Source separation via SVD-free rank-minimization in the hierarchical semi-separable representation

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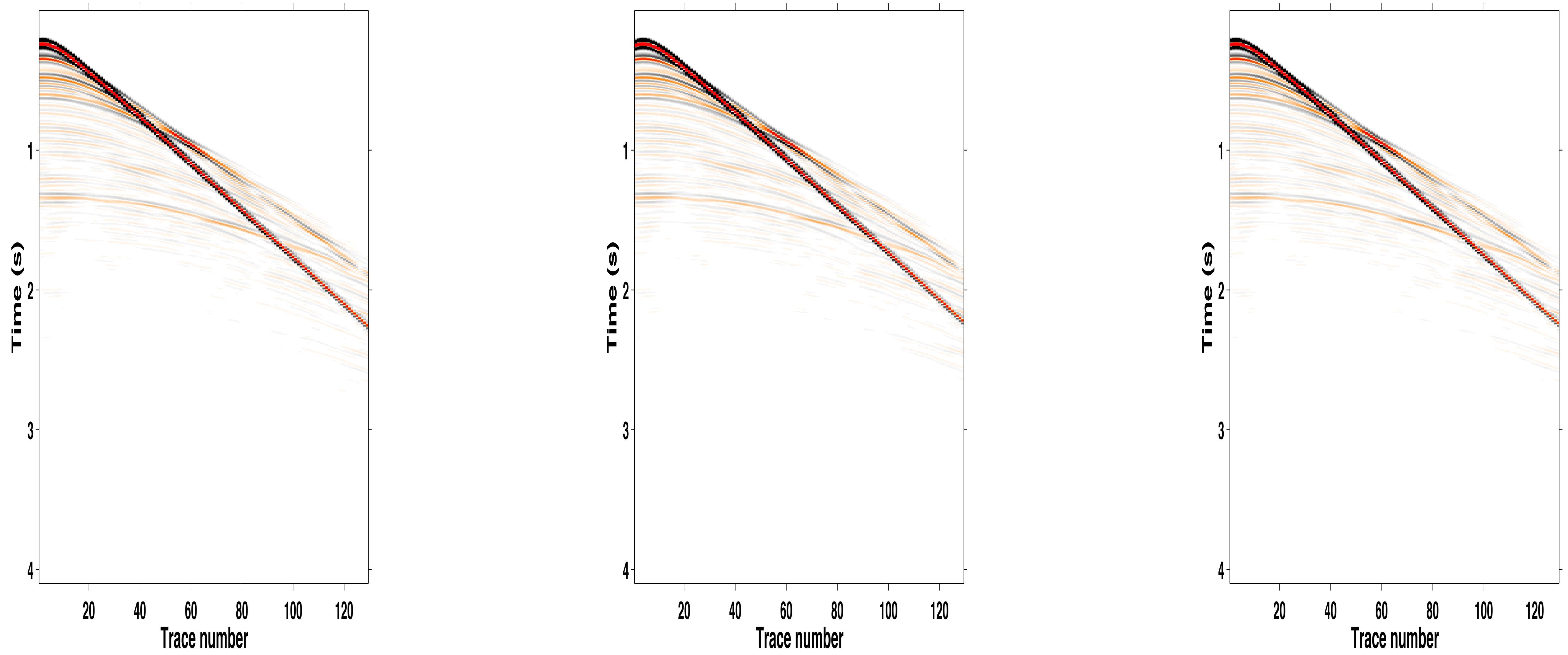
Conventional marine acquisition

★ source depth = 10 m

shot 1 shot 2 shot 3

regularly sampled spatial grid →

shot 1 shot 2 shot 3



Blended/Simultaneous marine acquisition

regularly sampled spatial grid

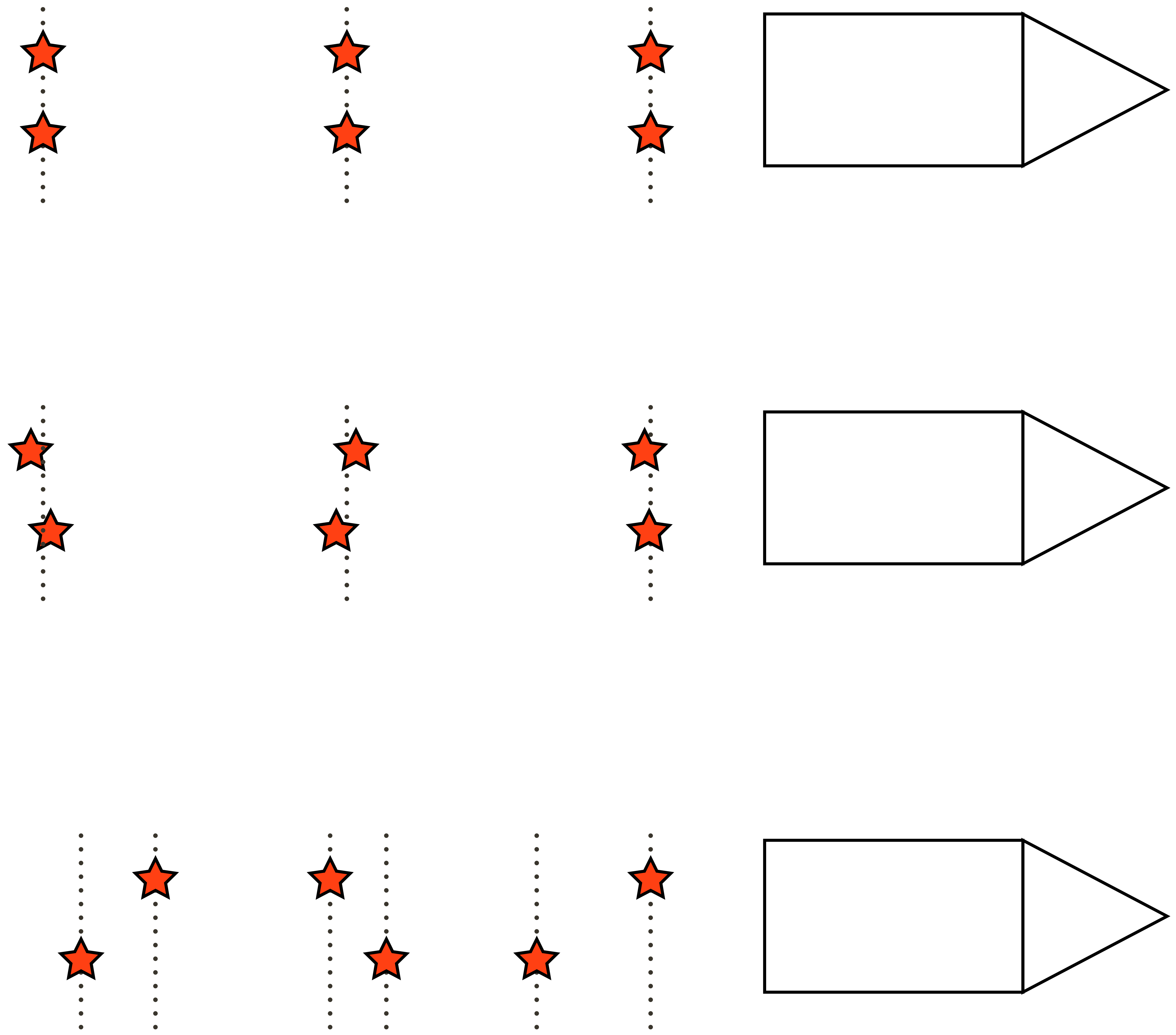
almost regularly sampled spatial grid

(over/under acquisition)

irregularly sampled spatial grid

(SLIM acquisition)

[Mansour et. al., 2012]
[Wason and Herrmann, 2013]



Blended/Simultaneous marine acquisition

[over/under acquisition]

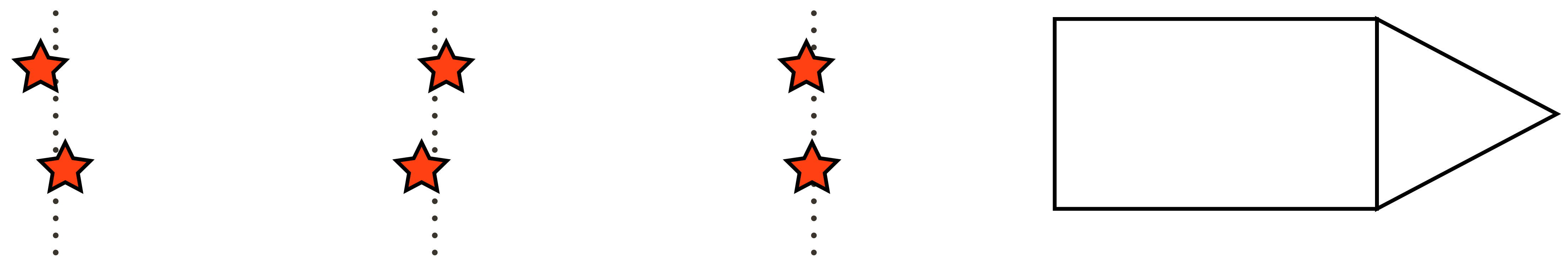
source1 depth = 10 m

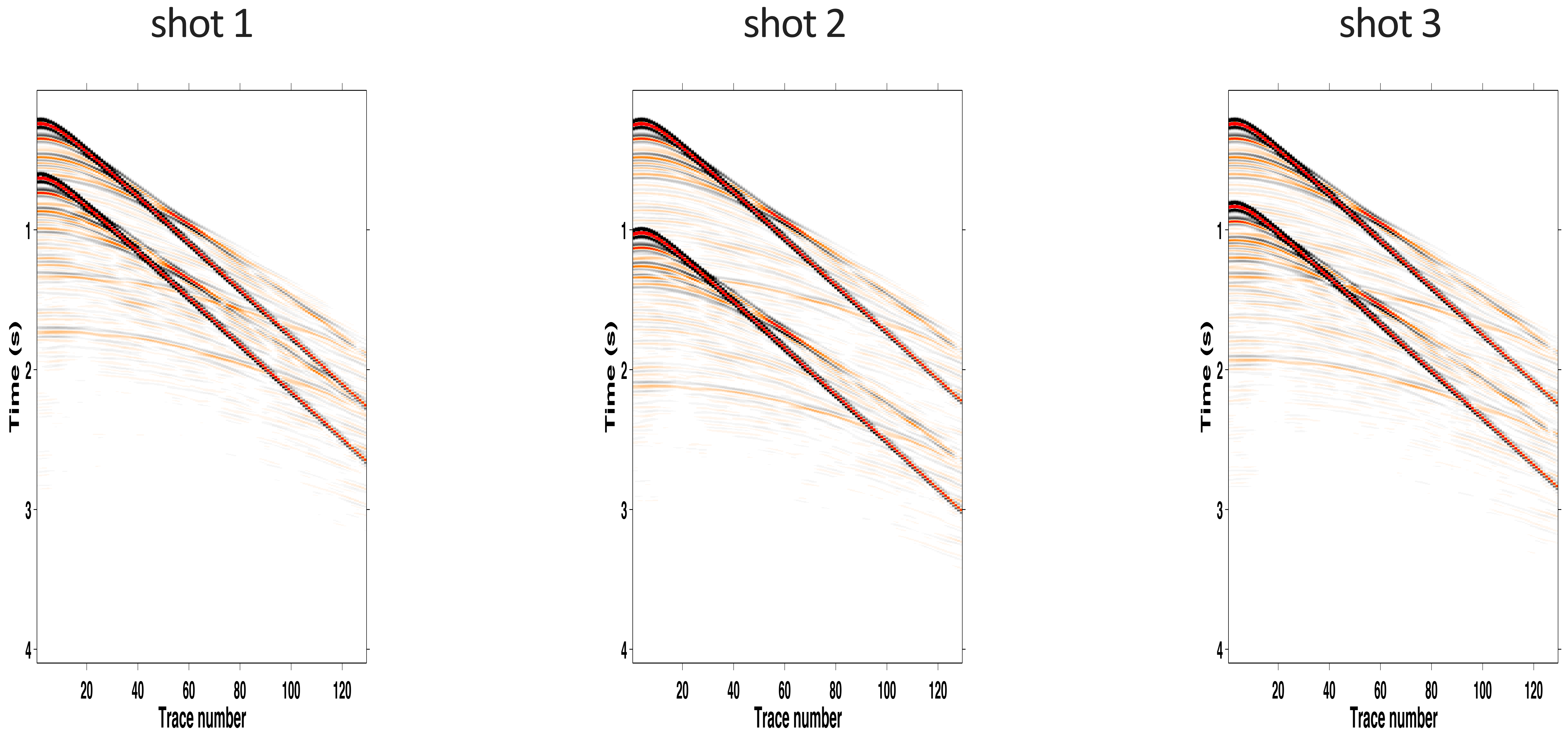
source2 depth = 15 m

regularly sampled spatial grid (almost)

shot-time randomness - ***LOW***

shot 1 shot 2 shot 3





Challenges

- Source separation (or deblending)
  - recover individual datasets
- Shot-time randomness
  - low

[Candès and Donoho, 2000; Herrmann, 2008]

Compressed sensing

Successful sampling & reconstruction scheme

- exploit *structure* via *sparsifying* transform
  - *fast decay* of "transform domain" coefficients
- sampling
  - randomly blended data *decreases* sparsity in "transform domain"
- optimization
  - via *sparsity-promotion*

