Time jittered marine acquisition:
a rank-minimization approach for 5D source separation

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Motivation

How to minimize costs of seismic acquisition?

Solution:

- randomize sampling w/ insights from Compressive Sensing to lower cost

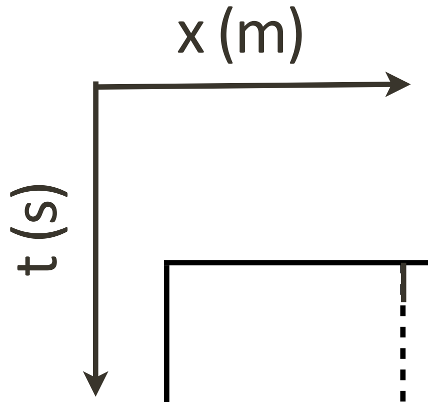
New paradigm:

- give up on dense acquisition
- sample coarsely at random
- works as long as we know where we were in the field

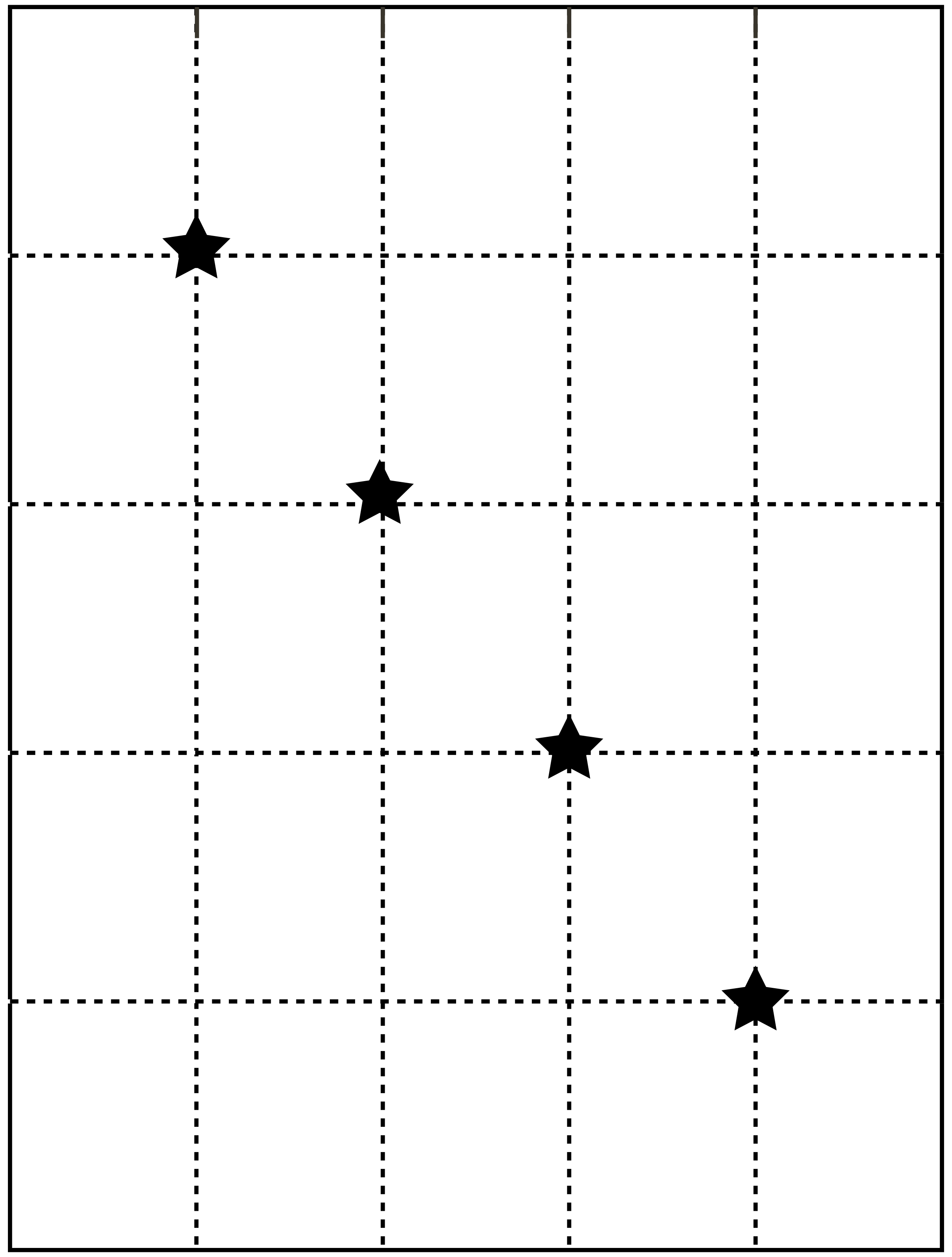
Compressive Sensing = increased acquisition productivity

Compressive time-lapse marine acquisition
W-13: Low cost geophysics: How to be creative in a cost-challenged environment

