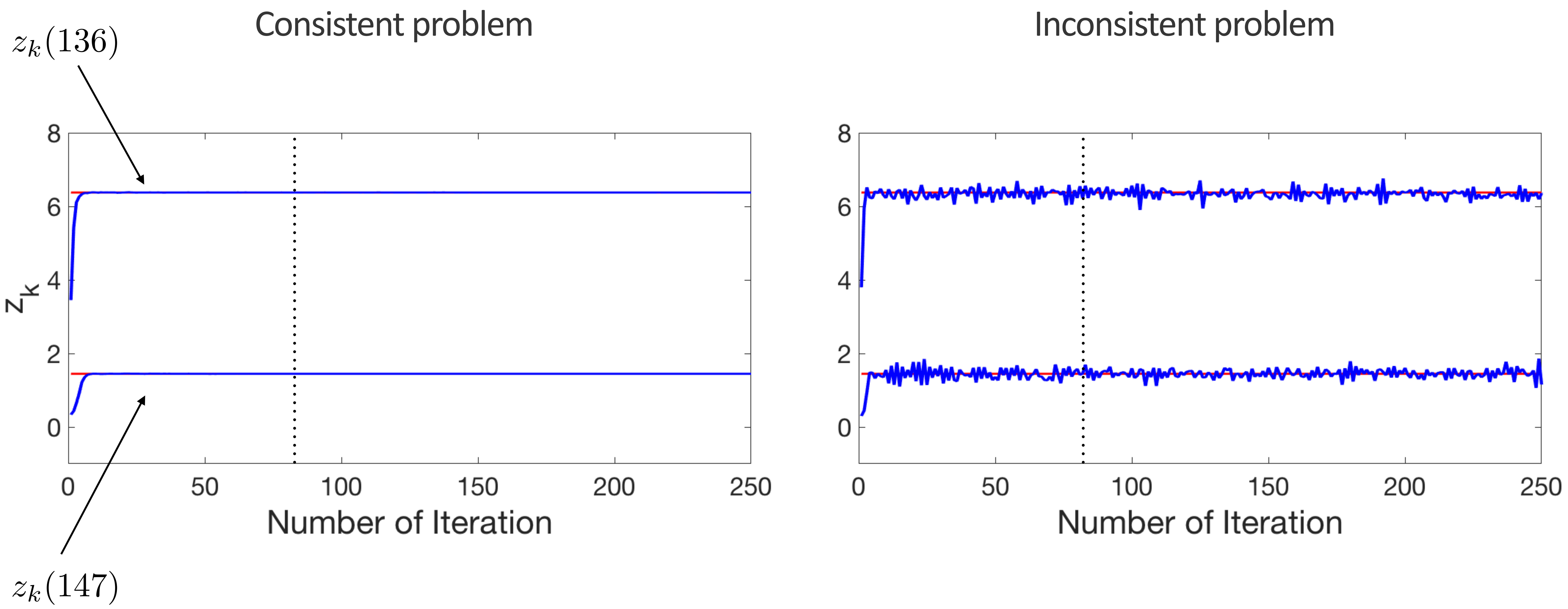


Weighted increment

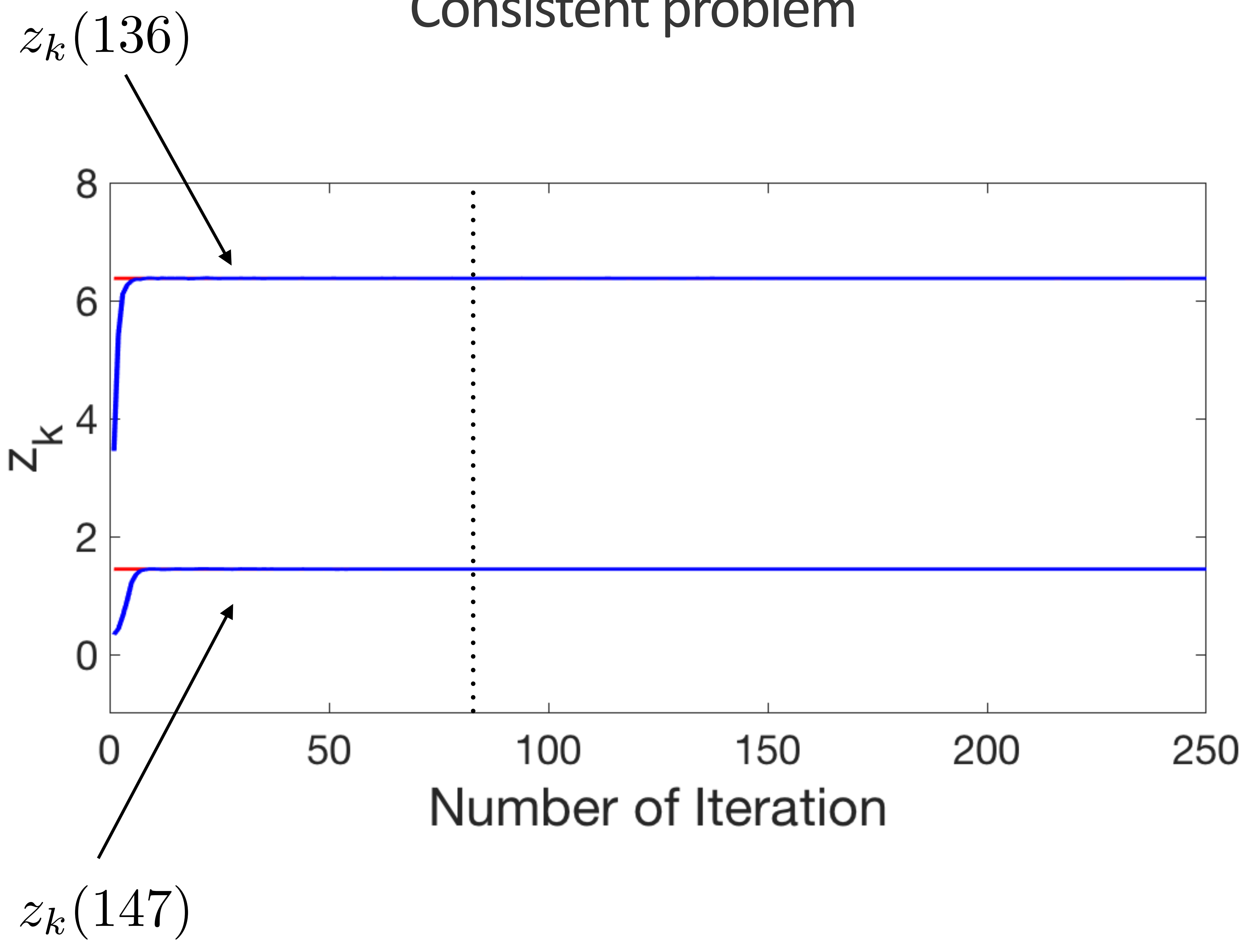


Intuition: Behaviour of the entries of the solution

