

Time domain sparsity promoting LSRTM with source estimation

Mengmeng Yang, Philipp Witte, Zhilong Fang & Felix J. Herrmann

Motivation

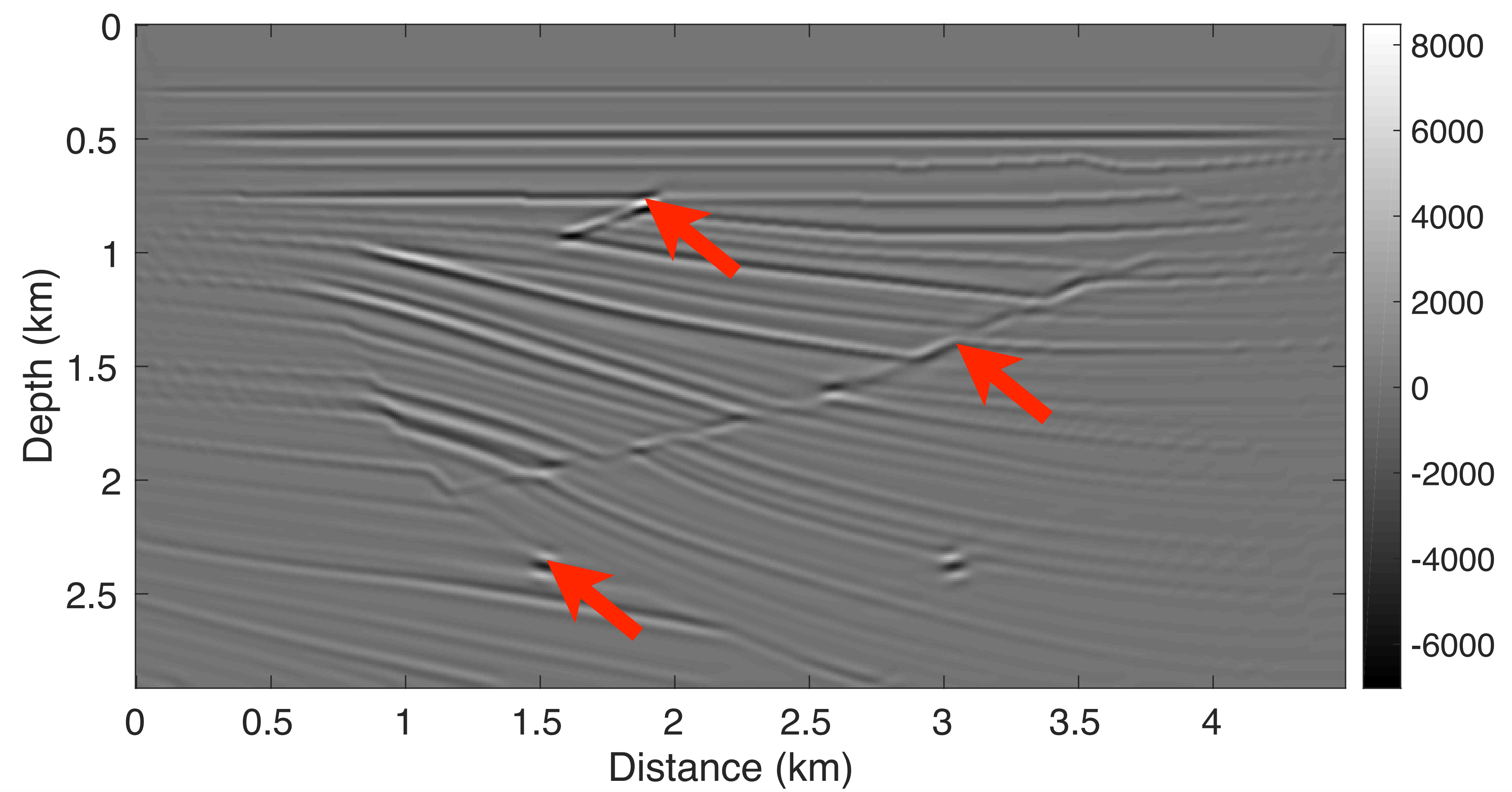
Features of RTM:

- ▶ pros
 - no dip limitation
 - strong lateral velocity variations
- ▶ cons
 - inaccurate amplitudes & low resolution

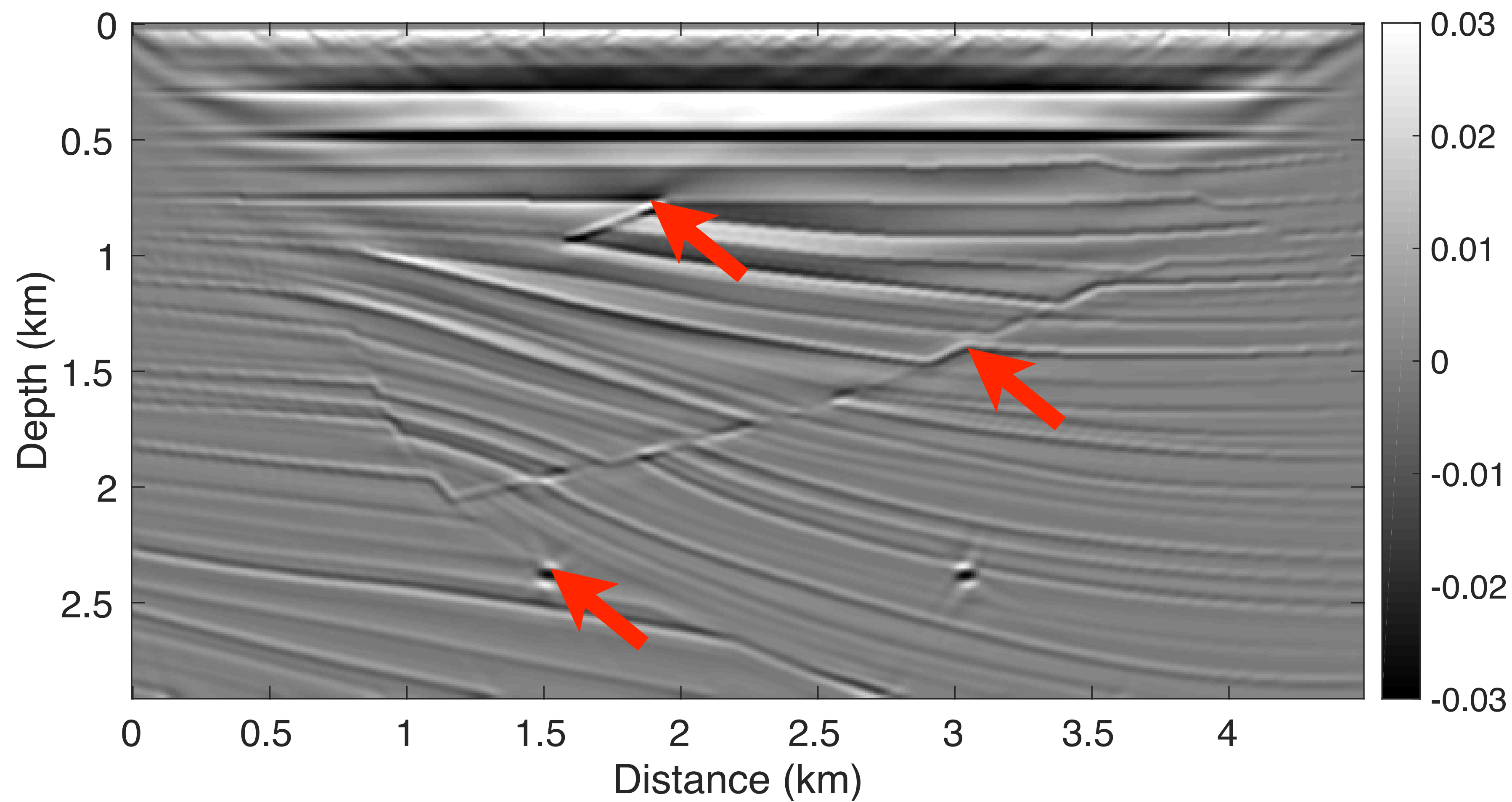
Problems of LS-RTM:

- ▶ iterations that touch all shots are too expensive
- ▶ data can be overfitted

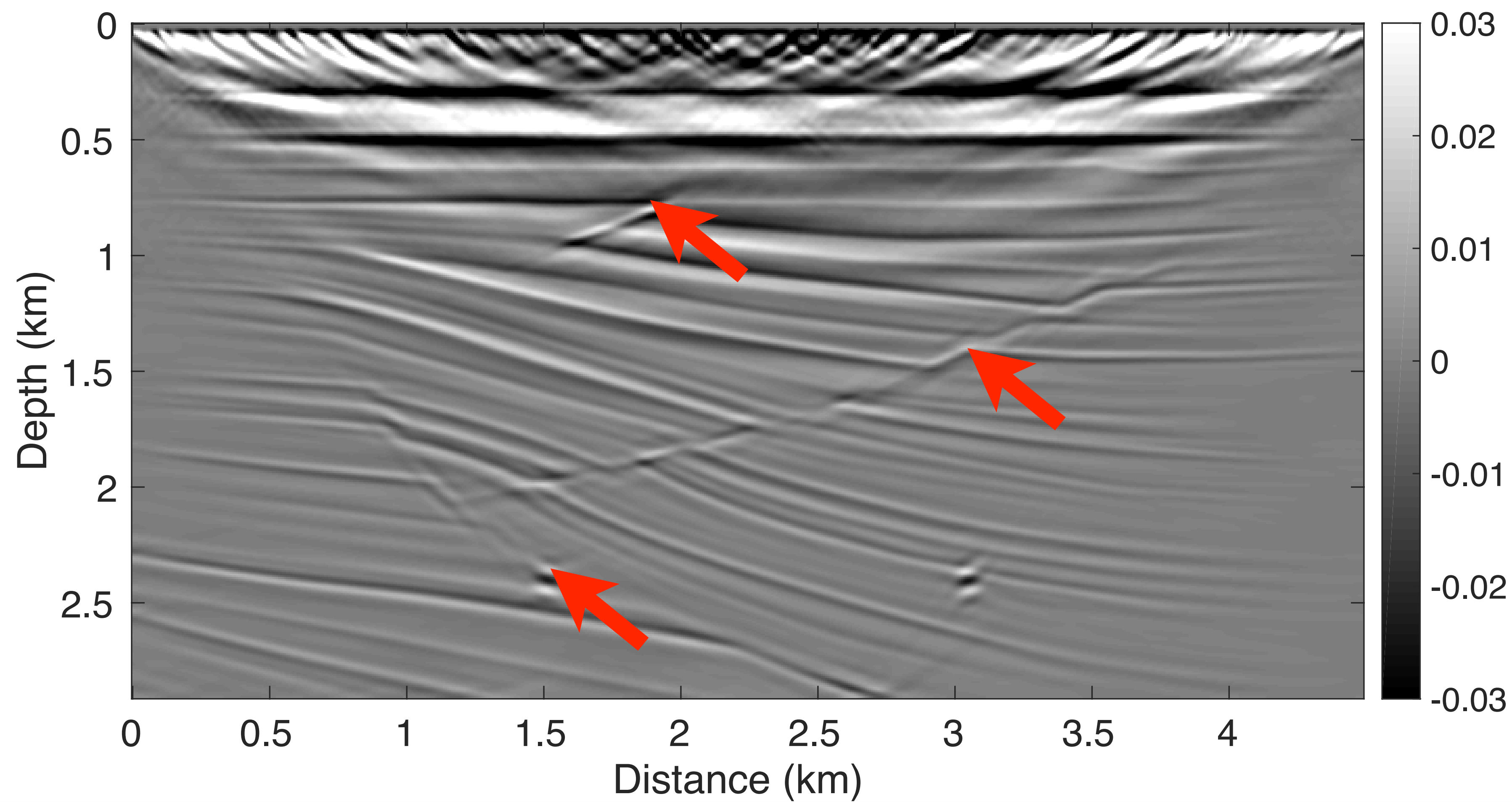
RTM w/ correct wavelet



Sparsity promoting LS-RTM w/ correct wavelet



Sparsity promoting LS-RTM w/ wrong wavelet



LS-RTM

$$\min_{\delta \mathbf{m}} \sum_{i=1}^{n_s} \|\mathbf{J}_i[\mathbf{m}_0, \mathbf{q}_i] \delta \mathbf{m} - \mathbf{b}_i\|^2$$

\mathbf{m}_0 : background model

\mathbf{J}_i : Born modelling operator for i^{th} shot

$\delta \mathbf{m}$: model perturbation

\mathbf{q}_i : source wavelet for i^{th} shot

\mathbf{b}_i : vectorized reflections for i^{th} shot

Sparsity promoting inversion

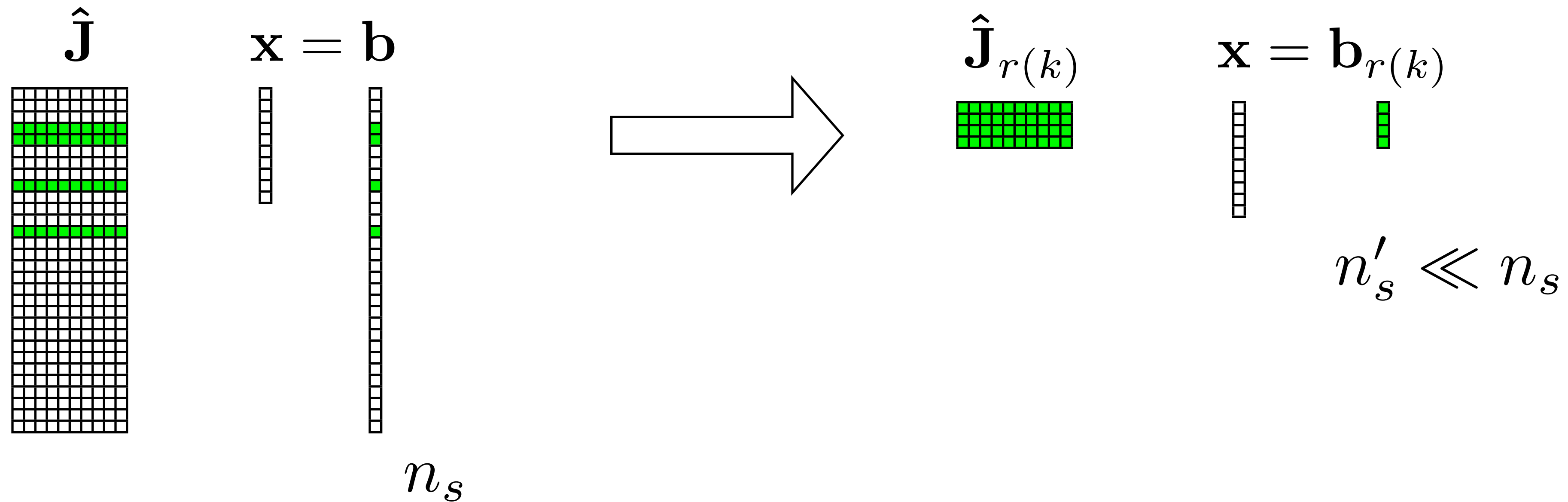
$$\begin{aligned} & \min_{\mathbf{x}} \|\mathbf{x}\|_1 \\ \text{s.t.} & \sum_{i=1}^{ns} \left\| \underbrace{\mathbf{J}_i[\mathbf{m}_0, \mathbf{q}_i] \mathbf{C}^*}_{\hat{\mathbf{J}}} \mathbf{x} - \underbrace{\mathbf{b}_i}_{\mathbf{b}} \right\|_2 \leq \sigma \end{aligned}$$