Randomized jitter sampling in marine

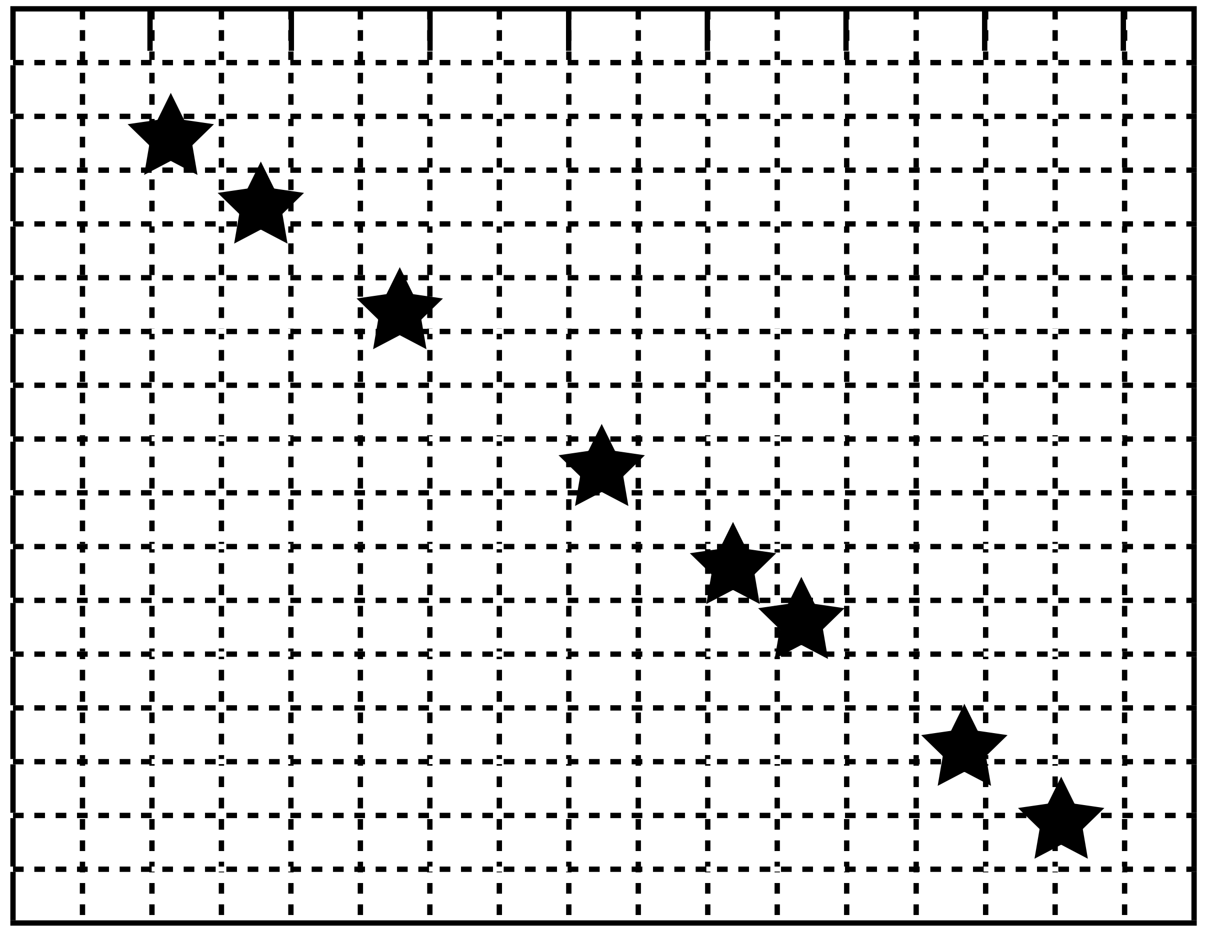


conventional



periodic–sparse–no overlap

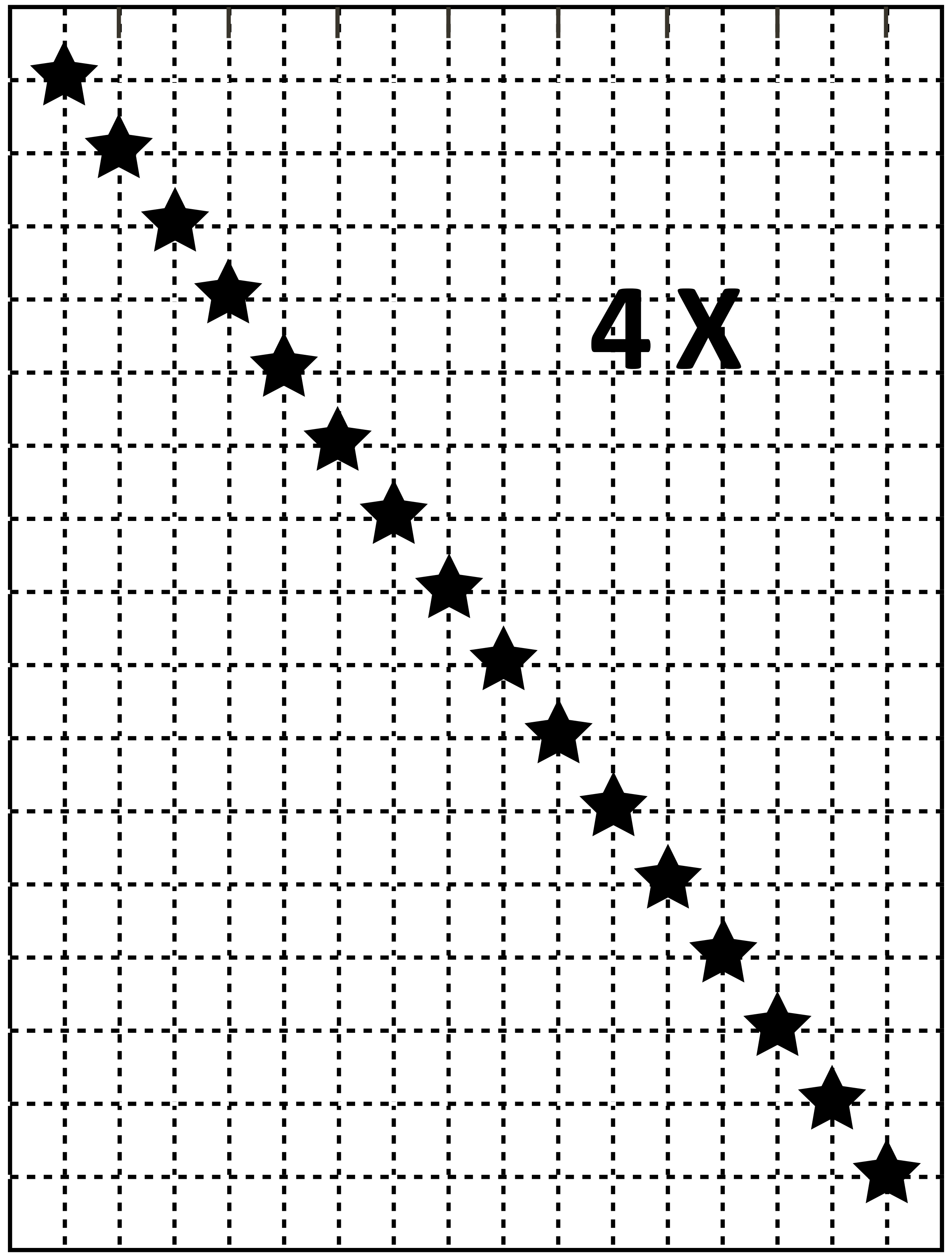
jittered



aperiodic
compressed
overlapping
irregular

separation + regularization
+ interpolation

(no overlap)
rank-minimization recovery



periodic & dense

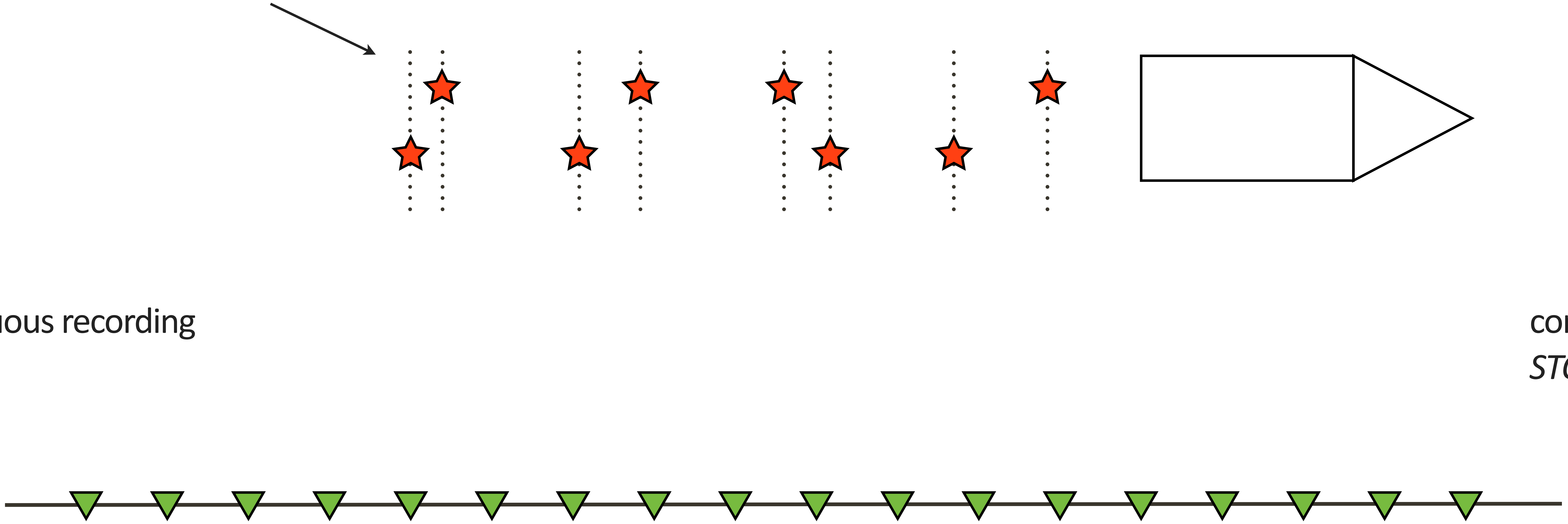
source sampling : 25 m (flip-flop) | 25 m (flip-flop) jittered | 6.25 m

Economical 3D OBN acquisition

Observed grid (m)	Recovered grid (m)	Subsampling %	Economical gain
25	12.5	50	**2X**
25	6.25	75	**4X**
25	3.125	90	**8X - 9X**

