Compressive Sensing in Exploration Seismology - where we came from, where we are now, and where we need to go

Felix J. Herrmann

SLIM
University of British Columbia

Motivation

Drivers:

- Wave-equation based inversions call for dense, wide-azimuth & long-offset surveys
- control on environmental impact
- economics

Solution:

- rethink sampling technologies for land & marine using insights from Compressive Sensing
- remove sub-sampling-related artifacts by carrying out structure-promoting inversionsCompressive Sensing = increased acquisition productivity

Mosher, C. C., Keskula, E., Kaplan, S. T., Keys, R. G., Li, C., Ata, E. Z., ... & Sood, S. (2012, November). Compressive Seismic Imaging. In *2012 SEG Annual Meeting*. Society of Exploration Geophysicists.

Randomized acquisition

examples from industry (ConocoPhilips)

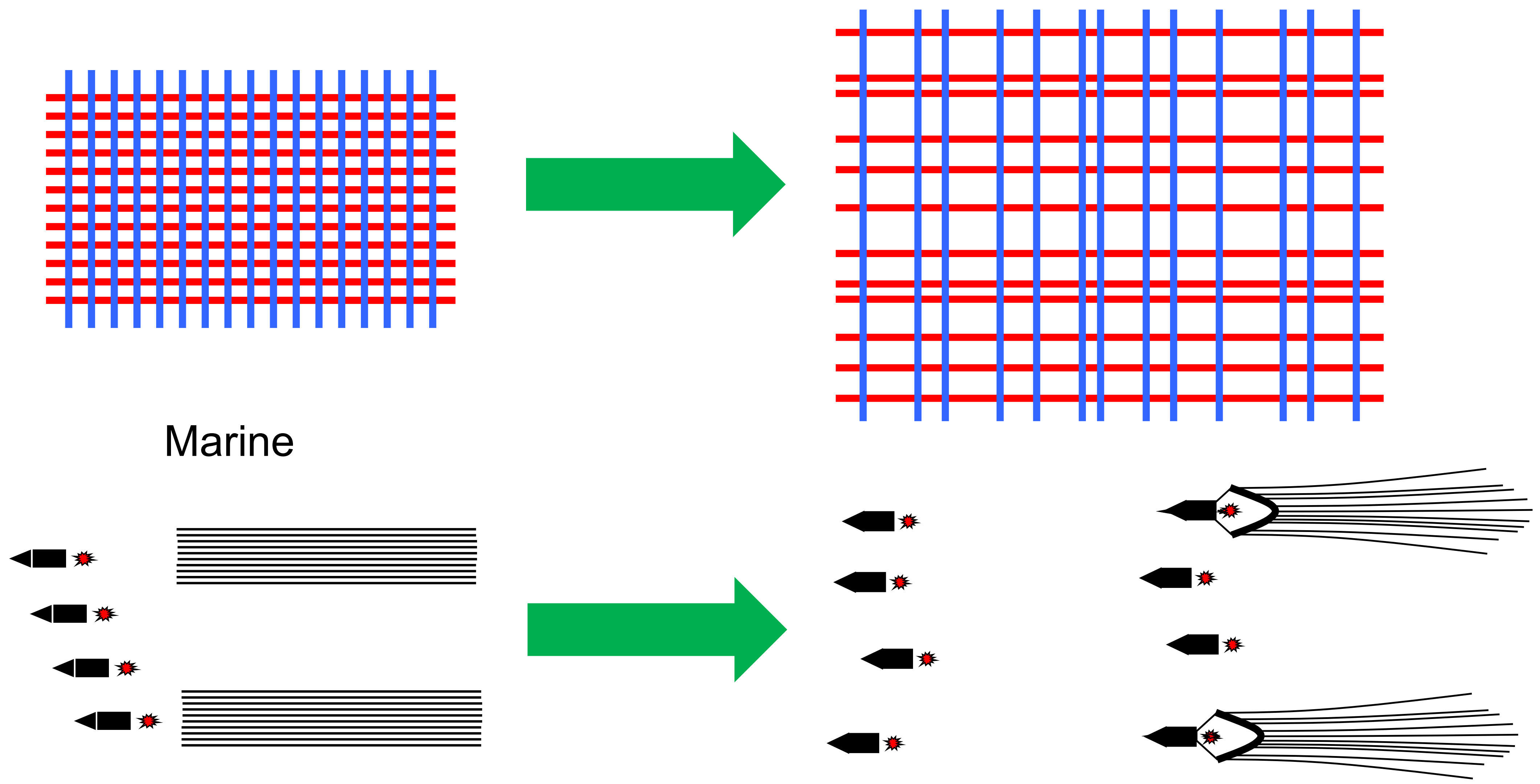
Deliberate & natural randomness in acquisition

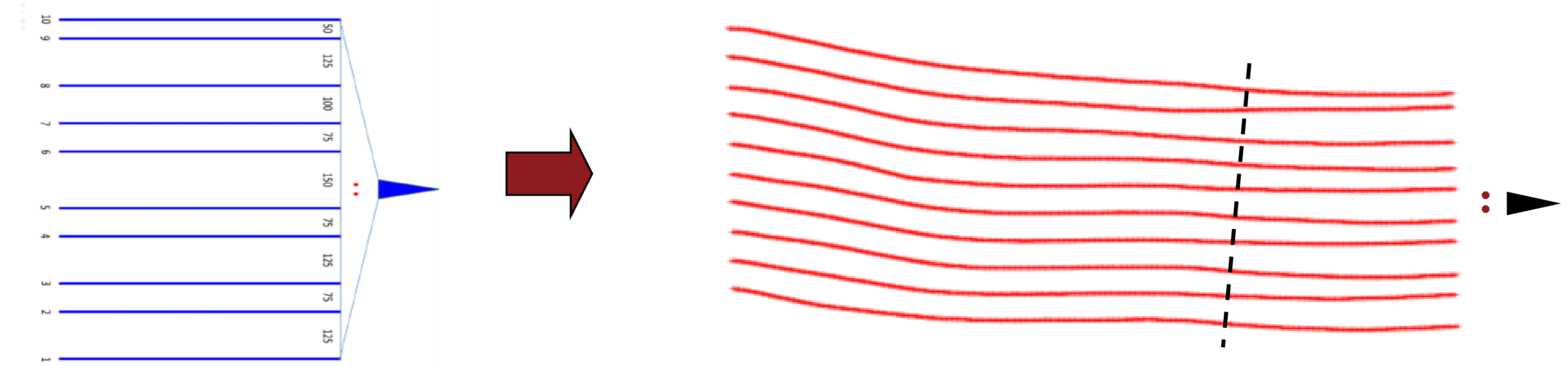
(thanks to Chuck Mosher)

Compressive Sensing = Acquisition Efficiency

Land / Node

Marine





Designed

Actually Acquired

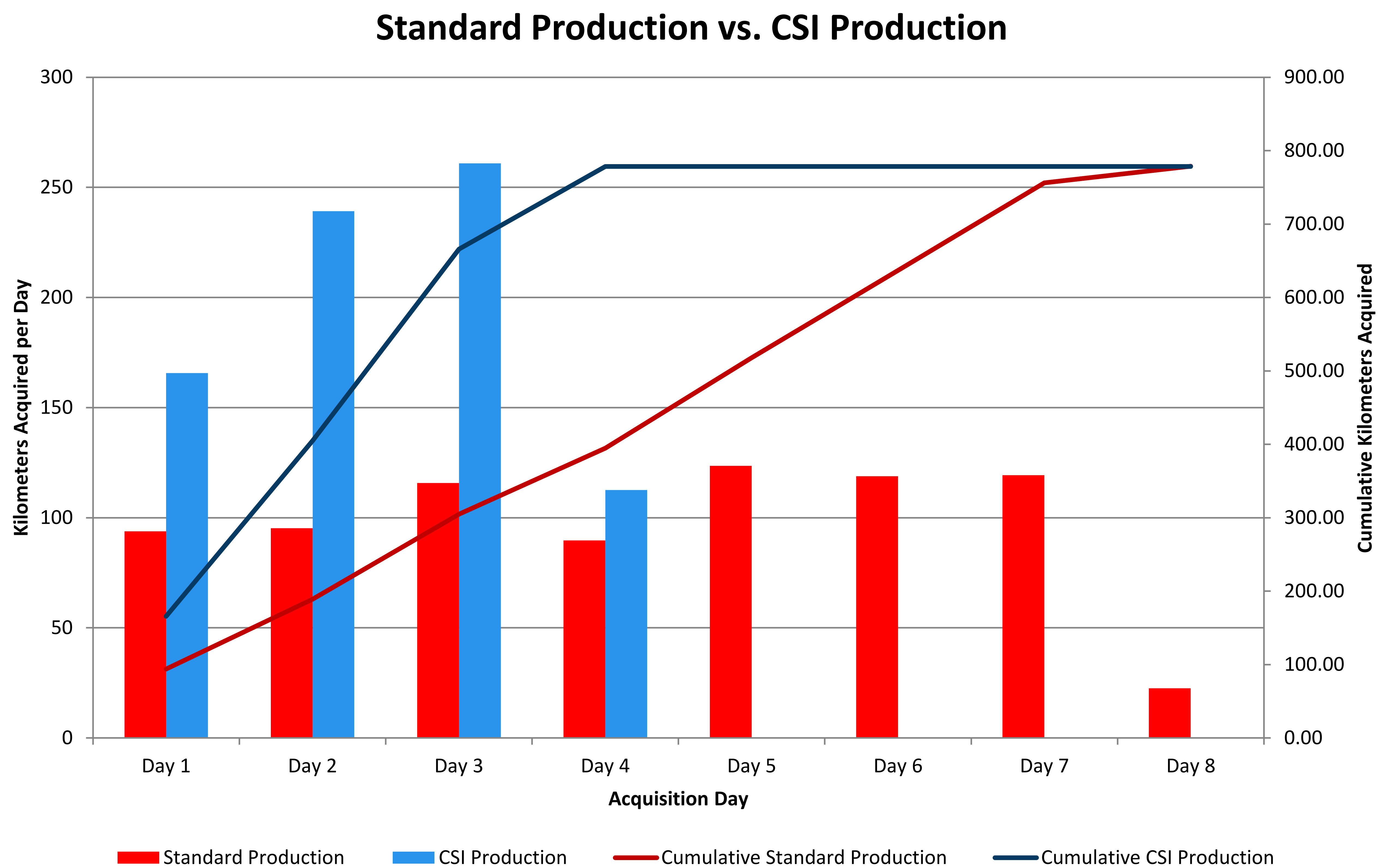
Bottom line
examples from industry (ConocoPhilips)

Randomized subsampling:

- exploits (natural) randomness & structure in seismic
- economic subsampled data
- recovers dense data via structure-promoting inversion

Output:

- improved quality artifact-fact free long-offset wide azimuth data
- 5 X – 10 X cost & environmental impact reduction



Compressive sensing in a nutshell

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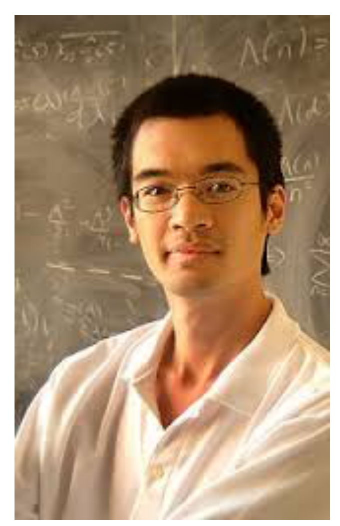
Brief history

Institute for Pure & Applied Mathematics
IPAM
University of California, Los Angeles

Compressive sensing was born during IPAM's program

Multiscale Geometry and Analysis in High Dimensions

- "Compressed sensing" by David Donoho
- "Signal recovery from incomplete and inaccurate measurements" by Emanuel Candés and J. Romberg and T. Tao



- > 10.000 papers but falls short of practical breakthroughs

2004

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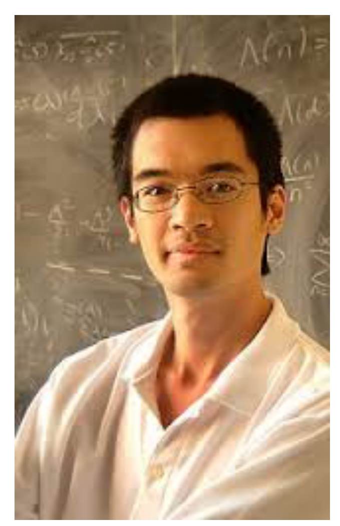


2004

- > 10.000 papers but falls short of practical breakthroughs

What is Compressive Sensing?

A new sampling paradigm:

- reconstructs *sparse* signals from *incoherent* subsampling
- involves inversions of underdetermined linear systems w/ (convex)optimization
- fewer samples then required by Shannon-Nyquist sampling theorem (sufficient but not necessary condition)

What it is not

CS is often misinterpreted

- as sparse (e.g. one-norm) minimization
- or as randomized sampling

CS is based on sparsity, sparsity promotion & mutual incoherence property.

Leads to fat matrices

- w/ random subsets of columns that act as near orthogonal bases
- favor inversion by sparsity promotion

The challenges

CS is not easy to implement in practice because

- assumes an idealized model for sampling, e.g. by Gaussian matrices
- can not easily be realized physically

We need to adapt CS to exploration seismology

- rethink (time-lapse) acquisition design
- incorporate incoherent samplings–e.g., via source location randomization
- come up with recovery algorithms that scale & can handle field practicalities

Resources

Papers:

Felix J. Herrmann, Michael P. Friedlander, and Ozgur Yilmaz, "**Fighting the Curse of Dimensionality: Compressive Sensing in Exploration Seismology**", *Signal Processing Magazine, IEEE*, vol. 29, p. 88-100, 2012

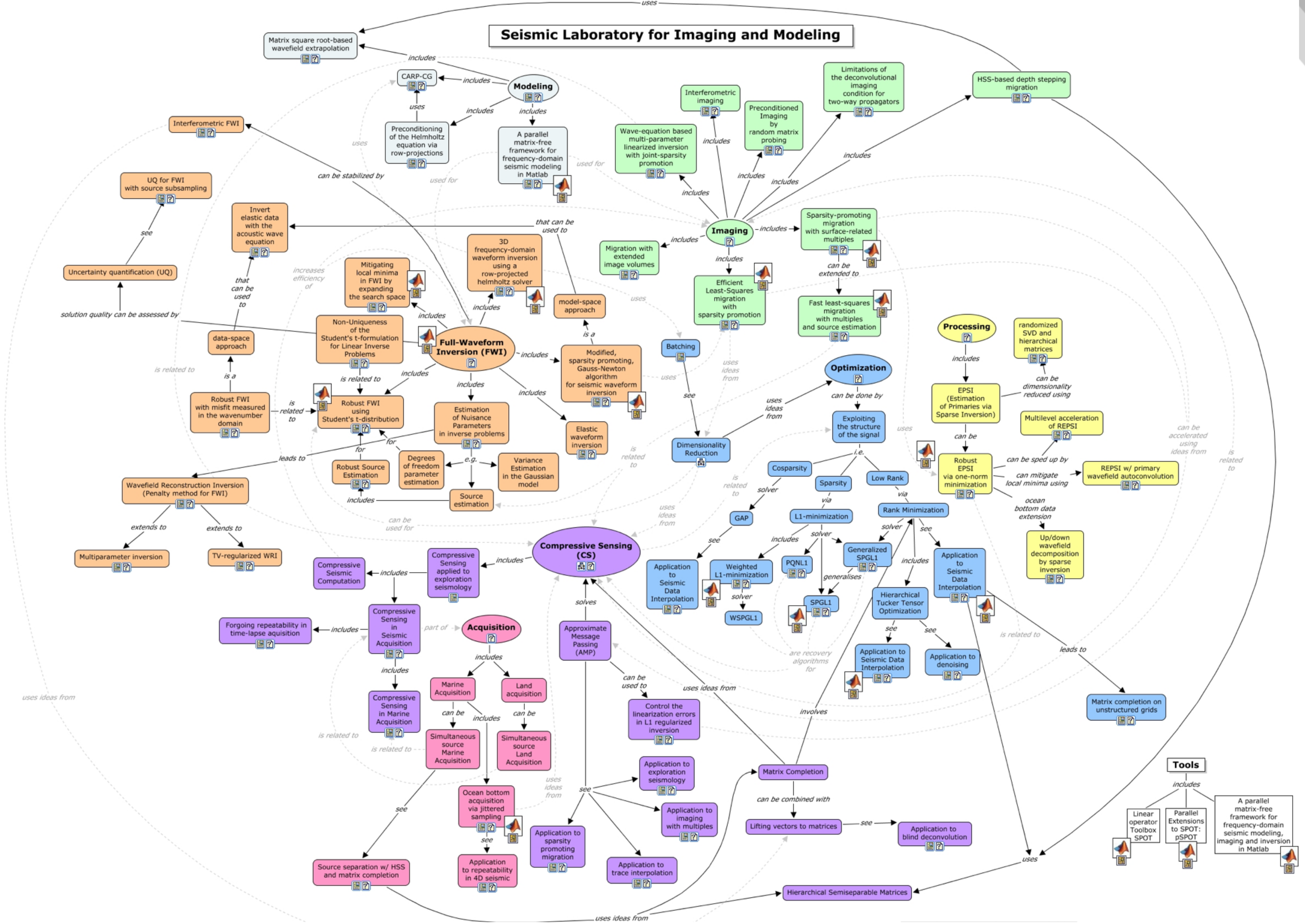
Felix J. Herrmann, "**Randomized sampling and sparsity: Getting more information from fewer samples**", *Geophysics*, vol. 75, p. WB173-WB187, 2010

Web:

https://www.slim.eos.ubc.ca/research/compressive-sensing

http://dsp.rice.edu/cs

http://nuit-blanche.blogspot.ca



Seismic Laboratory for Imaging and Modeling
uses
Matrix square root-based wavefield extrapolation
includes
CARP-CG
Modeling
uses
Preconditioning of the Helmholtz equation via row-projections
A parallel matrix-free framework for frequency-domain seismic modeling in Matlab
used for
Interferometric FWI
can be stabilized by
UQ for FWI with source subsampling
see
Uncertainty quantification (UQ)
solution quality can be assessed by
Invert elastic data with the acoustic wave equation
that can be used to
that can be used to
data-space approach
is a
Robust FWI with misfit measured in the wavenumber domain
is related to
Mitigating local minima in FWI by expanding the search space
increases efficiency of
3D frequency-domain waveform inversion using a row-projected helmholtz solver
model-space approach
Non-Uniqueness of the Student's t-formulation for Linear Inverse Problems
Full-Waveform Inversion (FWI)
Modified, sparsity promoting, Gauss-Newton algorithm for seismic waveform inversion
Robust FWI using Student's t-distribution
Estimation of Nuisance Parameters in inverse problems
Elastic waveform inversion
Robust Source Estimation
Degrees of freedom parameter estimation
Variance Estimation in the Gaussian model
e.g.
for
Source estimation
leads to
Wavefield Reconstruction Inversion (Penalty method for FWI)
extends to
Multiparameter inversion
TV-regularized WRI
can be used for
Interferometric imaging
Wave-equation based multi-parameter linearized inversion with joint-sparsity promotion
Preconditioned Imaging by random matrix probing
Limitations of the deconvolutional imaging condition for two-way propagators
HSS-based depth stepping migration
Imaging
Migration with extended image volumes
Sparsity-promoting migration with surface-related multiples
can be extended to
Efficient Least-Squares migration with sparsity promotion
Fast least-squares migration with multiples and source estimation
Batching
uses ideas from
Dimensionality Reduction
Optimization
can be done by
Exploiting the structure of the signal
i.e.
Cosparsity
Sparsity
Low Rank
solver
via
GAP
L1-minimization
Rank Minimization
Application to Seismic Data Interpolation
Weighted L1-minimization
PQNL1
Generalized SPGL1
generalises
SPGL1
WSPGL1
are recovery algorithms for
Hierarchical Tucker Tensor Optimization
Application to Seismic Data Interpolation
Application to denoising
Application to Seismic Data Interpolation
is related to
leads to
Matrix completion on unstructured grids
Processing
randomized SVD and hierarchical matrices
can be dimensionality reduced using
EPSI (Estimation of Primaries via Sparse Inversion)
Multilevel acceleration of REPSI
can be sped up by
Robust EPSI via one-norm minimization
can mitigate local minima using
REPSI w/ primary wavefield autoconvolution
ocean bottom data extension
Up/down wavefield decomposition by sparse inversion
can be accelerated using ideas from
Compressive Sensing (CS)
Compressive Sensing applied to exploration seismology
Compressive Seismic Computation
Compressive Sensing in Seismic Acquisition
part of
Acquisition
Forgoing repeatability in time-lapse aquisition
Compressive Sensing in Marine Acquisition
Marine Acquisition
Land acquisition
can be
Simultaneous source Marine Acquisition
Simultaneous source Land Acquisition
Ocean bottom acquisition via jittered sampling
Application to repeatability in 4D seismic
Source separation w/ HSS and matrix completion
solves
Approximate Message Passing (AMP)
can be used to
Control the linearization errors in L1 regularized inversion
Application to exploration seismology
Application to imaging with multiples
Application to sparsity promoting migration
Application to trace interpolation
Matrix Completion
involves
can be combined with
Lifting vectors to matrices
Application to blind deconvolution
Hierarchical Semiseparable Matrices
uses ideas from
Tools
Linear operator Toolbox SPOT
Parallel Extensions to SPOT: pSPOT
A parallel matrix-free framework for frequency-domain seismic modeling, imaging and inversion in Matlab

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Felix J. Herrmann, "Randomized sampling and sparsity: Getting more information from fewer samples", *Geophysics*, vol. 75, p. WB173-WB187, 2010

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Compressive sensing paradigm

Find representations that reveal structure

- transform-domain sparsity (e.g., Fourier, curvelets, etc.)
- rank revealing transforms (e.g. midpoint-offset domain)

Sample to break this structure

- randomized acquisition (e.g., jittered sampling, time dithering, source encoding, etc.)
- destroys sparsity or low-rank structure

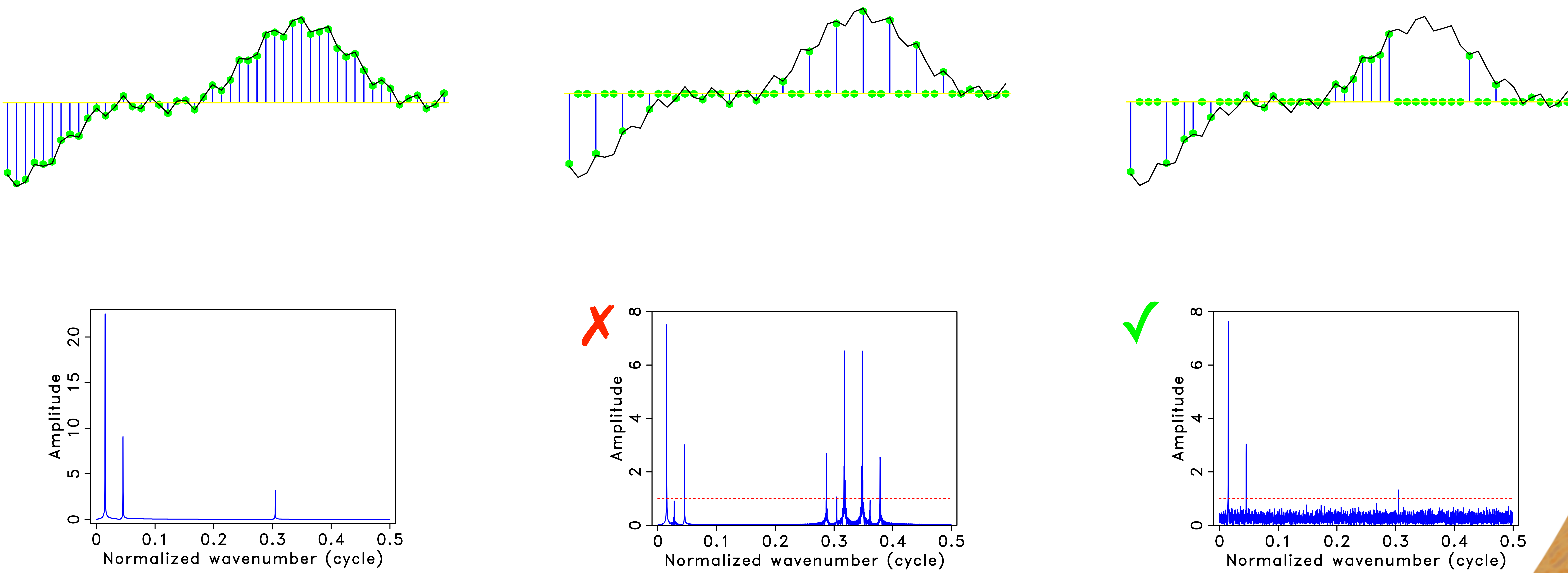
Recover this structure by promoting

- sparsity via one-norm minimization
- rank revealing nuclear-norm minimization (one-norm singular values)

Felix J. Herrmann and Gilles Hennenfent, "**Non-parametric seismic data recovery with curvelet frames**", *GJI*, vol. 173, p. 233-248, 2008.
Gilles Hennenfent and Felix J. Herrmann, "**Simply denoise: wavefield reconstruction via jittered undersampling**", *Geophysics*, vol. 73, p. V19-V28, 2008.
Felix J. Herrmann, "**Randomized sampling and sparsity: Getting more information from fewer samples**", *Geophysics*, vol. 75, p. WB173-WB187, 2010 .