\mathbf{C}^* : the transpose of Curvelet transform

\mathbf{x} : Curvelet coefficients

σ : tolerance for noise or modelling error

Randomized subsampling

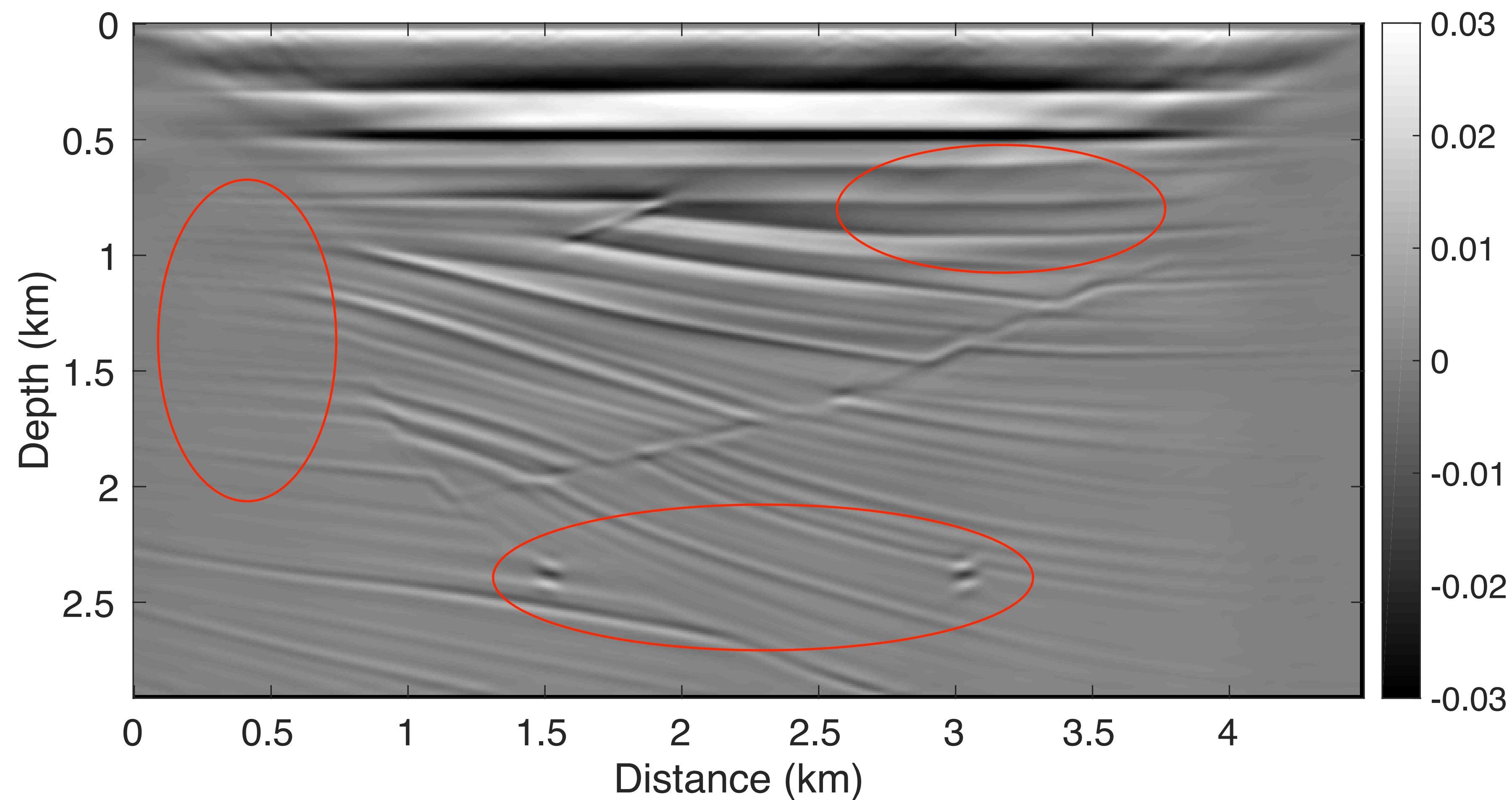


Solvers for sparsity promoting inversion

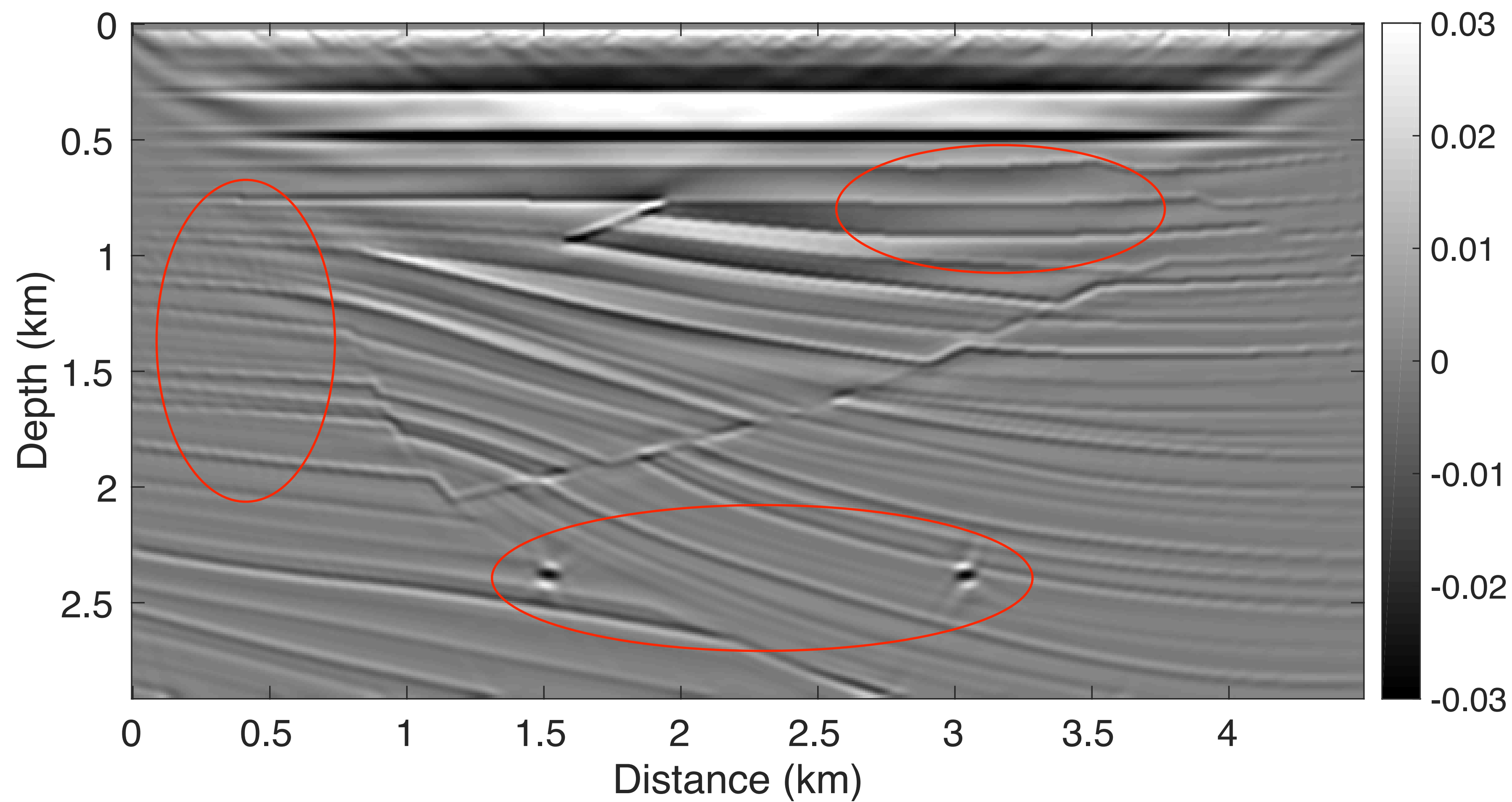
Many solvers for sparse. inversion:

- Iterative soft thresholding
(simple, but slow convergence, cooling of threshold ...)
- Spectral projected gradients w/ L1 constraint – SPGL1
(expensive, difficult to implement, slow convergence)
- Linearized Bregman (LB)
(easy to implement, proven convergence w/ subsampling)

Sparsity promoting LS-RTM w/ correct wavelet & SPGL1



Sparsity promoting LS-RTM w/ correct wavelet & LB



W, Yin. Analysis and generalizations of the linearized Bregman method. SIAM J. Imaging Sci., 3(4):856–877, 2010.

Herrmann F J, Tu N, Esser E. Fast “online” migration with Compressive Sensing[J].

Modification

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && \lambda \|\mathbf{x}\|_1 + \frac{1}{2} \|\mathbf{x}\|^2 \\ & \text{s.t.} && \|\hat{\mathbf{J}}\mathbf{x} - \mathbf{b}\|_2 \leq \sigma \end{aligned}$$

- strongly convex objective function because of additional 2-norm term
- for big enough λ solves BP problem

Workflow for LB

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && \lambda \|\mathbf{x}\|_1 + \frac{1}{2} \|\mathbf{x}\|^2 \\ & \text{s.t.} && \|\hat{\mathbf{J}}\mathbf{x} - \mathbf{b}\|_2 \leq \sigma \end{aligned}$$

1. **Initialize** $\mathbf{x}_0 = \mathbf{0}$, $\mathbf{z}_0 = \mathbf{0}$, q , λ , batchsize $n'_s \ll n_s$
2. **for** $k = 0, 1, \dots$
3. **Randomly choose shot subsets** $\mathcal{I} \in [1 \dots n_s]$, $|\mathcal{I}| = n'_s$
4. $\hat{\mathbf{J}}_k = \{\mathbf{J}_i(\mathbf{m}_0, q_i)\mathbf{C}^*\}_{i \in \mathcal{I}}$
5. $\mathbf{b}_k = \{\mathbf{b}_i\}_{i \in \mathcal{I}}$
6. $\mathbf{z}_{k+1} = \mathbf{z}_k - t_k \hat{\mathbf{J}}_k^T P_\sigma(\hat{\mathbf{J}}_k \mathbf{x}_k - \mathbf{b}_k)$
7. $\mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
8. **end**

note: $S_\lambda(\mathbf{z}_{k+1}) = \text{sign}(\mathbf{z}_{k+1}) \max\{0, |\mathbf{z}_{k+1}| - \lambda\}$
 $P_\sigma(\hat{\mathbf{J}}_k \mathbf{x}_k - \mathbf{b}_k) = \max\{0, 1 - \frac{\sigma}{\|\hat{\mathbf{J}}_k \mathbf{x}_k - \mathbf{b}_k\|}\} \cdot (\hat{\mathbf{J}}_k \mathbf{x}_k - \mathbf{b}_k)$

