Automatic salt flooding with constrained wavefield reconstruction inversion

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Strategy

Extend the search space

- “less” nonlinear (bi-linear)
- ensures data fit & avoids cycle skips

“Squeeze” the extension by

- enforcing the wave equation to compute model updates
- imposing *asymmetric* convex constraints that
  - encode “rudimentary” domain-specific properties of the geology
  - create hard reflectors w/ correct signs
- relaxing the convex constraints while stressing the physics

Wavefield Reconstruction Inversion – WRI

wave-equation X wavefield = source

versus

$$\begin{pmatrix} \text{wave-equation} \\ \text{-----------------} \\ \text{sampling operator} \end{pmatrix} \times \text{wavefield} = \begin{pmatrix} \text{source} \\ \text{-----------------} \\ \text{data} \end{pmatrix}$$

WRI – iterations

[van Leeuwen & FJH, ’13; ’15, Peters et. al. ’14]

WRI method

for each source i

$$\text{solve} \quad \begin{pmatrix} P_i \\ \lambda A_i(\mathbf{m}) \end{pmatrix} \mathbf{u}_{\lambda,i} \approx \begin{pmatrix} \mathbf{d}_i \\ \lambda \mathbf{q}_i \end{pmatrix}$$

$$\mathbf{g} = \mathbf{g} + \lambda^2 \omega^2 \text{diag}(\bar{\mathbf{u}}_{i,\lambda})^* (A(\mathbf{m})\bar{\mathbf{u}}_{i,\lambda} - \mathbf{q}_i)$$

end

$$\mathbf{m} = \mathbf{m} - \alpha \mathbf{g}$$

correlation proxy wavefield & PDE residual

Conventional method

for each source i

$$\text{solve} \quad A(\mathbf{m})\mathbf{u}_i = \mathbf{q}_i$$

$$\text{solve} \quad A(\mathbf{m})^* \mathbf{v}_i = P_i^* (P_i \mathbf{u}_i - \mathbf{d}_i)$$

$$\mathbf{g} = \mathbf{g} + \omega^2 \text{diag}(\mathbf{u}_i)^* \mathbf{v}_i$$

end

$$\mathbf{m} = \mathbf{m} - \alpha \mathbf{g}$$

correlation wavefield & data residual

[Aria Abubakar et. al. ’09]

Related work

Contrast-source formulation

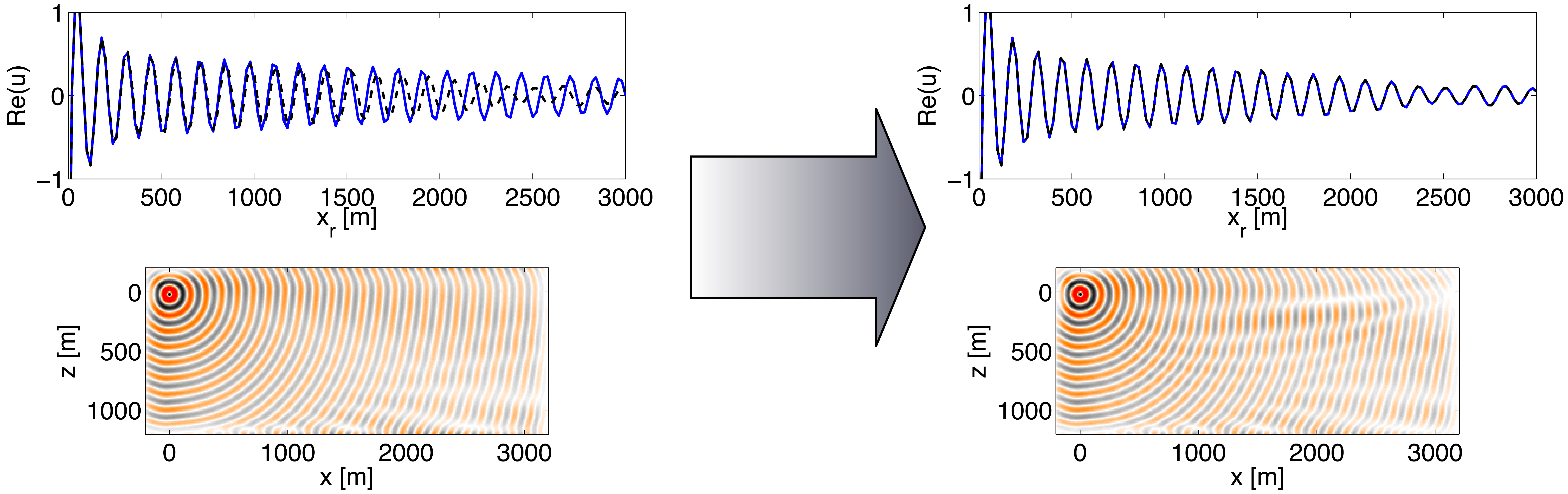
- combined objective is similar
- but does not eliminate wavefields via variable projection [Golub ’03, van Leeuwen & Aravkin ’12]
- **requires storage of wavefields for all sources**

Tomographic extensions

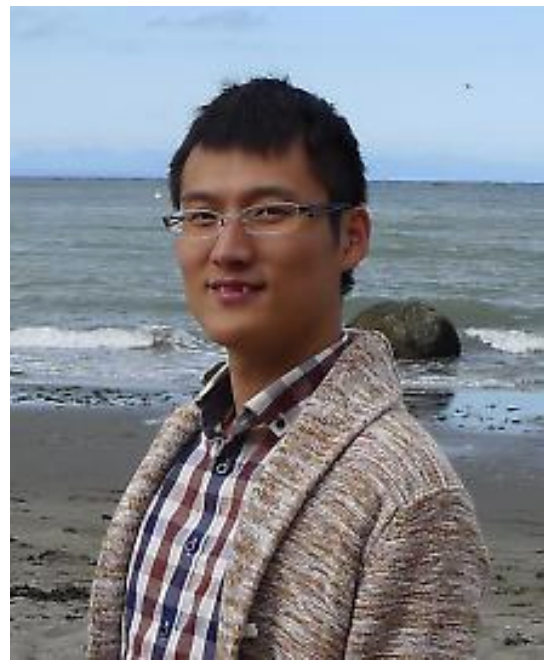
- **sensitivities to “noise” & relative strengths of events**
- WRI uses wave equation itself to “focus”

FWI vs WRI

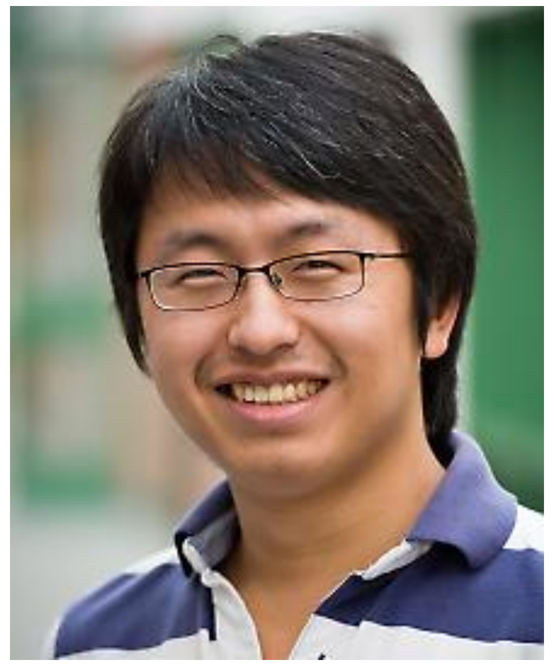
Solutions of data-augmented system force data fits… **no longer cycle skipped!**



Chevron blind test data



Zhilong Fang



Xiang Li

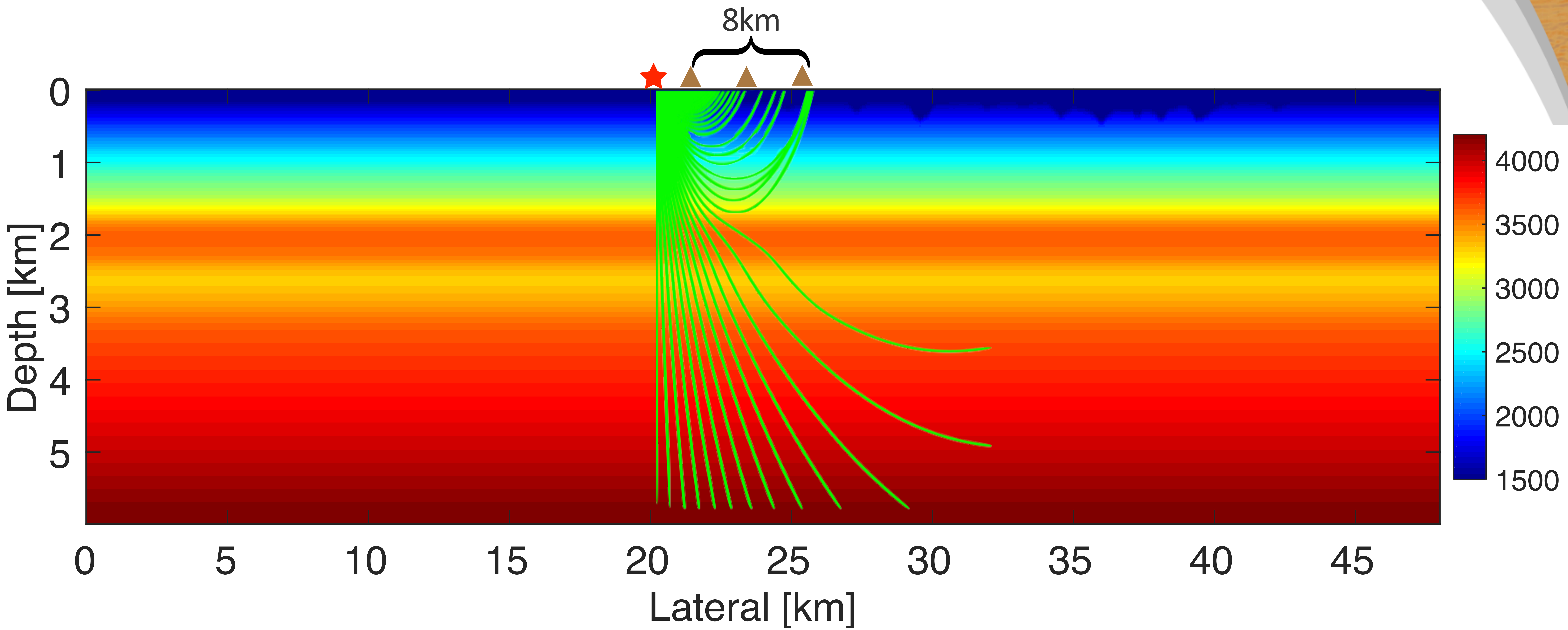
Bas Peters

Brendan Smithyman

Mengmeng Yang

Felix J. Herrmann

Initial model



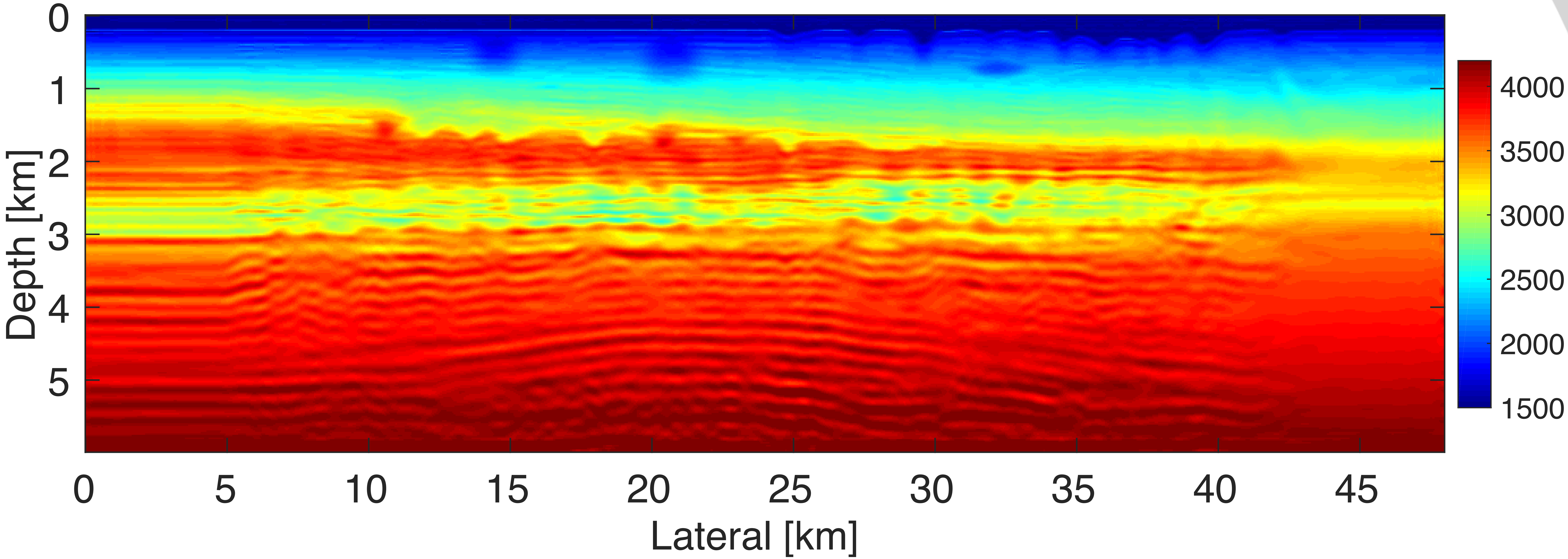
[van Leeuwen & FJH, ’13; ’15, Peters et. al. ’14]

Chevron blind test data

Inversion strategy:

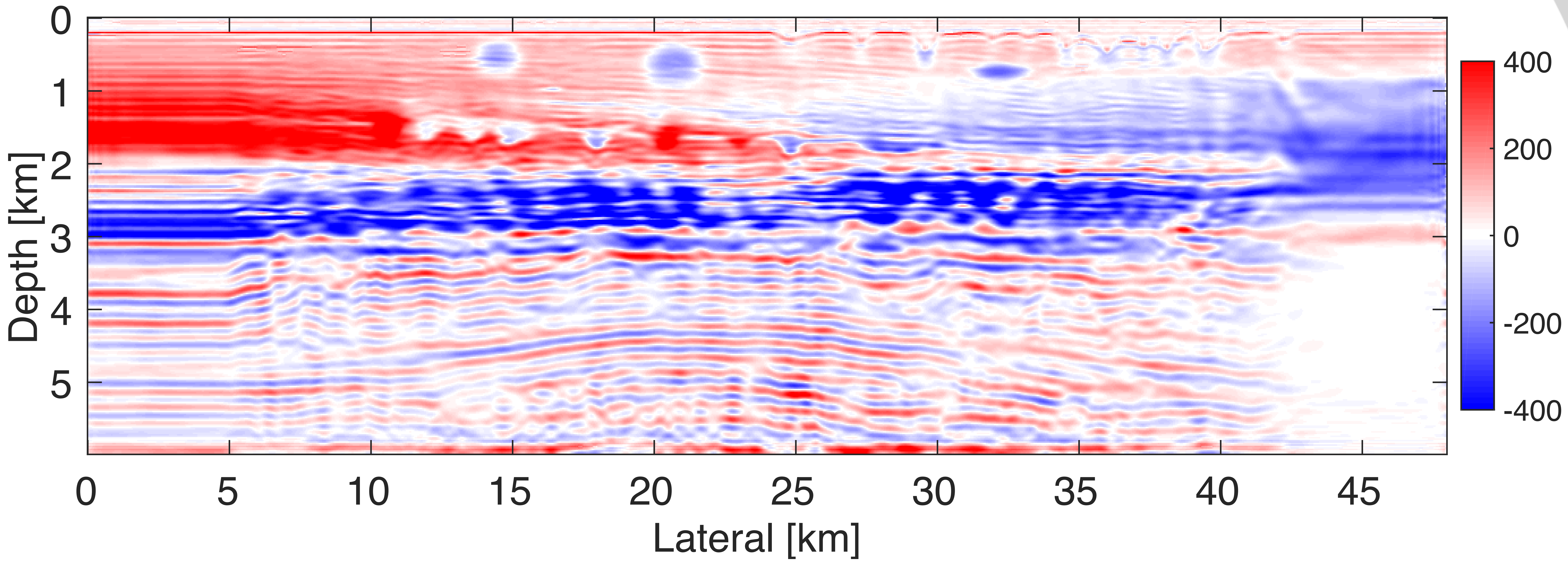
1. Frequency domain WRI with Source estimation;
2. Frequency bands: [3:0.2:5]Hz, [3:0.2:7]Hz, [3:0.2:9]Hz, [3:0.2:11]HzHz;
3. Batch sizes of random frequency subsets: 3, 6, 10, 10;
4. Batch size of random source subsets: 300;
5. Optimization solver: l-BFGS with 30 iterations per frequency band;
6. 2passes of WRI from frequency 3-11 Hz;
7. Grid size: 20m;
8. Minimum offset used: 1000m;
9. **No pre-processing !!!**