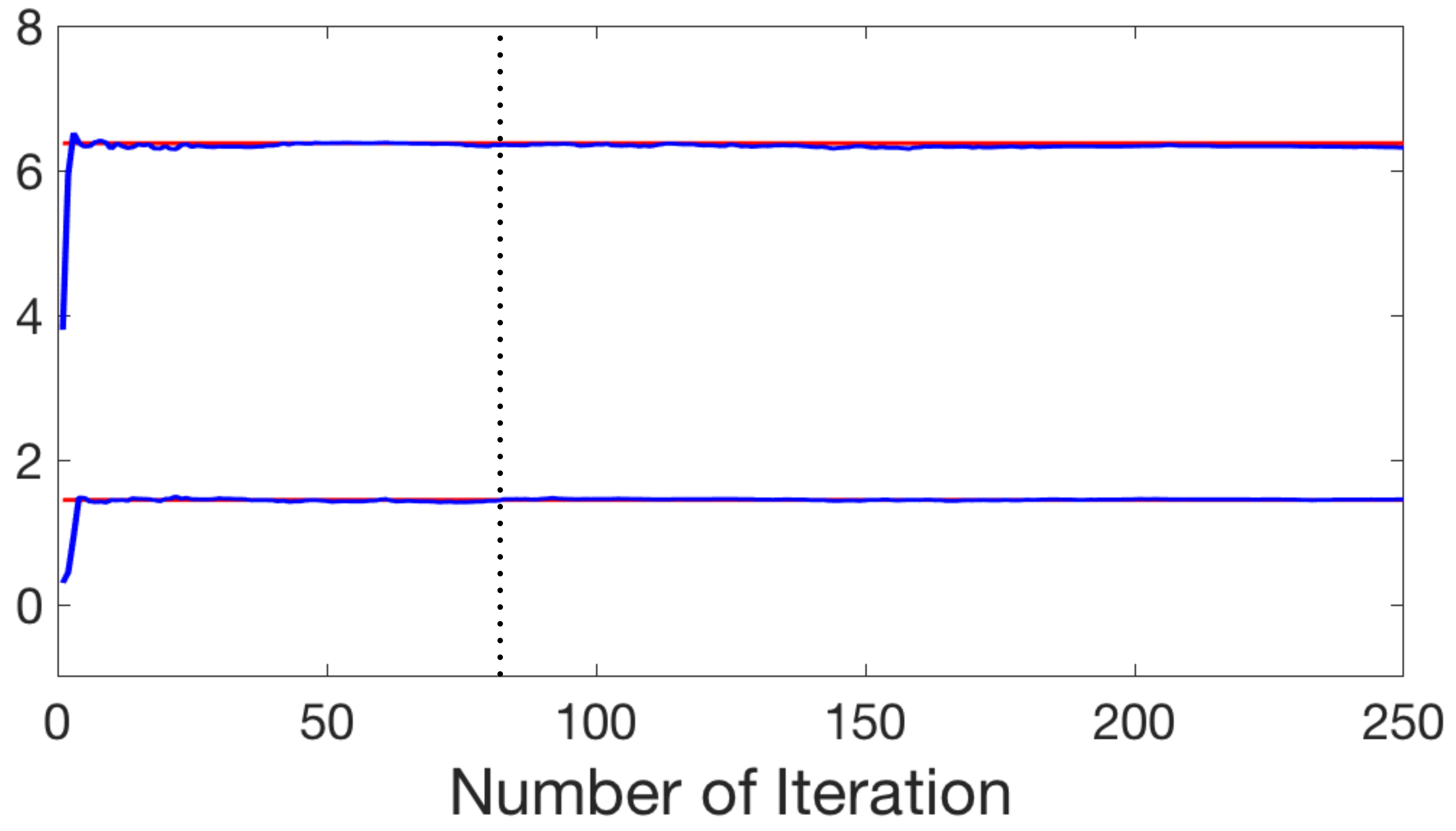


Intuition: Behaviour of the entries of the solution

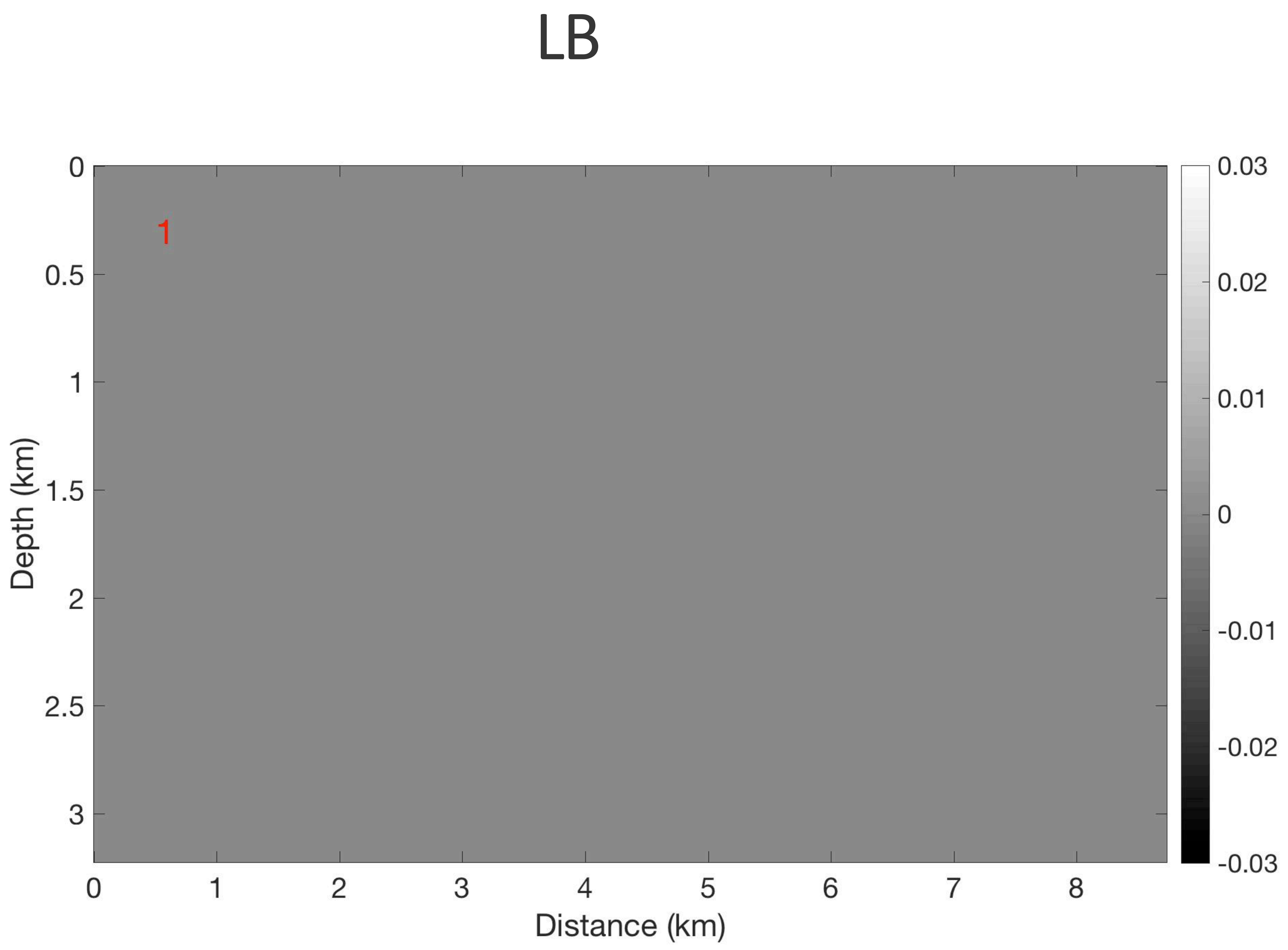
Consistent problem

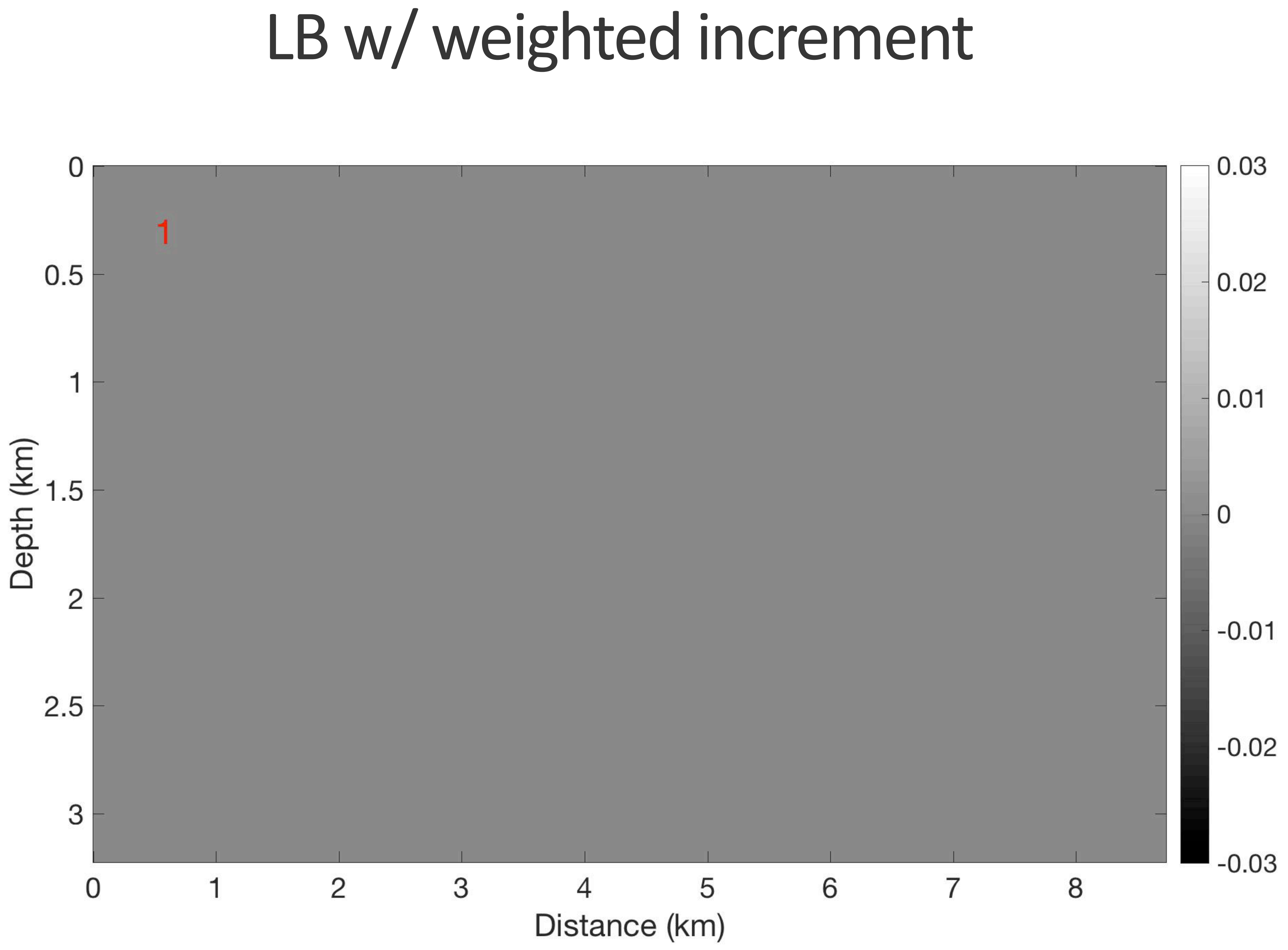