Time-jittered acquisition

regularly sampled spatial grid



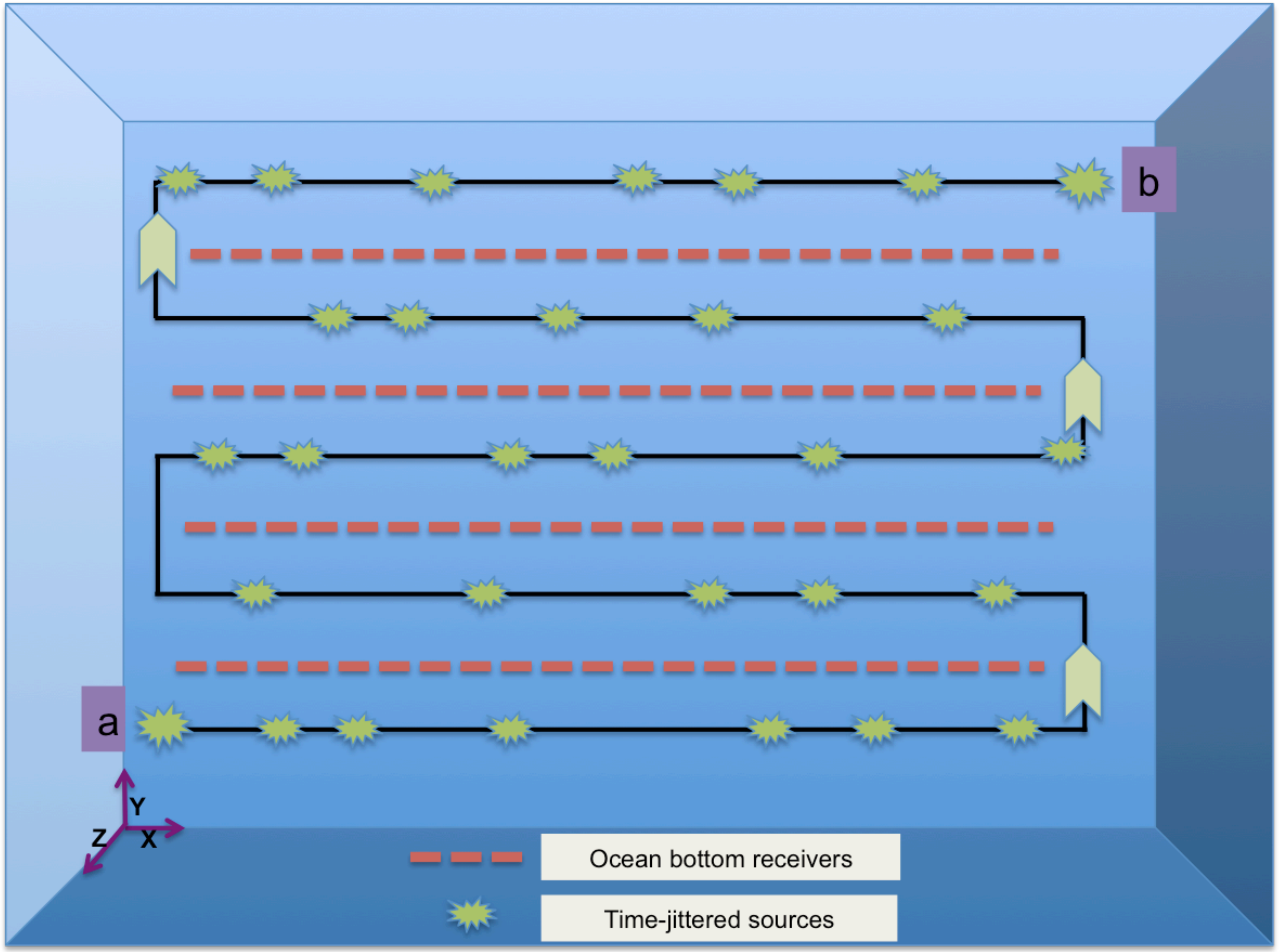
continuous recording
START

continuous recording
STOP

OBC / OBN

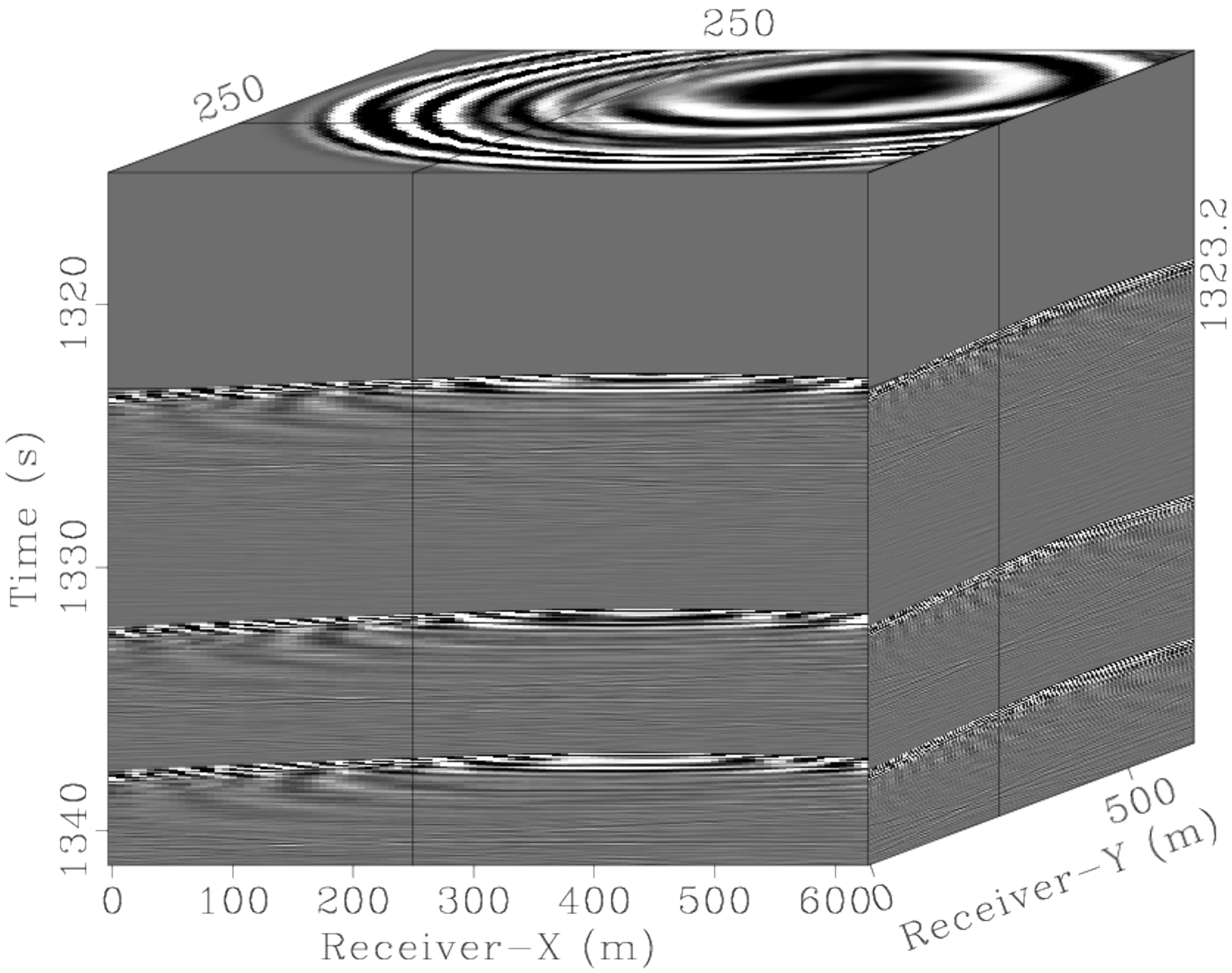
Acquisition setup

speed of source vessel = 5 knots ~ 2.5 m/s



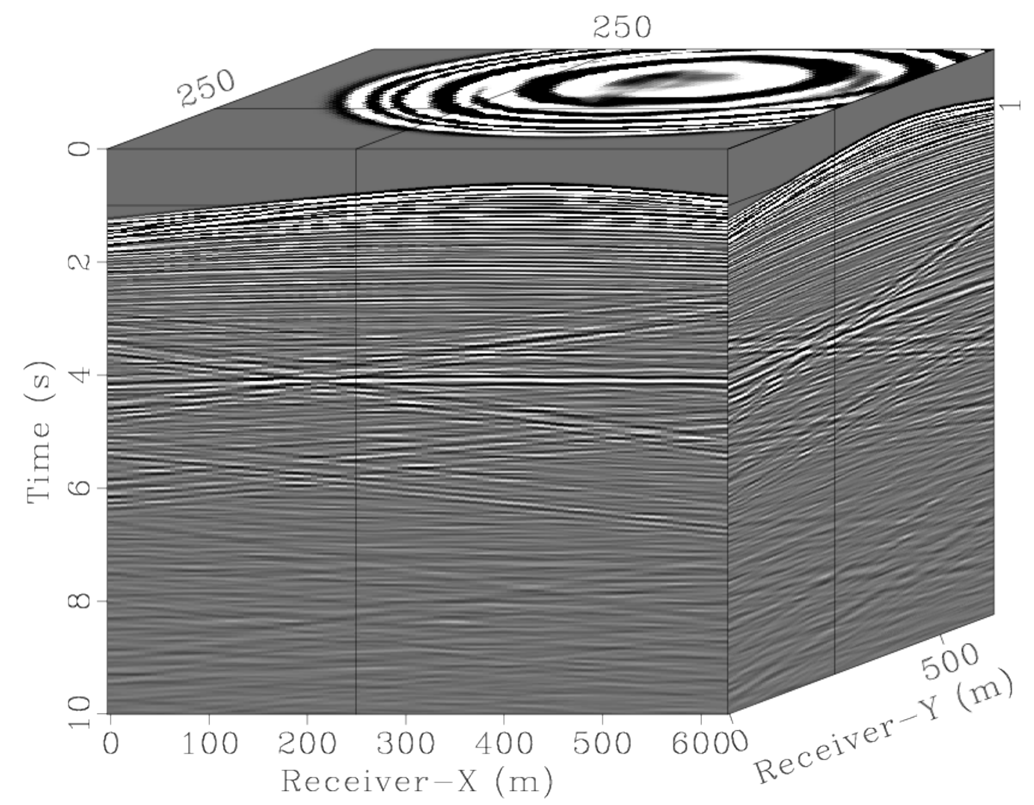
Observed v/s recovered

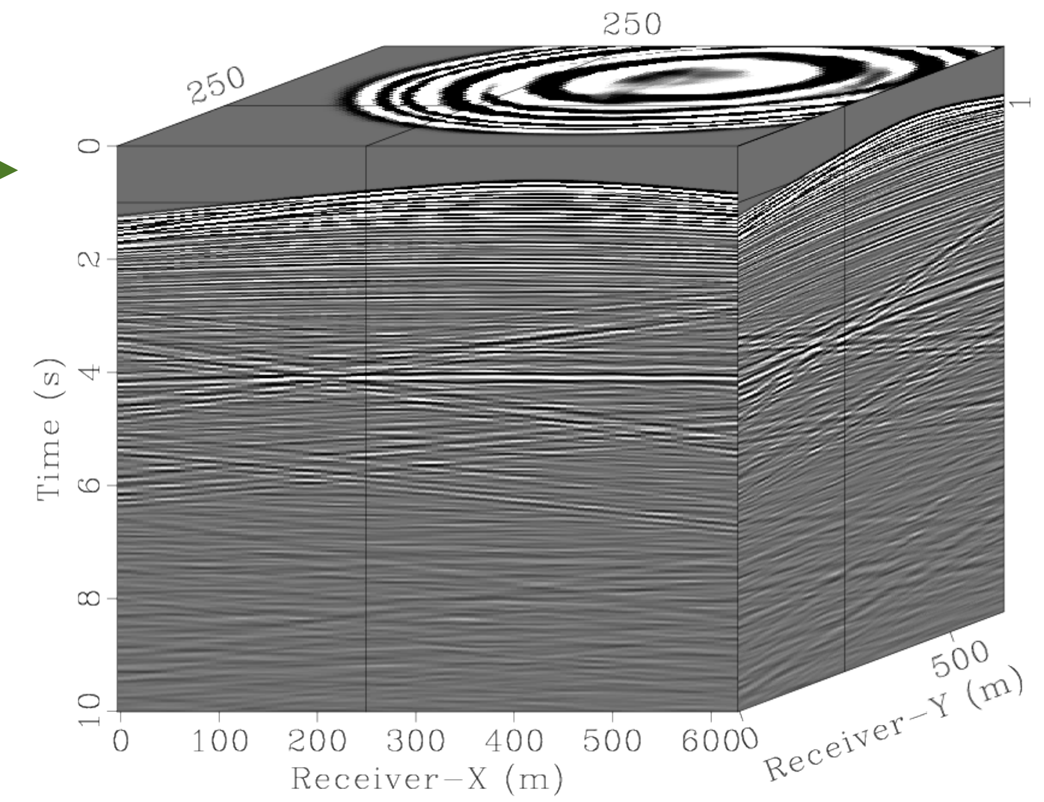
Observed data @ 25 m flip-flop
(overlapping & missing shots)

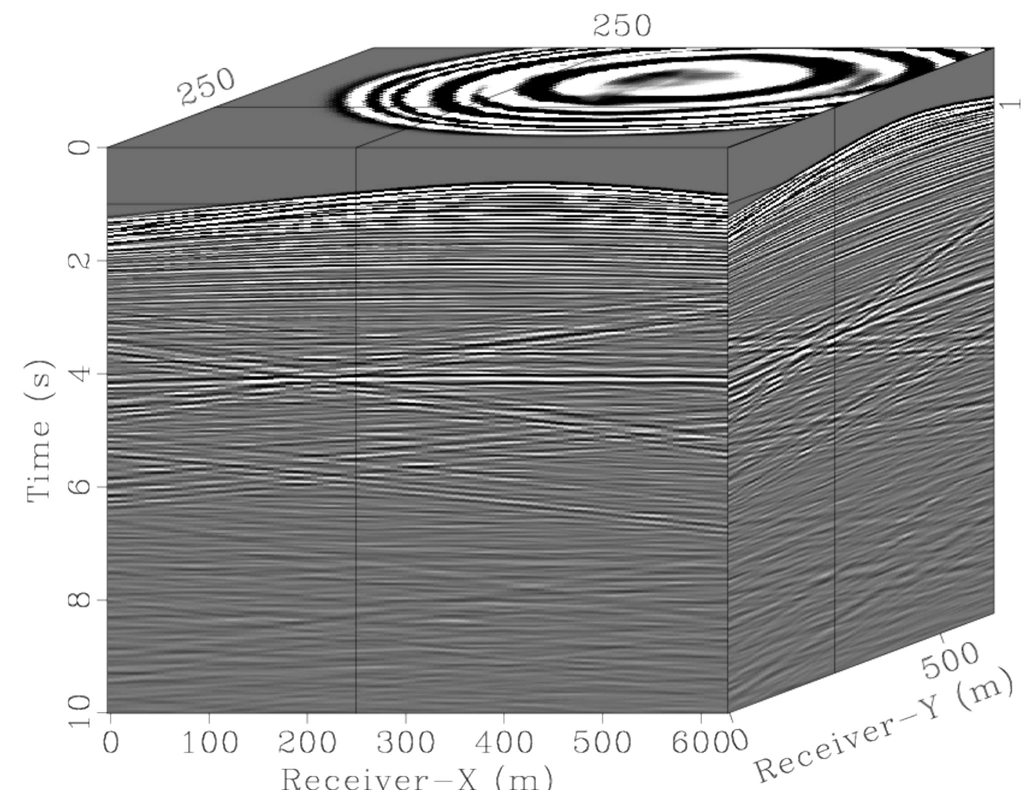


Recovery

Separation + Interpolation
(recovered grid @ 6.25m)







Methodology

[Candès and Plan, 2009]

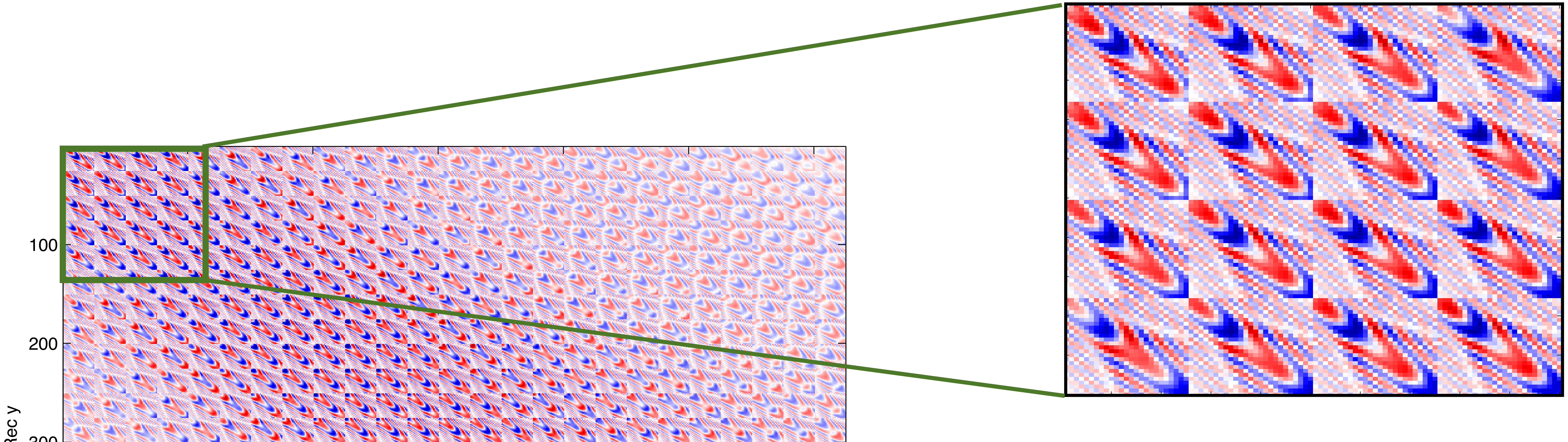
Matrix completion

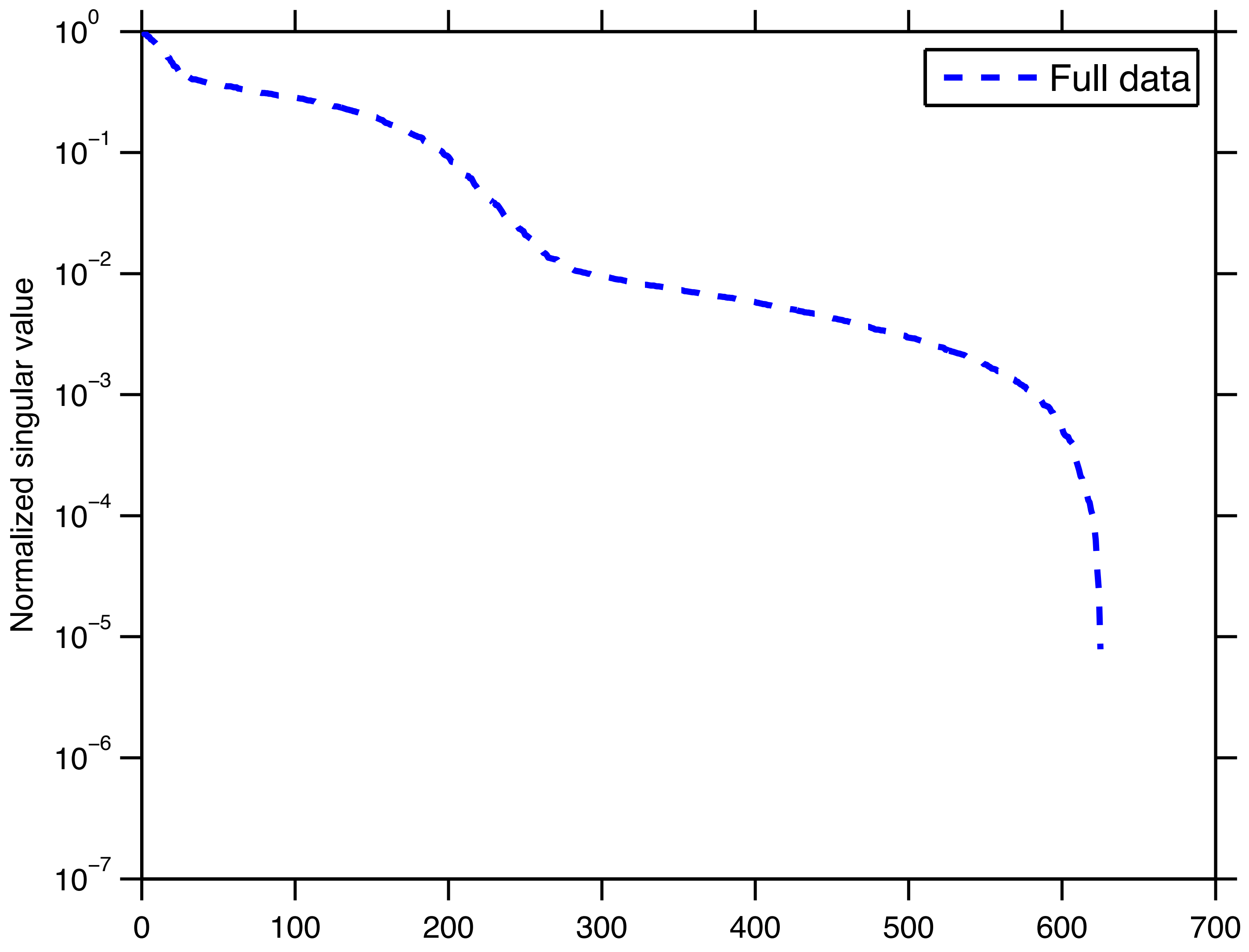
Successful reconstruction scheme

- exploit *structure*
  - *low-rank* / *fast decay* of singular values
- sampling
  - randomness *increases* rank in "transform domain"
- optimization
  - via *rank-minimization (nuclear norm-minimization)*