WRI

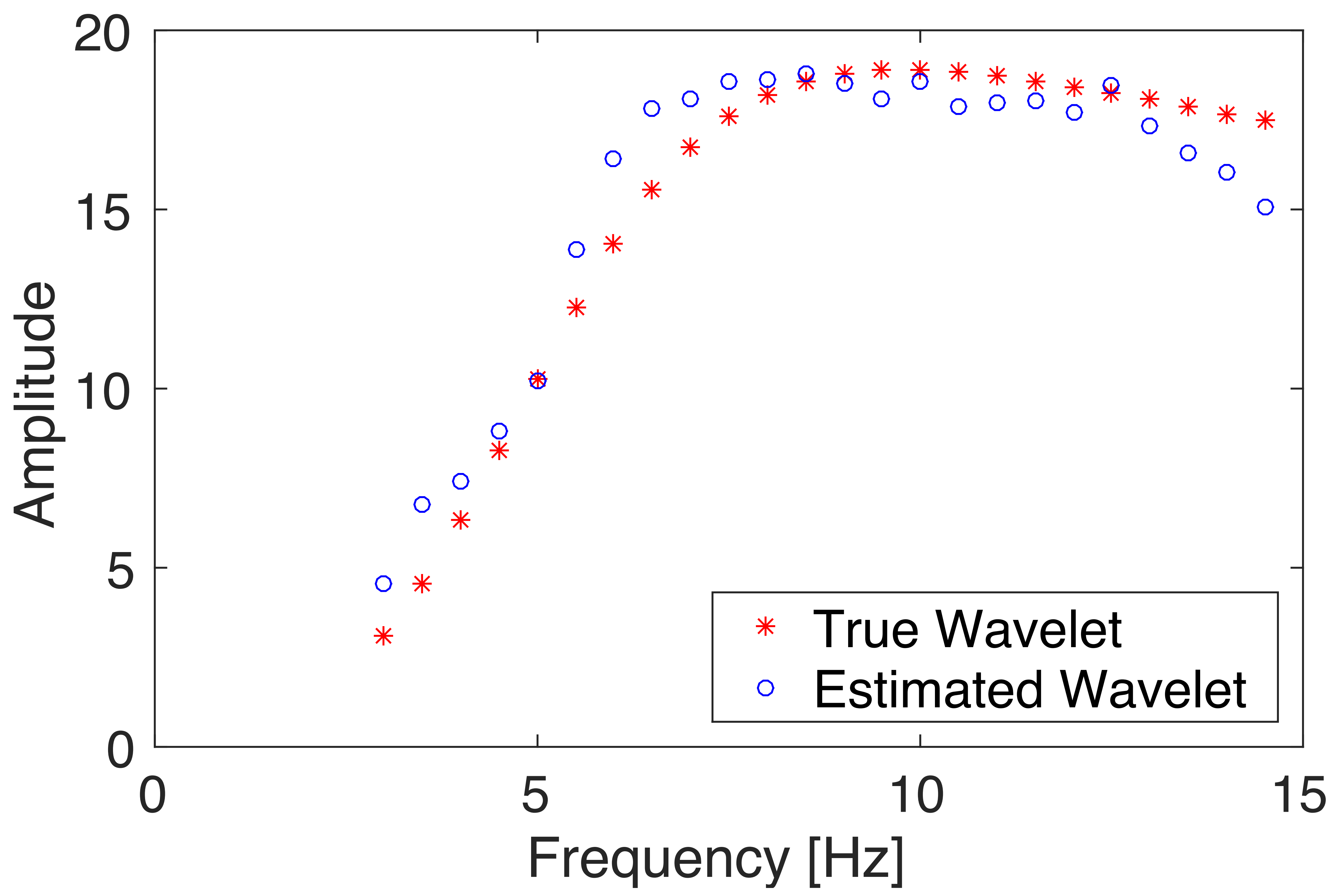


0
1
2
3
4
5
Depth [km]
0
5
10
15
20
25
30
35
40
45
Lateral [km]
4000
3500
3000
2500
2000
1500

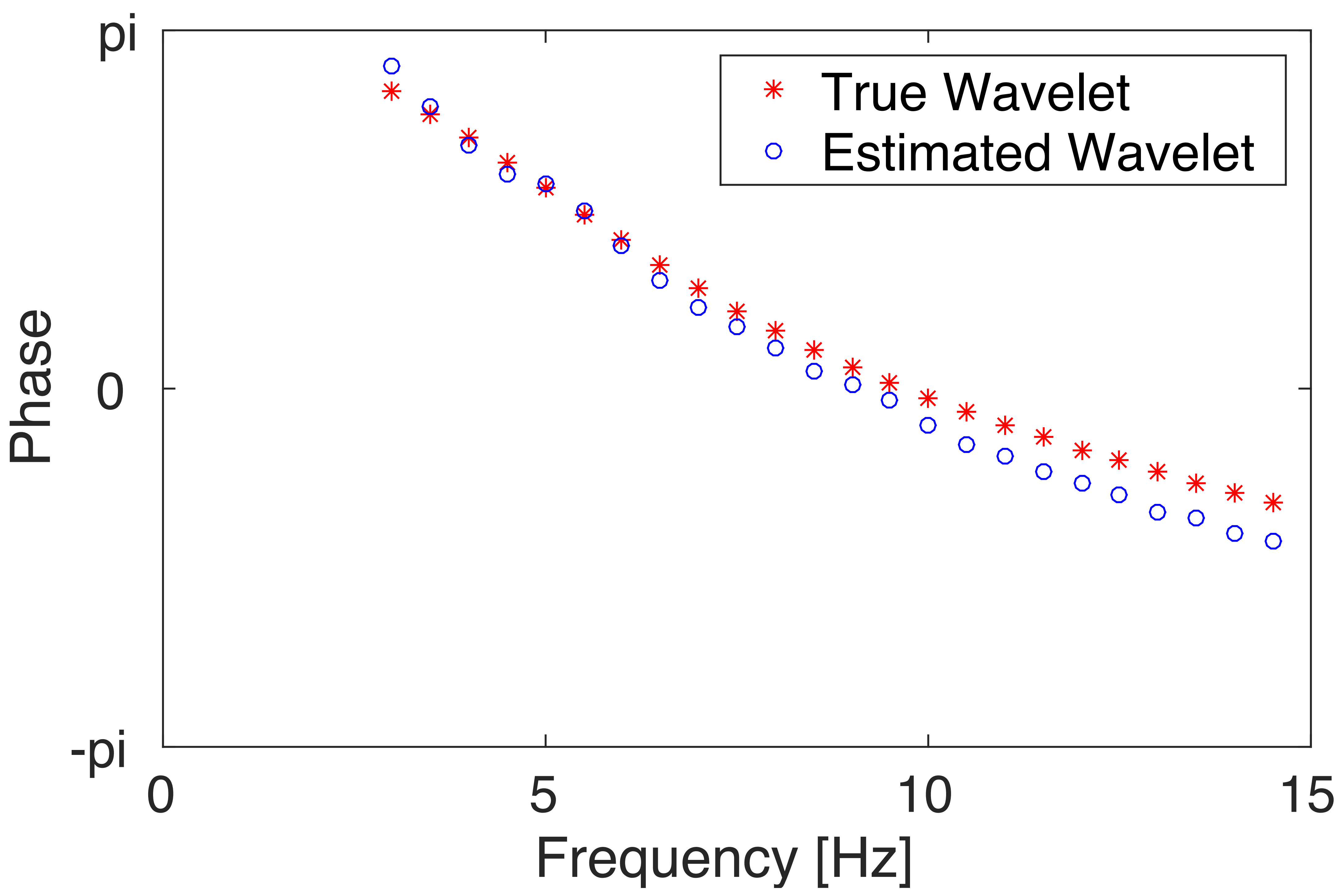
Model update



Source wavelet comparison

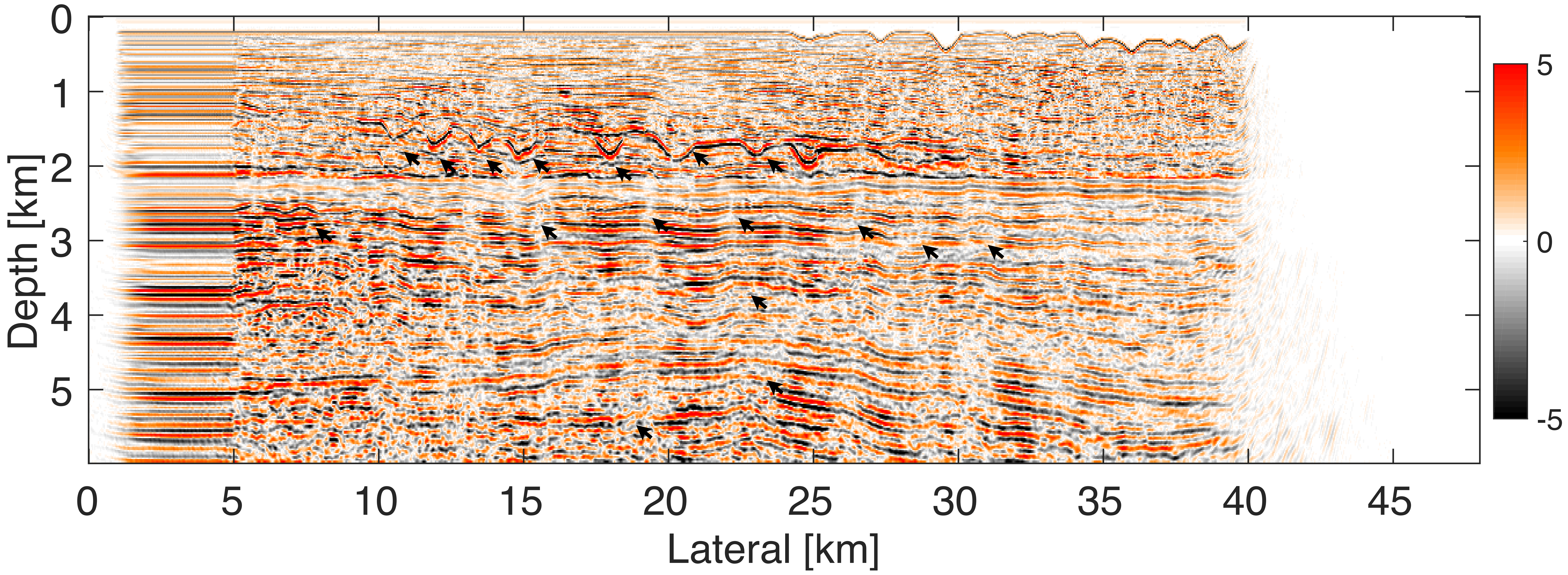


Amplitude



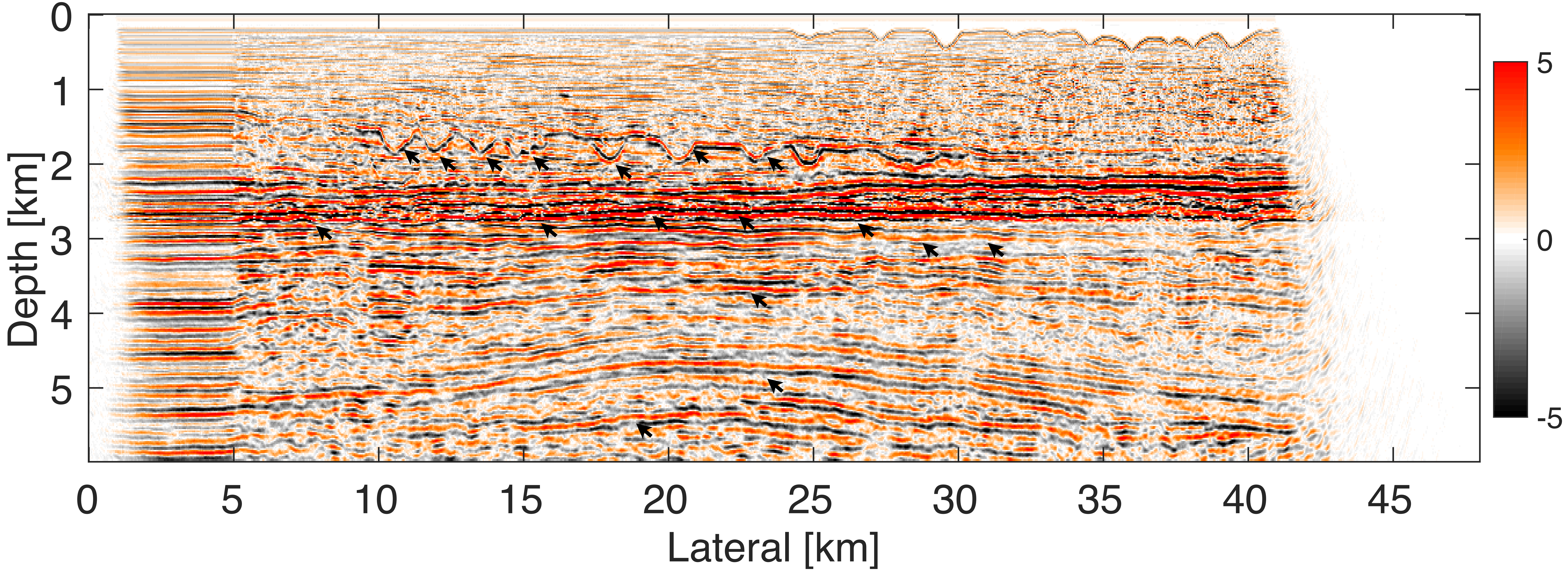
Phase

Kirchhoff migration
—Initial model



Kirchhoff migration

—Inversion result



Observations

WRI obtains a reasonable inversion results for the velocity & source

No data preprocessing or extensive handholding needed

How does this method hold up in cases where we have high-contrast high-velocity unconformities such as salt?