Toy example

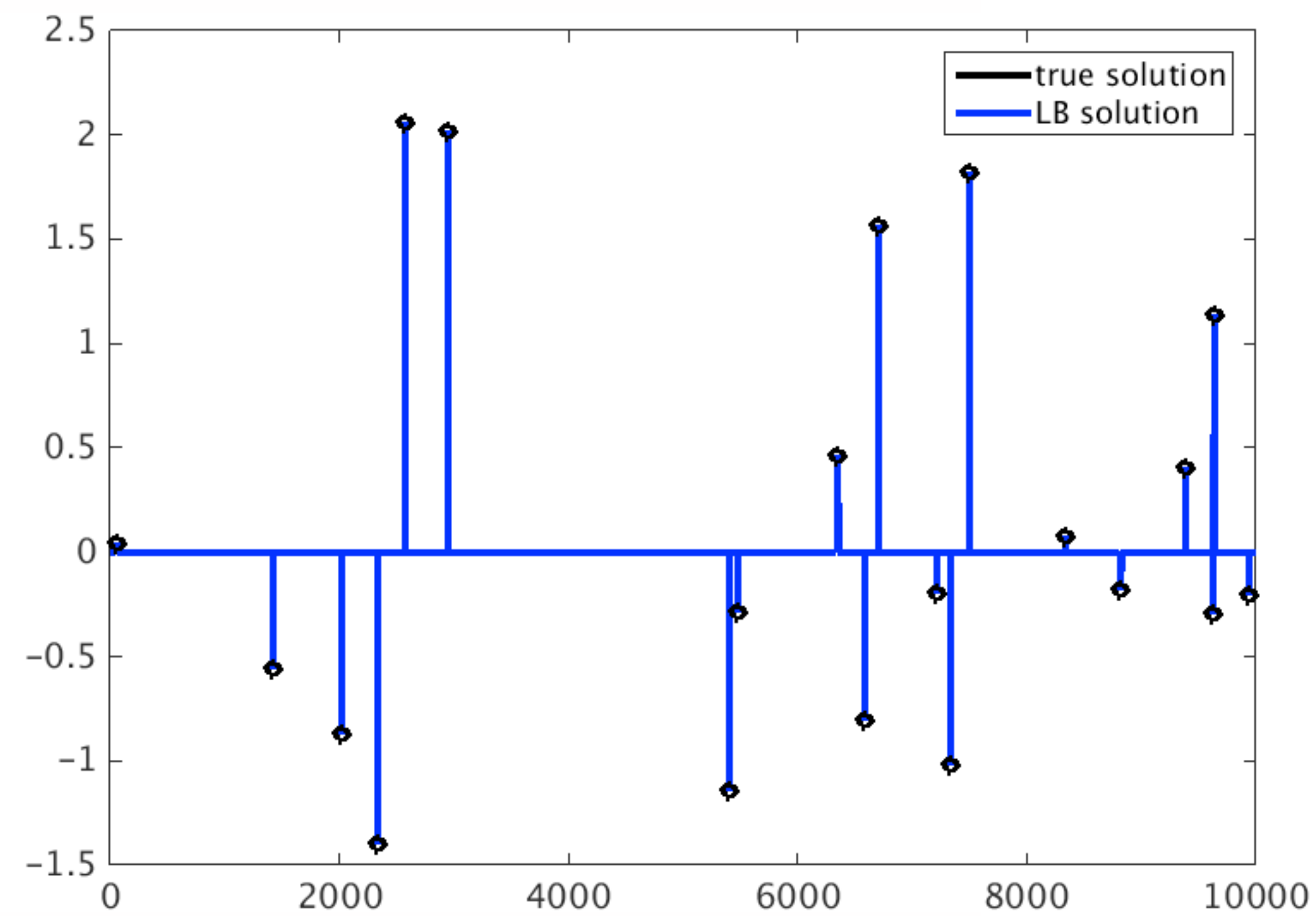
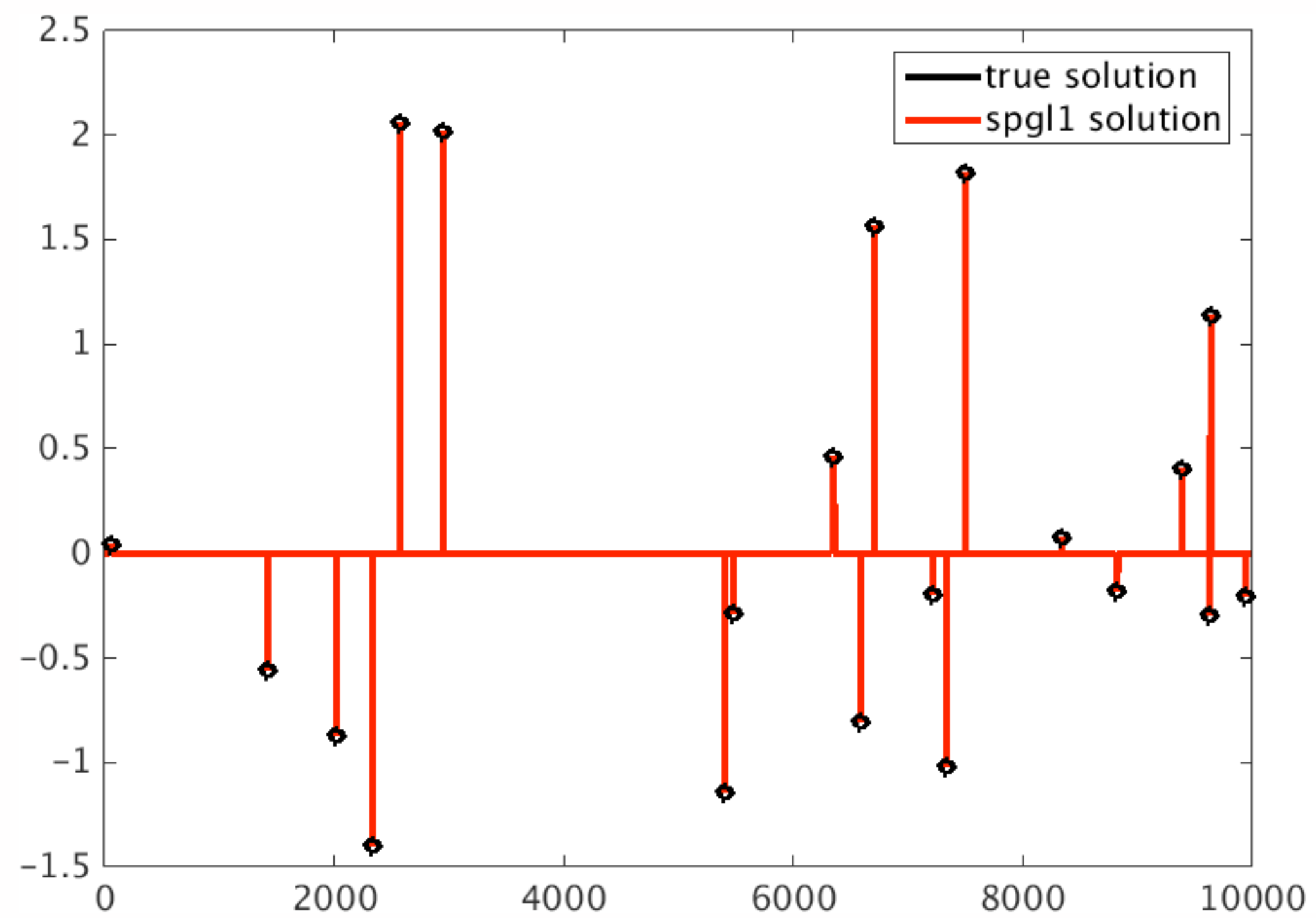
Sparsity recovery with tall ill-conditioned matrix

A: 20000 X 10000, with Rank 500

x: 10000 X 1, with 20 non-zeros

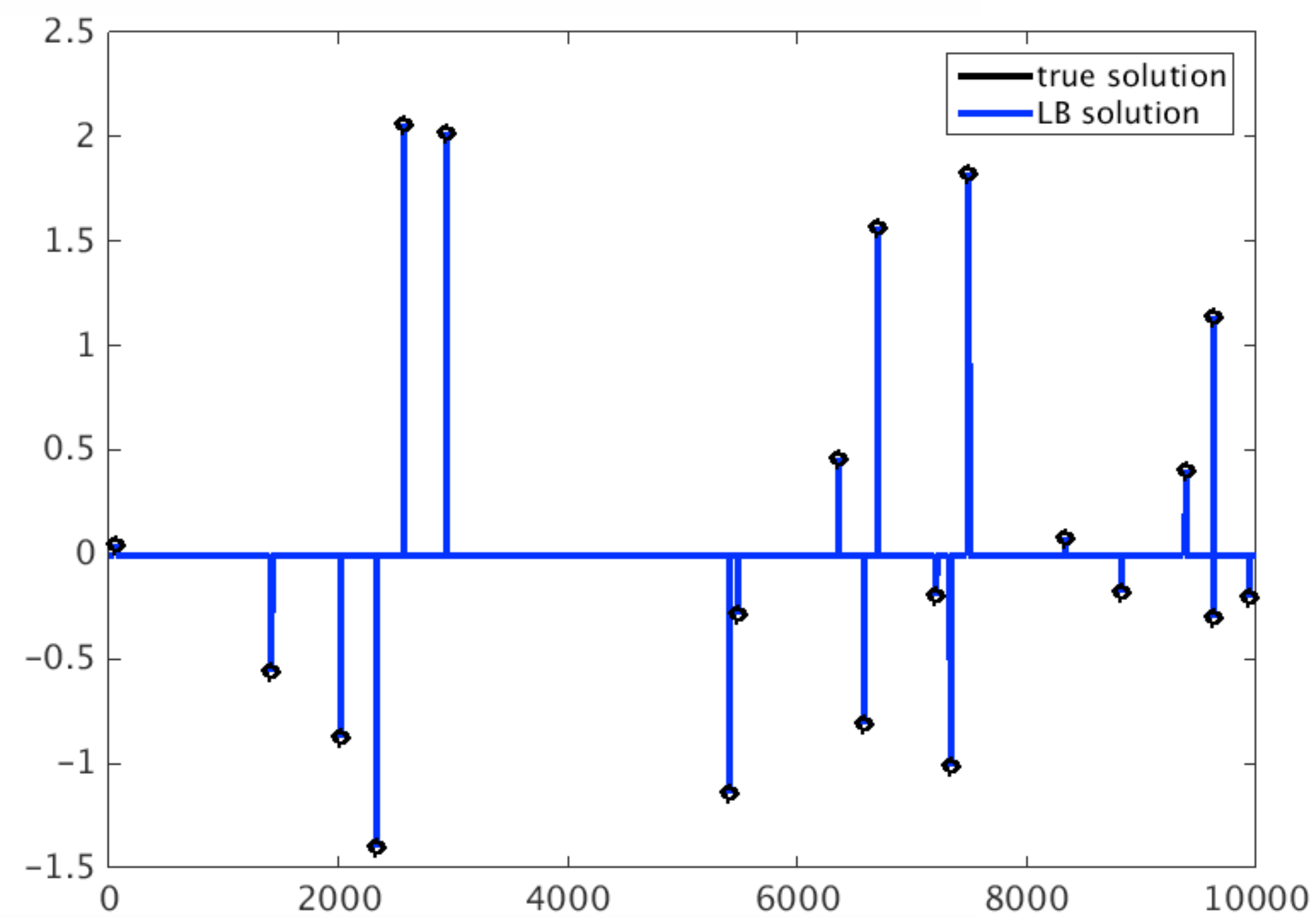
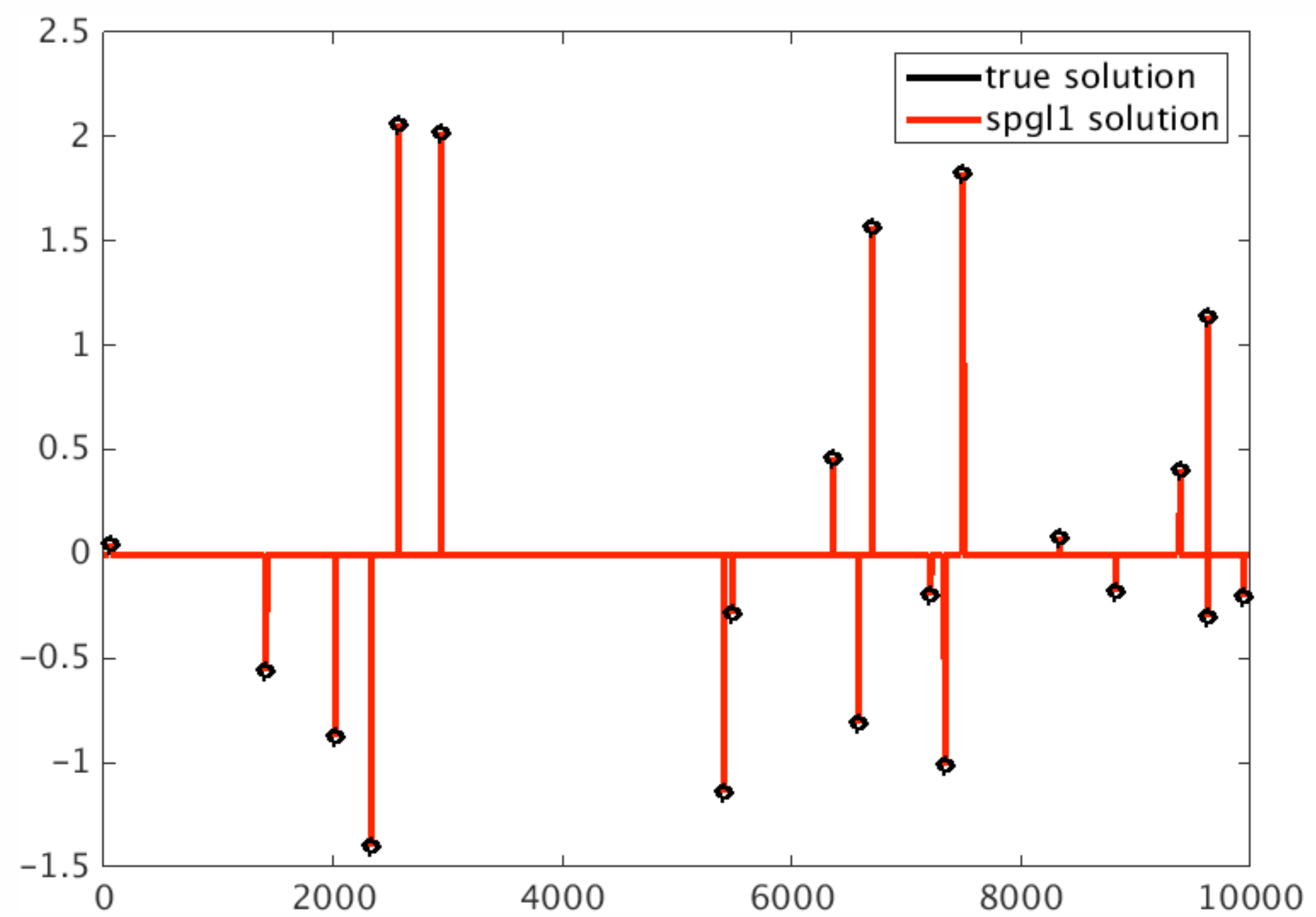
SPGL1 vs LB