Low-rank structure

conventional 5D data, monochromatic slice, Sx-Sy matricization





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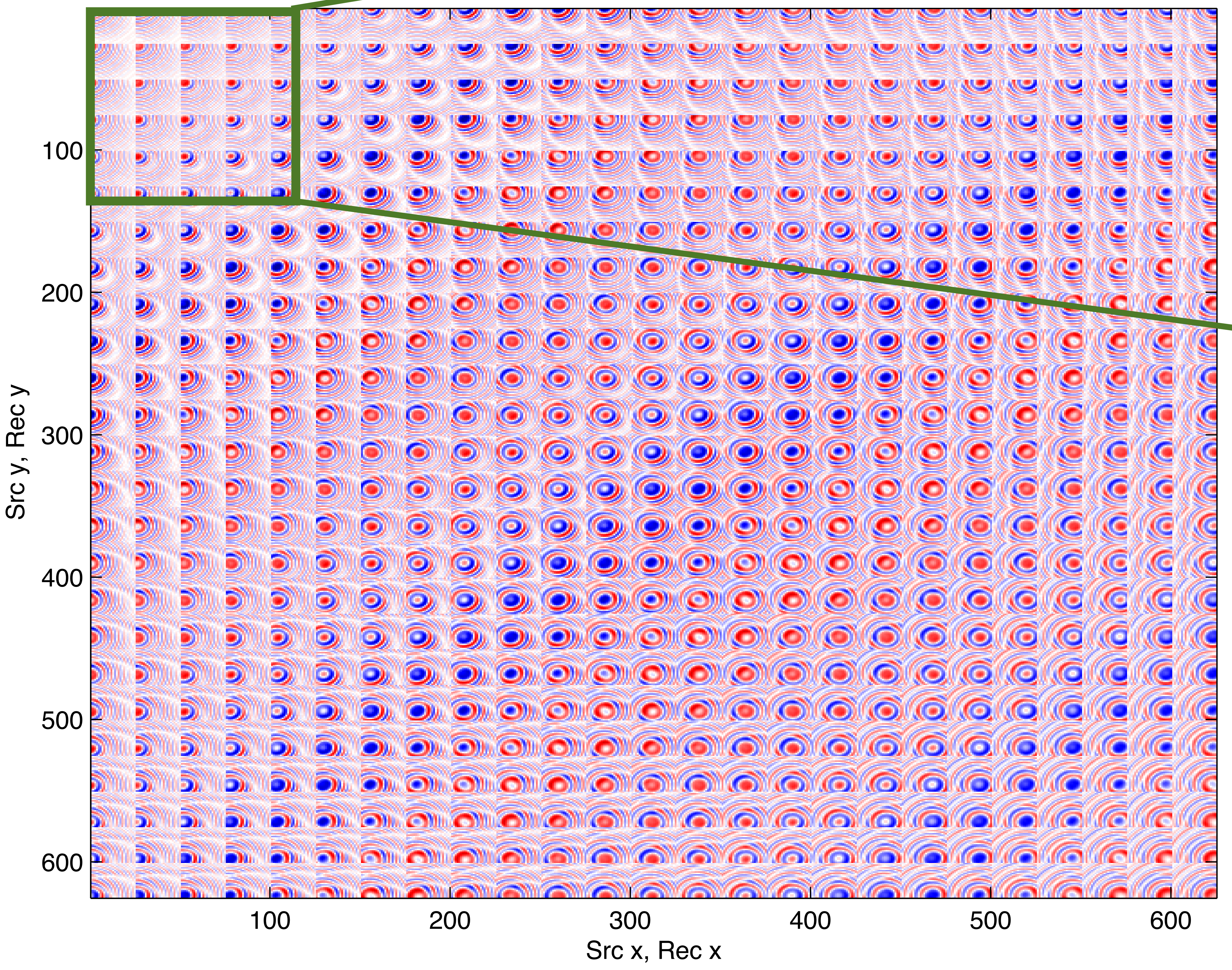

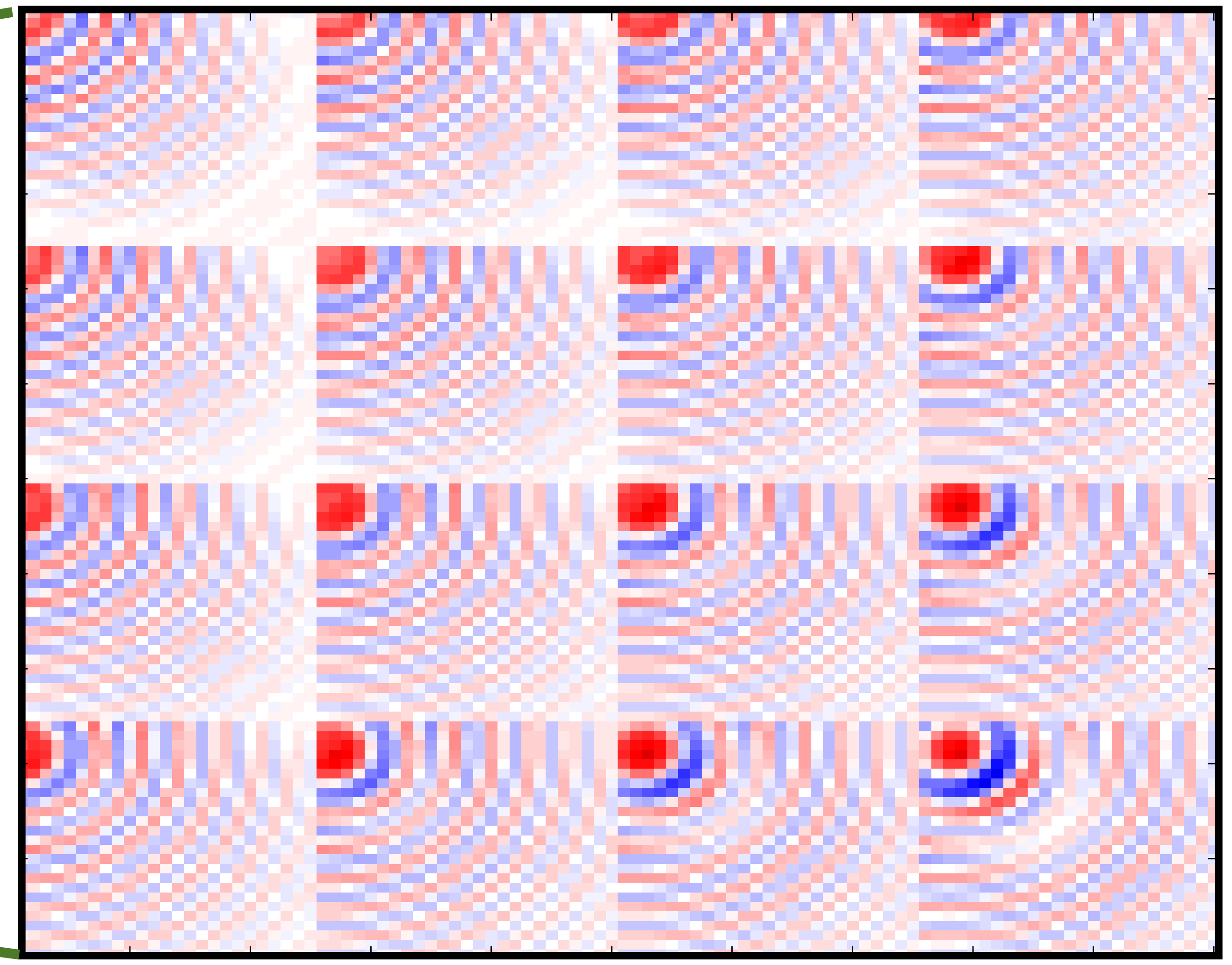

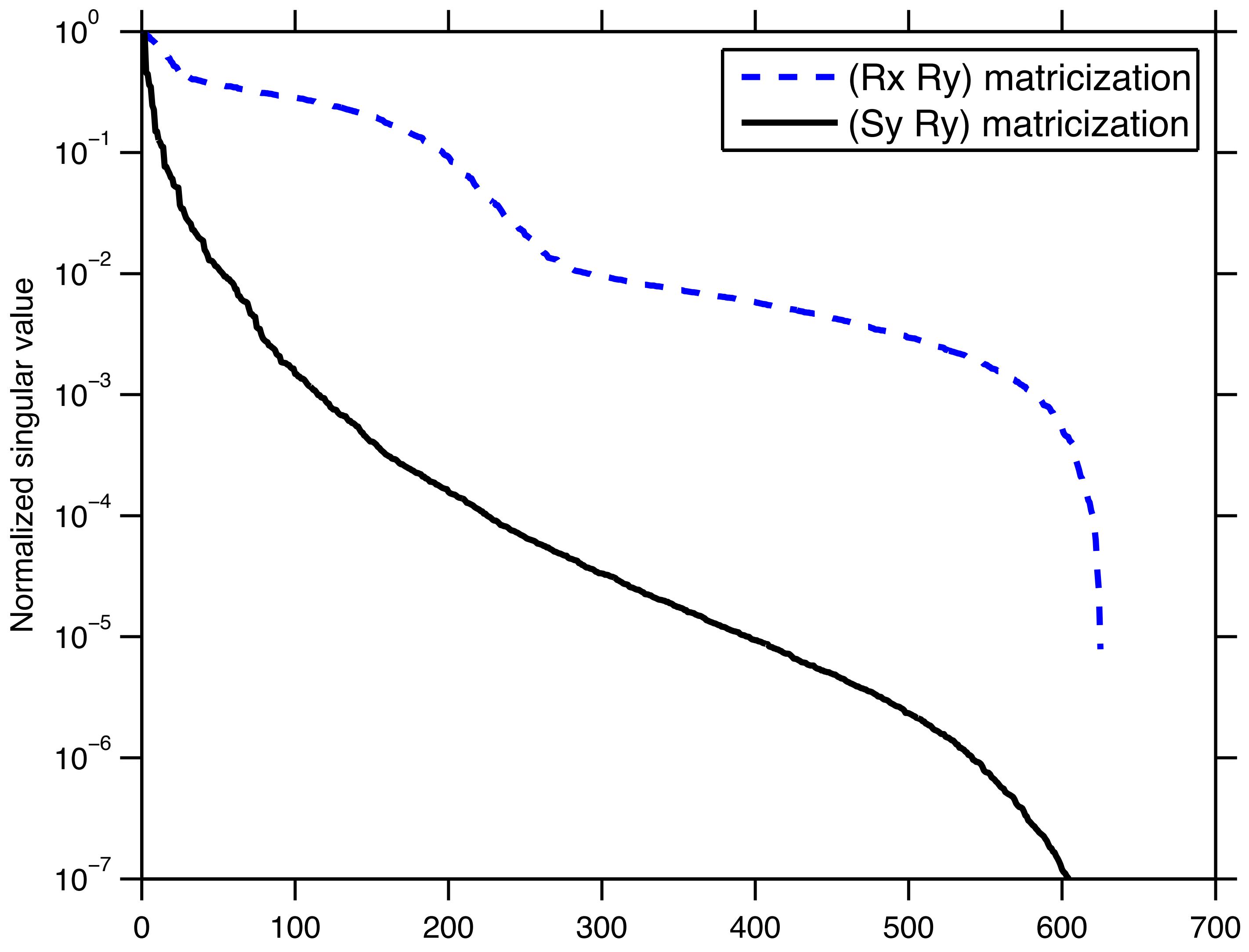

[Candès and Plan, 2009]

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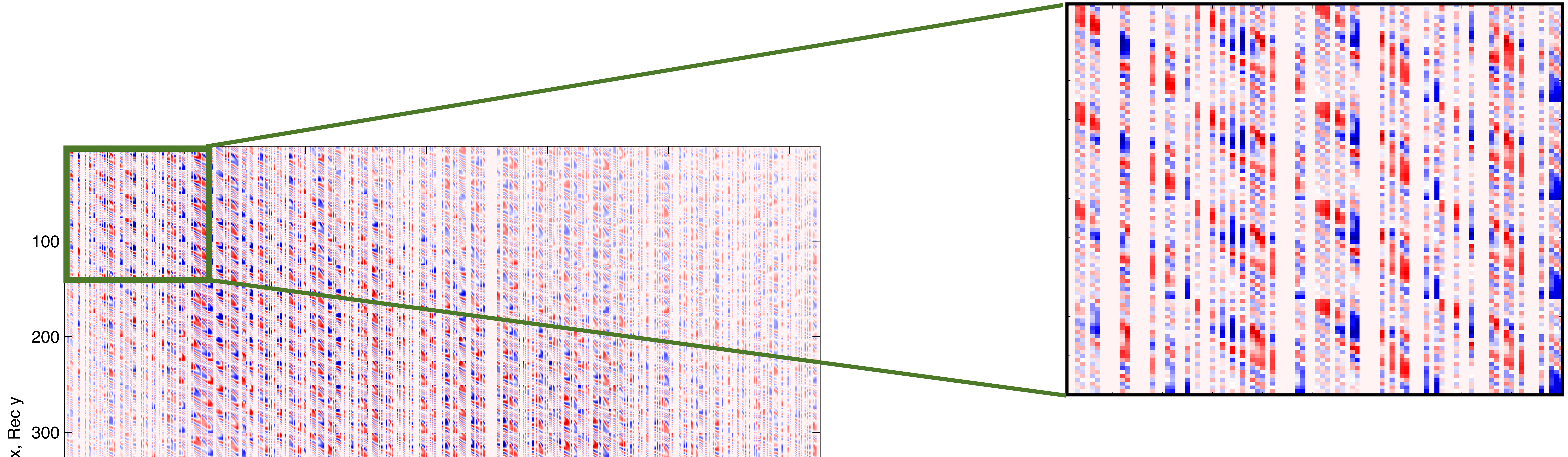
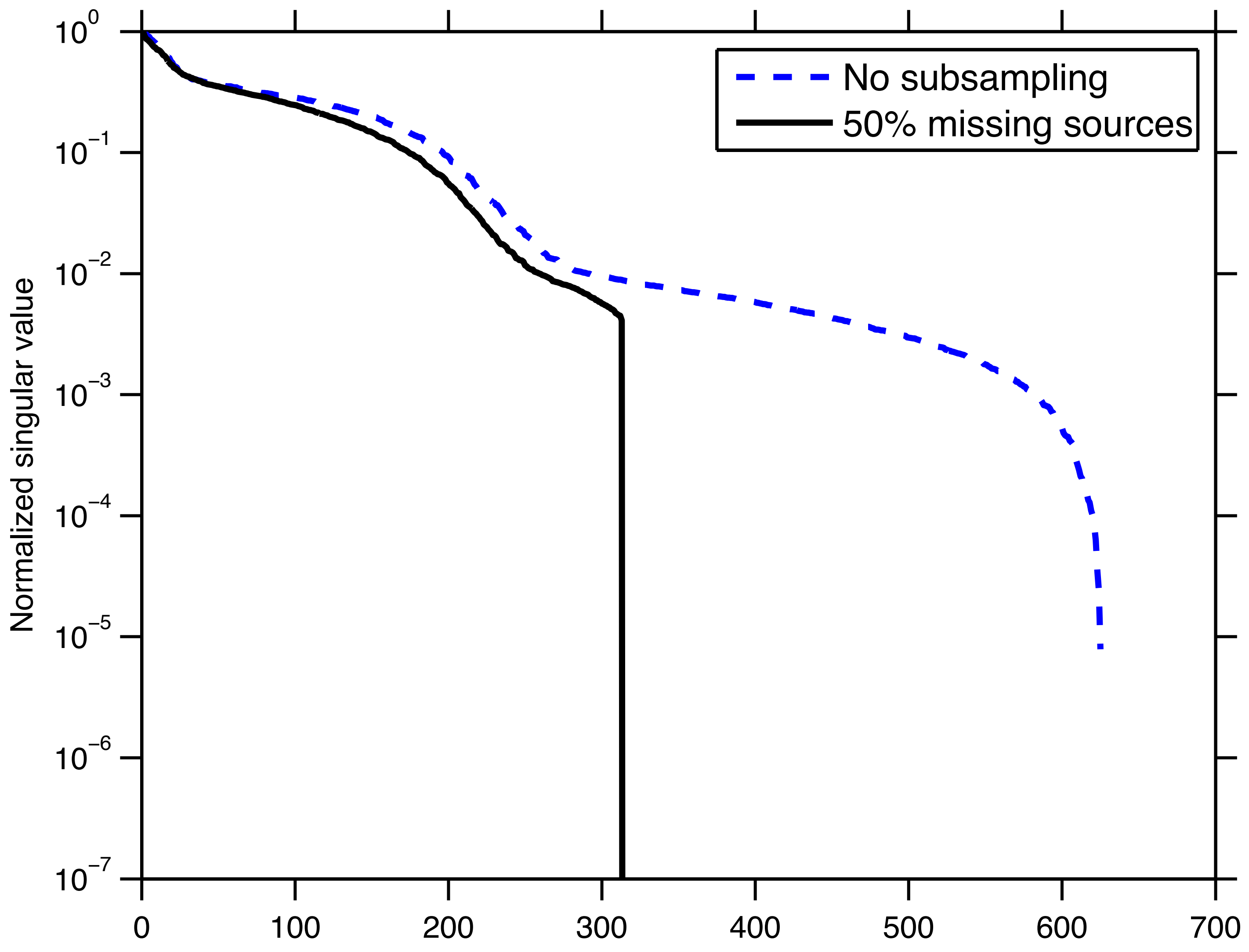
Low-rank structure