[Candès and Plan, 2010, Oropeza and Sacchi, 2011]

SLIM

Matrix completion

Successful reconstruction scheme

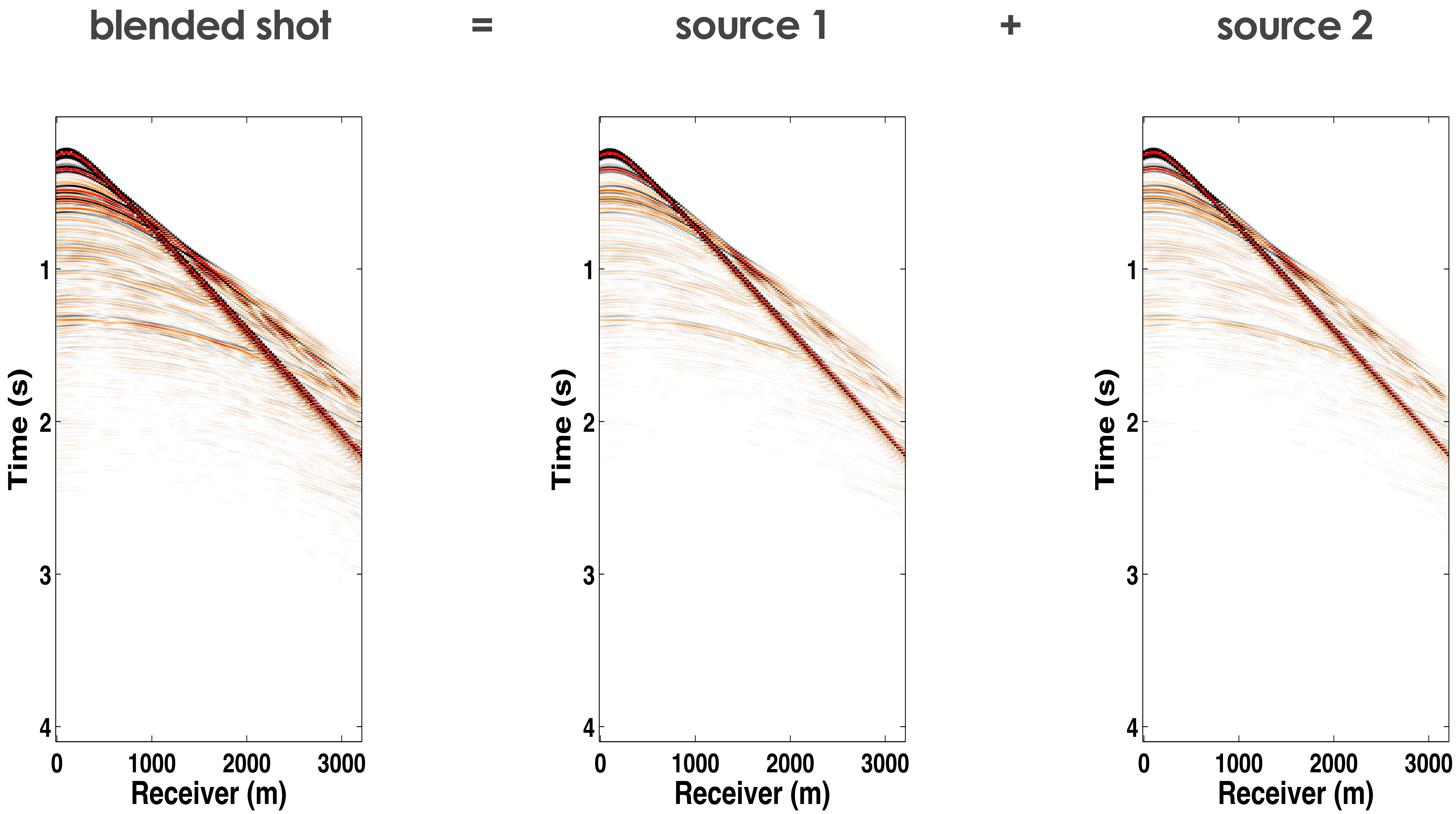
- exploit *structure*
  - *low-rank / fast decay* of singular values
- sampling
  - randomly blended data *increases* rank in “transform domain”
- optimization
  - via *rank-minimization (nuclear norm-minimization)*

Low-rank structure

In which domain?

source-receiver or *midpoint-offset*

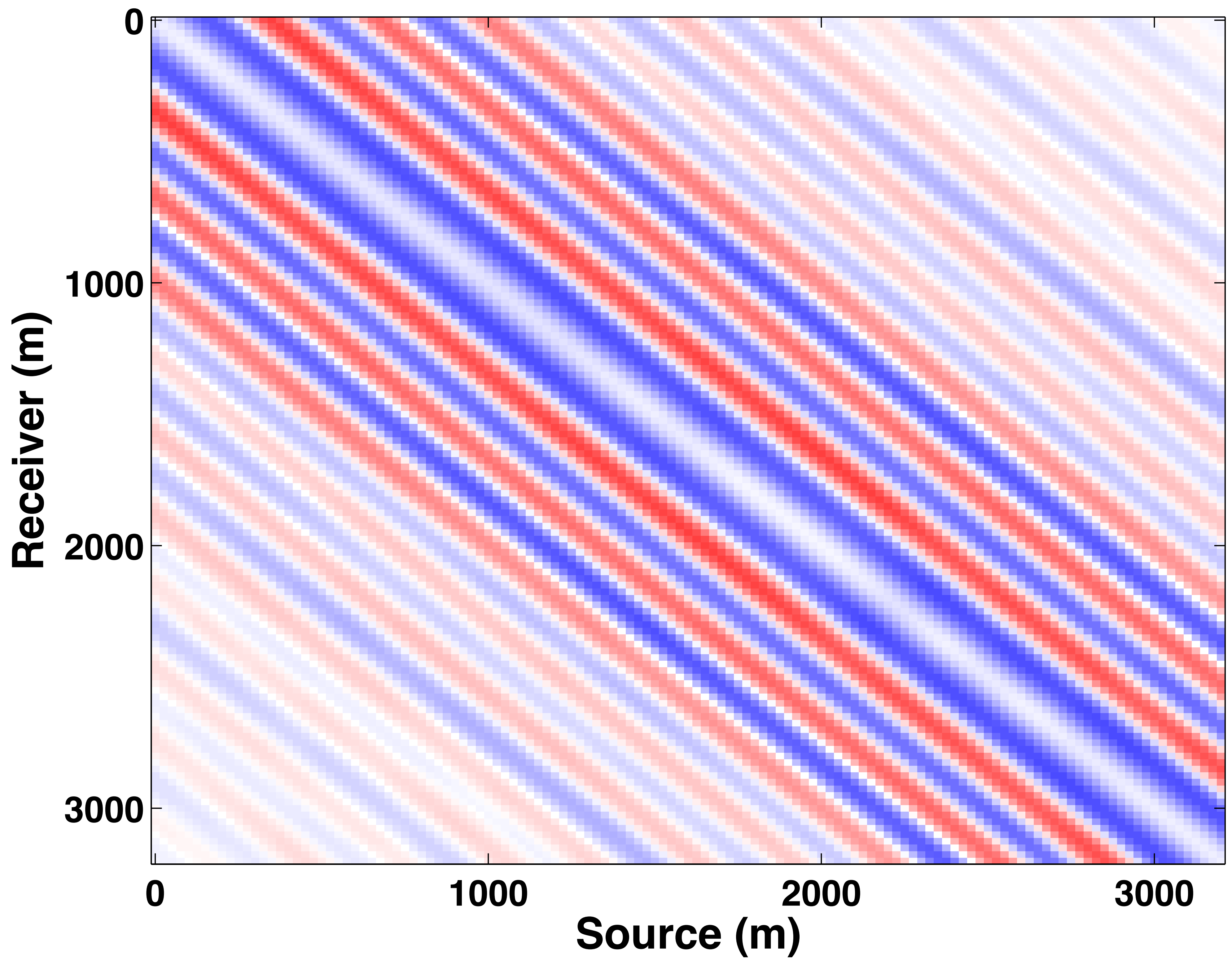
Blended data (w/o delay)



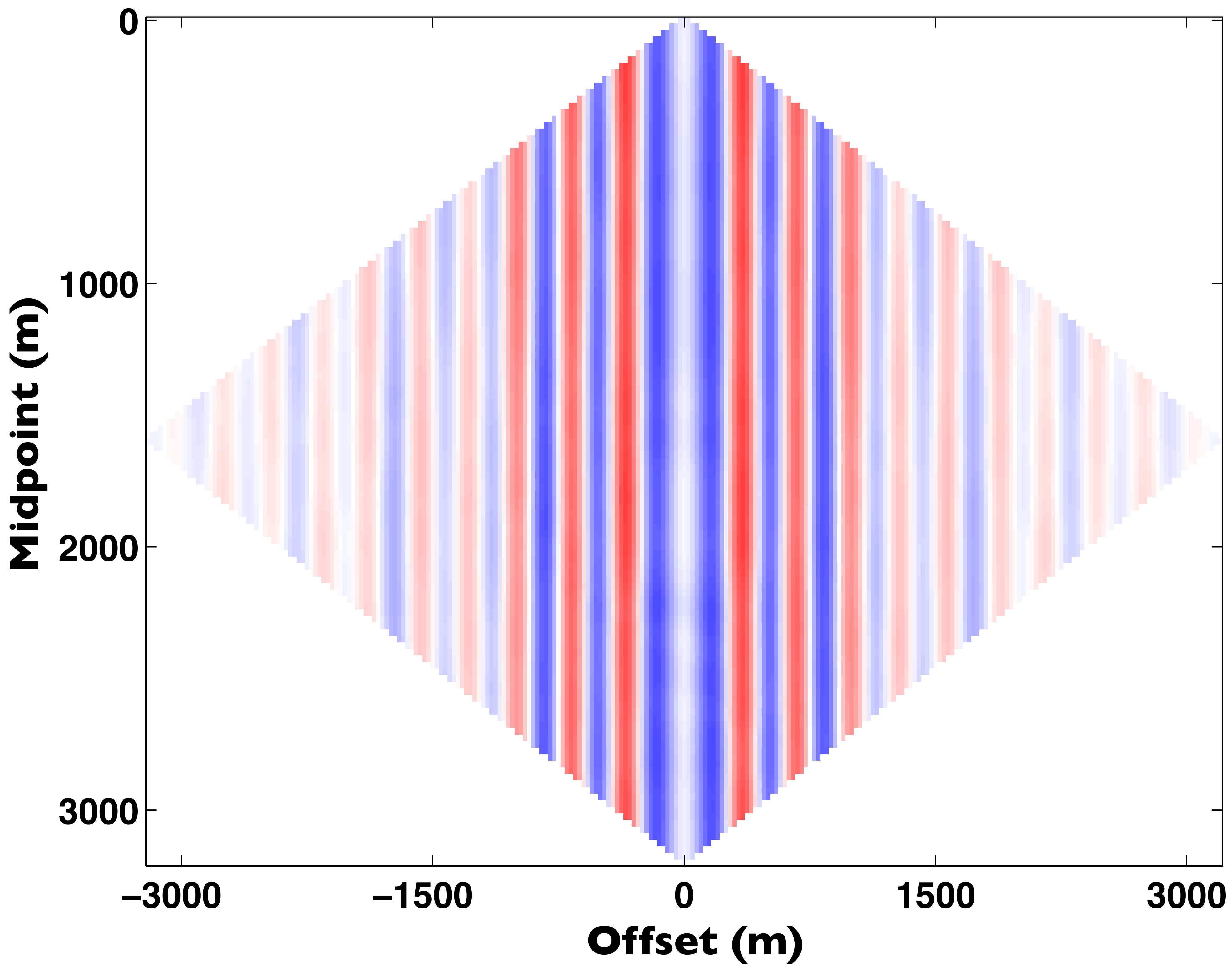
In which domain?