Inconsistent problem
(w/ weighted increment)



Effect on the LSRTM problem





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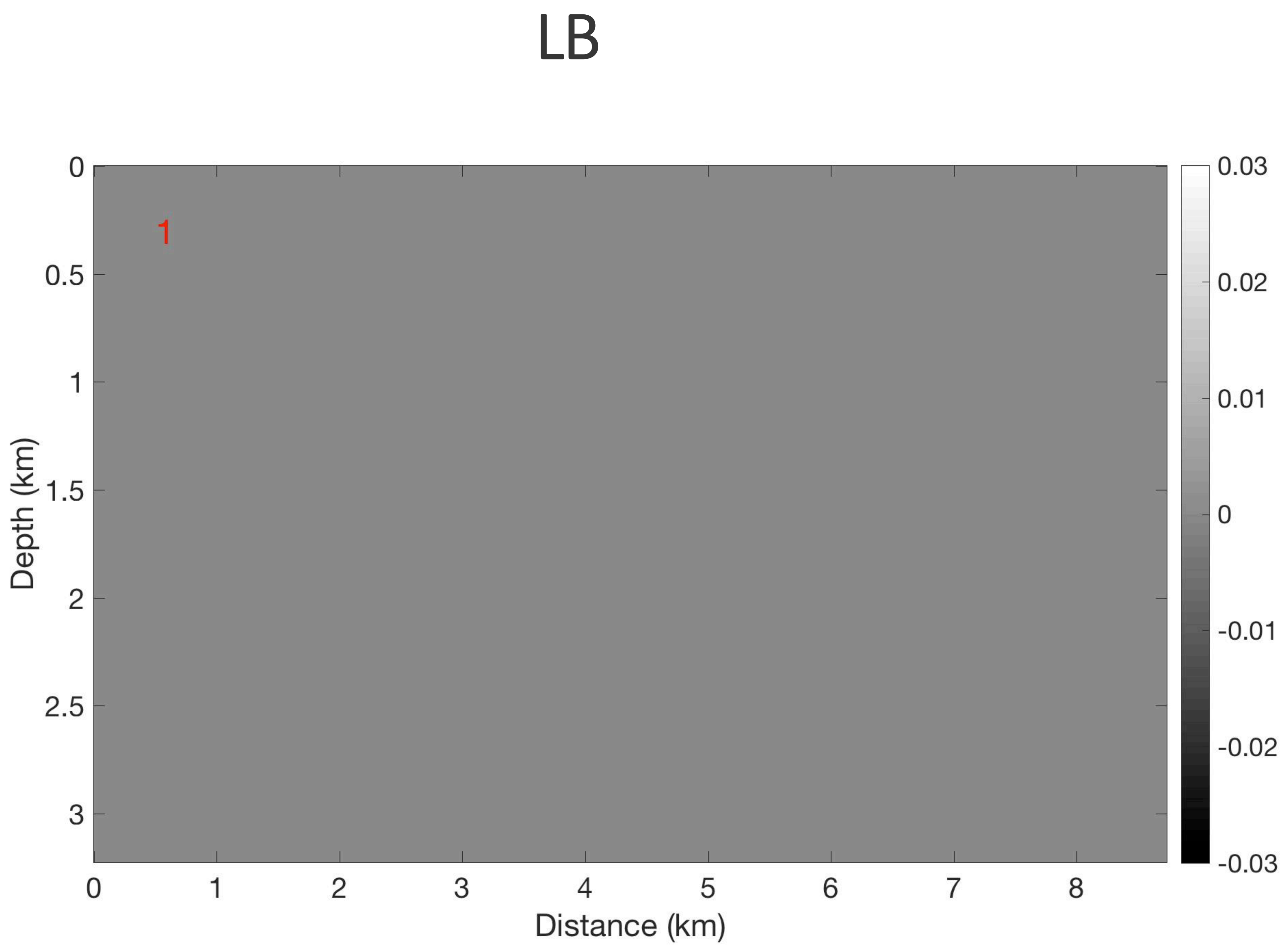