Periodic vs random subsampling

sparse time-harmonic signals



Gilles Hennenfent, Ewout van den Berg, Michael P. Friedlander, and Felix J. Herrmann, "New insights into one-norm solvers from the Pareto curve", Geophysics, vol. 73, p. A23-A26, 2008.

Sparsity-promoting recovery

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \underbrace{\|\mathbf{x}\|_1}_{\text{support detection}} \quad \text{subject to} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{b}}_{\text{data-consistent amplitude recovery}}$$

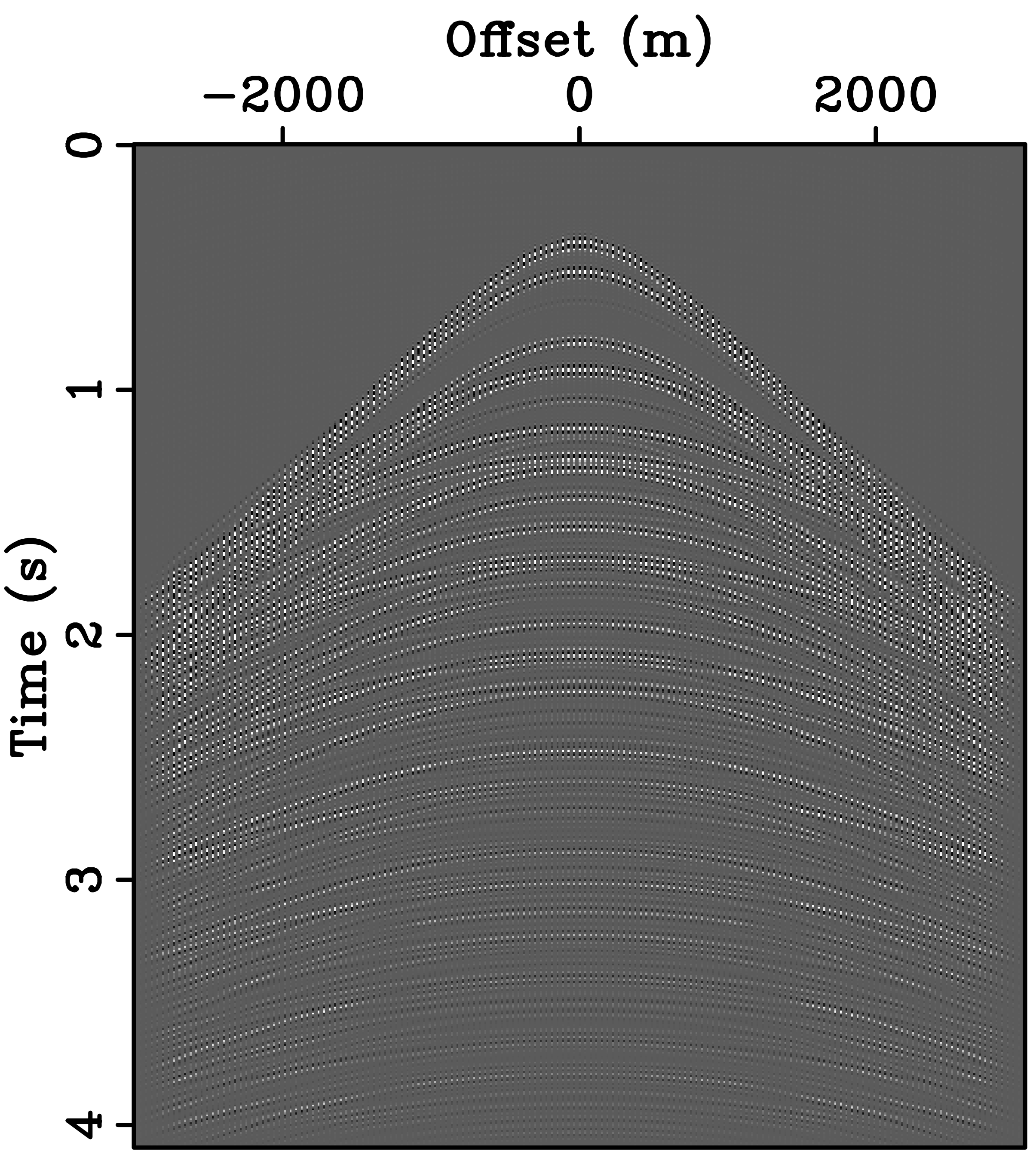
recovered data: $\tilde{\mathbf{d}} = \mathbf{S}^H \tilde{\mathbf{x}}$

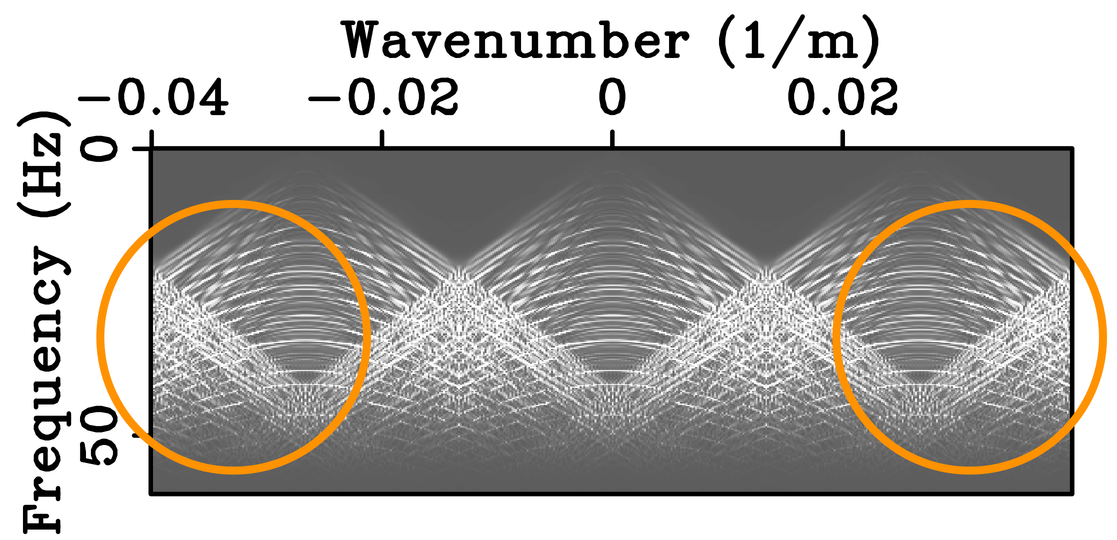
$\mathbf{S^H}$	transform domain synthesis matrix
$\mathbf{A}$	measurement matrix : $\mathbf{MS^H}$, $\mathbf{M}$ is a measurement matrix
$\mathbf{b}$	randomly sampled data
$\tilde{\mathbf{x}}$	estimated (curvelet) coefficients for recovered wavefields

Gilles Hennenfent, Lloyd Fenelon, and Felix J. Herrmann, "**Nonequispaced curvelet transform for seismic data reconstruction: a sparsity-promoting approach**", *Geophysics*, vol. 75, p. WB203-WB210, 2010.
Gilles Hennenfent and Felix J. Herrmann, "**Simply denoise: wavefield reconstruction via jittered undersampling**", *Geophysics*, vol. 73, p. V19-V28, 2008.