no subsampling



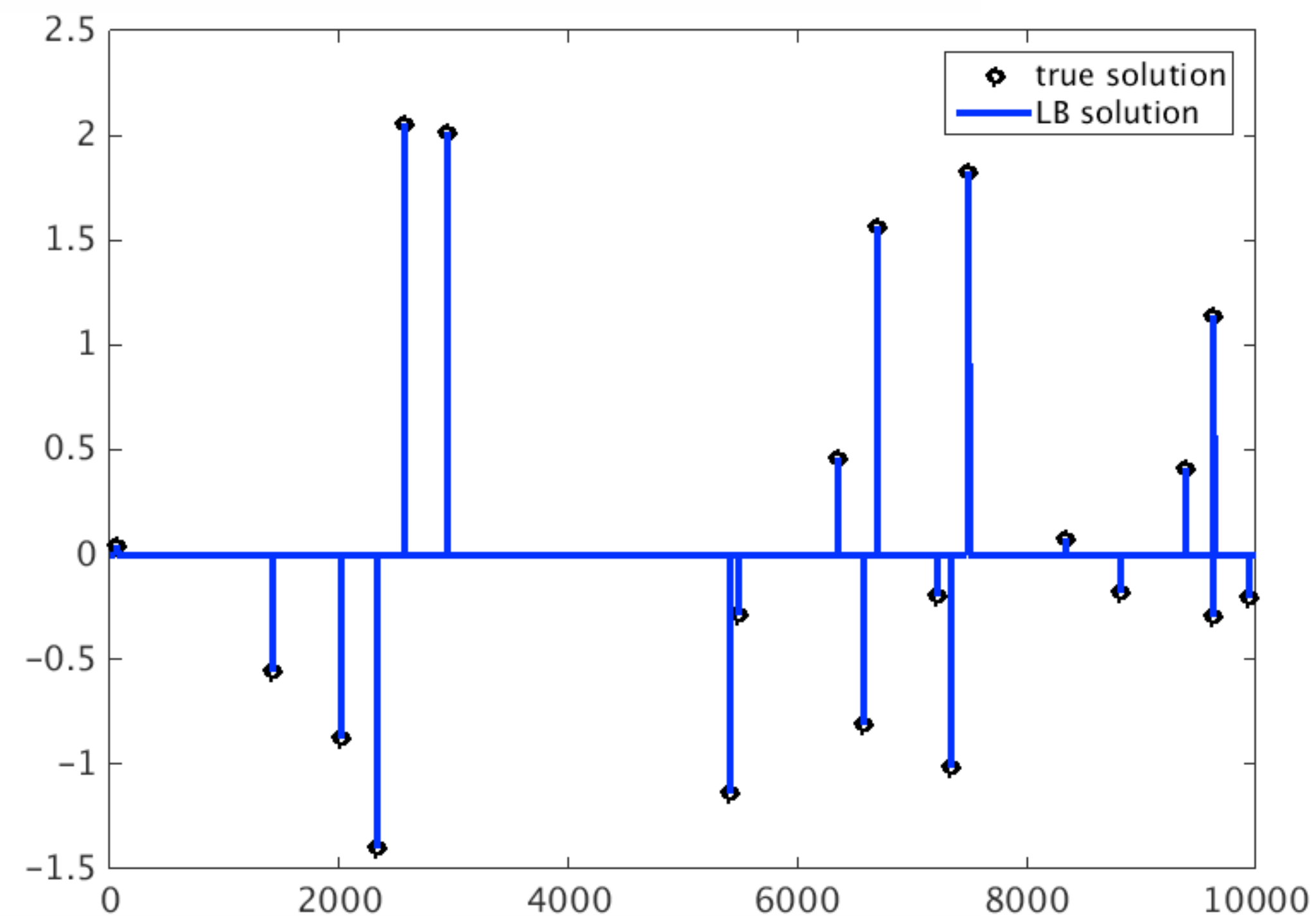
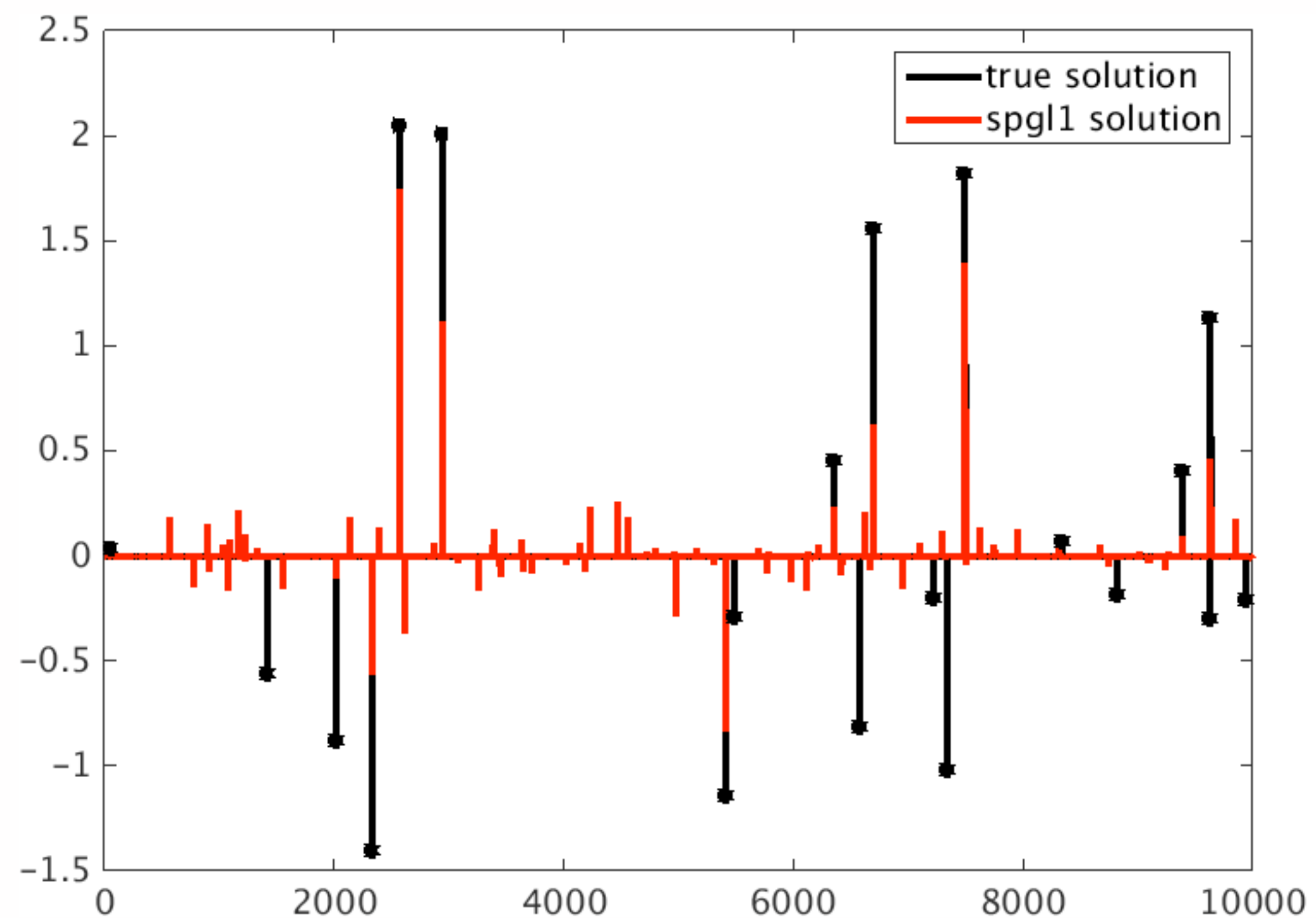
SPGL1 vs LB

50% subsampling



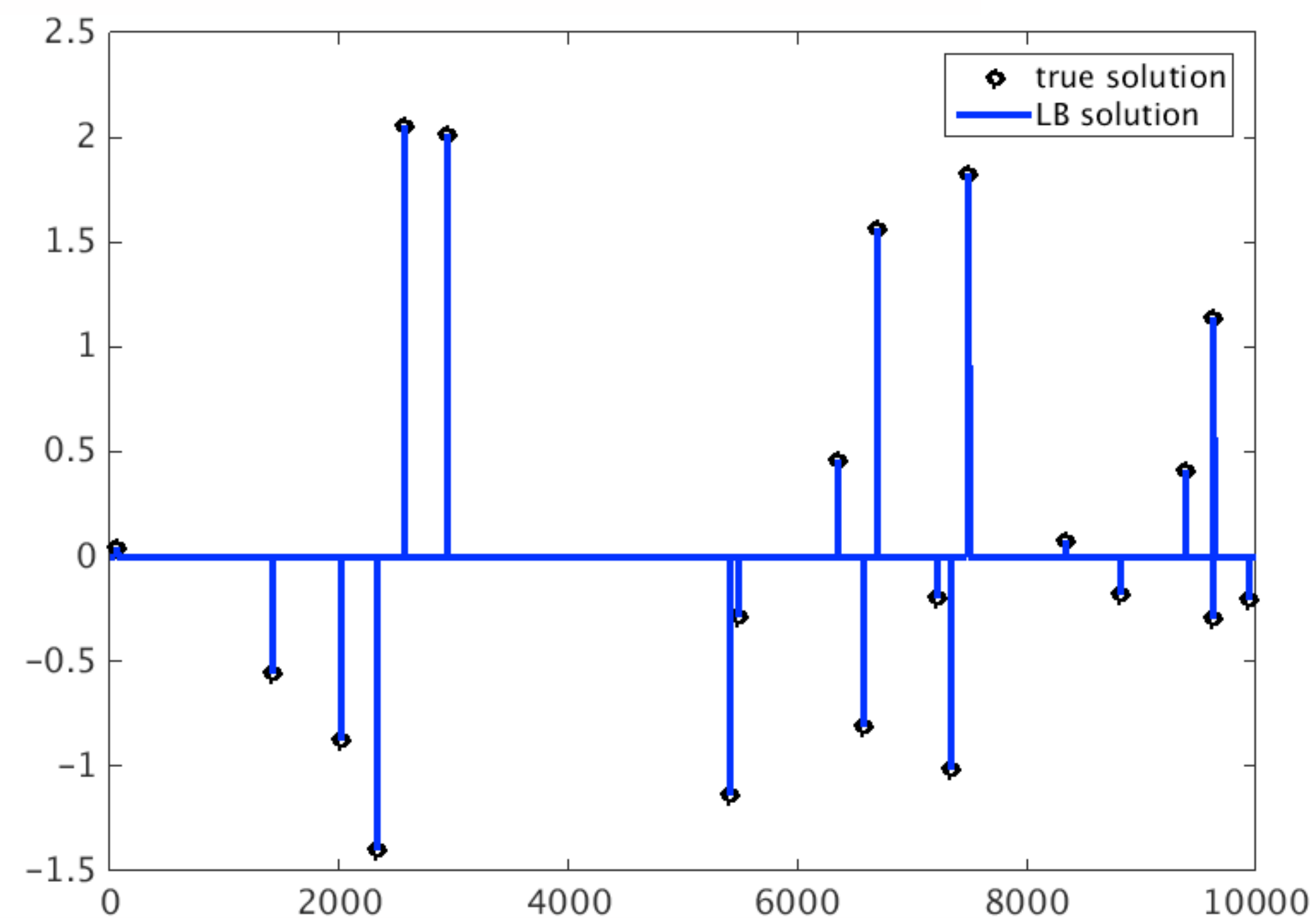
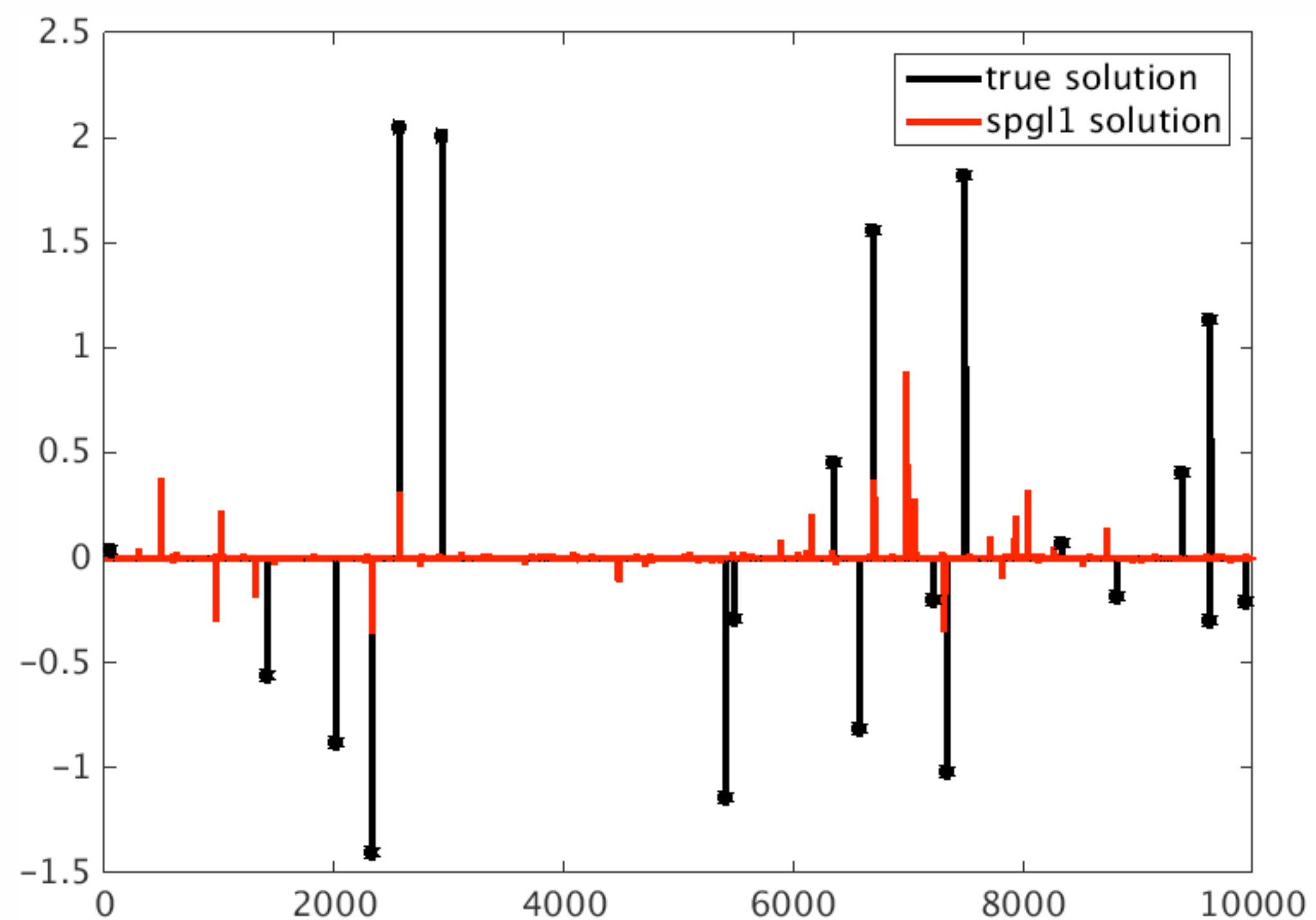
SPGL1 vs LB

80% subsampling



SPGL1 vs LB

90% subsampling



What if source signature is unknown?