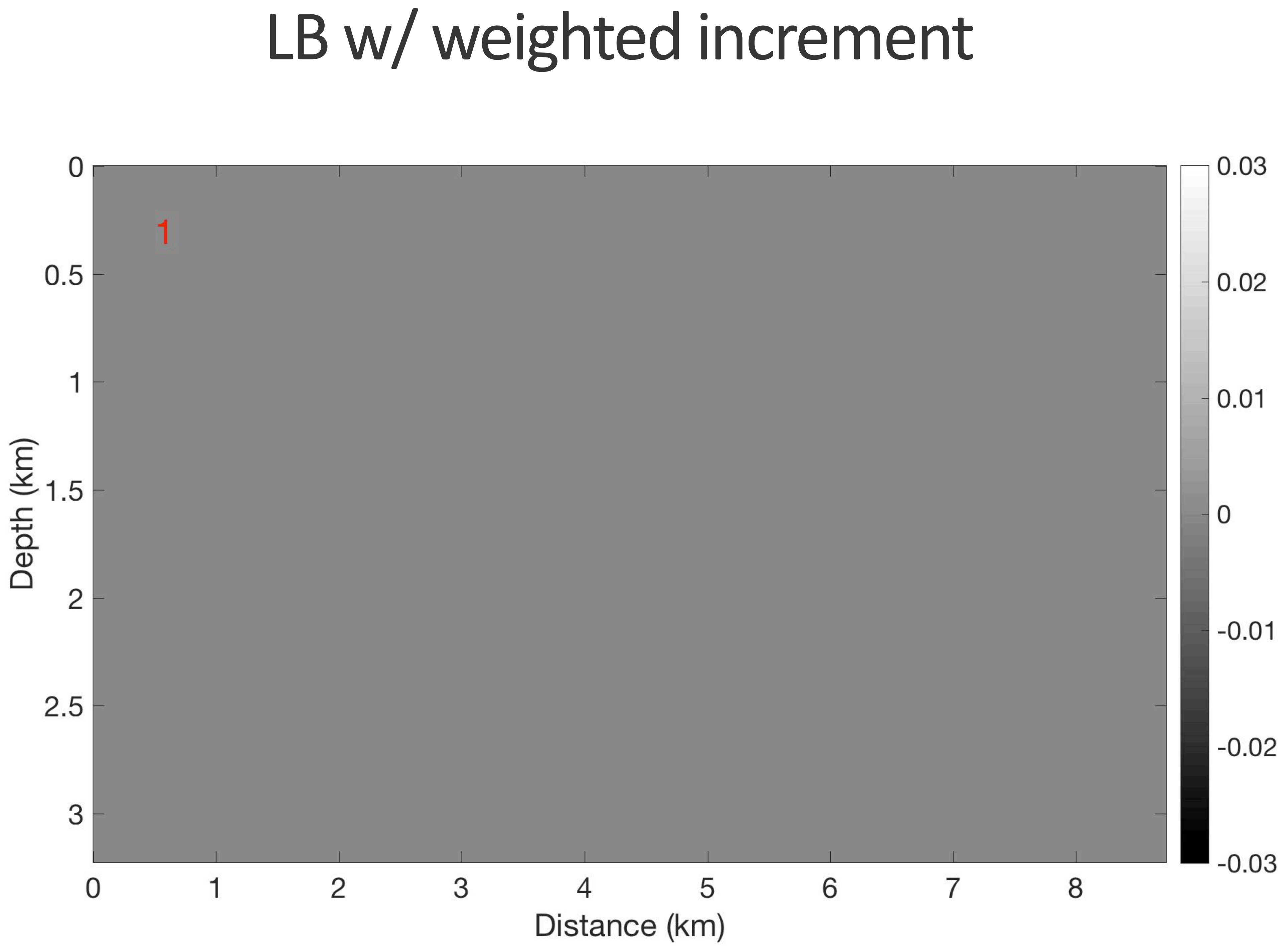

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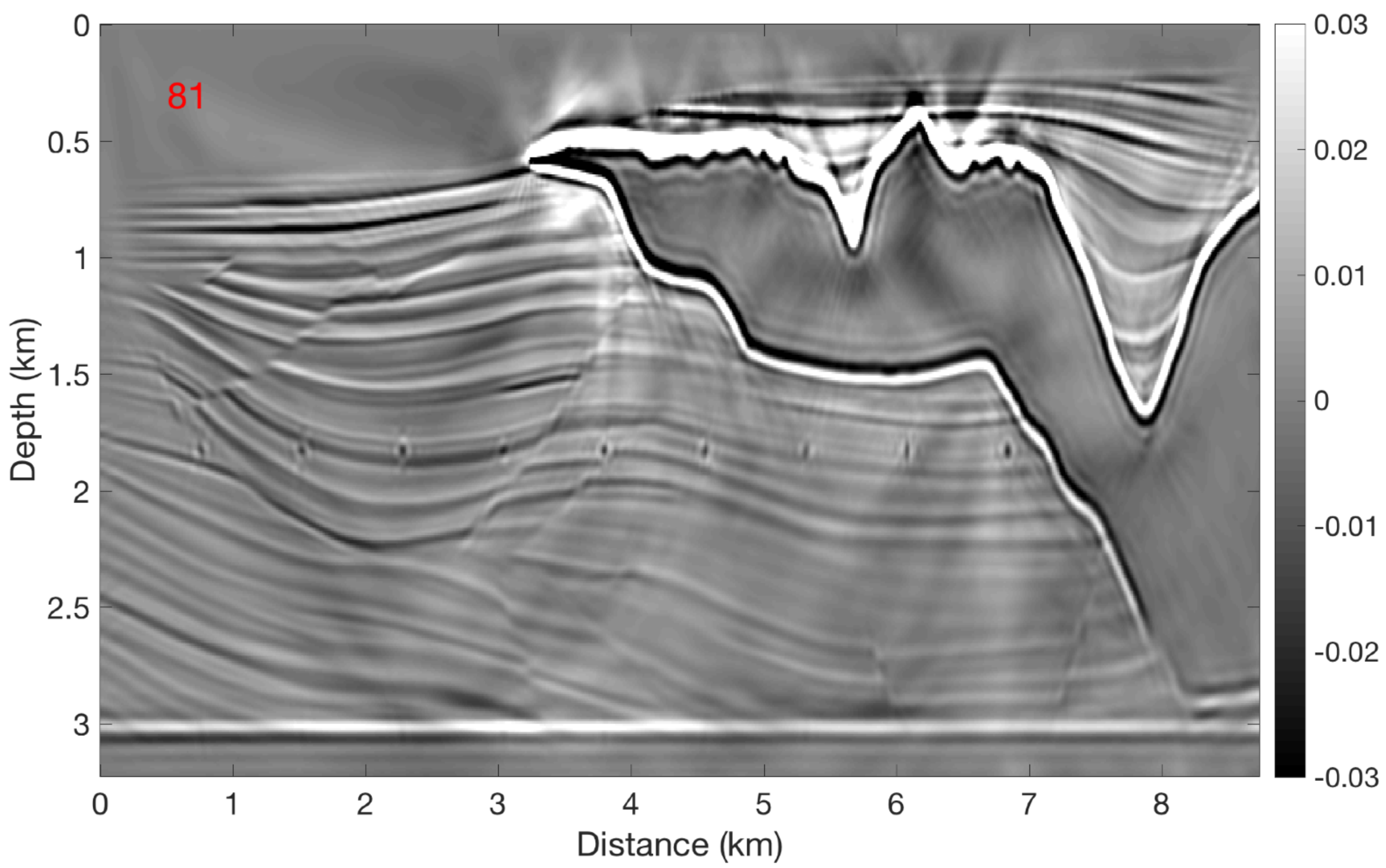

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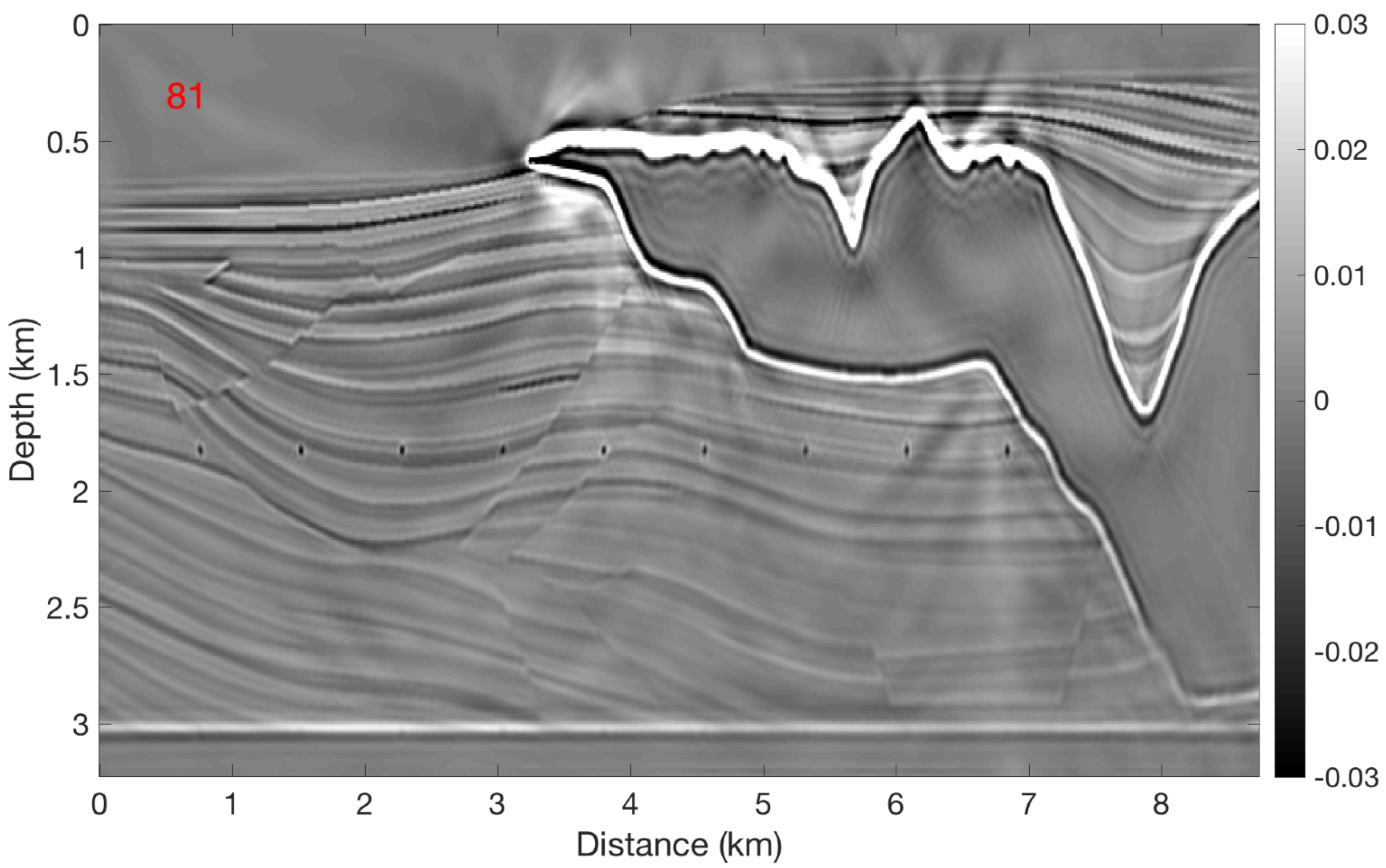