Automatic salt flooding with constrained wavefield reconstruction inversion

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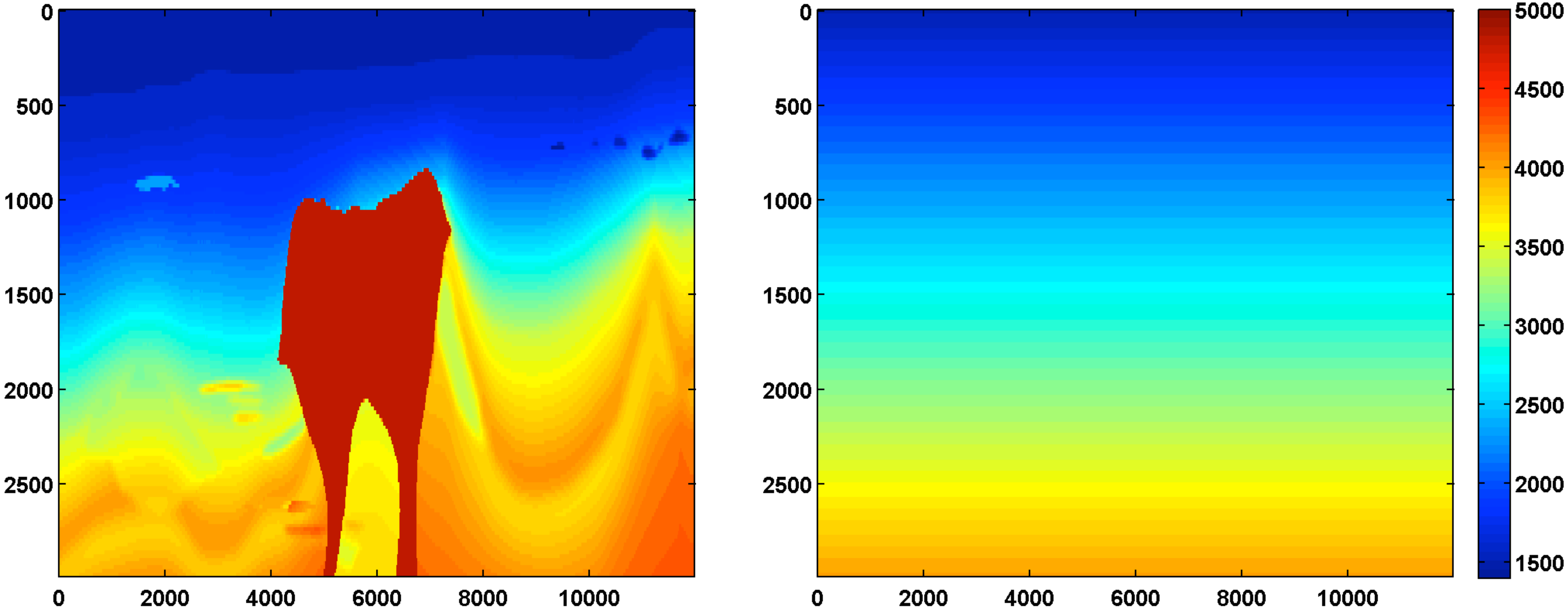
*Sub Salt Solutions Ltd

SLIM
University of British Columbia

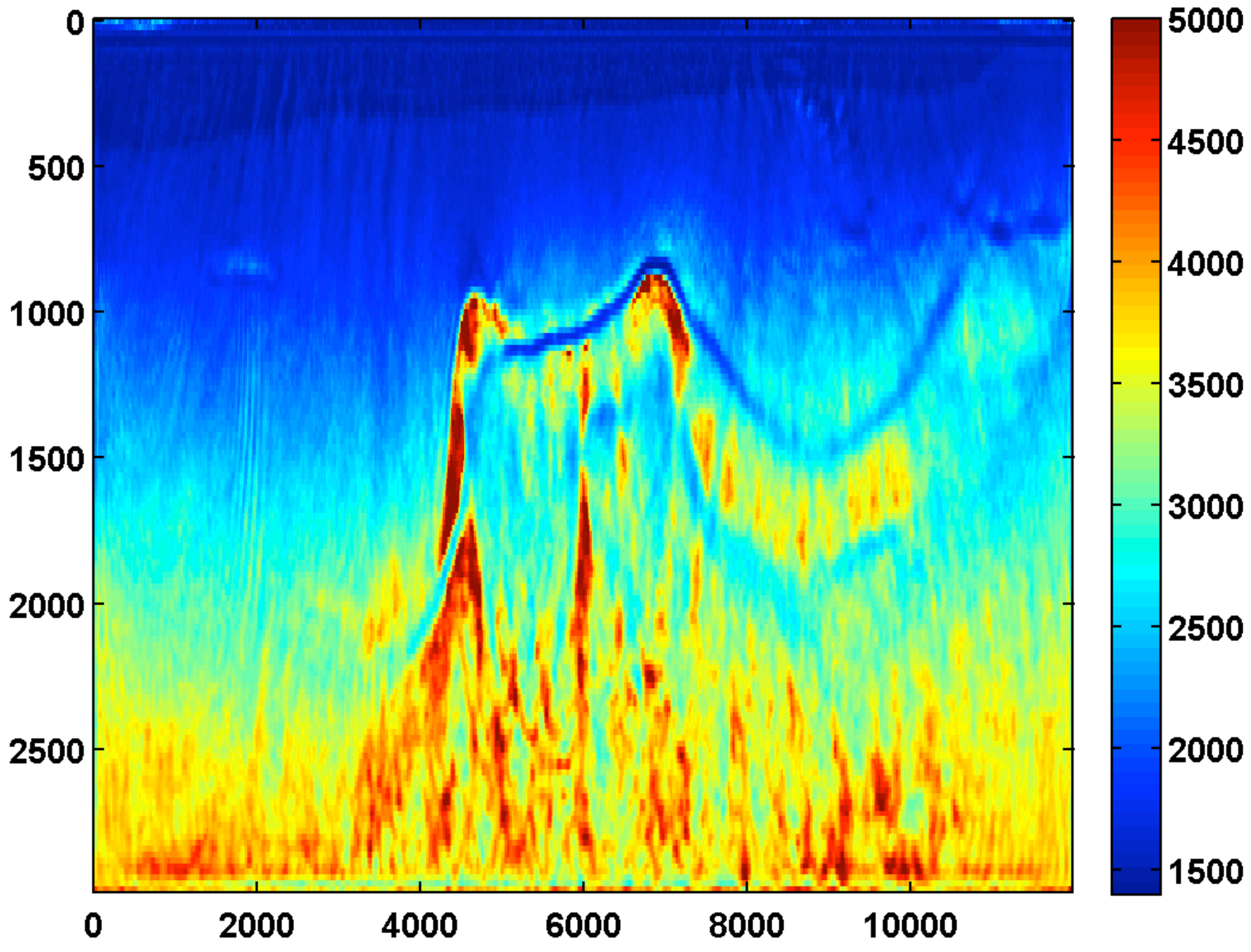
John “Ernie” Esser (May 19, 1980 – March 8, 2015)

Salt – poor starting model

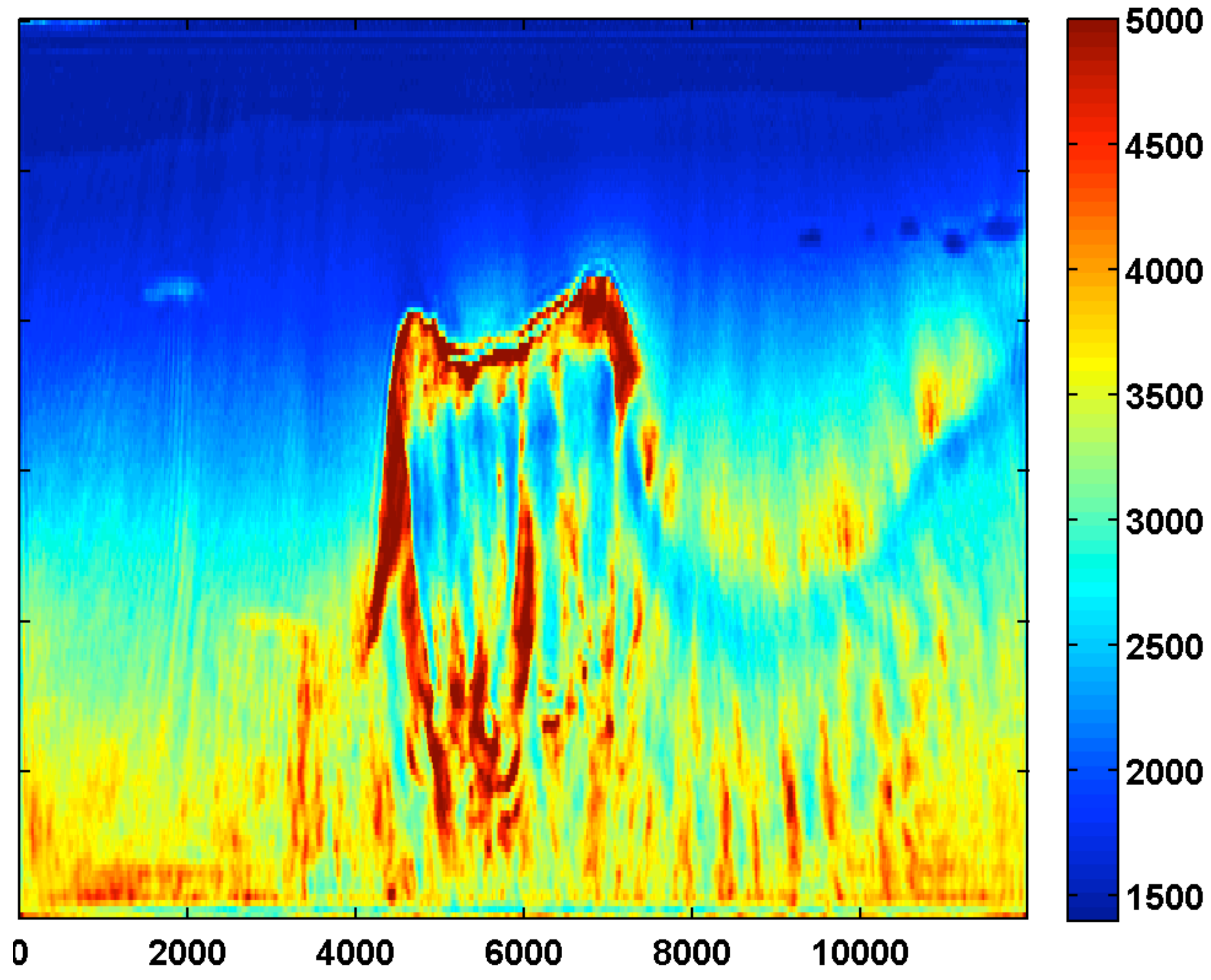


WRI results – w/o convex constraints

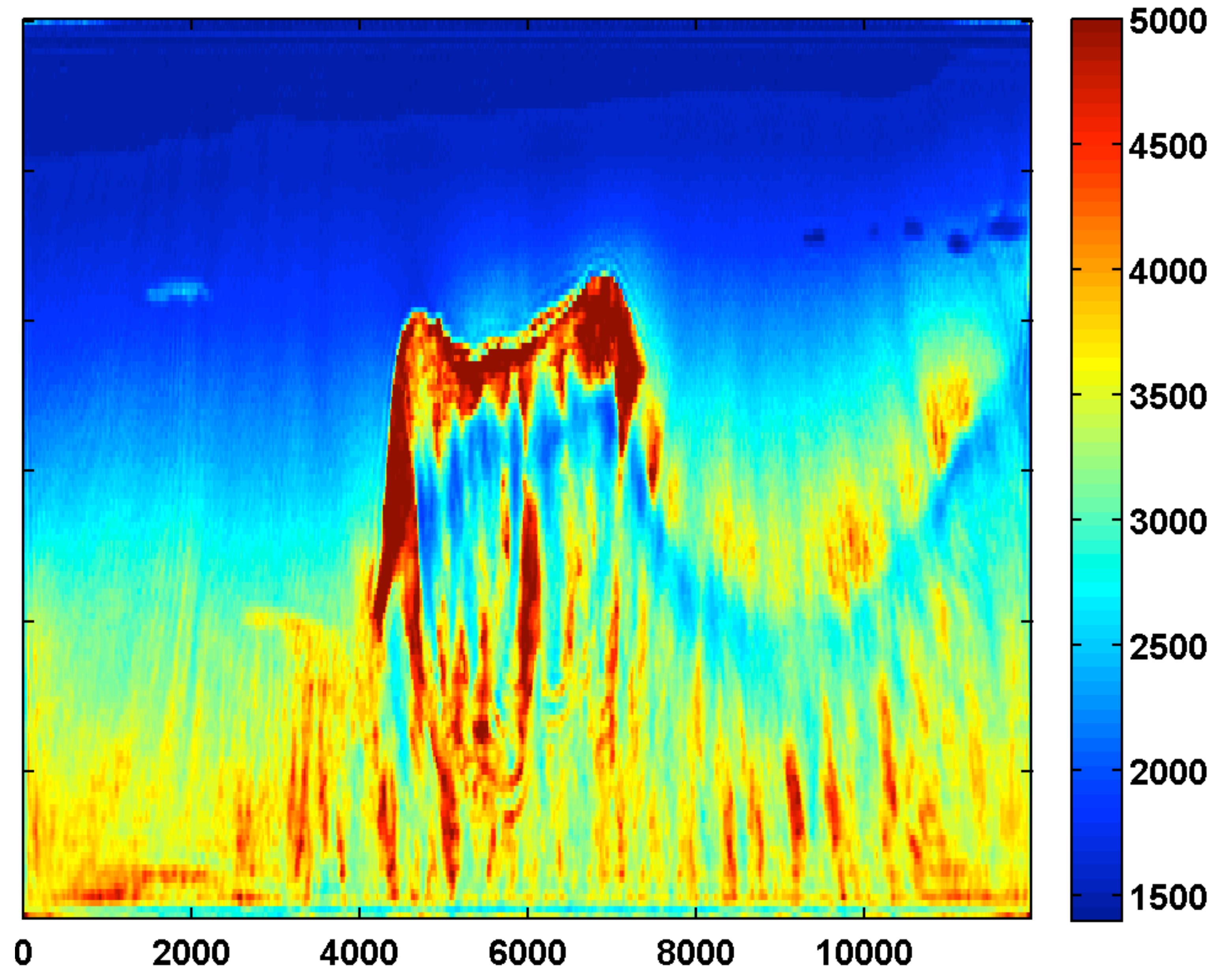
after one cycle through the frequencies



after two cycles through the frequencies



after three cycles through the frequencies



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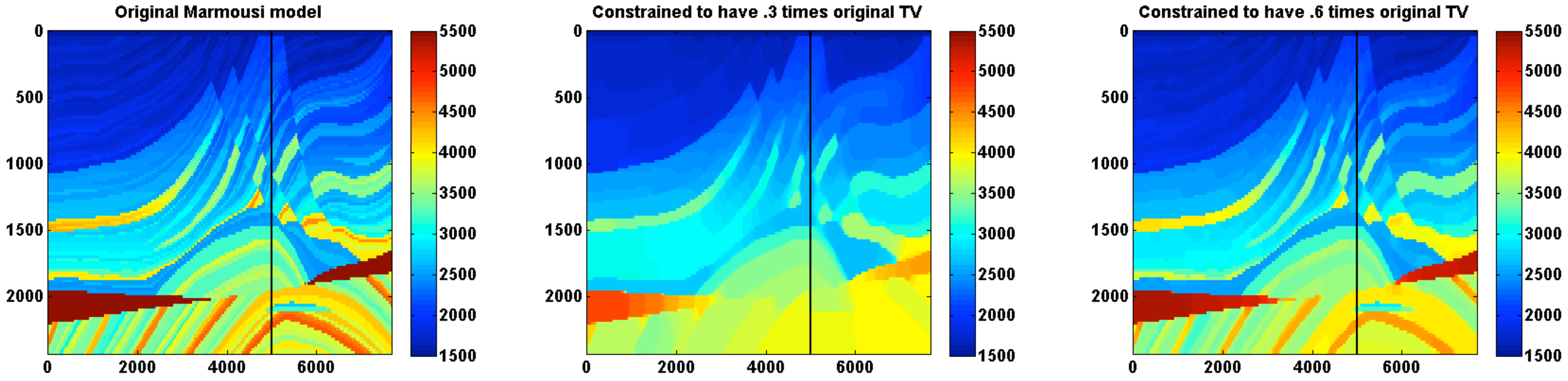
Projections onto convex sets

$v_{\min} = 1500,\ v_{\max} = 5500, \text{ and } \tau = \{0.3\tau_0,\ 0.6\tau_0\}$

$$\Pi_C(\mathbf{m}_0) = \arg\min_{\mathbf{m}} \frac{1}{2}\|\mathbf{m} - \mathbf{m}_0\|^2 \quad \text{subject to} \quad \mathbf{m}_i \in [B_i^l, B_i^u] \text{ and } \|\mathbf{m}\|_{TV} \leq \tau$$



