SVD-free matrix recovery in randomized marine acquisition

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SVD-free matrix recovery in randomized marine acquisition

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Motivation

- Acquisition challenges
  - conventional densely sampled data is costly
  - rethink acquisition design: compressed and/or subsampled
  - how to subsample?

- Data challenges
  - missing traces
  - simultaneous shot records

- Exploit *low-rank* structure of seismic data
  - *SVD-free* matrix reconstruction
  - trace interpolation
  - source separation (or *deblending*)

Outline

- Low-rank matrix recovery
  - *low-rank* structure of seismic data
  - *SVD-free* matrix reconstruction
- How should we subsample?
  - spectral gap analysis
- Experiments
  - trace interpolation
  - source separation

[Candès and Plan, 2009; Oropeza and Sacchi, 2011]

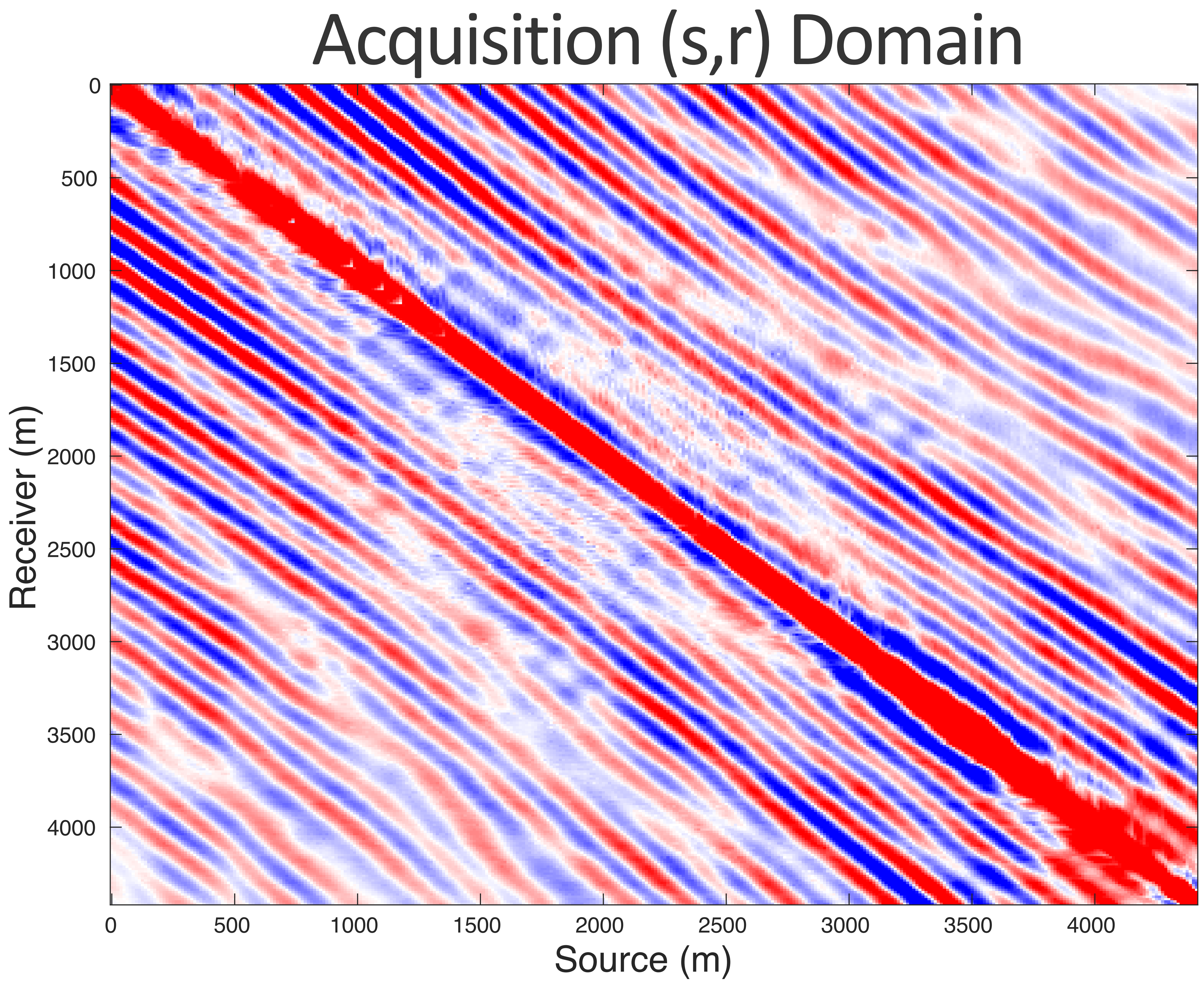
Matrix completion

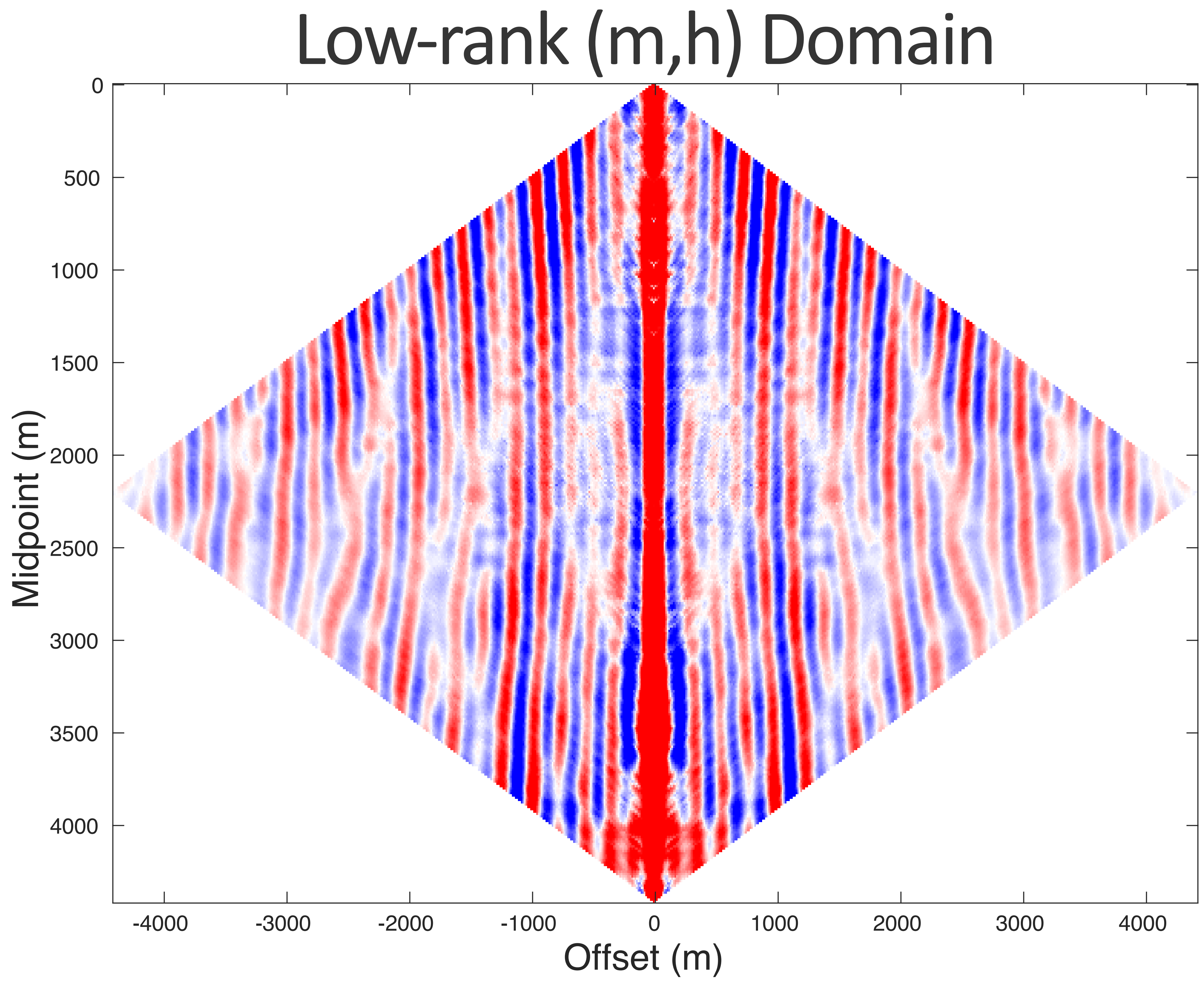
Successful reconstruction scheme

- exploit *structure*
  - *low rank / fast decay* of singular values
- sampling
  - missing traces *increase* rank in “transform domain”
- optimization
  - via *rank minimization (nuclear-norm minimization)*

2D low-rank structure

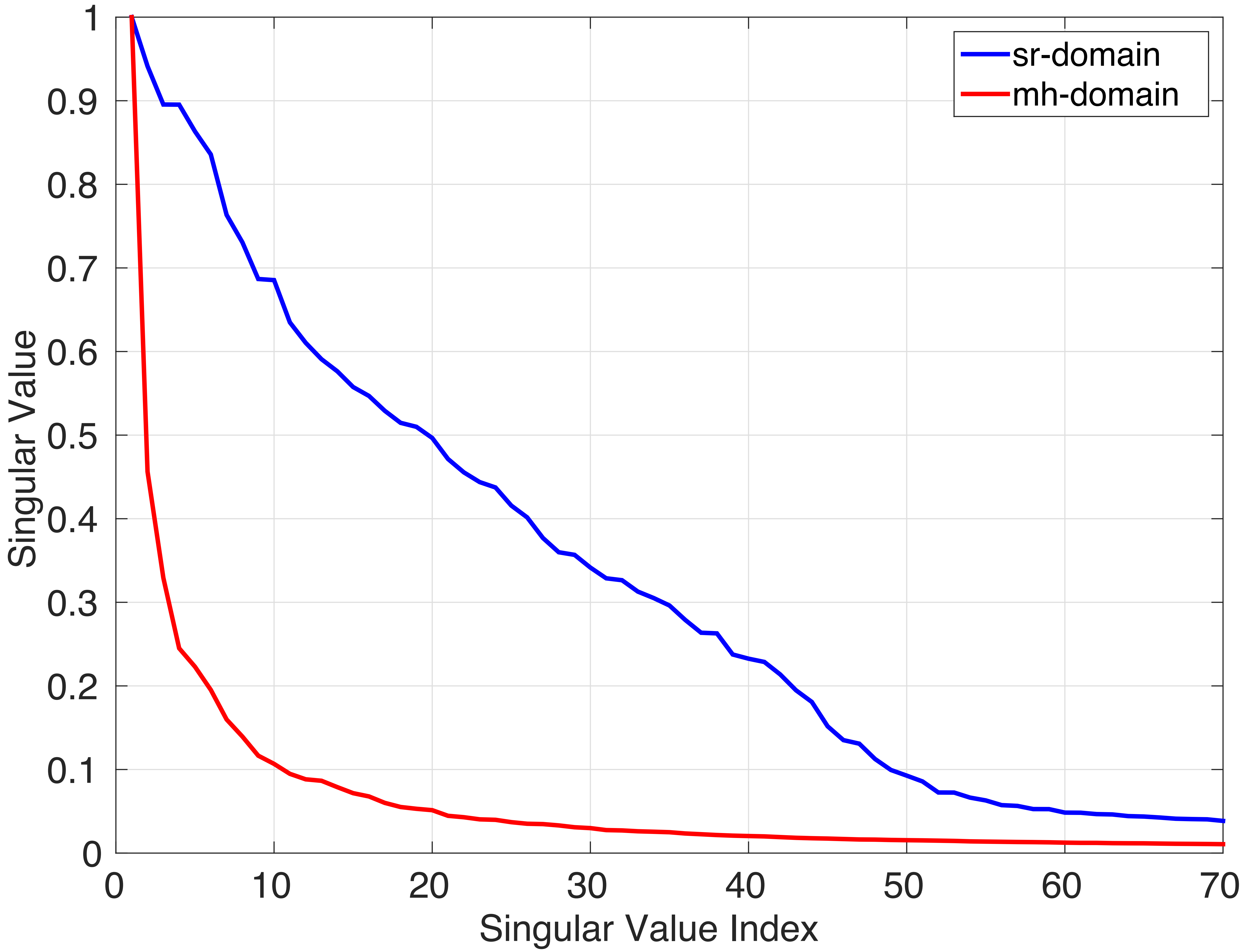
[Gulf of Suez data (OBC), frequency slice at 10 Hz]



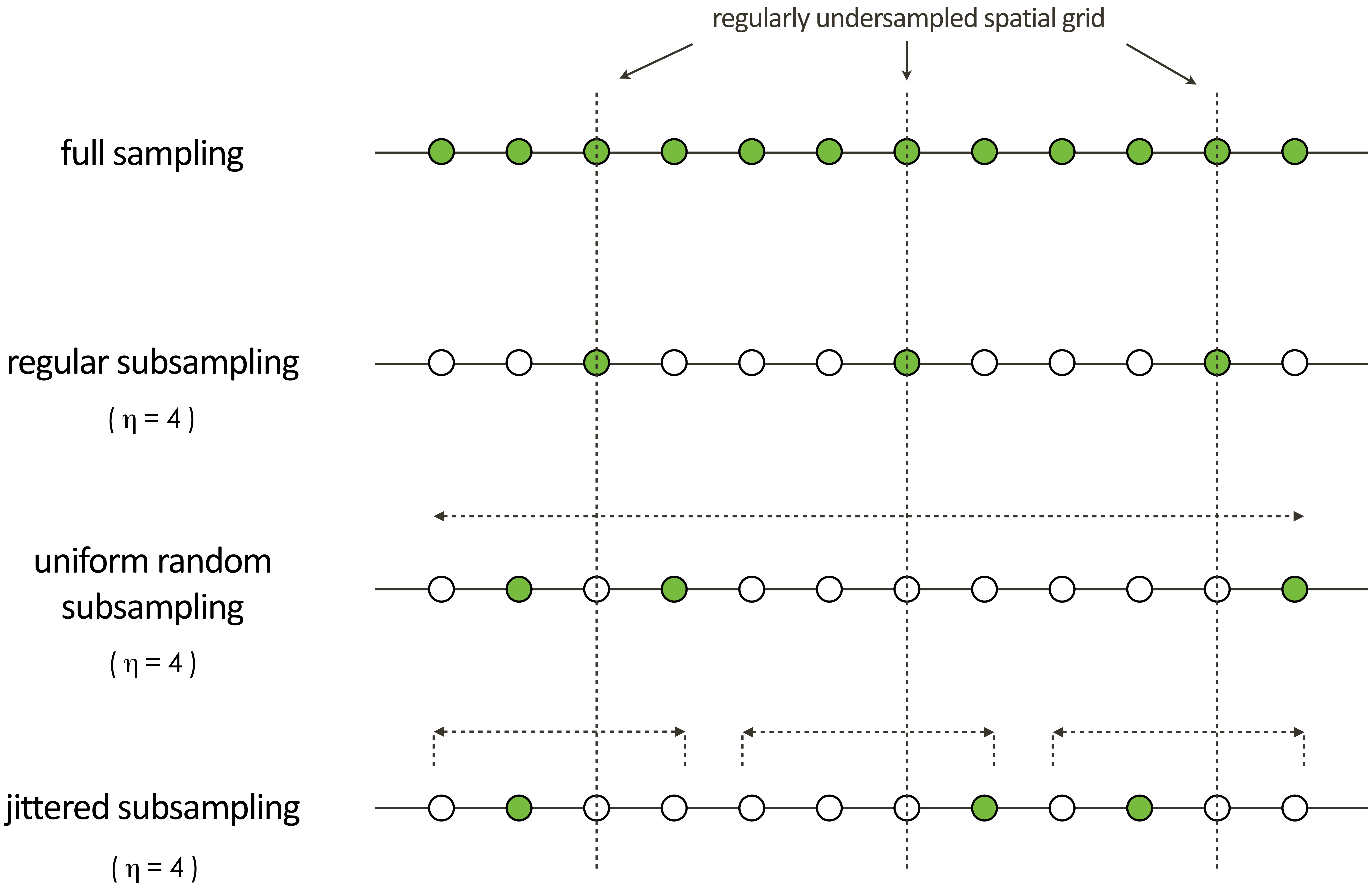


Singular value decay

s-r domain vs m-h domain



Sampling schemes

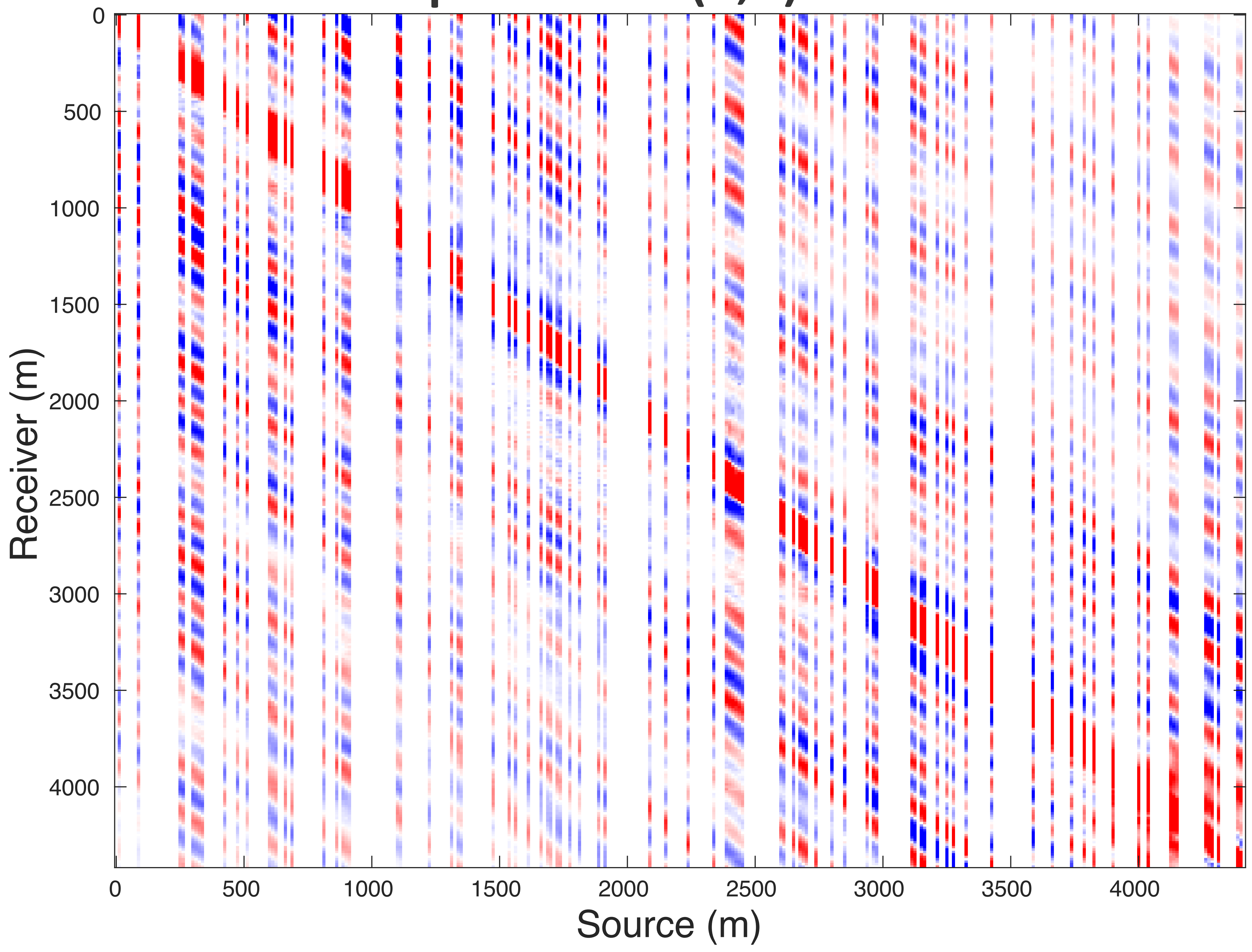


[Hennenfent and Herrmann, 2008]