[frequency slice at 5 Hz]

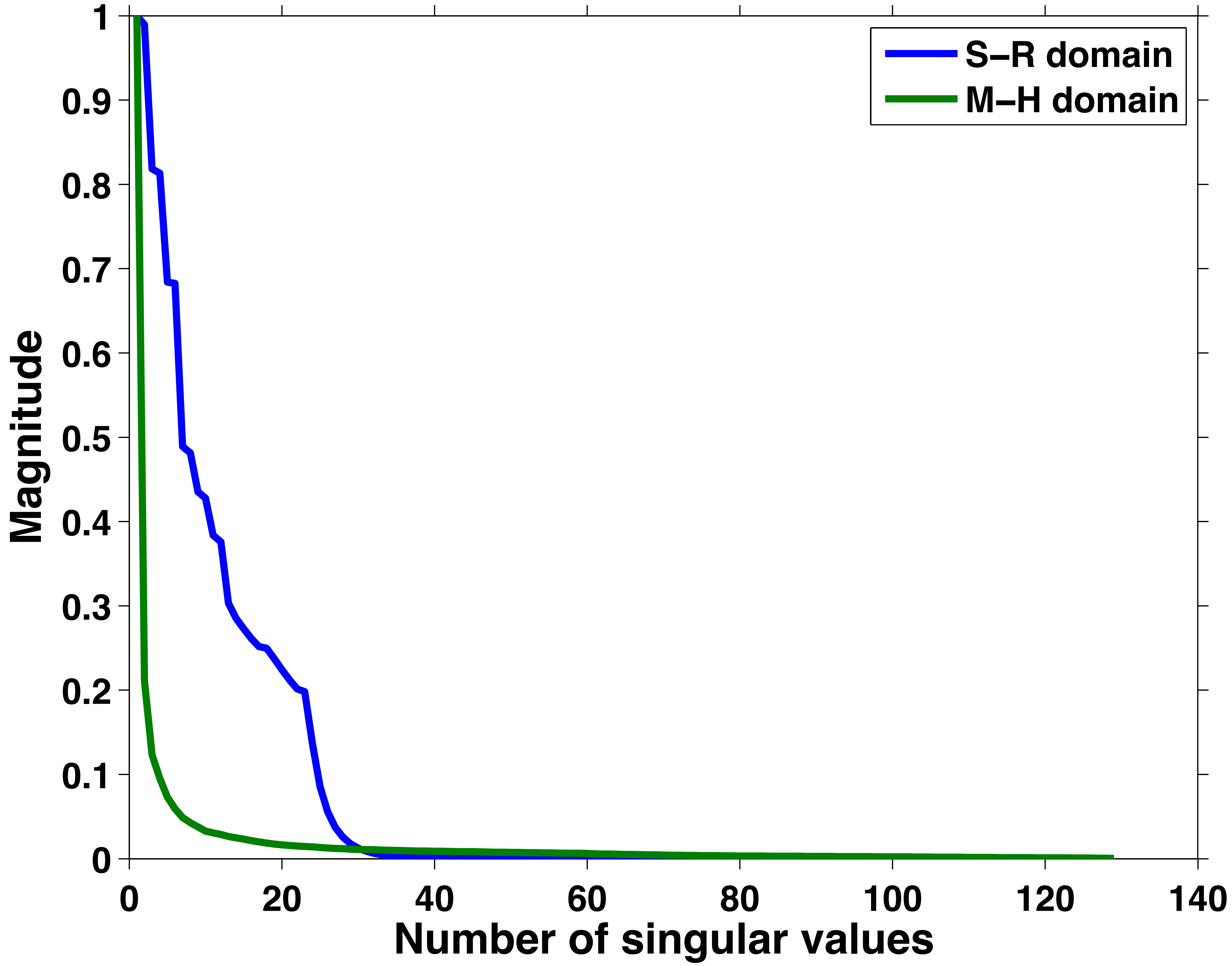
source-receiver domain



midpoint-offset domain



Decay of singular values



low-rank in midpoint-offset domain

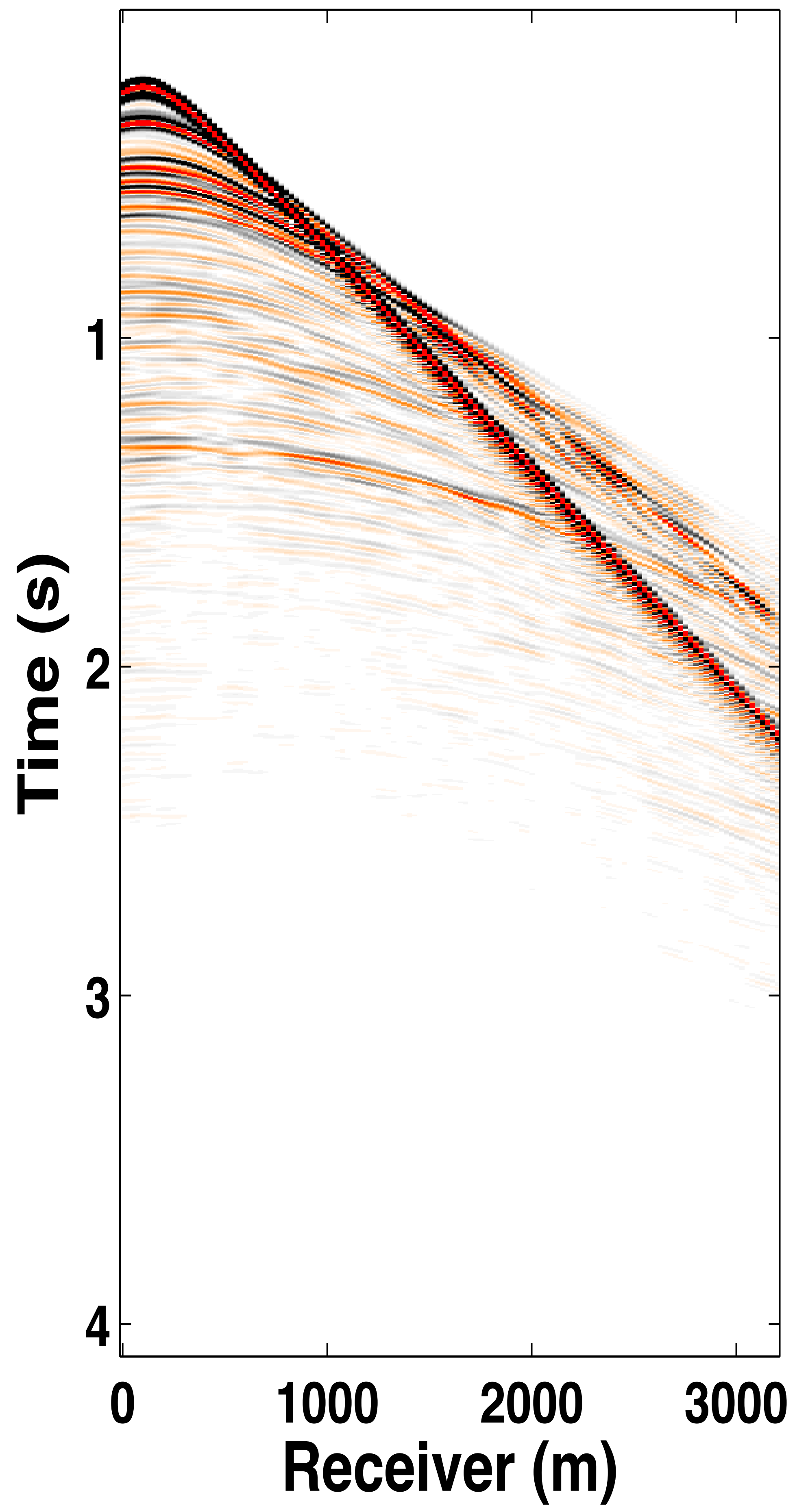
Sampling scheme

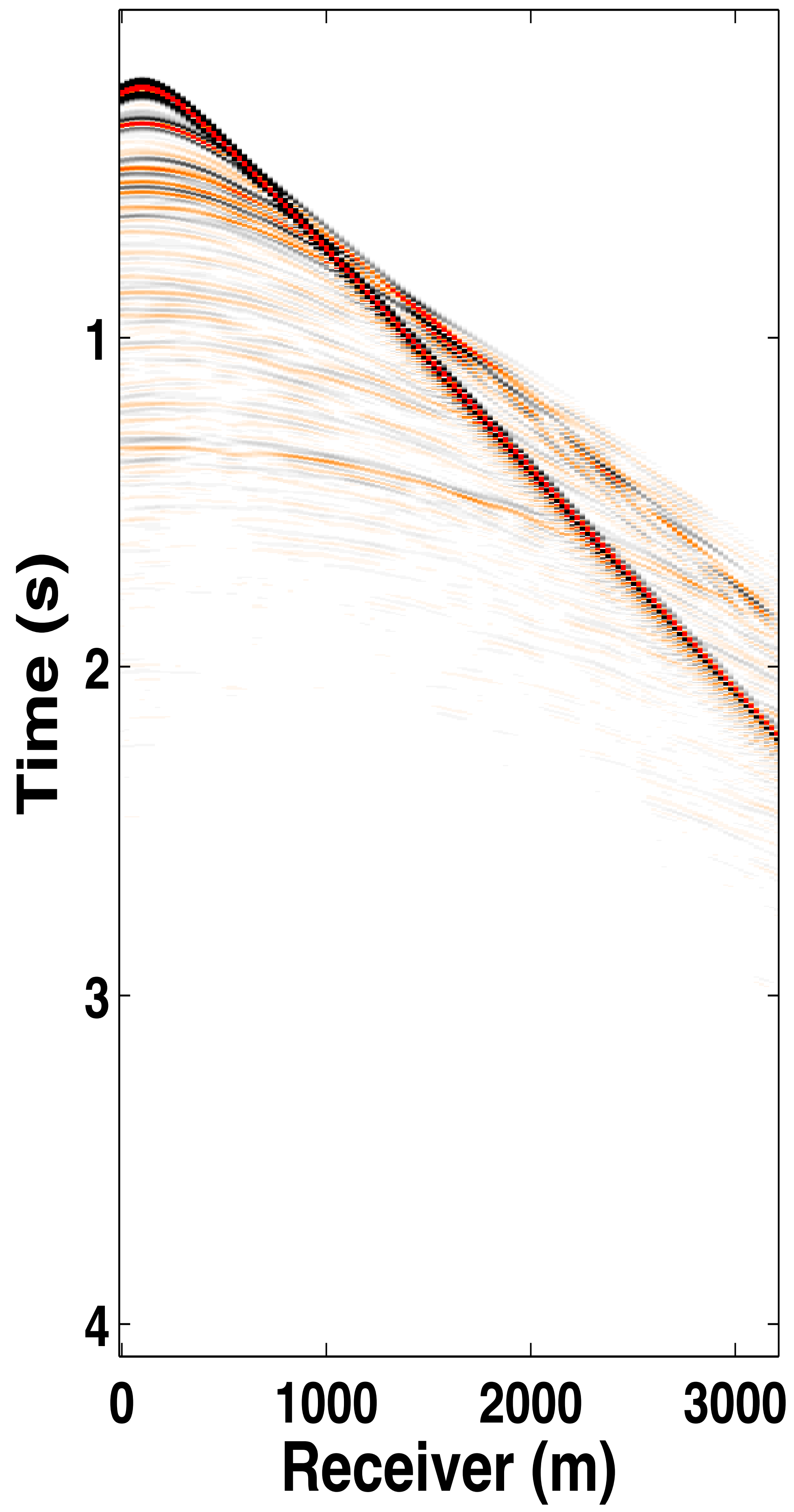
sample to *break* the structure

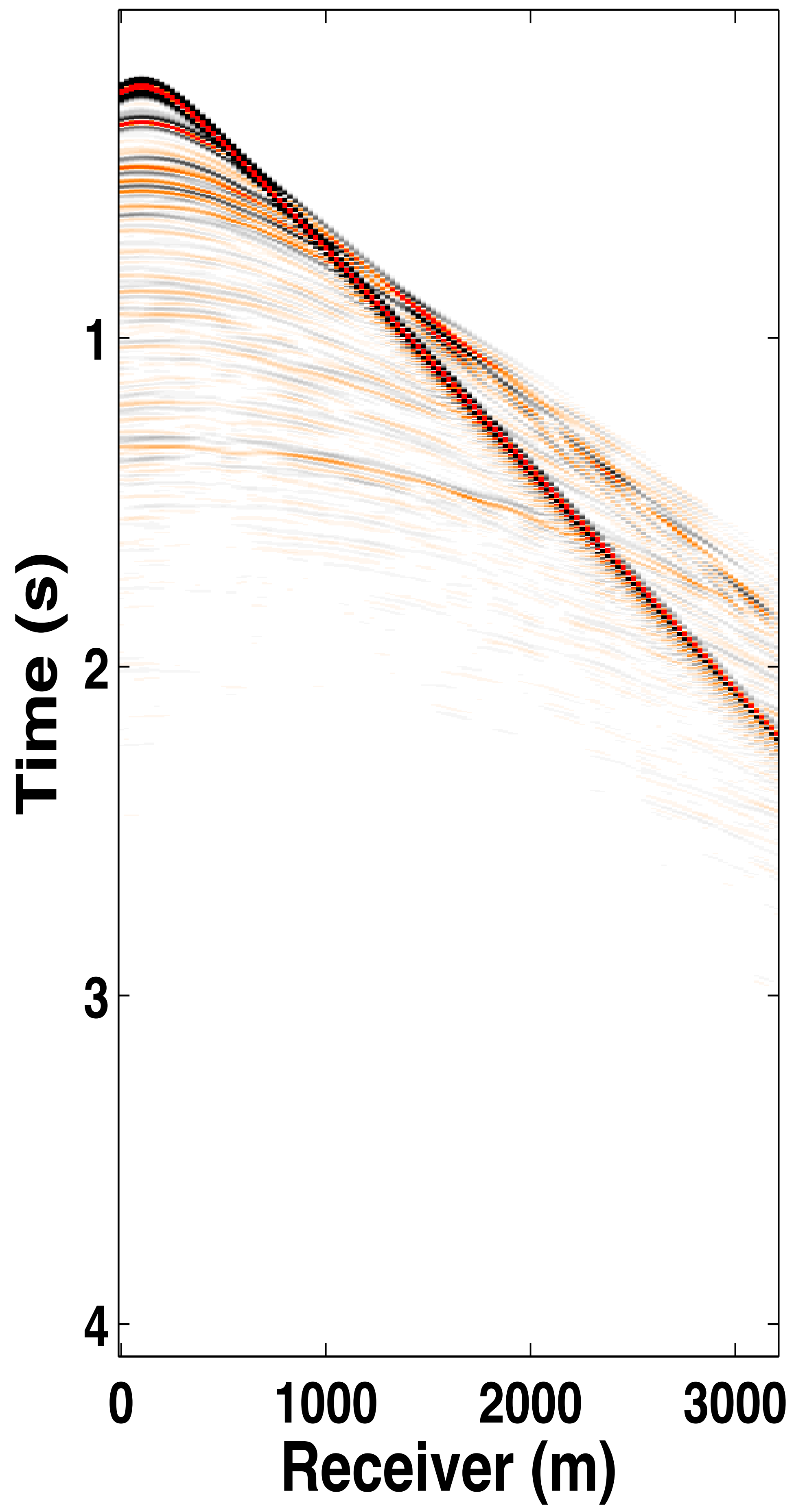
random time delays break the structure

Blended data (w/o delay)