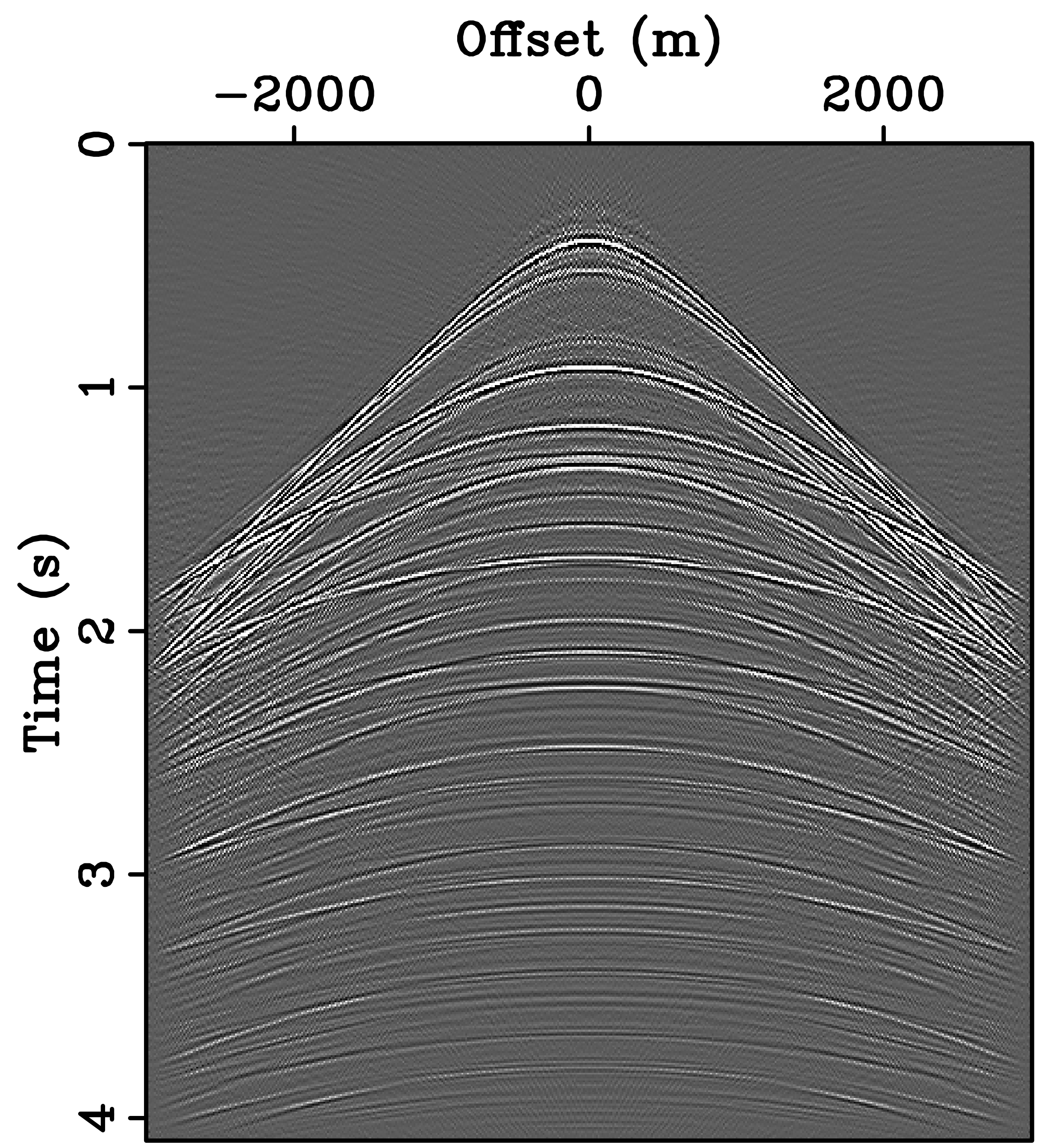
Jittered sampling

Periodic sampling

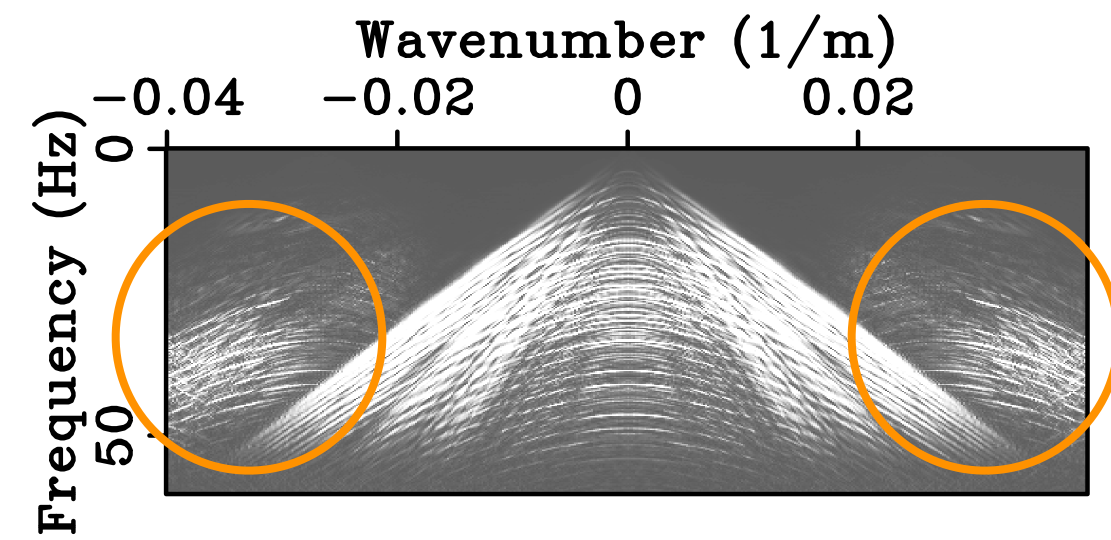




3-fold undersampled

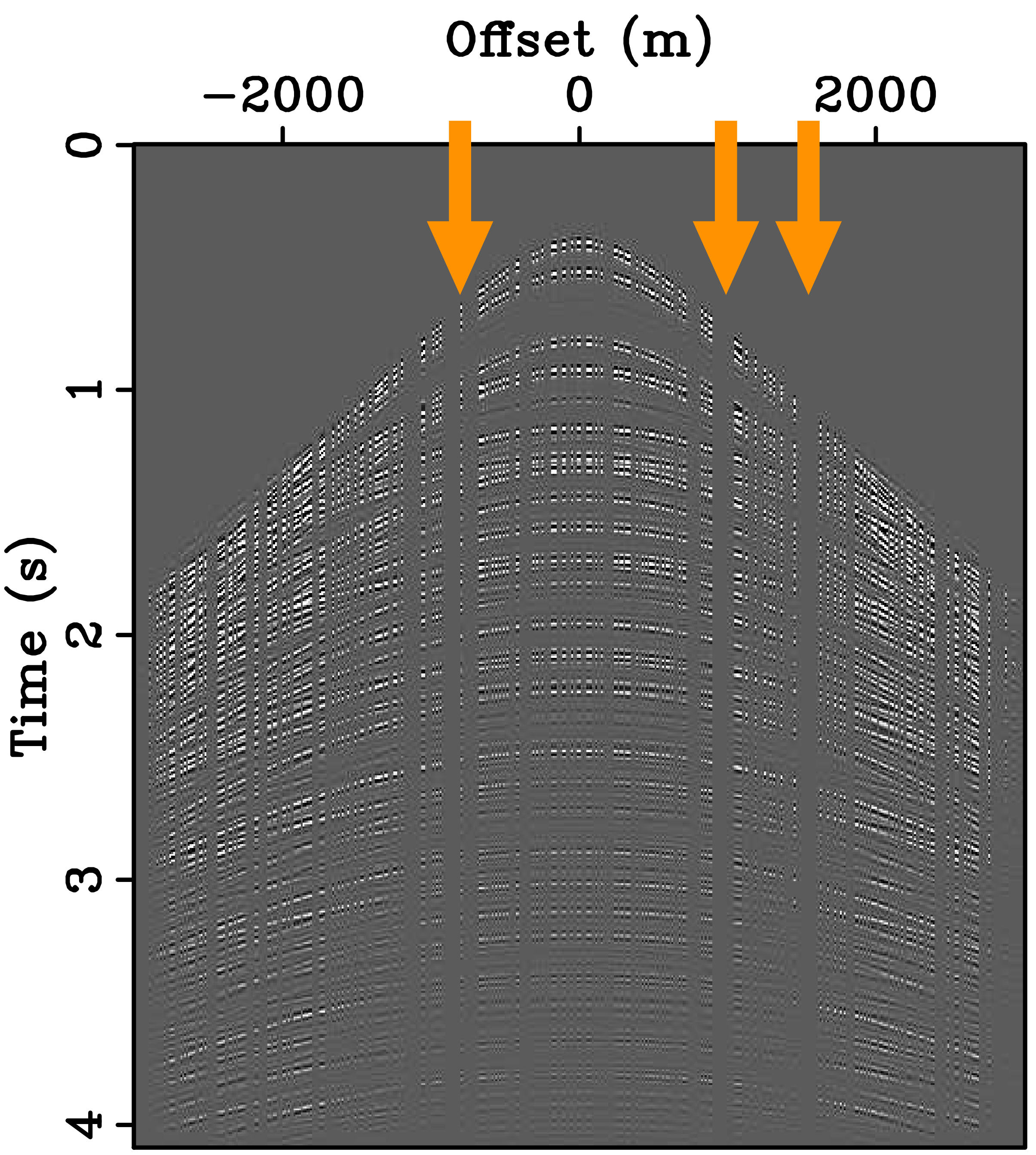


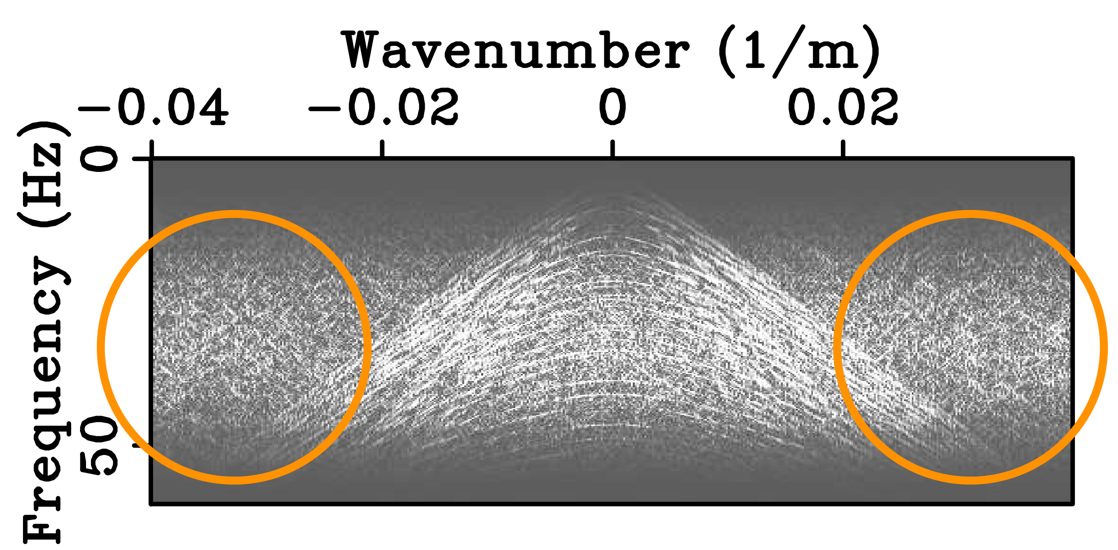
SNR = 6.92 dB

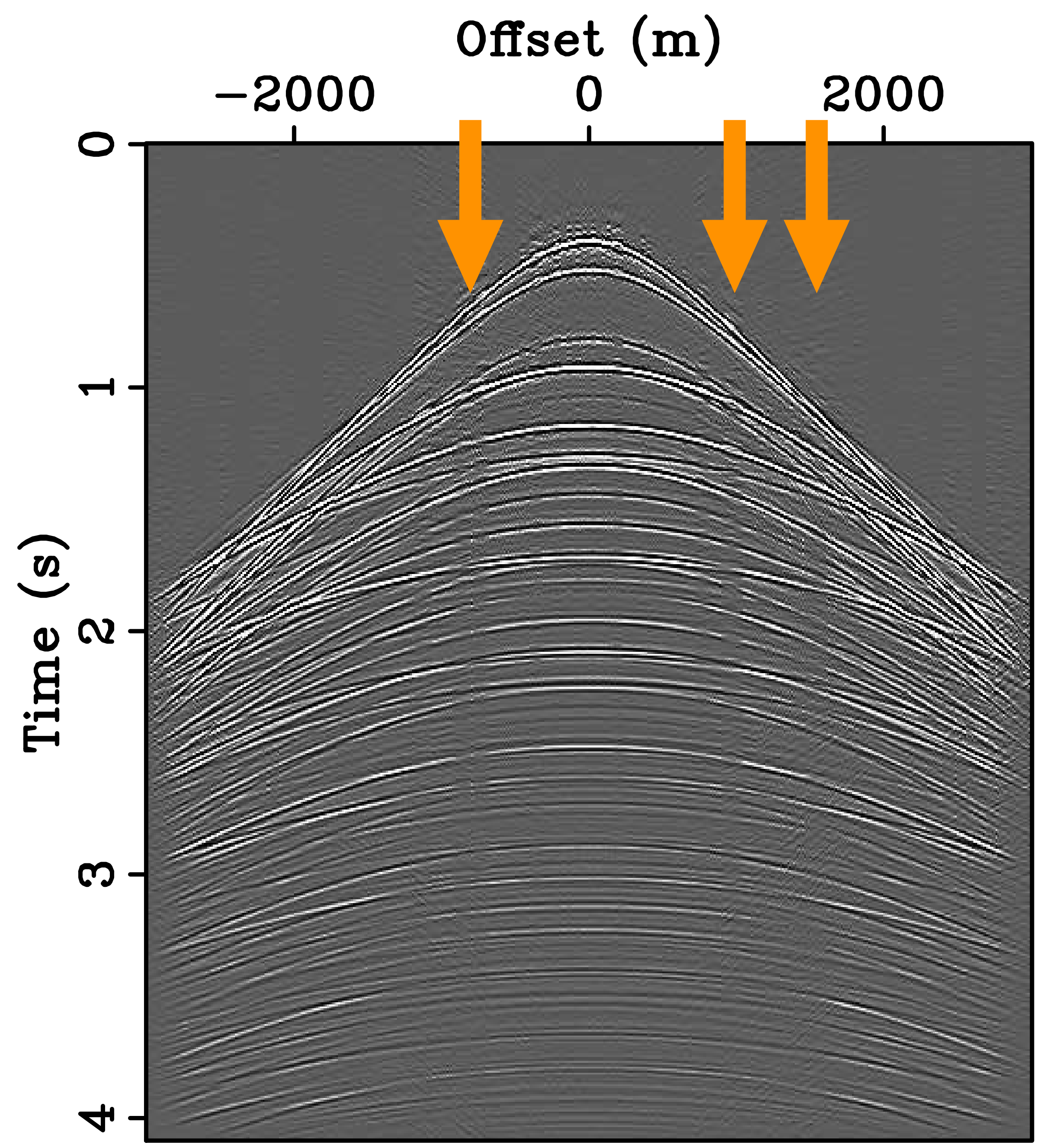


recovered

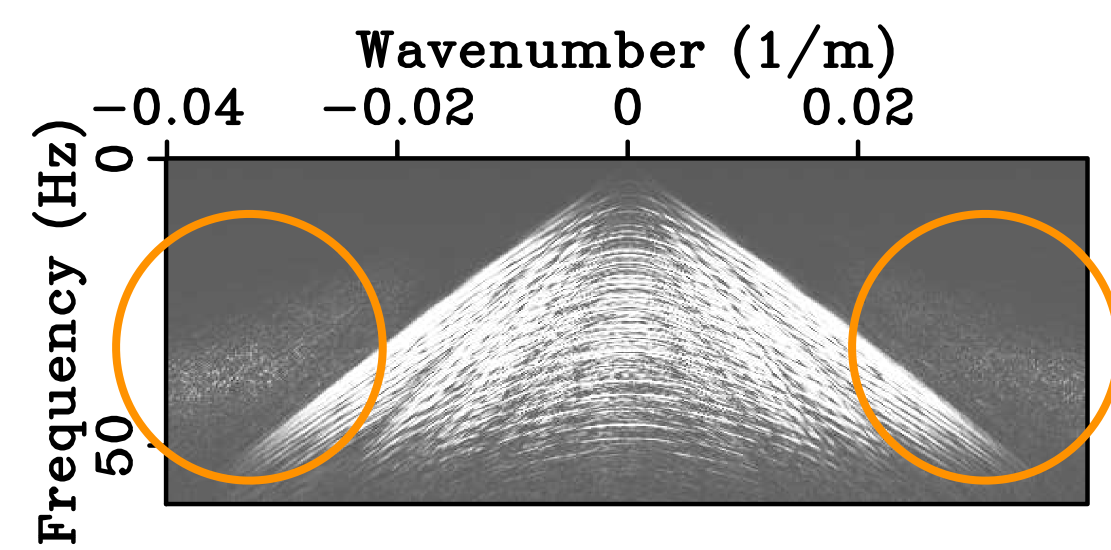
Uniform random sampling







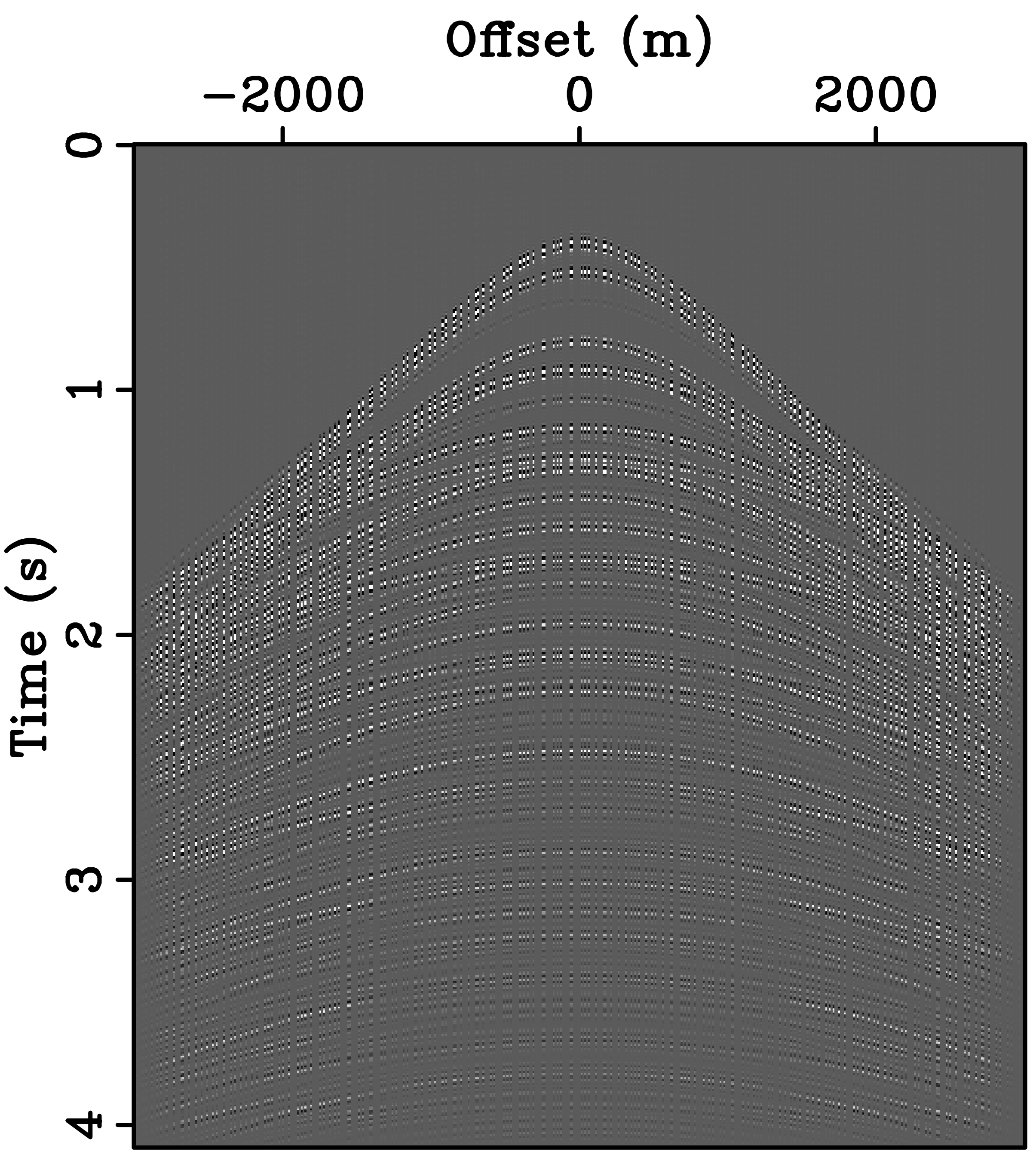
SNR = 9.72 dB

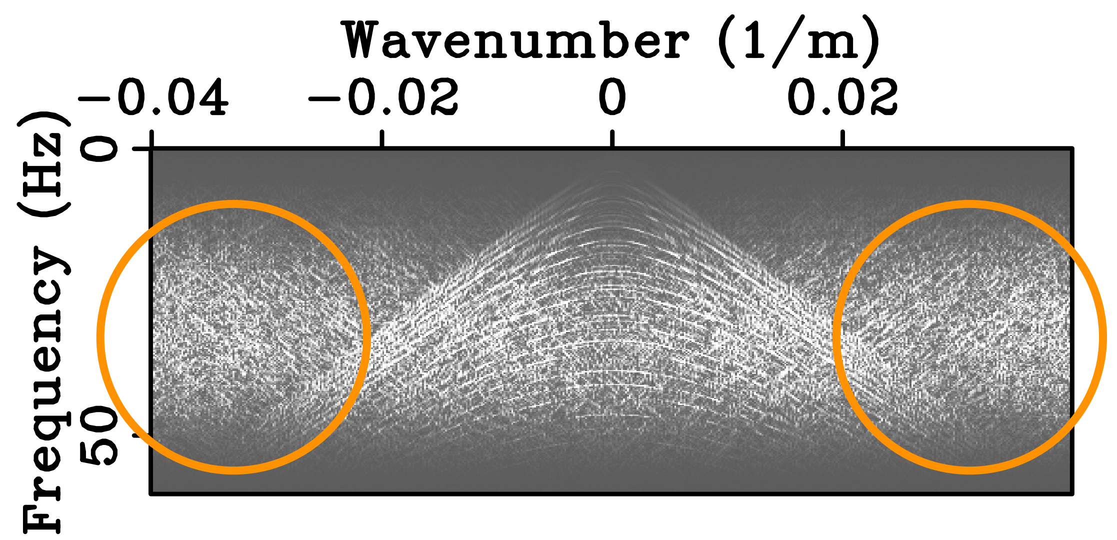


3-fold undersampled

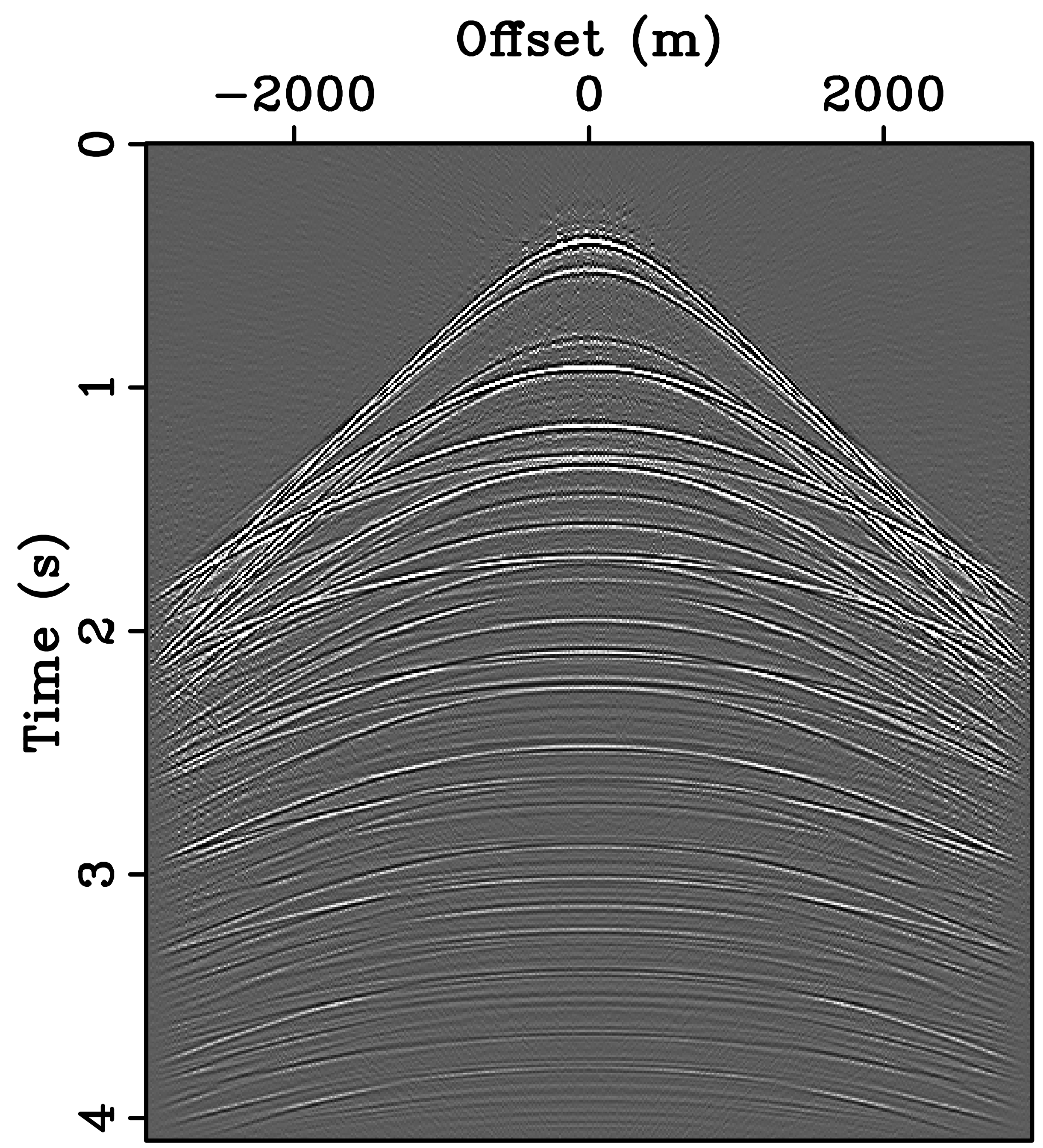
recovered

Jittered sampling

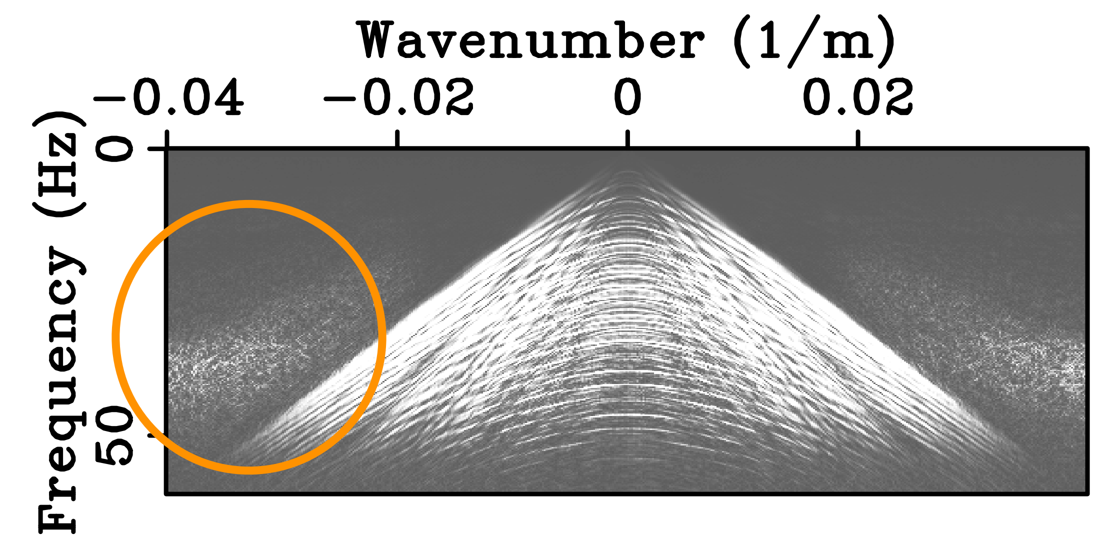




3-fold undersampled



SNR = 10.42 dB



recovered

Time-jittered marine acquisition

Haneet Wason & Rajiv Kumar

SLIM
University of British Columbia

Objective

Shorten **marine acquisition** ***times*** **&** ***increase*** **source sample** ***density*****.**

Periodic vs. jittered marine acquisition

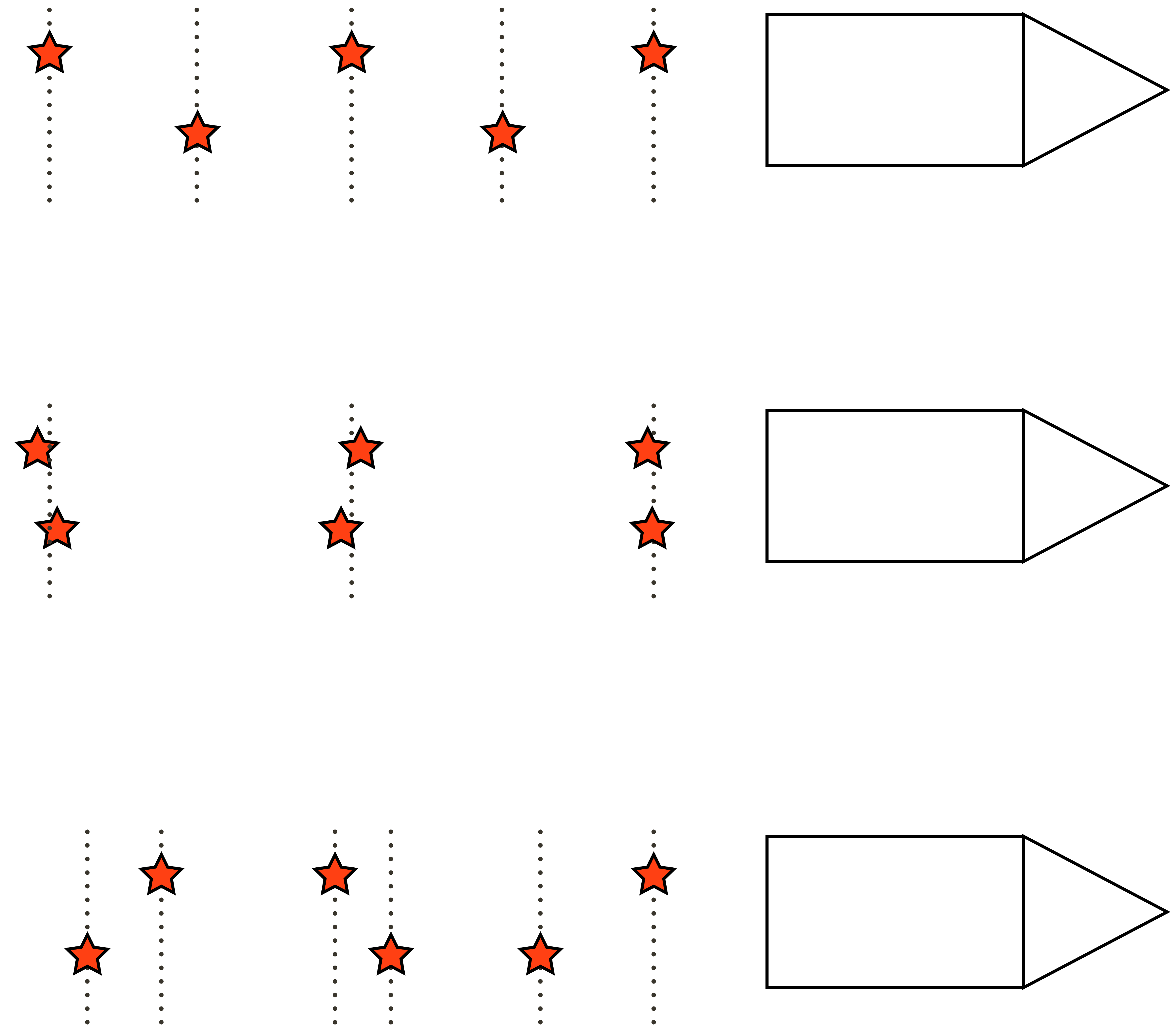
shot-time randomness

periodically sampled spatial grid — NONE

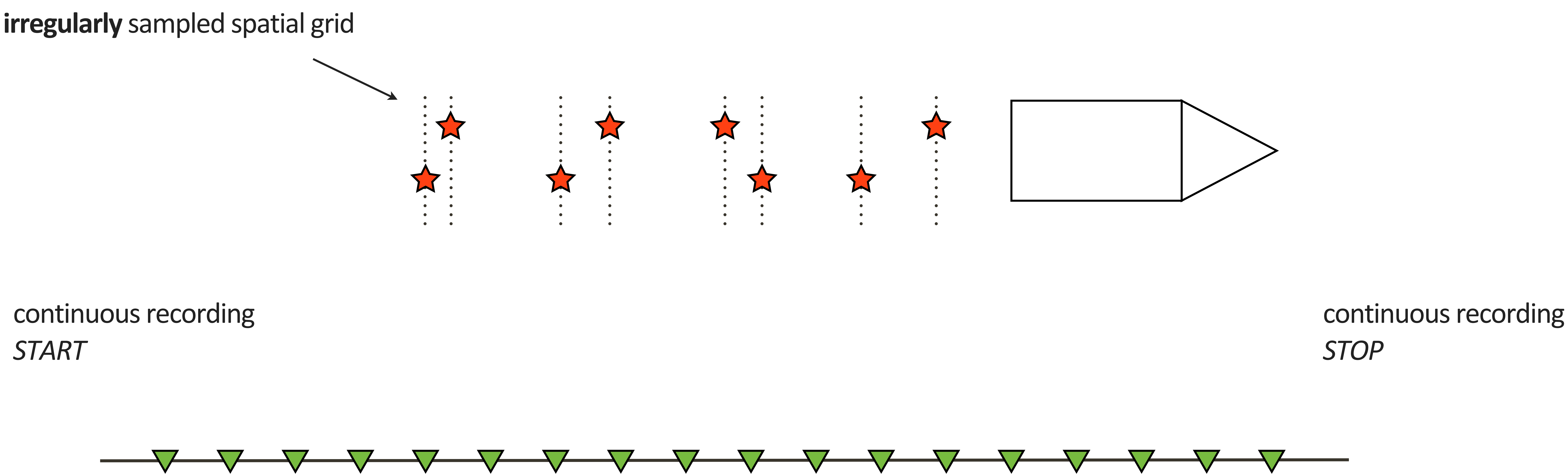
almost periodically sampled spatial grid
(over/under acquisition, towed arrays) — LOW

randomly jittered sampled spatial grid
(Time-jittered acquisition, OBC/OBN) — HIGH

