Time compressively sampled full-waveform inversion with stochastic optimization

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Time domain Full-waveform Inversion (FWI) via the adjoint state method is a broadly used method for seismic inversion due to its efficient and easy to implement time-marching structure. The main challenge of this method is the the implementation of the imaging condition involving cross-correlation of wavefields computed in reverse order. We present here a method allowing to reduce the memory cost without loosing any computational efficiency or accuracy.

Adjoint-state time domain inversion

The adjoint-state inversion problem can be formulated as follow:

$$\underset{\mathbf{m}}{\text{minimize}}\, \Phi_s(\mathbf{m}) = \frac{1}{2}\left\|\mathbf{P}_r A^{-1}(\mathbf{m})\mathbf{q}_s - \mathbf{d}\right\|_2^2$$

where

$\mathbf{P}_r$: the discrete projection operator restricting the synthetic wavefield to the receiver locations

$A(\mathbf{m})$: the discretized wave equation matrix

$\mathbf{q}$: the discrete source

$\mathbf{d}$: the measured data

$\mathbf{m} = \{v_i^{-2}\}_{i=1\cdots N}$: the square slowness with N the size of the discretization of the model

and the gradient of the FWI objective function is defined as:

$$\nabla\Phi_s(\mathbf{m}) = -\sum_{t=1}^{n_t}\left\{((\mathbf{D}\mathbf{u})[t])^T \operatorname{diag}(\mathbf{v}[t])\right\} = \mathbf{J}^T\delta\mathbf{d}$$

The main challenge of time-domain adjoint state is the necessity to store or recompute the full time history of the source wavefield (computed forward in time) during the back-propagation of the receiver wavefield (computed backward in time). Methods have been proposed to reduce the memory cost such as optimal checkpoint (Symes 2009) or boundaries methods (Clapp 2009).

Examples : 2D BG Compass model

Data :

- Ricker wavelet at 15Hz, 2.4s recording at 4ms sampling rate
- 51 sources at 100m interval
- 201 receivers at 25m interval