Estimate source by solving least-square problem:

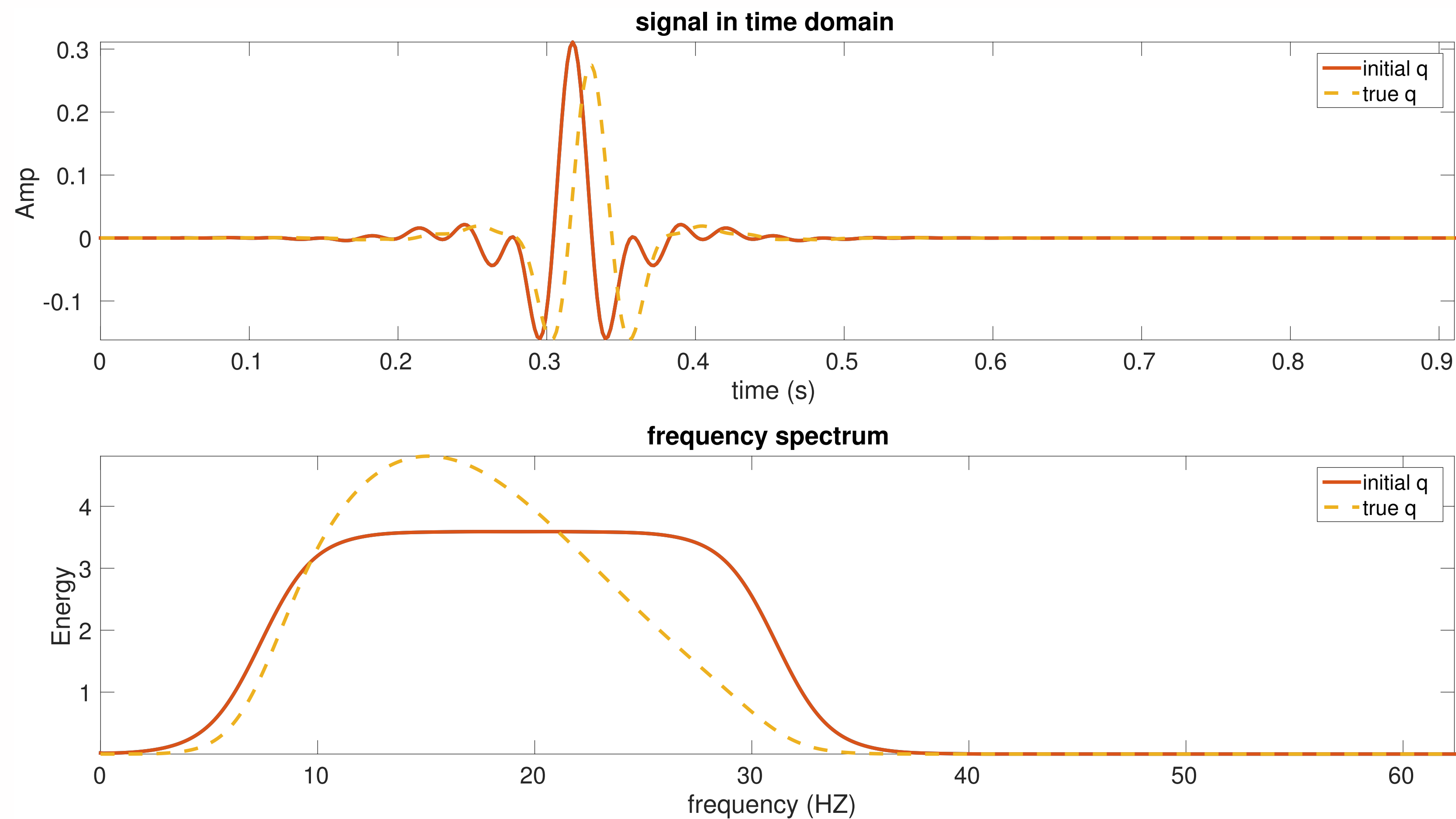
$$\min_{\mathbf{w}} \sum_j^{N_{tr}} \|\mathbf{w} * \tilde{\mathbf{b}}_j - \mathbf{b}_j\|^2 \quad \text{s.t.} \|\mathbf{w}\|^2 = 1$$

where $\tilde{\mathbf{b}} = \mathbf{J}(\mathbf{m}_0, \mathbf{q}_0) \delta \tilde{\mathbf{m}}$

Suppose that $\mathbf{q} = \mathbf{w} * \mathbf{q}_0$, and \mathbf{q} is the same for all shots

\mathbf{q}_0 is the initial guess of \mathbf{q}

Initial wavelet setting

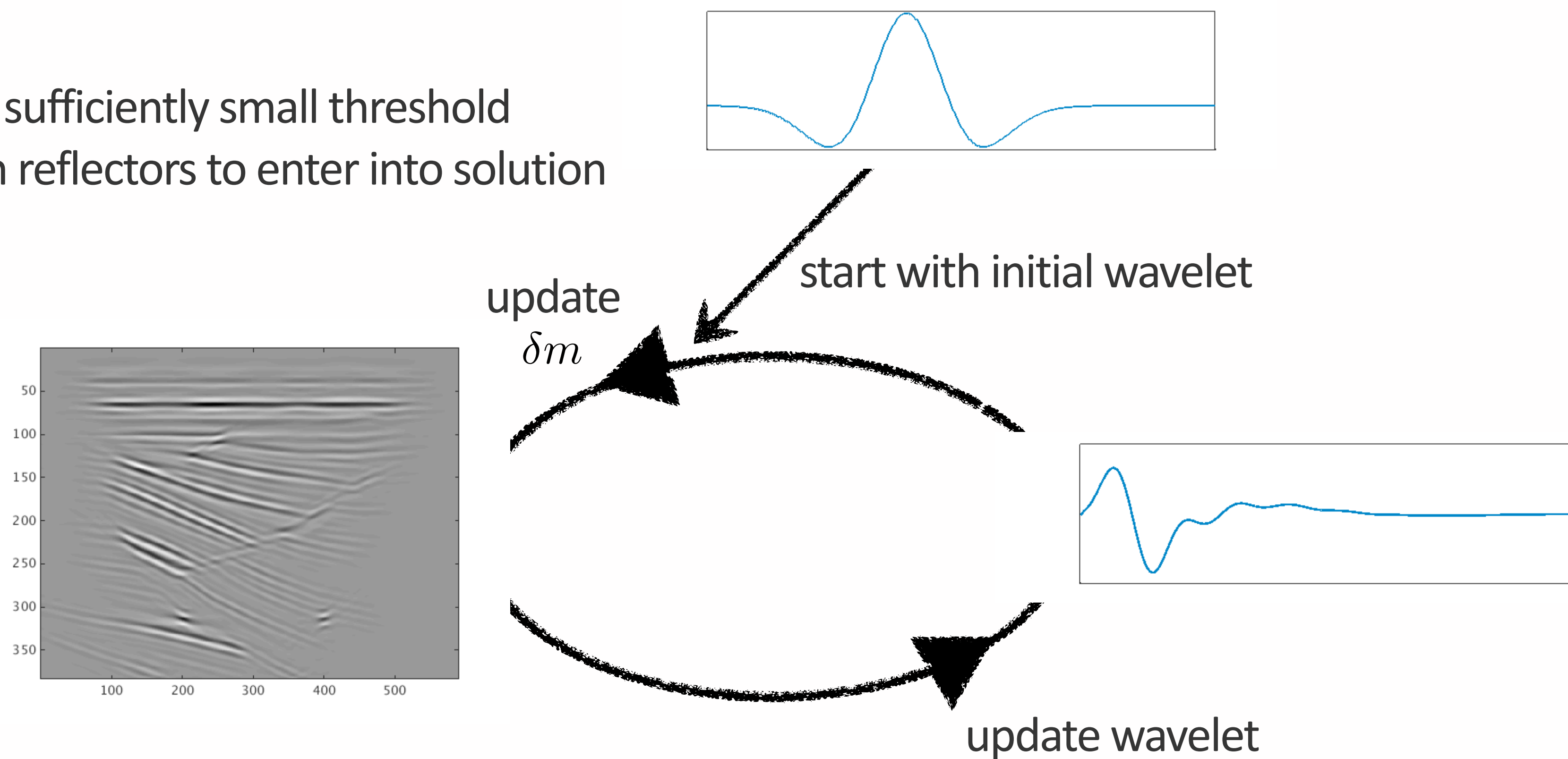


approximate
duration

frequency
bandwidth wider
due to factorization

Combine the image inversion & source estimation

start w/ sufficiently small threshold
to allow main reflectors to enter into solution



Workflow for sparsity-promoting LS-RTM w/ source estimation

1. Initialize $\mathbf{x}_0 = \mathbf{0}$, $\mathbf{z}_0 = \mathbf{0}$, \mathbf{q}_0 , λ , λ_2 , batchsize $n'_s \ll n_s$, weights r
2. for $k = 0, 1, \dots$
3. Randomly choose shot subsets $\mathcal{I} \in [1 \dots n_s]$, $|\mathcal{I}| = n'_s$
4. $\hat{\mathbf{J}}_k = \{\mathbf{J}_i(\mathbf{m}_0, q_0)\mathbf{C}^*\}_{i \in \mathcal{I}}$
5. $\mathbf{b}_k = \{\mathbf{b}_i\}_{i \in \mathcal{I}}$
6. $\tilde{\mathbf{b}}_k = \hat{\mathbf{J}}_k \mathbf{x}_k$
7. $\mathbf{w}_k = \arg \min_{\mathbf{w}} \sum_{\mathcal{I}} \|\mathbf{w} * \tilde{\mathbf{b}}_k - \mathbf{b}_k\|^2 + \|r(\mathbf{w} * \mathbf{q}_0)\|^2 + \lambda_2 \|\mathbf{w} * \mathbf{q}_0\|^2$
8. $\mathbf{z}_{k+1} = \mathbf{z}_k - t_k \hat{\mathbf{J}}_k^* \left(\mathbf{w}_k \star P_\sigma(\mathbf{w}_k * \tilde{\mathbf{b}}_k - \mathbf{b}_k) \right)$
9. $\mathbf{x}_{k+1} = S_\lambda(\mathbf{z}_{k+1})$
10. end

Experiments

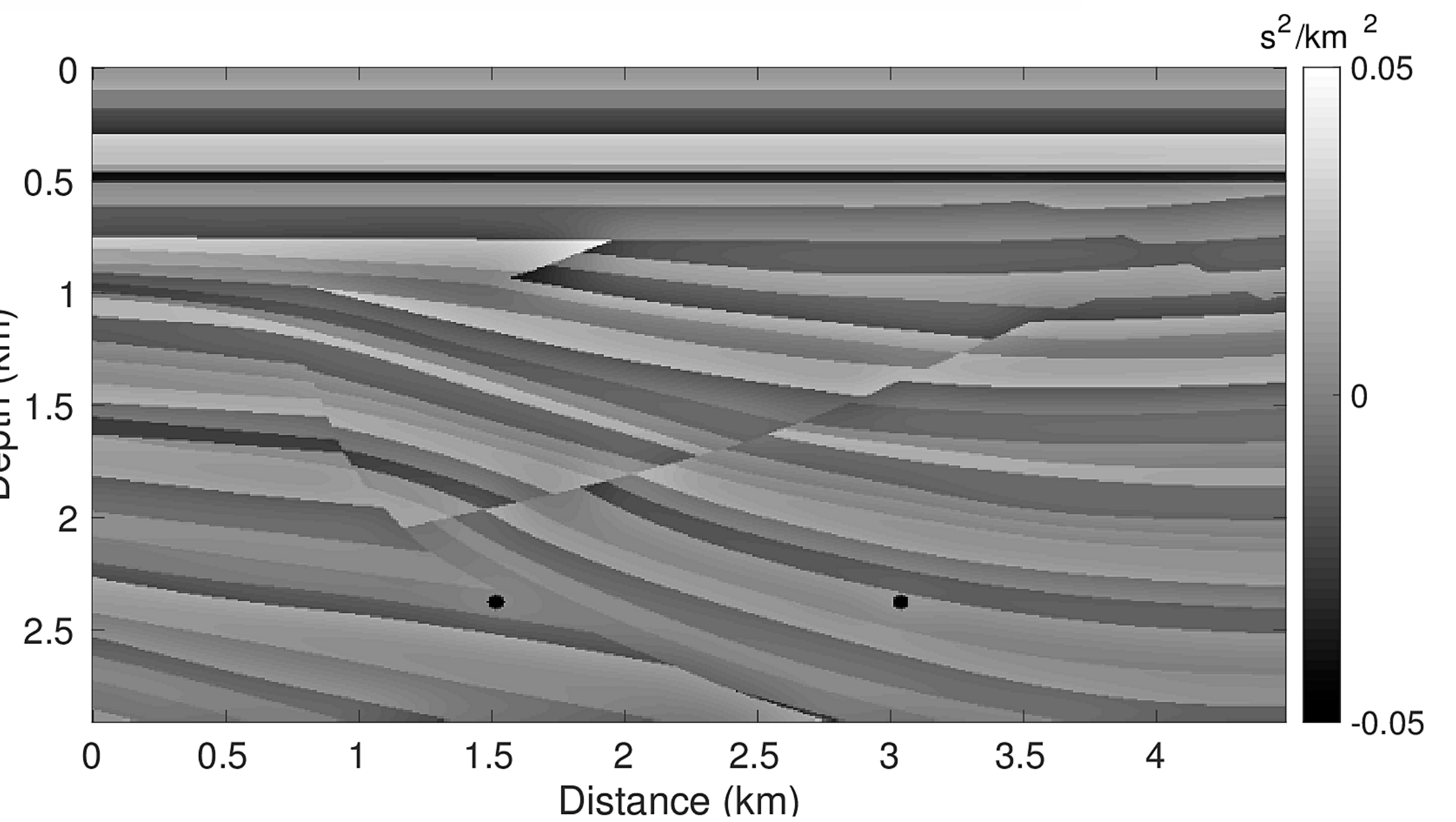
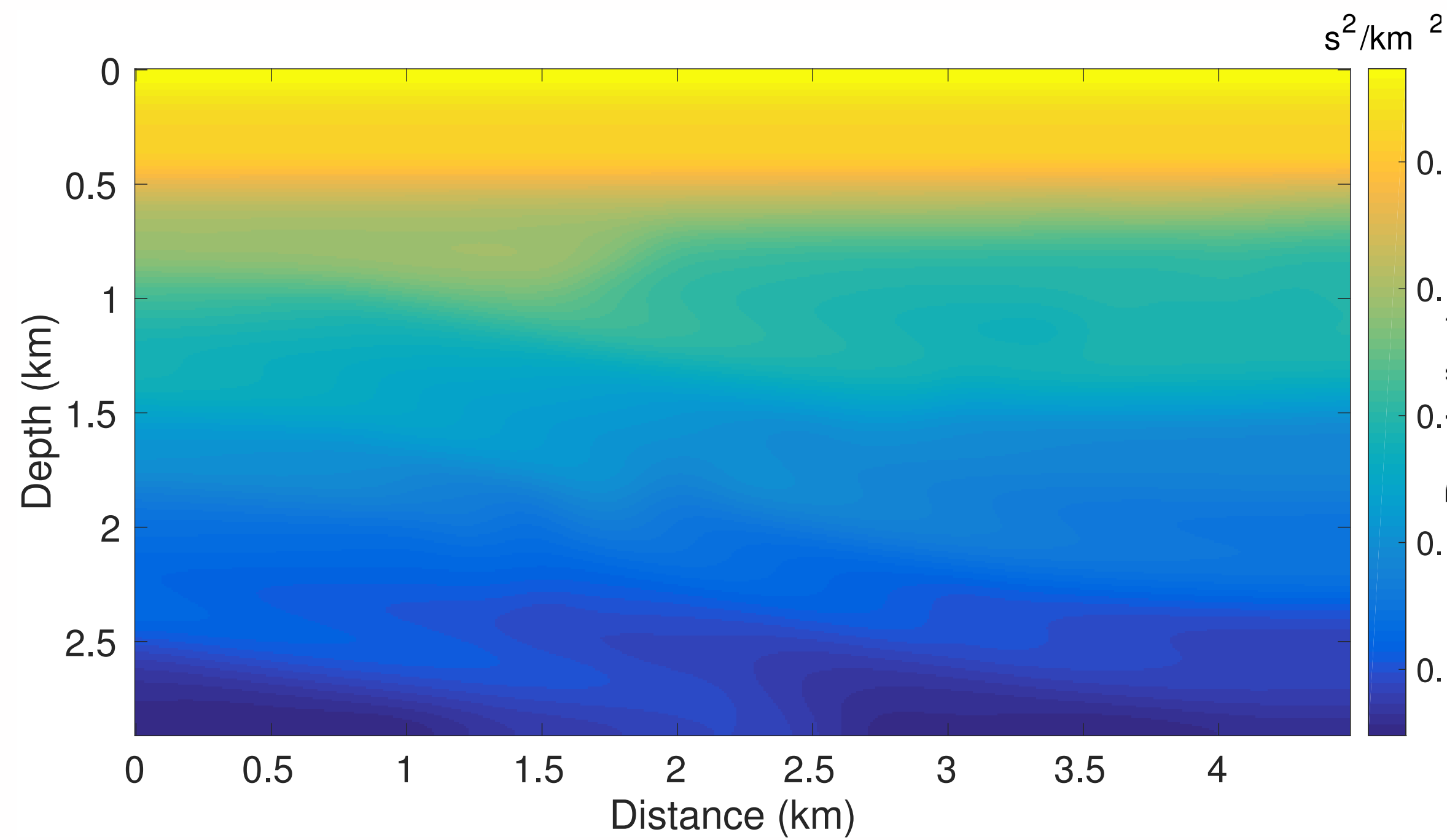
Data:

- 295 shots with shot interval 15m
- 295 receivers with receiver interval 15m
- 4s record, 15Hz peak frequency designed wavelet
- synthetic linearized data

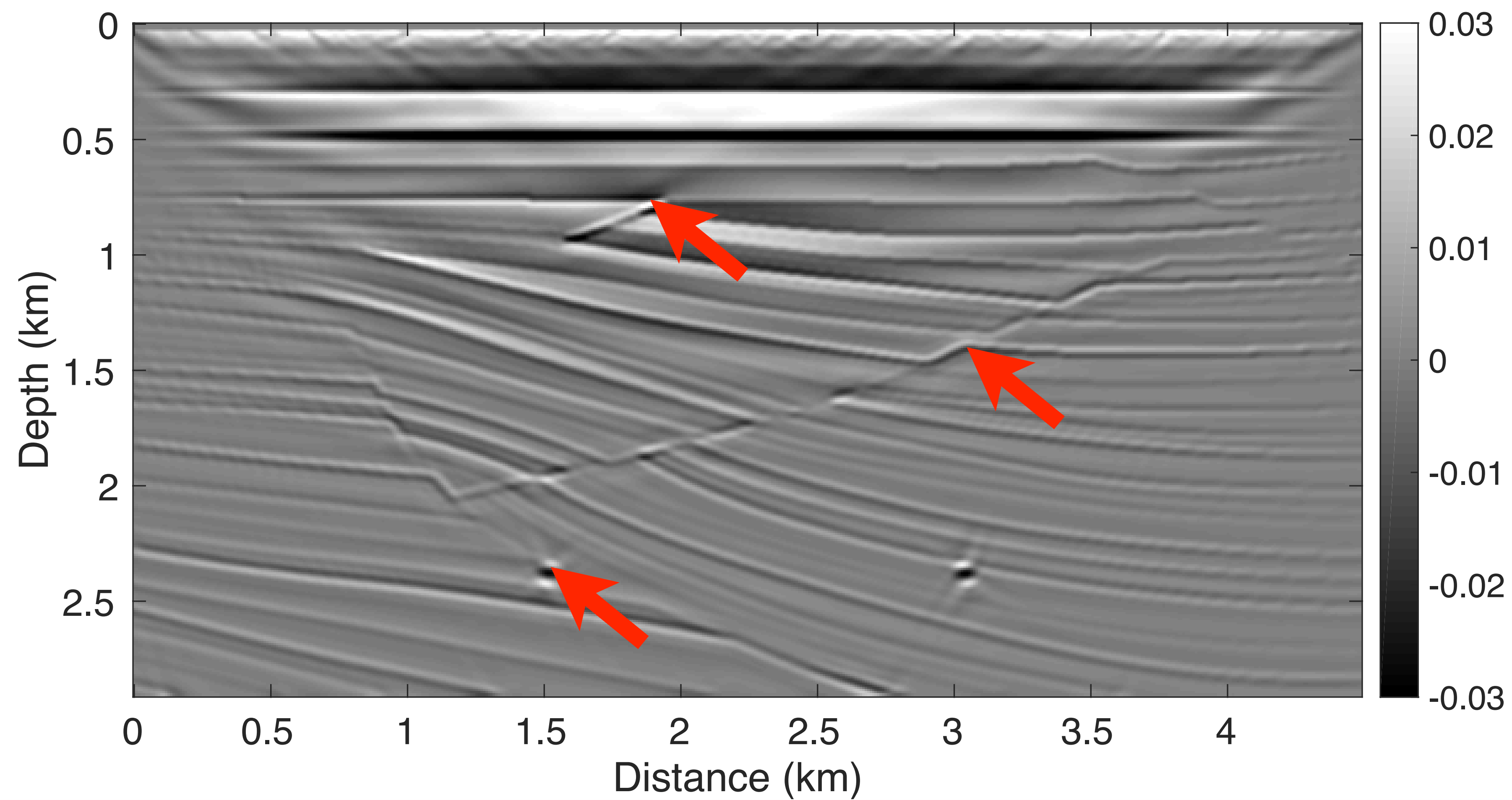
Experiments:

- one pass through the data with batch sizes 2.5% data
- randomized subset of shots
- normalized true source wavelet & initial guessed wavelet

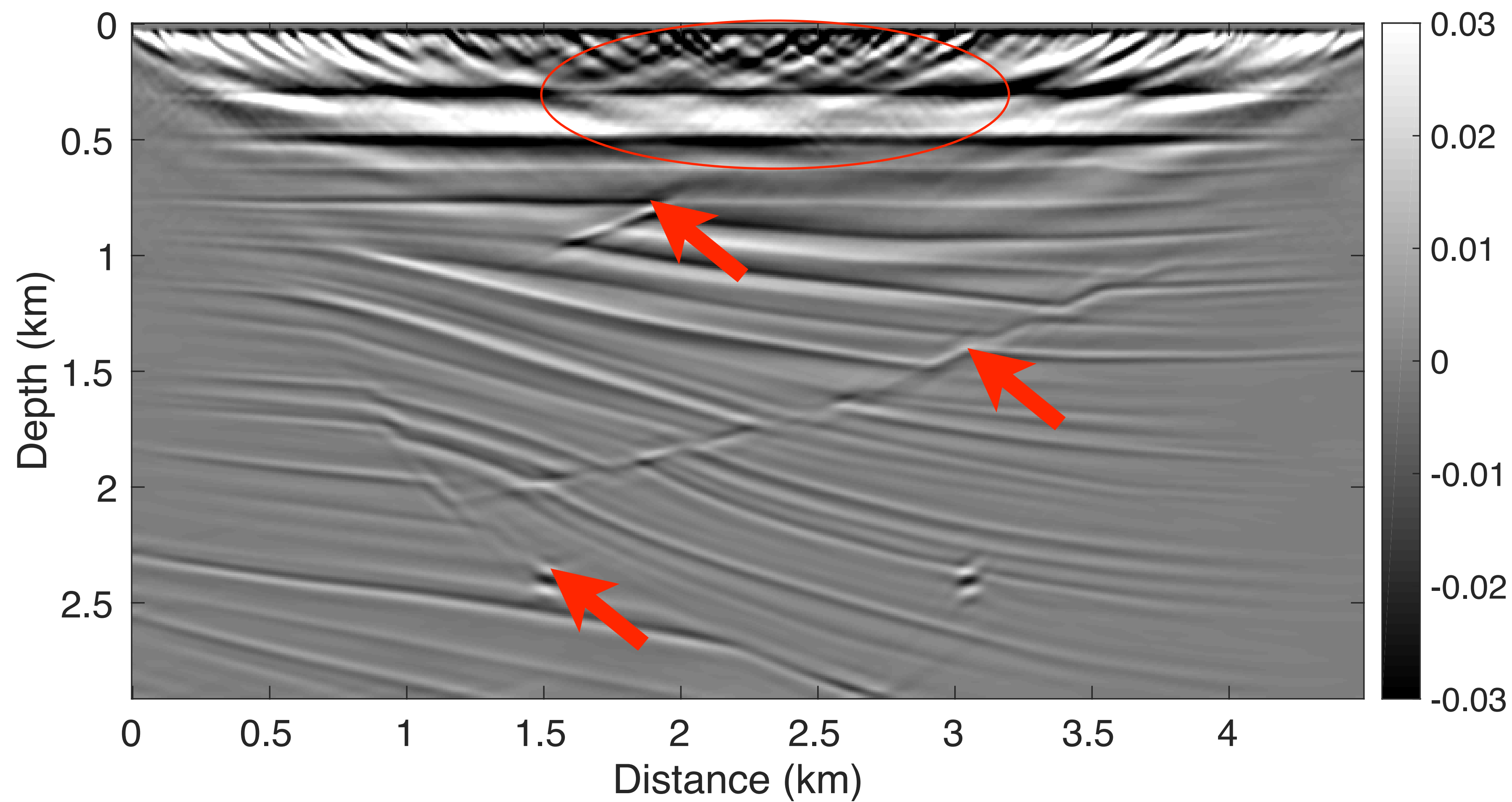
background model and model perturbation



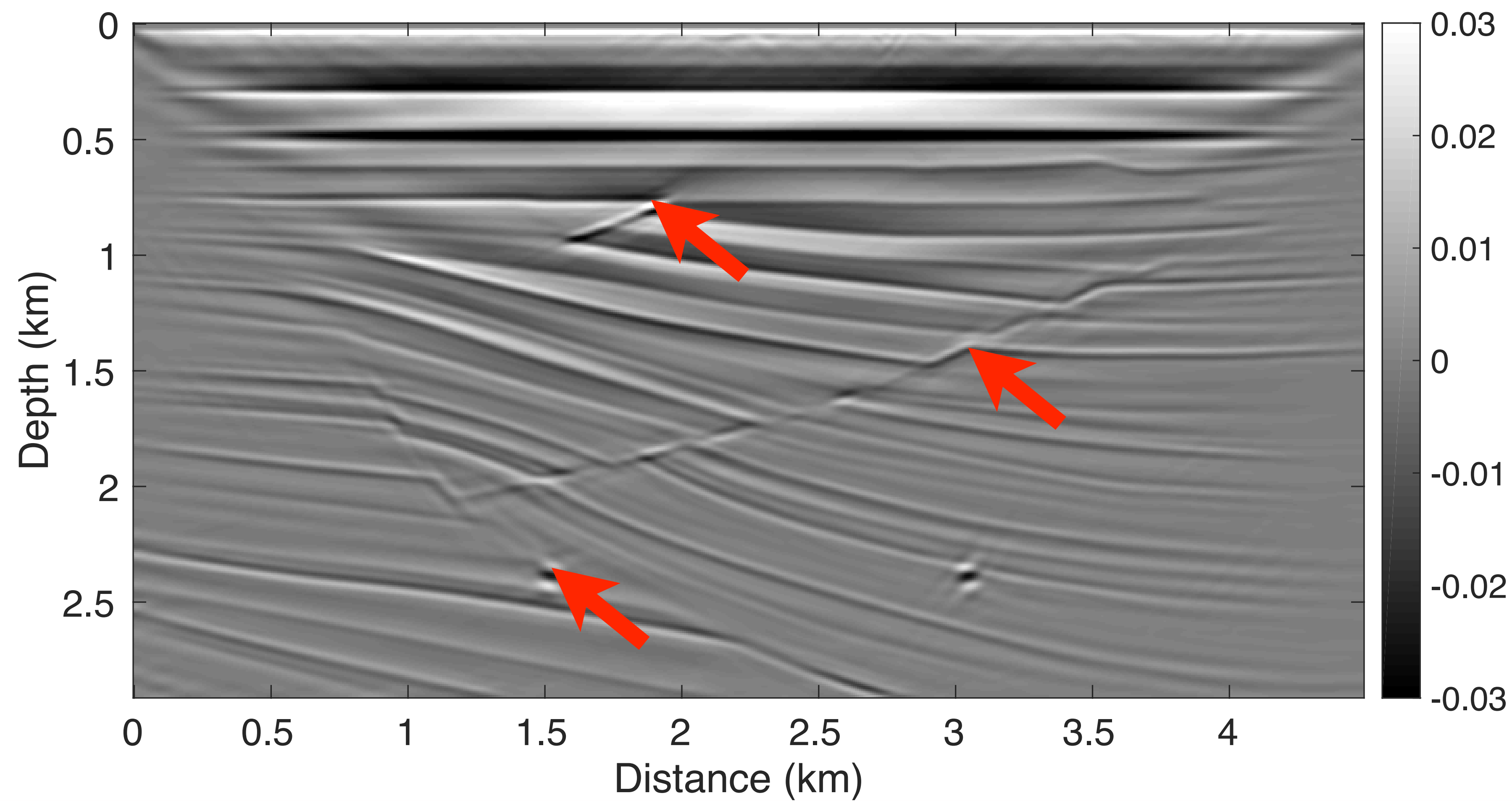
Sparsity promoting LS-RTM w/ correct wavelet & LB