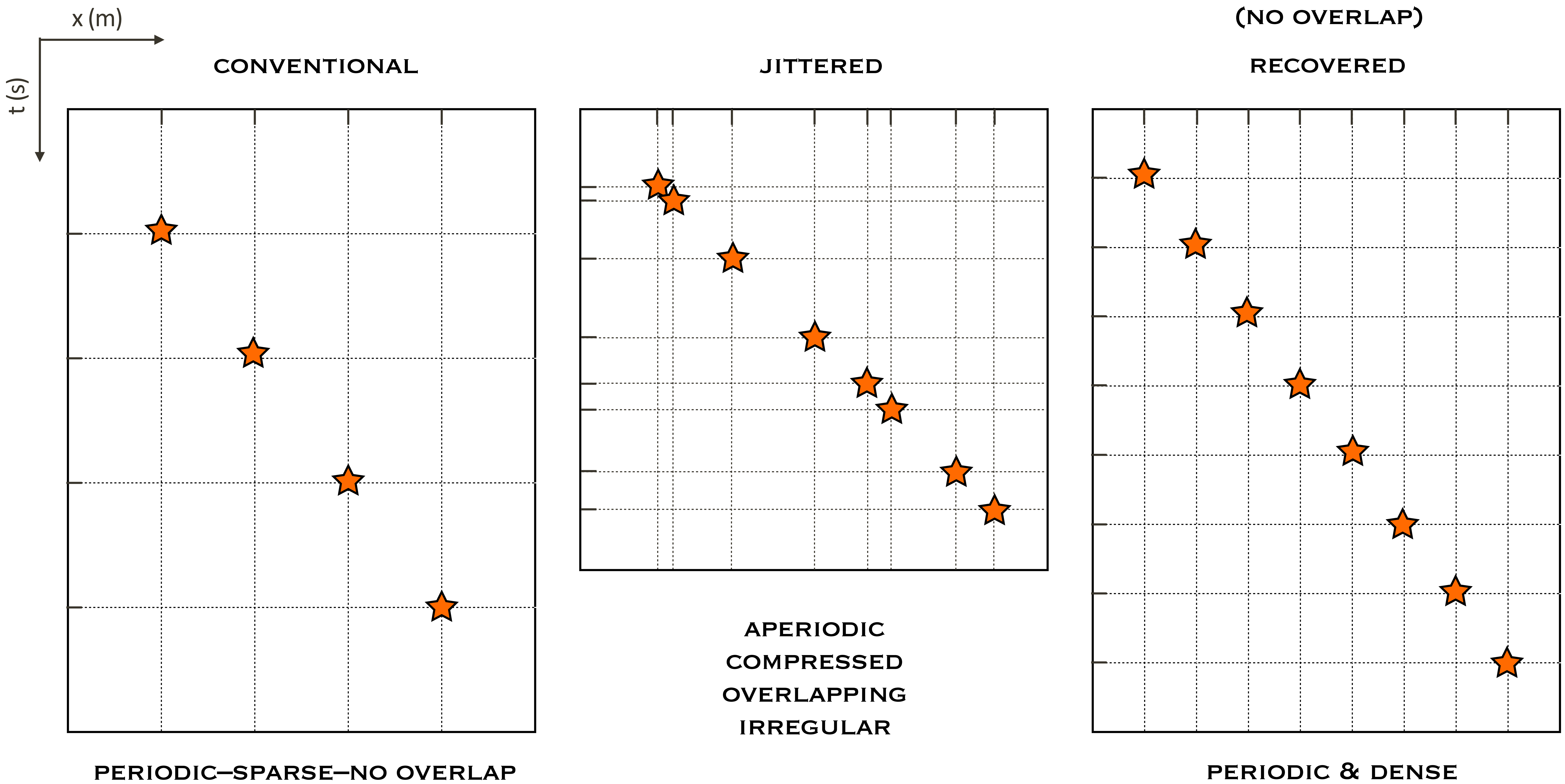
[Wason and Herrmann, 2013]
[Mansour et. al., 2012]



Time-jittered marine acquisition



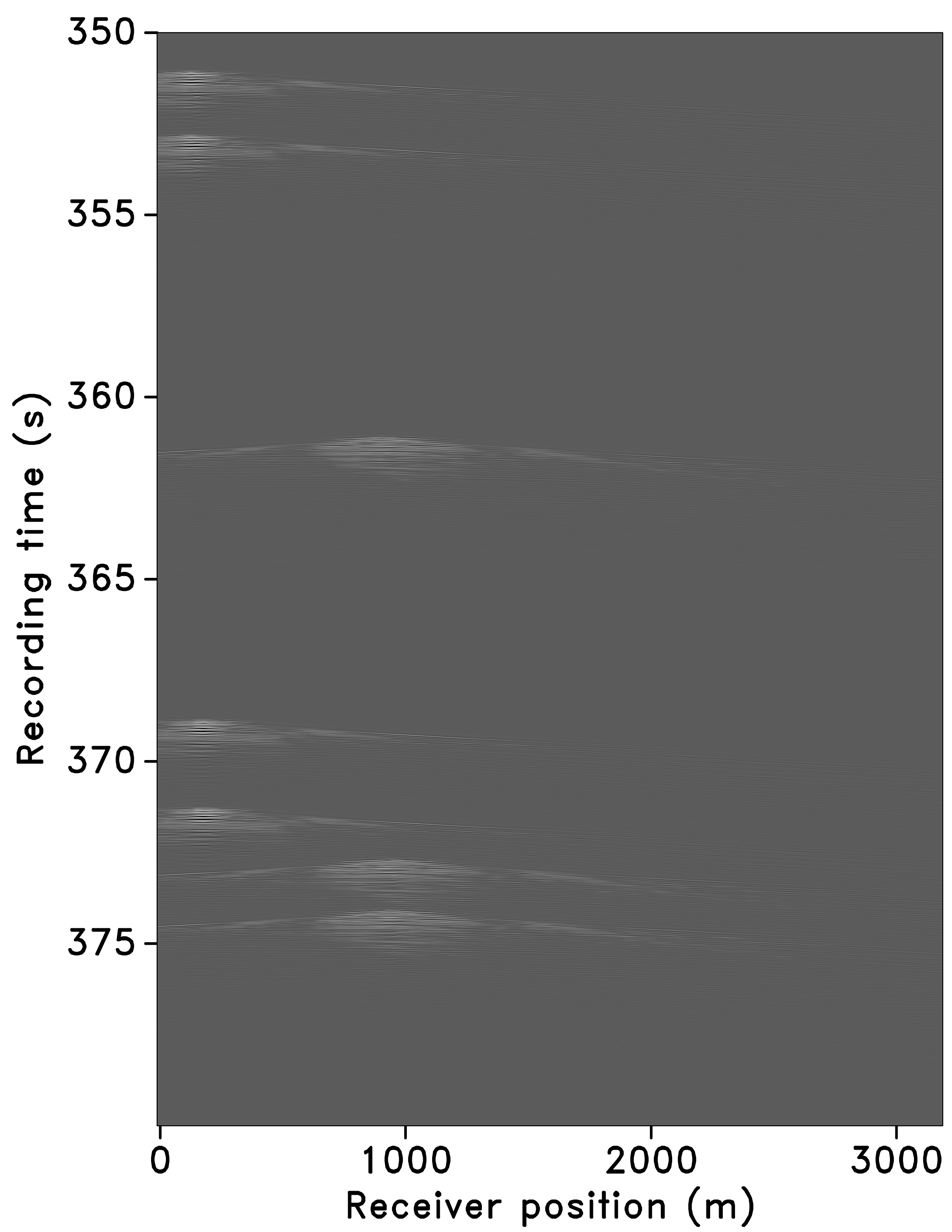
Randomized jitter sampling in marine
– continuous recording w/ OBC/OBN

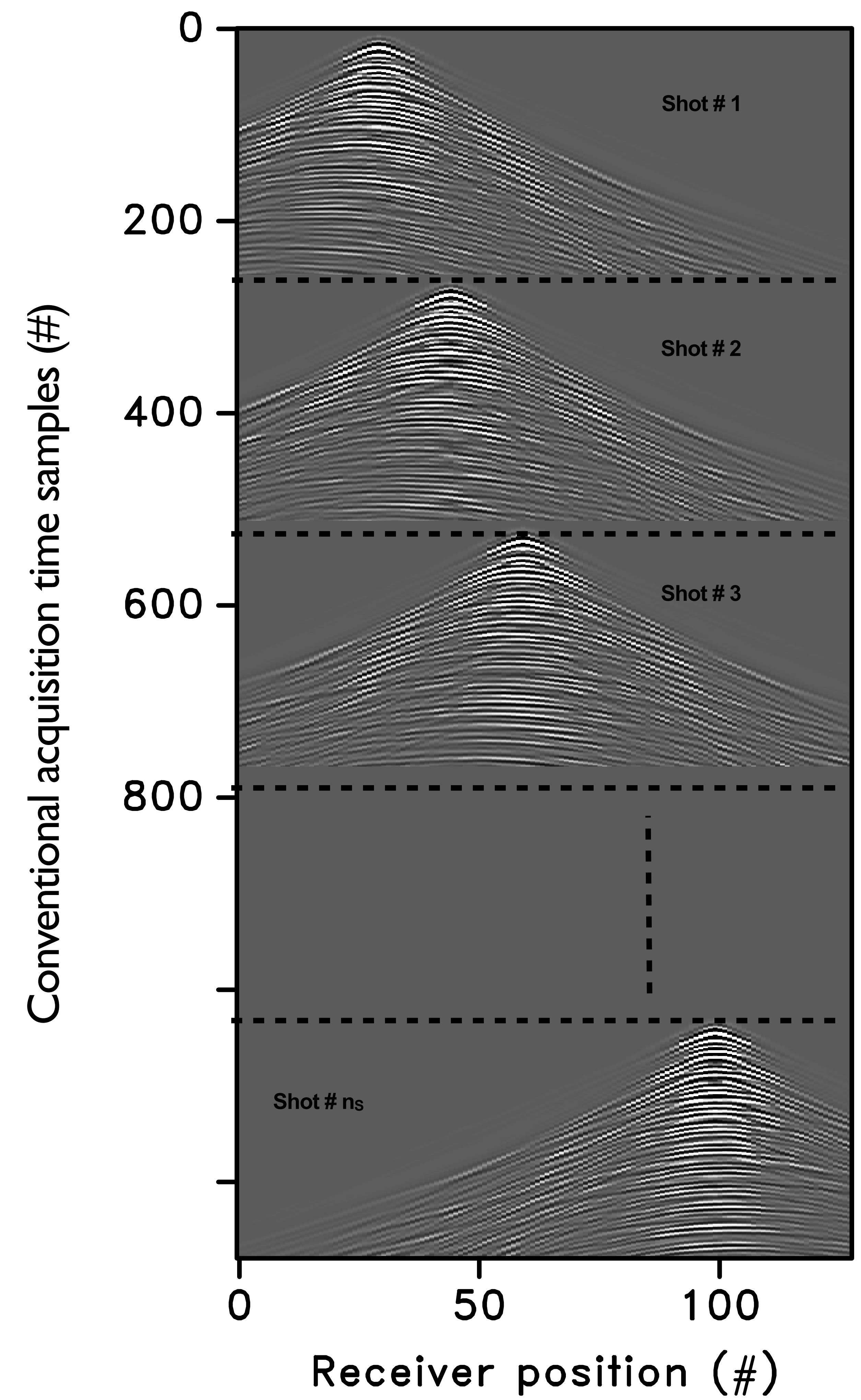


acquire in the field on *irregular* grid (*subsampled* shots *w/ overlap* between shot records)

would like to have on *regular* grid (*all* shots *w/o overlaps* between shot records)