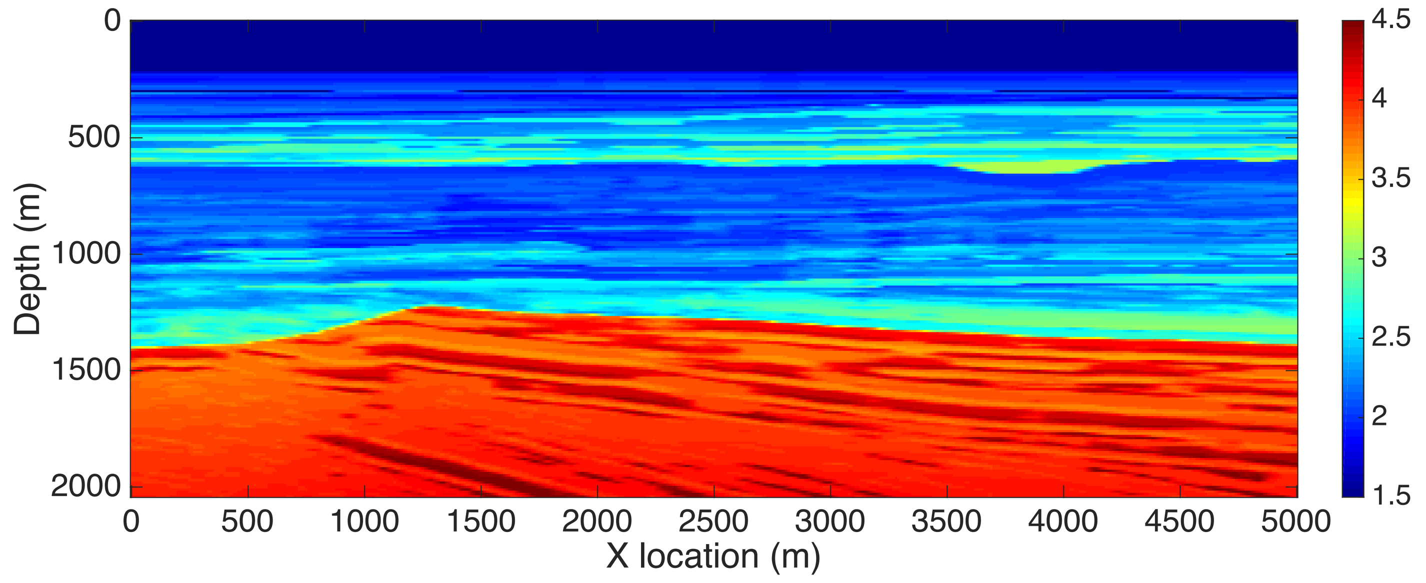

Figure 1: True model

Time compressed adjoint state

We propose a method that only uses a randomly selected partial history of the wavefields and compute the cross-correlations for this subset. A independent subset is redrawn for each source experiment at each iteration in order to obtain a full coverage of the time history in average. We define a subsampling ratio

$$r = \frac{n_{\text{corr}}}{n_{\text{Nyquist}}}$$

The new subsampled gradient

$$\tilde{\nabla}\Phi_s(\mathbf{m}) = -\sum_{t\in\tilde{I}}\left[\operatorname{diag}(\mathbf{u}[t])(\mathbf{D}^T\mathbf{v}[t])\right] = \tilde{\mathbf{J}}^T\delta\mathbf{d}$$

With

$$\operatorname{size}(\tilde{I}) = n_{\text{corr}} << n_{\text{Nyquist}}$$

In the following examples we use a subsampling ratio of **r=0.05** (5% of the time history at Nyquist sampling rate) **equivalent to 0.6% of the full time history** for the given setup. It corresponds to one wavefield every 80ms in average.

Compared to	Speed-up	Memory reduction factor
Full storage FWI	slightly faster no I/O	20 (Nyquist) 150 (Full history)
Basic checkpointing FWI	10 times faster	2-4
Optimal checkpointing FWI	3 times faster	2-4