Projections on the convex sets

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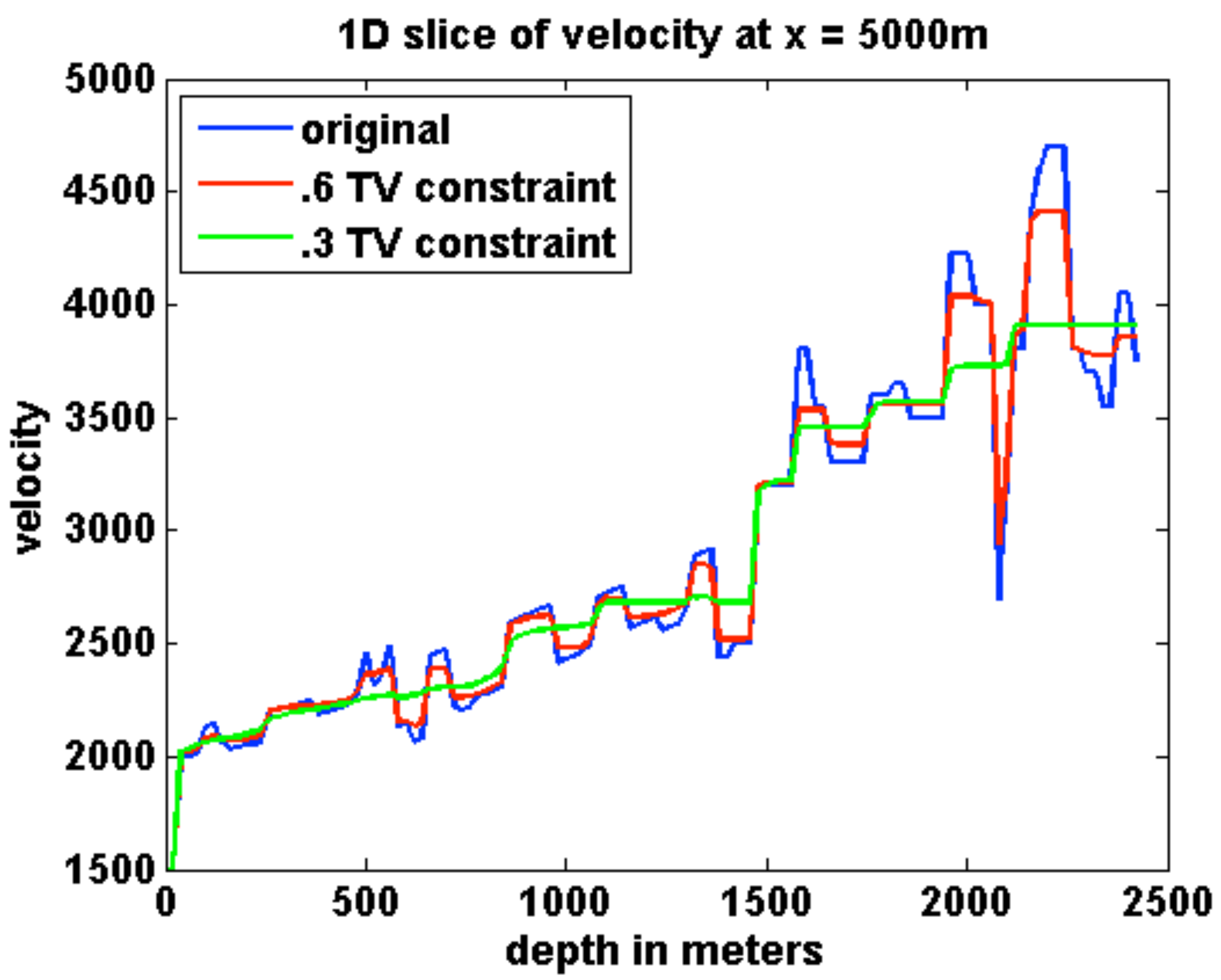

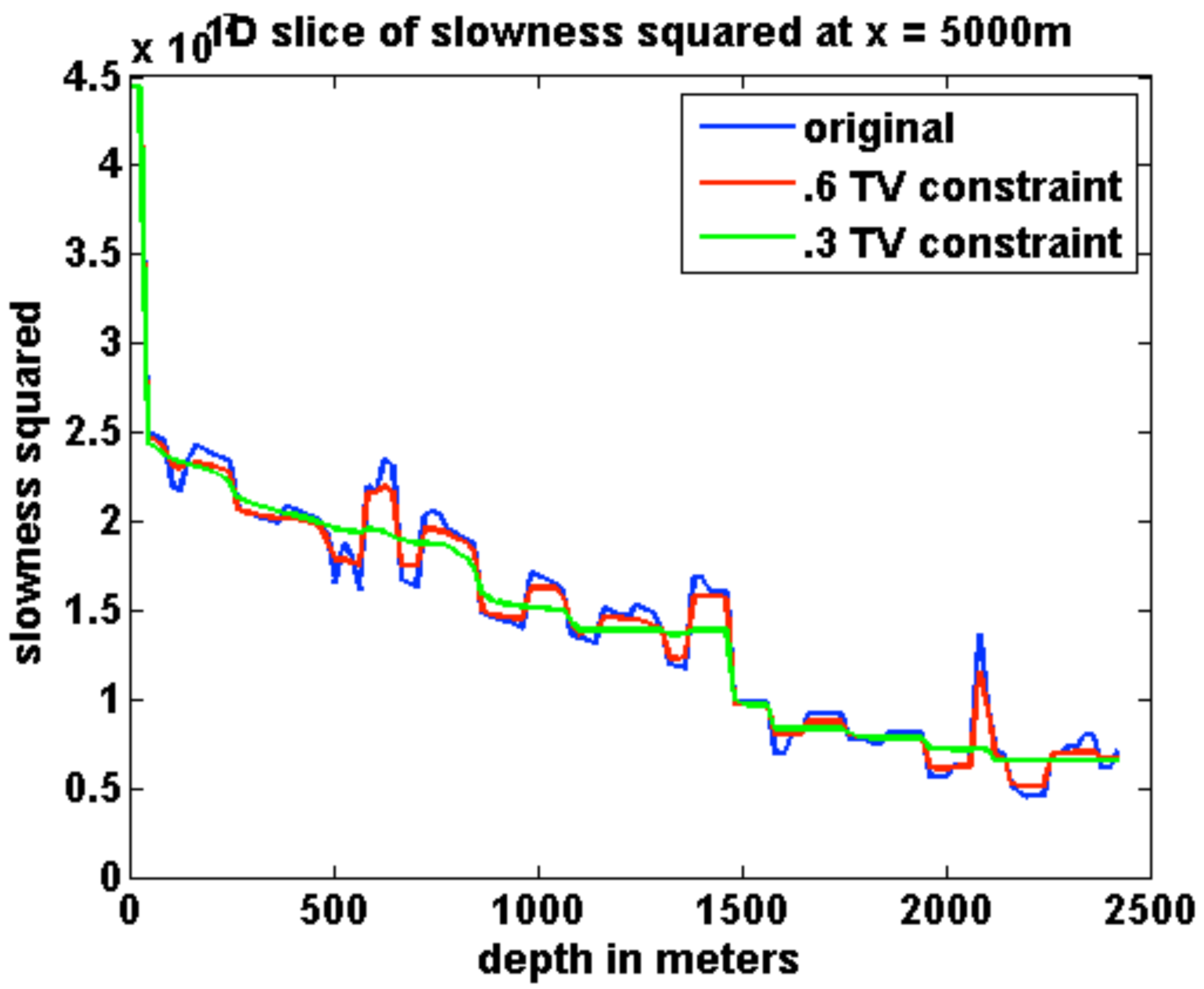

- preserves the reflectors (blocky)
- relaxing the constraint allows for more detail

WRI – outer iterations

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WRI method

for each source i

$$\text{solve} \quad \begin{pmatrix} P_i \\ \lambda A_i(\mathbf{m}) \end{pmatrix} \mathbf{u}_{\lambda,i} \approx \begin{pmatrix} \mathbf{d}_i \\ \lambda \mathbf{q}_i \end{pmatrix}$$

$$\mathbf{g} = \mathbf{g} + \lambda^2\omega^2 \text{diag}(\bar{\mathbf{u}}_{i,\lambda})^*(A(\mathbf{m})\bar{\mathbf{u}}_{i,\lambda} - \mathbf{q}_i)$$

$$H_{GN} = H_{GN} + \lambda^2\omega^4 \text{diag}(\mathbf{u}_i)^* \text{diag}(\mathbf{u}_i)$$

end

$$\mathbf{m} = \mathbf{m} - \alpha H_{GN}^{-1}\mathbf{g}$$

diagonal Hessian
=
pseudo Hessian

replace by inner loop that imposes convex constraints

Conventional method

for each source i

$$\text{solve} \quad A(\mathbf{m})\mathbf{u}_i = \mathbf{q}_i$$

$$\text{solve} \quad A(\mathbf{m})^*\mathbf{v}_i = P_i^*(P_i\mathbf{u}_i - \mathbf{d}_i)$$

$$\mathbf{g} = \mathbf{g} + \omega^2 \text{diag}(\mathbf{u}_i)^*\mathbf{v}_i$$

end

$$\mathbf{m} = \mathbf{m} - \alpha\mathbf{g}$$

dense Hessian
&
too expensive

[Oldenburg ’83; Akçelik ’08; Anagaw ’11; Maharramov ’14; Esser & FJH ’14]

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Total-variation regularization
– w/ bound constraints

Promote physical models w/ sharp boundaries via

$$\mathbf{m}^{n+1} = \mathbf{m}^n + \Delta\mathbf{m} \quad \text{subject to} \quad \mathbf{m}^{n+1} \in C_{\text{box}} \cap C_{\text{TV}}$$

where $C_{\text{TV}} = \{\mathbf{m} : \|\mathbf{m}\|_{\text{TV}} \leq \tau\}$ and

$$\begin{aligned}\|\mathbf{m}\|_{TV} &= \frac{1}{h}\sum_{ij}\sqrt{(m_{i+1,j}-m_{i,j})^2+(m_{i,j+1}-m_{i,j})^2} \\ &= \sum_{ij}\frac{1}{h}\left\|\begin{bmatrix}(m_{i,j+1}-m_{i,j}) \\ (m_{i+1,j}-m_{i,j})\end{bmatrix}\right\| \\ &= \|D\mathbf{m}\|_{1,2} := \sum_{l=1}^{N}\|(D\mathbf{m})_l\| \ .\end{aligned}$$

Ernie Esser, Xiaoqun Zhang, and Tony F. Chan. A General Frame- work for a Class of First Order Primal-Dual Algorithms for Convex Optimization in Imaging Science. SIAM Journal on Imaging Sciences, 3(4):1015–1046, 2010.

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Proposed algorithm
– via scaled gradient projections

Solve at the n[th] outer iteration

$$\underset{\mathbf{m}}{\text{minimize}}\, \Phi(\mathbf{m}) \quad \text{subject to} \quad \mathbf{m}^{n+1} \in C_{\text{box}} \cap C_{\text{TV}}$$

by iterating cheaply on

expensive but fixed

diagonal

$$\Delta\mathbf{m} = \arg\min_{\Delta\mathbf{m}} \Delta\mathbf{m}^T \mathbf{g}^n + \frac{1}{2}\Delta\mathbf{m}^T (H^n + c_n \mathrm{I})\Delta\mathbf{m}$$

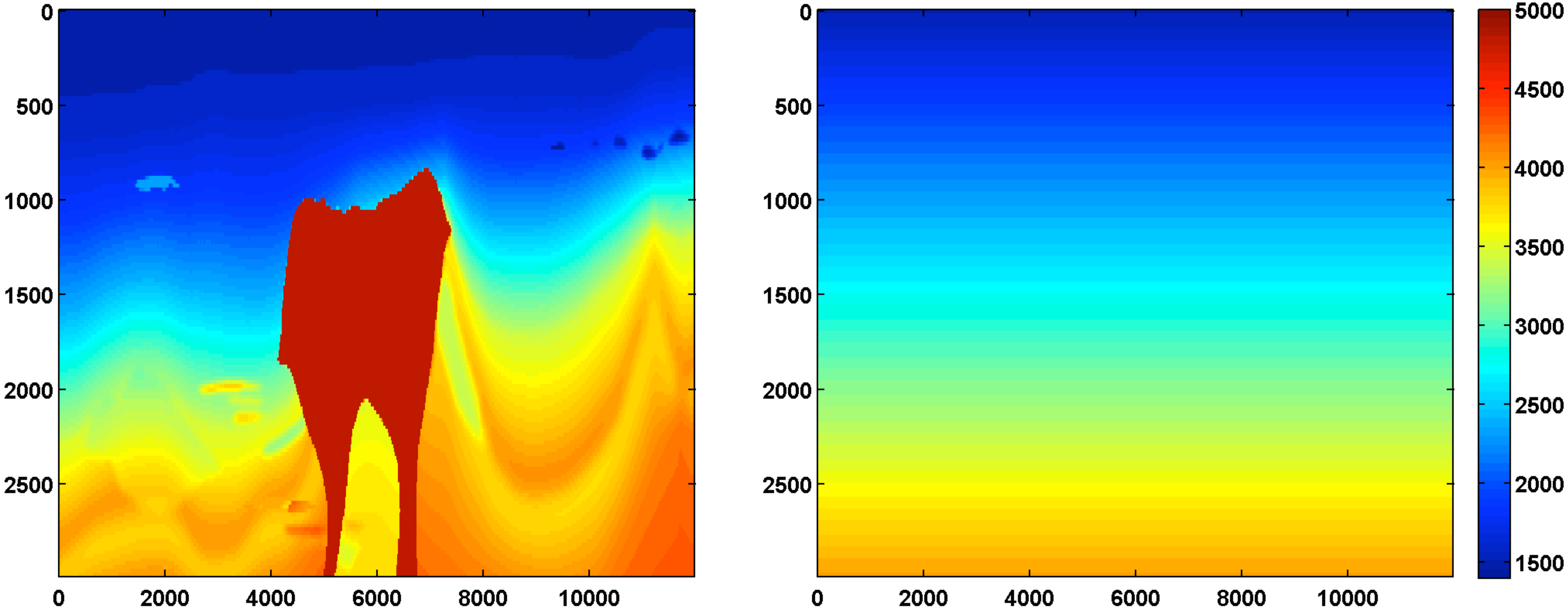
$$\text{subject to} \quad \mathbf{m}_i^n + \Delta\mathbf{m}_i \in [B_i^l, B_i^u] \text{ and } \|\mathbf{m}^n + \Delta\mathbf{m}\|_{TV} \leq \tau$$

$$\mathbf{m}^{n+1} = \mathbf{m}^n + \Delta\mathbf{m}$$

BP model

- number of sources: 126
- number of receivers: 299
- frequency continuation over 3-20Hz in overlapping batches of 2
- maximum number of outer iterations per frequency batch: 25
- maximum number of inner iterations for convex subproblems: 2000
- known Ricker wavelet sources with 15Hz peak frequency
- two simultaneous shots with Gaussian weights w/ redraws
- no added noise