blended shot = source 1 + source 2

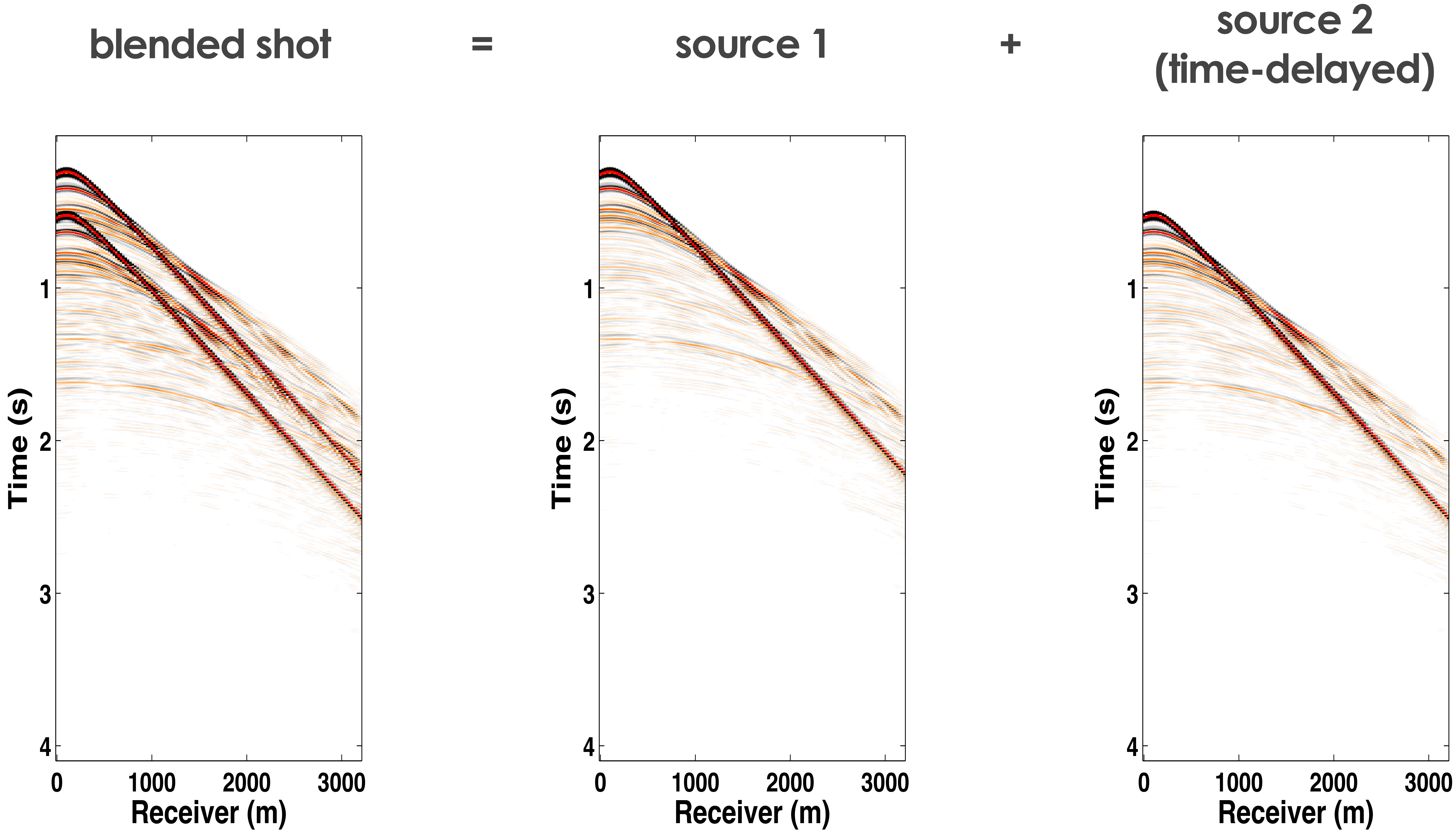






Blended data (w/ delay)

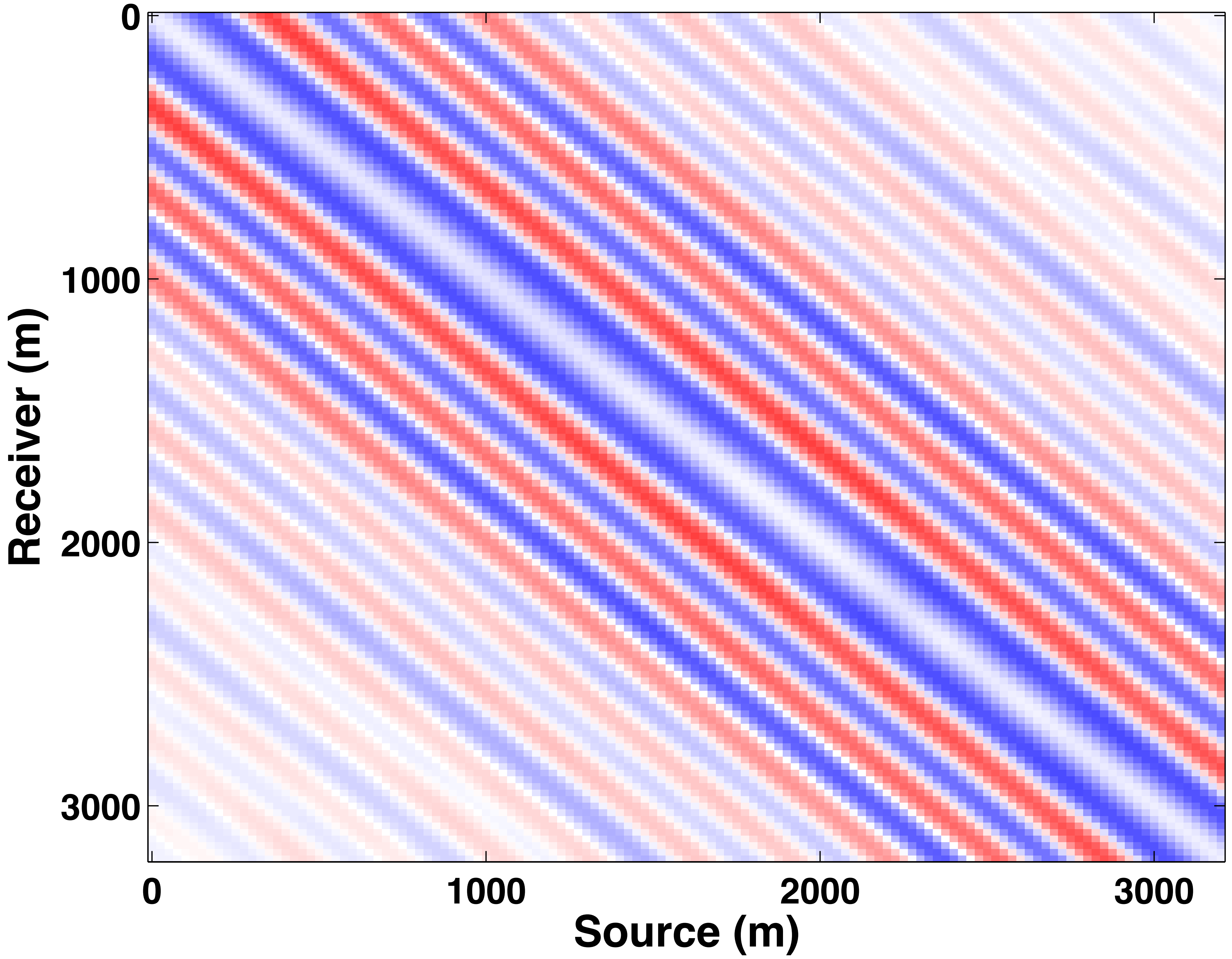
[random time delays applied to source 2]



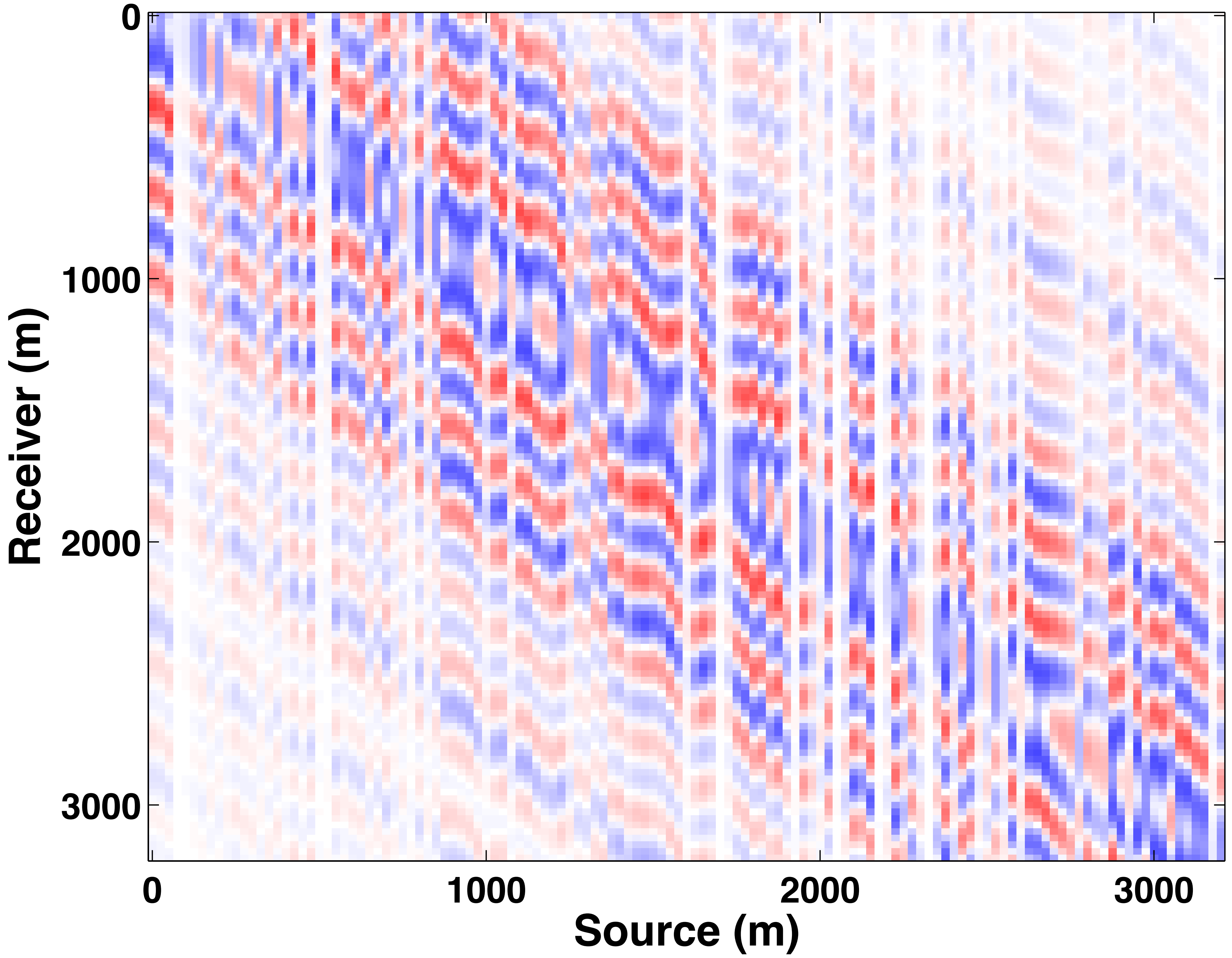
Source-receiver domain

[frequency slice at 5 Hz]