$$\mathbf{b} = \boxed{\mathbf{M}} \; \mathbf{d}$$

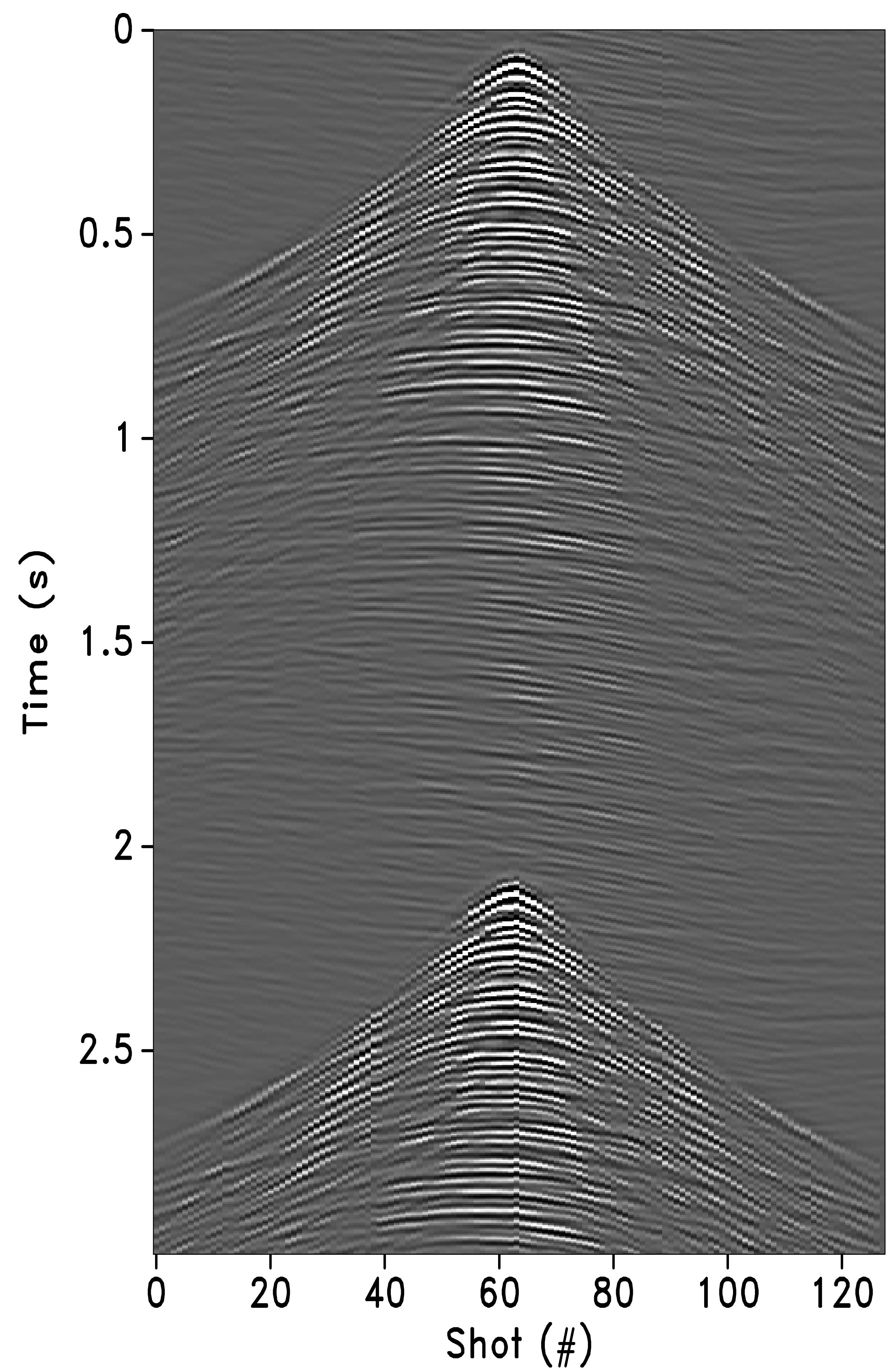




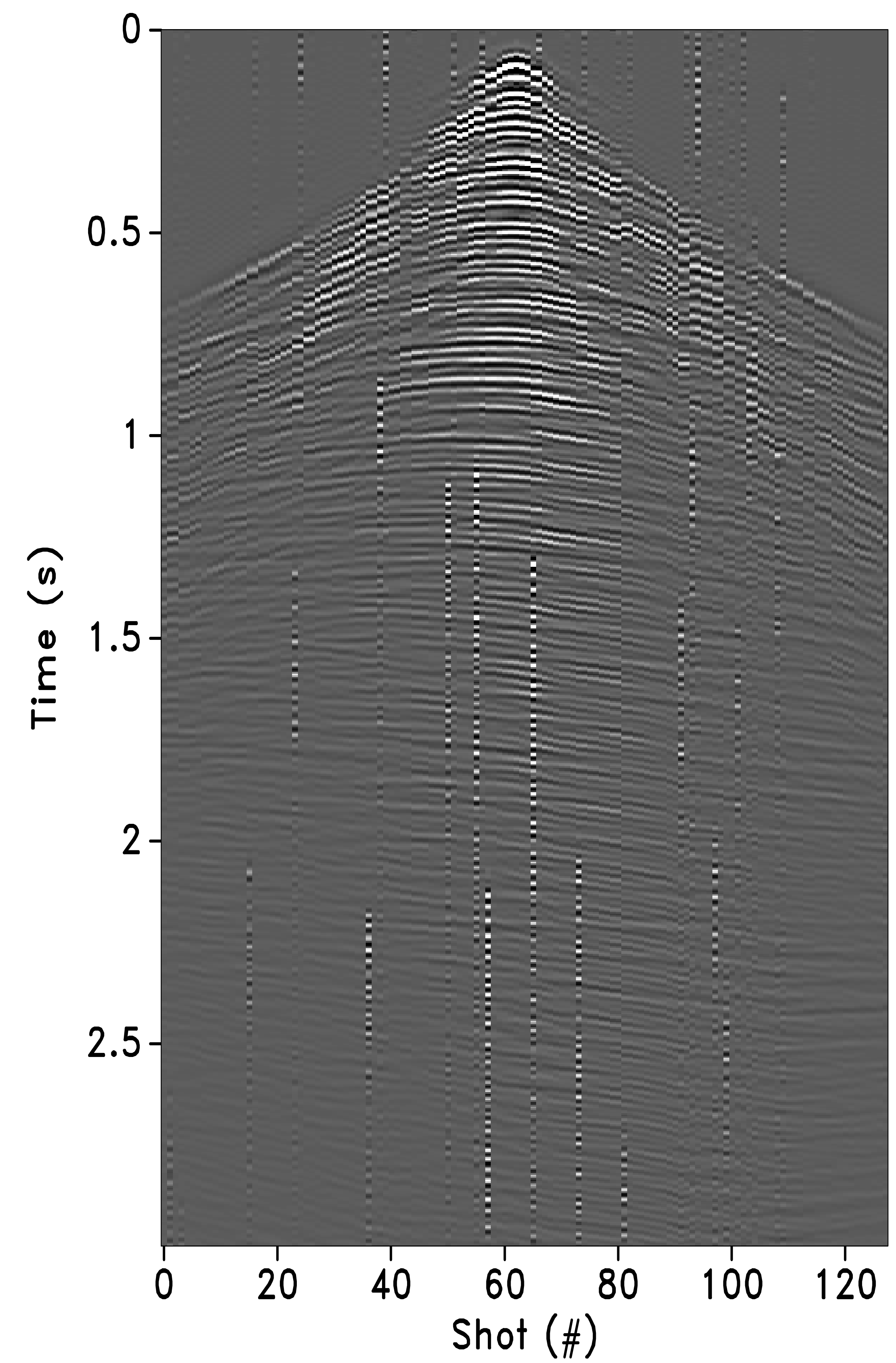
Interferences

source-crosstalk for common receiver

periodic

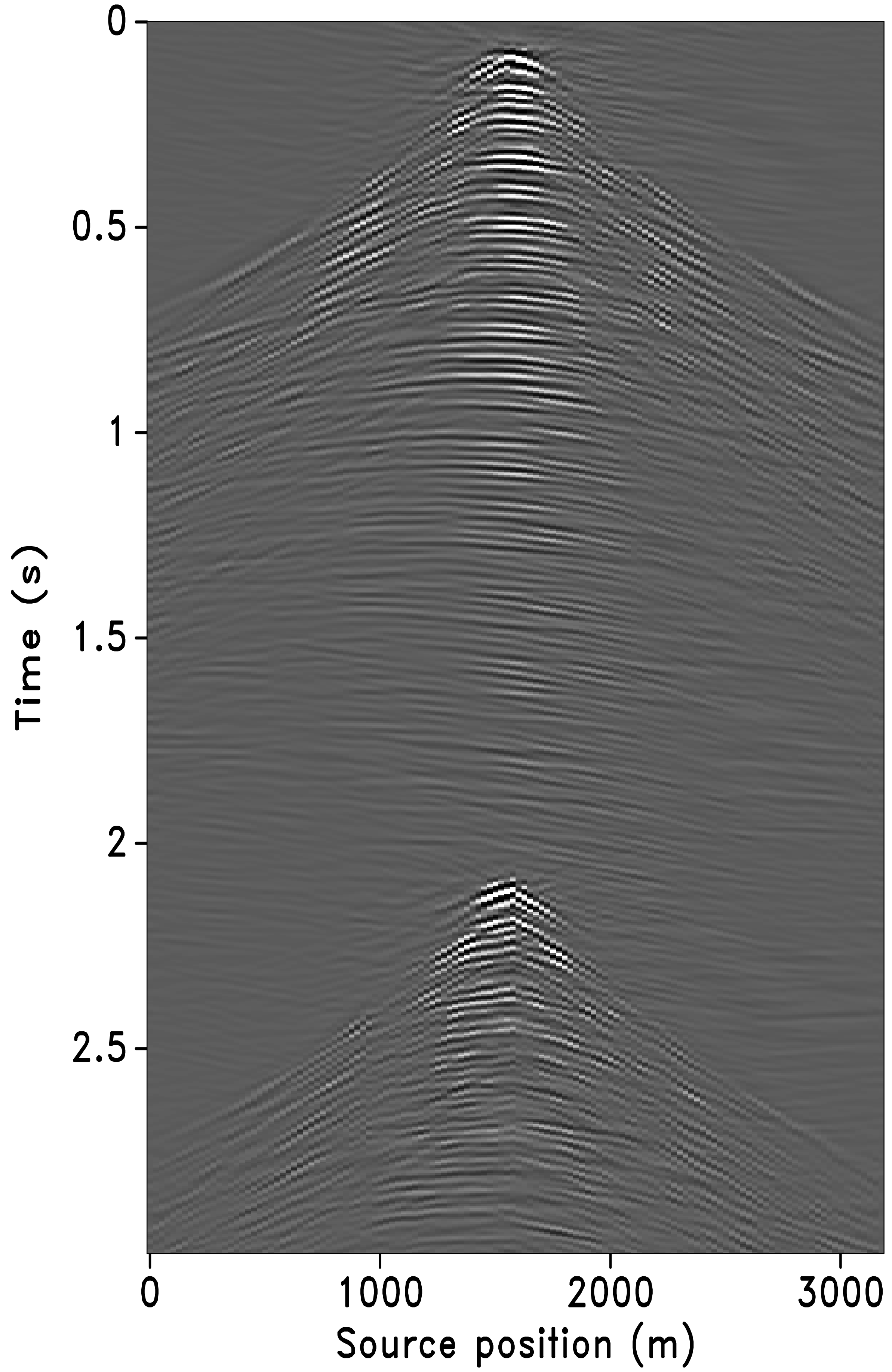


jittered

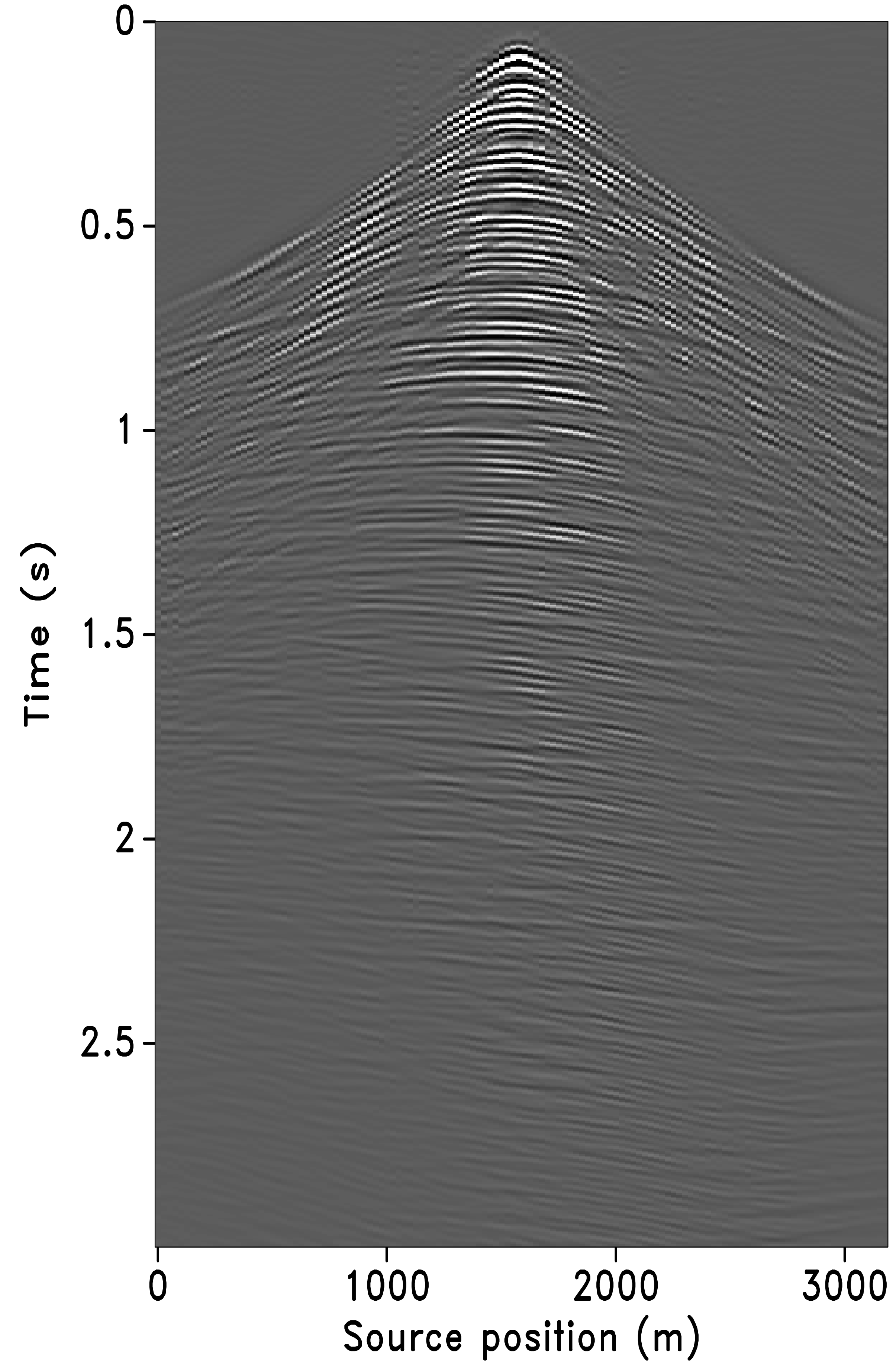


Recovery
via sparsity promotion

periodic
(3.6 dB)



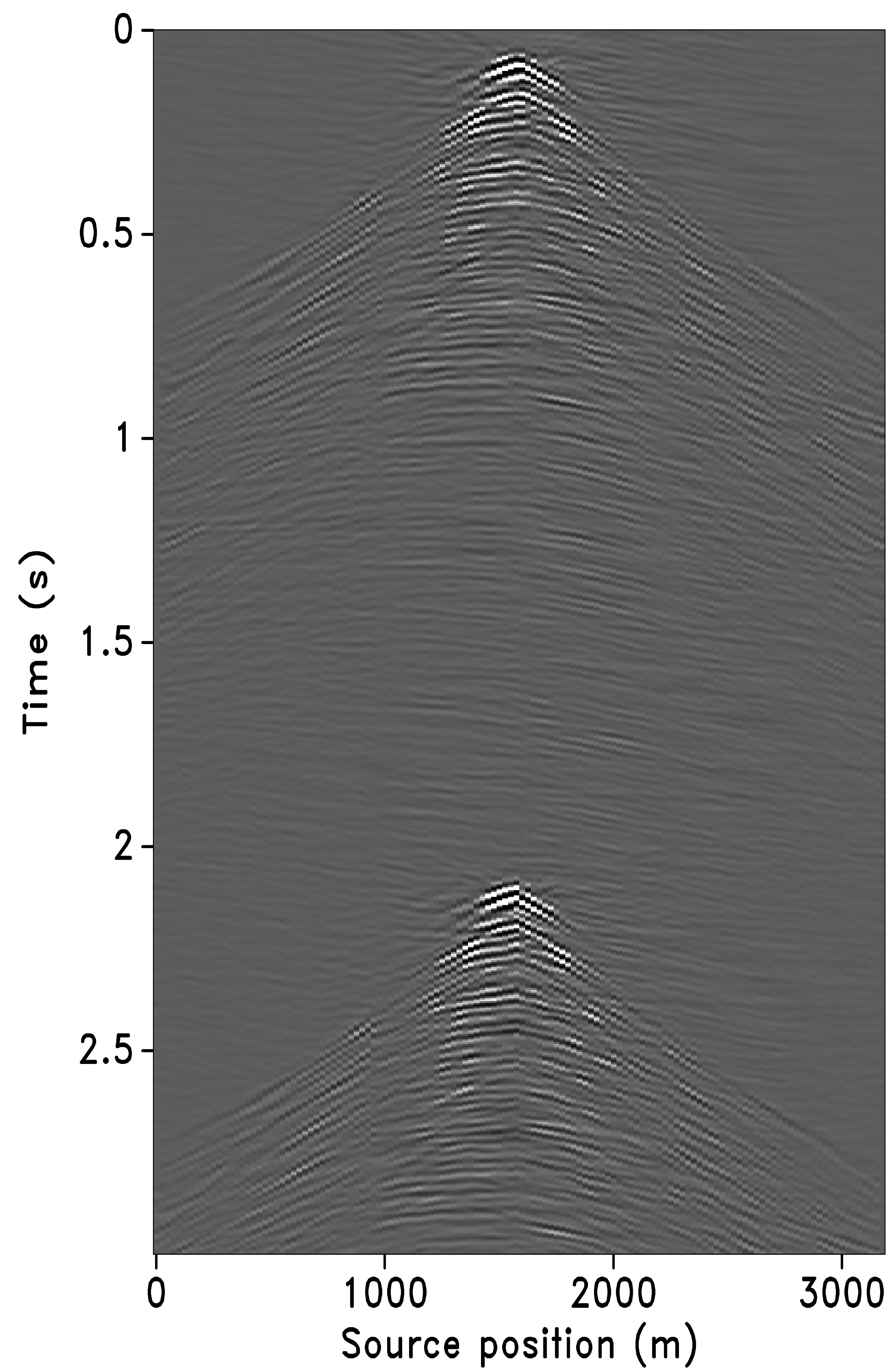
jittered
(16.5 dB)



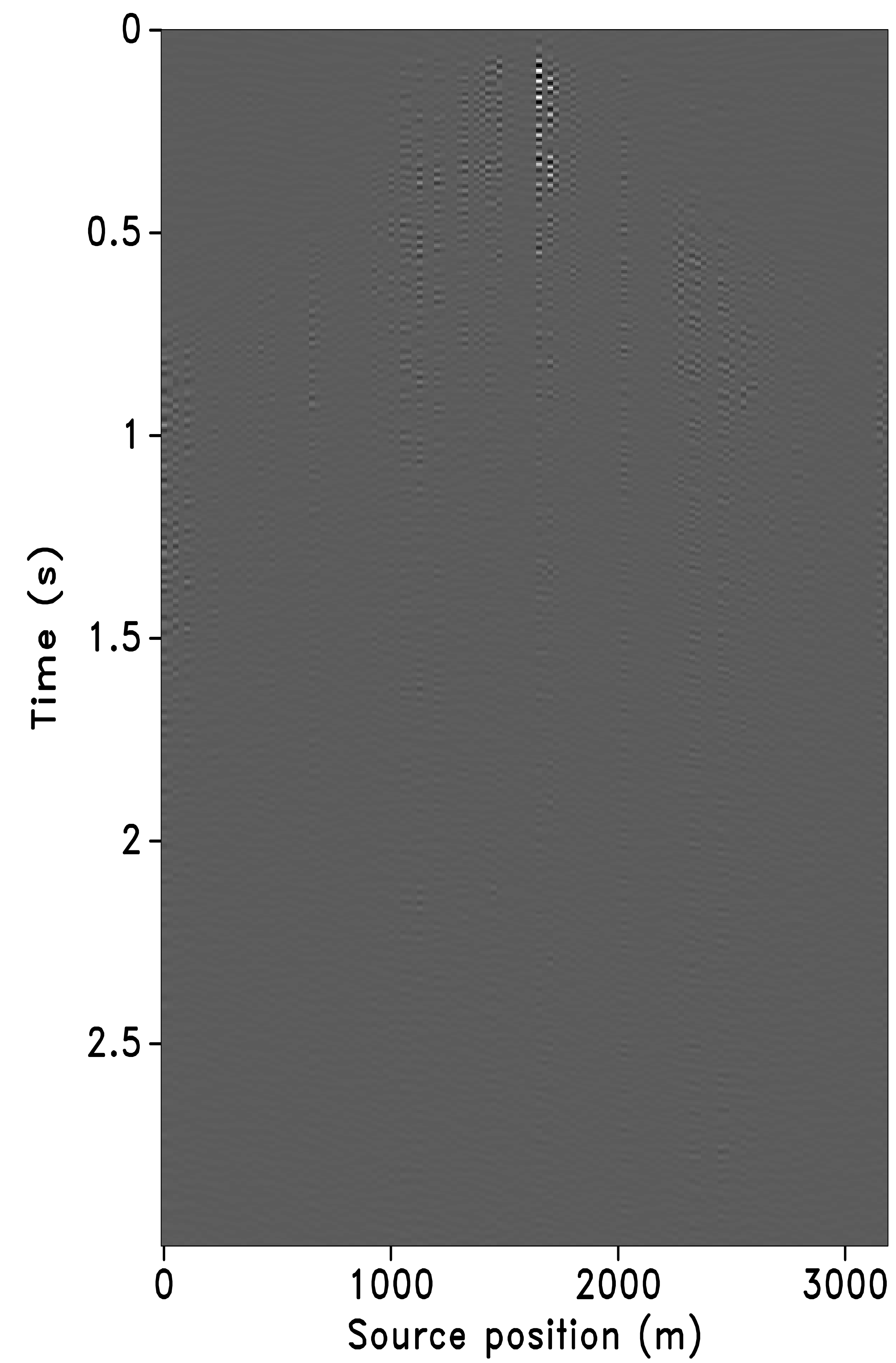
Difference

SLIM

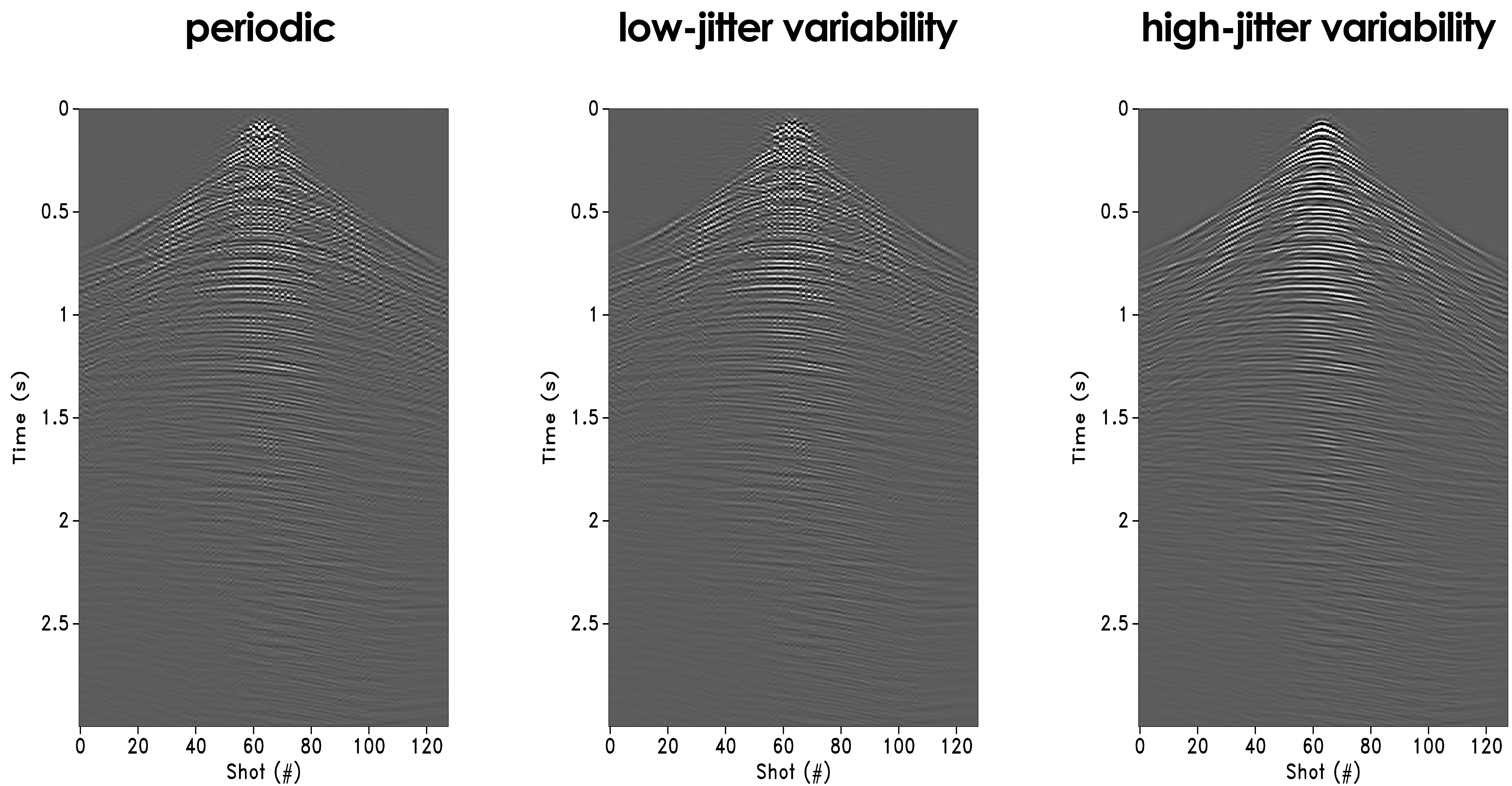
periodic



jittered



Recovery


periodic
low-jitter variability
high-jitter variability
Time (s)
0
0.5
1
1.5
2
2.5
Shot (#)
0 20 40 60 80 100 120

Observations

Transform-based CS works well for large variability

- limited to static geometries such as OBC / OBN

Can we relax requirement of large variability?

- enabler for dynamic geometries such as towed arrays
- over-under w/ random delays < 1S
- shot-by-shot source-separation

Simultaneous marine acquisition

[over/under acquisition, towed arrays]

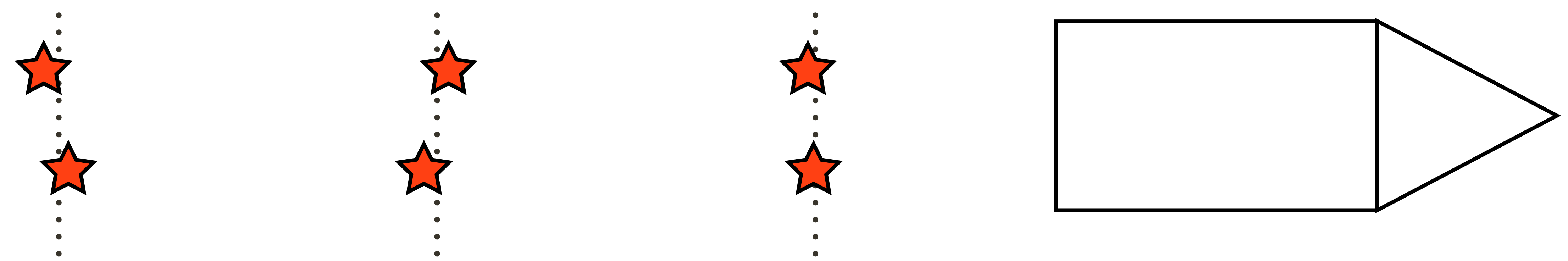
★ source depth 1

★ source depth 2

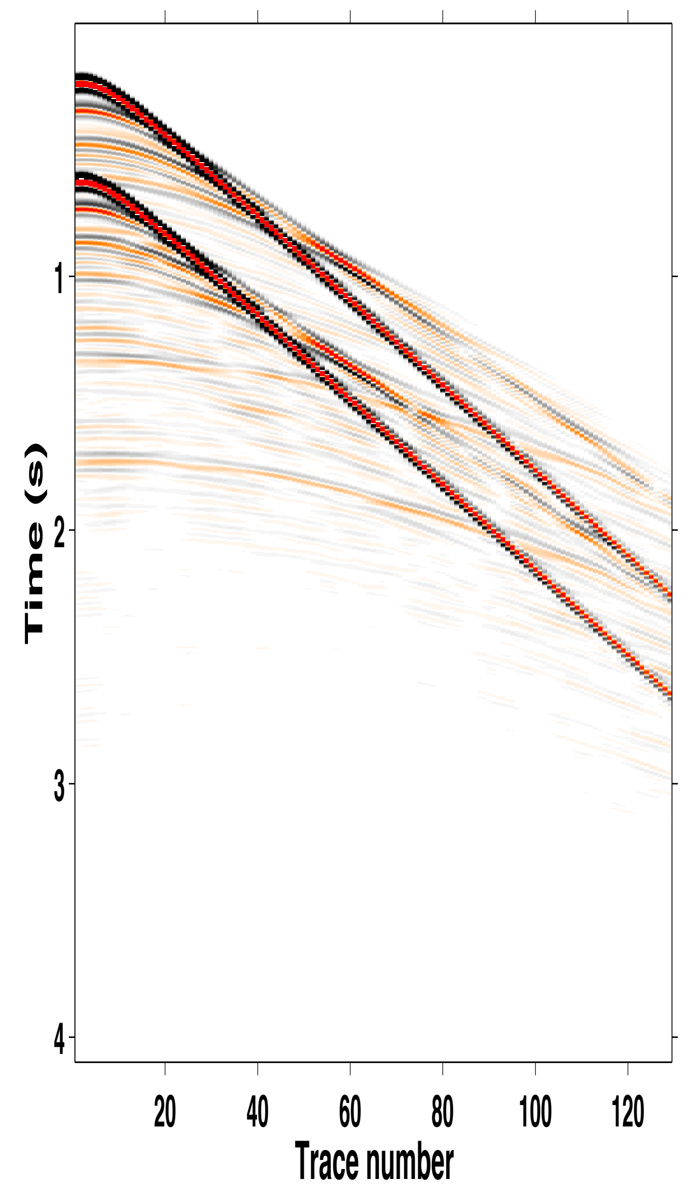
almost periodically sampled spatial grid