Table 1: Efficiency comparison

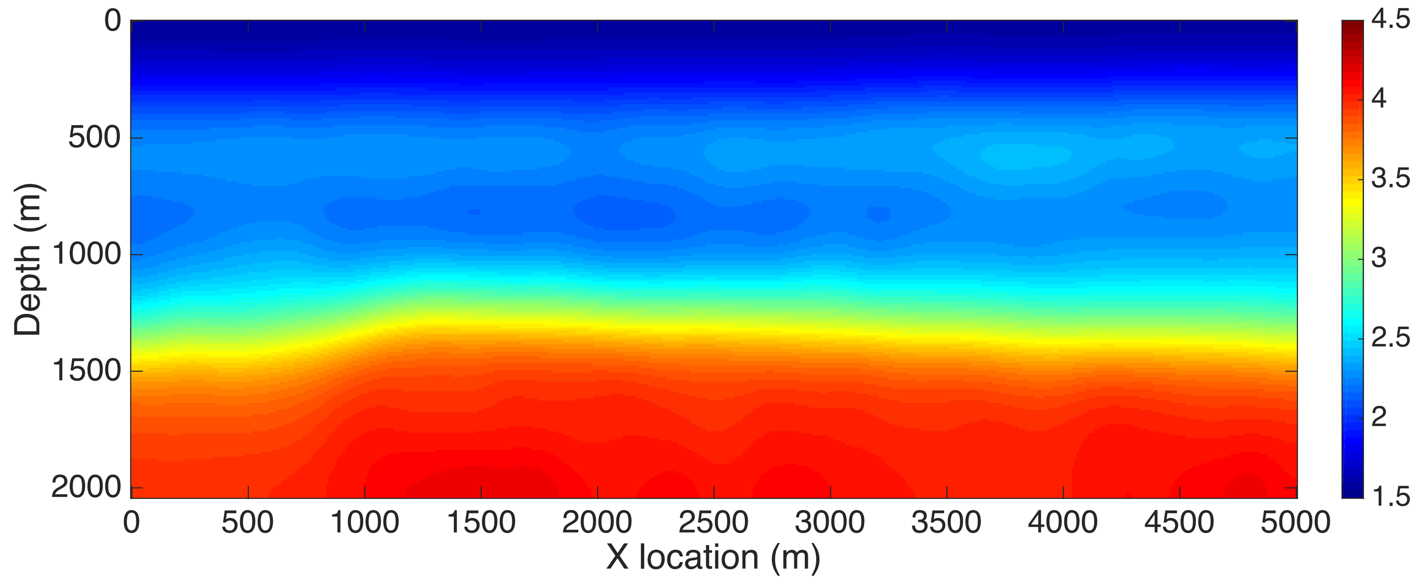

Figure 2: Initial model

On Figure 4, we see that randomly (in a jittered way) subsampling the wavefield gives us a velocity model equivalent to usual FWI (Figure 3) with little artifact and doesn't contains any strong subsampling artifacts periodic subsampling shows in FIgure 5.