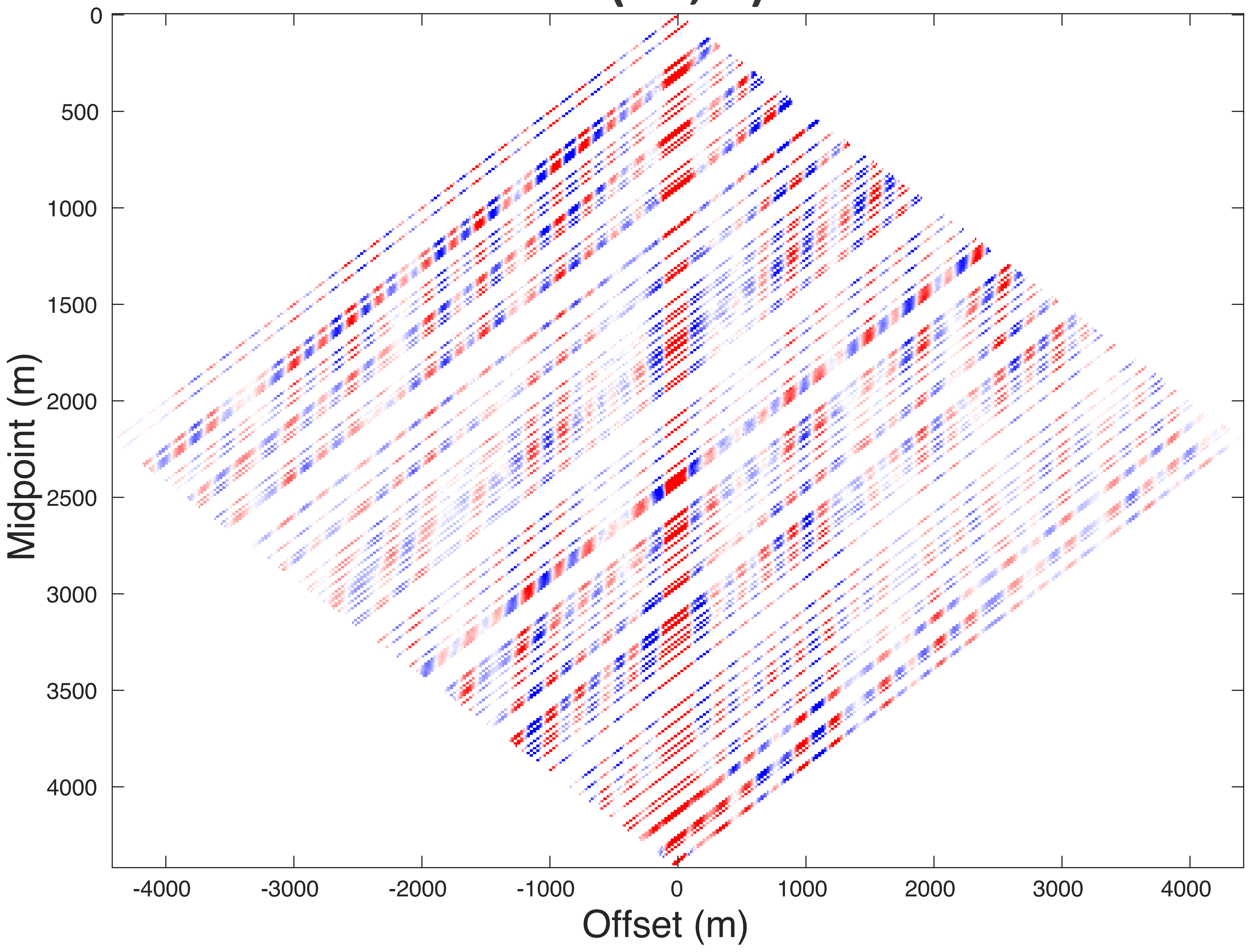
2D seismic subsampling (jittered random)

[Gulf of Suez data (OBC), frequency slice at 10 Hz]

Acquisition (s,r) Domain

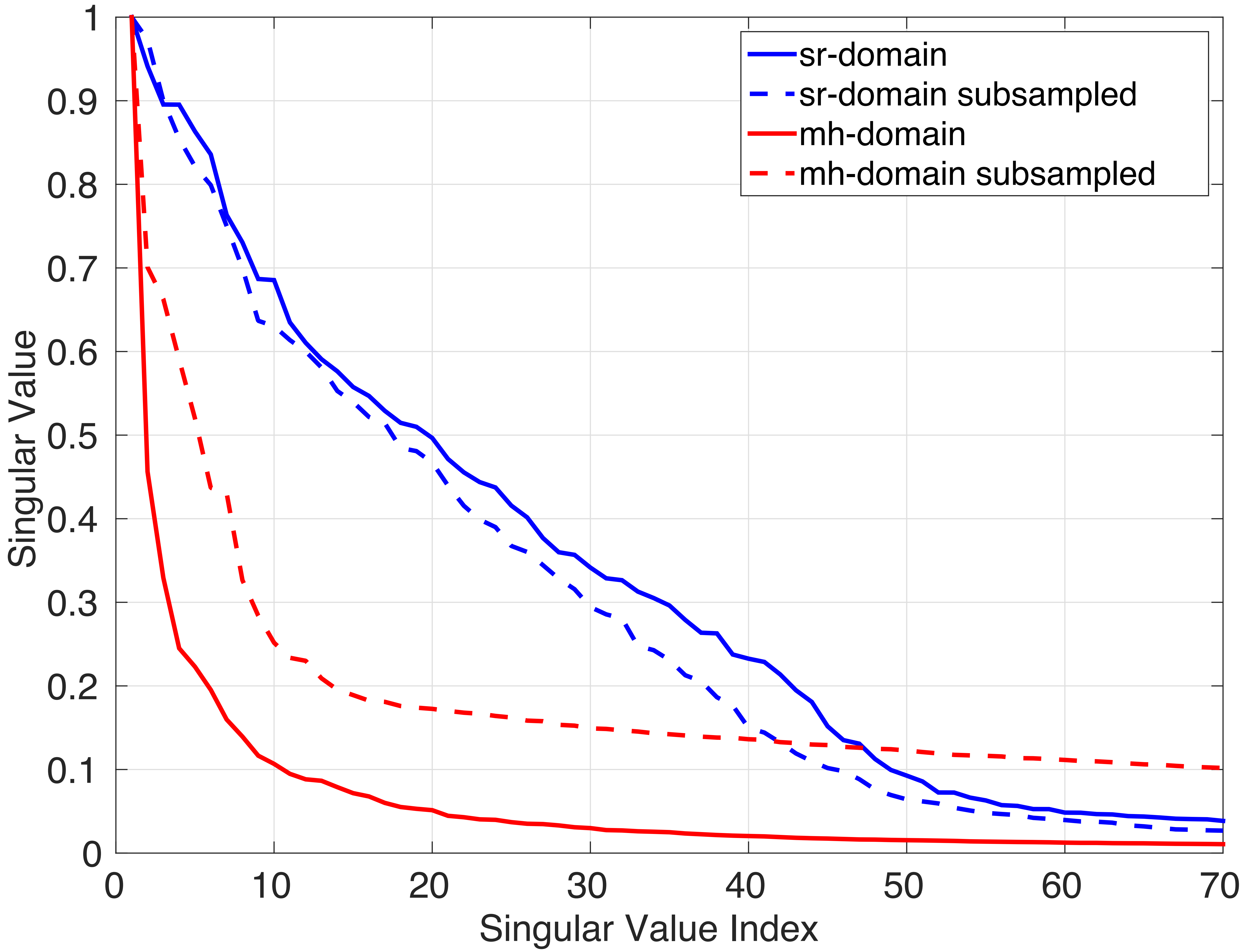


Low-rank (m,h) Domain



Singular value decay

full data vs subsampled data



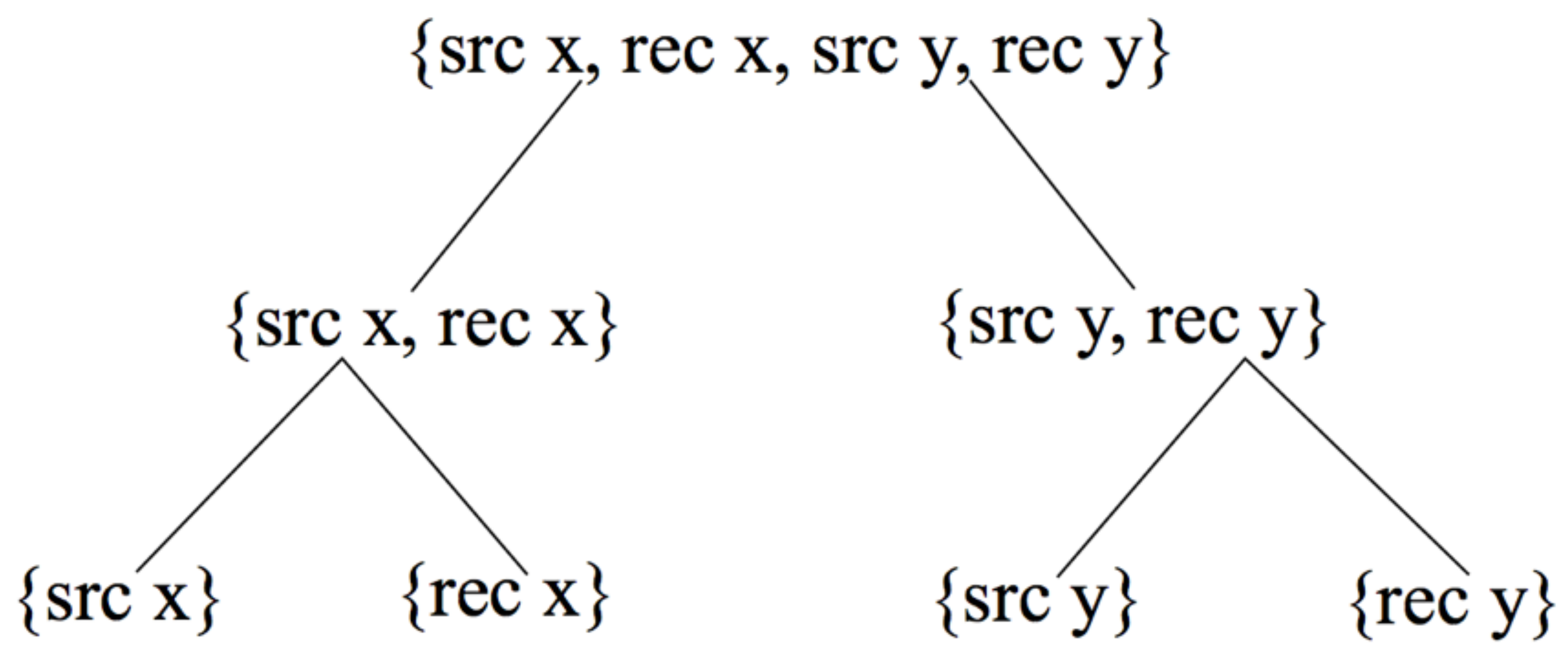
[Kreimer et. al., 2013]

3D seismic data low-rank structure

- Consider a 3D seismic survey with coordinates (src x, src y, rec x, rec y, time)
- Take a Fourier transform in time and restrict ourselves to a single frequency slice
- Many options on how to matricize

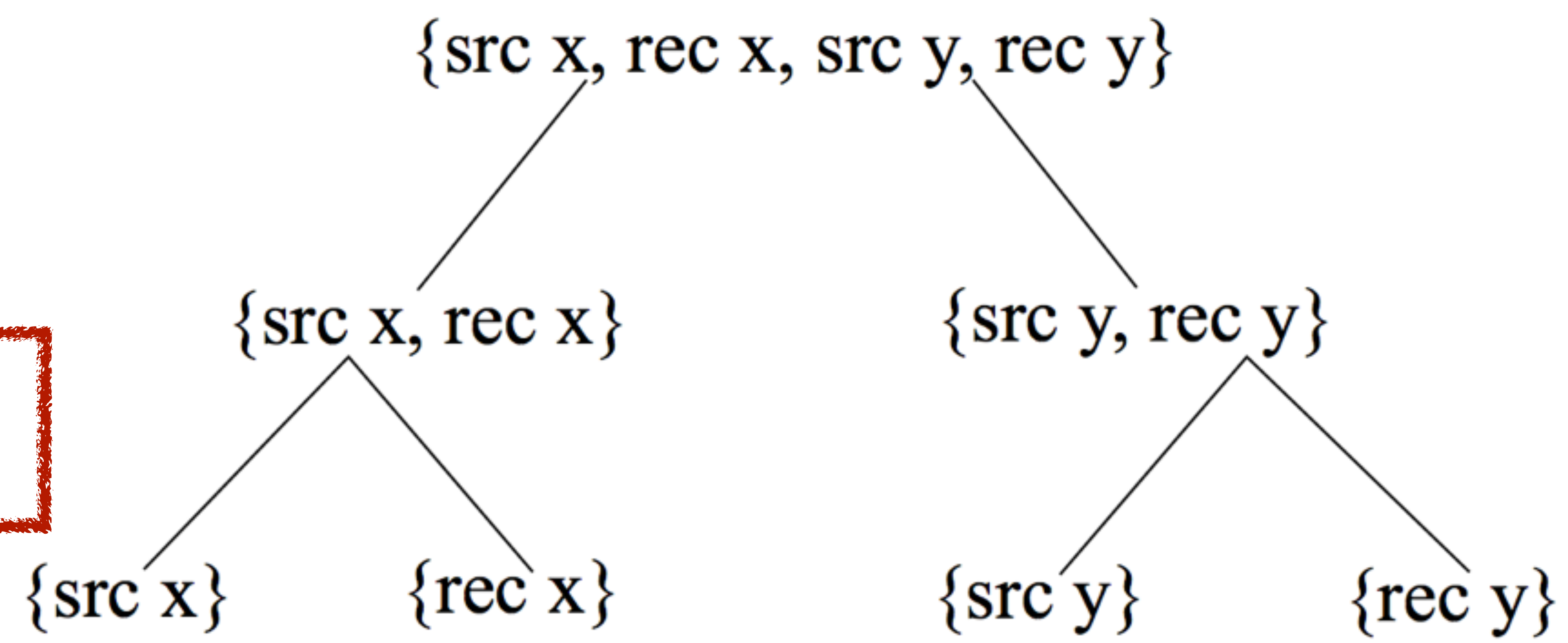
[Da Silva and Herrmann, 2015]
[Kumar et. al., 2015a]