shot-time randomness - **LOW**

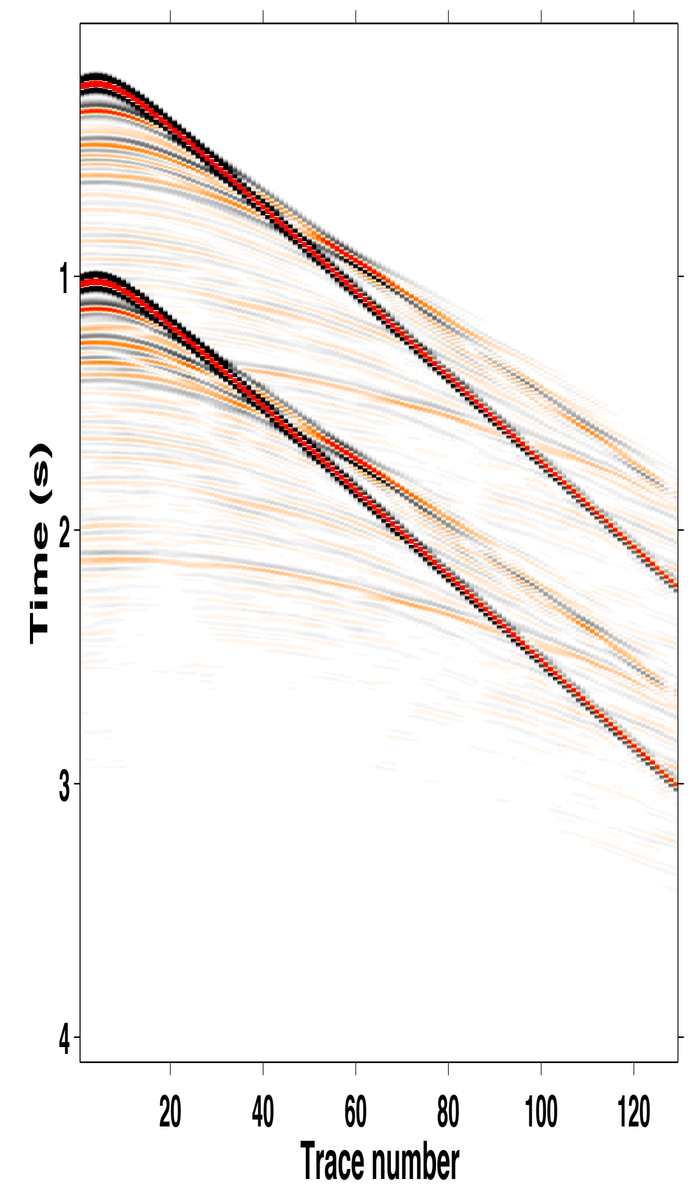
shot 1 shot 2 shot 3



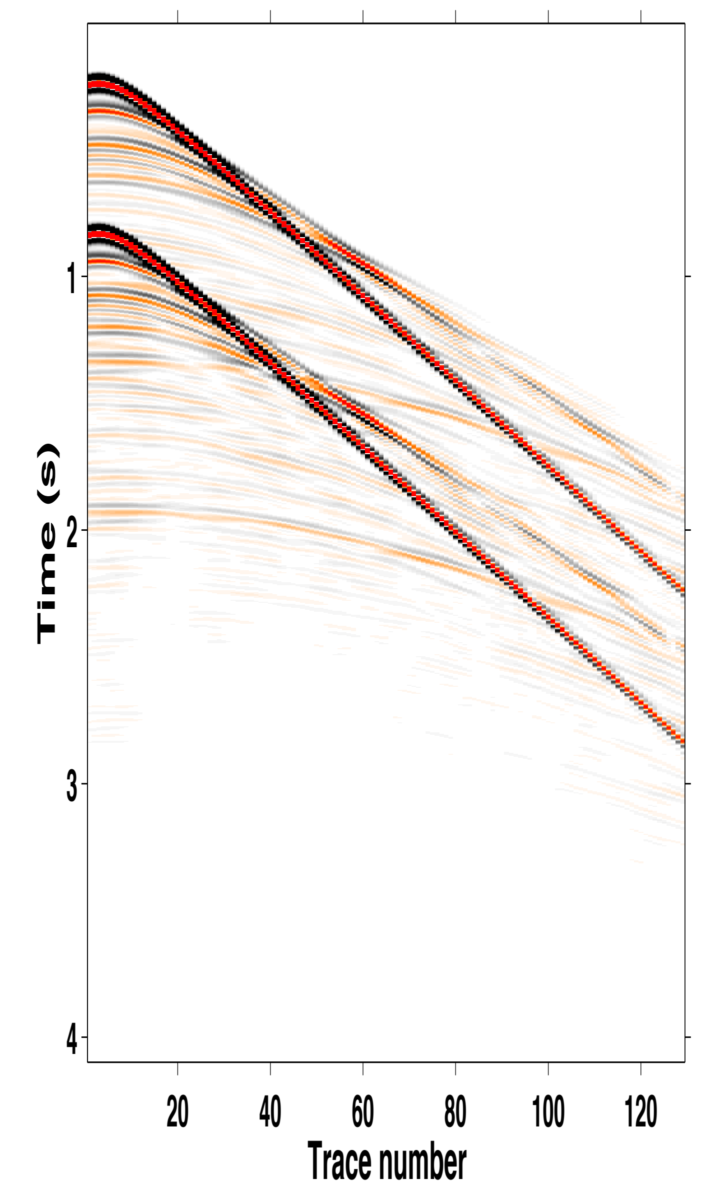
shot 1



shot 2



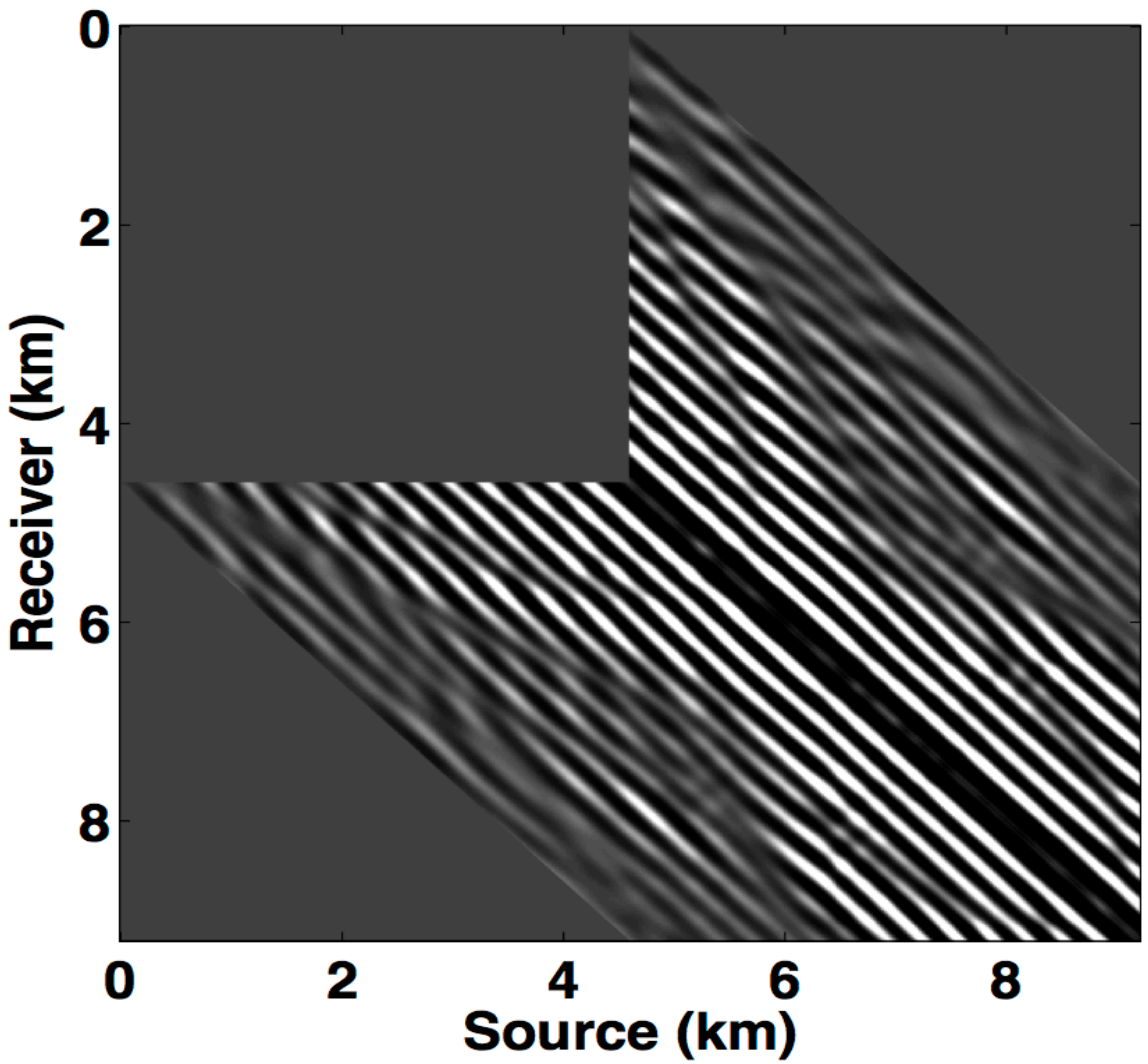
shot 3



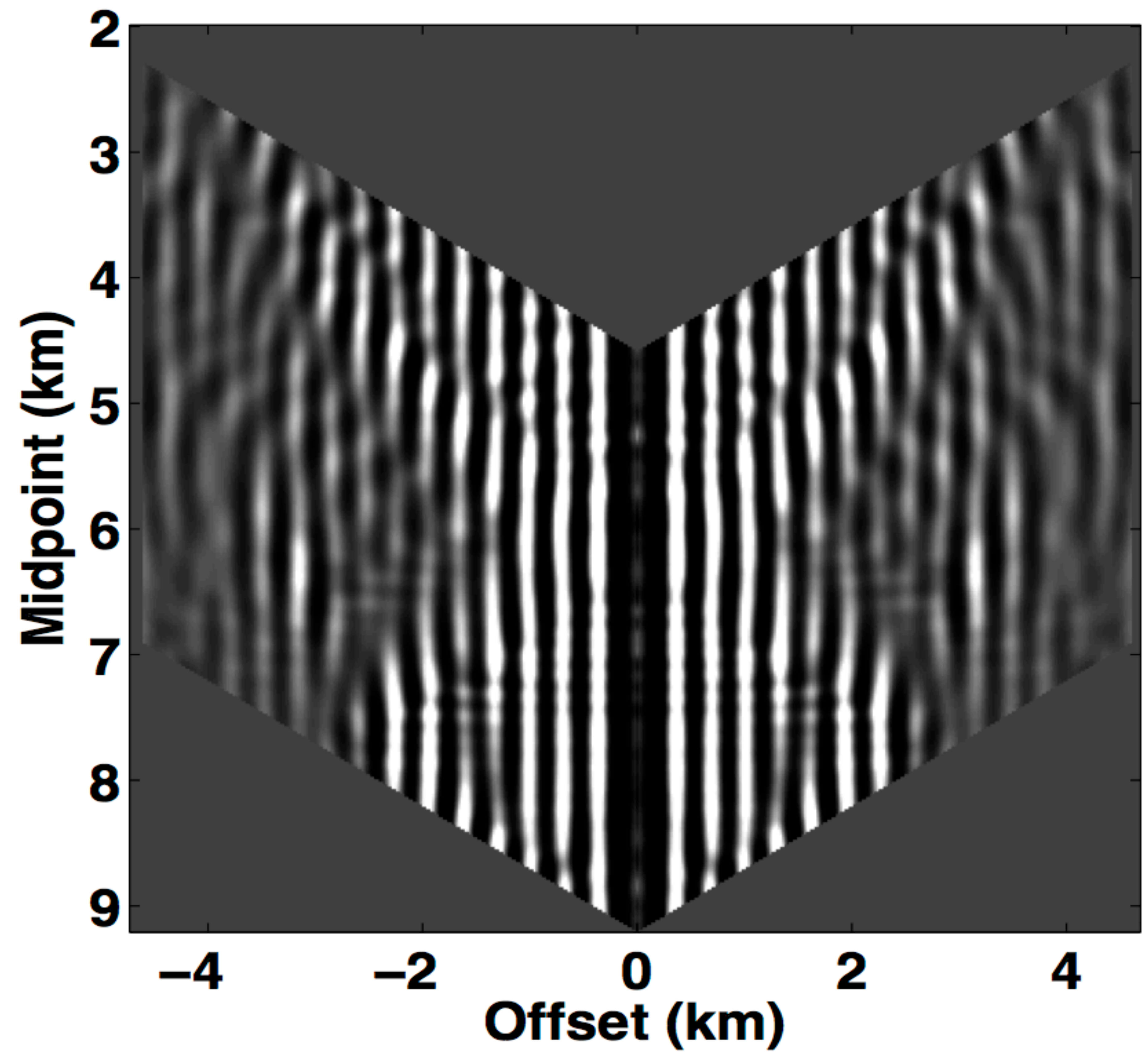
In which domain?

frequency slice at 5 Hz

source-receiver domain
(with reciprocity)



midpoint-offset domain
(with reciprocity)

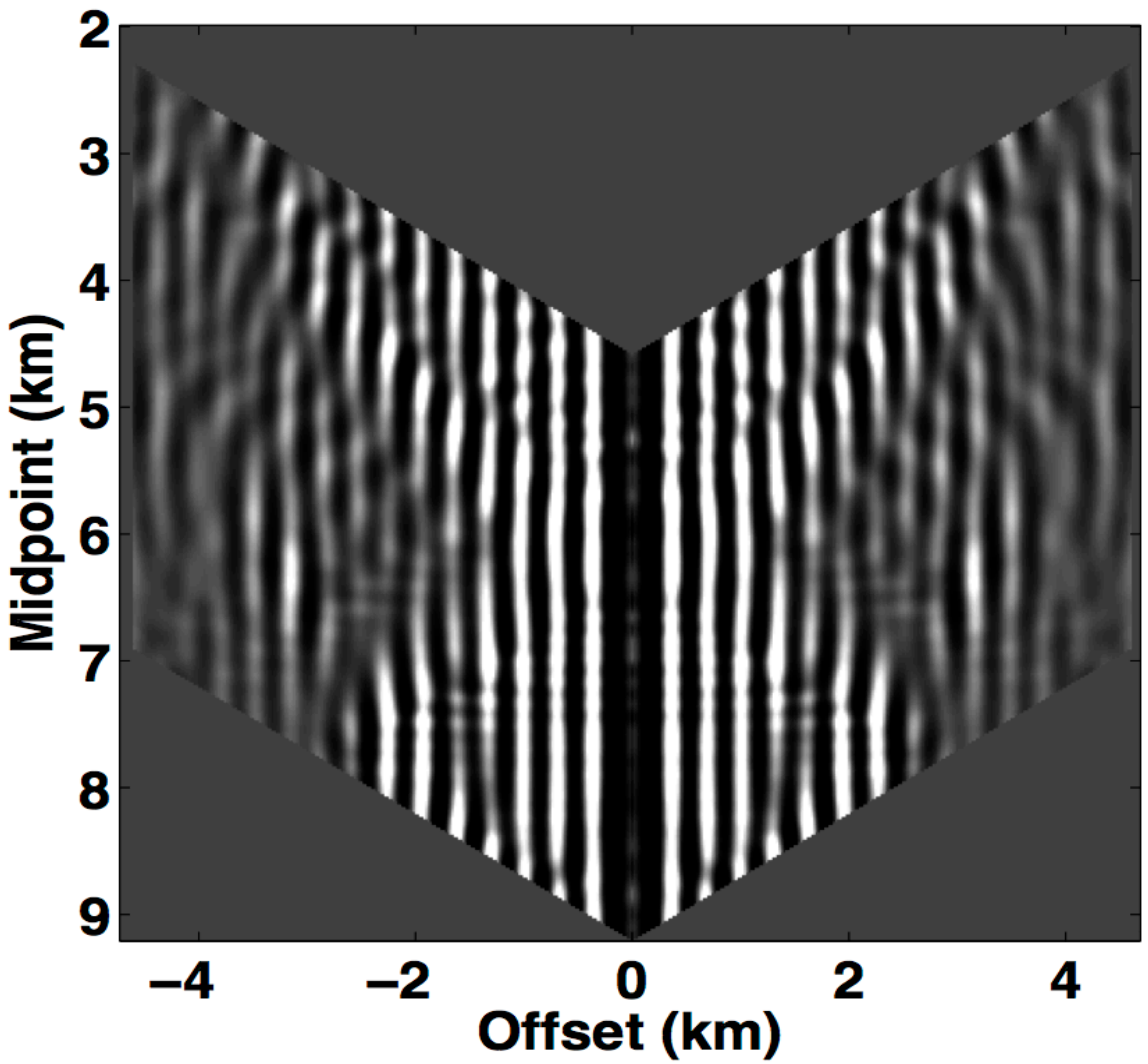


How to destroy the structure?

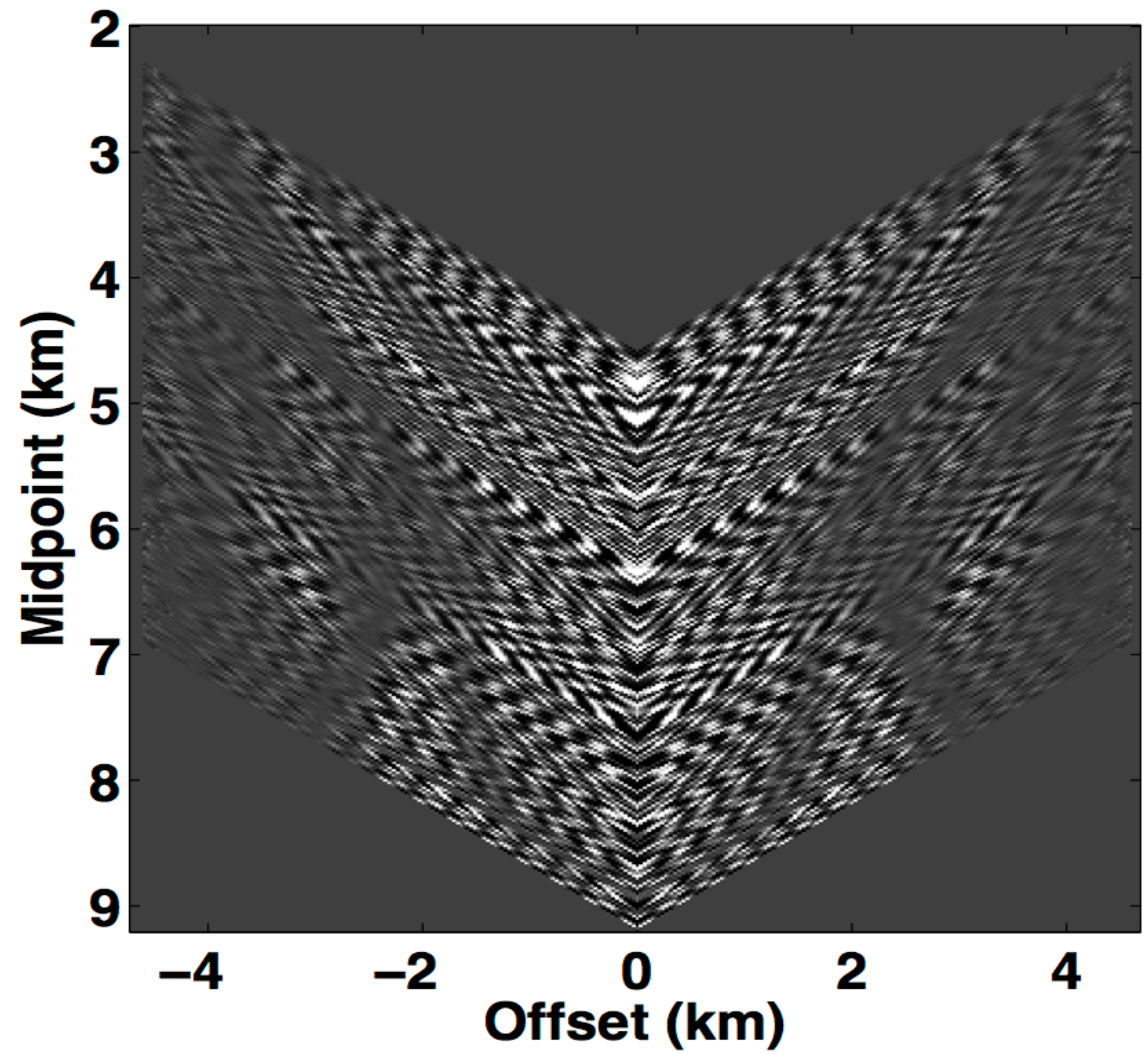
add random time delays

no missing traces!

without delays

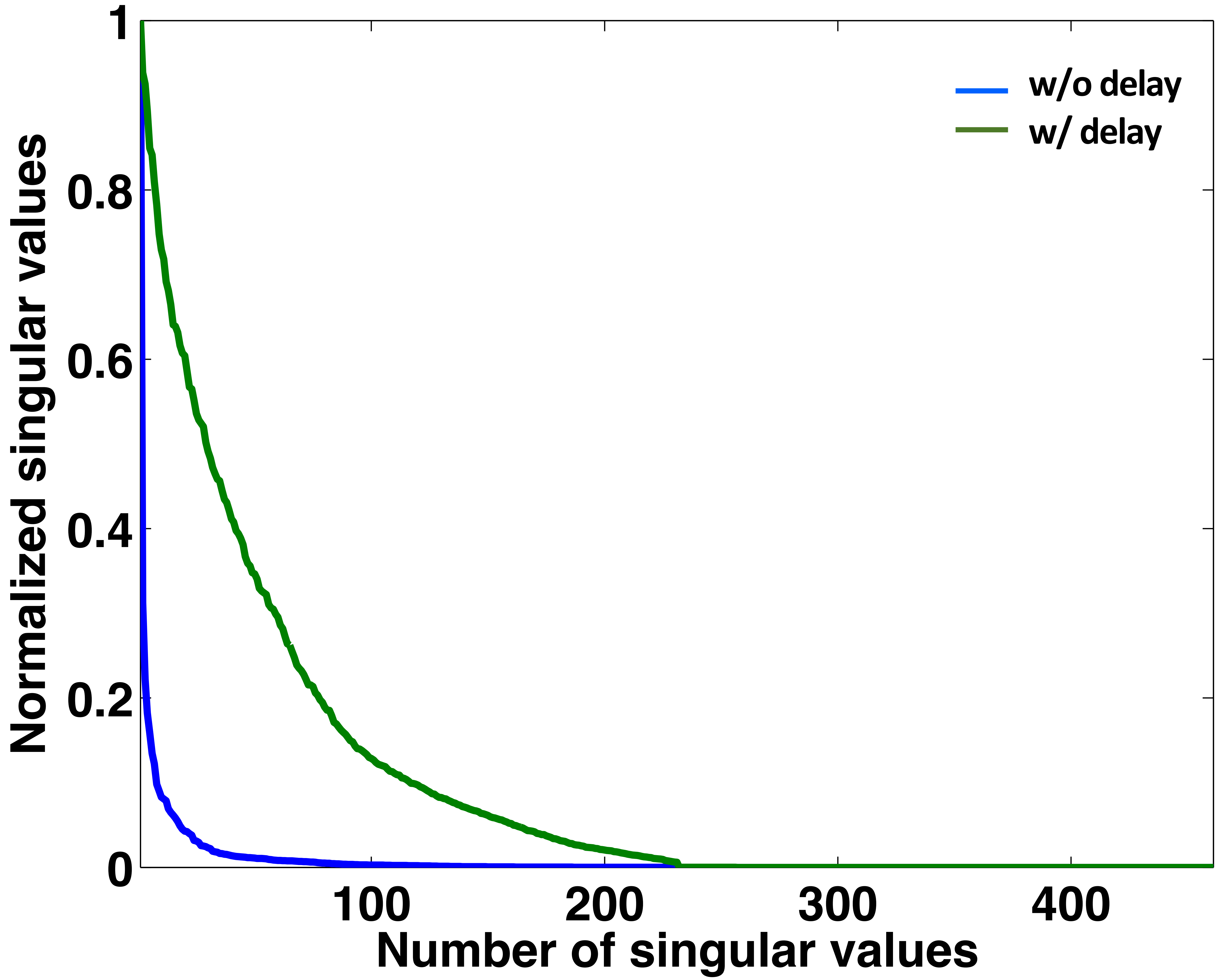


with random delays (< 1s)



Decay of singular values

midpoint-offset domain



random time delays increase the rank

Rank minimization

$$\min_{\mathbf{X}} \quad \underbrace{\operatorname{rank}(\mathbf{X})}_{\text{number of singular values of } \mathbf{X}} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

for blended acquisition:

unblended data matrix

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \begin{array}{l} \longleftarrow \text{ source 1} \\ \longleftarrow \text{ source 2} \end{array}$$

$\mathbf{b}$: blended data

$$\mathcal{A} := \begin{bmatrix} \mathbf{M}\mathbf{T_1}\mathbf{S^H} & \mathbf{M}\mathbf{T_2}\mathbf{S^H} \end{bmatrix}$$

time delay matrices

Rank minimization

expective

(search over all possible values of rank)

$$\min_{\mathbf{X}} \quad \underbrace{\operatorname{rank}(\mathbf{X})} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

Nuclear-norm minimization

convex relaxation of rank-minimization

[Recht et. al., 2010]