Gradient descent : 50 iterations

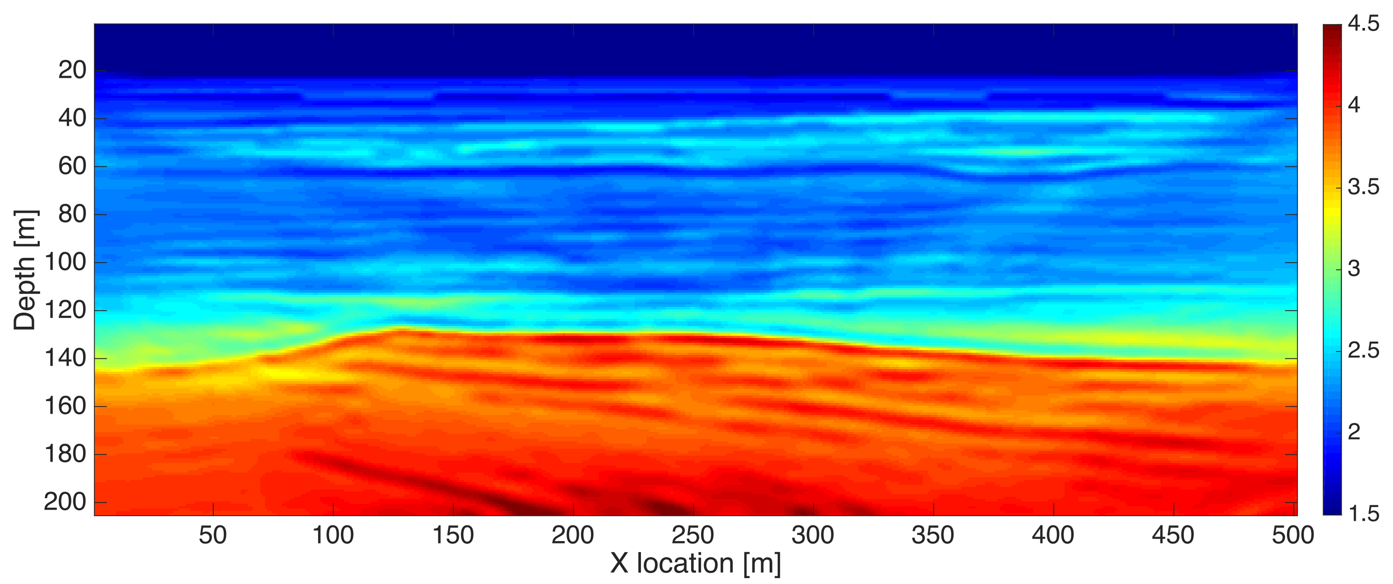

Figure 3: Usual FWI

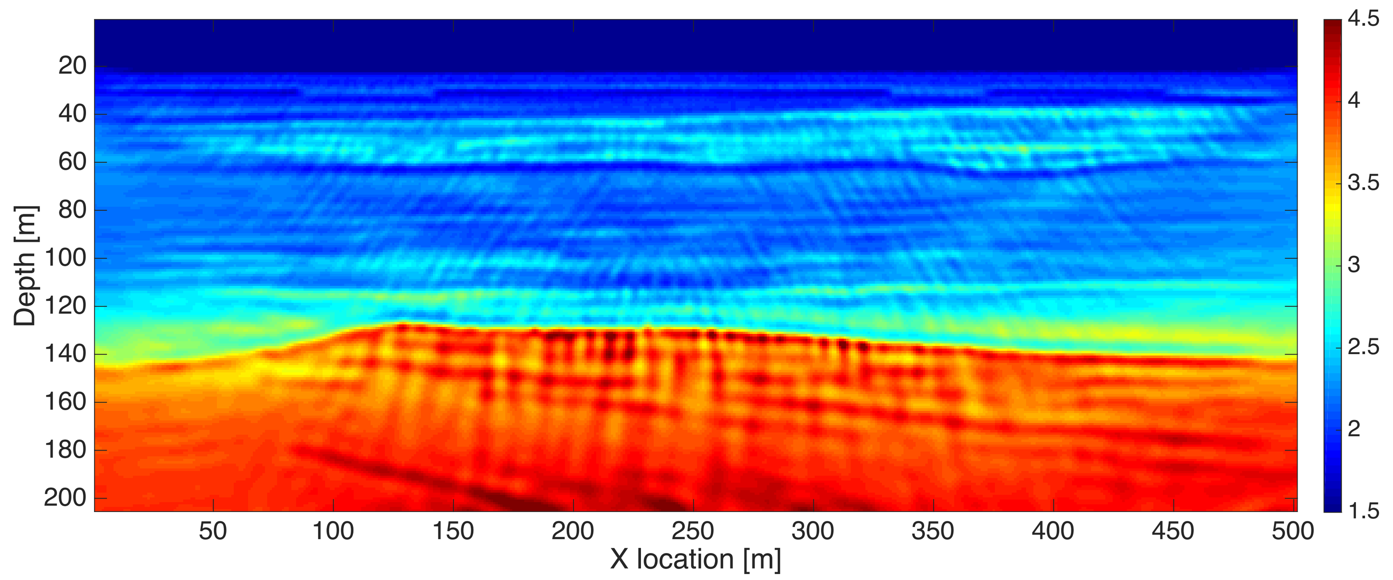

Figure 4: Time compressed FWI

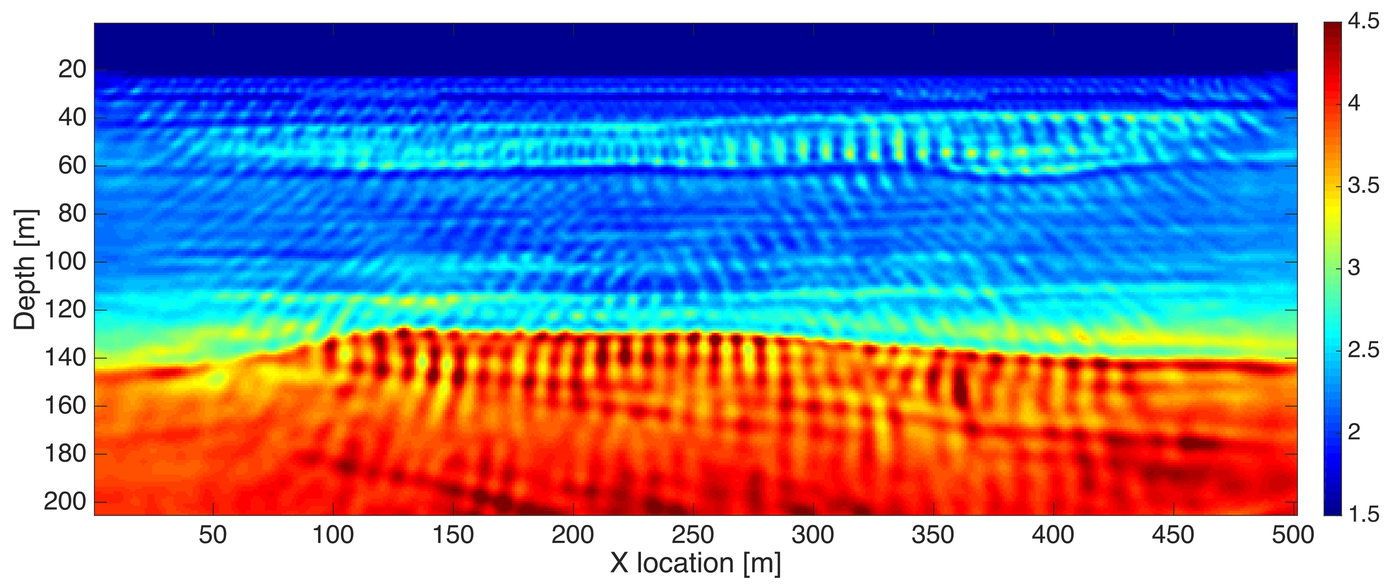

Figure 5: Periodic subsampled (Time compressed ratio)

Conclusion

We showed that with a good memory and computational cost reduction (compared to checkpointing or other methods) and less I/O necessary than full history storage, we obtain an inversion result comparable with usual FWI in the case of first order method, and equivalent to usual FWI with second order method. The subsampling artifact are