without delay



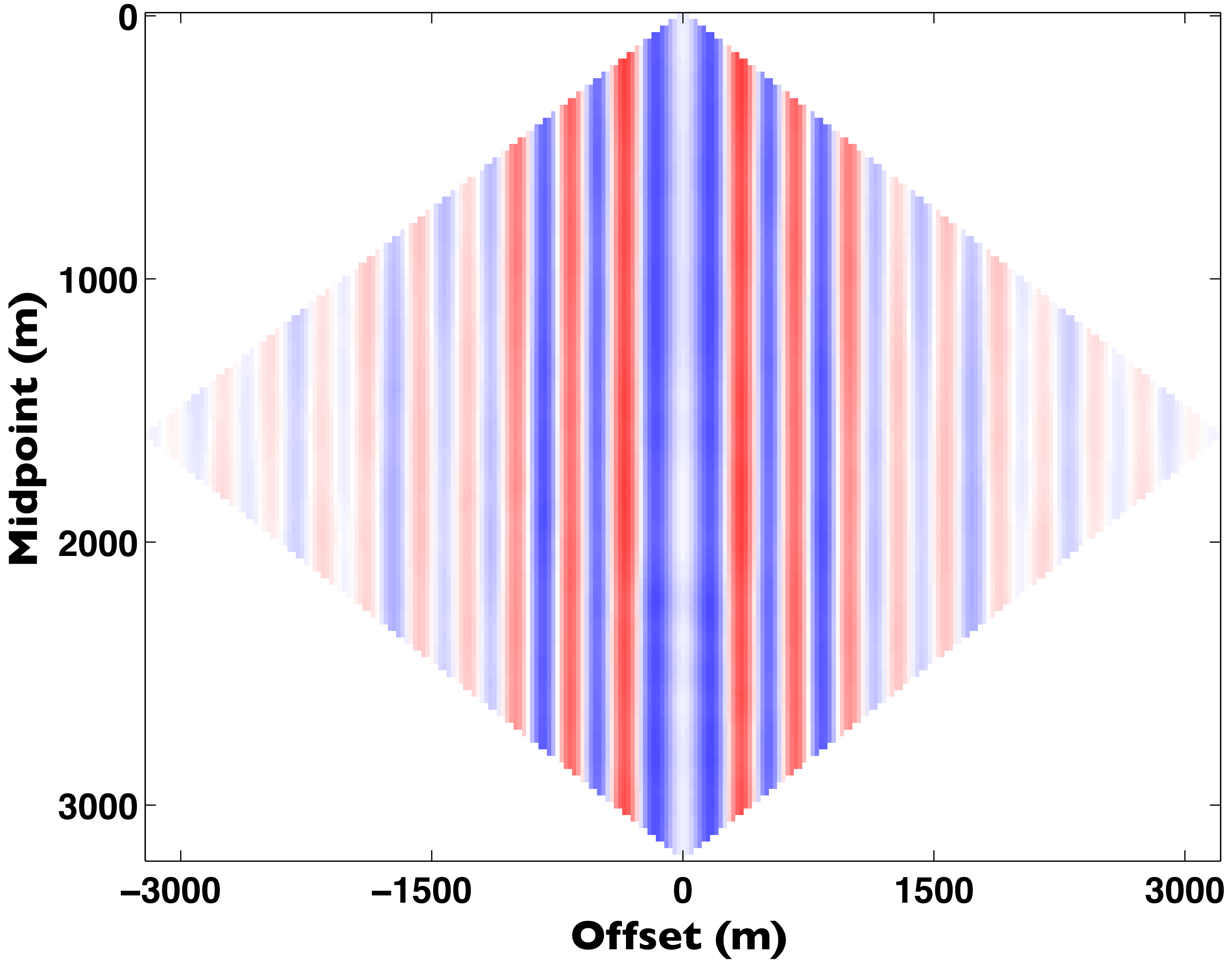
with delay



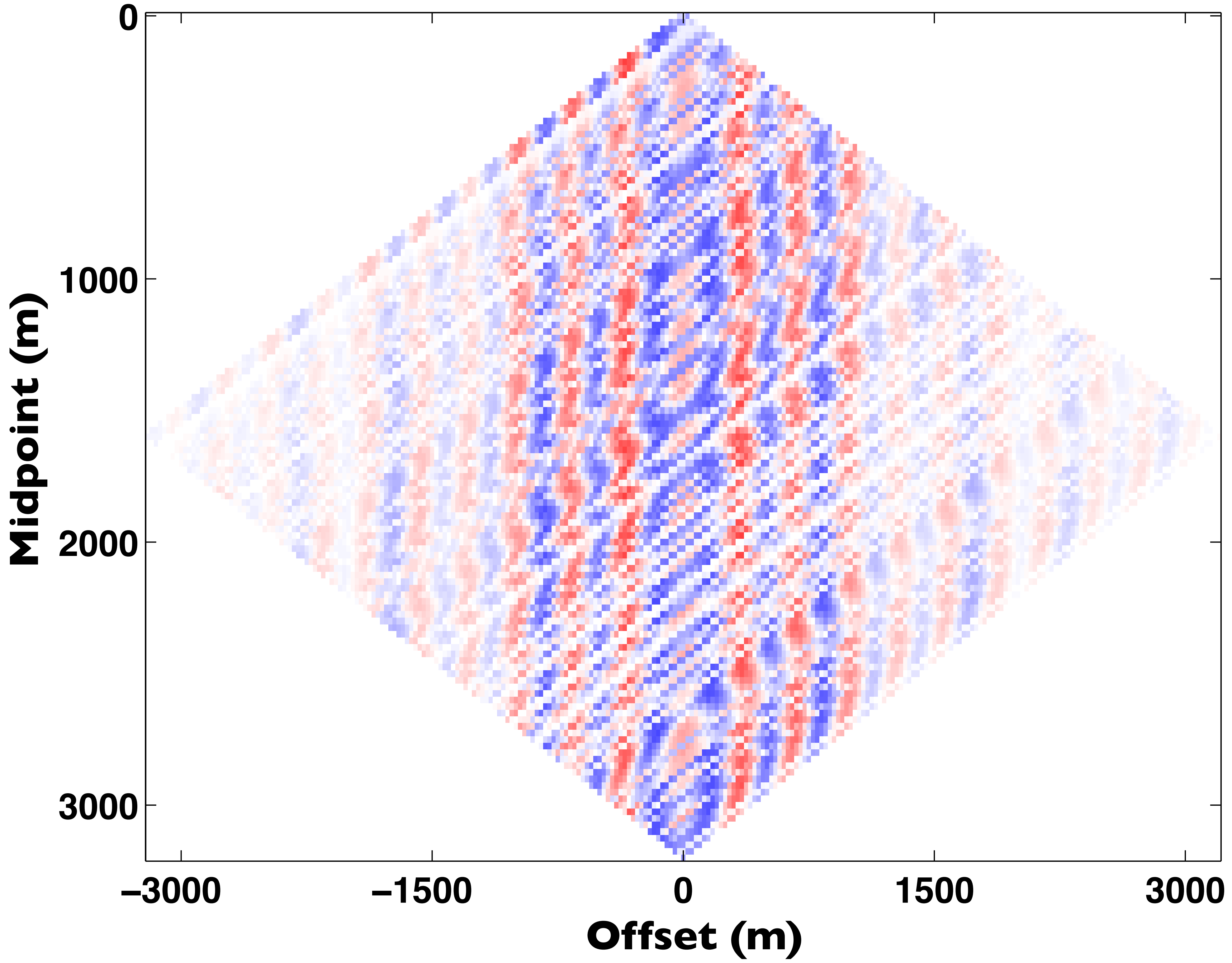
Midpoint-offset domain

[frequency slice at 5 Hz]