3D data: matricized



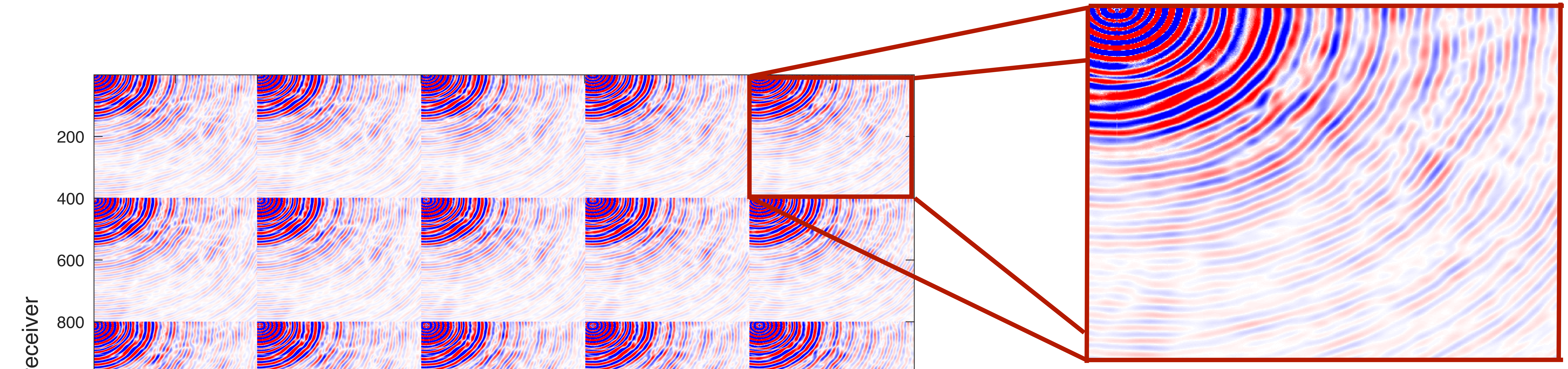
3D data: matricized

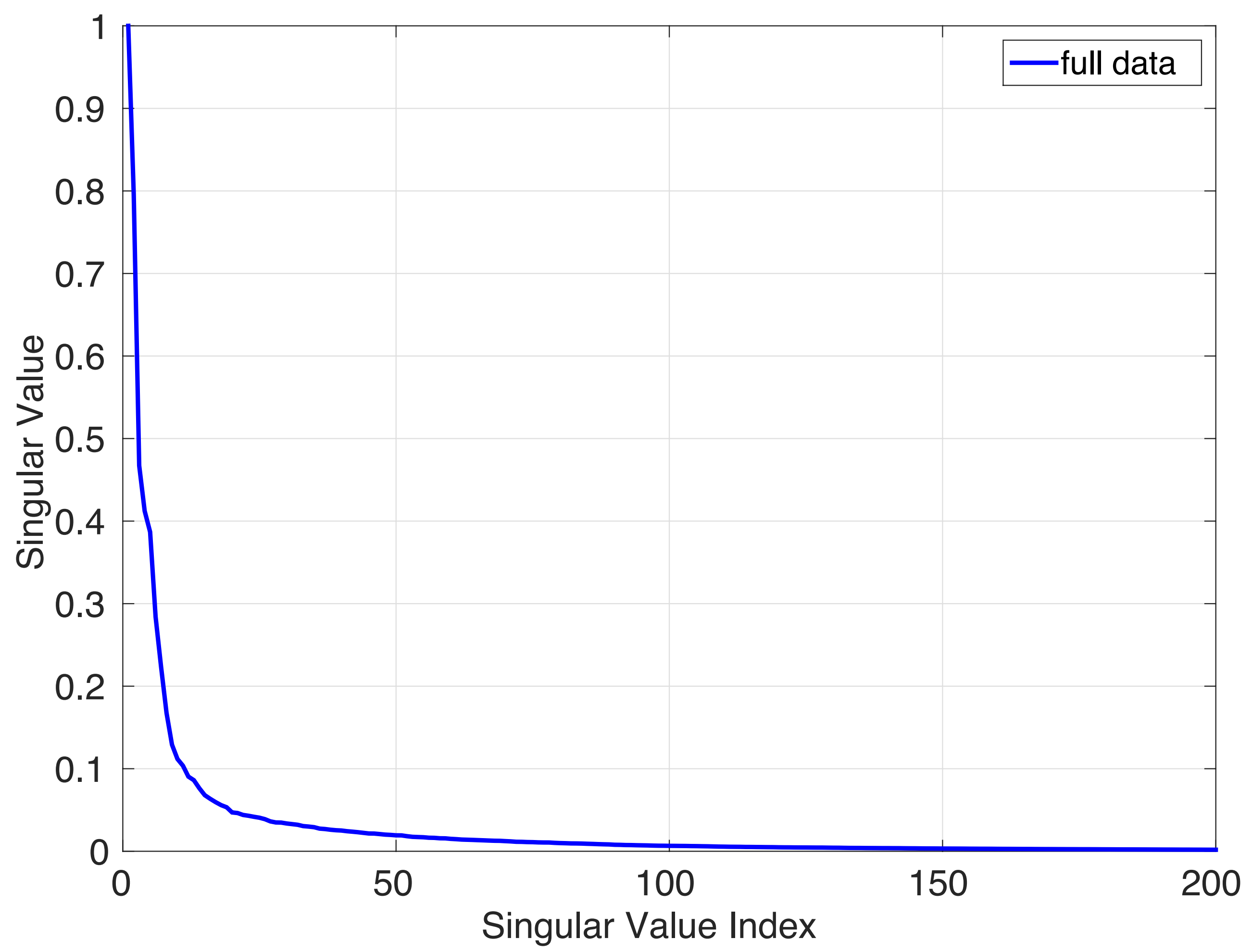
Option 1: (rec,rec) - form



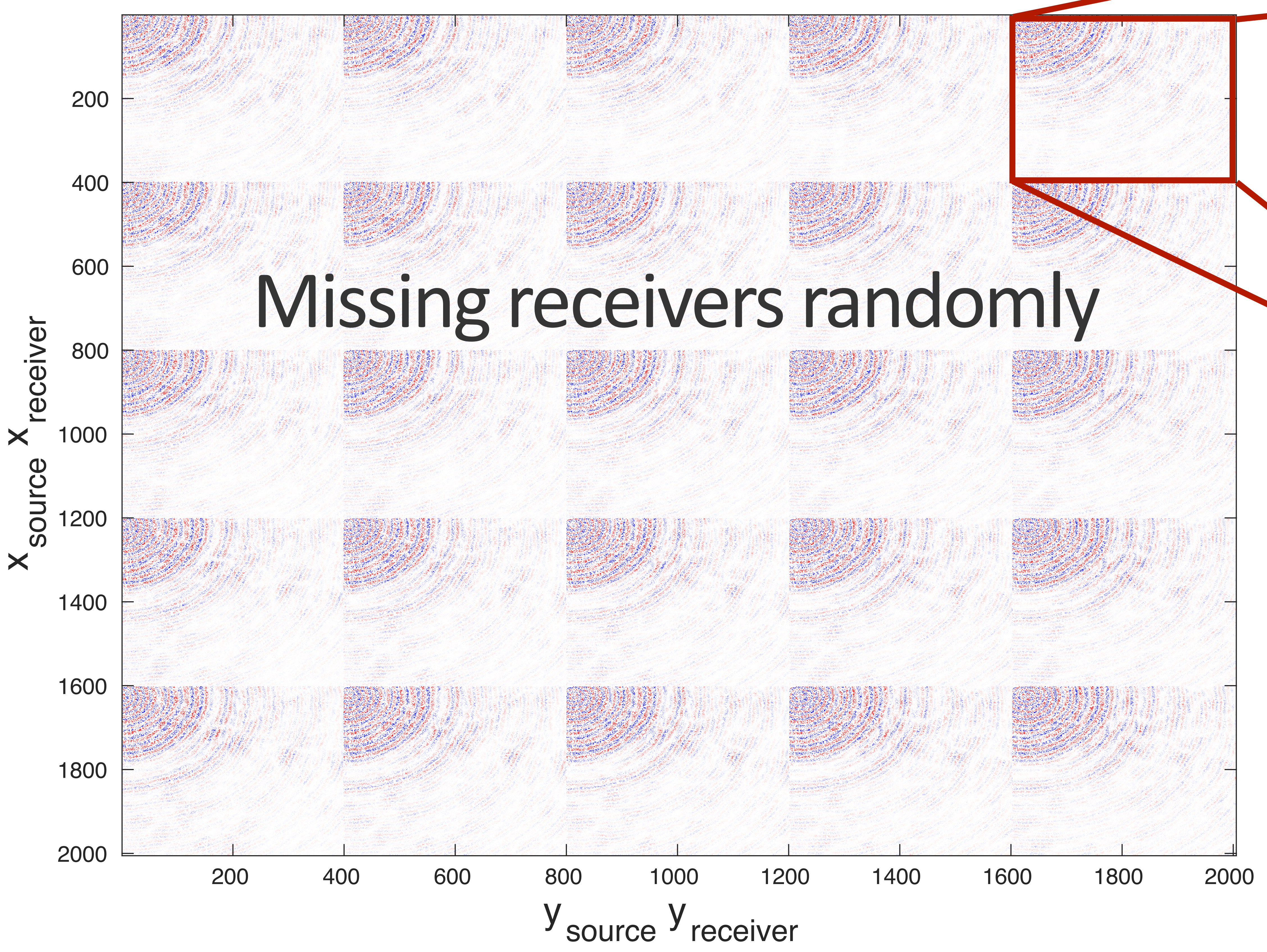
Make receiver by receiver blocks

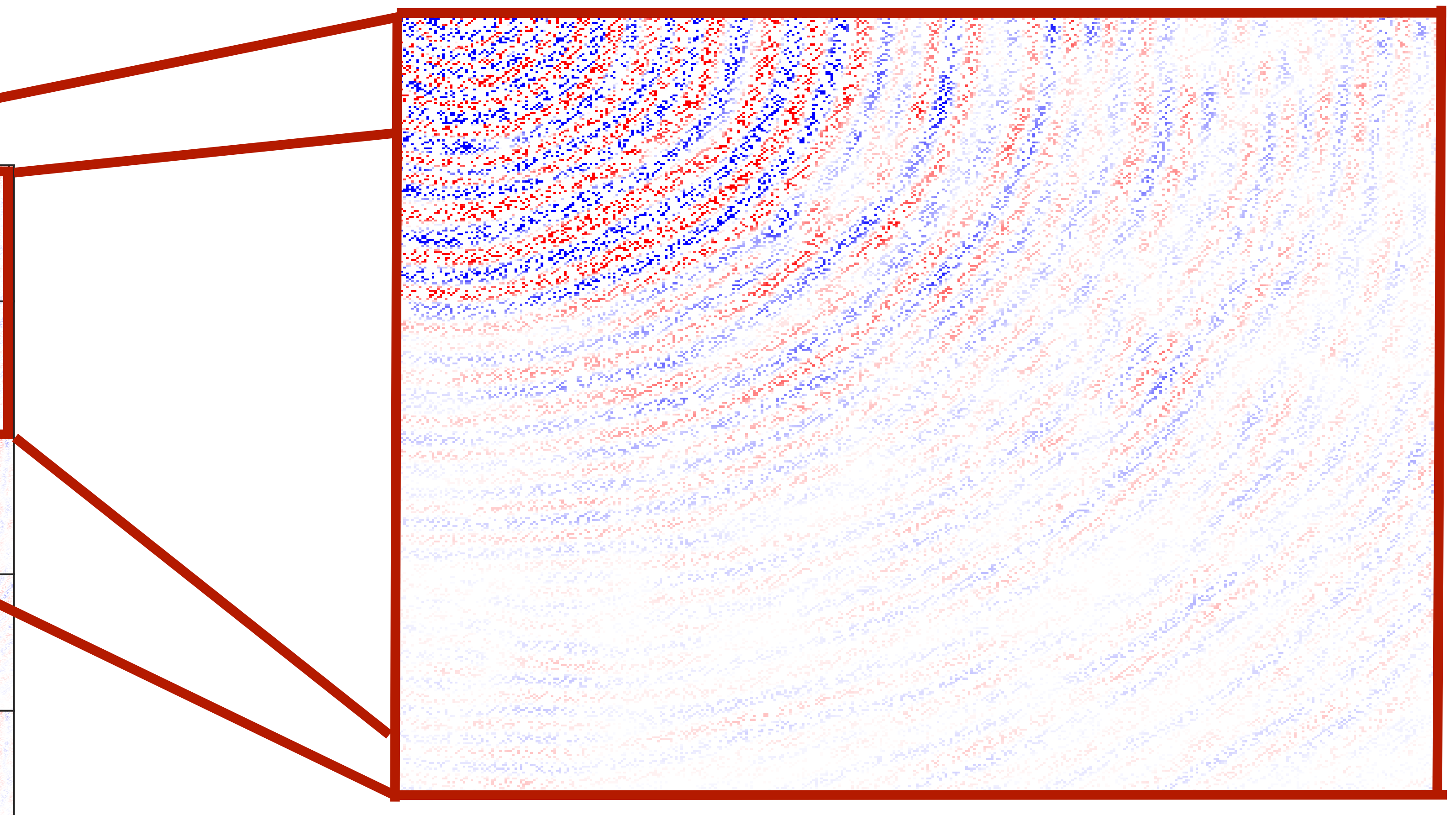
3D data: matricized in (rec,rec) form

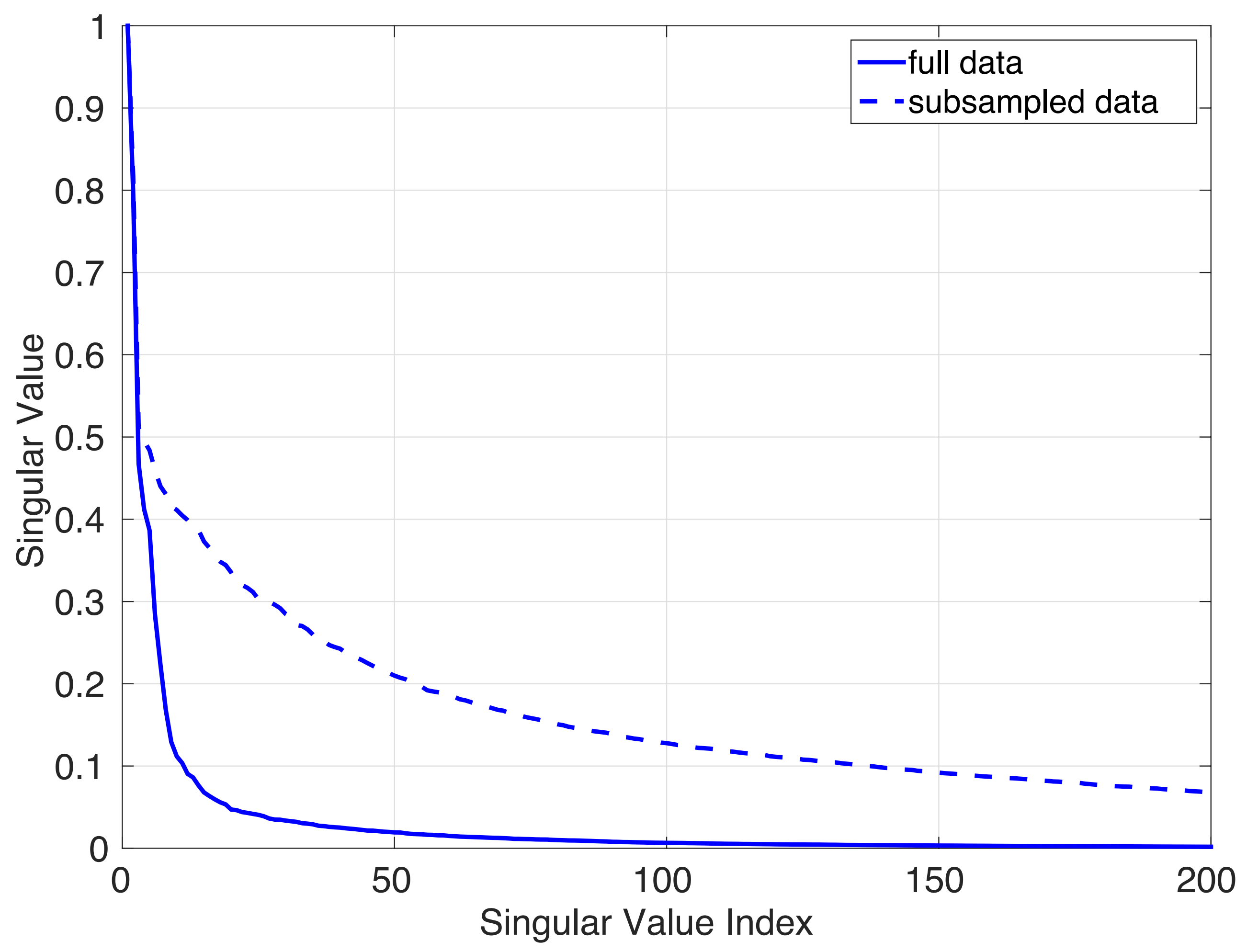




3D data: matricized in (rec,rec) form







[Candès and Plan, 2009]

Methodology: penalize rank

- want to minimize nuclear norm

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} ||\mathbf{X}||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}\|_* = \sum_{k=1}^{m} \lambda_k$$

where λ_k are the *singular* values

Factorized formulation

$$\mathbf{X} \in \mathbb{R}^{n\times m} = \mathbf{L} \in \mathbb{R}^{n\times r} \times \mathbf{R}^H \in \mathbb{R}^{r\times m}$$

$$\mathbf{X} = \mathbf{L}\mathbf{R}^{\mathbf{H}}$$

Factorized formulation

$$\mathbf{X} \in \mathbb{R}^{n \times m} = \mathbf{L} \in \mathbb{R}^{n \times r} \times \mathbf{R}^H \in \mathbb{R}^{r \times m}$$

$$\boxed{\text{Choose factorization parameter } \; r \ll \min(n, m)}$$

$$\mathbf{X} = \mathbf{L}\mathbf{R}^{\mathbf{H}}$$

[Recht et. al., 2010]

Nuclear-norm minimization - factorized form

- equally can solve

$$\min_{\mathbf{L}\in\mathbb{R}^{n\times r},\mathbf{R}\in\mathbb{R}^{m\times r}} \frac{1}{2}(||\mathbf{L}||_F^2 + ||\mathbf{R}||_F^2) \ \text{ s.t. } \ ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - b||_F^2 \leq \sigma$$

with sampling operator:

$$\mathcal{A} = \mathbf{R}\mathbf{M}\mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator

$\mathbf{M}$: measurement operator

$\mathcal{S}^H$: transform operator

[Lopez et. al., 2015, Off the grid low rank matrix recovery (submitted)]

Nuclear-norm minimization - factorized form

- equally can solve

$$\min_{\mathbf{L}\in\mathbb{R}^{n\times r},\mathbf{R}\in\mathbb{R}^{m\times r}} \frac{1}{2}(||\mathbf{L}||_F^2 + ||\mathbf{R}||_F^2) \;\; \text{s.t.} \;\; ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - b||_F^2 \leq \sigma$$

with sampling operator:

$$\mathcal{A} = \mathbf{RMN}\mathcal{S}^H$$

incorporate unstructured grid

where

$\mathbf{R}$: restriction operator

$\mathbf{M}$: measurement operator

$\mathcal{S}^H$: transform operator

$\boxed{\mathbf{N} \text{ : regularization operator}}$

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- Low-rank matrix recovery
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  - spectral gap analysis
- Experiments
  - trace interpolation
  - source separation

How should we subsample?

Consider our sampling mask

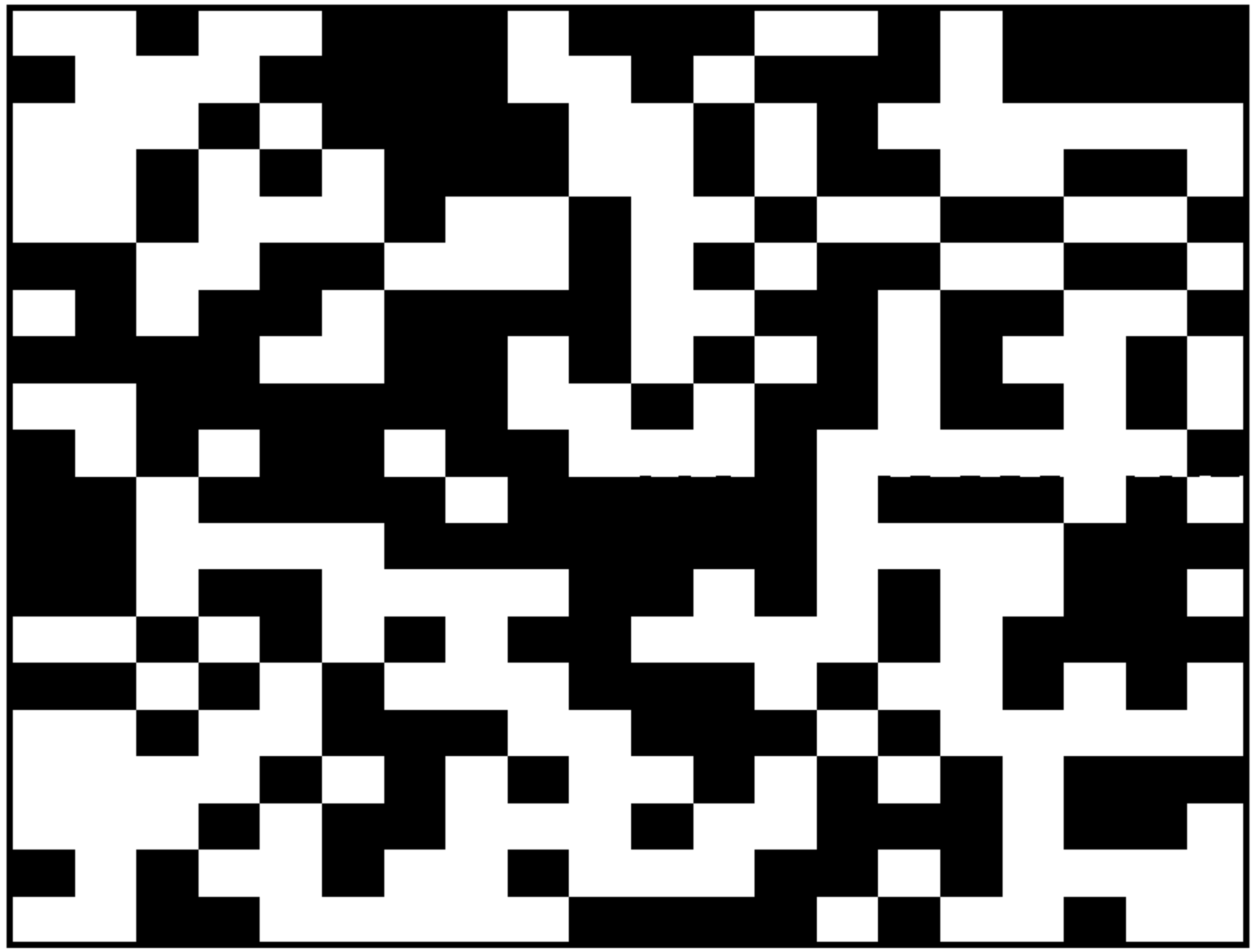
$$\mathbf{A}_{ij} = \begin{cases} 1 & \text{if entry } (i,j) \text{ observed} \\ 0 & \text{otherwise} \end{cases}$$

What determines if $\mathbf{A}$ is good for matrix completion?

Example: ideal mask

Samples chosen uniformly at random

$$\mathbf{A} =$$



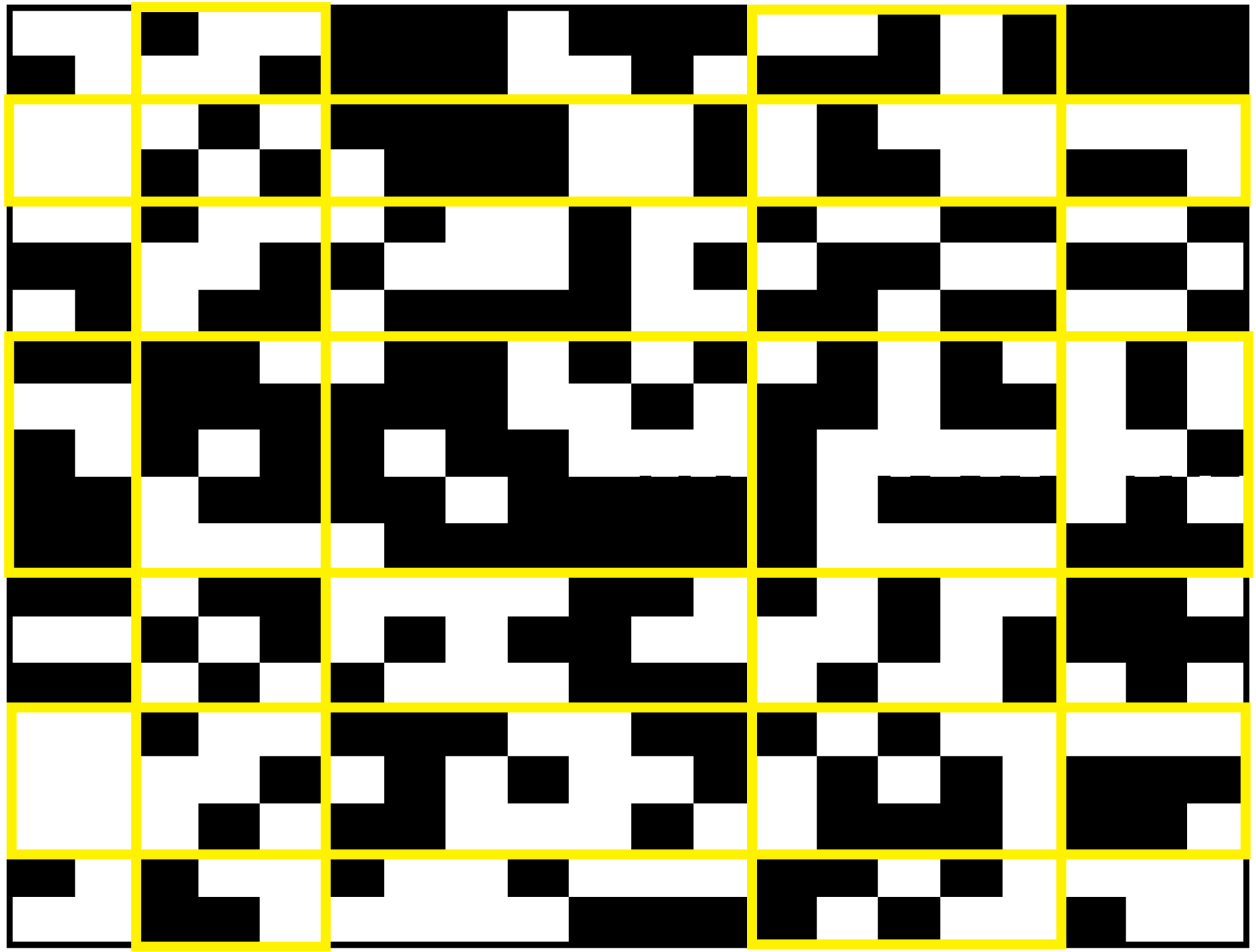
$\blacksquare = 0$

$\square = 1$

Example: ideal mask

Choose any sub matrix

$\mathbf{A} =$



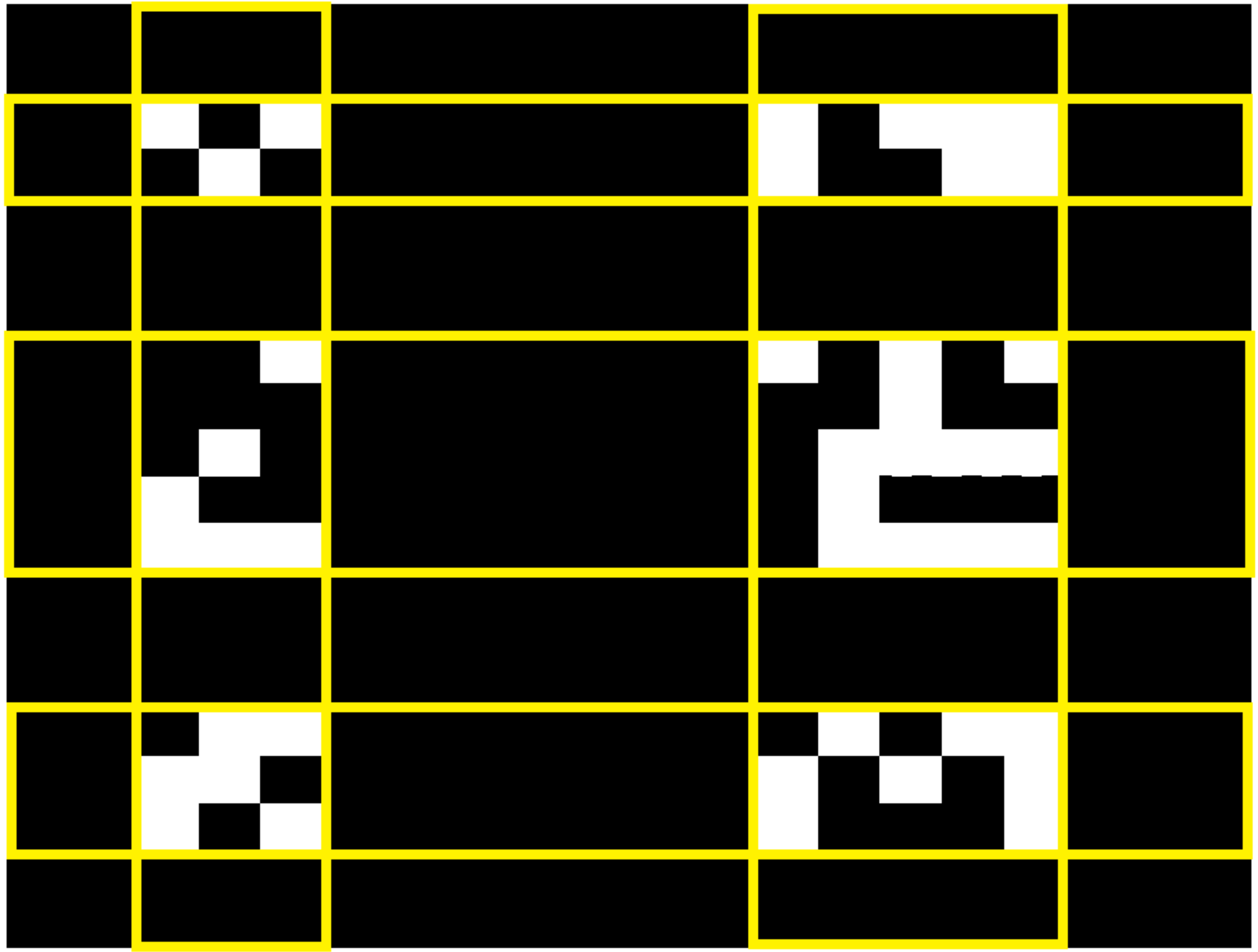
$\blacksquare = 0$

$\square = 1$

Example: ideal mask

All sub matrices are nicely sampled!