time-jittered data, monochromatic slice, Sx-Sy matricization





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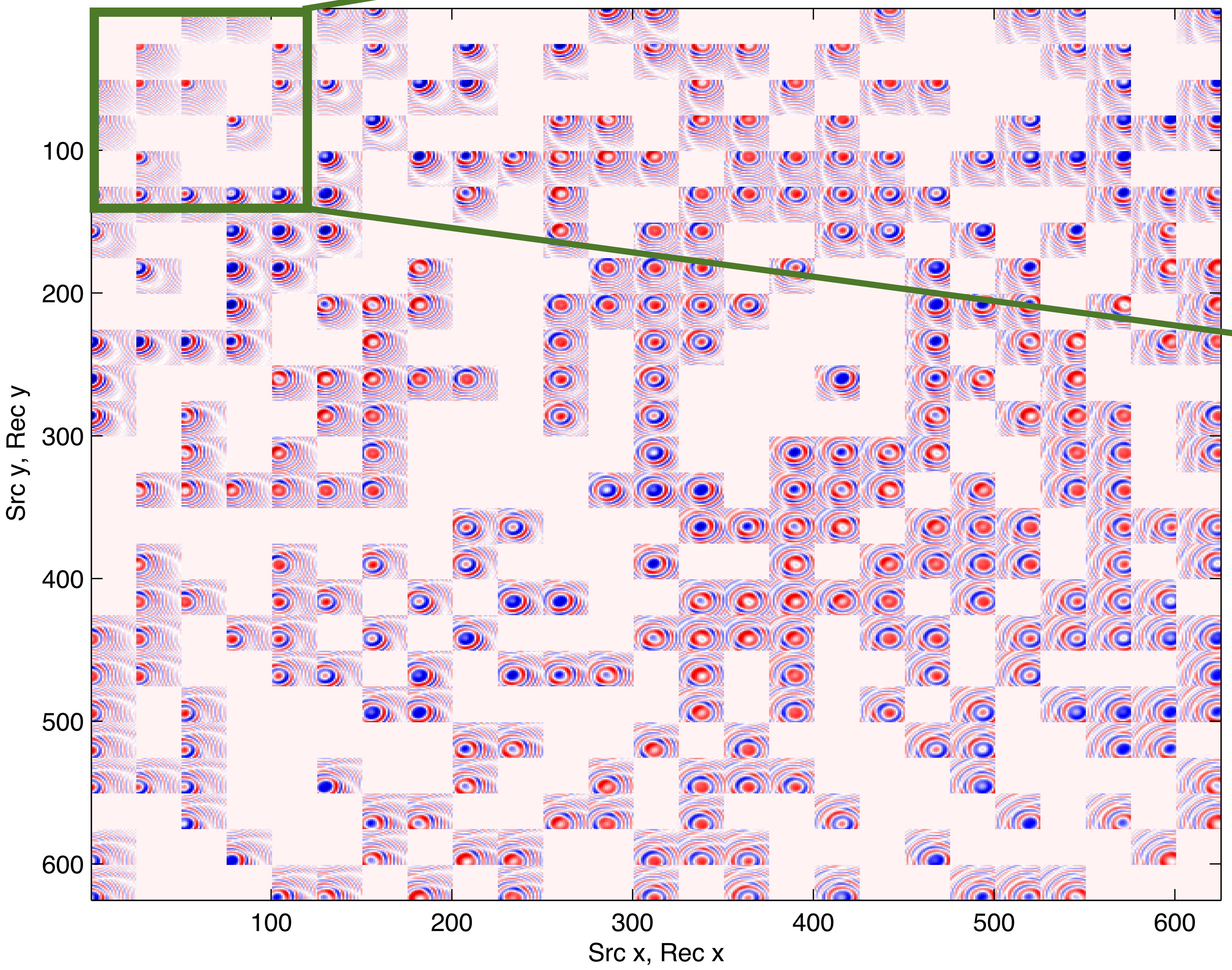
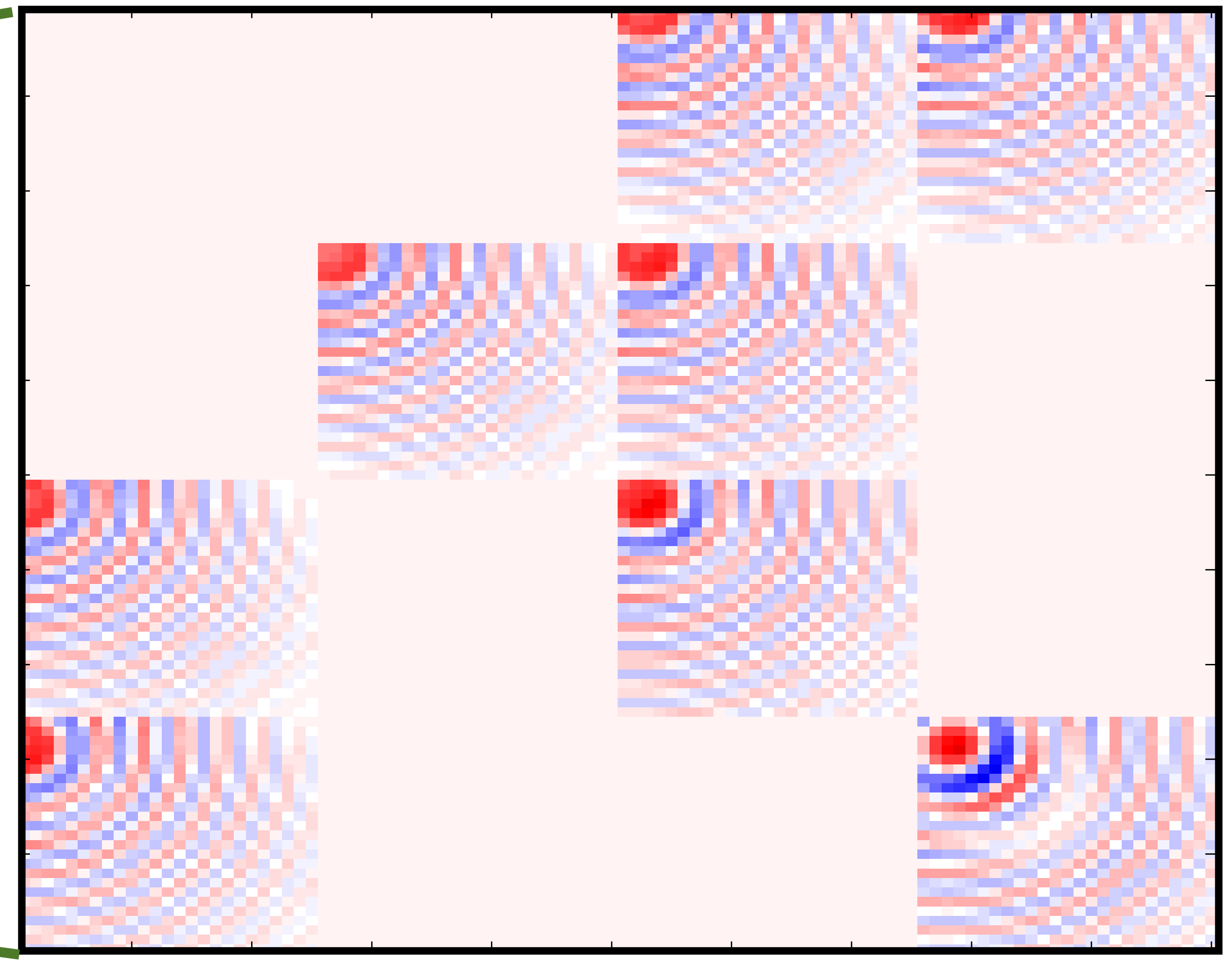
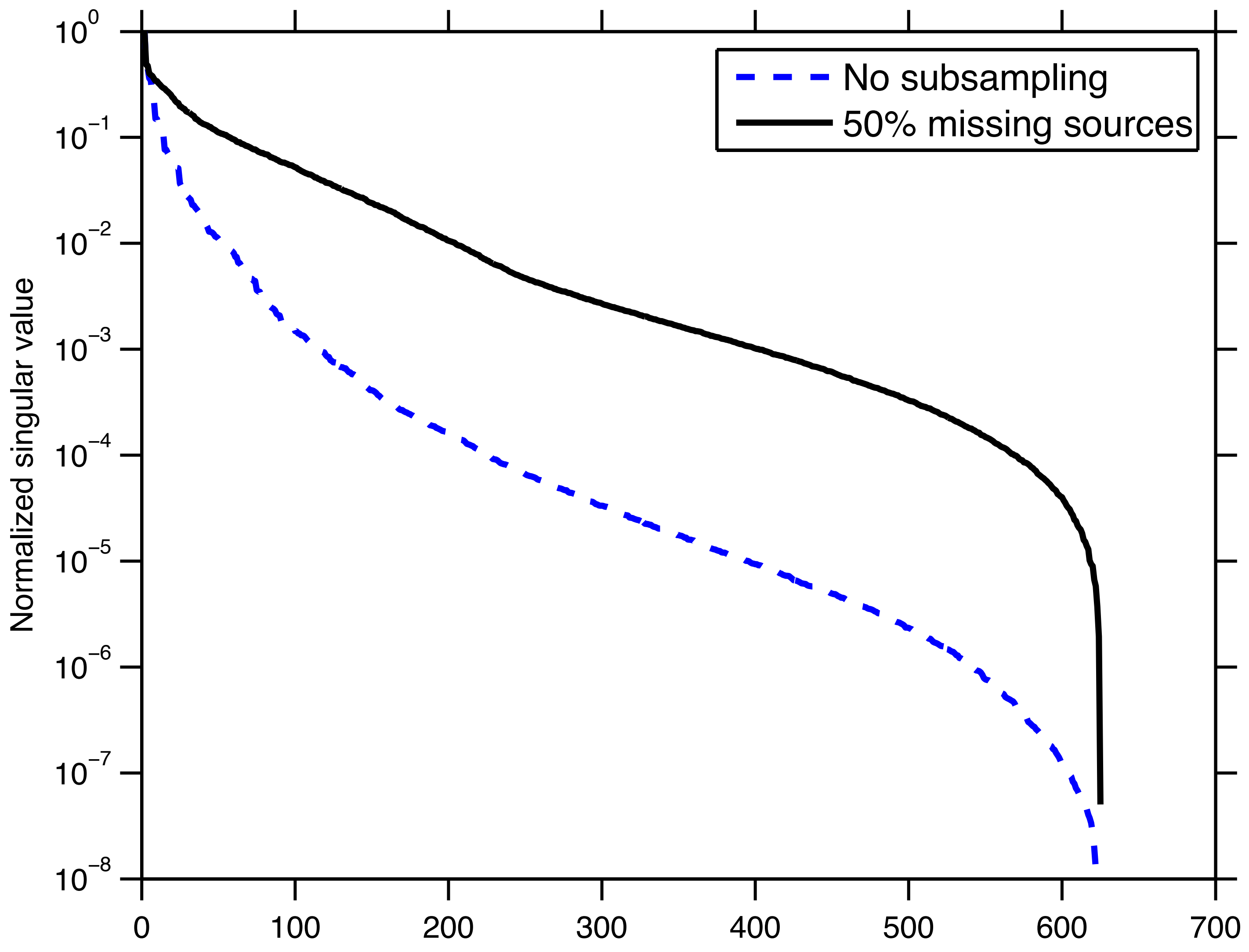
time-jittered data, monochromatic slice, Sx-Rx matricization