without delay

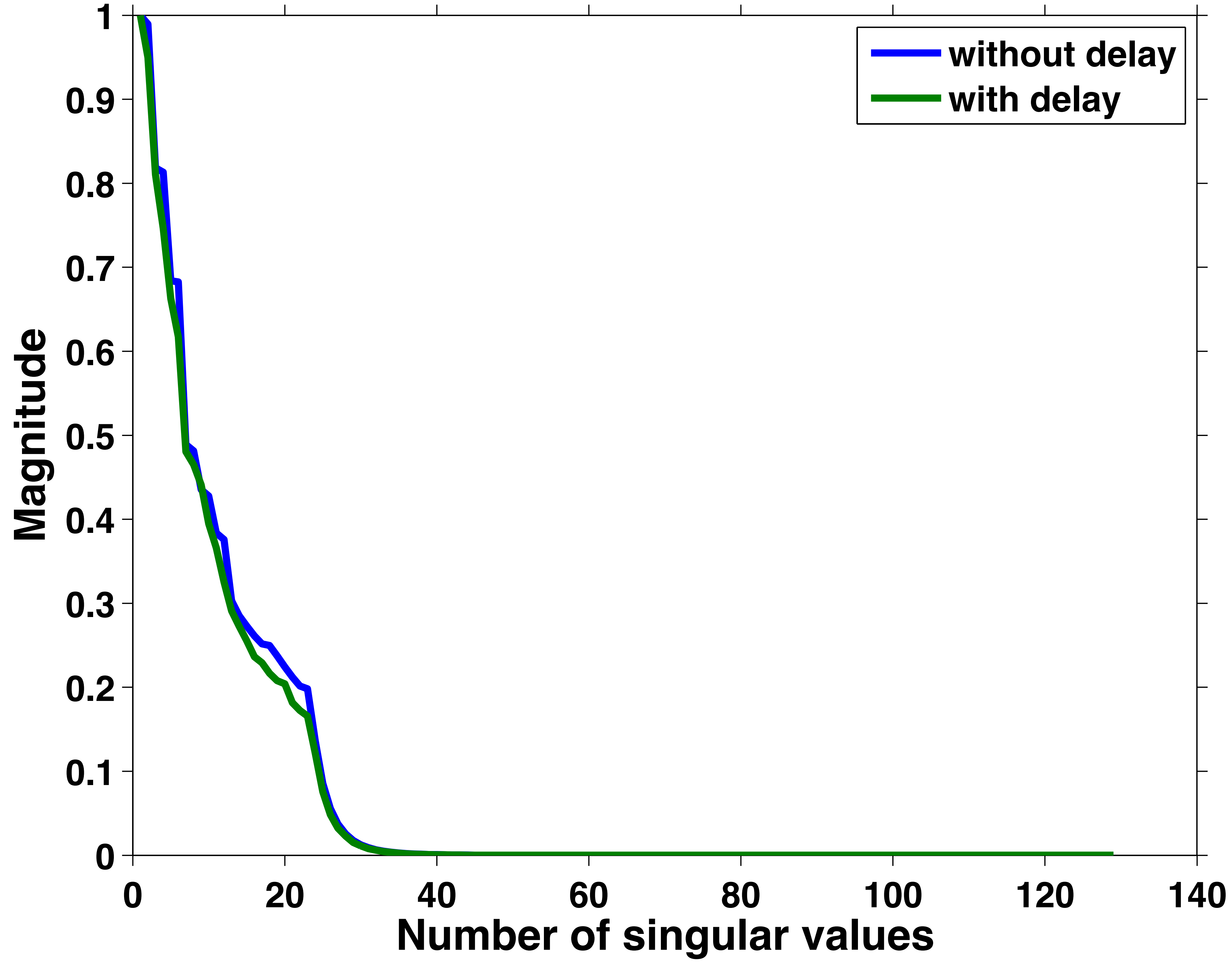


with delay

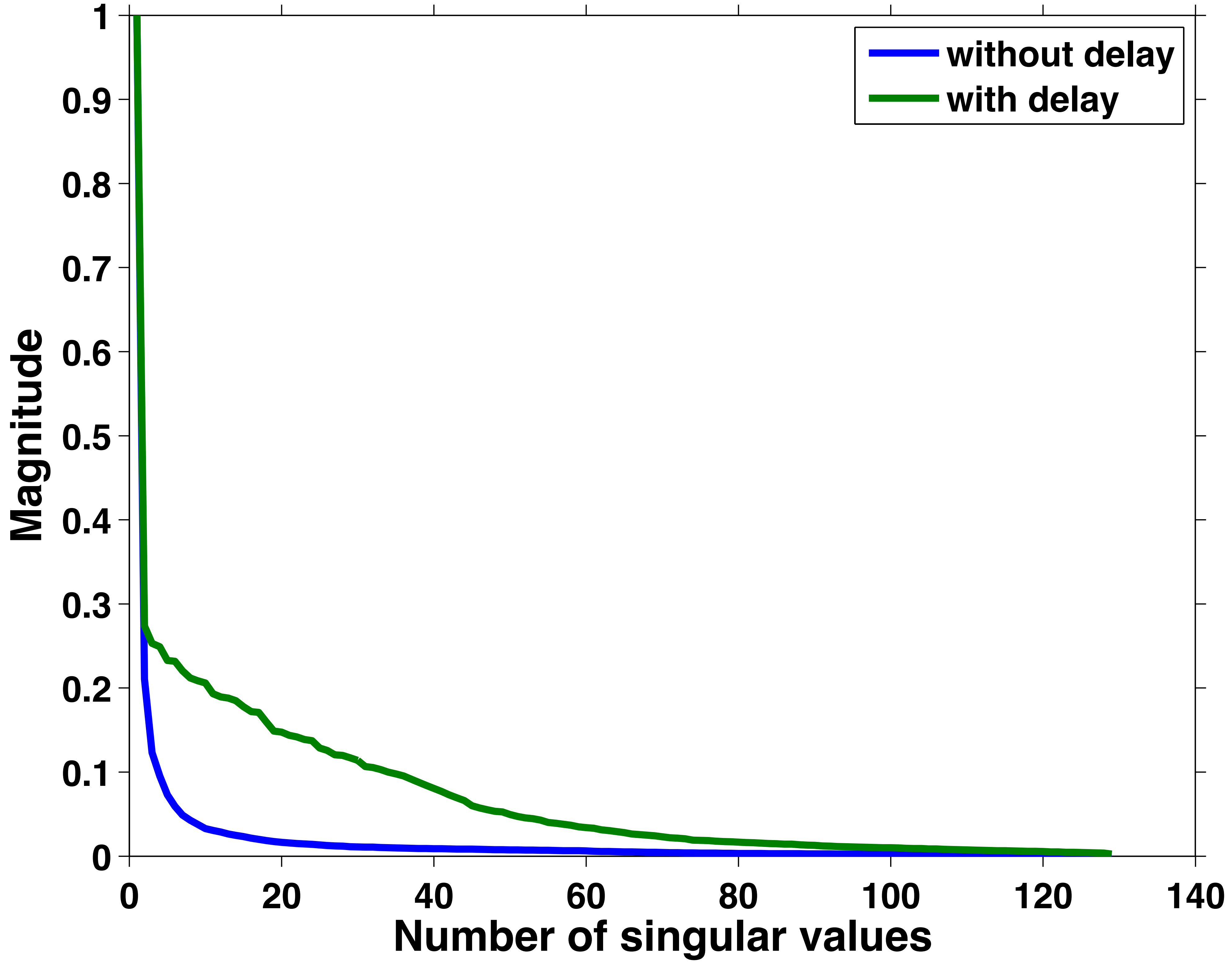


Decay of singular values

source-receiver domain

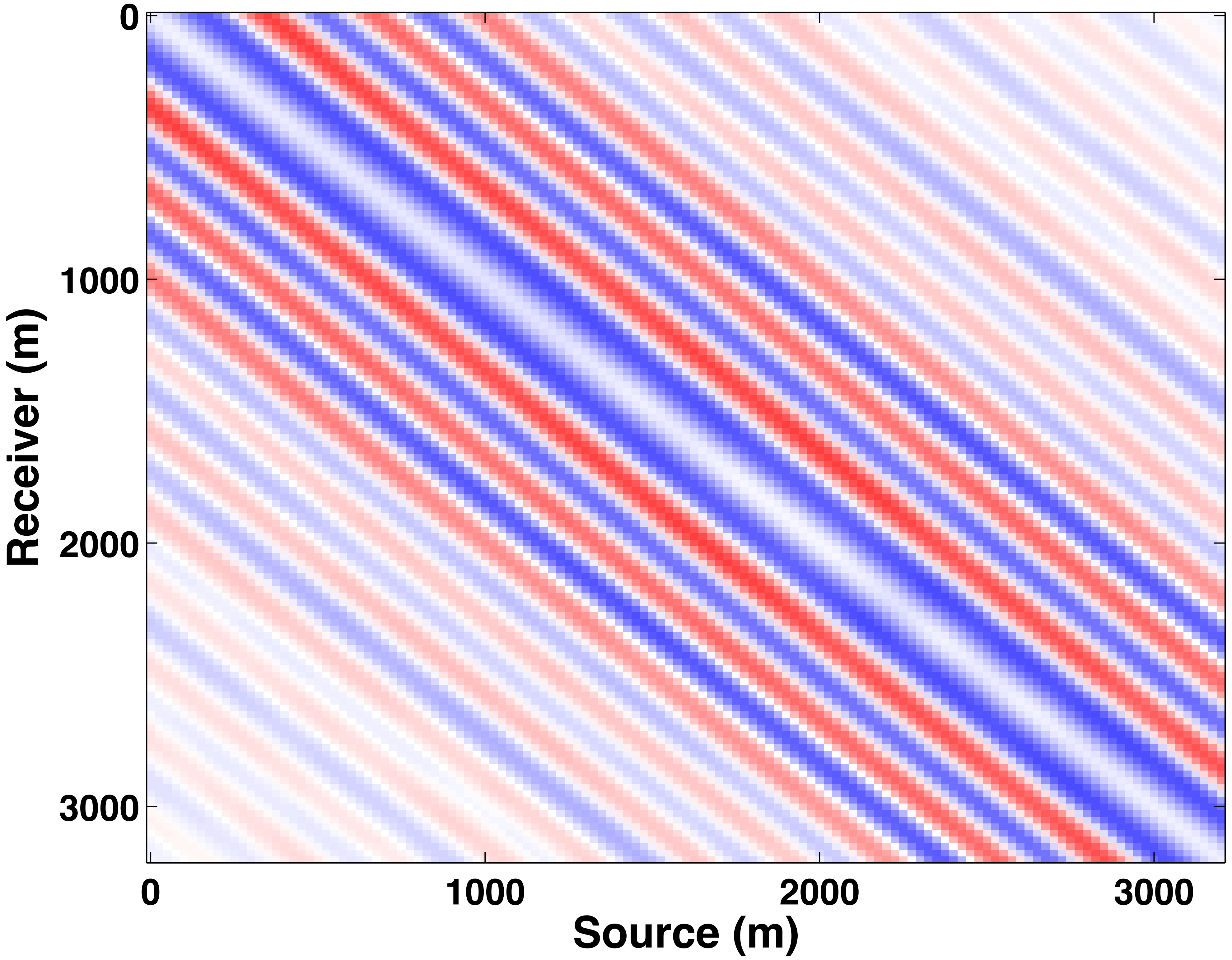


midpoint-offset domain

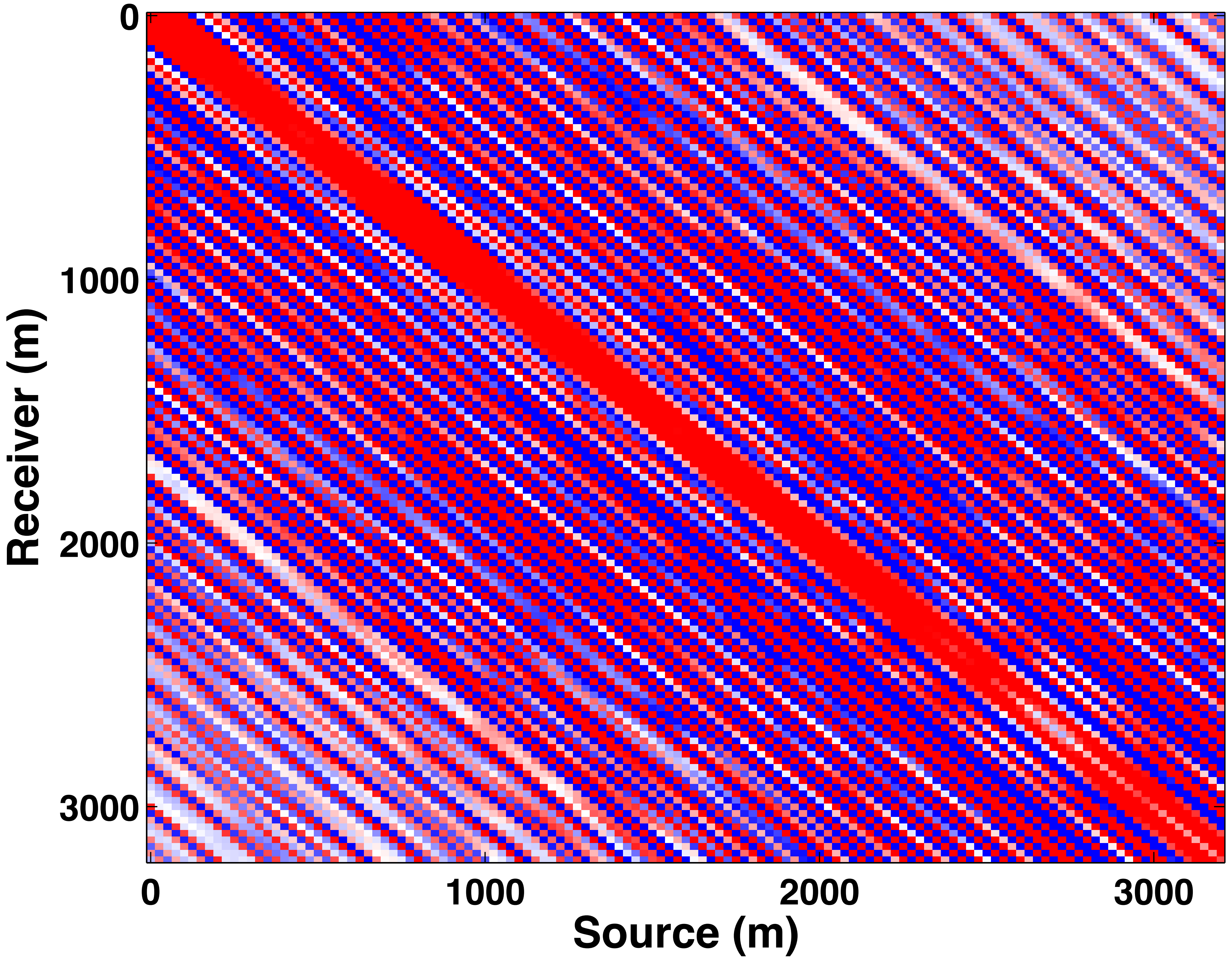


Are high frequencies low-rank?