Salt – poor starting model

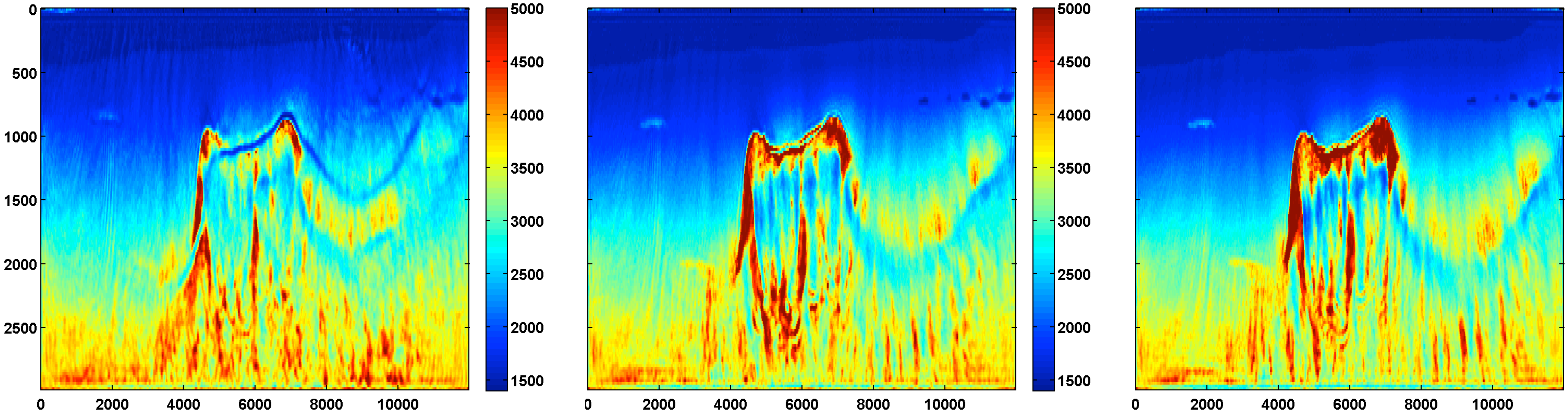


Results w/o TV

after one cycle through the frequencies

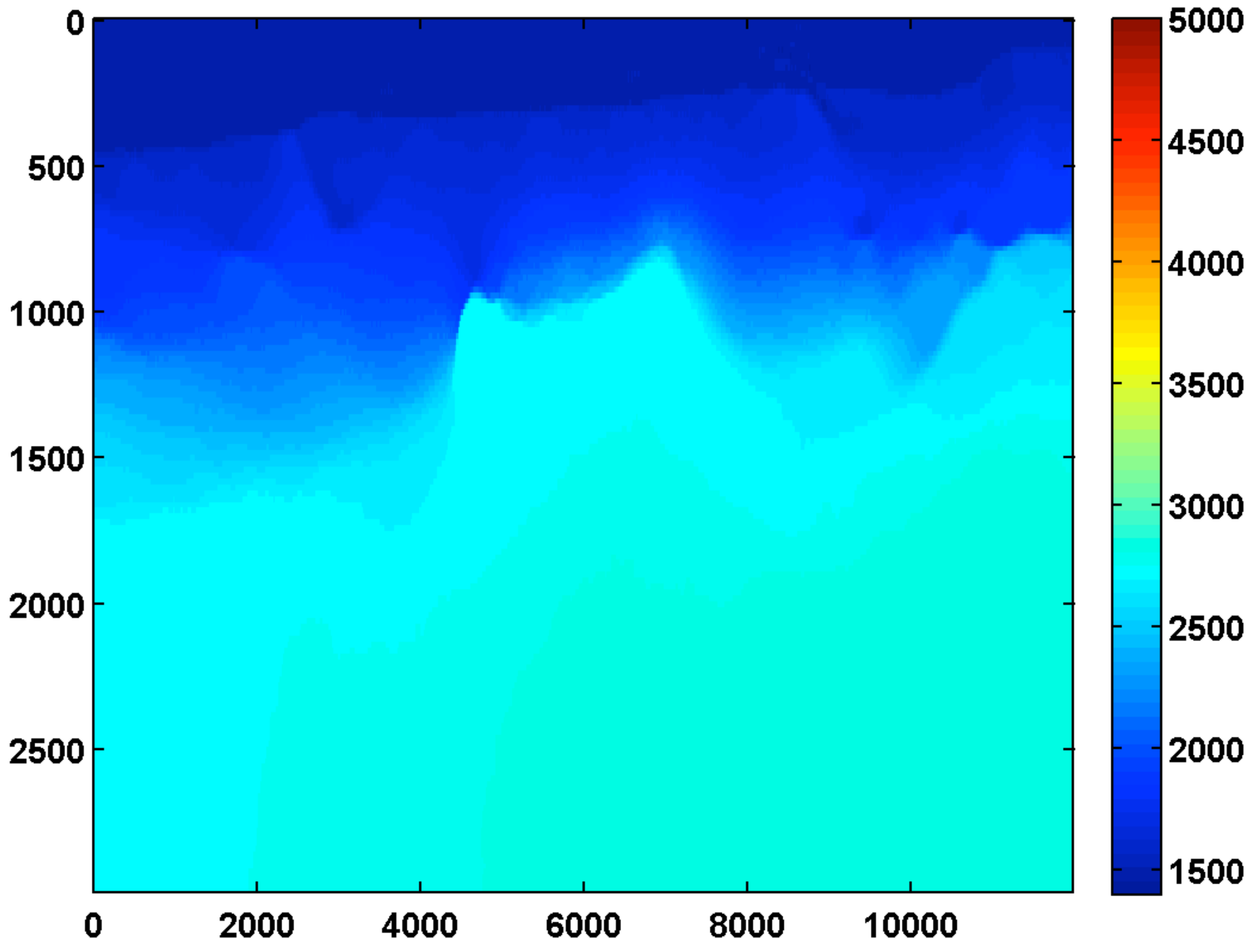
after two cycles through the frequencies

after three cycles through the frequencies

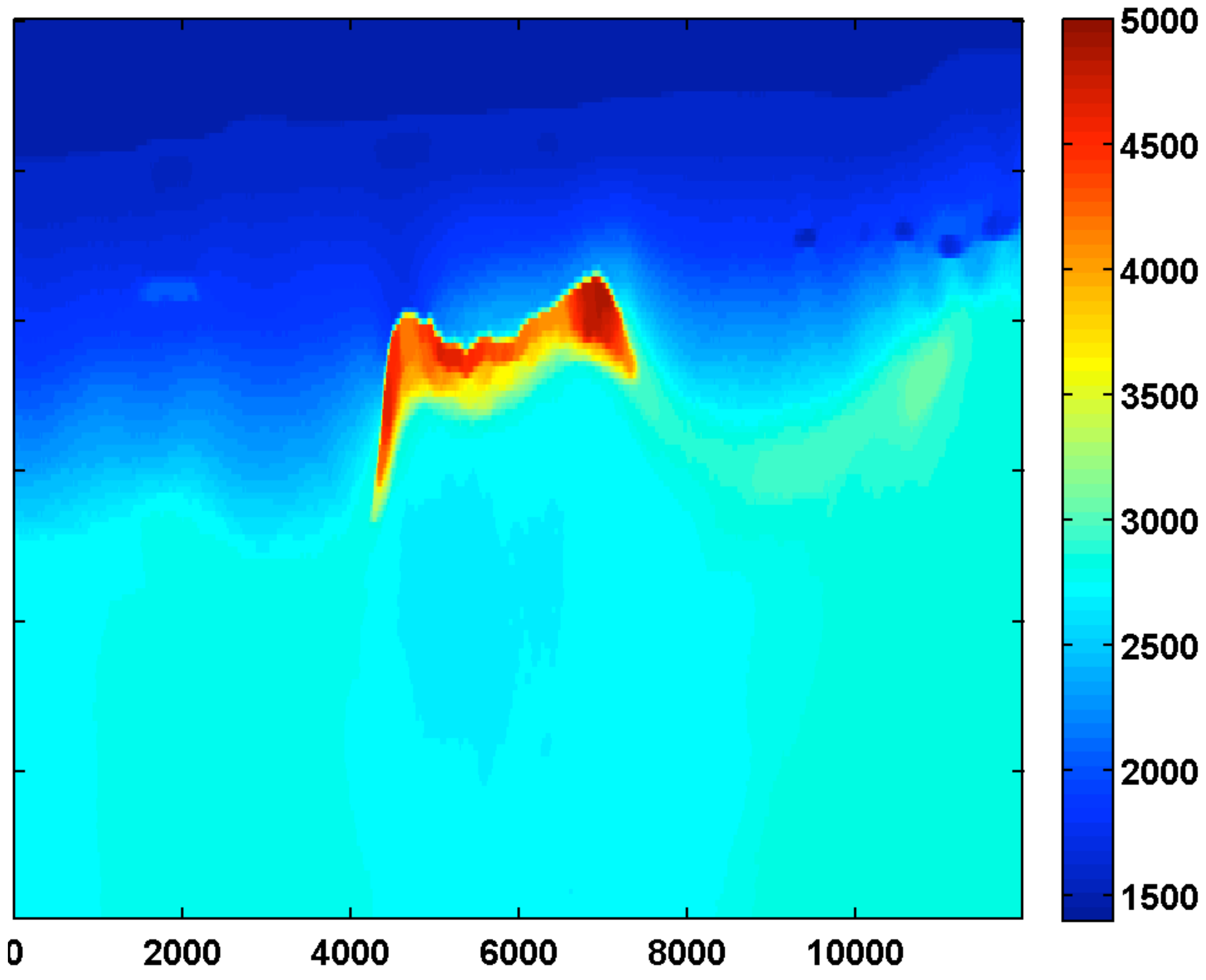


Results w/ TV

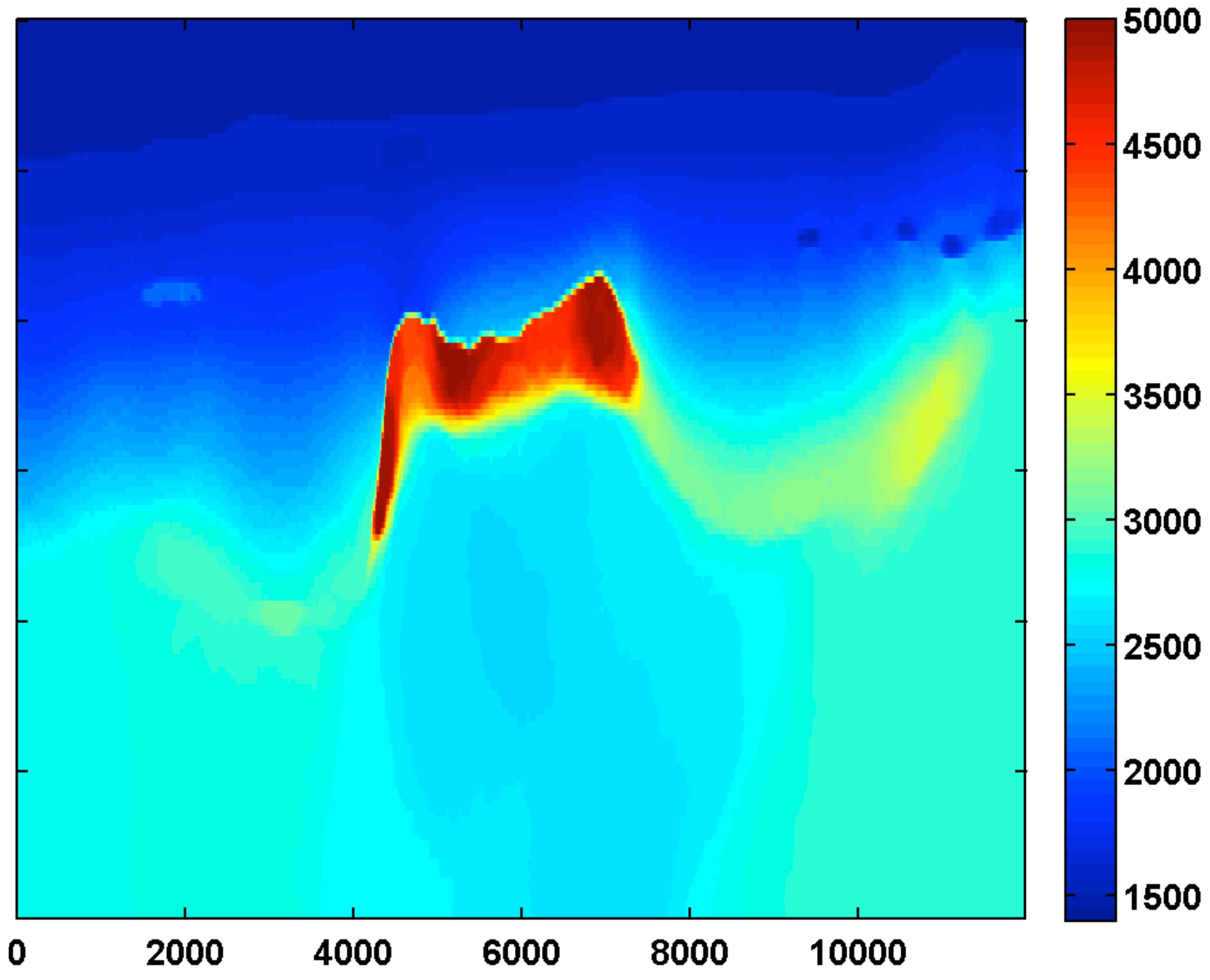
after one cycle through the frequencies



after two cycles through the frequencies



after three cycles through the frequencies



Hinge loss
– one-sided TV constraint

Mitigate erroneous velocity model updates by using the fact that

- vertical slowness profiles tend to decrease w/ depth
- makes it less probable that velocities jump down along the vertical

Mathematically expressed as the one-norm of a hinge-loss function

$$\|\max(0, D_z\mathbf{m})\|_1 \leq \xi$$

- for ξ small slowness is unlikely to step up
- extended to a weighted directional gradient
- combined w/ omni-directional TV and bound constraints

Ernie Esser, Lluís Guasch, Tristan van Leeuwen, Aleksandr Y. Aravkin, and Felix J. Herrmann, “Total variation regularization strategies in full waveform inversion for improving robustness to noise, limited data and poor initializations”. 2015.

Scaled-gradient projections
– w/ convex total-variation, box, & hinge-loss constraints

Solve for given $\bar{\mathbf{u}}_\lambda$

$$\min_{\mathbf{m}} \phi(\mathbf{m}, \bar{\mathbf{u}}_\lambda) \quad \text{subject to} \quad \begin{cases} m_i \in [B_1, B_2] \\ \|\mathbf{m}\|_{\mathrm{TV}} \leq \tau \\ \|\mathbf{m}\|_{\mathrm{Hinge}} \leq \xi \end{cases}$$

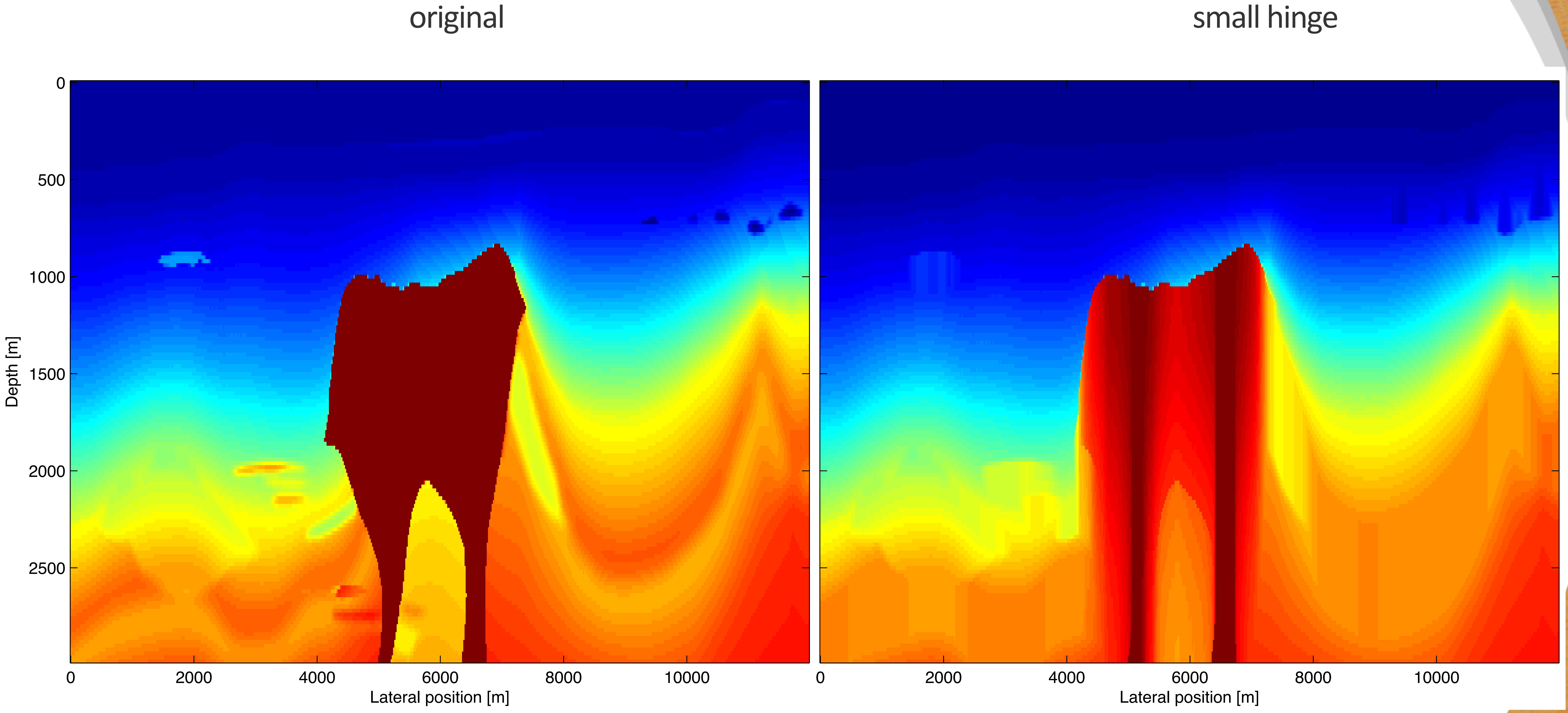
with

$$\|\mathbf{m}\|_{TV} = \sum_{ij} \frac{1}{h} \left\| \begin{bmatrix} (m_{i,j+1} - m_{i,j}) \\ (m_{i+1,j} - m_{i,j}) \end{bmatrix} \right\|$$