[Candès and Plan, 2009]

Matrix completion

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Rank minimization

expensive
(search over all possible values of rank)

$$\min_{\mathbf{X}} \quad \underbrace{\mathrm{rank}(\mathbf{X})} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

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Rank minimization

expensive
(search over all possible values of rank)

$$\min_{\mathbf{X}} \quad \underbrace{\operatorname{rank}(\mathbf{X})}_{} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

Nuclear-norm minimization

convex relaxation of rank-minimization

[Recht et. al., 2010]

$$\min_{\mathbf{X}} \quad \underbrace{||\mathbf{X}||_*}_{} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

sum of singular values of $\mathbf{X}$

Matrix-Completion framework

- Restriction operator is constant across frequencies
- Perform matrix-completion across frequencies in parallel

5D Jittered marine acquisition

- Restriction operator is non-separable
  - combination of time-shifting and shot-jittered operator

- Can’t perform matrix-completion over independent frequencies
  - reformulate nuclear-norm minimization over temporal-frequency domain

Rank-minimization problem

- Let $\mathbf{X} \in \mathbb{C}^{\mathbf{n_f} \times \mathbf{n_{rx}} \times \mathbf{n_{sx}} \times \mathbf{n_{ry}} \times \mathbf{n_{sy}}}$ be the conventional 5D seismic data volume represented as a tensor.

- Given a set of measurements $\mathbf{b}$, aim is to solve

$$\min_{\mathbf{X}_f} \sum_f \|\mathbf{X}_f\|_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{X}_f) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}_f\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

Sampling-measurement operator

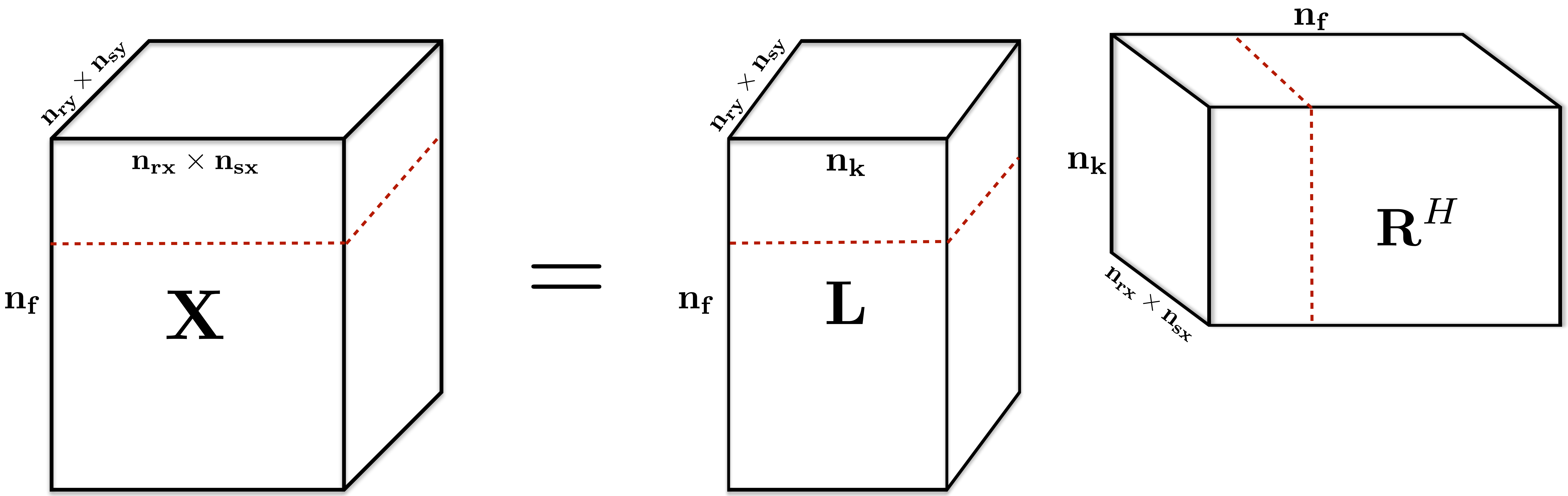
- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A}(.) = \mathbf{M}\mathbf{F}^H\mathcal{S}^H(.)$$

$\mathbf{M}$	time-jittered operator
$\mathbf{F}^H$	inverse Fourier transform along frequency axis
$\mathcal{S}^H$	rank-revealing transform domain

Factorized formulation

$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$



$\mathbf{X} \in \mathbb{C}^{\mathbf{n_f} \times \mathbf{n_{rx}} \times \mathbf{n_{sx}} \times \mathbf{n_{ry}} \times \mathbf{n_{sy}}}$ $\mathbf{L} \in \mathbb{C}^{\mathbf{n_f} \times \mathbf{n_{rx}} \times \mathbf{n_{sx}} \times \mathbf{n_k}}$ $\mathbf{R} \in \mathbb{C}^{\mathbf{n_f} \times \mathbf{n_{ry}} \times \mathbf{n_{sy}} \times \mathbf{n_k}}$

Factorized formulation

- Costly SVD's

- Nuclear norm satisfies

$$\sum_{j}^{n_f} \|\mathbf{D}_{\mathbf{j}}^{(\mathbf{i})}\|_* \leq \sum_{\mathbf{j}}^{\mathbf{n_f}} \frac{\mathbf{1}}{\mathbf{2}} \|\mathbf{L}_{\mathbf{j}}^{(\mathbf{i})} \mathbf{R}_{\mathbf{j}}^{(\mathbf{i})}\|_{\mathbf{F}}^{\mathbf{2}}$$

[Rennie and Srebro 2005]

where $\|\cdot\|_F^2$ is sum of squares of all entries

- Choose rank $\mathbf{k}$ *explicitly* & avoid costly *SVD's*

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How to choose the rank parameter?

Typical abridged result from low-rank matrix recovery theory:

If $\mathcal{A} : \mathbb{C}^{n\times m} \mapsto \mathbb{C}^k$ is a random linear operator (e.g., Ω chosen randomly, subgaussian), then we can recover a rank-r matrix via nuclear norm minimization if

$$k \geq Cr\max(n,m)\log(\max(n,m))$$

[Candes and tao 2009]

with high probability.

How to choose the rank parameter?

$$k \geq Cr \max(n,m) \log(\max(n,m))$$

In our case: $k = .25 \cdot nm$, where 0.25 is subsampling ratio,

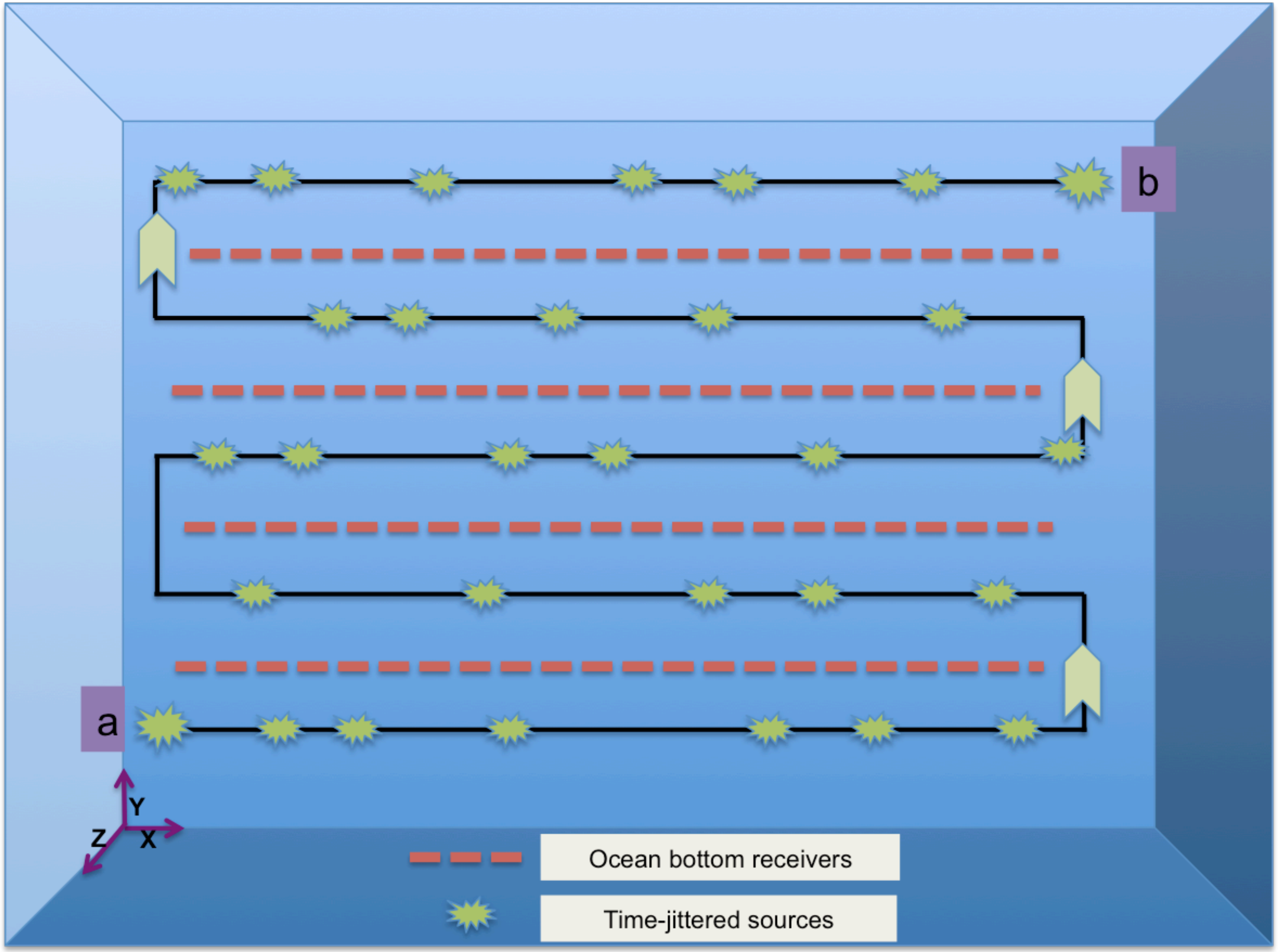
$$n = m = 4141$$

(with $C = 1$ and rounding) $\Longrightarrow r \leq 100$

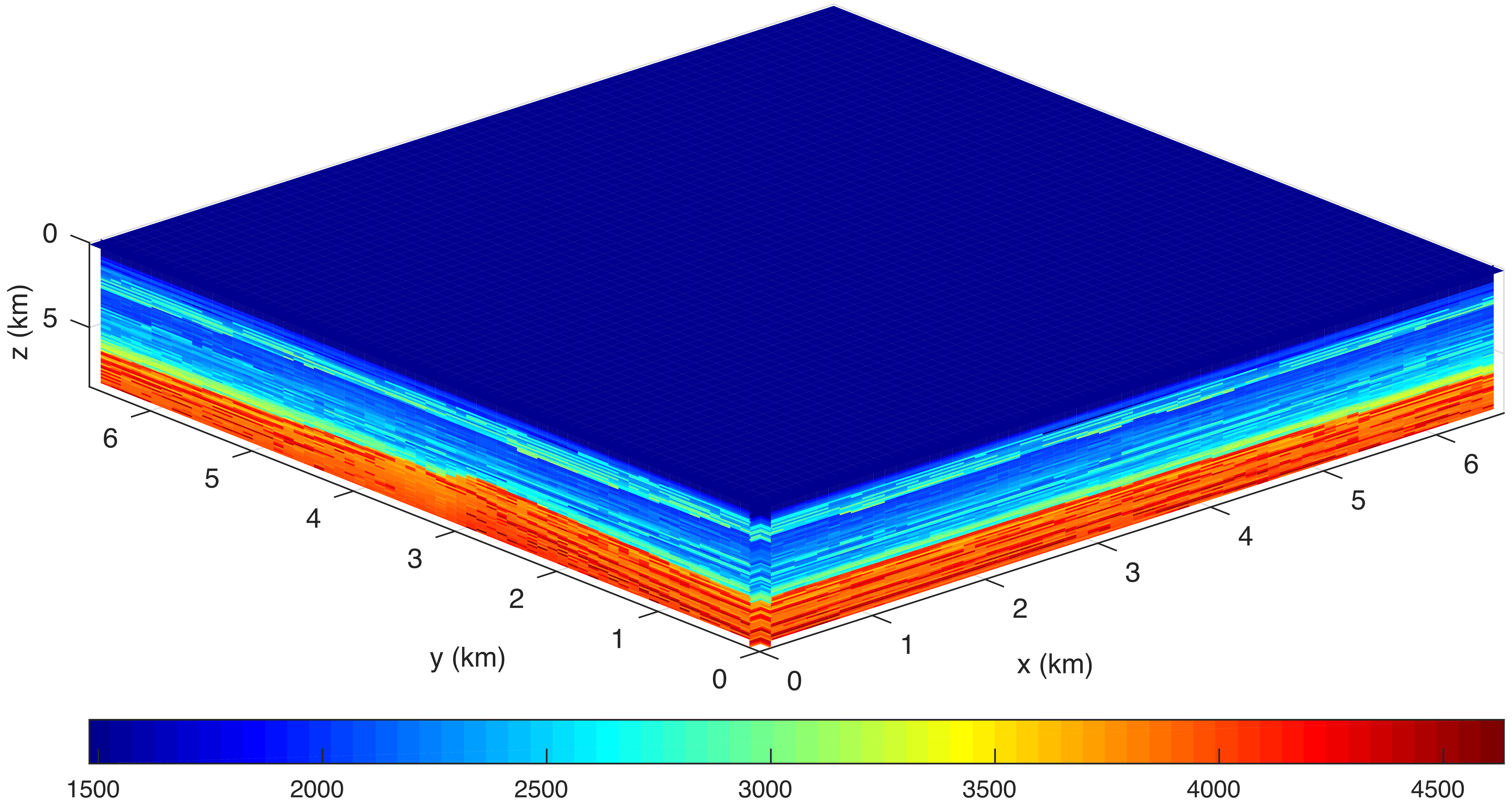
Choose upper bound as rank.