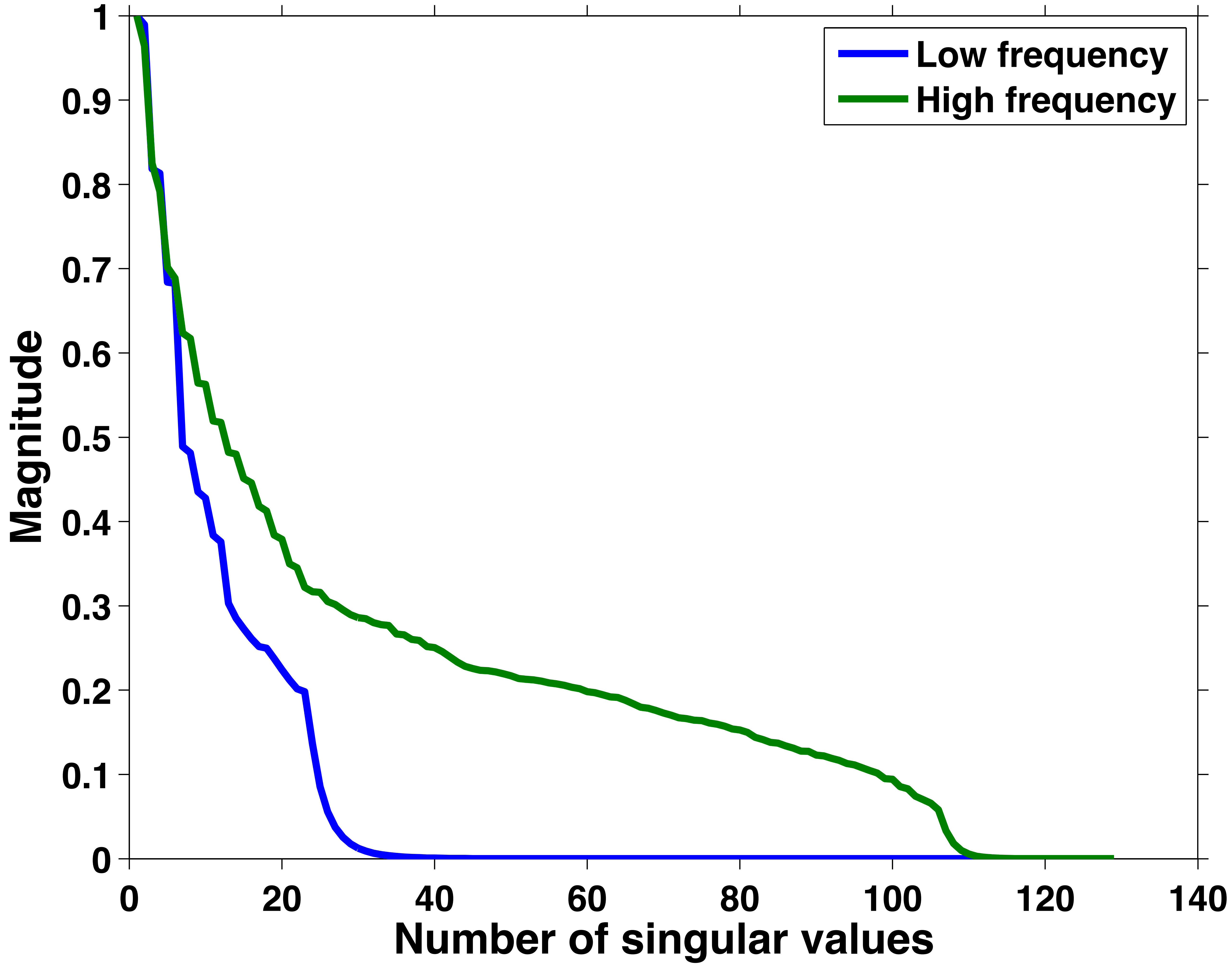
low frequency



high frequency



Decay of singular values



high frequencies
do NOT have
low-rank structure

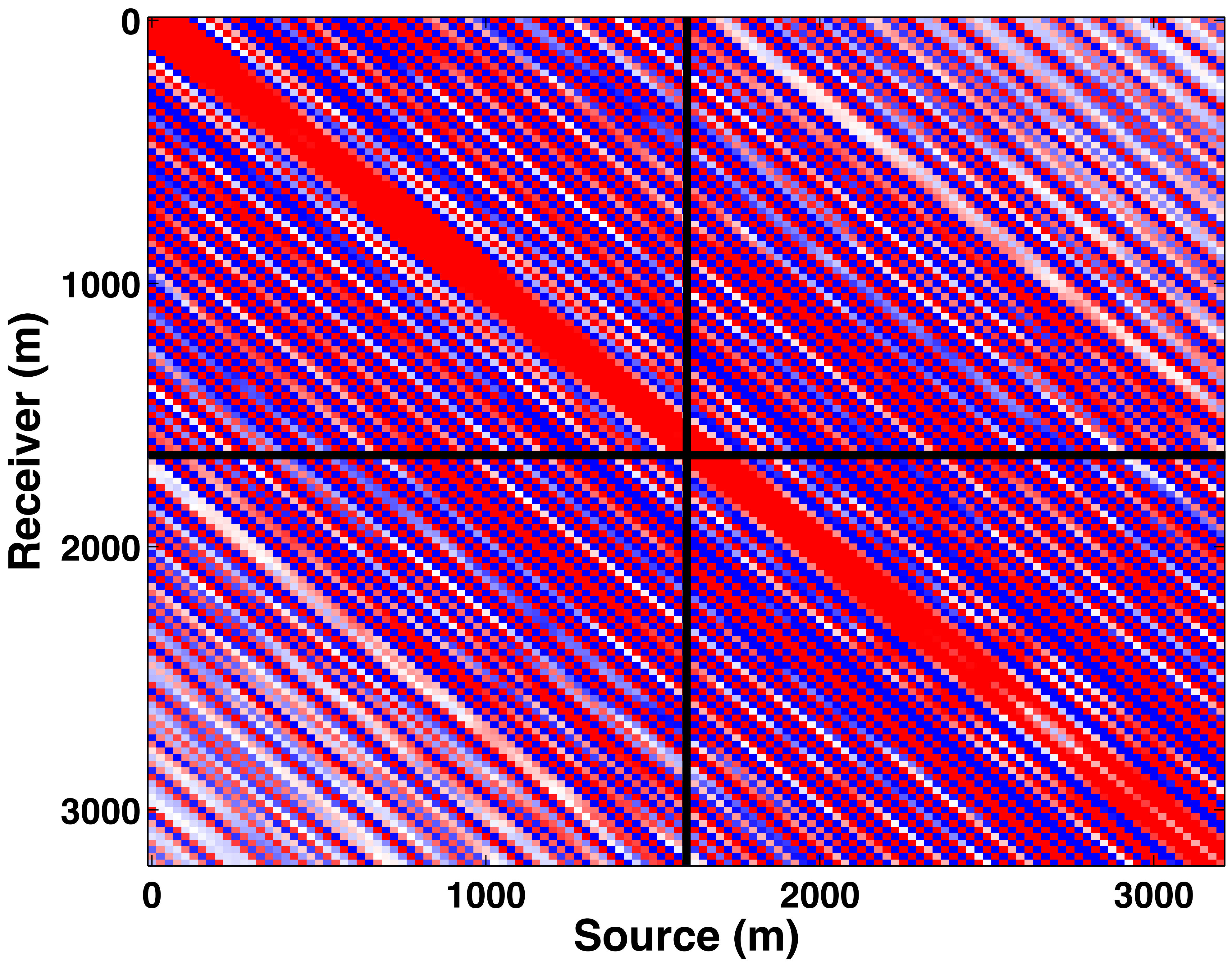
Hierarchical semi-separable (HSS) representation

HSS representation

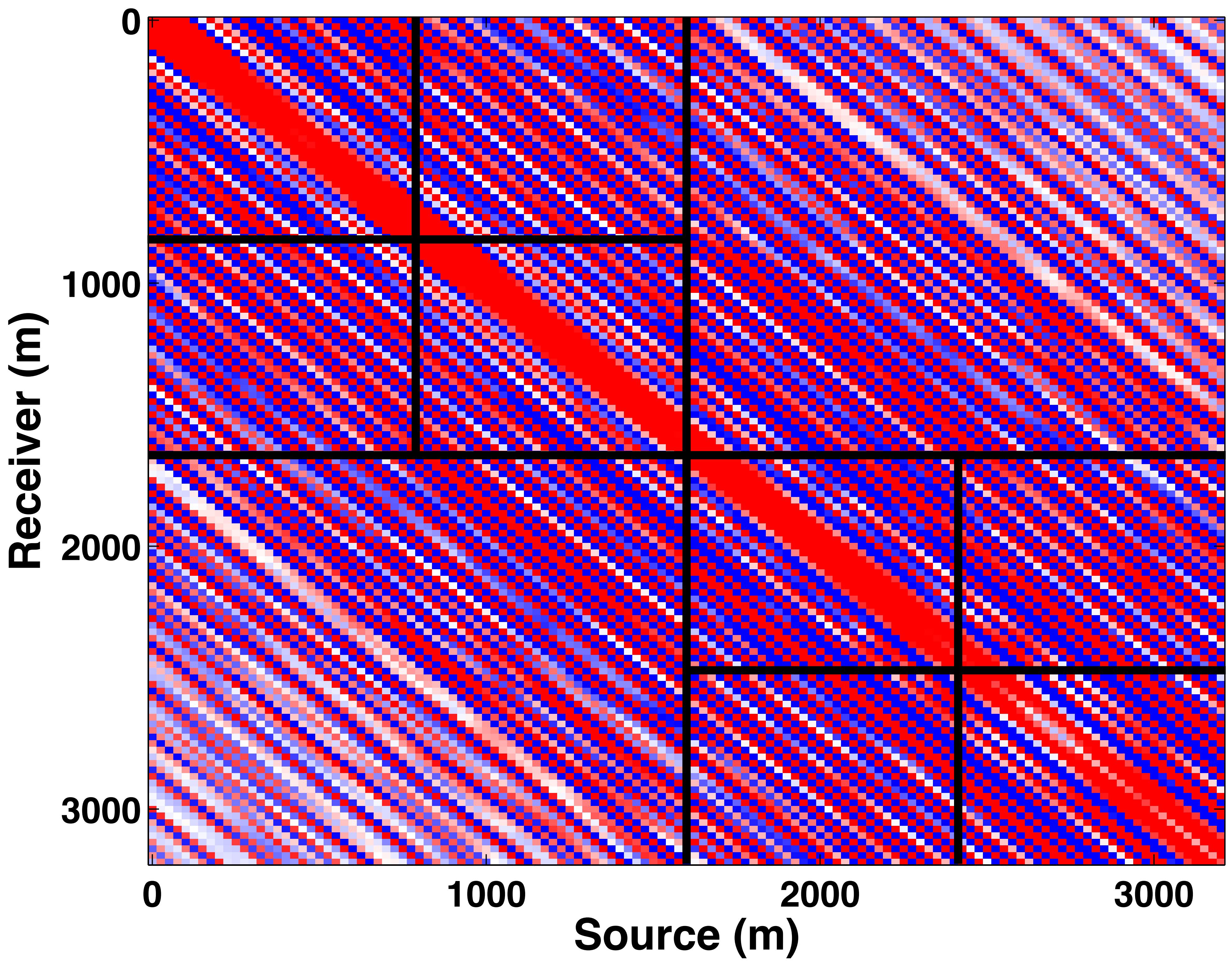
[Chandrasekaran, et. al., 2006]

off-diagonals are low-rank

level - 1



level - 2

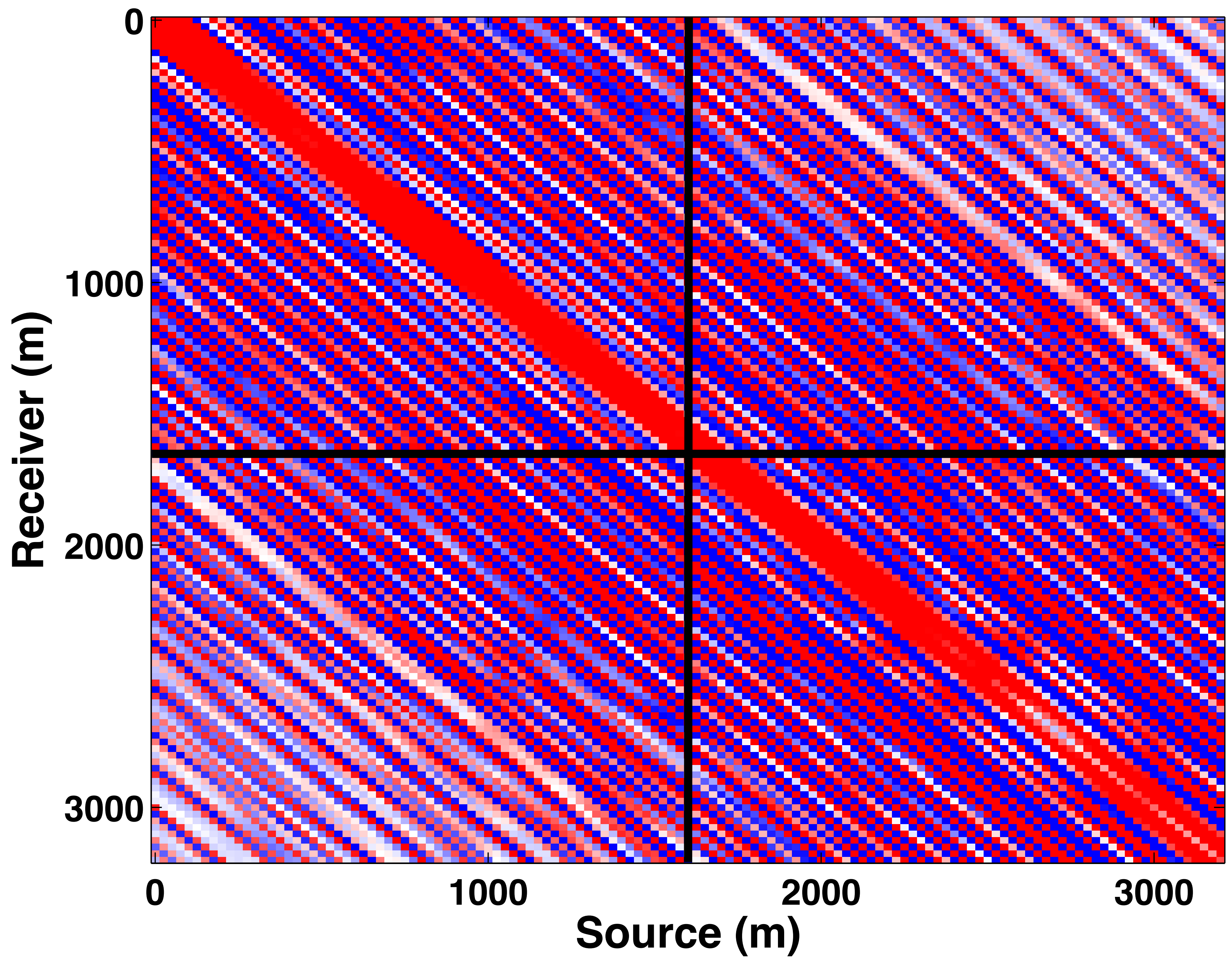


HSS representation

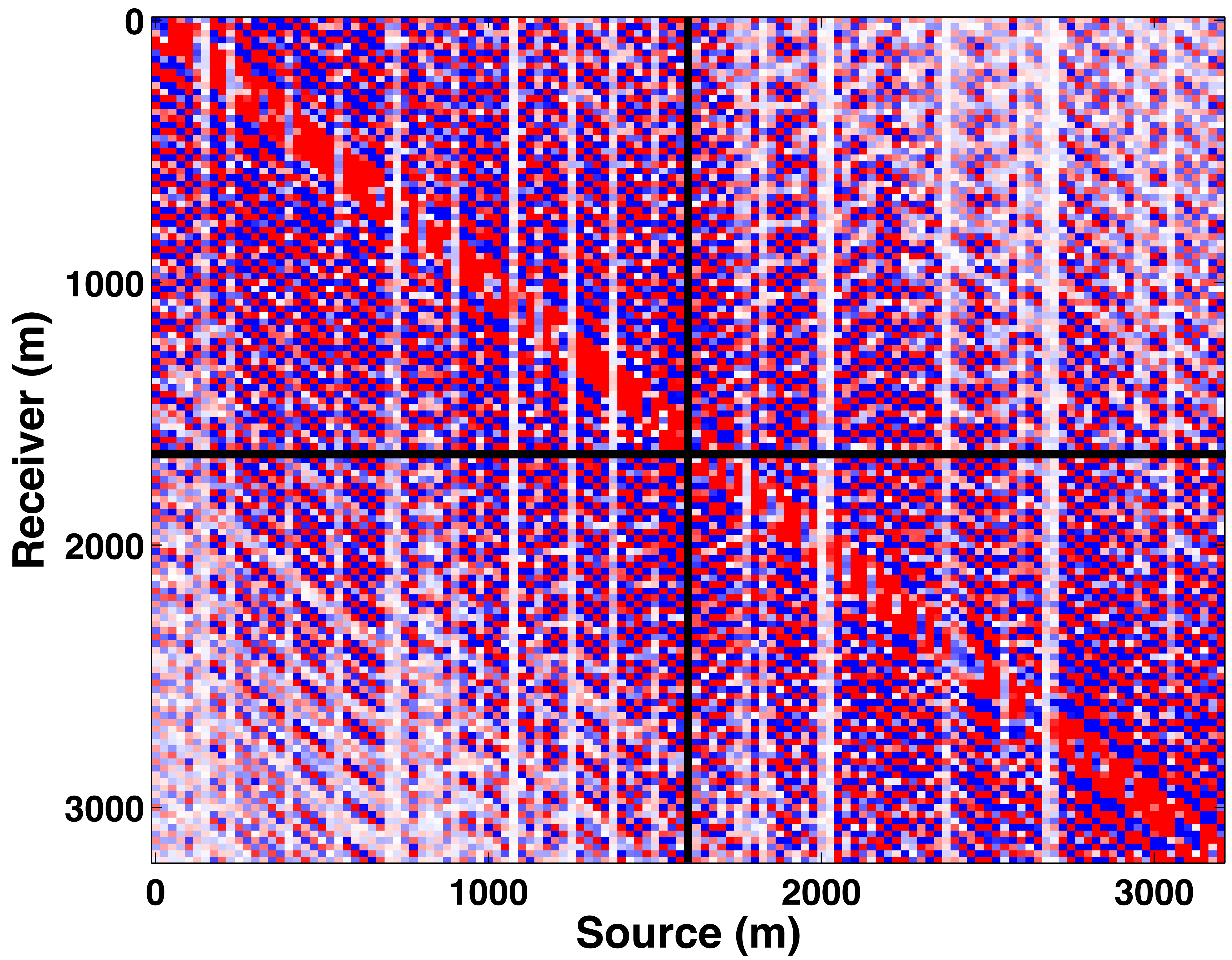
[Chandrasekaran, et. al., 2006]

level - 1 (applied)

without delay



with delay



Rank-minimization

Nuclear norm-minimization

Factorized formulation ("*SVD-free*")