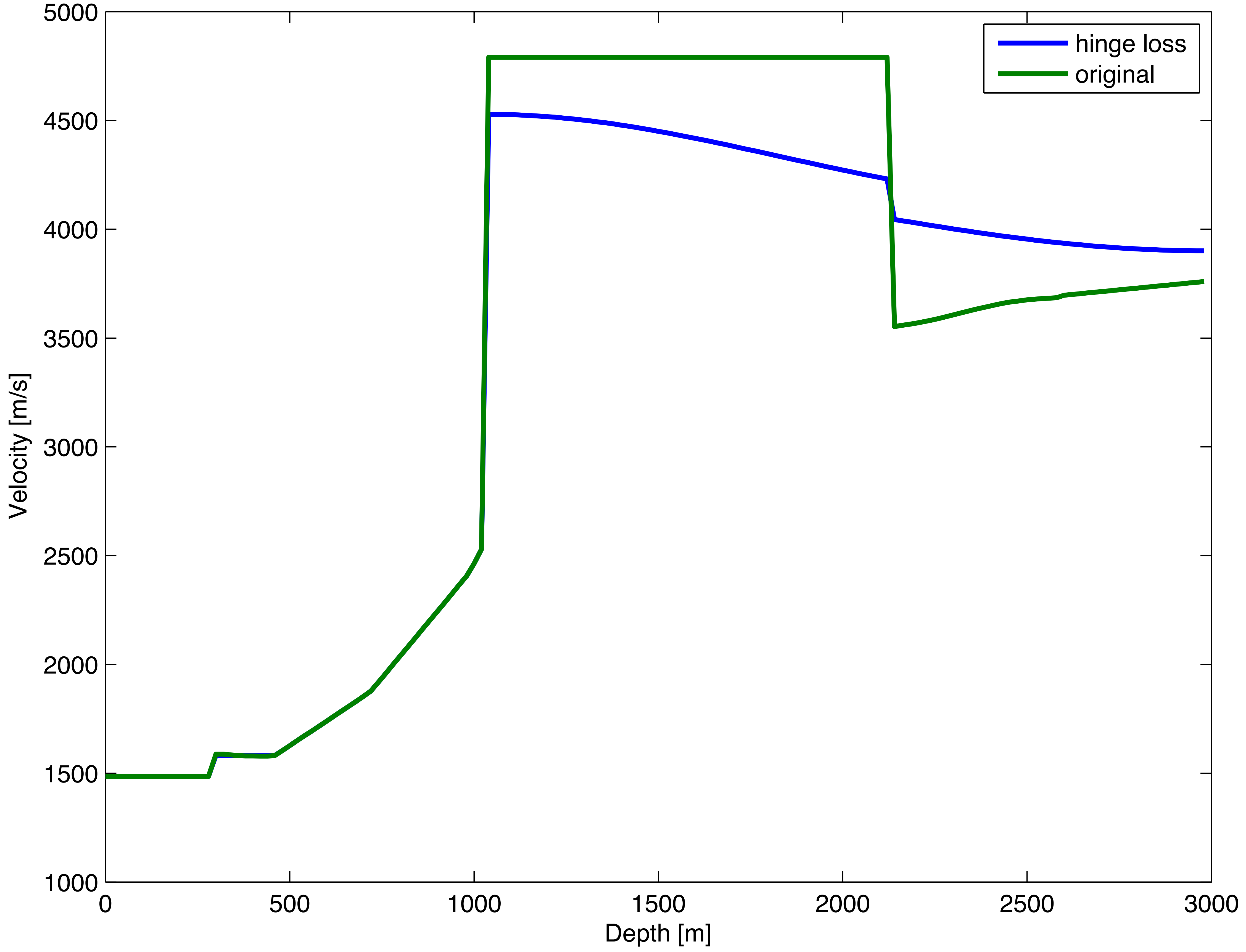
and

$$\|\mathbf{m}\|_{\mathrm{Hinge}} = \|\max\left(0, D_z \mathbf{m}\right)\|_1$$

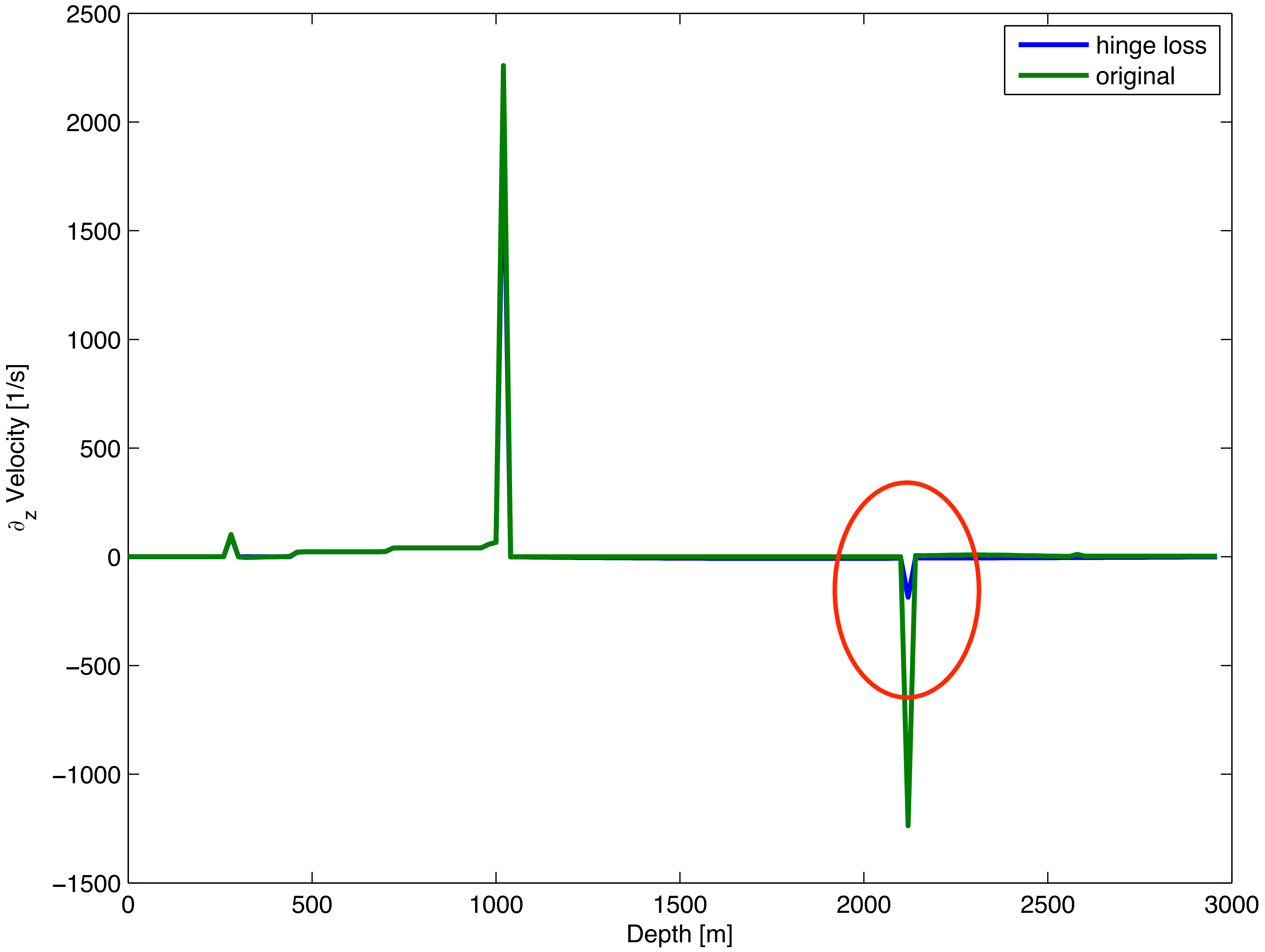
Hinge loss


original
small hinge
0
500
1000
1500
2000
2500
Depth [m]
0
2000
4000
6000
8000
10000
Lateral position [m]
0
2000
4000
6000
8000
10000
Lateral position [m]

Hinge loss



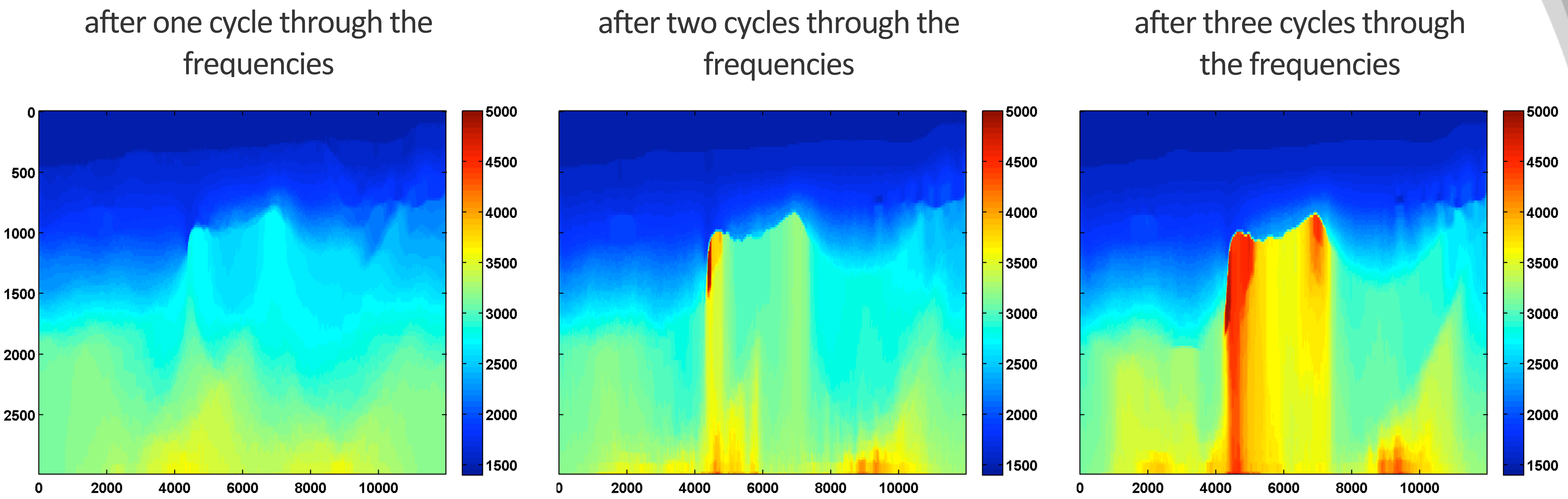
hinge loss
original
Velocity [m/s]
Depth [m]
5000
4500
4000
3500
3000
2500
2000
1500
1000
0
500
1000
1500
2000
2500
3000



hinge loss
original
∂z Velocity [1/s]
Depth [m]
2500
2000
1500
1000
500
0
−500
−1000
−1500
0
500
1000
1500
2000
2500
3000

Results w/ hinge loss continuation

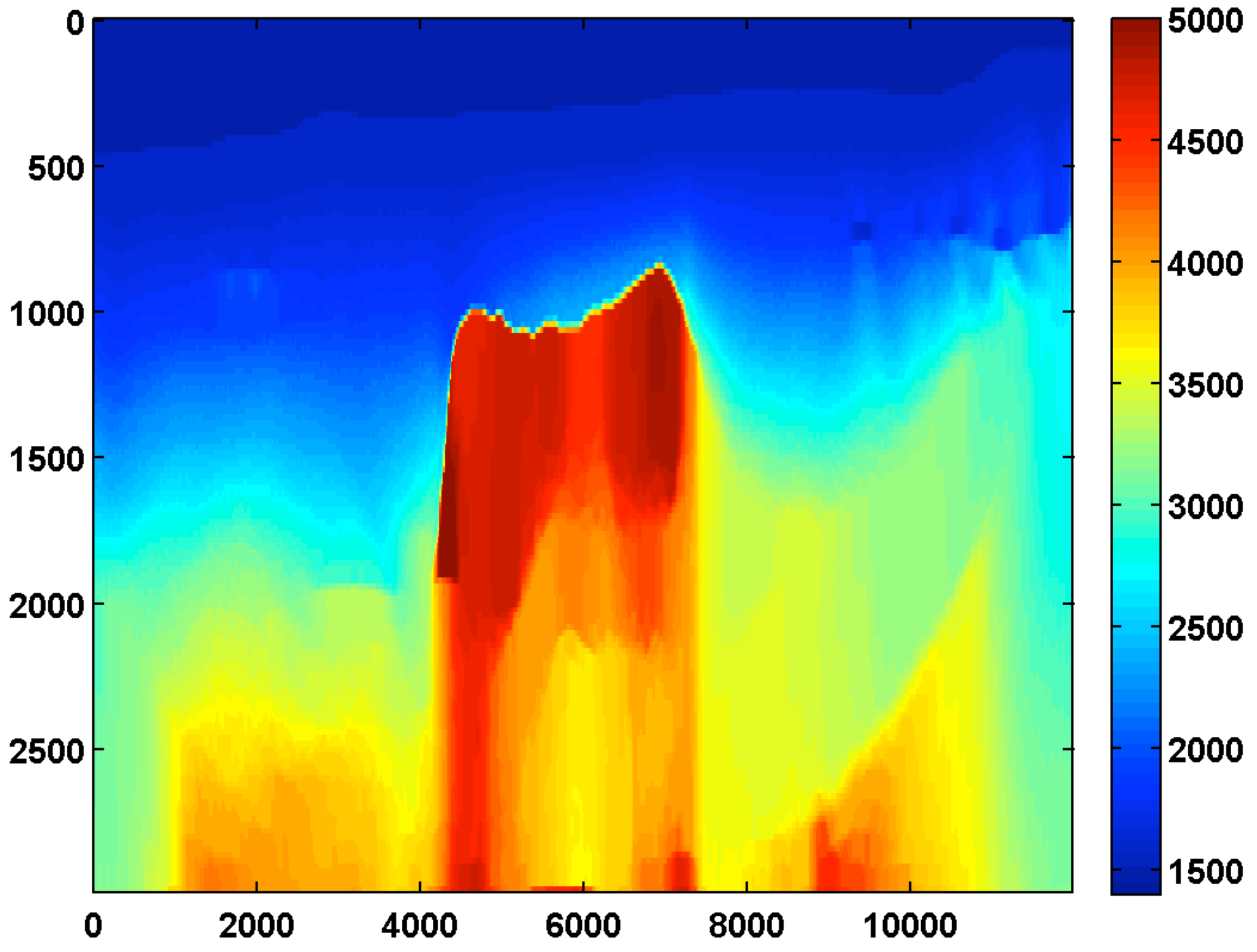
$$\frac{\xi}{\xi_{\text{true}}} = \{.01, .05, .10\}$$