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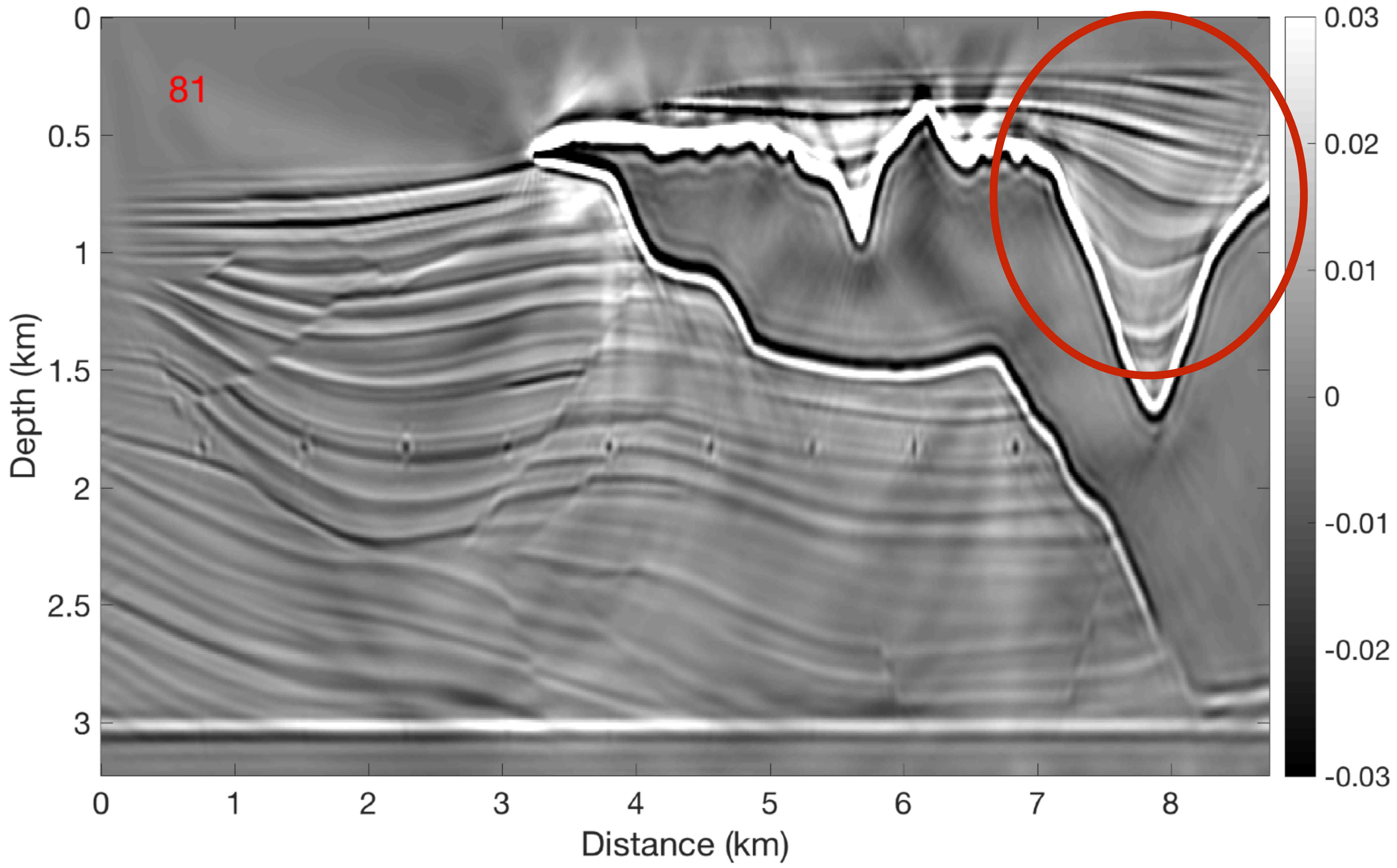

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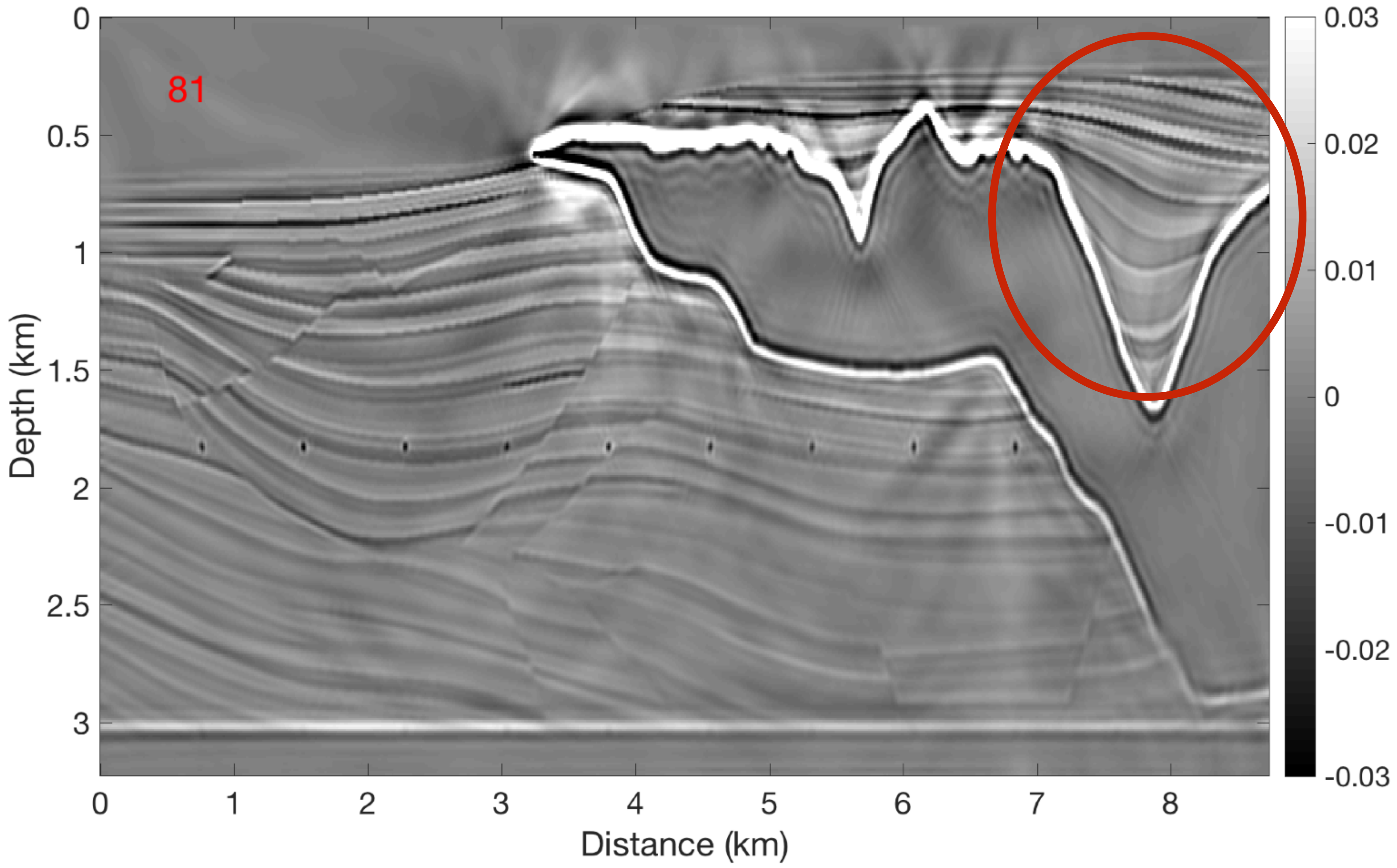

