Reconstruction of S-waves from low-cost randomized and simultaneous acquisition by joint sparse inversion

Ali M. Alfaraj, Rajiv Kumar, Felix J. Herrmann

SEG Houston

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Outline

- Advantages of S-waves
- Why S-wave is not commonly used in practice?
- Elastic decomposition
- Jittered undersampling
- Single component reconstruction
- Various joint reconstruction formulations w/ sparsity promotion
- Conclusions
Advantages of S-waves

• Imaging through gas chimneys
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
- Reservoir detection & monitoring
Advantages of S-waves

• Imaging through gas chimneys
• High resolution imaging (thin layers)
• Reservoir detection & monitoring
• Elastic rock properties
Advantages of S-waves

- Imaging through gas chimneys
- High resolution imaging (thin layers)
- Reservoir detection & monitoring
- Elastic rock properties
- Improve accuracy & confidence
Why S-wave is not commonly used in practice?
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Low S-wave velocity
Why S-wave is not commonly used in practice?

Low S-wave velocity

Nyquist criterion
Why S-wave is not commonly used in practice?

- Low S-wave velocity
- Nyquist criterion
- Denser sampling
- Higher acquisition costs
Why S-wave is not commonly used in practice?

Low S-wave velocity

- Nyquist criterion
- Denser sampling
- Higher acquisition costs

Solution
Why $S$-wave is not commonly used in practice?

- Low $S$-wave velocity
- Higher acquisition costs
- Denser sampling
- Nyquist criterion

Solution: Compressive sensing
Why S-wave is not commonly used in practice?

Low S-wave velocity

- Nyquist criterion
- Denser sampling
- Higher acquisition costs

Solution

- Compressive sensing
- Randomized undersampling
- Lower acquisition costs
Ocean bottom acquisition

\[ \phi^+ \]
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Ocean bottom acquisition
Wavefield decomposition
Elastic wavefield decomposition

\[ d = Nq \]
Elastic wavefield decomposition

\[ d = Nq \]

\[ \begin{pmatrix} \phi^+ \\ \psi^+_y \\ \phi^- \\ \psi^-_y \end{pmatrix} = \begin{pmatrix} N_1^+ & N_2^+ \\ N_1^- & N_2^- \end{pmatrix} \begin{pmatrix} -\tau_{xz} \\ -\tau_{zz} \\ v_x \\ v_z \end{pmatrix} \]
Elastic wavefield decomposition

\[ \mathbf{d} = \mathbf{Nq} \]

\[
\begin{pmatrix}
\phi^+ \\
\psi^+_y \\
\phi^- \\
\psi^-_y
\end{pmatrix}
= \begin{pmatrix}
N_1^+ & N_2^+ \\
N_1^- & N_2^-
\end{pmatrix}
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix}
\]

At the ocean bottom:

\[ \tau_{xz} = 0 \quad \tau_{zz} = -p \]
Elastic wavefield composition

\[ q = Ld \]

\[
\begin{pmatrix}
-\tau_{xz} \\
-\tau_{zz} \\
v_x \\
v_z
\end{pmatrix} = \begin{pmatrix}
L_1^+ & L_1^- \\
L_2^+ & L_2^-
\end{pmatrix} \begin{pmatrix}
\phi^+ \\
\psi^+_y \\
\phi^- \\
\psi^-_y
\end{pmatrix}
\]
Multicomponent data

$P$, $V_x$, $V_z$
Elastic decomposition

\[ \Psi^+ \]

\[ \Psi^- \]
Can’t afford dense acquisition
40 m source interval receiver gathers

- P
- $V_x$
- $V_z$
f-k spectrum, 40 m source interval
Decomposed S-waves
f-k spectrum, 40 m source interval
Can’t afford dense acquisition
Jittered under-sampled acquisition

Jittered under-sampled acquisition

Jittered under-sampled acquisition

Gaps

single component reconstruction w/ sparsity promotion
Single component reconstruction

(1) Interpolation
Single component reconstruction

(1) Interpolation

(2) Decomposition
Single component reconstruction w/ sparsity promotion

$$\min_{x} \|x\|_1 \text{ subject to } \|Ax - b\|_2 \leq \sigma \quad \text{(BPDN}_\sigma\text{)}$$

$x$: curvelet coefficients

$$A = MS^H$$
75% jittered subsampling

Observation example
f-k spectrum, 75% jittered subsampling
Reconstructed S-waves

$\Psi^+$

$\Psi^-$

offset [m]

time [s]
Densely sampled S-waves
Residual

\[ \Psi^+ \]

\[ \Psi^- \]
f-k spectrum, reconstructed S-waves
f-k spectrum, true S-waves
Marmousi II data

\( \rho \)