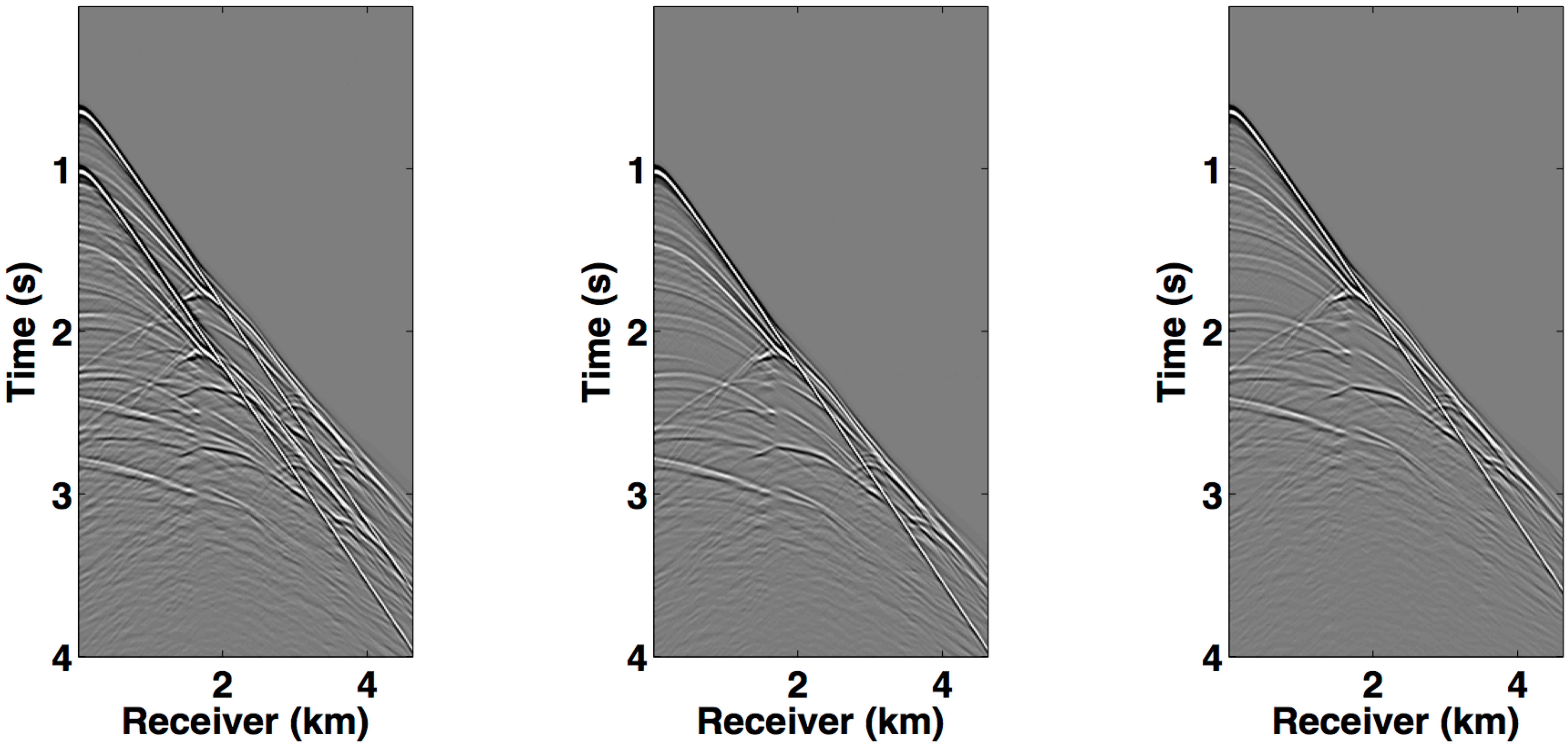
$$\min_{\mathbf{X}} \quad \underbrace{||\mathbf{X}||_*} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

sum of singular values of $\mathbf{X}$

Blended data

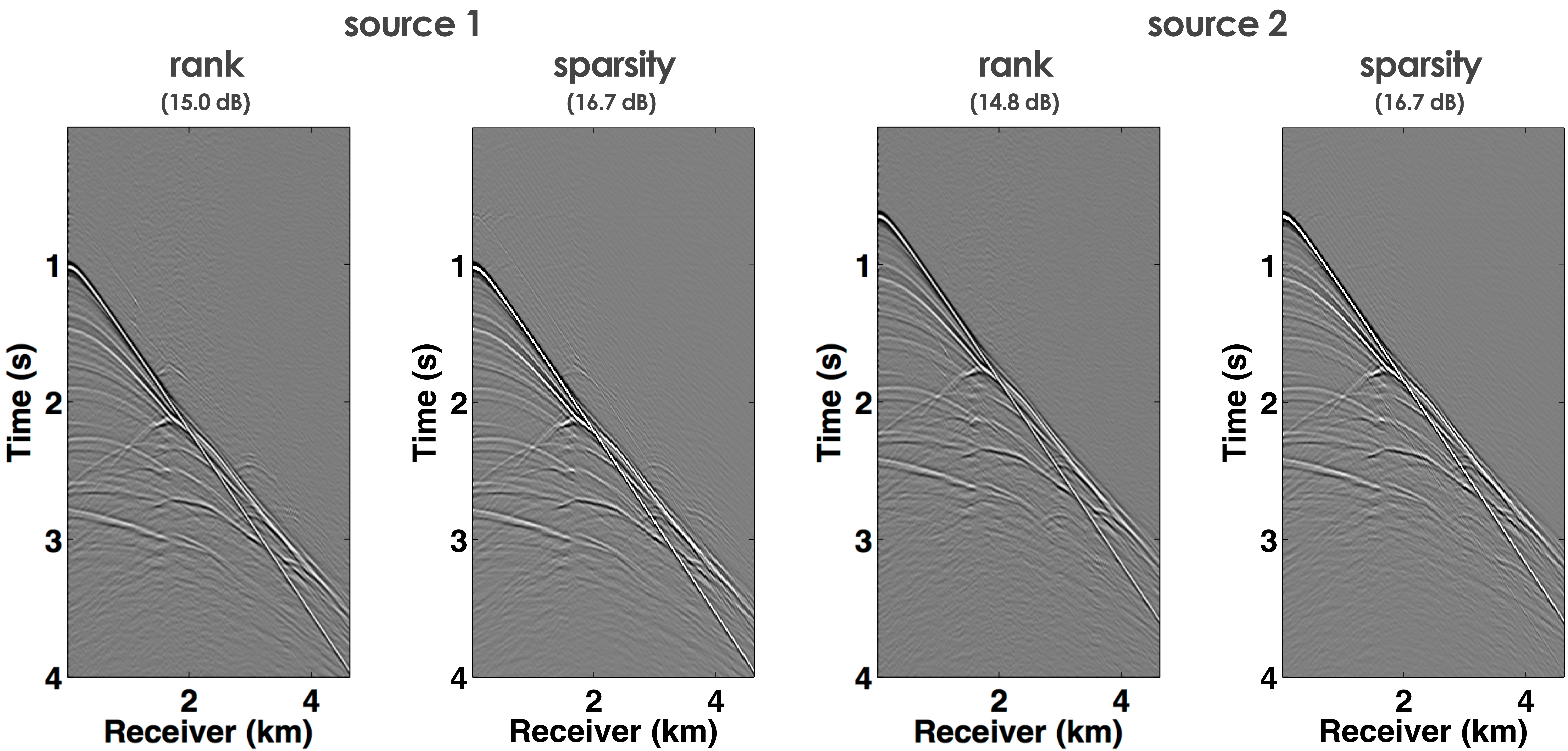
random time delays (< 1 sec) applied to both sources

blended shot = source 1 + source 2



Source separation - rank vs. sparsity

memory usage = 2.8 vs. 7.0 GB



Randomized time-lapse acquisition

Haneet Wason & Rajiv Kumar & Felix Oghenekohwo

SLIM
University of British Columbia

Motivation

Seemingly *innocent* remark by Craig J. Beasley at SBGf meeting:

"Should we repeat in randomized marine acquisition?"

Dror Baron, Marco F. Duarte, Shriram Sarvotham, Michael B. Wakin, Richard G. Baraniuk, "**An Information-Theoretic Approach to Distributed Compressed Sensing**" (2005)

SLIM

Distributed compressed sensing

joint recovery model (JRM)

$$\overbrace{\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{A}_2 \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}}^{\mathbf{z}} = \overbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}}^{\mathbf{b}}$$

baseline

monitor

vintages

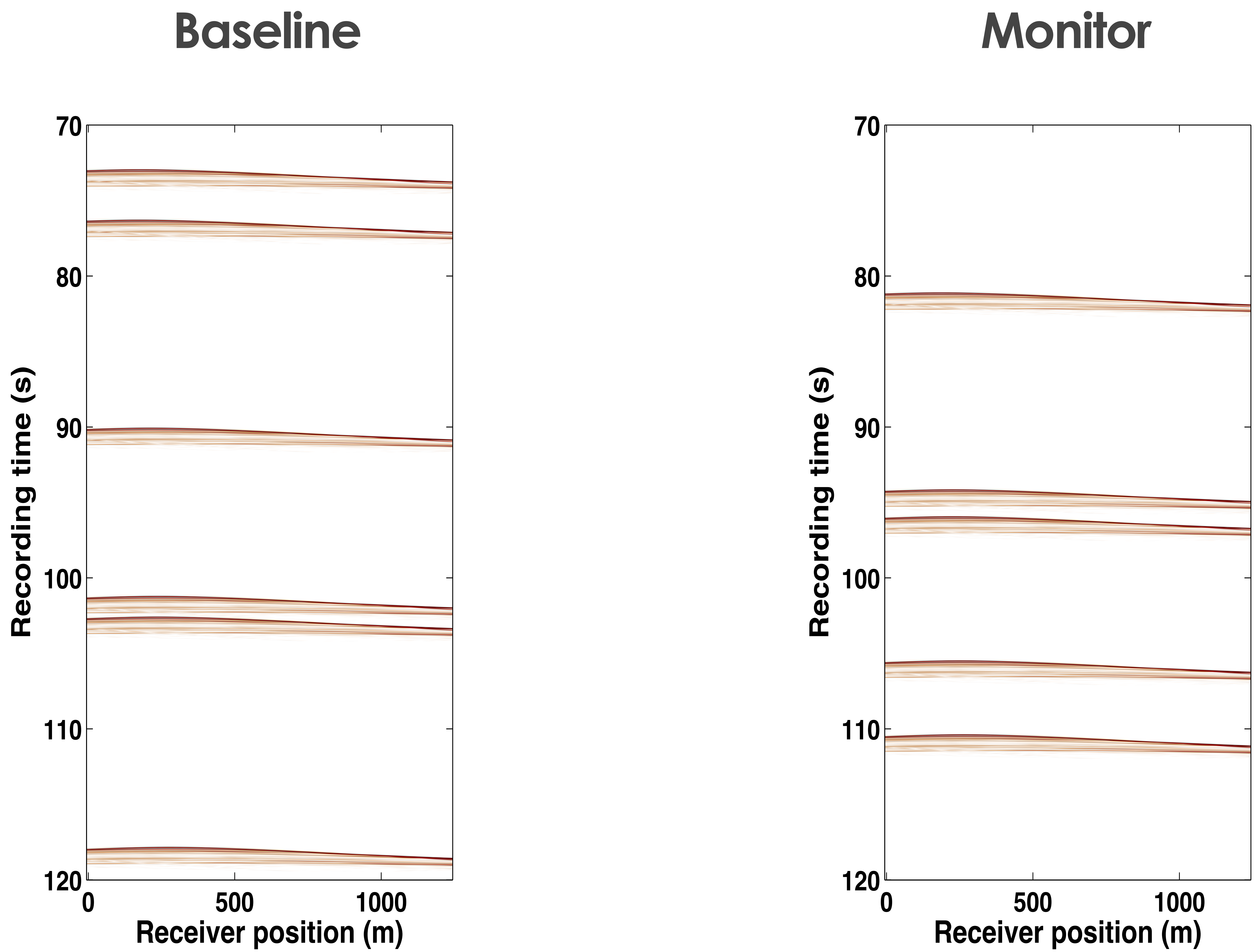
$$\mathbf{x}_1 = \mathbf{z}_0 + \mathbf{z}_1$$
$$\mathbf{x}_2 = \mathbf{z}_0 + \mathbf{z}_2$$

differences

common component

Different vintages share common information!

Measurements
subsampled & blended



Context

Acquire economic randomized subsamplings for baseline & monitor surveys

Aim: recovery of pre-stack vintages & post-stack time-lapse attributes

Questions:

- Process/recover independently or jointly to exploit common features w/i surveys?
- Should we repeat the surveys when doing randomized subsampling?

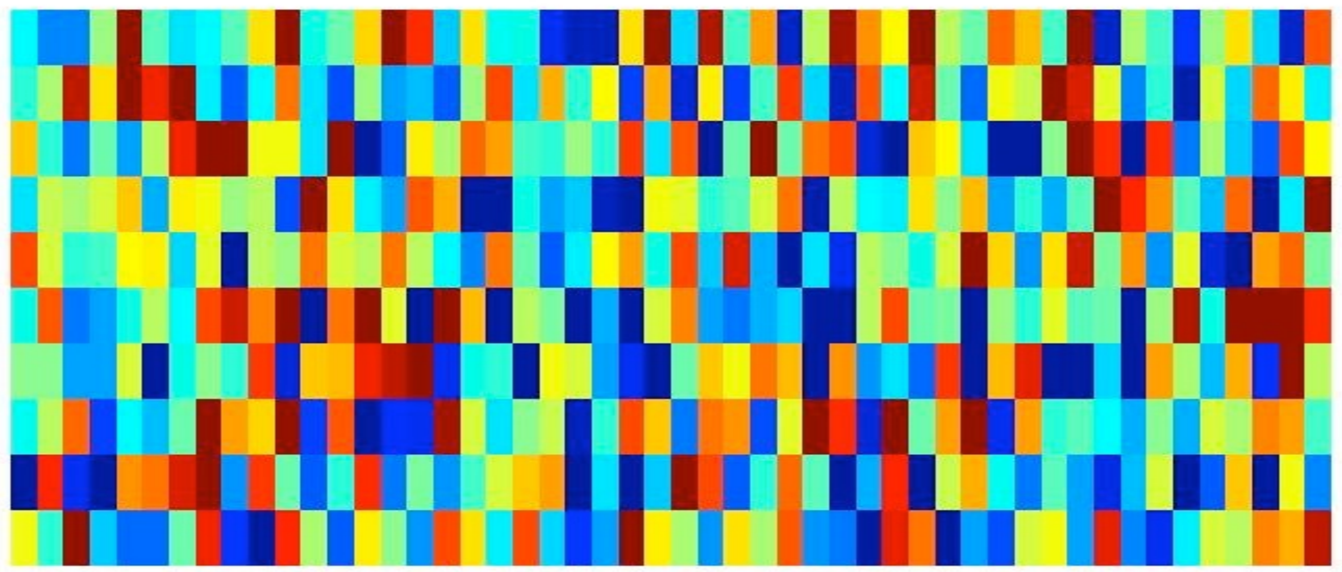
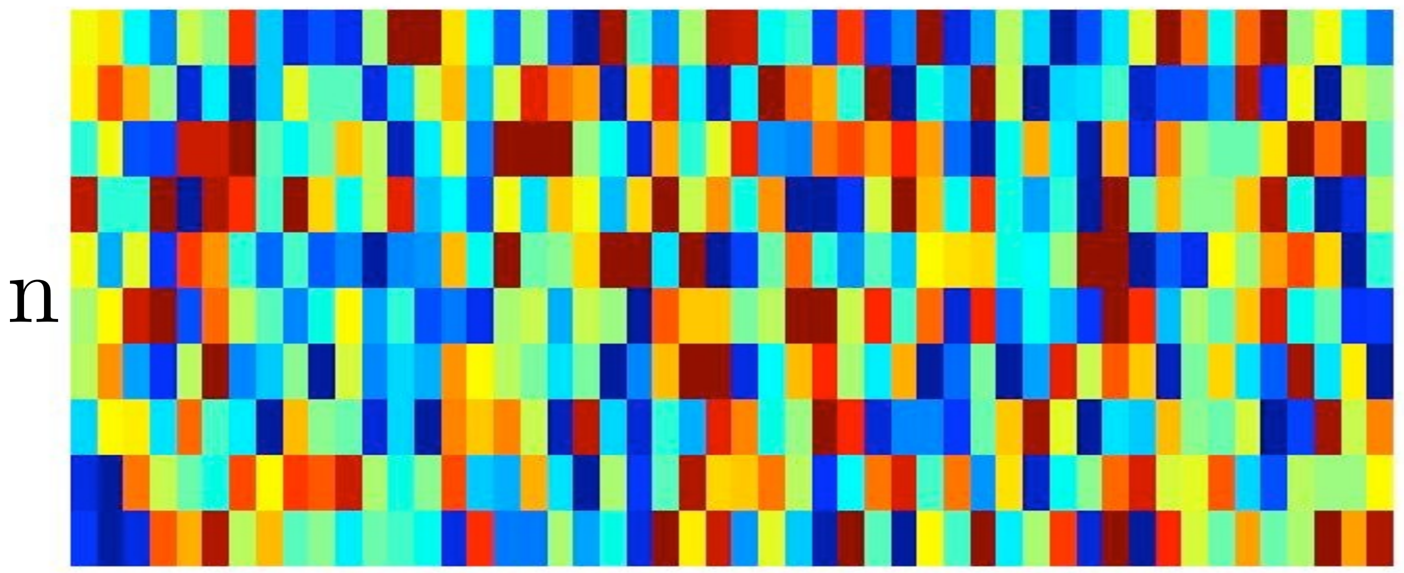
Stylized experiments

Stylized experiments

Conduct *many* CS experiments to compare

- *joint* vs *parallel* recovery of signals and the difference
- recovery with *same*, *partially* or *completely* independent matrices
- *random* acquisition with different numbers of samples

$\mathbf{A}_1$ $\quad n = 10 \quad$ $\mathbf{A}_2$

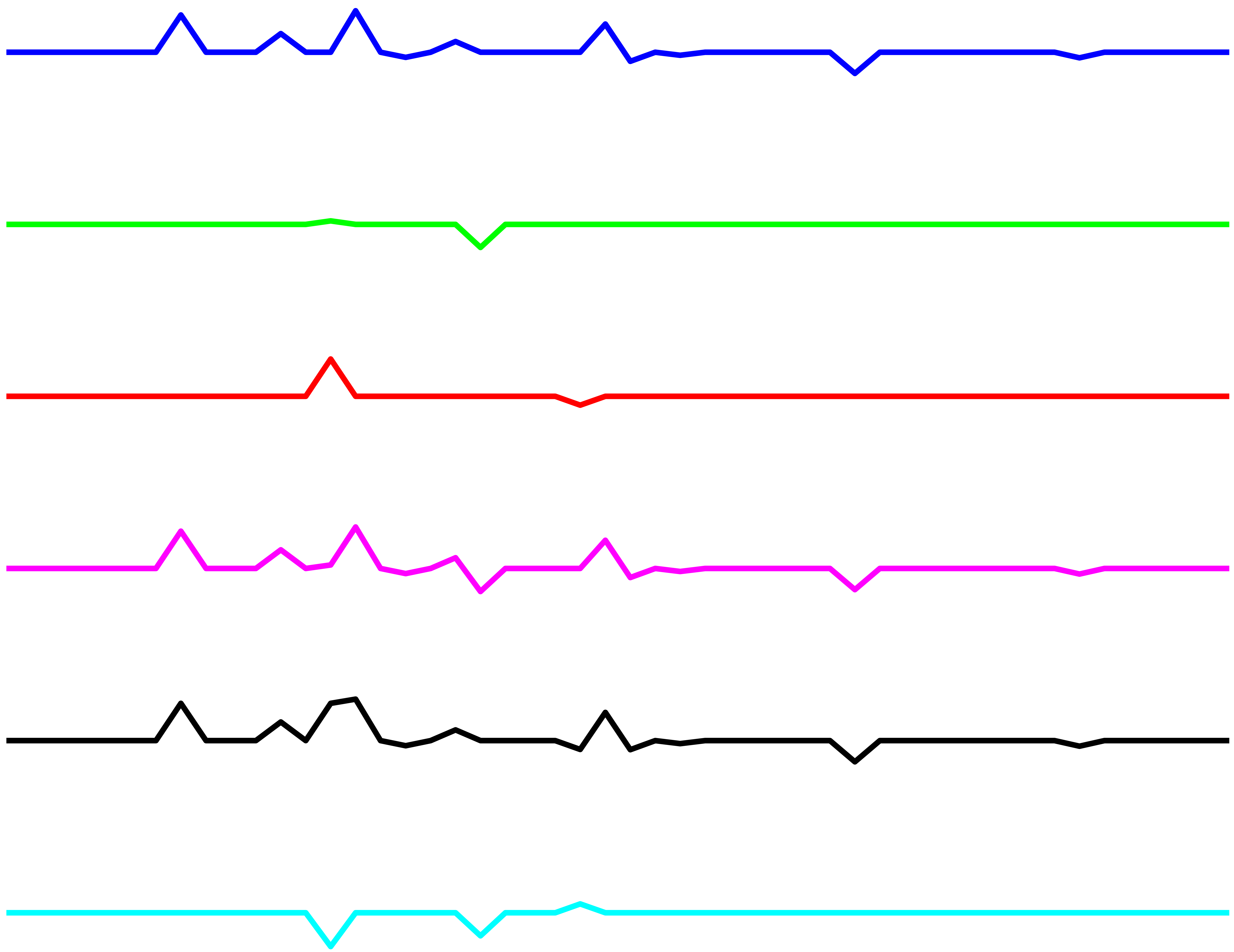
n  n 

N $\quad$ N

$\mathbf{b}_1 = \mathbf{A}_1 \mathbf{x}_1$ $\qquad$ $\mathbf{b}_2 = \mathbf{A}_2 \mathbf{x}_2$

Run 2000 different experiments & compute probability of recovery.