$\mathbf{A} =$



$\blacksquare = 0$

$\square = 1$

[Bhojanapalli and Jain, 2014, “Universal Matrix Completion” ICML]

Spectral gap

Consider the gap between the two largest singular values of $\mathbf{A}$

$$\frac{\sigma_2}{\sigma_1} = \begin{cases} \approx 1 & \text{if small spectral gap} \\ << 1 & \text{if large spectral gap} \end{cases}$$

where σ_i is the i-th largest singular value of $\mathbf{A}$

[Bhojanapalli and Jain, 2014, "Universal Matrix Completion" ICML]

Spectral gap

$$\frac{\sigma_2}{\sigma_1} = \begin{cases} \approx 1 & \text{if small spectral gap} \\ << 1 & \text{if large spectral gap} \end{cases}$$

From graph theory literature:

$\mathbf{A}$ with Large Spectral Gap $\implies$ all "sub matrices" are nicely sampled

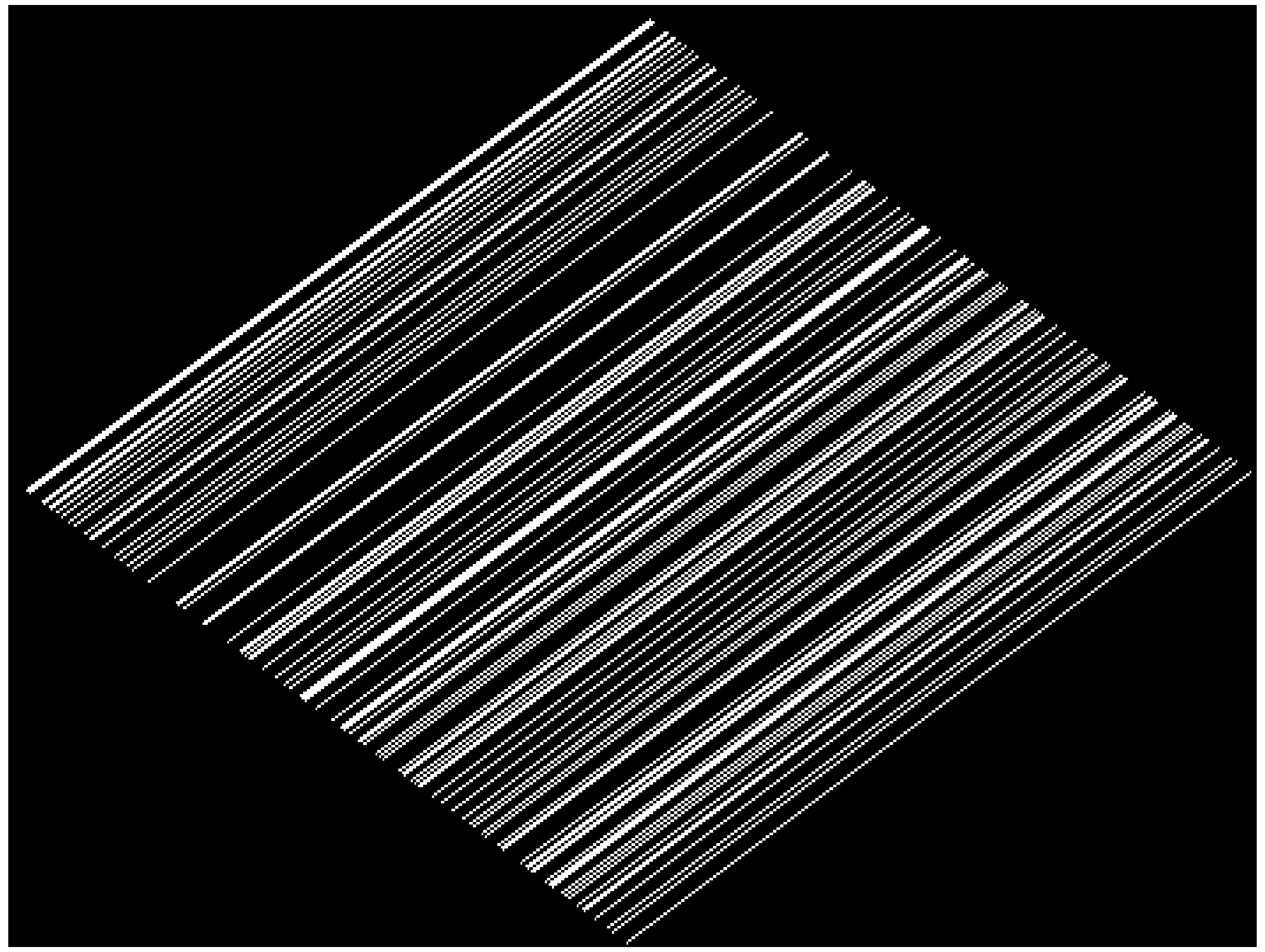
$\implies$ better results for matrix completion

2D interpolation experiments

Generate 2D seismic masks with increasing spectral gap

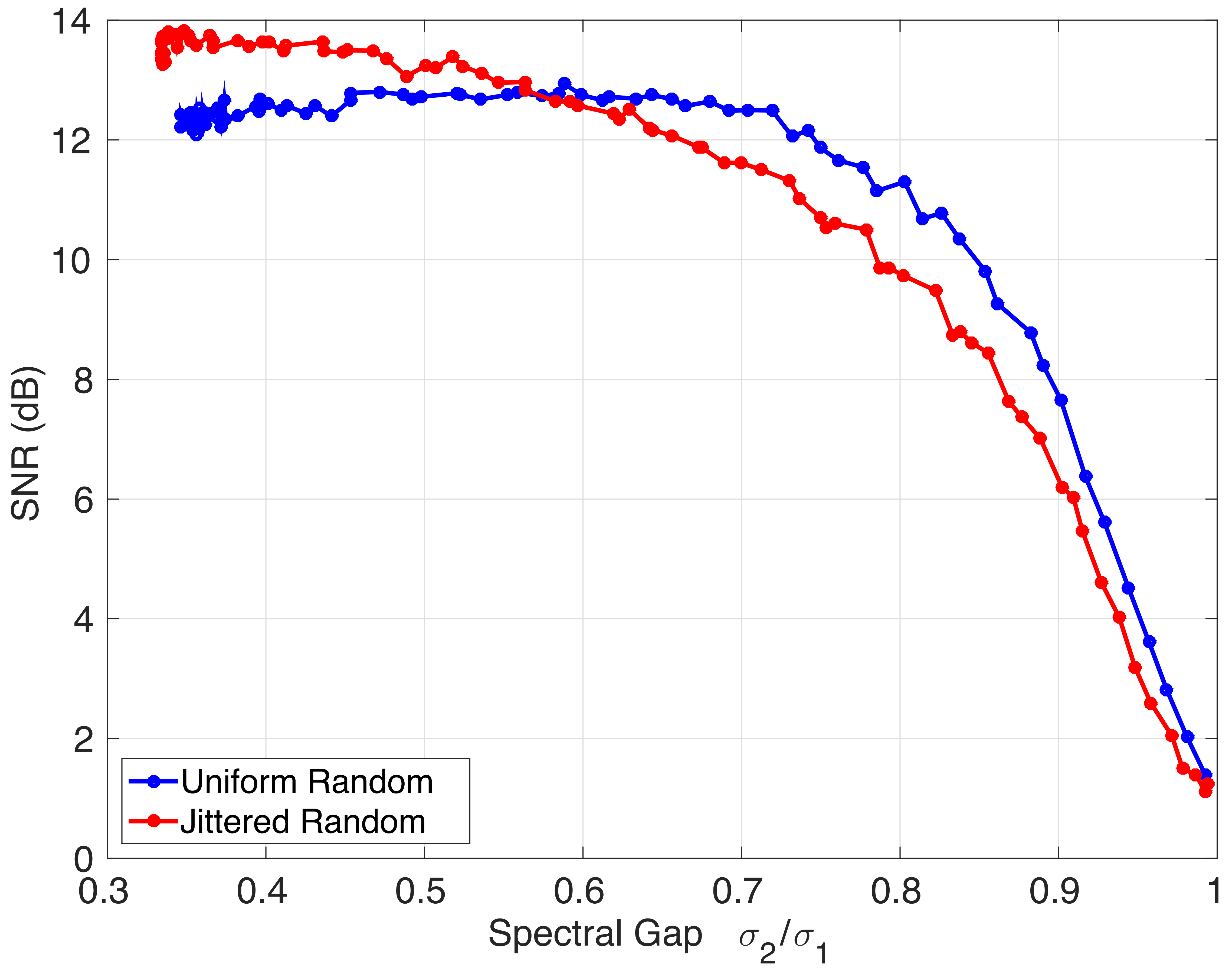
Plot correlation with reconstruction SNR

Uniform random vs Jittered random



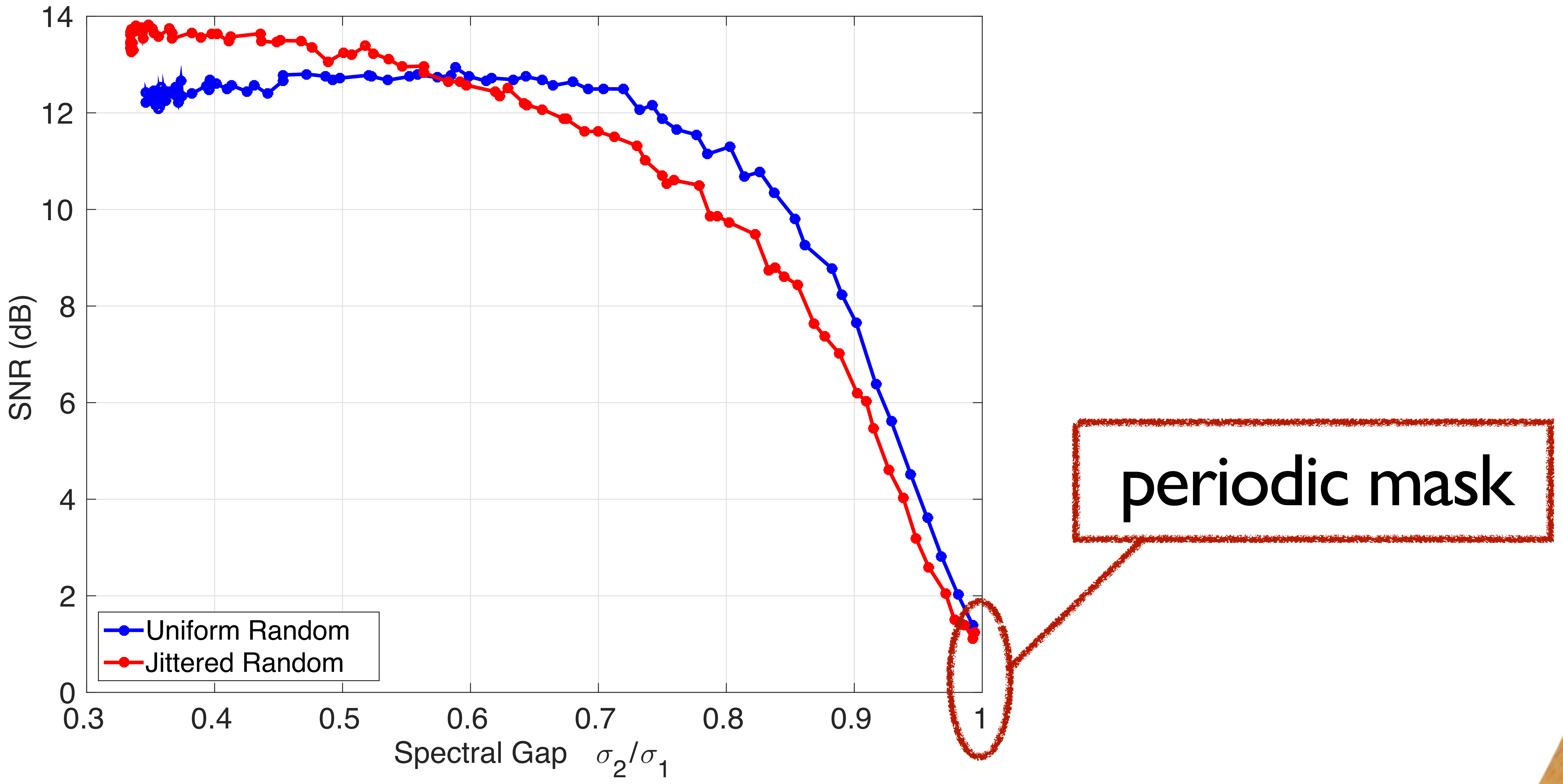
[Gulf of Suez data (OBC), frequency slice at 10 Hz]

2D interpolation experiments: 75% missing sources



[Gulf of Suez data (OBC), frequency slice at 10 Hz]

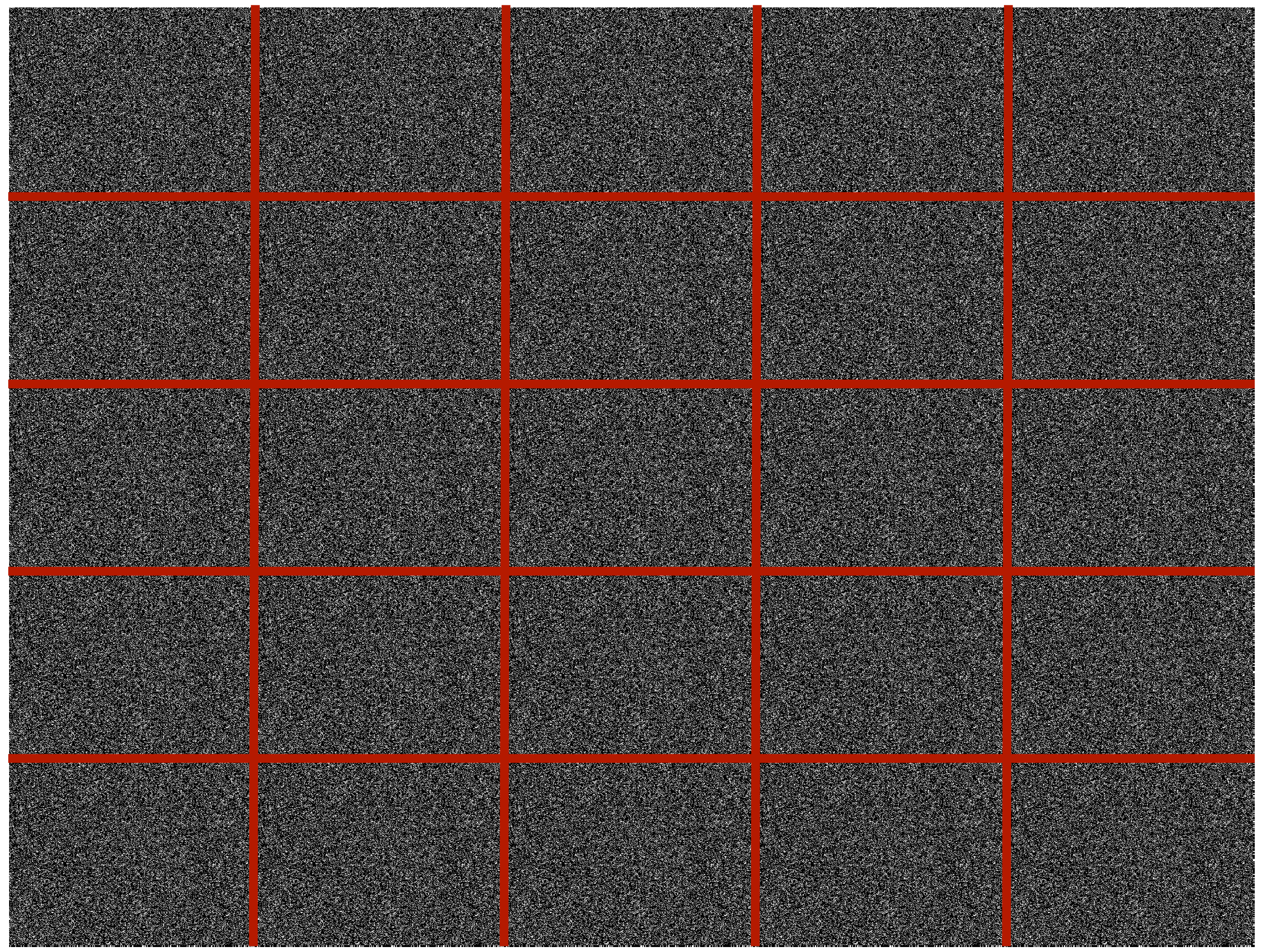
2D interpolation experiments: 75% missing sources