L-BFGS : 10 iterations

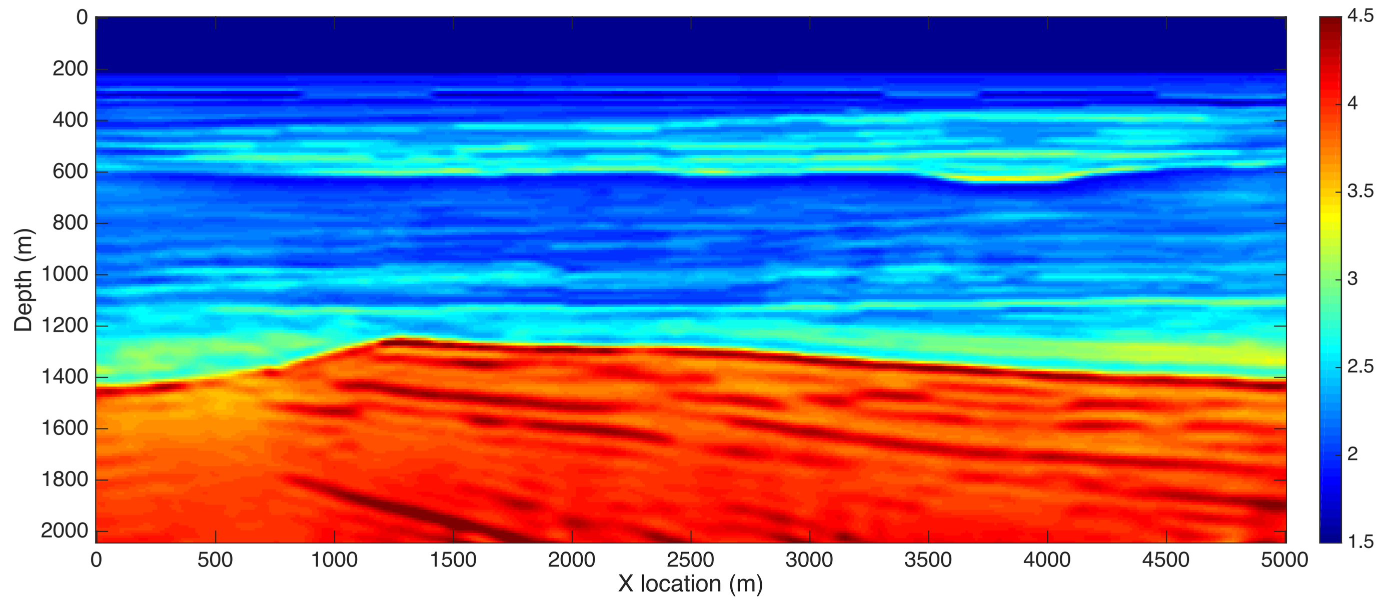

Figure 6: Usual FWI

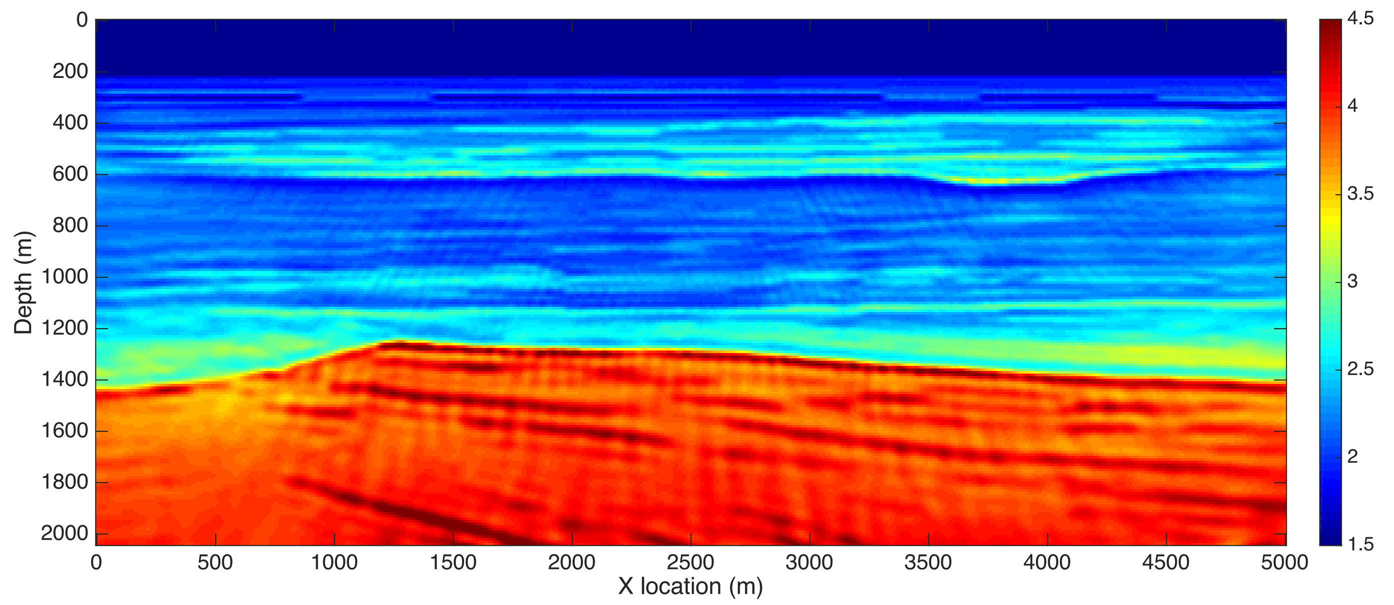

Figure 7: Time compressed FWI

mostly removed by redrawing a new set of time indexes for each source experiment at every iteration, and using a partial history of the updates with l-BFGS remove the few artifacts present with a gradient descent. New methods (Peters 2015) are really promising to regularize and should remove completely any subsampling artifacts

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