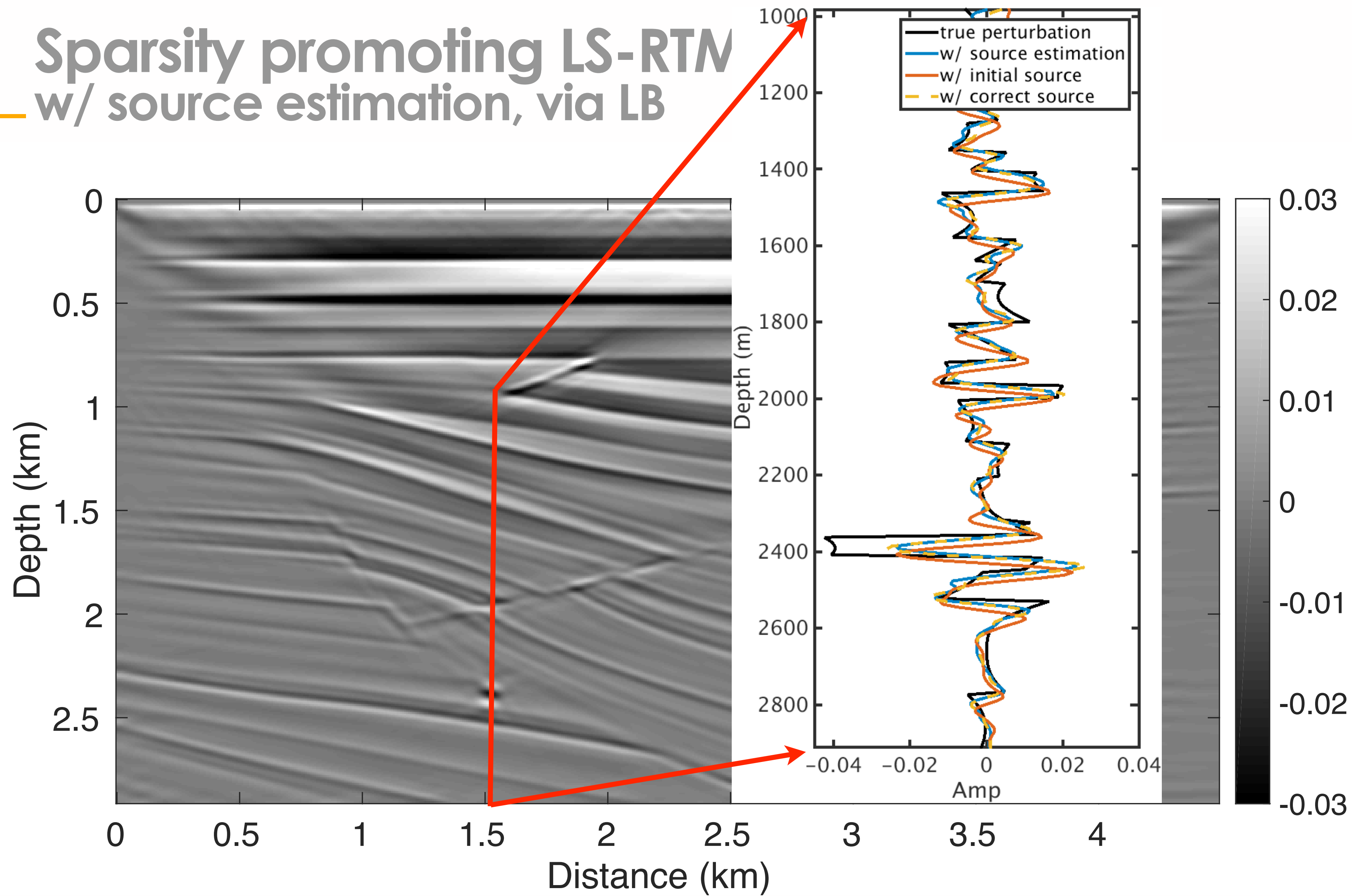
Sparsity promoting LS-RTM w/ wrong wavelet & LB



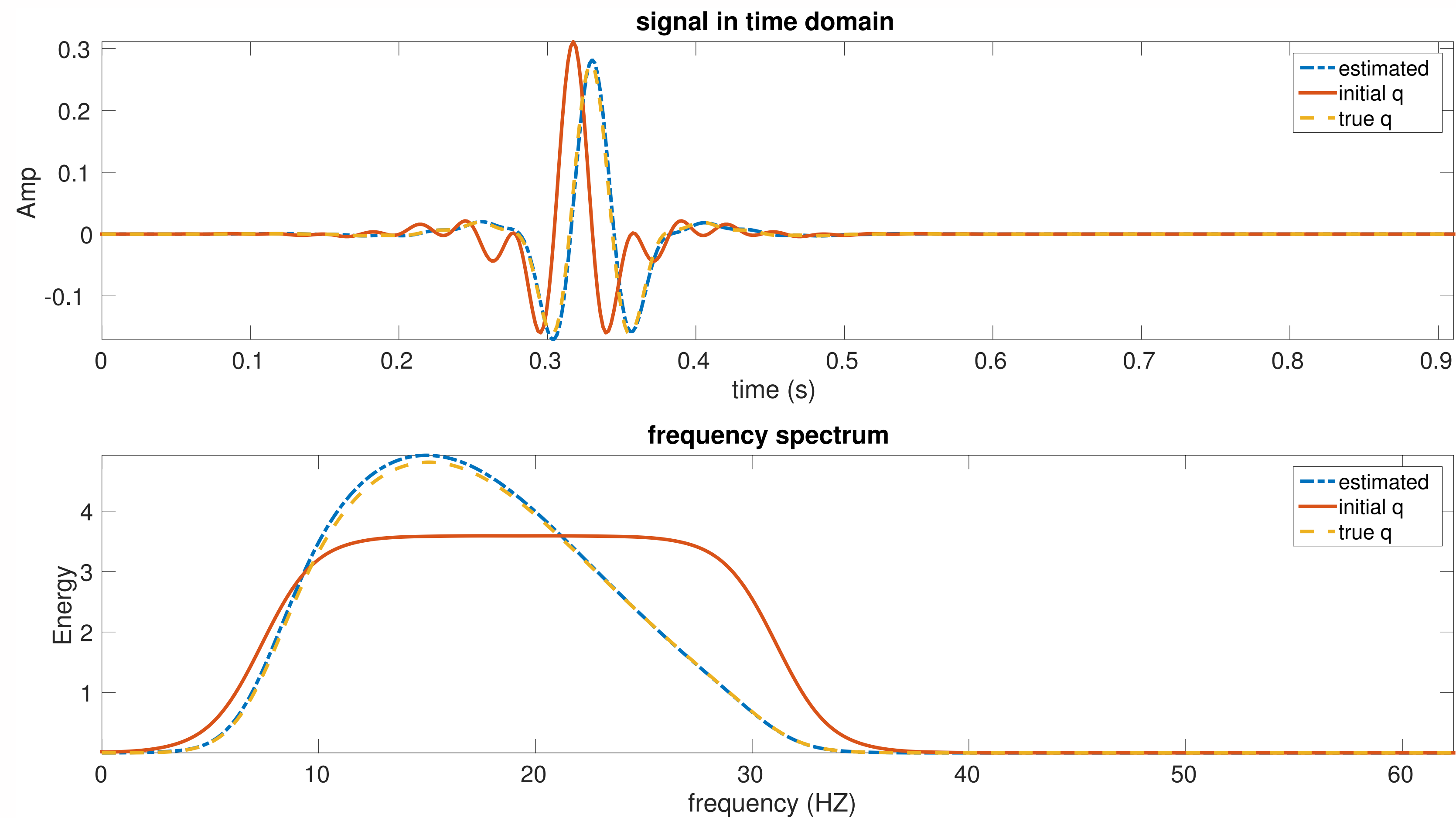
Sparsity promoting LS-RTM w/ source estimation w/ LB



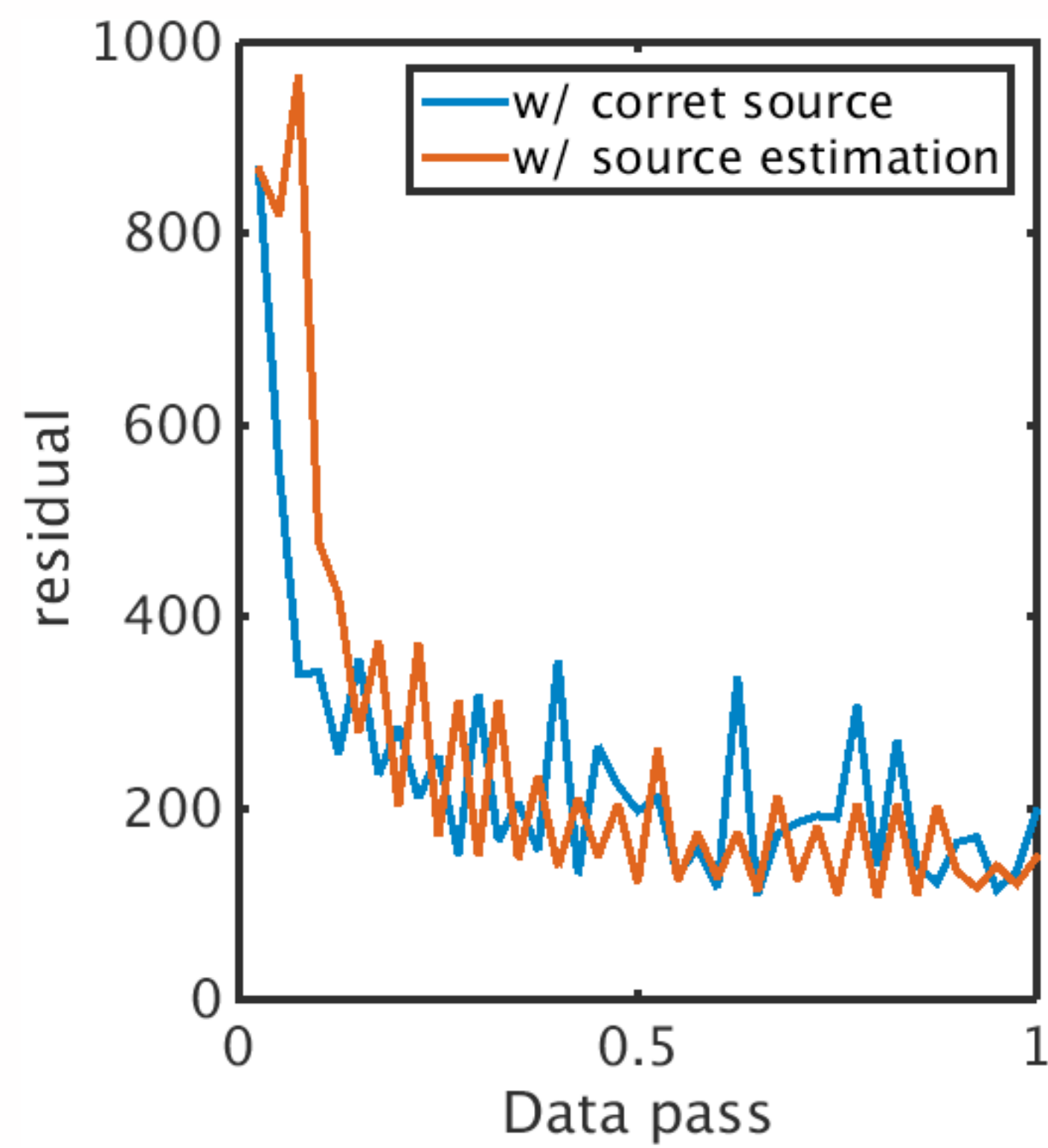
Sparsity promoting LS-RTN w/ source estimation, via LB