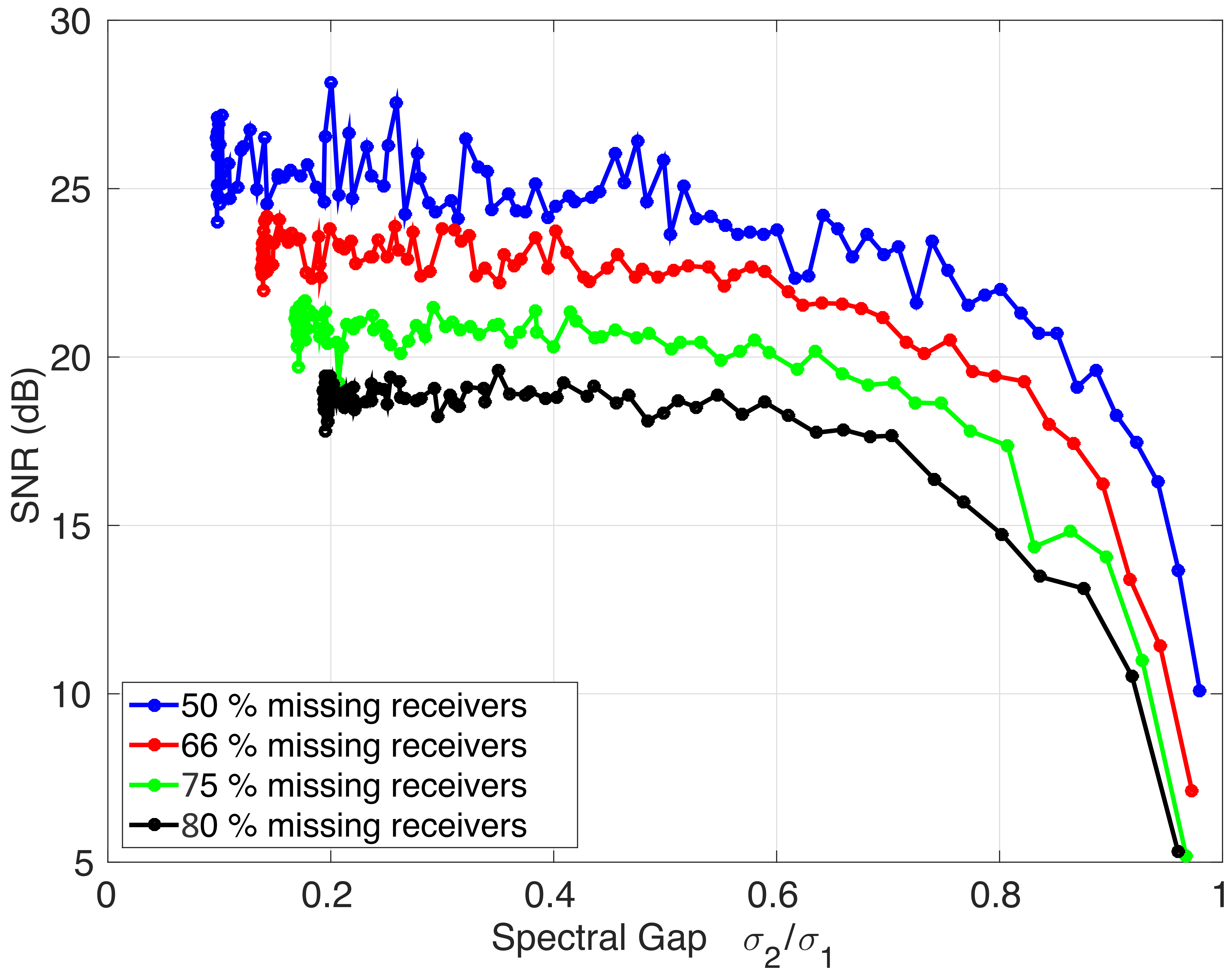
3D interpolation experiments

Generate 3D seismic masks with increasing spectral gap

Plot correlation with reconstruction SNR



3D interpolation experiments



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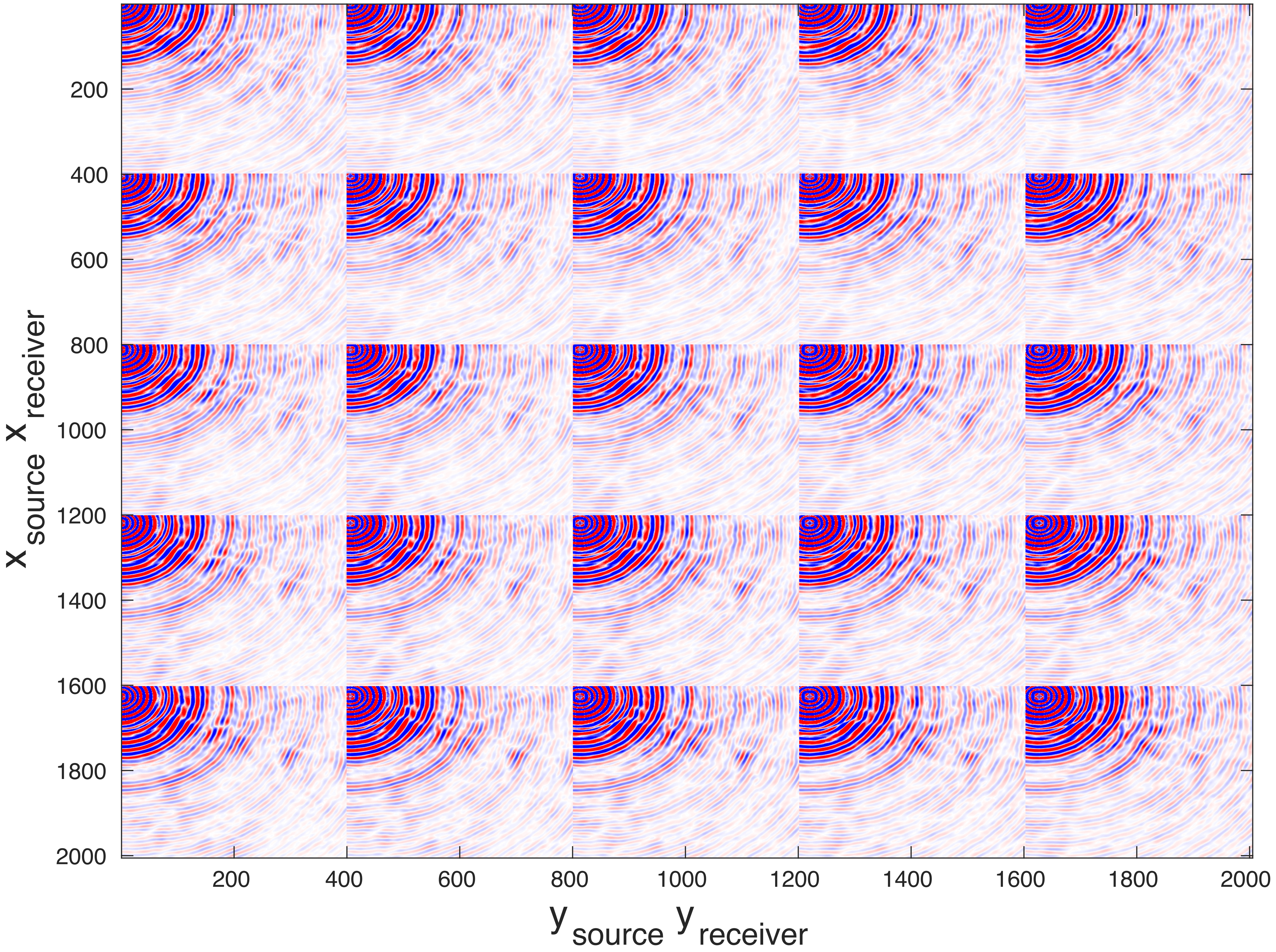
- How should we subsample?
  - spectral gap analysis

- Experiments
  - trace interpolation
  - source separation

3D interpolation results



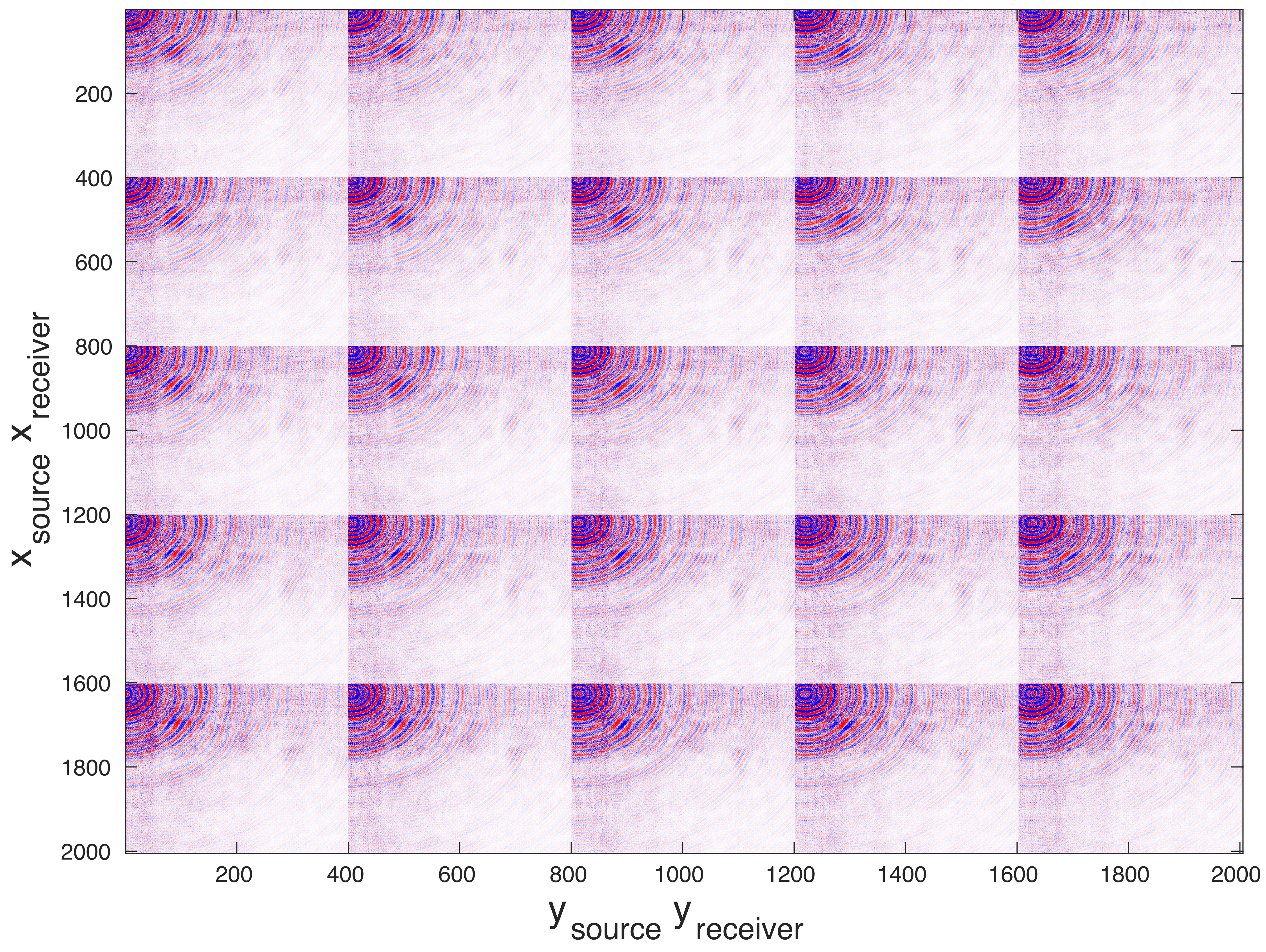
fully sampled data

size: 2005 x 2005

remove 75% of receivers

[BG data (OBN), frequency slice at 75 Hz]

3D interpolation results: bad recovery

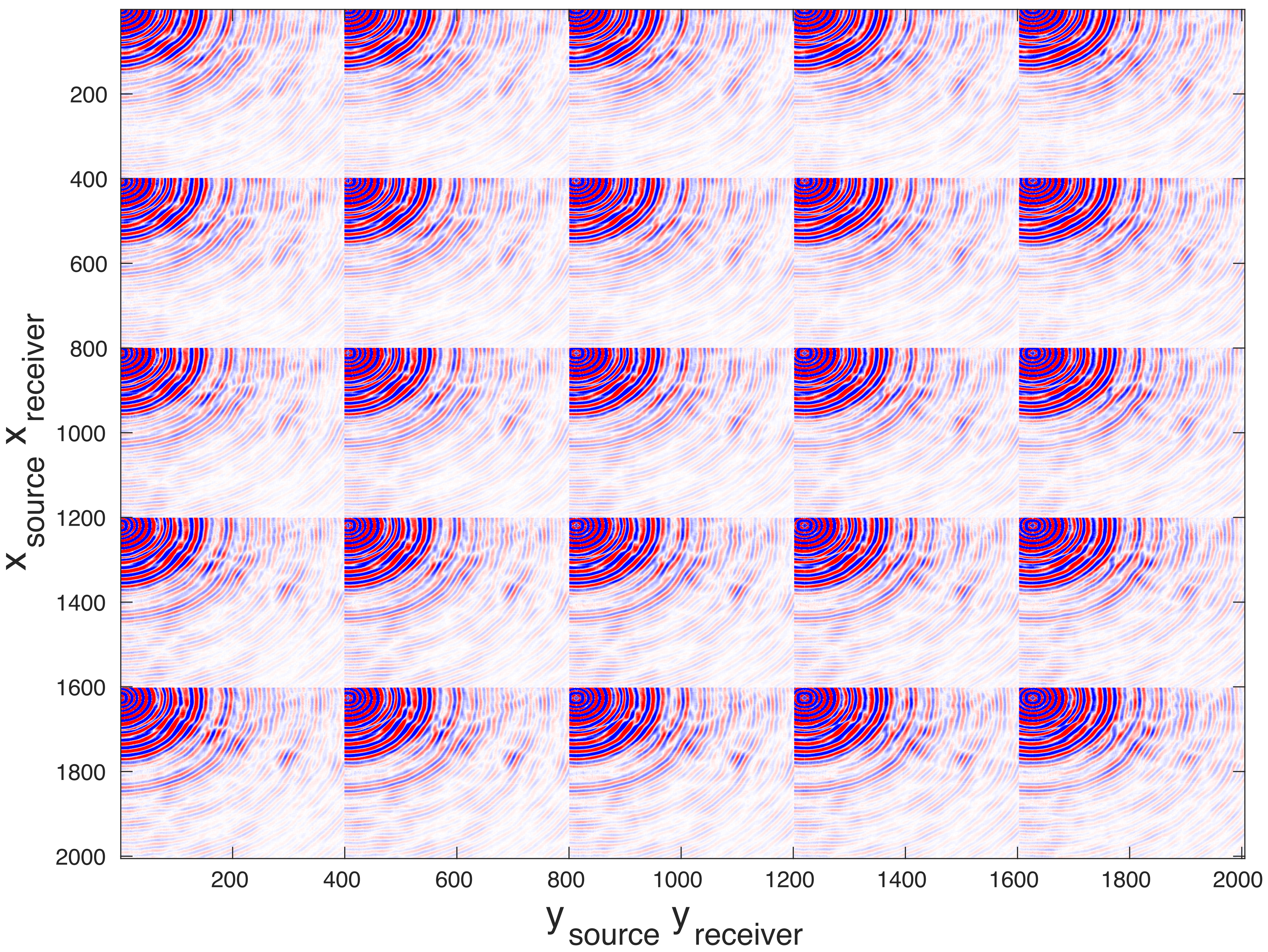


SNR : 3.5 dB

$$\frac{\sigma_2}{\sigma_1} : 0.9828$$

[BG data (OBN), frequency slice at 75 Hz]

3D interpolation results: good recovery



SNR : 20.7 dB

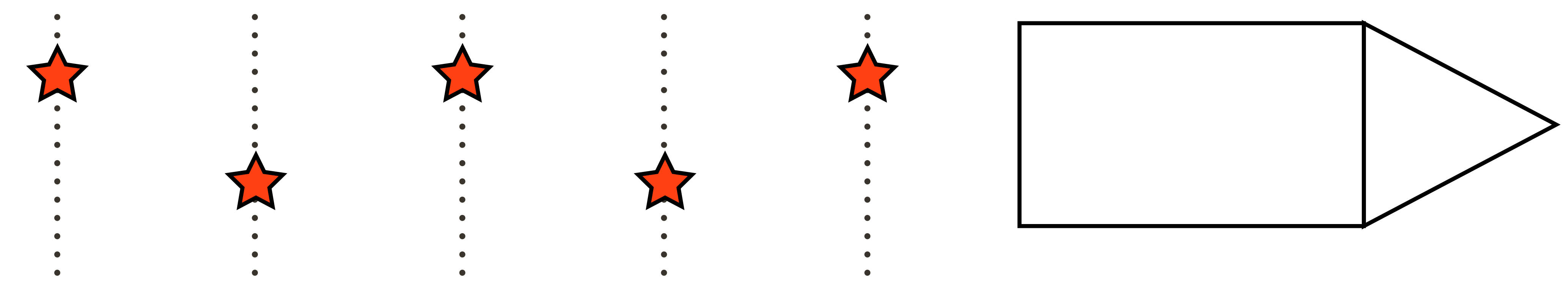
$$\frac{\sigma_2}{\sigma_1} : 0.1796$$

Source separation

Periodic vs. jittered marine acquisition

variability in firing times

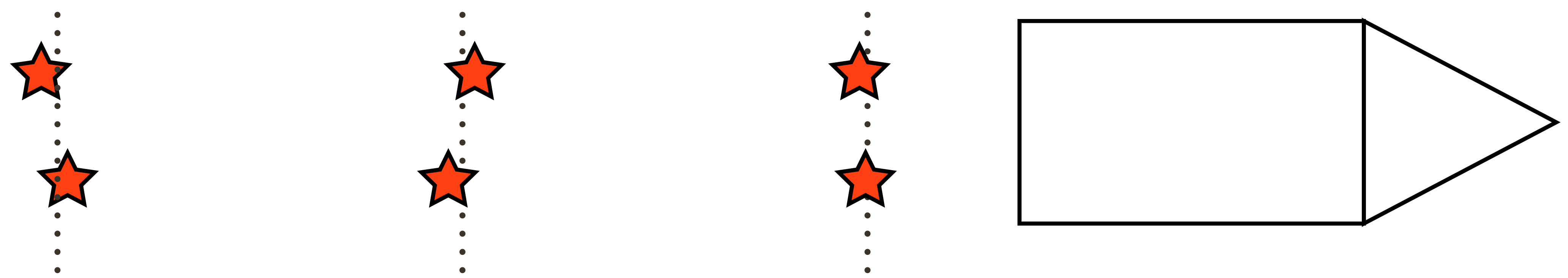
periodically sampled spatial grid



NONE

almost periodically sampled spatial grid

(over/under acquisition, towed arrays)

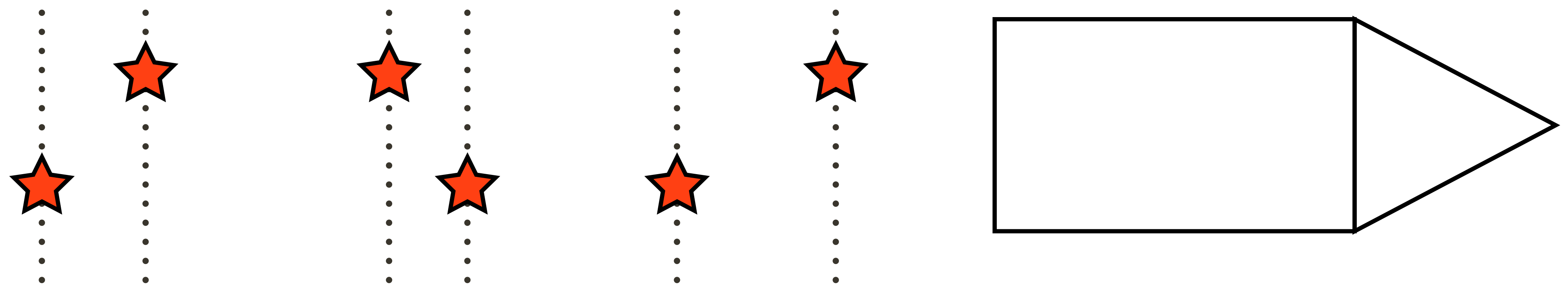


LOW

randomly jittered sampled spatial grid

(Time-jittered acquisition, OBC/OBN)

[Wason and Herrmann, 2013]
[Mansour et. al., 2012]