\( C_p \)

\( C_s \)
Densely sampled data

\( V_x \)

\( V_z \)

\( T_{zz} \)
75% jittered subsampling
Reconstructed data
Densely sampled data
Residual

$\Phi^-$

$\Phi^+$

$\Psi^-$

$\Psi^+$
Joint interpolation decomposition
Joint interpolation decomposition

(1) Interpolation

(2) Decomposition

\[ P \xrightarrow{V_x} V_z \]

\[ P \xrightarrow{V_x} V_z \]

\[ \Psi^+ \quad \Psi^- \]
Joint interpolation decomposition

(1) Interpolation

(2) Decomposition

\[ P \rightarrow V_x \rightarrow V_z \]

\[ P \rightarrow V_x \rightarrow V_z \rightarrow \Psi^+ \rightarrow \Psi^- \]
Joint interpolation decomposition w\ curvelets

\[
\min_x \|x\|_1 \quad \text{subject to} \quad \|Ax - b\|_2 \leq \sigma \quad \text{(BPDN}_\sigma) \\
\]

\(x\): coefficients of the decomposed data

Sparsifying transform:

\[A_c = MF^H LFS^H\]
75% jittered subsampling

- **P**
- **V_x**
- **V_z**
Joint interpolation decomposition in the curvelet domain

\[ \Psi^+ \]

\[ \Psi^- \]
Densely sampled S-waves
Residual

\[\Psi^+\]

\[\Psi^-\]
Marmousi II Densely sampled data

\[ V_x \]

\[ V_z \]

\[ T_{zz} \]
75% jittered subsampling

$V_x$

$V_z$

$T_{zz}$
Reconstructed data
Densely data
Residual

\[ \Phi^+ \]

\[ \Phi^- \]

\[ \Psi^+ \]

\[ \Psi^- \]
Joint interpolation decomposition in the f-k domain
Joint interpolation decomposition, f-k

\[
\min_x ||x||_1 \text{ subject to } ||Ax - b||_2 \leq \sigma \quad \text{(BPDN}_\sigma) \\
\]

\(\mathbf{x}\): coefficients of the decomposed data

Sparsifying transform:

\[
\mathbf{A}_c = \mathbf{MF}^H \mathbf{LFS}^H
\]
Joint interpolation decomposition, f-k

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2 \leq \sigma \quad \text{(BPDN}_\sigma)$$

\(\mathbf{x}\): coefficients of the decomposed data

Sparsifying transform:

$$\mathbf{A}_c = \mathbf{MF}^H \mathbf{LFS}^H$$

$$\mathbf{A}_{fk} = \mathbf{MF}^H \mathbf{L}$$
Reconstructed data, f-k
Reconstructed data, curvelet
Residual

\( \Phi^- \)

\( \Phi^+ \)

\( \Psi^- \)

\( \Psi^+ \)
Why curvelets are better?

- Better at capturing curve-like events.
Why curvelets are better?

- Better at capturing curve-like events.
- Sparser representation.

![Coefficients of decomposed data](image)
Joint source separation decomposition
Jittered continuous recording, 1 boat, 2 air guns
Joint source separation decomposition

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2 \leq \sigma \quad (\text{BPDN}_\sigma)$$

\(\mathbf{x}\): curvelet coefficients of the decomposed data

\(\mathbf{A}_c = \mathbf{M}\mathbf{F}^H\mathbf{L}\mathbf{F}\mathbf{S}^H\)

\(\mathbf{M}\): blending matrix
Reconstructed data
Densely data
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
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- Avoid multi stage processing & artifacts.
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- Minimize parameters selection.
Advantages of the joint formulations

- Use all the multicomponent data in one optimization problem.
- Avoid multi stage processing & artifacts.
- Minimize parameters selection.
- Ensure preservation of amplitude ratios.
Conclusions

- Acquisition of S-waves is prohibitively expensive with conventional dense acquisition designs.
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- Coarse regular sampling results in aliasing of the S-waves.
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- Acquisition of S-waves is prohibitively **expensive** w/ conventional dense acquisition designs.
- **Coarse regular** sampling results in **aliasing** of the S-waves.
- Using low-cost **jittered under-sampling & simultaneous acquisition w/ joint interpolation source separation decomposition**, S-waves become feasible to acquire & utilize in practice.
Conclusions

- Acquisition of S-waves is prohibitively expensive with conventional dense acquisition designs.
- Coarse regular sampling results in aliasing of the S-waves.
- Using low-cost jittered under-sampling & simultaneous acquisition with joint interpolation source separation decomposition, S-waves become feasible to acquire & utilize in practice.
- Utilize the multicomponent data to its available full extent at a lower cost compared with conventional acquisition.
References

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Thank you for your attention!