Reconstruction of S-waves from low-cost randomized and simultaneous acquisition by joint sparse inversion

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Outline

- Advantages of S-waves
- Why S-wave is not commonly used in practice?
- Elastic decomposition
- Jittered undersampling
- Single component reconstruction
- Various joint reconstruction formulations w/ sparsity promotion
- Conclusions
Advantages of S-waves

- Imaging through gas chimneys
Advantages of S-waves

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- High resolution imaging (thin layers)
Advantages of S-waves

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- High resolution imaging (thin layers)
- Reservoir detection & monitoring
Advantages of S-waves

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- High resolution imaging (thin layers)
- Reservoir detection & monitoring
- Elastic rock properties
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
- Reservoir detection & monitoring
- Elastic rock properties
- Improve accuracy & confidence
Why S-wave is not commonly used in practice?
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Low S-wave velocity
Why S-wave is not commonly used in practice?

- Low S-wave velocity
- Nyquist criterion
Why S-wave is not commonly used in practice?

- Low S-wave velocity
  - Nyquist criterion
  - Denser sampling
- Higher acquisition costs
Why S-wave is not commonly used in practice?

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Solution
Why S-wave is not commonly used in practice?

- Low S-wave velocity
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Solution

Compressive sensing
Why S-wave is not commonly used in practice?

- Low S-wave velocity
- Nyquist criterion
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- Solution
- Compressive sensing
- Randomized undersampling
- Lower acquisition costs
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Wavefield decomposition
Elastic wavefield decomposition

\[ d = Nq \]
Elastic wavefield decomposition

\[ \mathbf{d} = \mathbf{Nq} \]

\[
\begin{pmatrix}
\phi^+ \\
\psi_{y}^+ \\
\phi^- \\
\psi_{y}^-
\end{pmatrix}
= 
\begin{pmatrix}
\mathbf{N}_1^+ & \mathbf{N}_2^+
\end{pmatrix}
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix}
\]
Elastic wavefield decomposition

\[ \mathbf{d} = \mathbf{Nq} \]

\[
\begin{pmatrix}
\phi^+ \\
\psi^+_y \\
\phi^- \\
\psi^-_y
\end{pmatrix}
= 
\begin{pmatrix}
\mathbf{N}^+_1 & \mathbf{N}^+_2 \\
\mathbf{N}^-_1 & \mathbf{N}^-_2
\end{pmatrix}
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix}
\]

At the ocean bottom:

\[ \tau_{xz} = 0 \quad \tau_{zz} = -p \]
Elastic wavefield composition

\[
q = Ld \\
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix} = \begin{pmatrix}
L_1^+ & L_1^- \\
L_2^+ & L_2^-
\end{pmatrix} \begin{pmatrix}
\phi^+ \\
\psi^+_y \\
\phi^- \\
\psi^-_y
\end{pmatrix}
\]
Multicomponent data
Elastic decomposition

\[ \Psi^+ \]

\[ \Psi^- \]
Can’t afford dense acquisition
40 m source interval receiver gathers
f-k spectrum, 40 m source interval
Decomposed S-waves
f-k spectrum, 40 m source interval
Can’t afford dense acquisition
Jittered under-sampled acquisition

Jittered under-sampled acquisition
Jittered under-sampled acquisition

single component reconstruction w/ sparsity promotion
Single component reconstruction

\[ P \xrightarrow{\text{Interpolation}} V_x \xrightarrow{} V_z \]

(1) ...
Single component reconstruction

\[ P, V_x, V_z \]

(1) \--------
Interpolation

(2) \--------
Decomposition

\[ \Psi^+, \Psi^- \]
Single component reconstruction w/ sparsity promotion

\[
\min_x \|x\|_1 \quad \text{subject to} \quad \|Ax - b\|_2 \leq \sigma \quad \text{(BPDN}_\sigma) \]

\(x\): curvelet coefficients

\(A = MS^H\)
75% jittered subsampling

Observation example
f-k spectrum, 75% jittered subsampling
Reconstructed S-waves

$$\Psi^+$$

$$\Psi^-$$
Densely sampled S-waves
Residual

\[ \Psi^+ \]

\[ \Psi^- \]
f-k spectrum, reconstructed S-waves
f-k spectrum, true S-waves

\[ \Psi^+ \]

\[ \Psi^- \]
Marmousi II data

\[ \rho \]

\( C_p \)