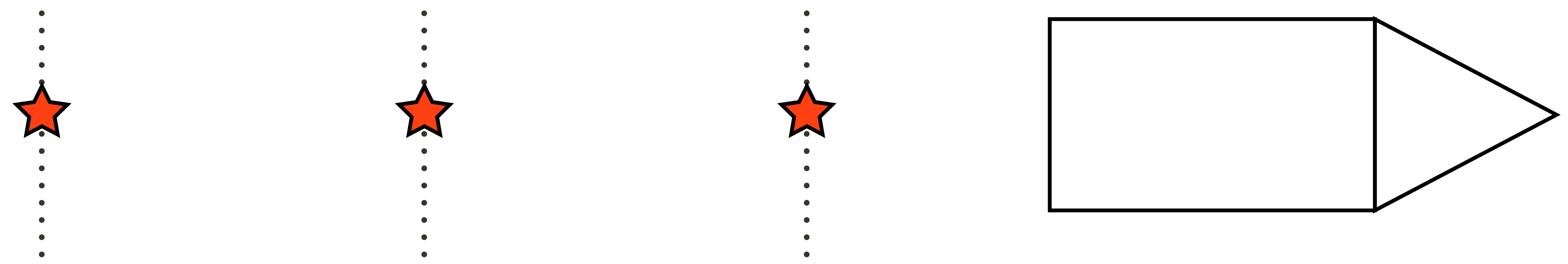


HIGH

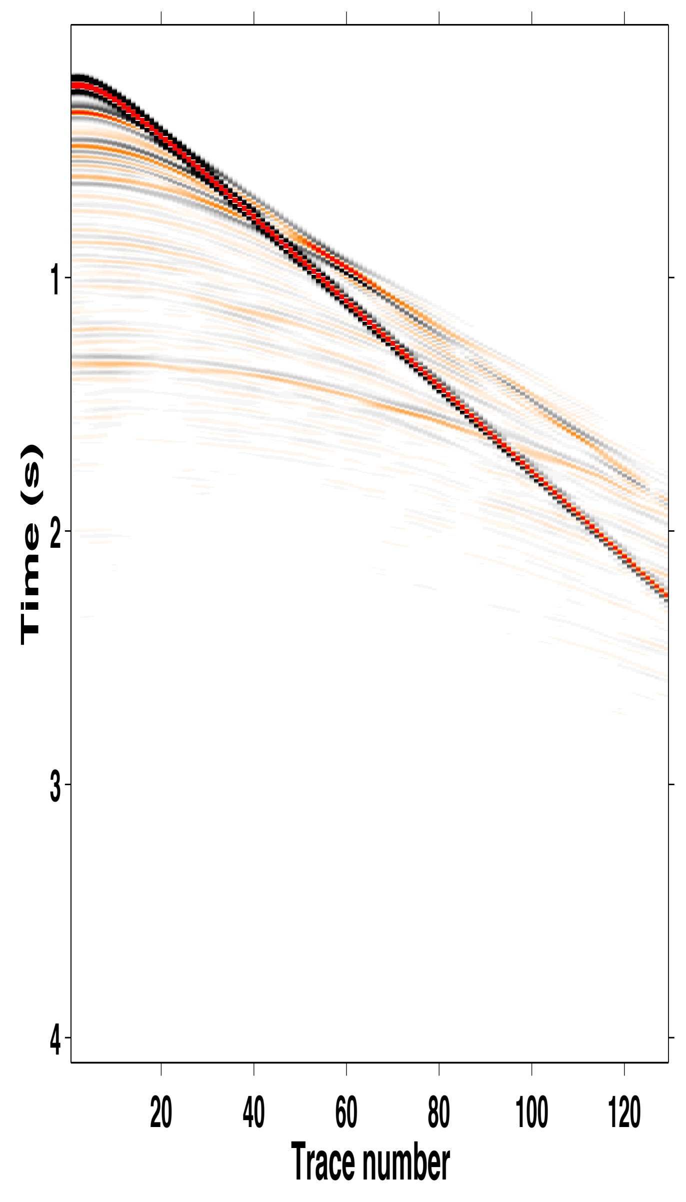
Conventional marine acquisition

shot 1 shot 2 shot 3

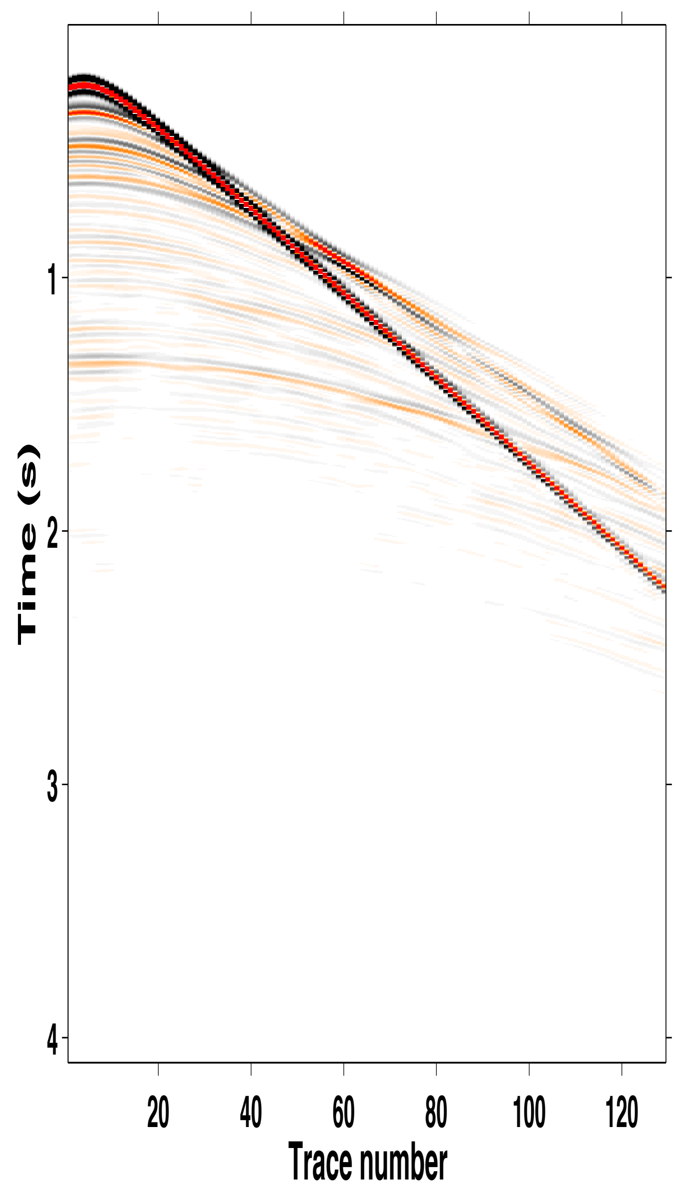
periodically sampled spatial grid



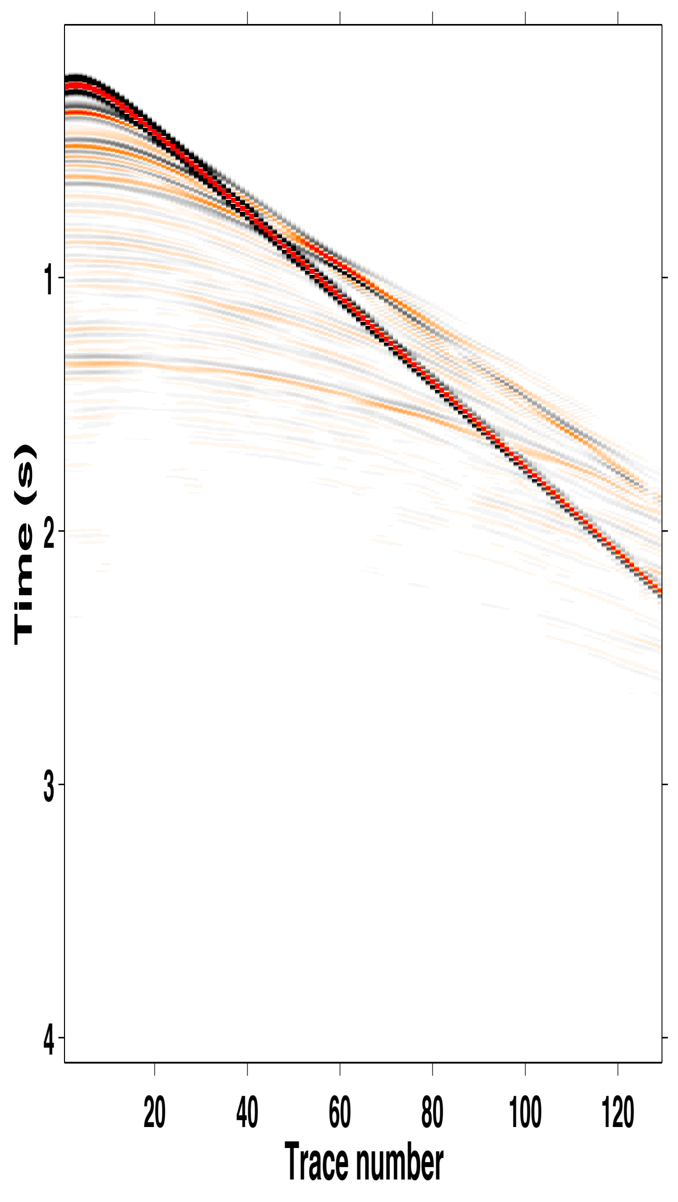
shot 1



shot 2



shot 3



Simultaneous marine acquisition

[over/under acquisition, towed arrays]

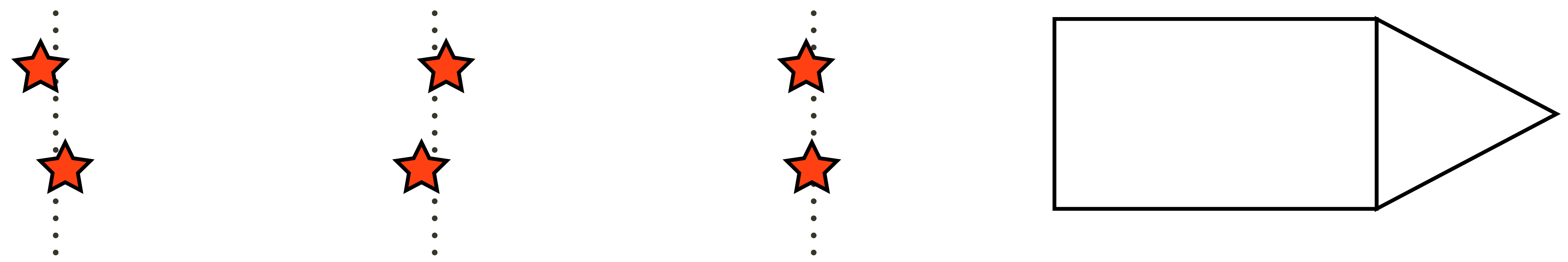
source depth 1

source depth 2

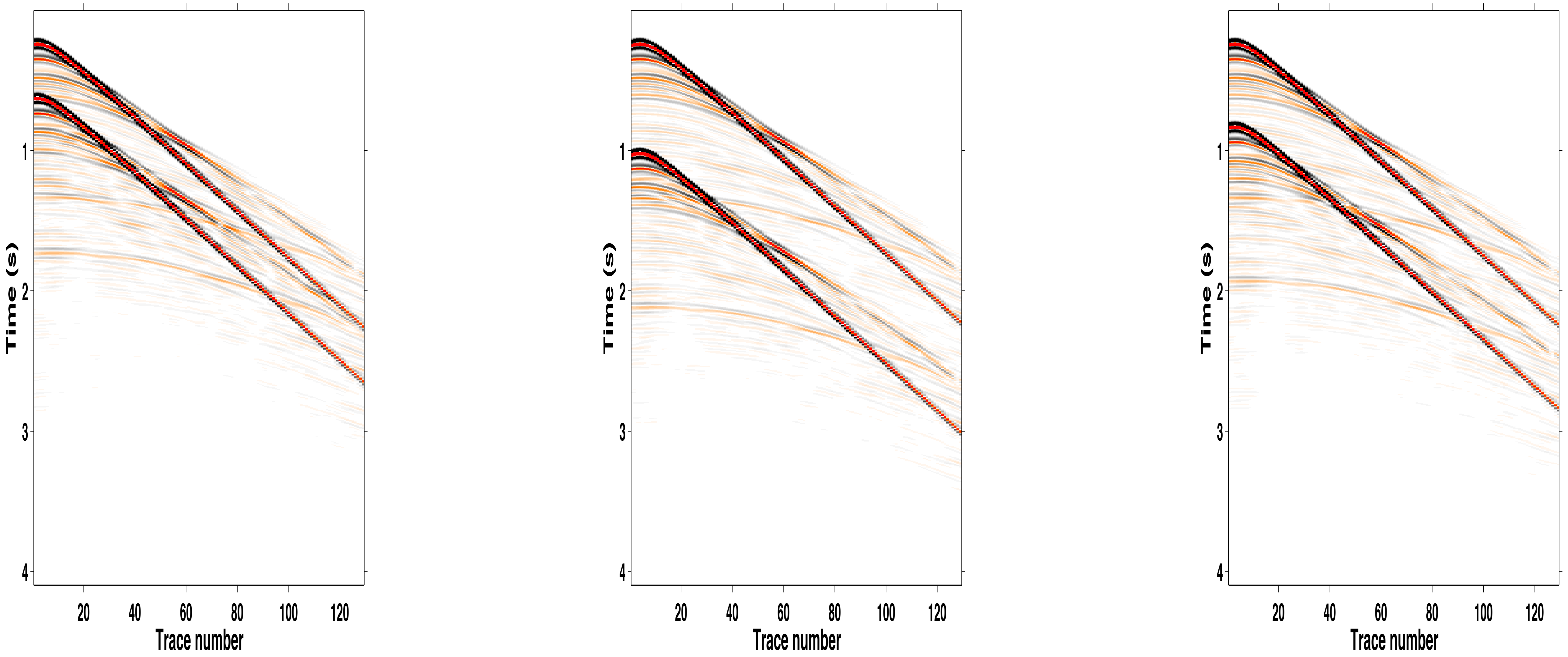
shot 1 shot 2 shot 3

almost periodically sampled spatial grid

shot-time randomness - **LOW**



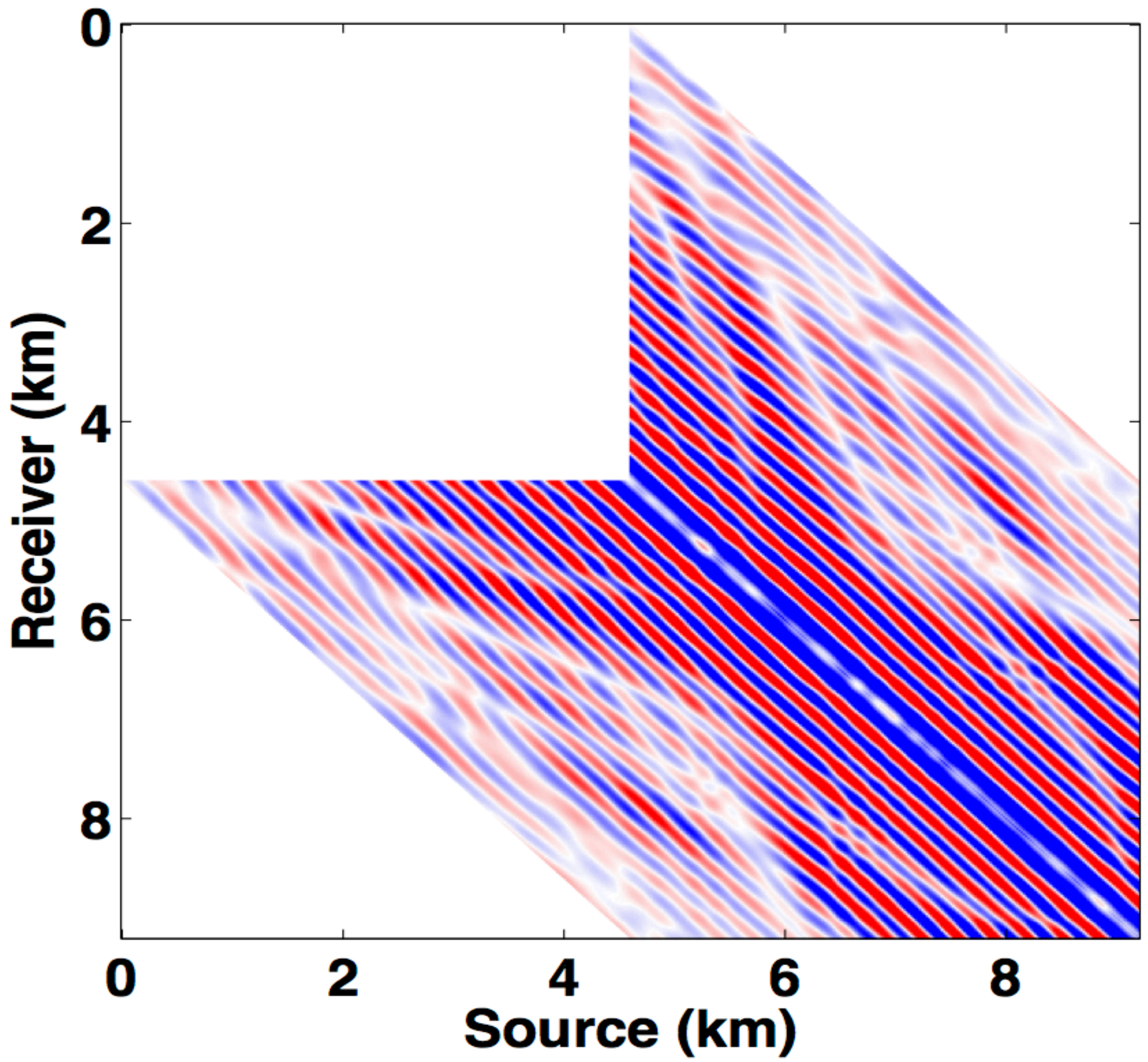
shot 1 shot 2 shot 3



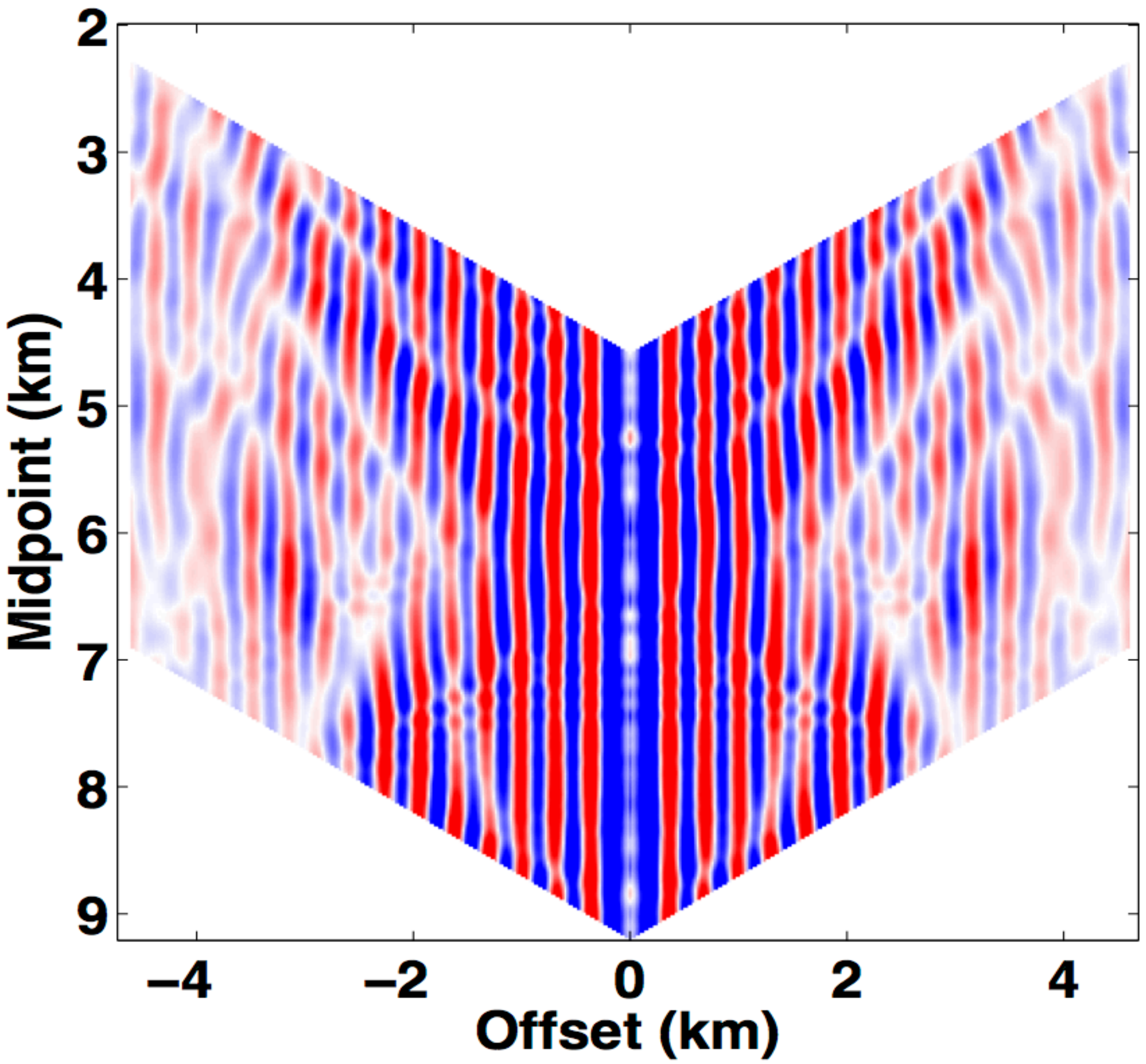
Low-rank structure in which domain?