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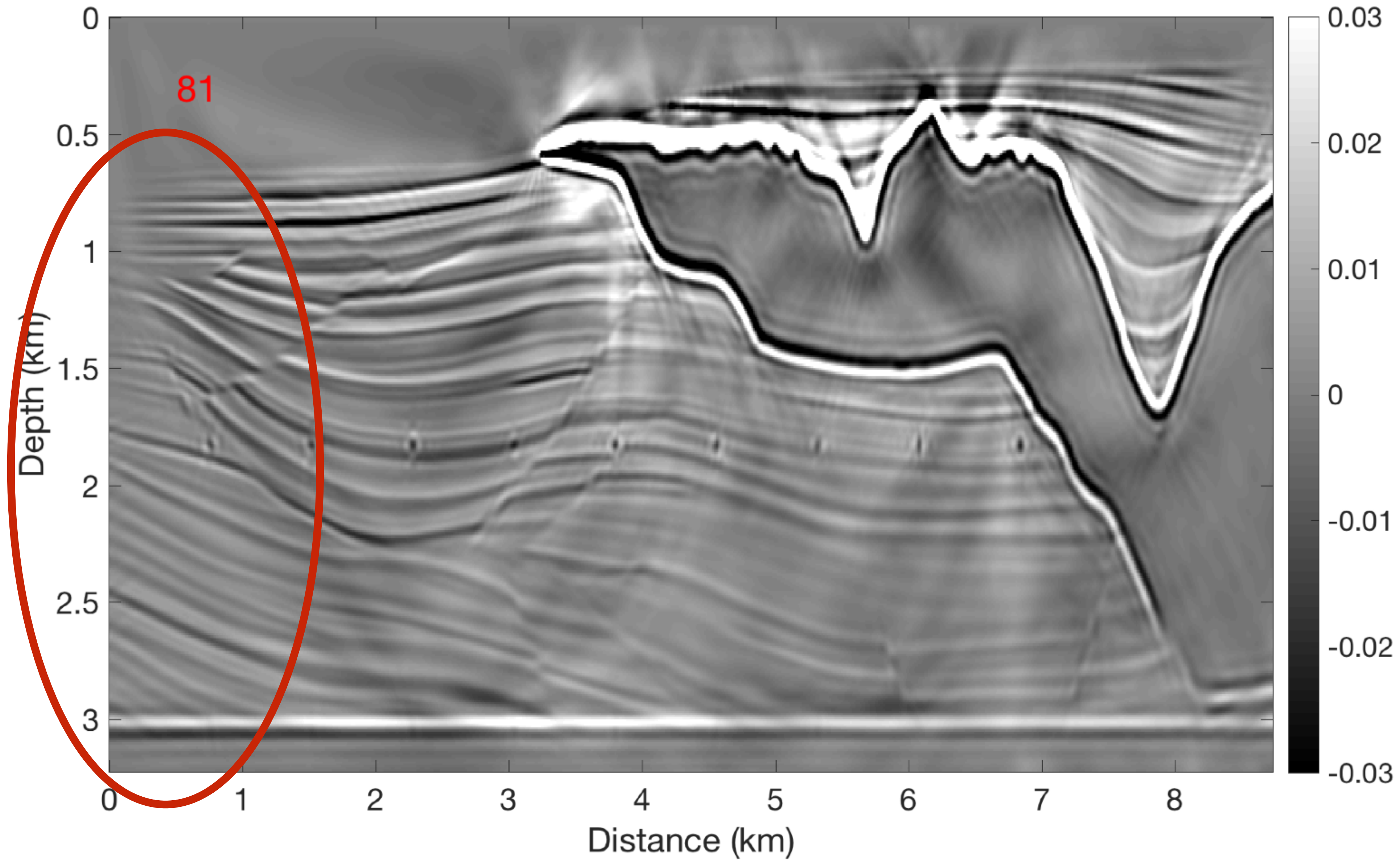

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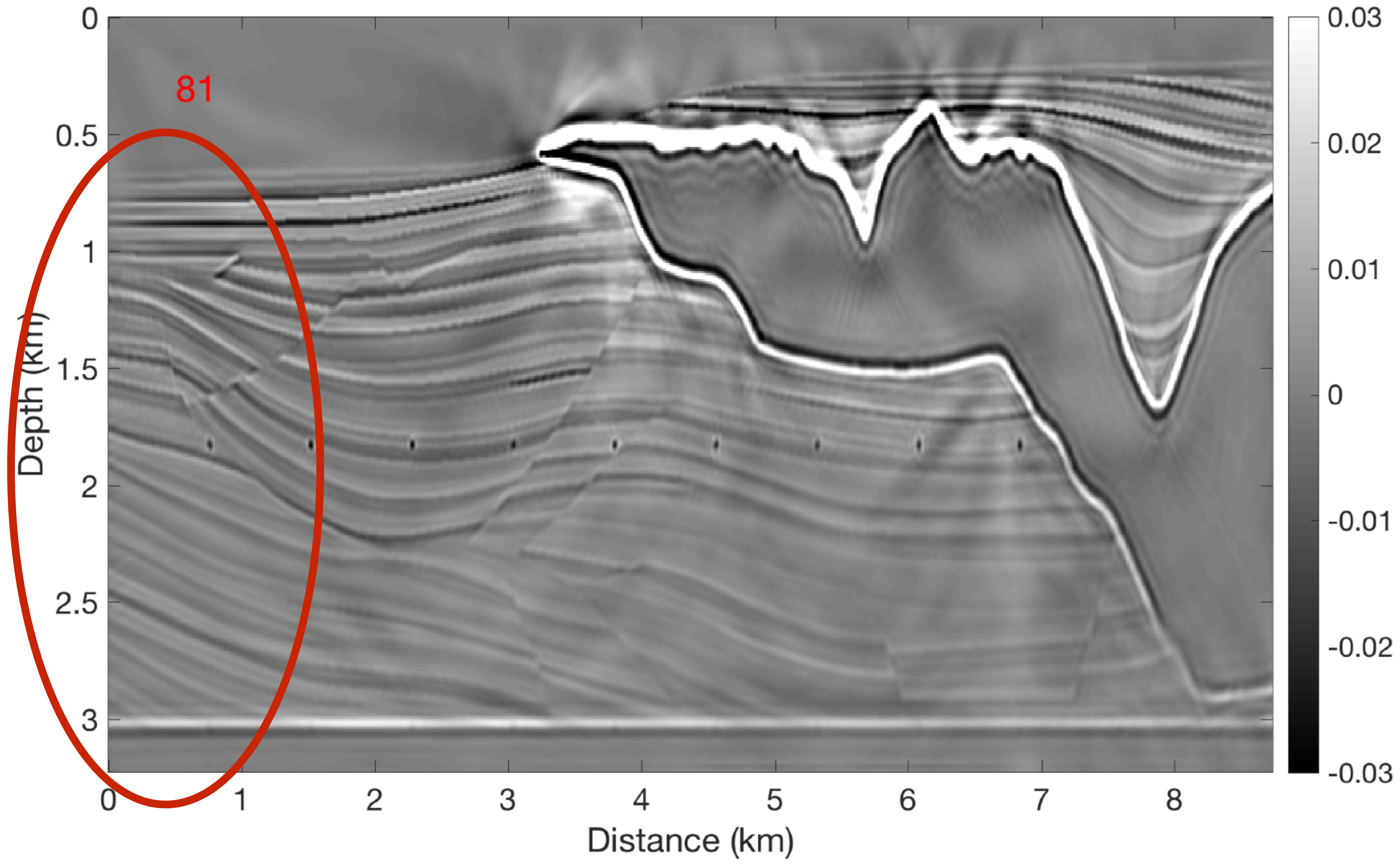