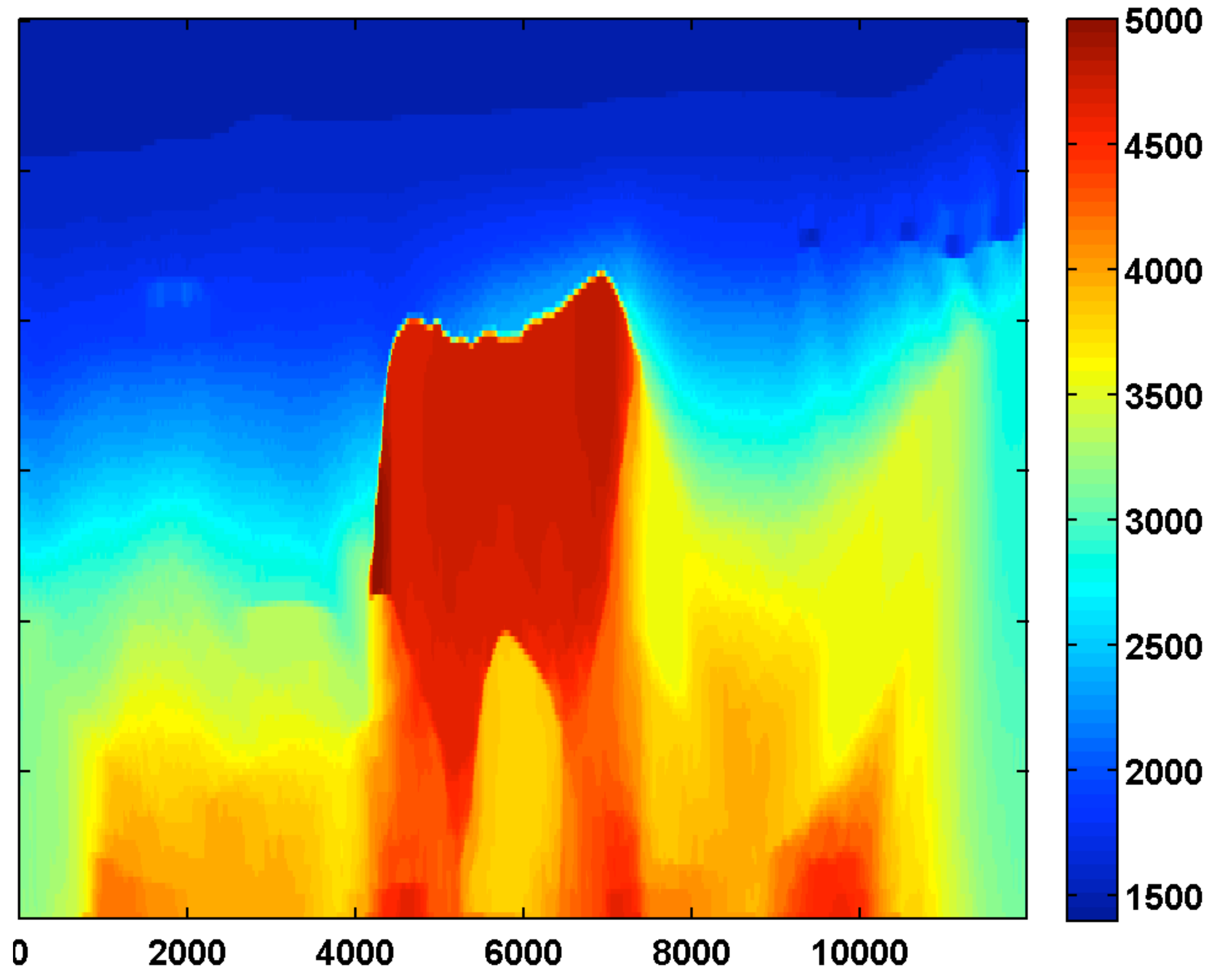
Results w/ hinge loss continuation

$$\frac{\xi}{\xi_{\text{true}}} = \{.15, .20, .25\}$$

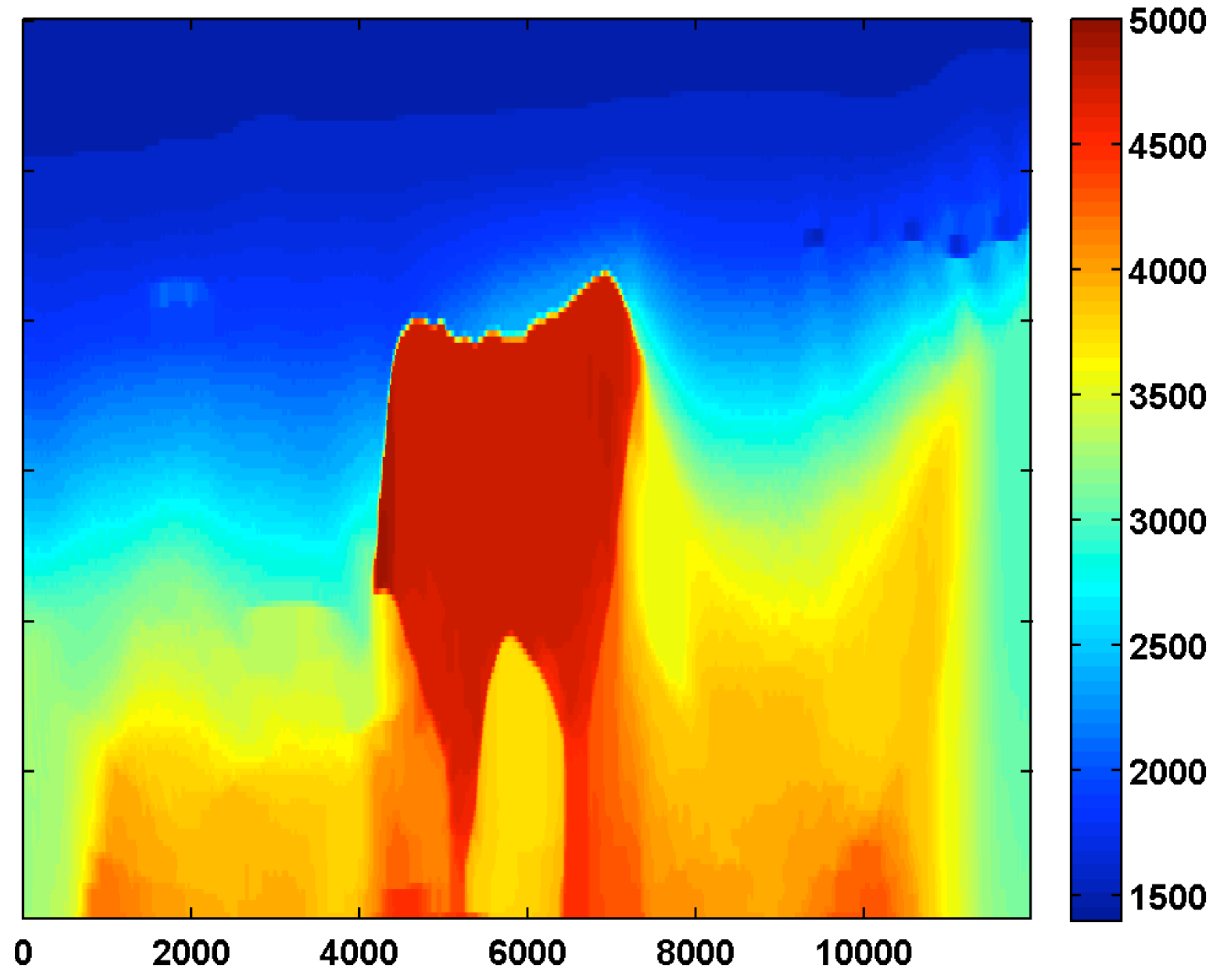
after four cycles through the frequencies



after five cycles through the frequencies

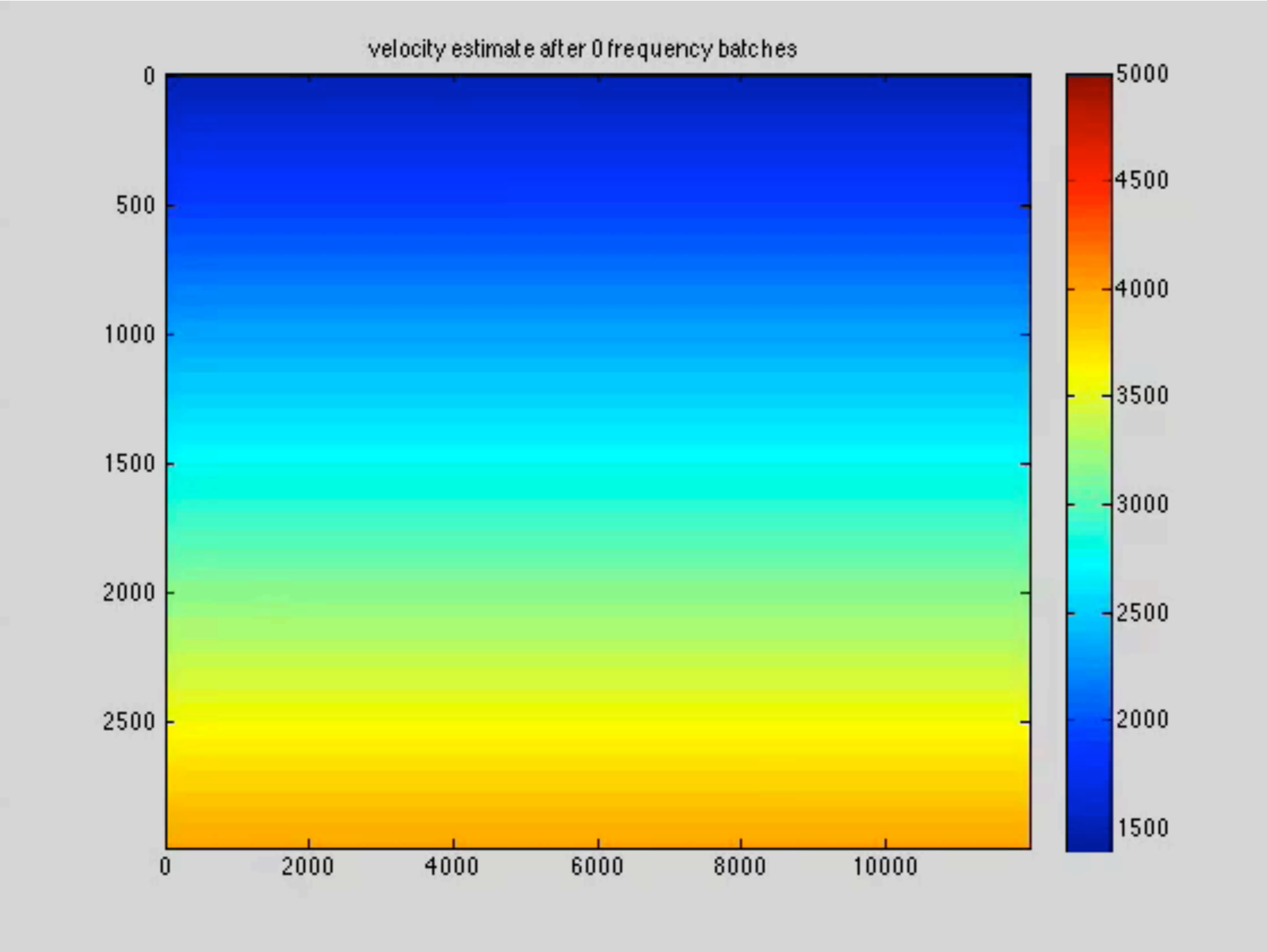


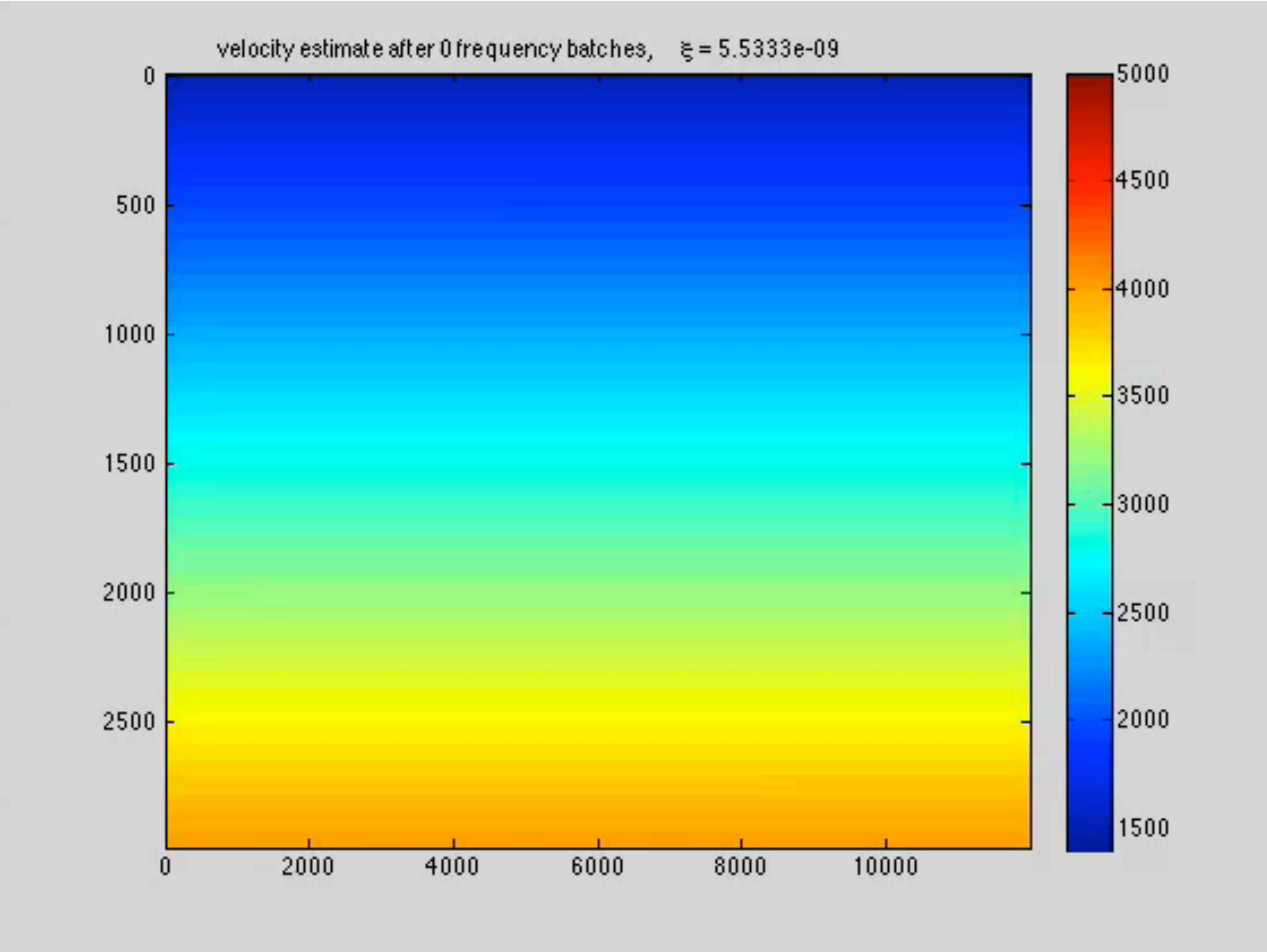
after six cycles through the frequencies



WRI

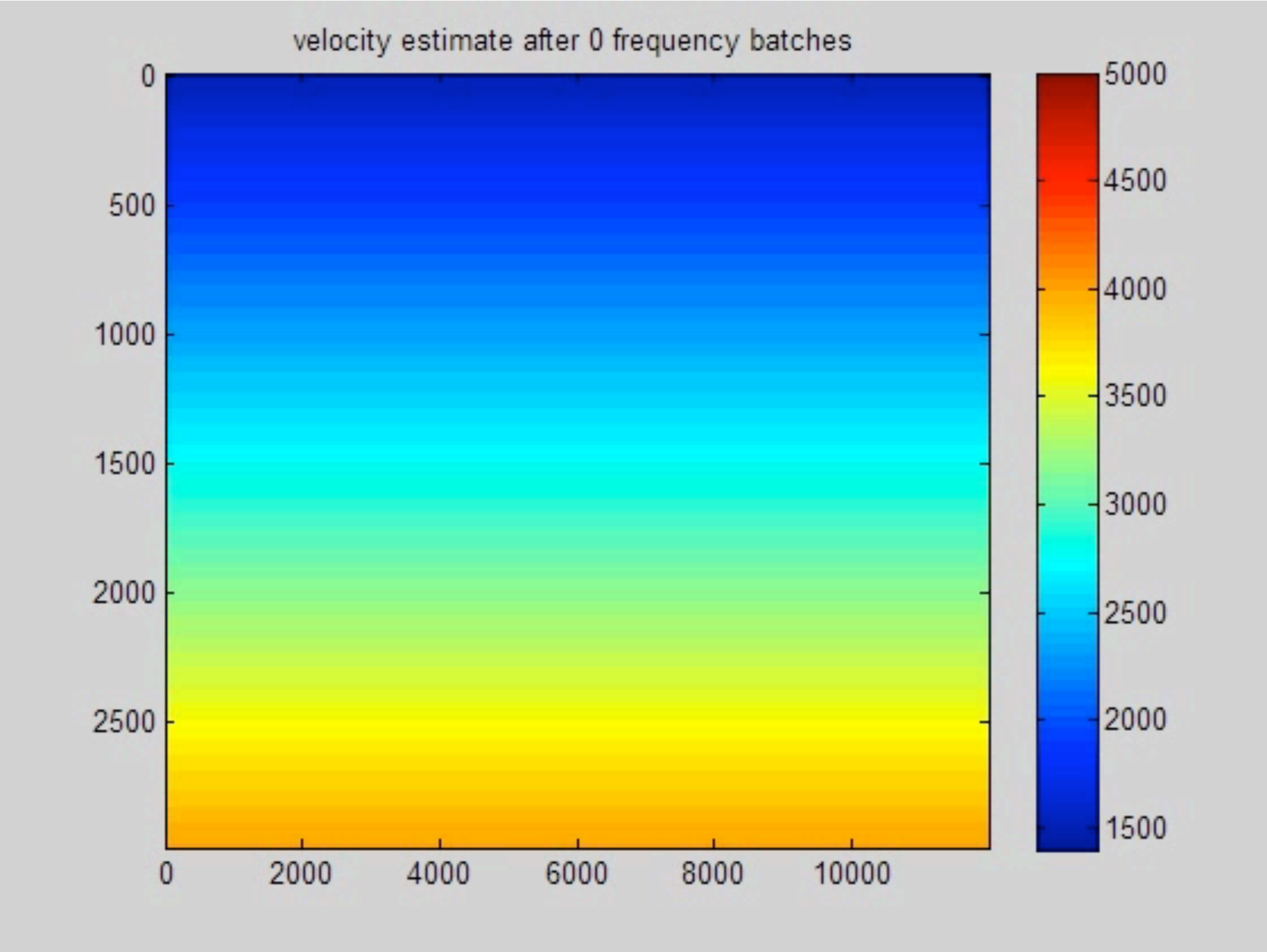
w/ or w/o TV-norm & hinge-loss projections & poor starting model