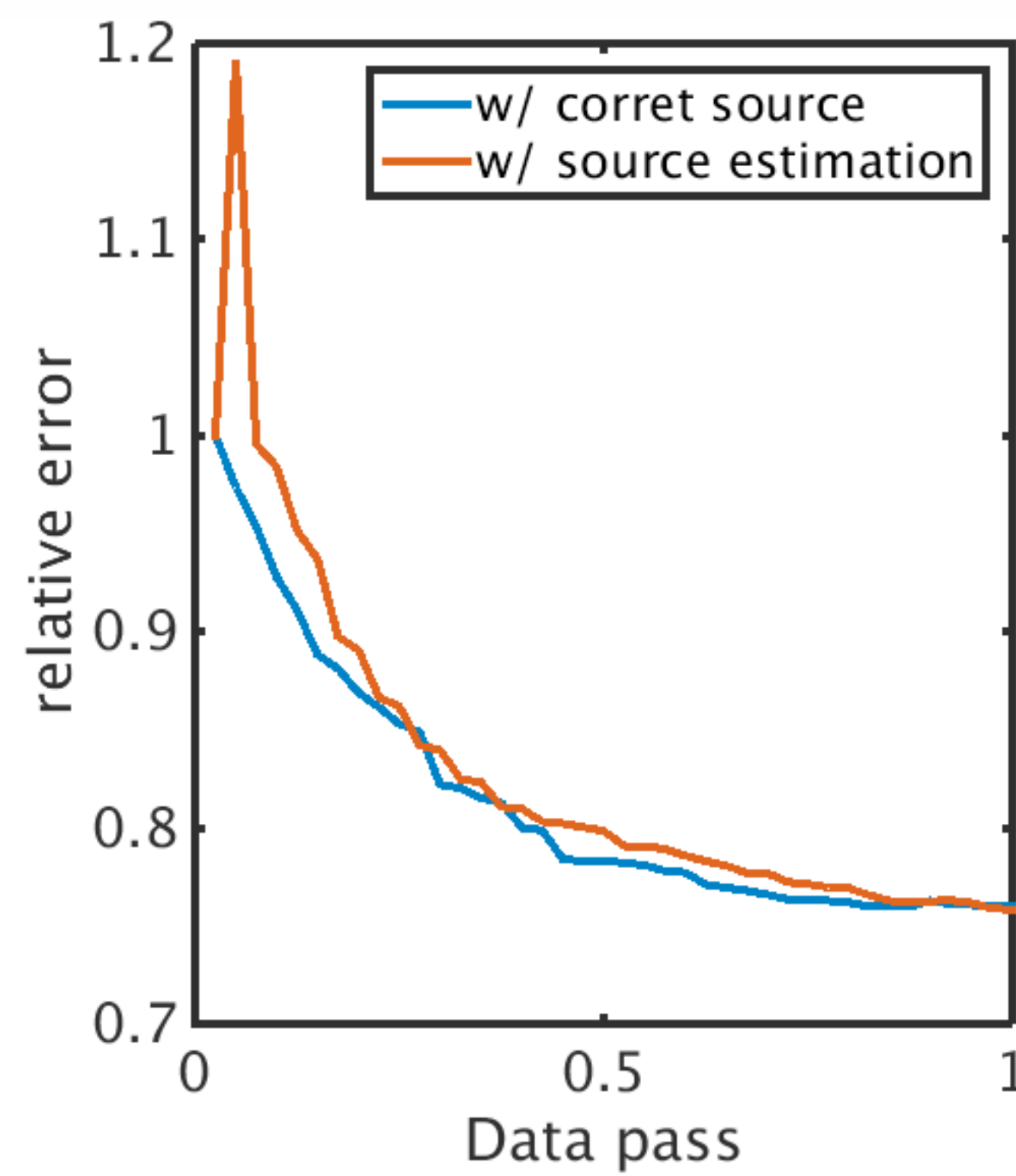
Sparsity promoting LS-RTM w/ source estimation



Residual & model error

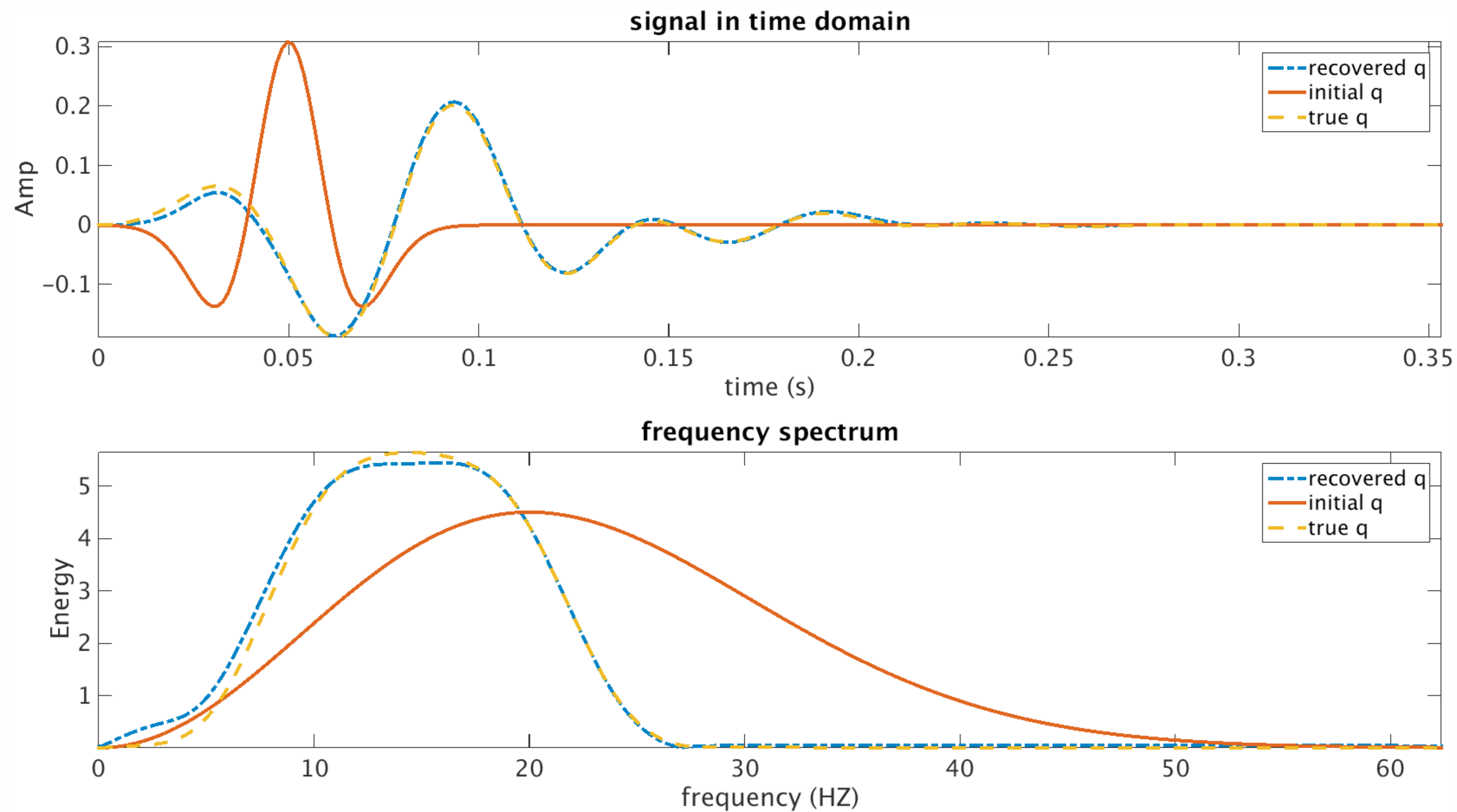


Residuals

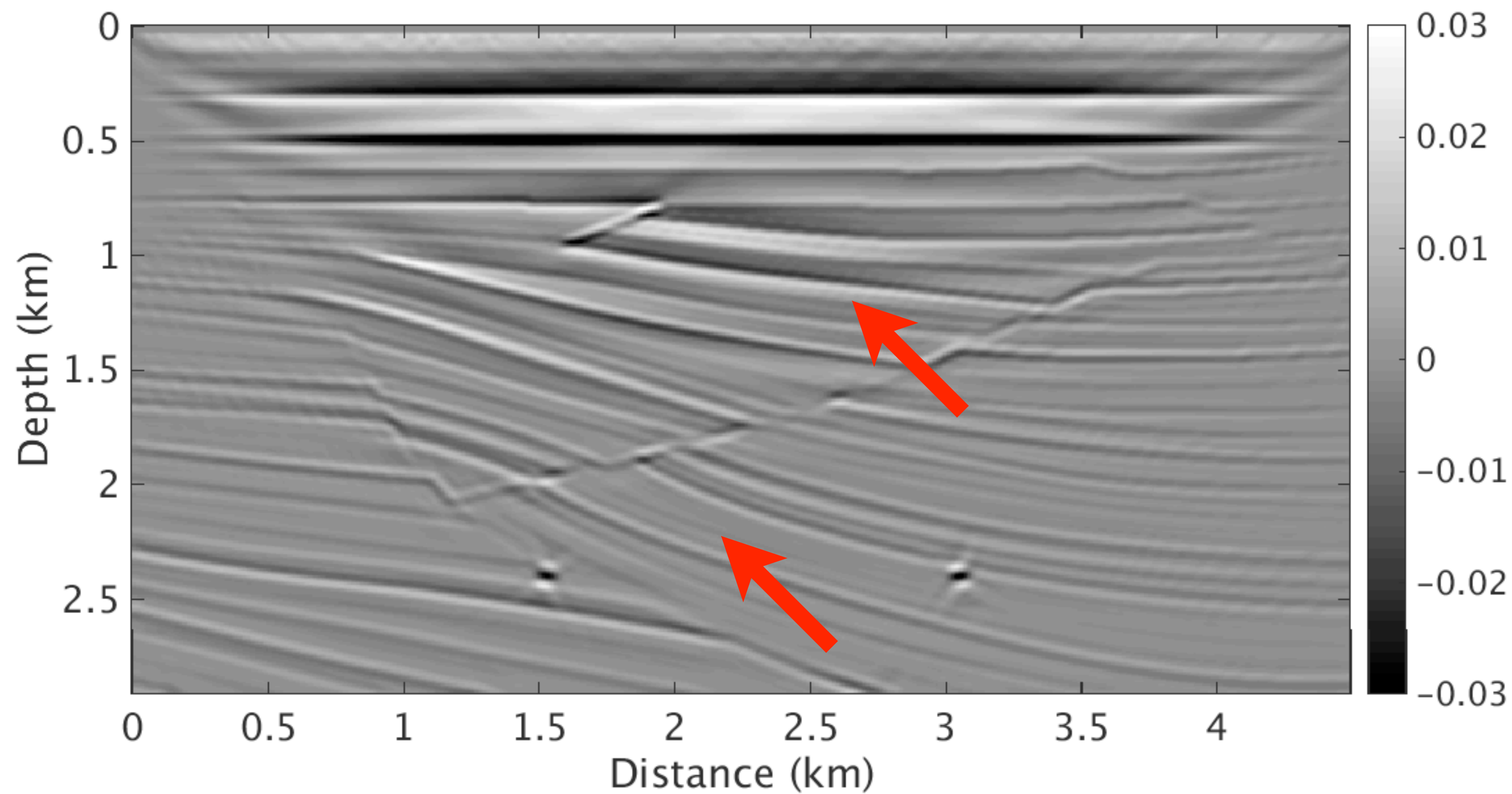


Relative model error

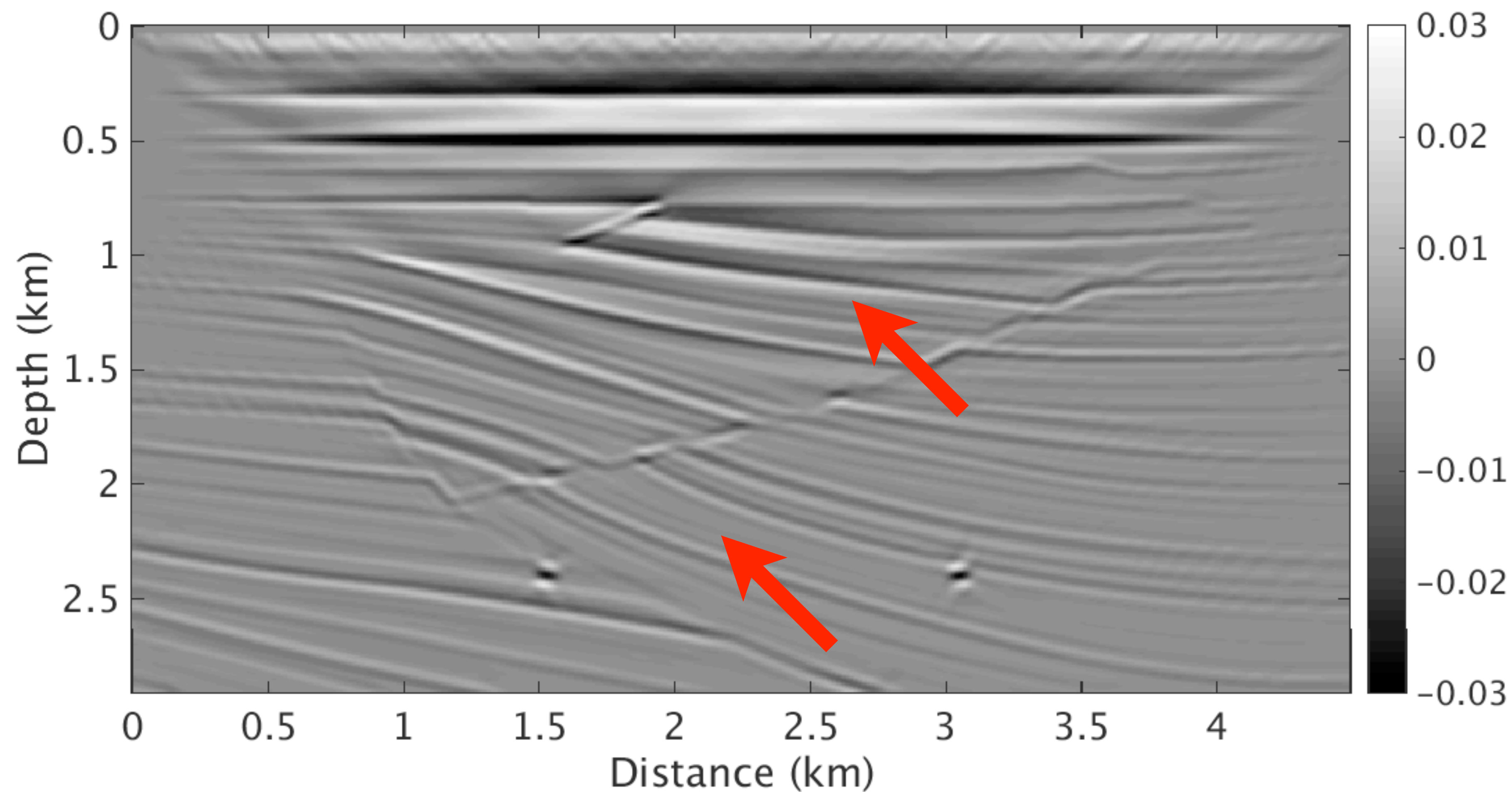
Robustness of source estimation starting w/ zero-phase wavelet



Sparsity promoting LS-RTM w/ correct wavelet & LB



Sparsity promoting LS-RTM w/ source estimation & LB



Conclusions

- LB with correct source signature gives image with sharp interfaces w/ correct amplitudes
- Computational complexity is controlled to ~ 1 RTM w/ randomized source subsampling
- LB improves inversion results compared to other one-norm solvers
- LB can be combined w/ on-the-fly source estimation w/o a large computational overhead

Acknowledgements

This research was carried out as part of the SINBAD project with the support of the member organizations of the SINBAD Consortium.