\( C_s \)
Densely sampled data

$V_x$

$V_z$

$T_{zz}$
75% jittered subsampling

\( V_x \)

\( V_z \)

\( T_{zz} \)
Reconstructed data
Densely sampled data
Residual

\[ \Phi^- \]

\[ \Phi^+ \]

\[ \Psi^- \]

\[ \Psi^+ \]
Joint interpolation decomposition
Joint interpolation decomposition

(1) Interpolation

(2) Decomposition
Joint interpolation decomposition

(1) Interpolation

(2) Decomposition
Joint interpolation decomposition w\ curvelets

\[
\min_x \|x\|_1 \quad \text{subject to} \quad \|Ax - b\|_2 \leq \sigma \quad (\text{BPDN}_\sigma)
\]

\(x\): coefficients of the decomposed data

Sparsifying transform:

\[
A_c = MF^H LFS^H
\]
75% jittered subsampling

- $P$
- $V_x$
- $V_z$
Joint interpolation decomposition in the curvelet domain
Densely sampled S-waves

\[ \Psi^+ \]

\[ \Psi^- \]
Residual

\[ \Psi^+ \]

\[ \Psi^- \]
Marmousi II Densely sampled data

- $V_x$
- $V_z$
- $T_{zz}$
75% jittered subsampling

$V_x$

$V_z$

$T_{zz}$
Reconstructed data
Densely data

\[ \Phi^- \]

\[ \Phi^+ \]

\[ \Psi^- \]

\[ \Psi^+ \]
Residual

\begin{align*}
\Phi^- & \quad \Phi^+ \\
\Psi^- & \quad \Psi^+
\end{align*}
Joint interpolation decomposition in the f-k domain
Joint interpolation decomposition, f-k

\[
\min_{\mathbf{x}} \| \mathbf{x} \|_1 \quad \text{subject to} \quad \| \mathbf{A} \mathbf{x} - \mathbf{b} \|_2 \leq \sigma \quad \text{(BPDN}_\sigma \text{)}
\]

\( \mathbf{x} \): coefficients of the decomposed data

Sparsifying transform:

\[
\mathbf{A}_c = \mathbf{M} \mathbf{F}^H \mathbf{L} \mathbf{F}_S^H
\]
Joint interpolation decomposition, f-k

\[
\min_x \|x\|_1 \text{ subject to } \|Ax - b\|_2 \leq \sigma \quad \text{(BPDN}_\sigma)\\
\]

\(x\): coefficients of the decomposed data

Sparsifying transform:

\[
A_c = MF^H LFS^H \\
A_{fk} = MF^H L
\]
Reconstructed data, f-k

\[ \Phi^- \]

\[ \Phi^+ \]

\[ \Psi^- \]

\[ \Psi^+ \]
Reconstructed data, curvelet
Residual

\[
\Phi^-, \Phi^+, \Psi^-, \Psi^+
\]
Why curvelets are better?

- Better at capturing curve-like events.
Why curvelets are better?

- Better at capturing curve-like events.
- Sparser representation.
Joint source separation decomposition
Jittered continuous recording, 1 boat, 2 air guns
Joint source separation decomposition

\[
\min_x ||x||_1 \text{ subject to } \|Ax - b\|_2 \leq \sigma \quad \text{(BPDN}_\sigma)\\
\]

\(x\): curvelet coefficients of the decomposed data

\[
A_c = MF^H LFS^H
\]

\(M\): blending matrix
Reconstructed data
Densely data
Residual
Advantages of the joint formulations

• Use all the multicomponent data in one optimization problem.
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
- Avoid multi stage processing & artifacts.
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
- Avoid multi stage processing & artifacts.
- Minimize parameters selection.
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
- Avoid multi stage processing & artifacts.
- Minimize parameters selection.
- Ensure preservation of amplitude ratios.
Conclusions

- Acquisition of S-waves is prohibitively expensive with conventional dense acquisition designs.
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- Using low-cost jittered under-sampling & simultaneous acquisition with joint interpolation source separation decomposition, S-waves become feasible to acquire & utilize in practice.
Conclusions

- Acquisition of S-waves is prohibitively expensive with conventional dense acquisition designs.
- Coarse regular sampling results in aliasing of the S-waves.
- Using low-cost jittered under-sampling & simultaneous acquisition with joint interpolation source separation decomposition, S-waves become feasible to acquire & utilize in practice.
- Utilize the multicomponent data to its available full extent at a lower cost compared with conventional acquisition.
References

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Thank you for your attention!