Experimental results

Acquisition setup



3D BG Compass model



Acquisition information

- 10s temporal length
- 25 m flip-flop shooting
  - source-sampling ranges from 25 m to 175 m
  - effective 50 m source sampling for each airgun array
  - acquired 400 sources
- 10201 receivers
- Ricker wavelet with central frequency of 20 Hz
- size of the recovered 5D seismic data volume is 0.5 TB

Optimization information

- Parallelized factorization framework over sources and receivers
- 200 iterations, computational time 42 hours
- Separation + interpolation @ 6.25 m grid
  - recovered 1600 sources

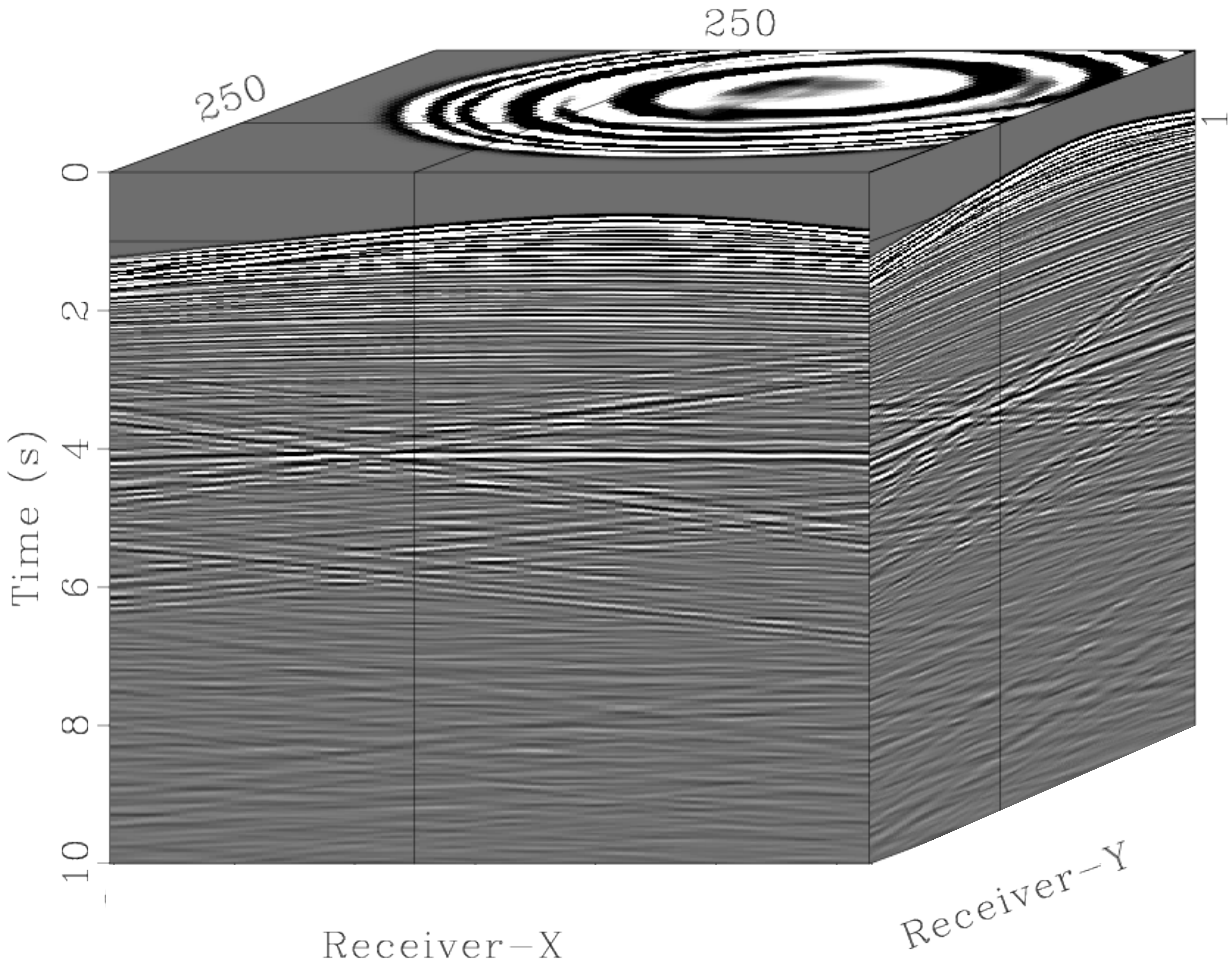
Computational Environment

SENAI Yemoja cluster

- 30 nodes, 128 GB RAM each, 20-core processors
- 300 Parallel Matlab workers (10 per node), multithread - full core utilization

Conventional data

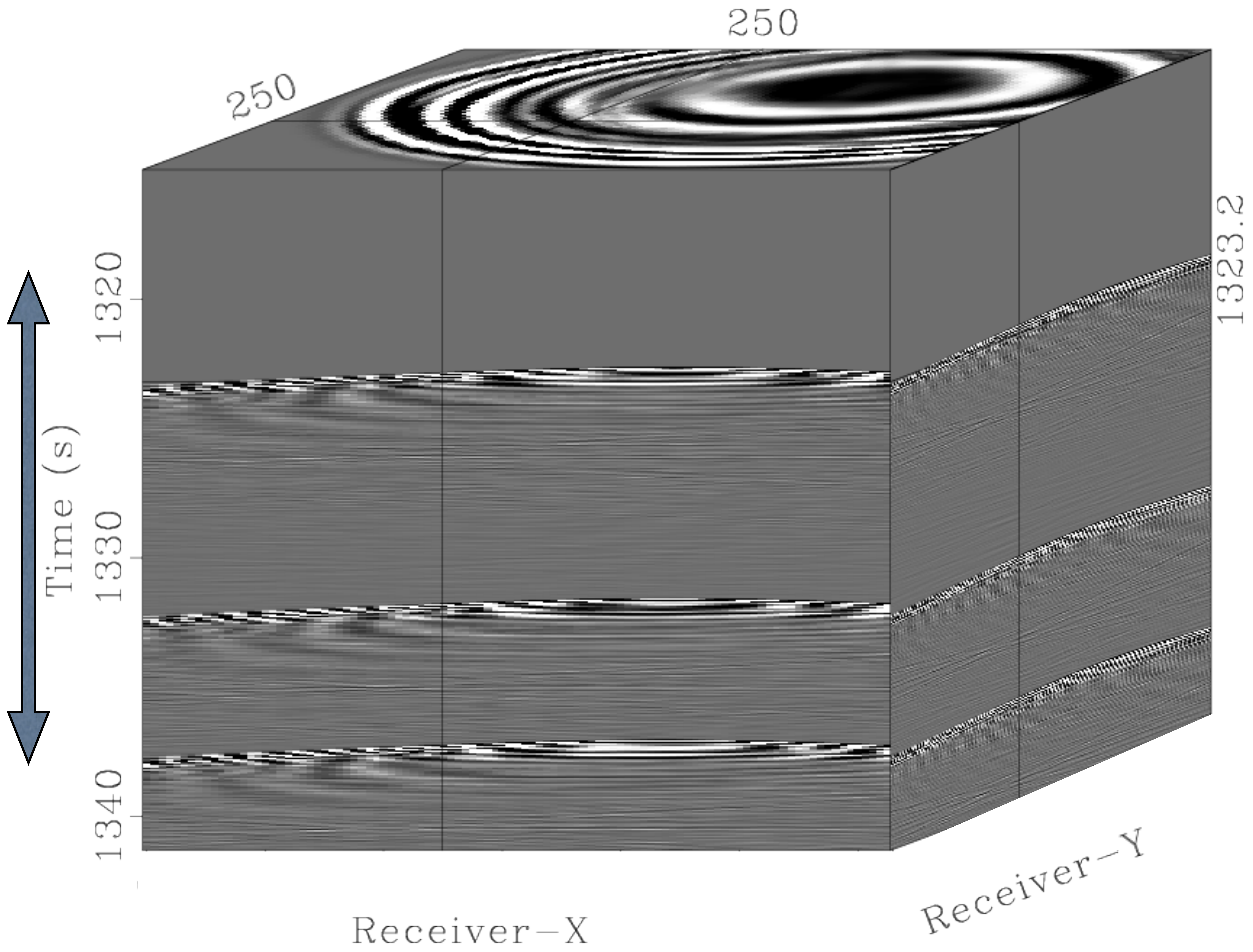
common-shot gather, @6.25 m source sampling



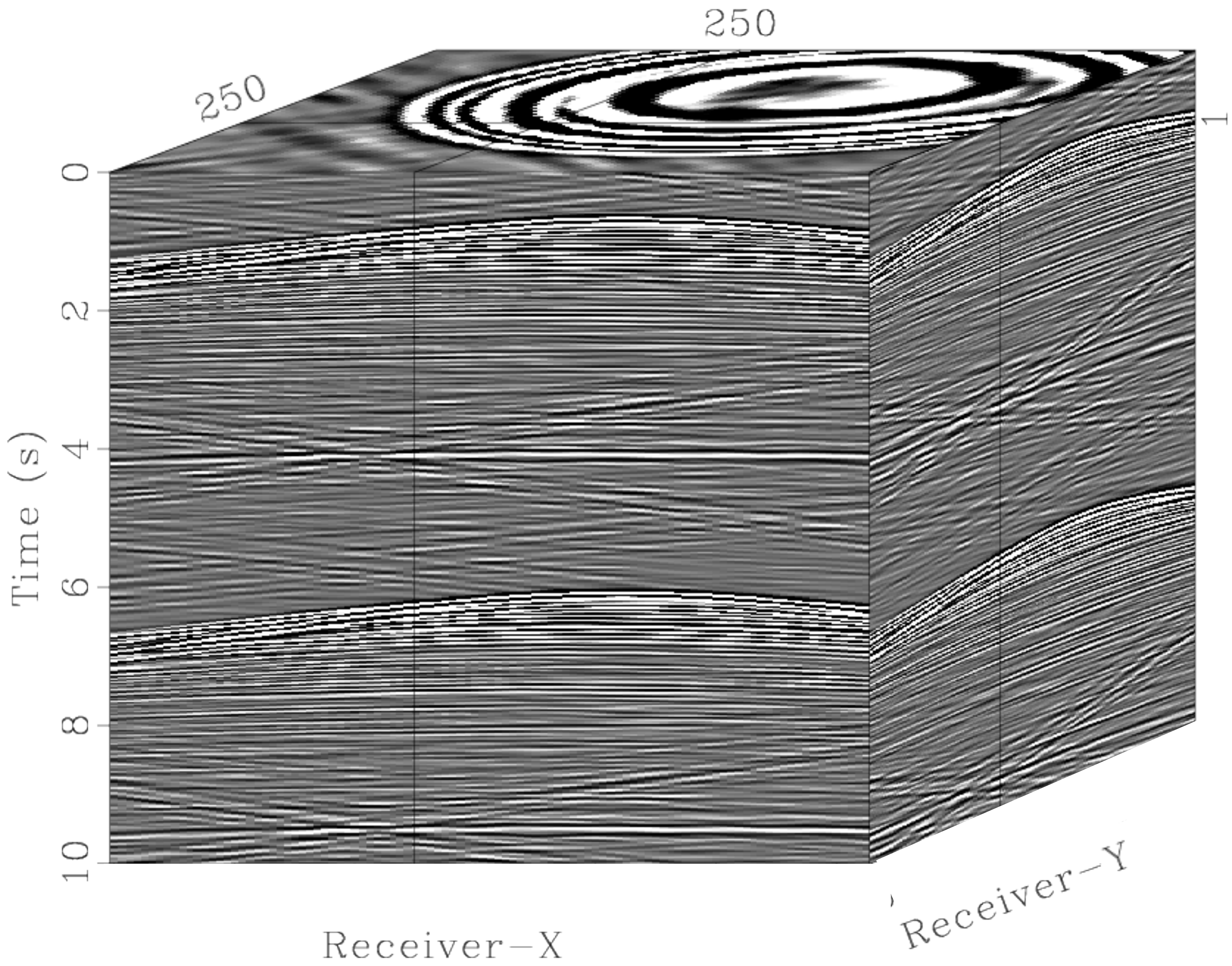
Time-jittered continuous record

@ 25m flip-flop shooting, blended & missing shots

continuous data
(30 s)