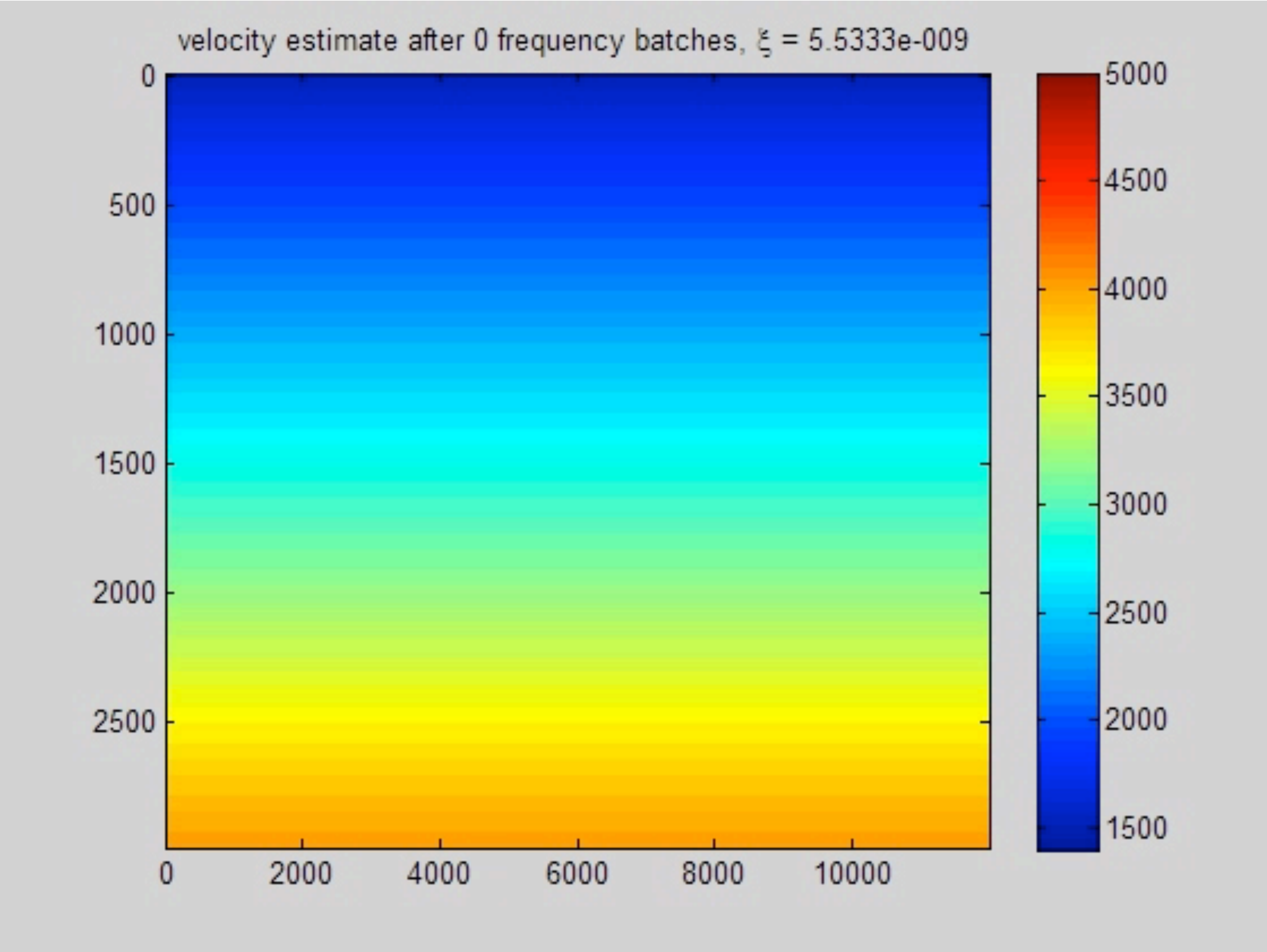




Adjoint-state FWI

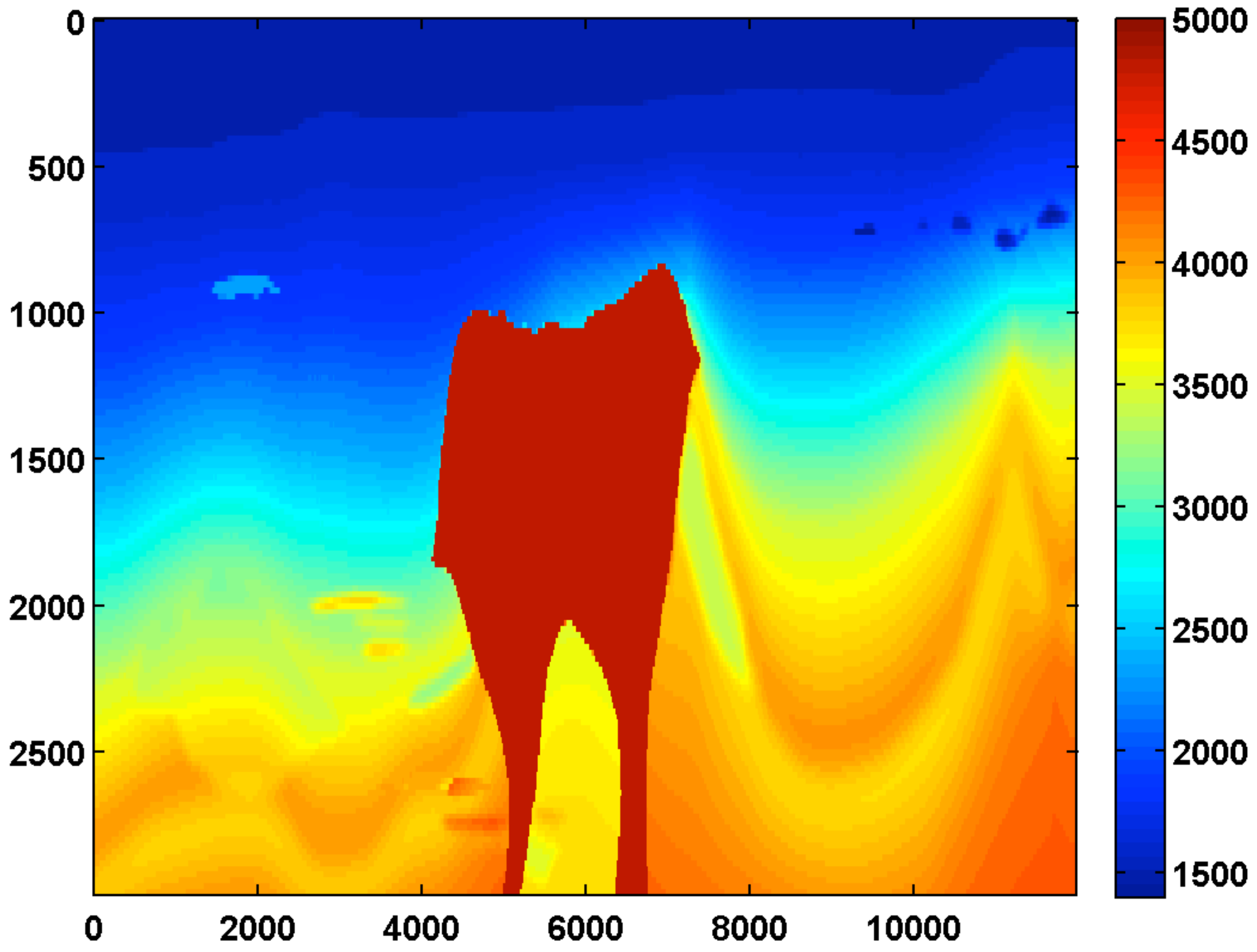
w/ or w/o TV-norm & hinge-loss projections & poor starting model



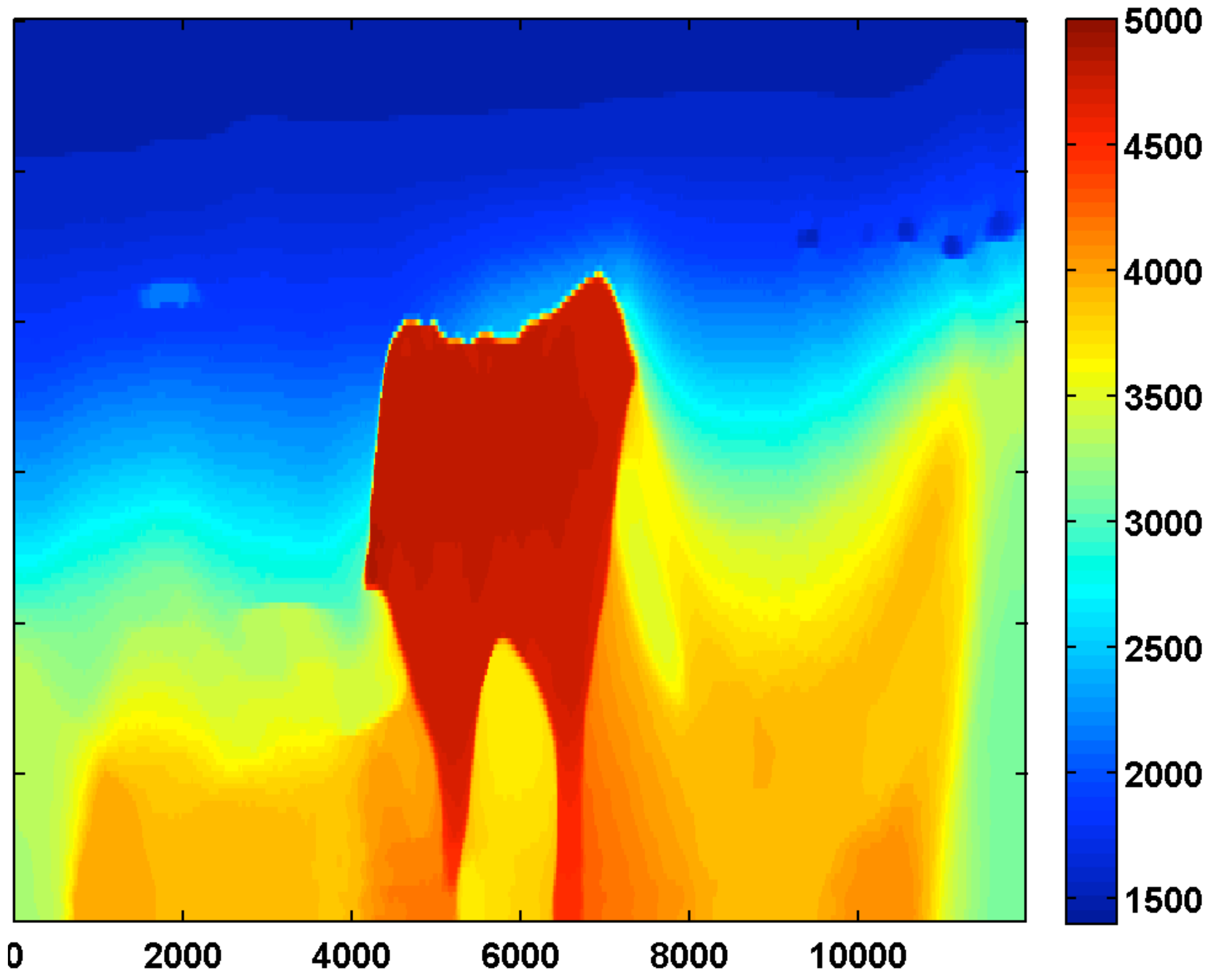


WRI vs adjoint-state

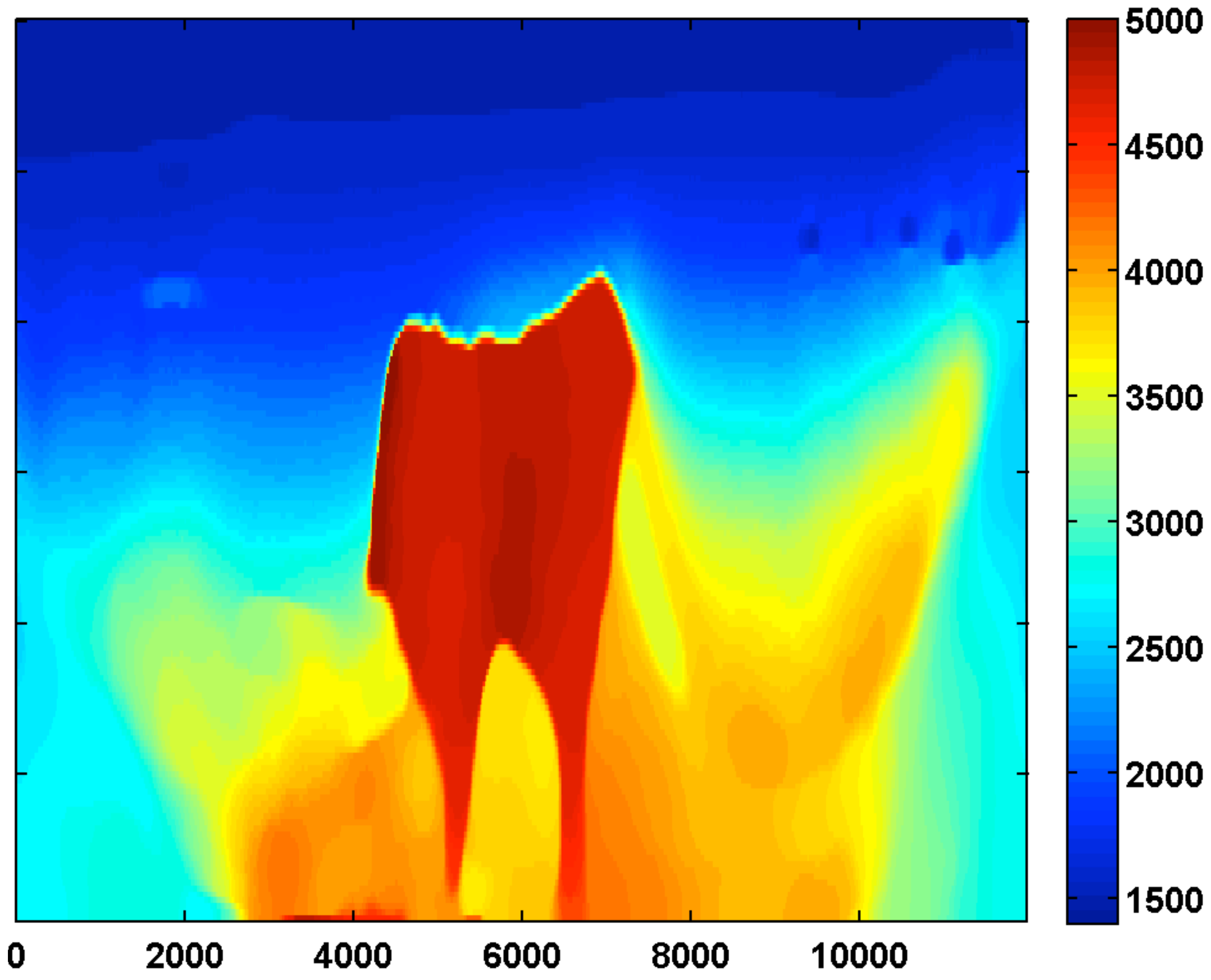
initial model



WRI



adjoint-state



Conclusions

New method for regularizing wave-equation based inversion benefits from

- combination of convex constraints
- multiple frequency sweeps w/ warm starts & relaxing of the constraints
- a hinge-loss function, which plays a critical role

Works for both WRI & adjoint-state FWI

Development of automatic continuation strategies for relaxing the constraints is ongoing.

Candidate for “automatic” salt flooding...

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

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