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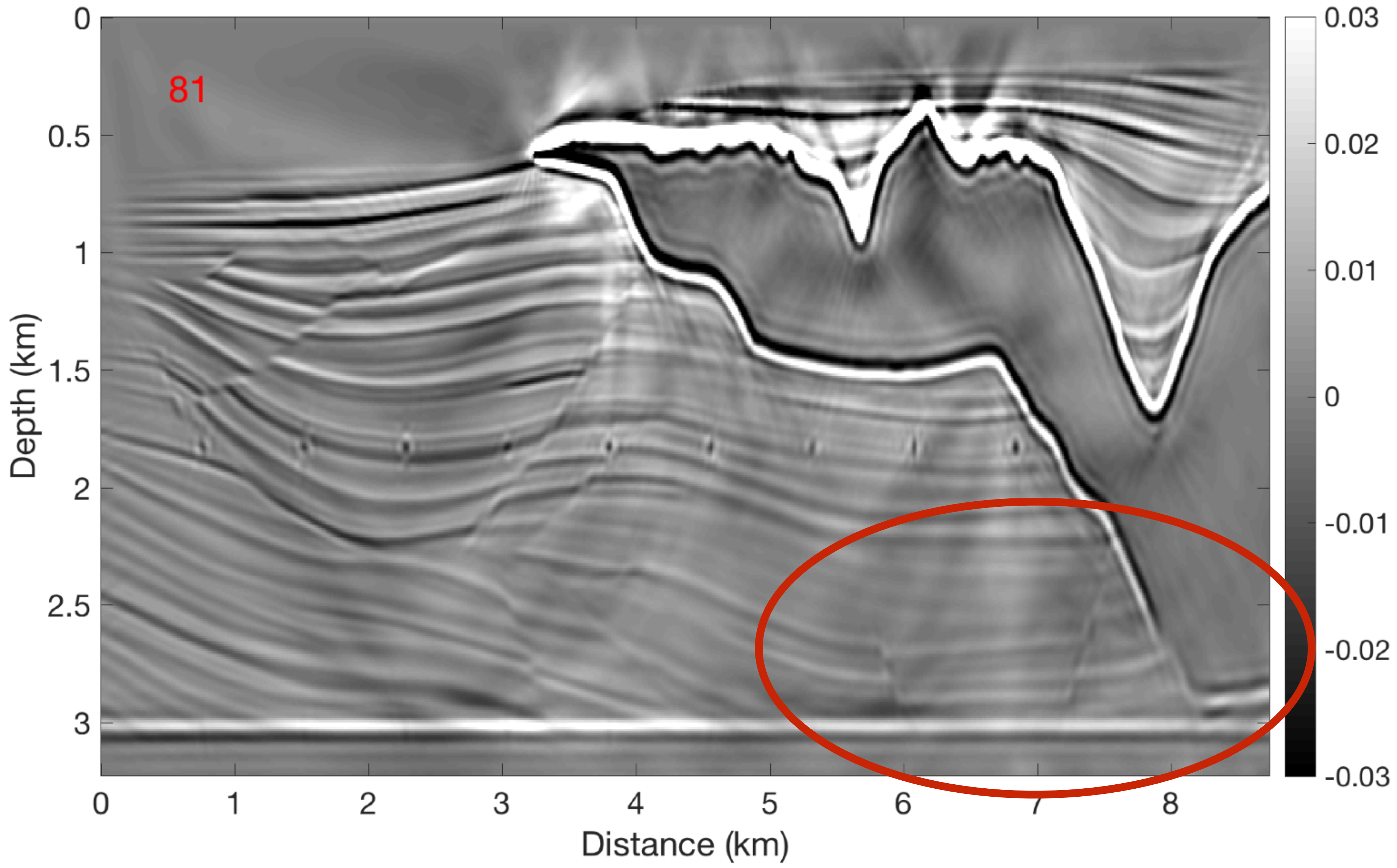

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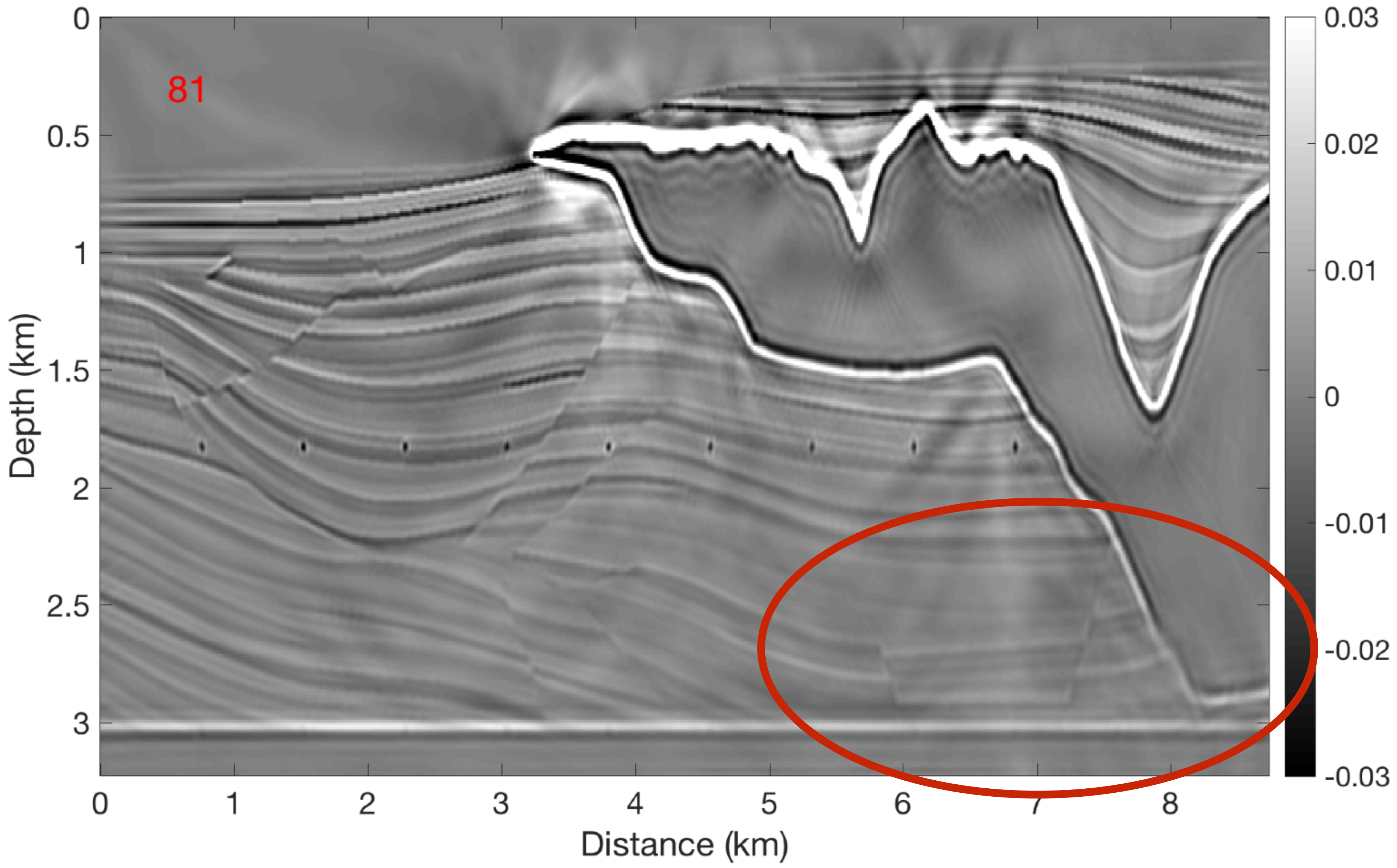
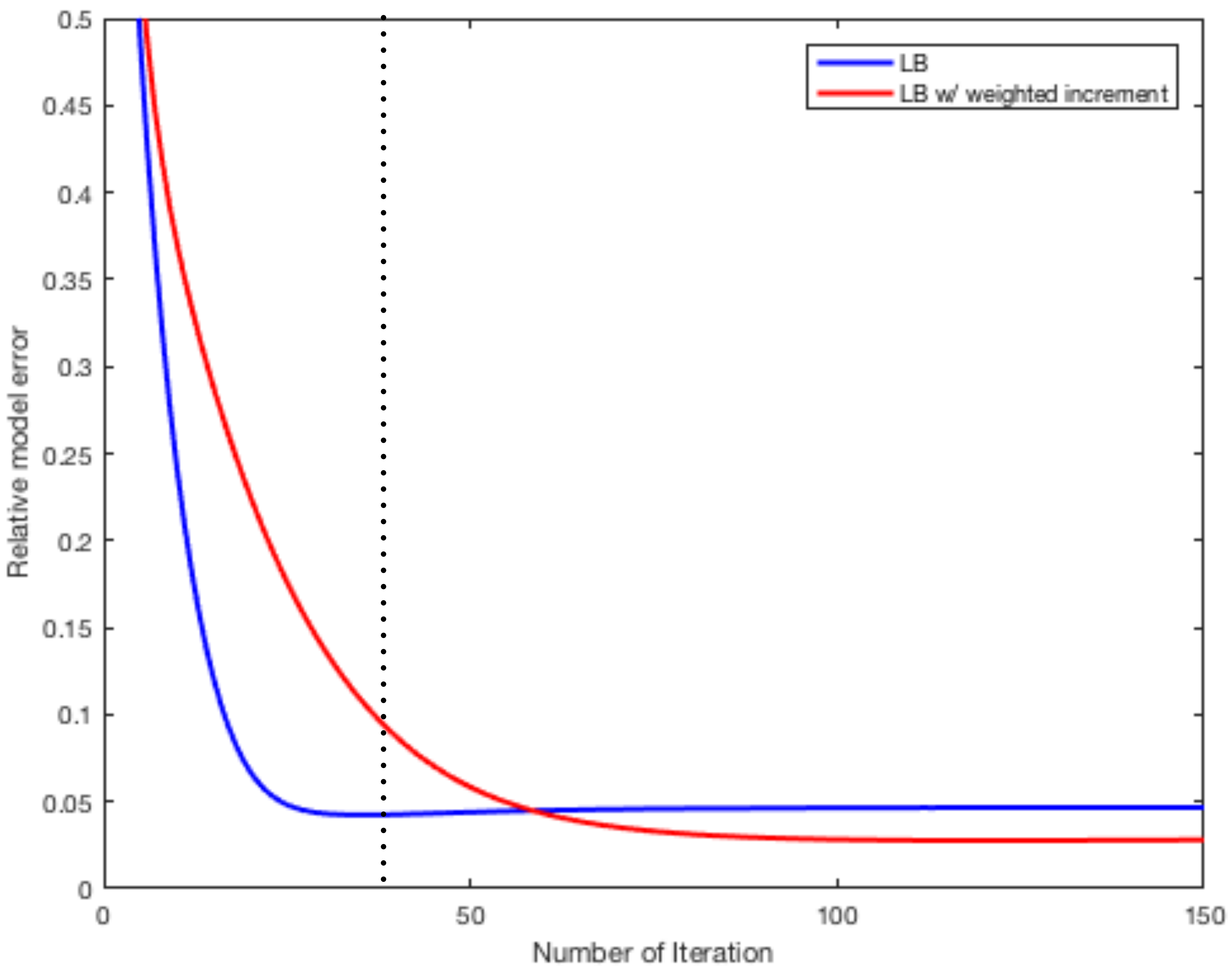