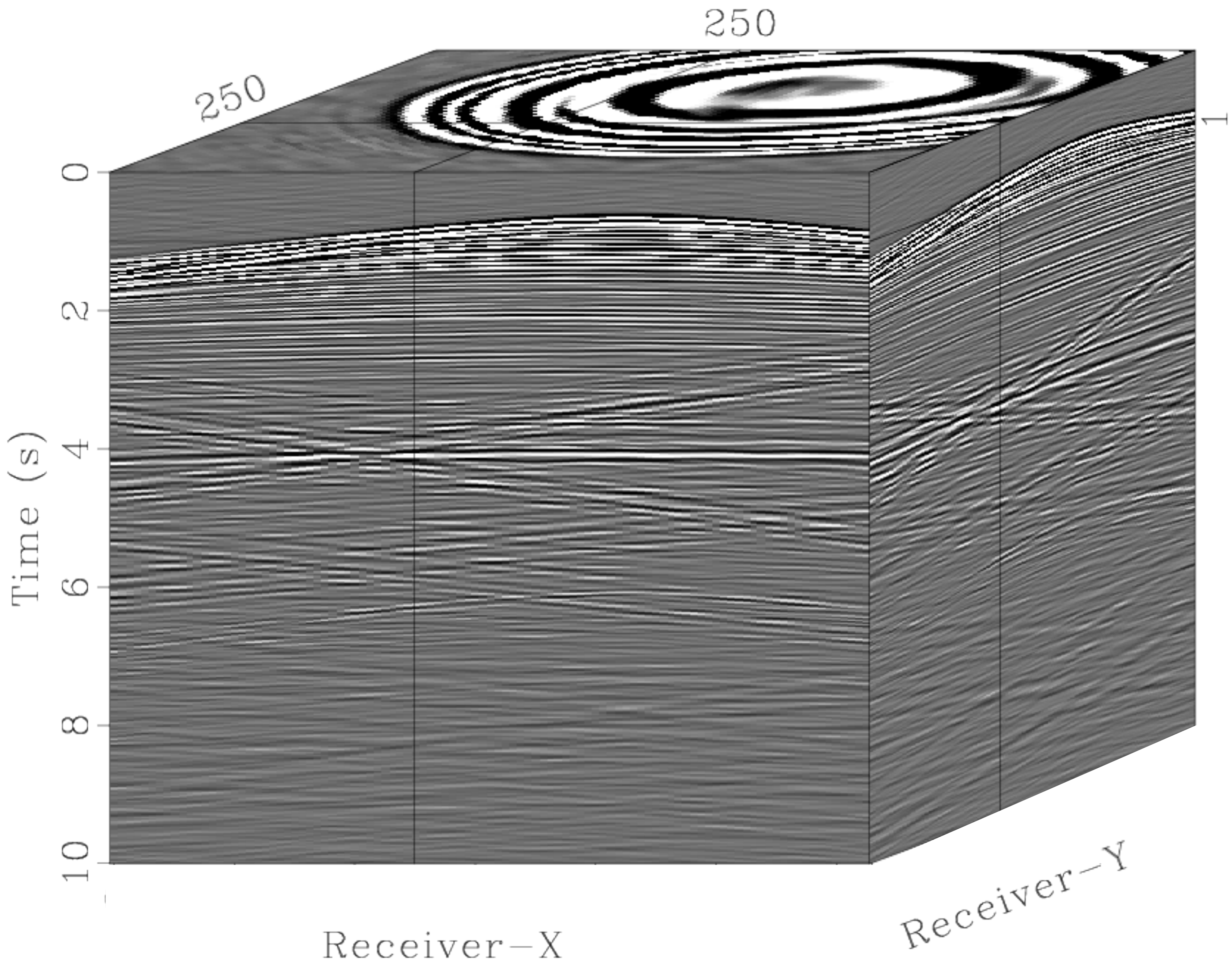


Adjoint of sampling-operator
common-shot gather



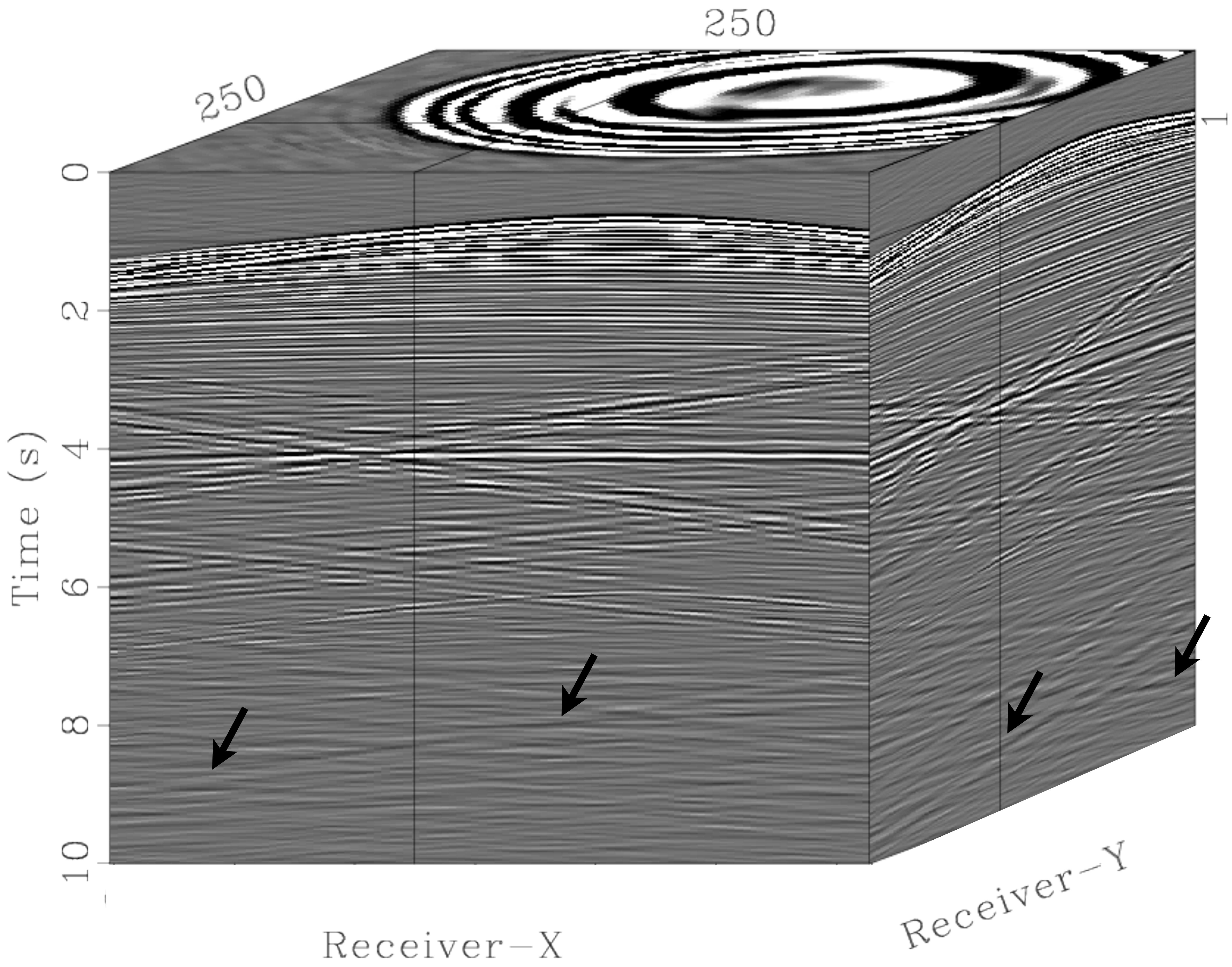
After Source-Separation

common-shot gather, 21dB signal-to-noise ratio



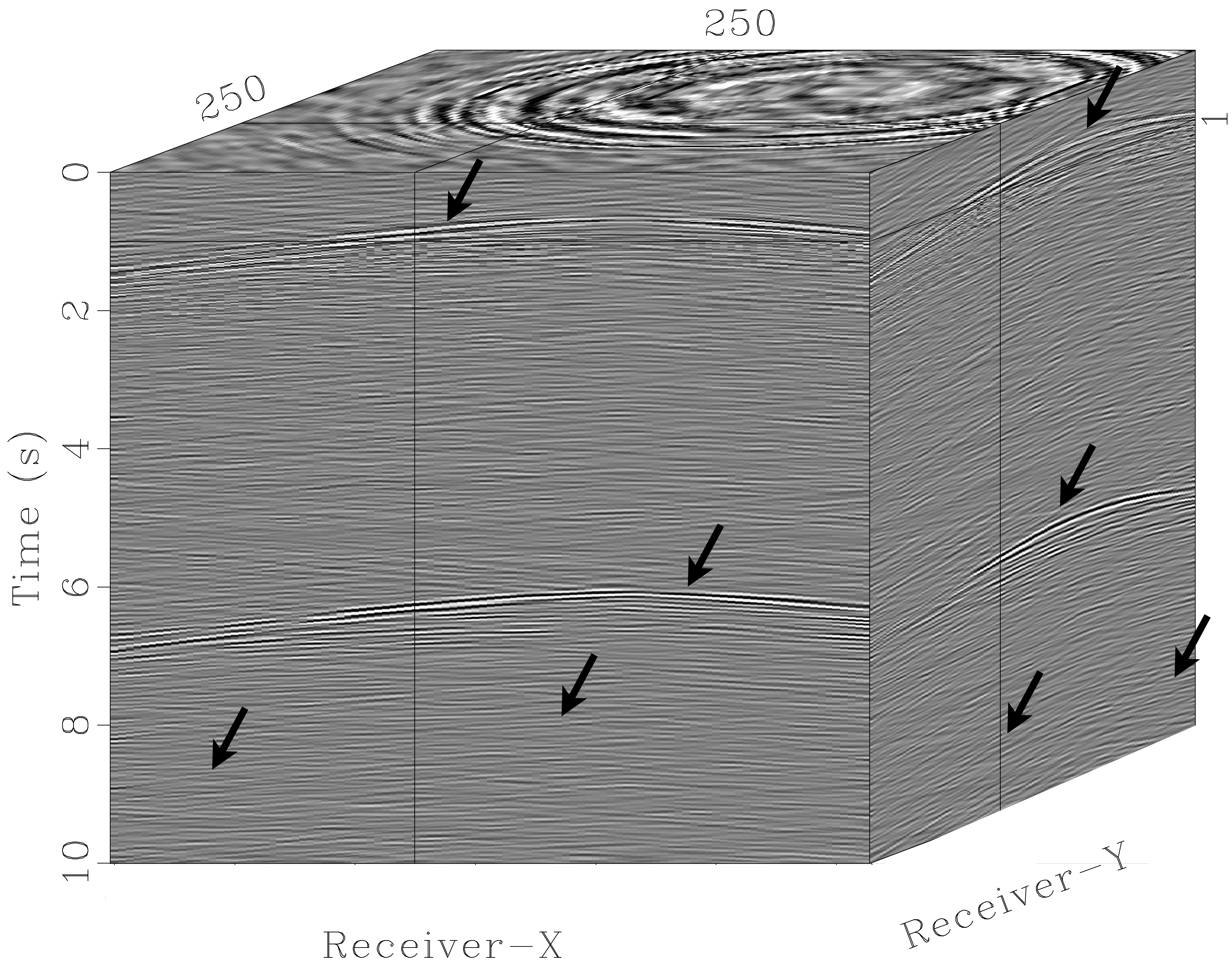
After Source-Separation

preserved late-arrivals energy



Residual

coherent energy can be reconstructed using 2nd pass over data



10x magnified

reconstructed late-arrivals

Take-away message

- **4X** up-sampling (@ 6.25m) & saving in acquisition time
- size of final recovered data volume is **0.5 TB**
  - no need to save fully sampled seismic data volume
- save $\mathbf{L}$ and $\mathbf{R}$ factors
  - compression rate is **98%**
  - size of final compressed 5D seismic volume is **~13 GB**

Conclusions

- Low-cost 3D OBN acquisition
- expandable to time-lapse OBN acquisition
- Factorization based rank-minimization framework can handle large-scale seismic data
- Embarrassingly parallel framework

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Thank you for your attention