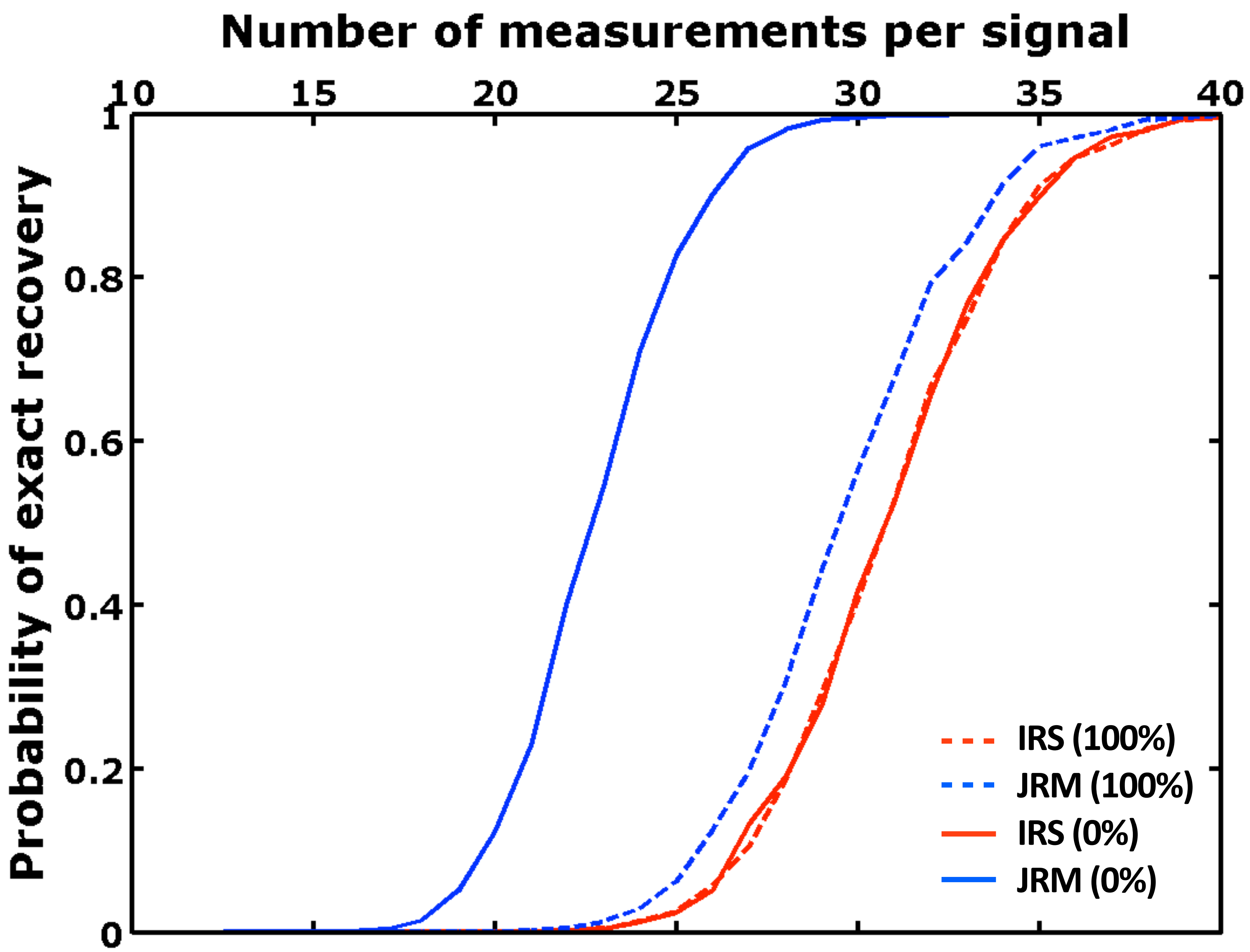
Sparse signals



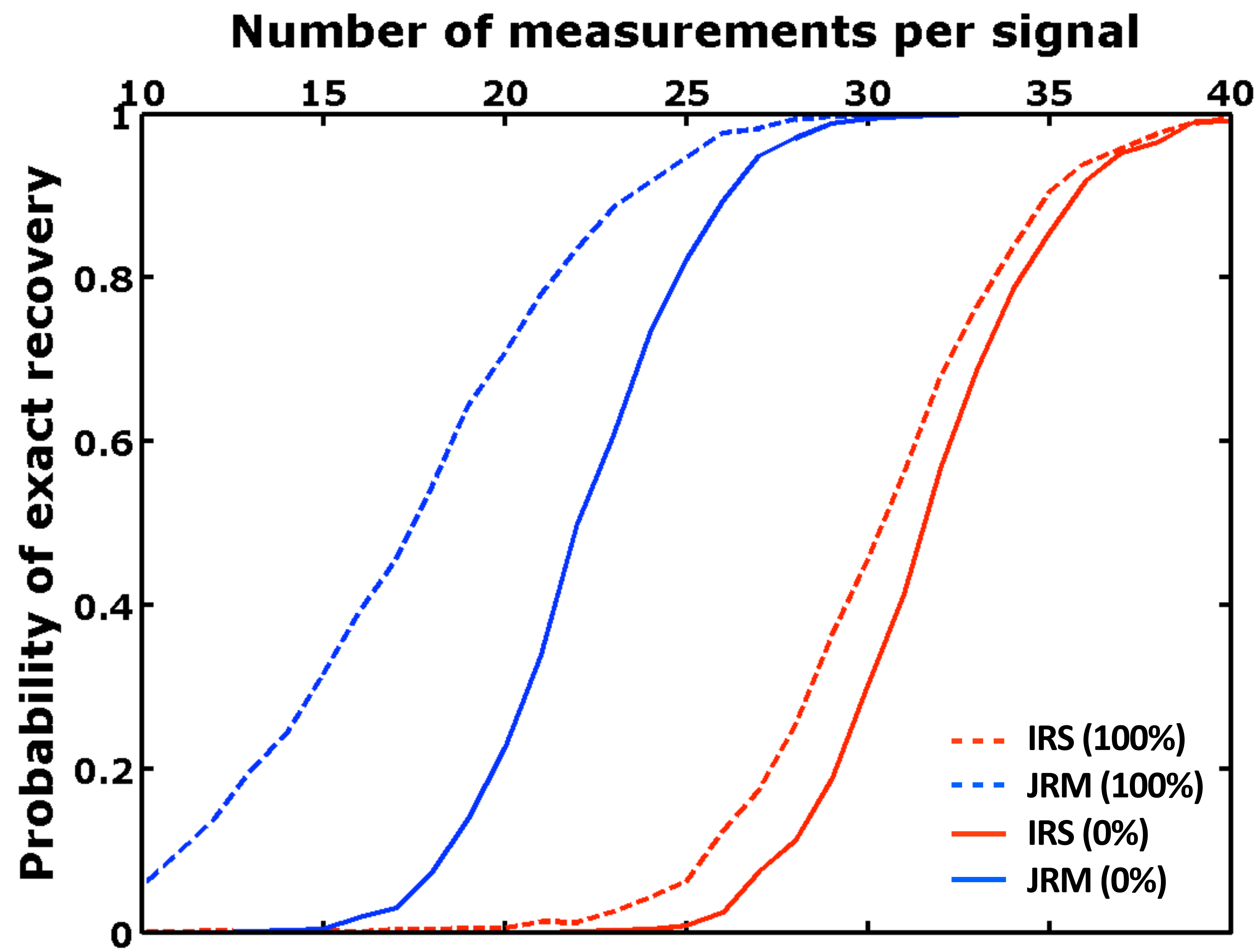
Independent vs. joint recovery

100% & **0%** overlap in acquisition matrices

100% => "exact" repeatability (difficult to achieve in practice)



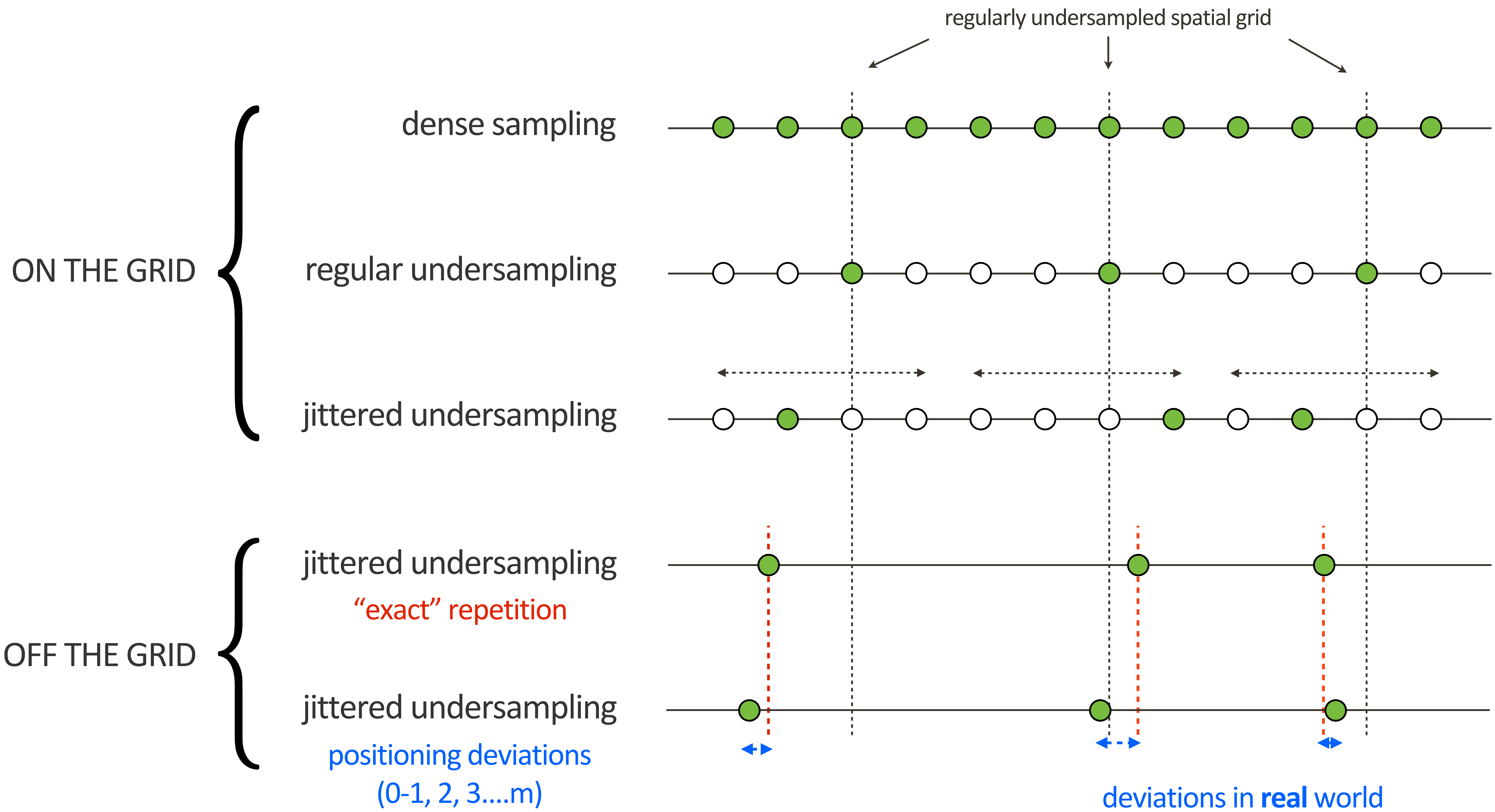
Vintages



4-D signal

Off-the-grid synthetic marine seismic

Randomized sampling in marine

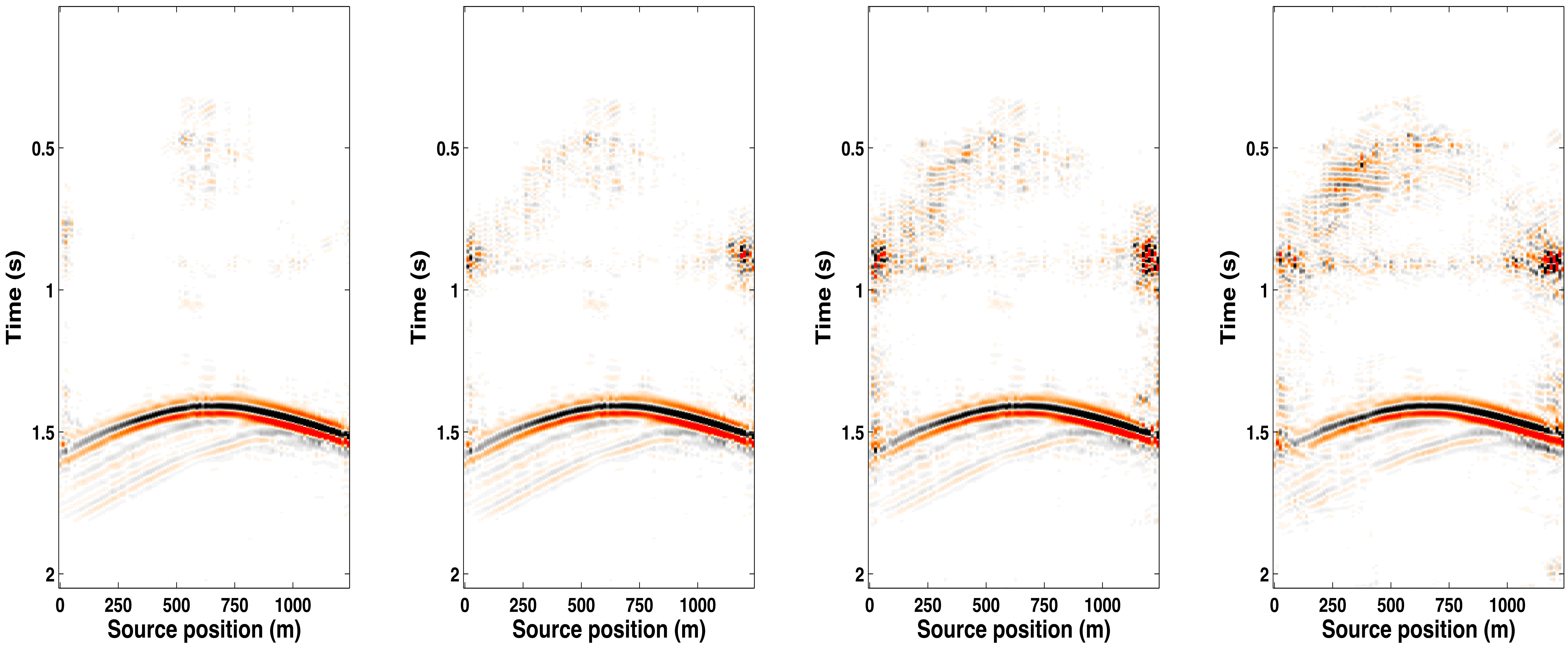


4-D recovery - JRM

50% overlap in acquisition matrices

0% overlap

no deviation	deviation ≈ 1.0 m	deviation ≈ 2.8 m	
[12.2 dB]	[8.5 dB]	[3.8 dB]	[2.0 dB]



On the contrary,

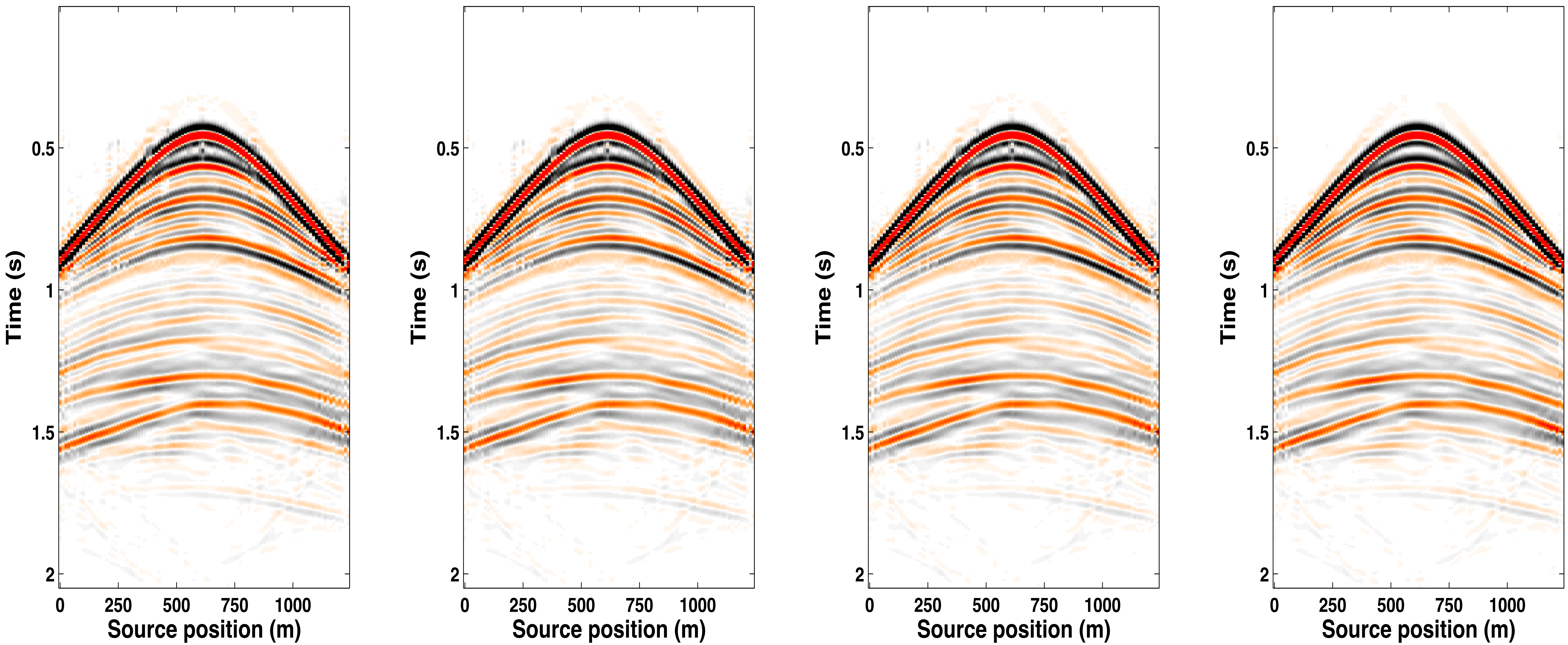
calibration errors improve recovery of the vintages!

Monitor recovery - JRM

50% overlap in acquisition matrices

0% overlap

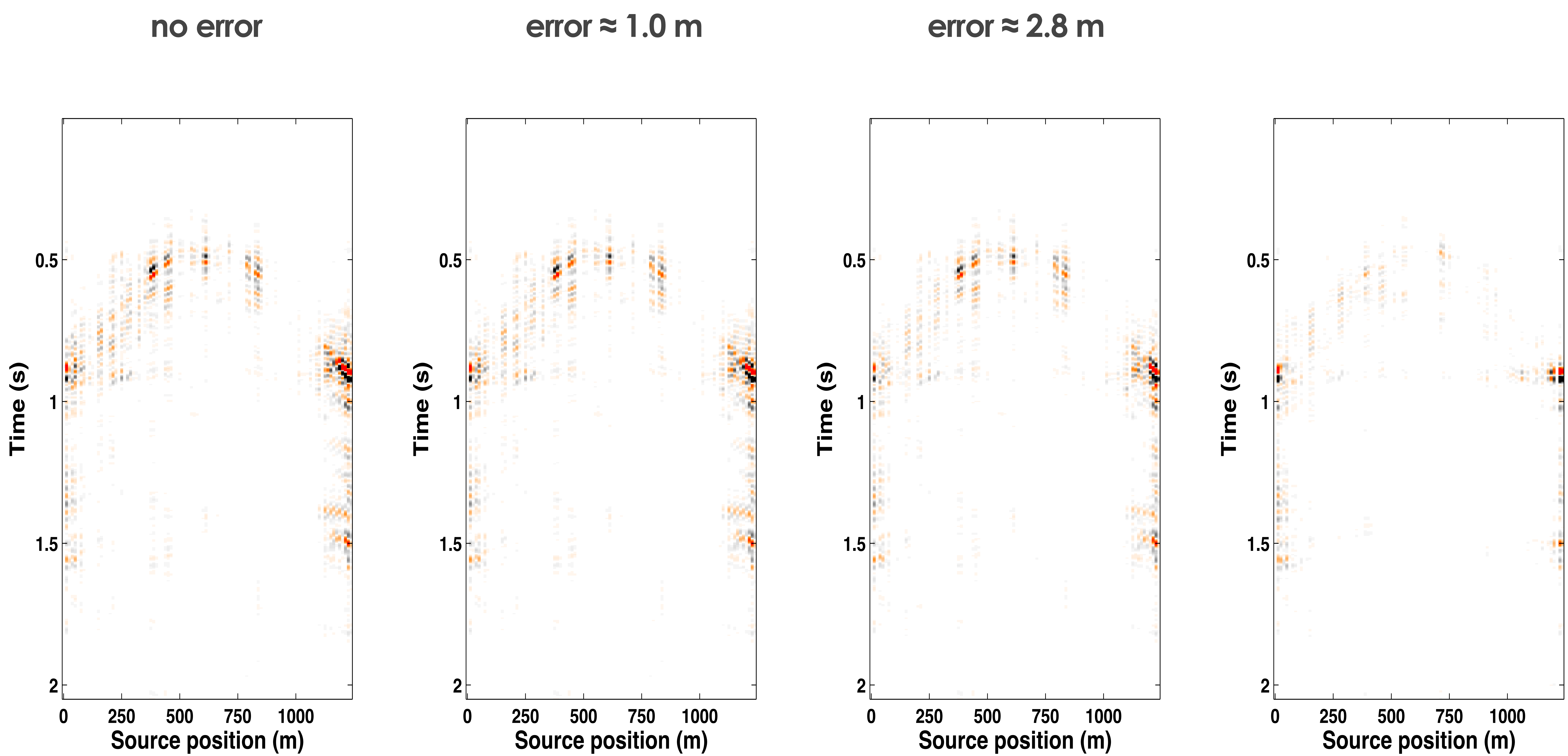
no deviation	deviation ≈ 1.0 m	deviation ≈ 2.8 m	
[13.9 dB]	[14.5 dB]	[15.5 dB]	[18.3 dB]



Monitor residual - JRM

50% overlap in acquisition matrices

0% overlap



Haneet Wason, Felix Oghenekohwo, and Felix J. Herrmann, “**Compressed sensing in 4-D marine—recovery of dense time-lapse data from subsampled data without repetition**”, in *EAGE Annual Conference Proceedings*, 2015.

Observations

In the given context of randomized subsampling, deviations in the shot locations

- deteriorate recovery of the pre-stack time-lapse signal
- improve recovery of the pre-stack vintages

“Exact” repeatability is a red herring

- unfeasible in the field
- pre-stack vintages are used to compute post-stack time-lapse attributes

Conclusion: there is no need to repeat in the field as long as you know where you where…

Where we need to go

CS in Exploration Seismology:

- step change in economics & sampling density
- but relies on calibration – need to know where you were
- also role of noise is not well understood

Future CS is in need of

- practical quantitative design principles
- more “adaptive” samplings
- quantitative assessment of risk

Conclusions

CS corresponds to an acquisition design problem

- validated in the field
- dense surveys (static & dynamic geometries) from economic randomly subsampled data
- provides fundamental new insights how to acquire seismic wavefield by exploring structure

Bottom line: Randomized sampling reduces

- acquisition costs (5 X – 10 X)
- environmental imprint
- improved data quality

Acknowledgements

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