Effect on large problems

Problem a



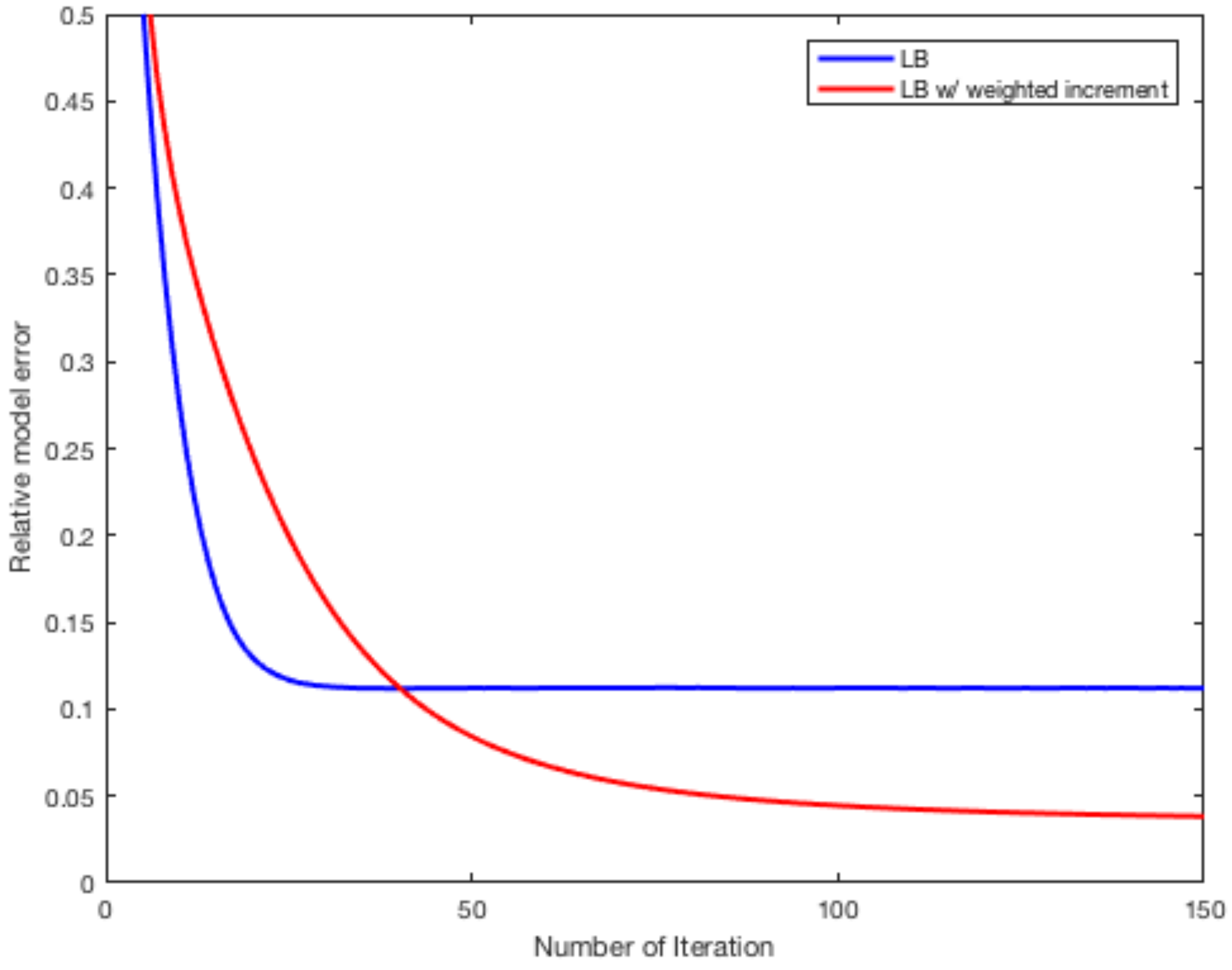
Problem a: $A \in \mathbb{R}^{10000000 \times 2040739}$

Problem b: $A \in \mathbb{R}^{70000000 \times 2040739}$

- The vector x corresponds to a known vector of curvelet coefficients
- $A_k \in \mathbb{R}^{500000 \times 2040739}$
- The signal to noise ratio for the data in both problems is the same.

Effect on large problems

Problem b



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SLIM

Future goals

- Using the Weighted Increment in other iterative methods (SGD, Kaczmarz etc) is possible.
- Improve the convergence speed of Weighted Increment.

Acknowledgement

This research was carried out as part of the SINBAD project with the support of the member organizations of the SINBAD Consortium.

SINBAD