[frequency slice at 5 Hz]

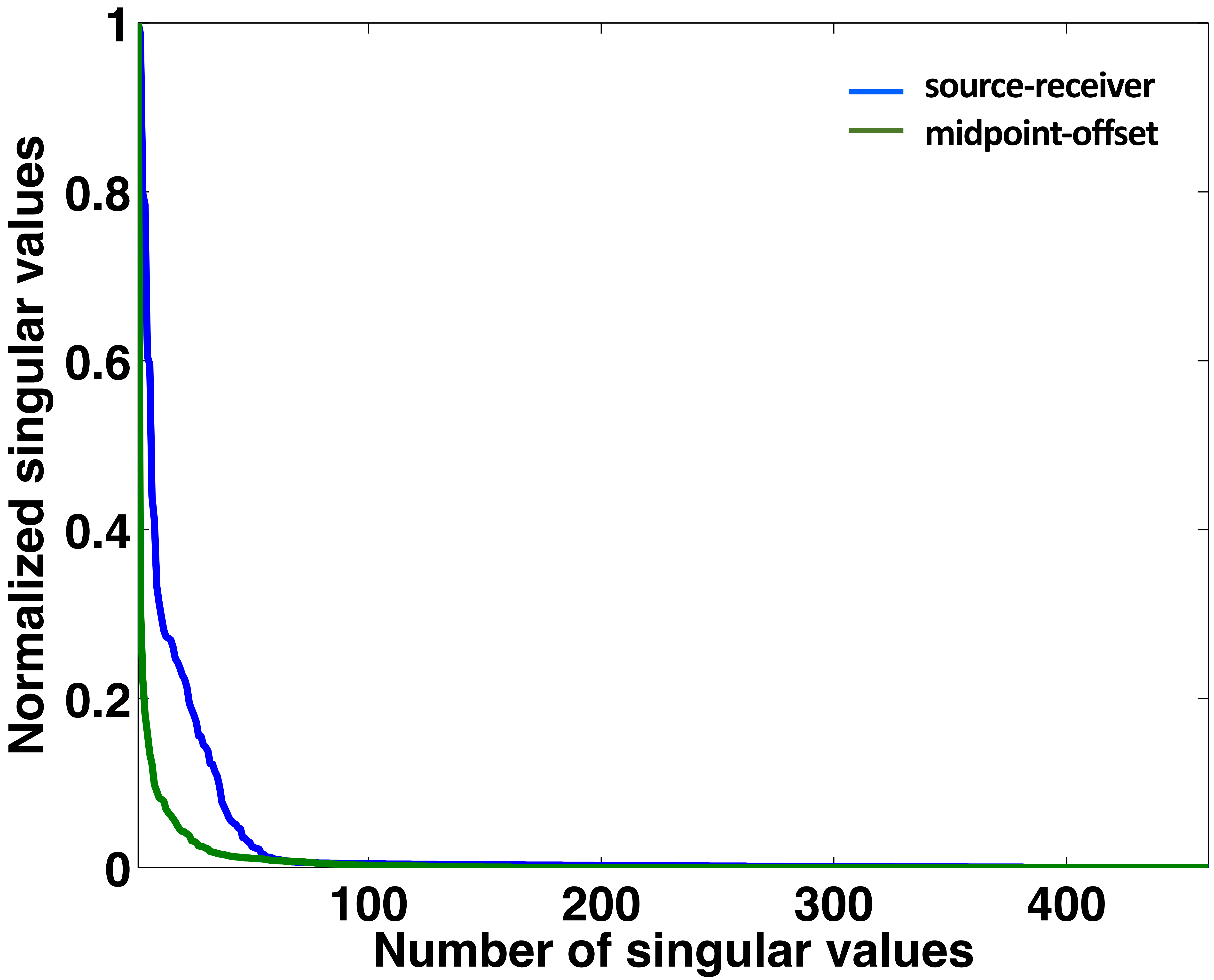
source-receiver domain
(with reciprocity)



midpoint-offset domain
(with reciprocity)



Decay of singular values



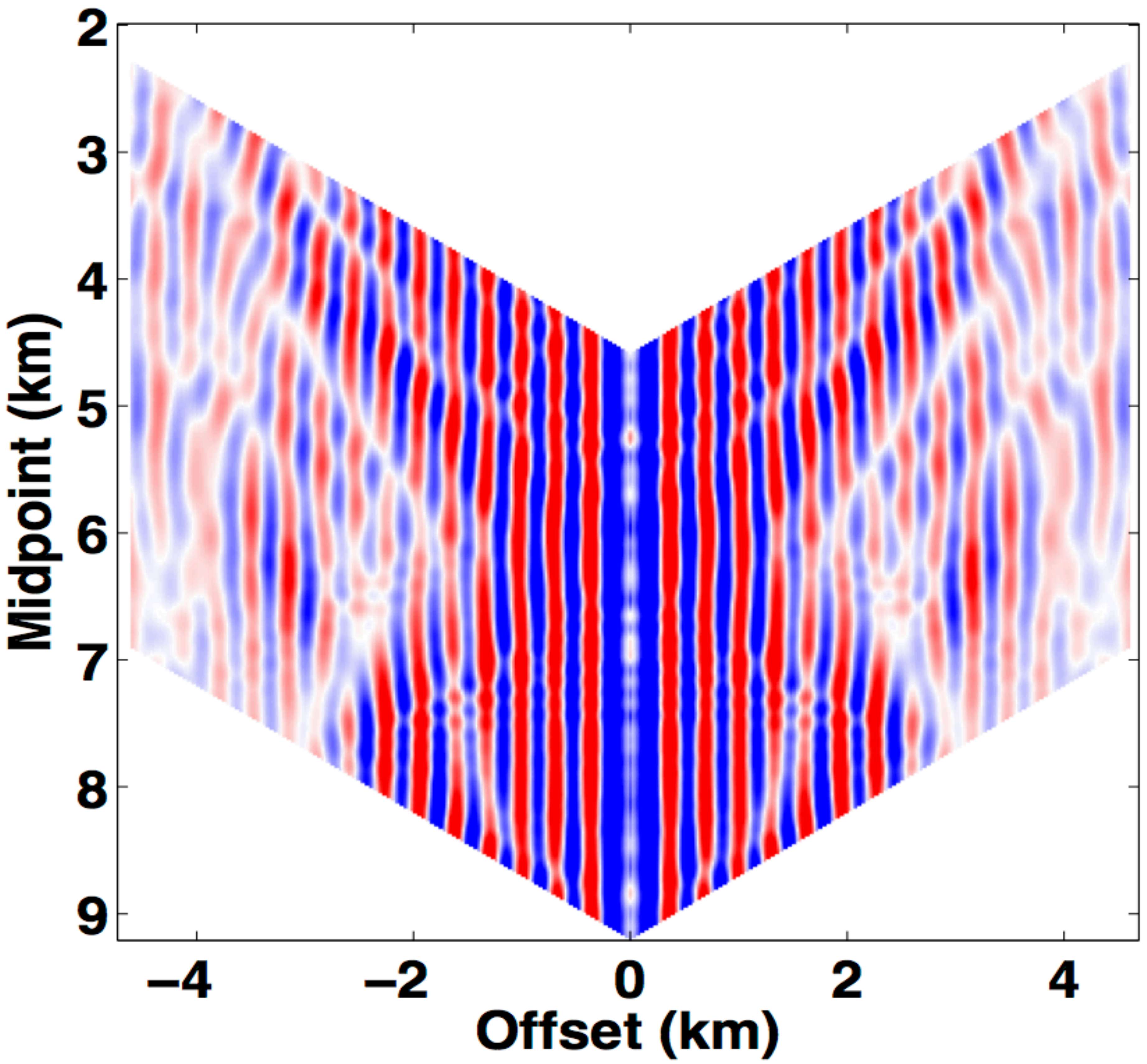
low-rank in midpoint-offset domain

How to destroy the structure?

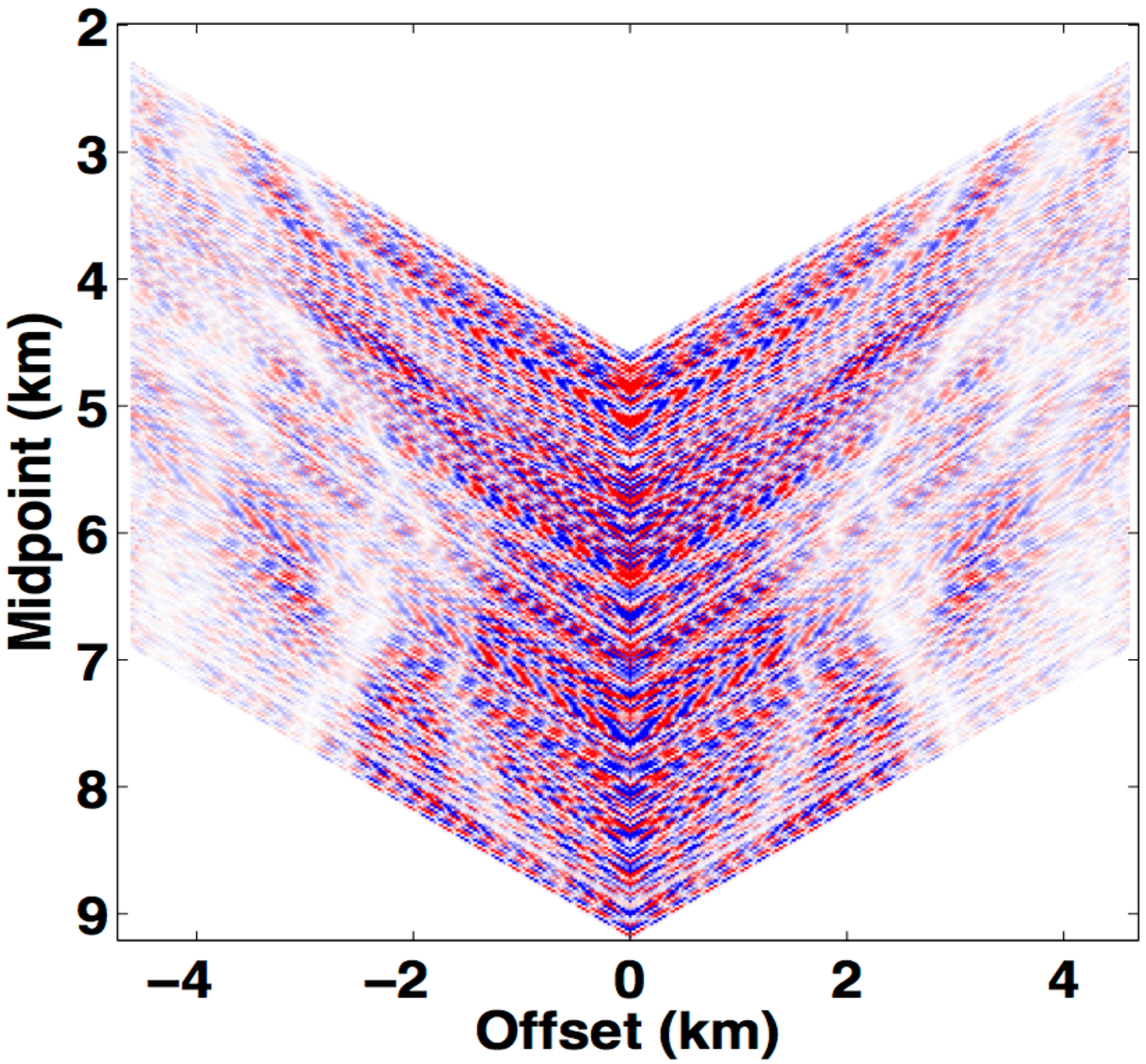
[add random time delays]

no missing traces!

without delays

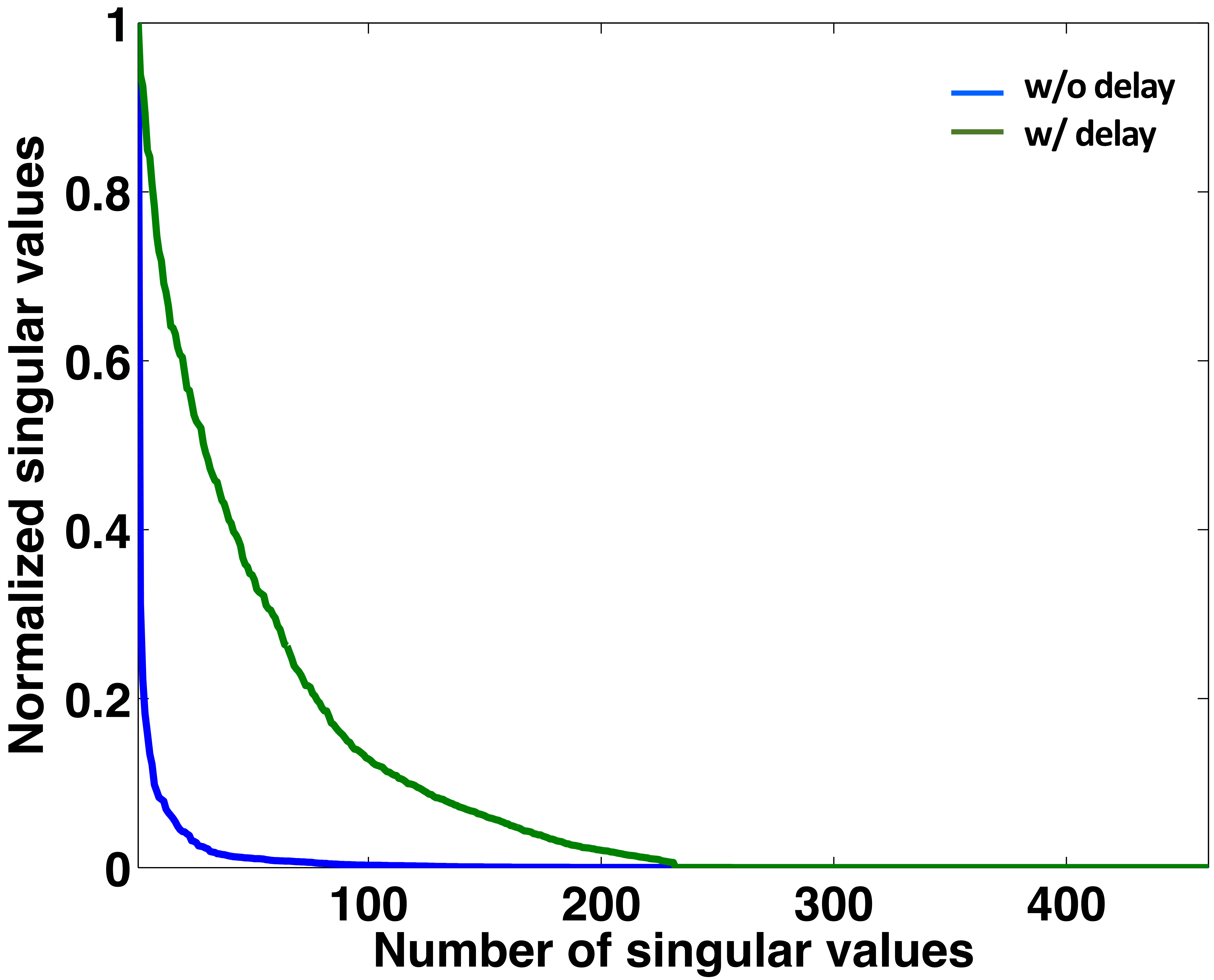


with random delays (< 1s)



Decay of singular values

[midpoint-offset domain]



random time delays increase the rank

Nuclear-norm minimization

[Recht et. al., 2010]

$$\min_{\mathbf{X}} \quad \underbrace{||\mathbf{X}||_*}_{\text{sum of singular values of } \mathbf{X}} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

for blended acquisition:

unblended data matrix

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \begin{array}{l} \longleftarrow \text{source 1} \\ \longleftarrow \text{source 2} \end{array}$$

$\mathbf{b}$: blended data

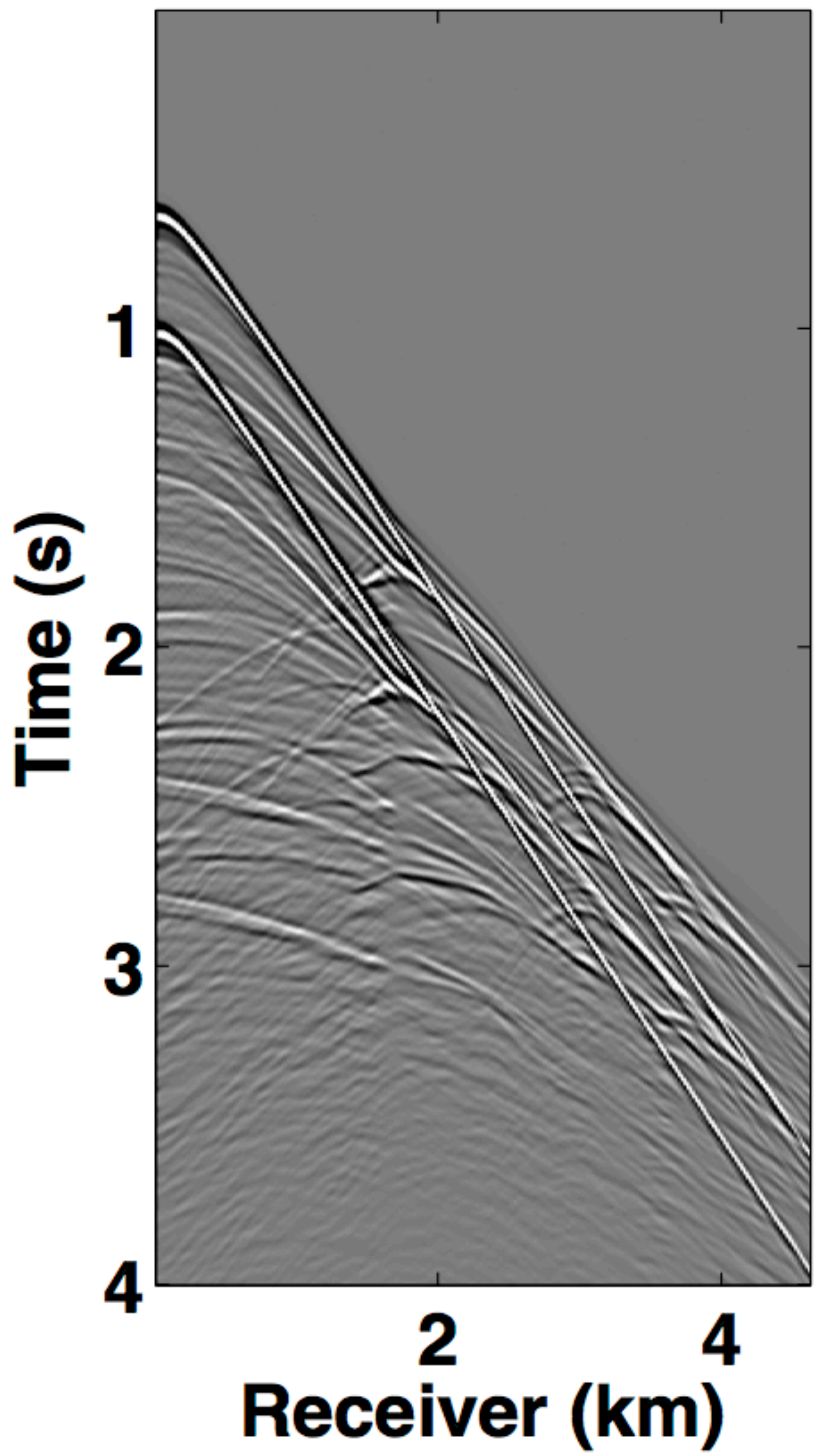
$$\mathcal{A} := [\mathbf{M}\mathbf{T}_1\mathbf{S}^{\mathbf{H}} \quad \mathbf{M}\mathbf{T}_2\mathbf{S}^{\mathbf{H}}]$$

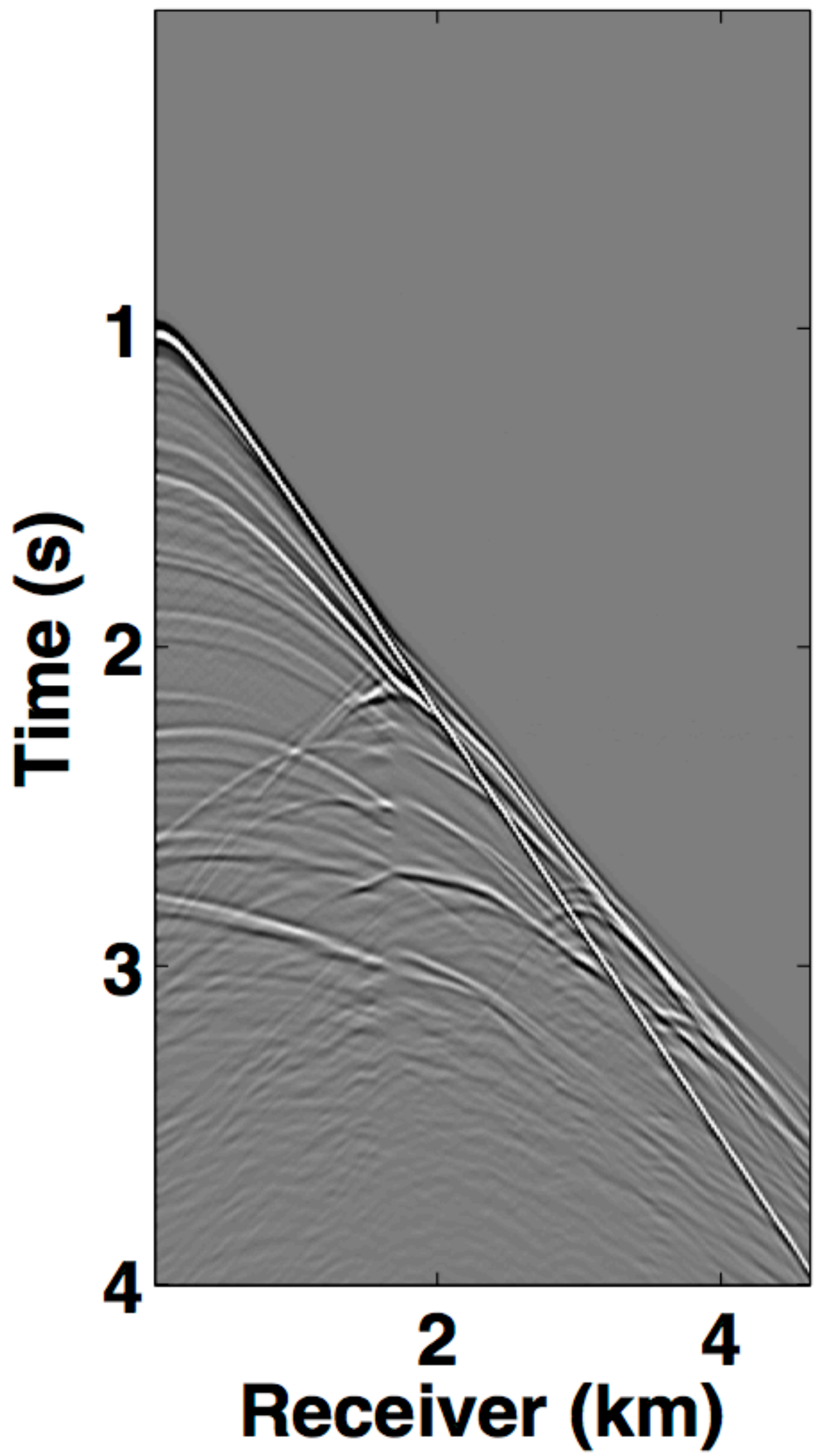
time delay matrices

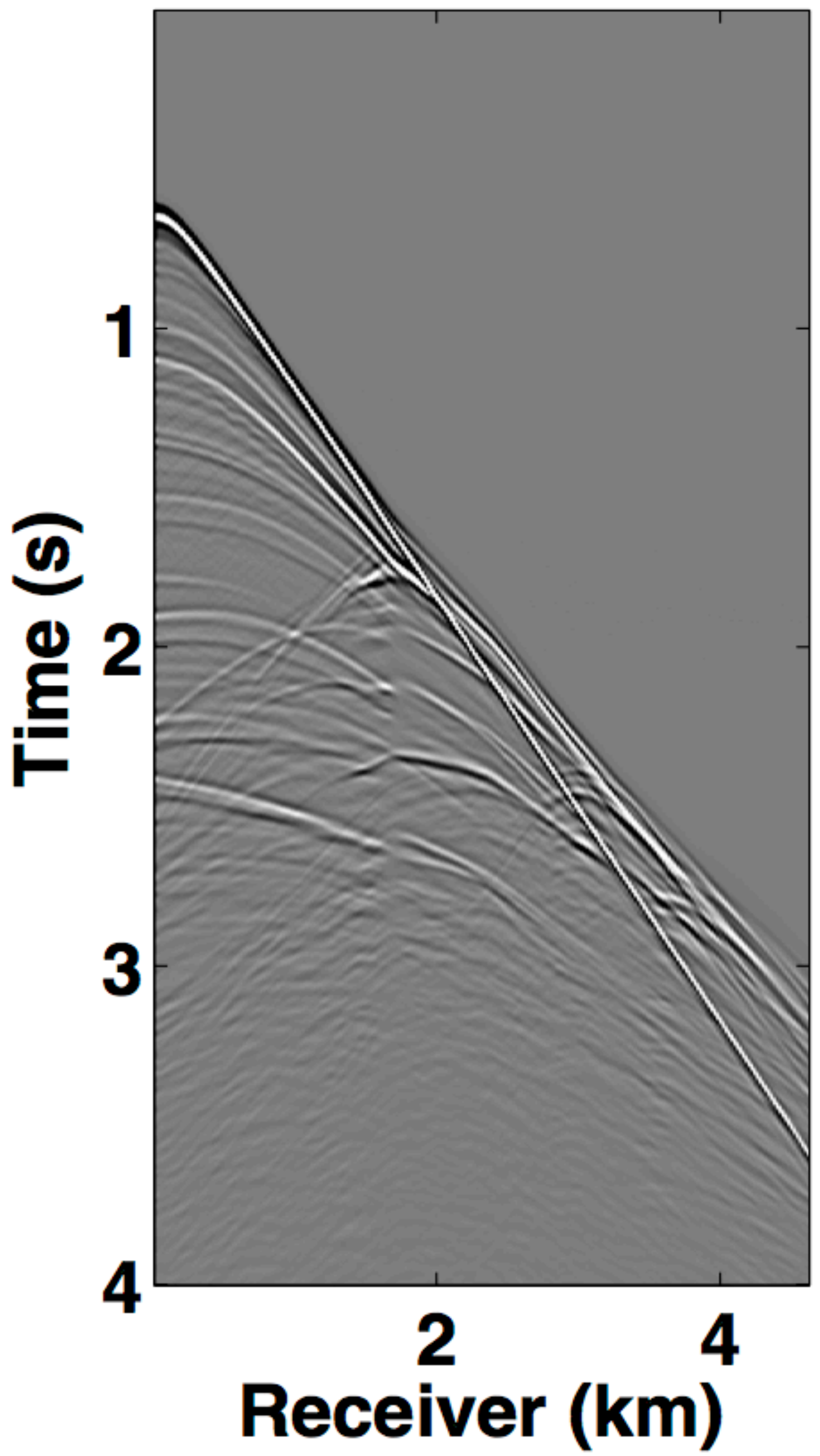
Blended data (w/ delay)

random time delays (< 1 sec) applied to both sources

blended shot = source 1 + source 2







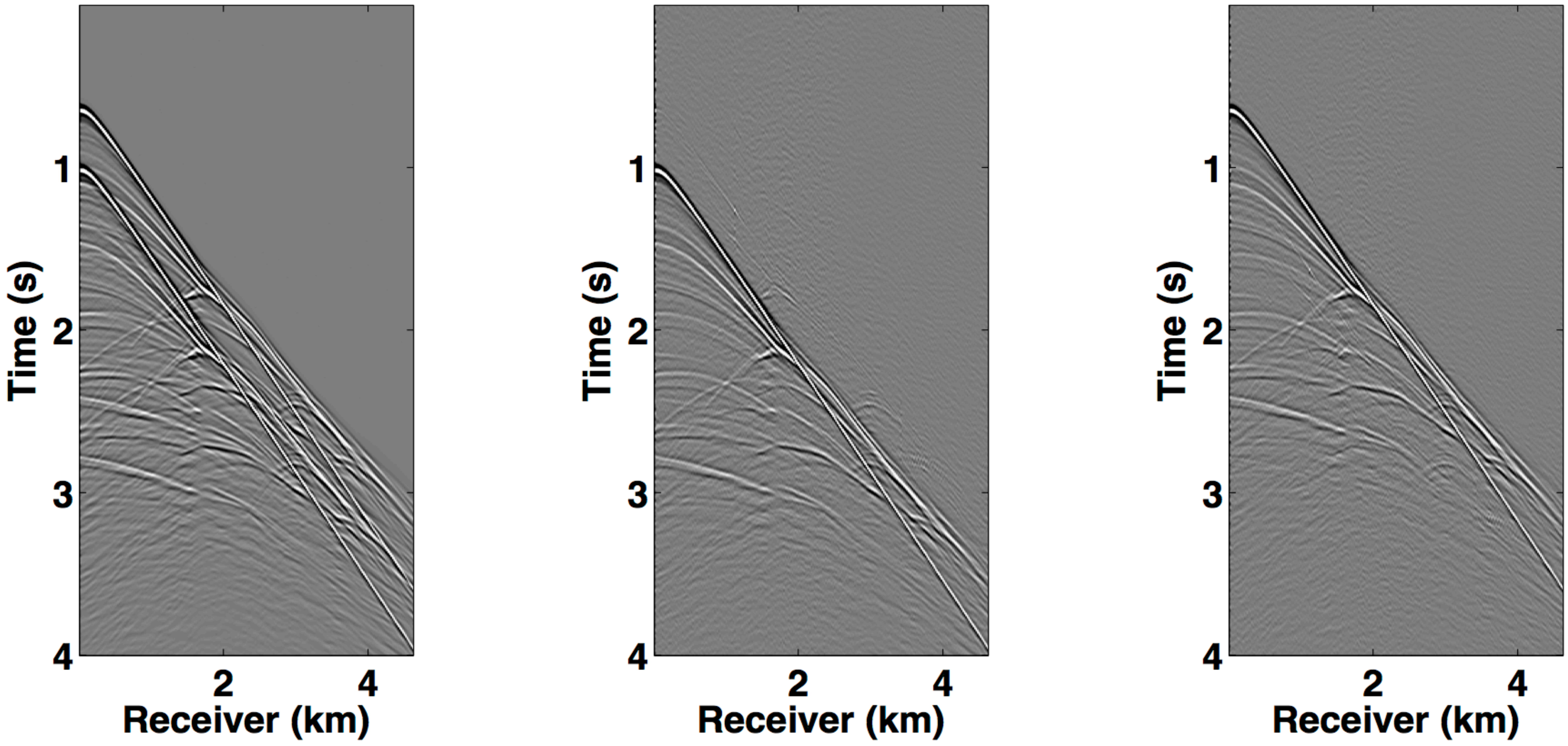
Source separation results

[Kumar et. al., 2015b]

blended shot

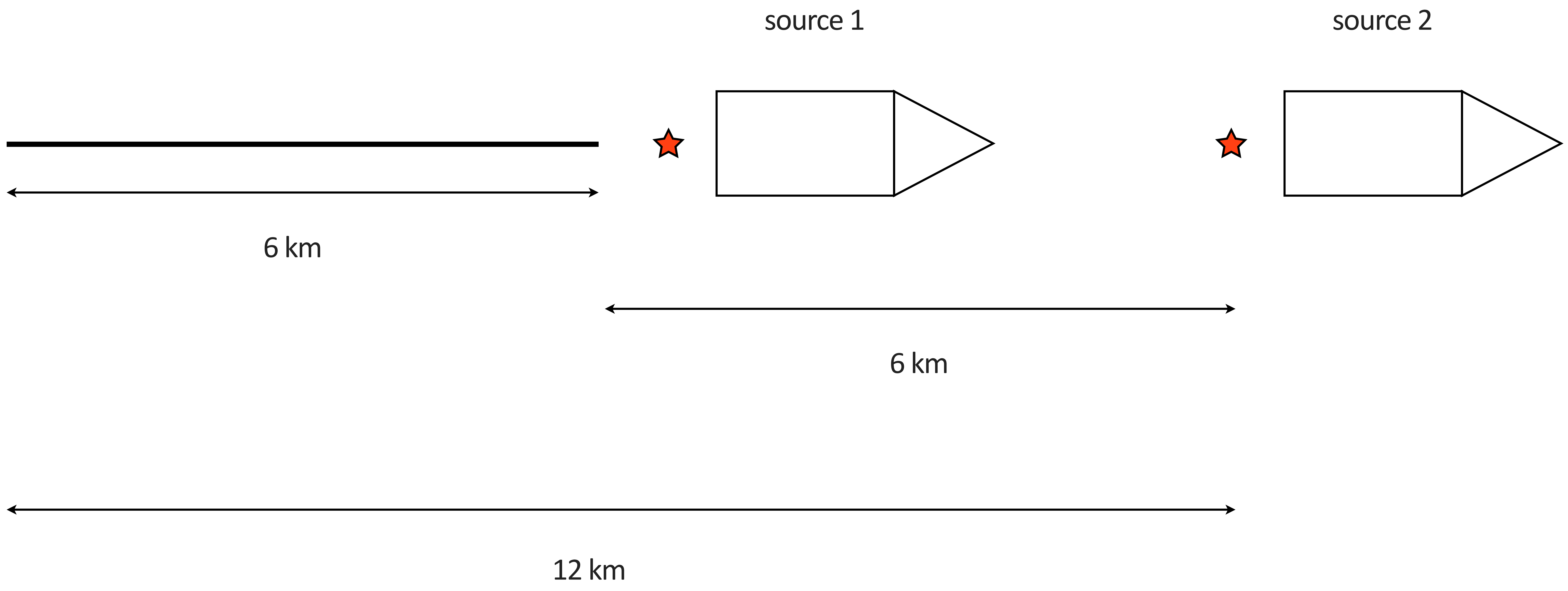
source 1
(15.0 dB)

source 2
(14.8 dB)



Simultaneous long offset acquisition

[adapted from Long et. al., 2013]

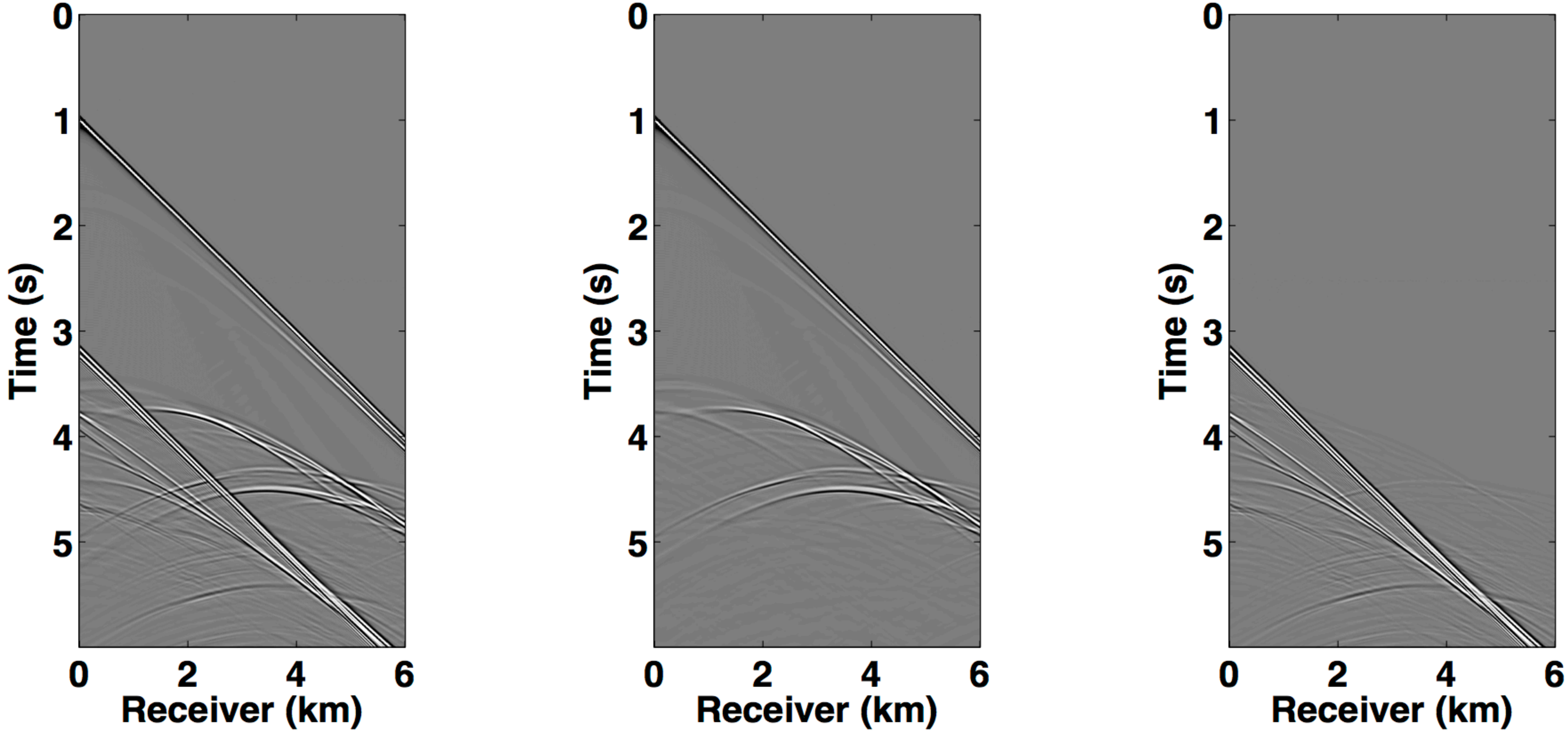


A. S. Long, et. al., “**Simultaneous long offset (SLO) towed streamer seismic acquisition**”, presented at the *75th EAGE Conference and Exhibition*, June 2013.

Blended data (w/ delay)

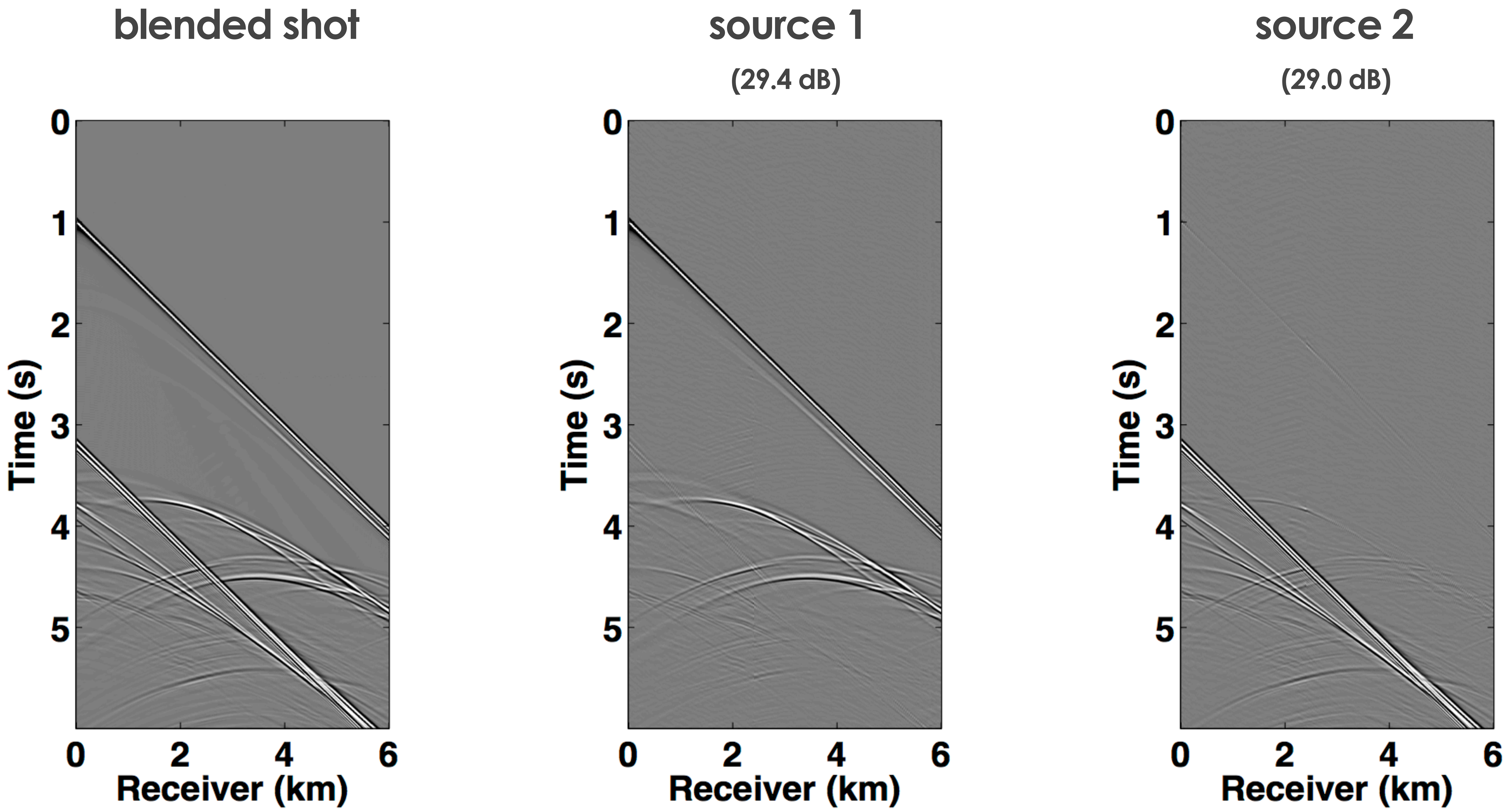
random time delays (< 1 sec) applied to both sources

blended shot = source 1 + source 2



Source separation results

[Kumar et. al., 2015b]



Summary

Good understanding of how to subsample

Simple procedure to quantify acquisition design

- compute only σ_1, σ_2 of sampling mask
- critical tools for 3D acquisition

Source separation for towed-streamer acquisition scenarios can be treated as a rank-minimization problem

References

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