Rank-minimization

$$\min_{\mathbf{X}} \quad \underbrace{\operatorname{rank}(\mathbf{X})} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

Rank-minimization

$$\min_{\mathbf{X}} \quad \underbrace{\operatorname{rank}(\mathbf{X})} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

for blended acquisition:

unblended data matrix

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} \begin{matrix} \longleftarrow \text{source 1} \\ \longleftarrow \text{source 2} \end{matrix}$$

$\mathbf{b}$: blended data

$$\mathcal{A} := \begin{bmatrix} \mathbf{M}\mathbf{S}^{\mathbf{H}} & \mathbf{M}\mathbf{T}\mathbf{S}^{\mathbf{H}} \end{bmatrix}$$

↑ time delay matrix

Rank-minimization

expensive
(search over all possible values of rank)

$$\min_{\mathbf{X}} \quad \underbrace{\mathrm{rank}(\mathbf{X})} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

Rank-minimization

expensive
(search over all possible values of rank)

$$\min_{\mathbf{X}} \quad \underbrace{\mathrm{rank}(\mathbf{X})} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

Nuclear norm-minimization

convex relaxation of rank-minimization

[Recht, et. al., 2010]

$$\min_{\mathbf{X}} \quad \underbrace{||\mathbf{X}||_*} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

sum of singular values of $\mathbf{X}$

Rank-minimization

expensive
(search over all possible values of rank)

$$\min_{\mathbf{X}} \quad \underbrace{\text{rank}(\mathbf{X})} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

number of singular values of $\mathbf{X}$

Nuclear norm-minimization

convex relaxation of rank-minimization

[Recht, et. al., 2010]

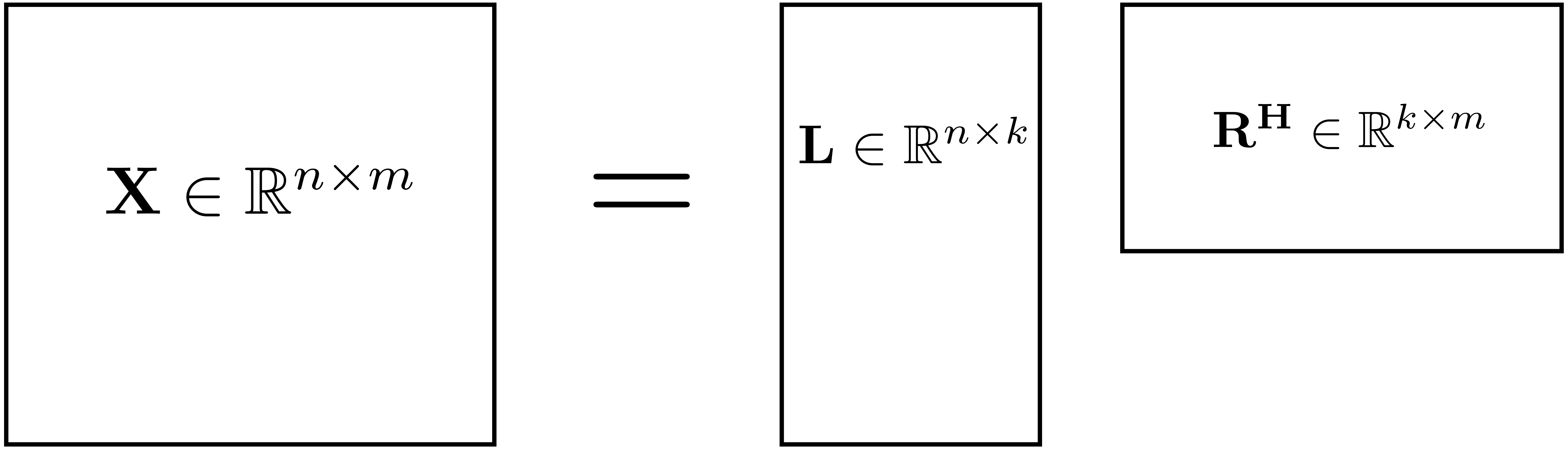
$$\min_{\mathbf{X}} \quad \underbrace{||\mathbf{X}||_*} \quad \text{s.t.} \quad \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2 \leq \epsilon$$

sum of singular values of $\mathbf{X}$

however ...
requires repeated application of SVD

Factorized formulation ("*SVD-free*")

[Rennie and Srebro, 2005; Lee et. al., 2010; Recht and Re, 2011]



Upper-bound on nuclear norm:

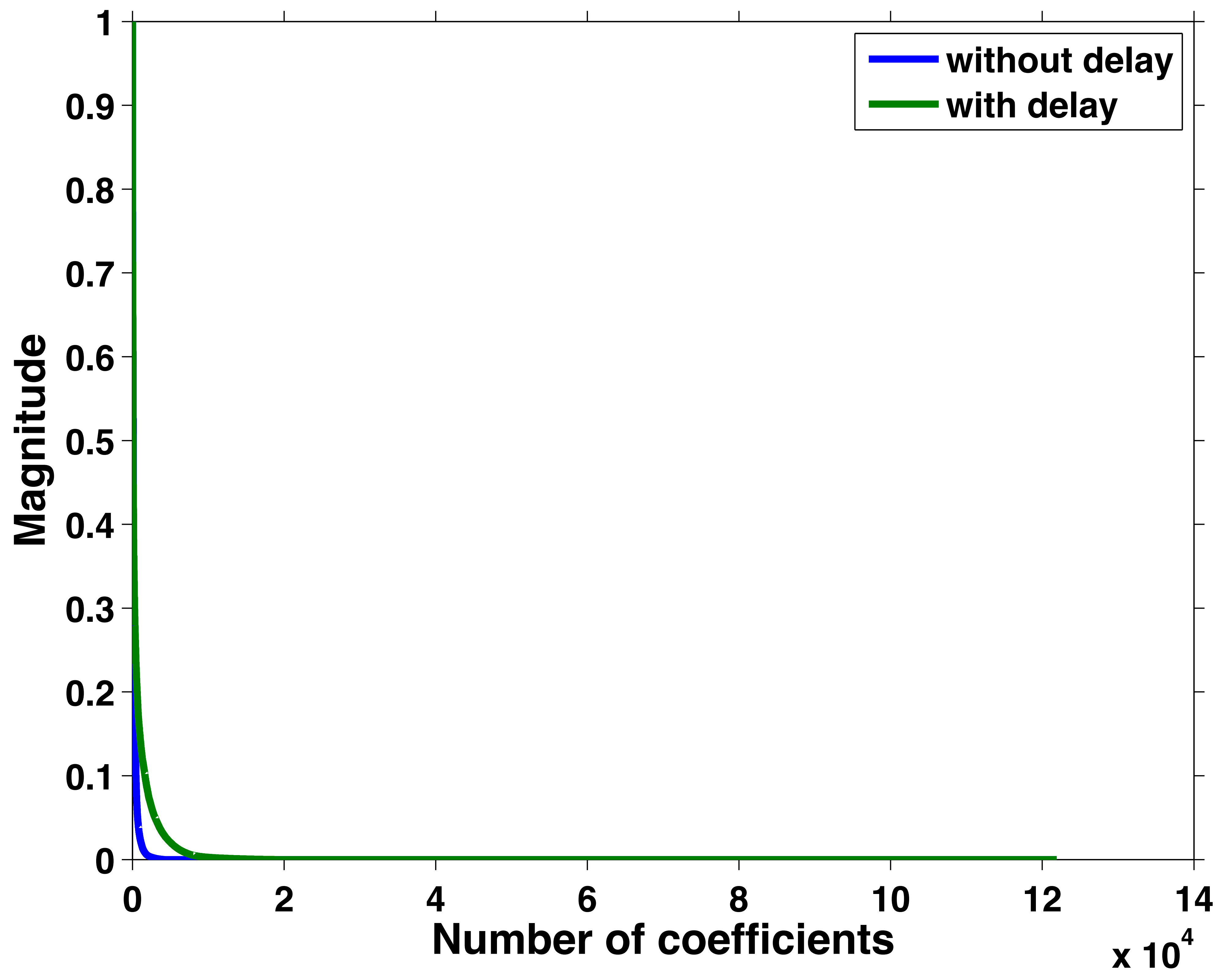
$$\|\mathbf{X}\|_* \leq \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}_1\\ \mathbf{R}_1\end{bmatrix}\right\|_F^2 + \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}_2\\ \mathbf{R}_2\end{bmatrix}\right\|_F^2 =: \Phi(\mathbf{L}_1, \mathbf{R}_1, \mathbf{L}_2, \mathbf{R}_2)$$

Sparsity-promotion

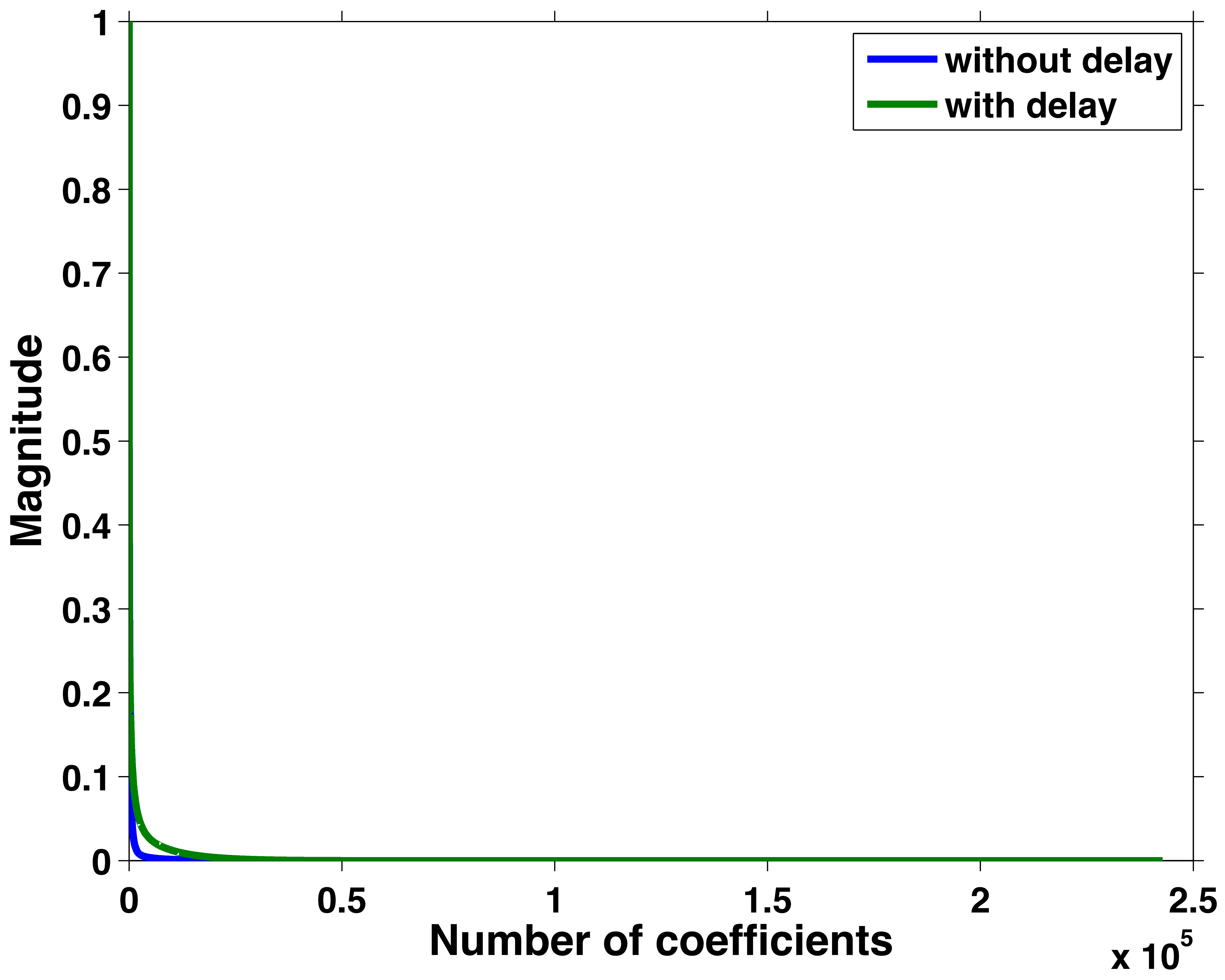
one norm-minimization

Decay of curvelet coefficients

source-receiver domain



midpoint-offset domain

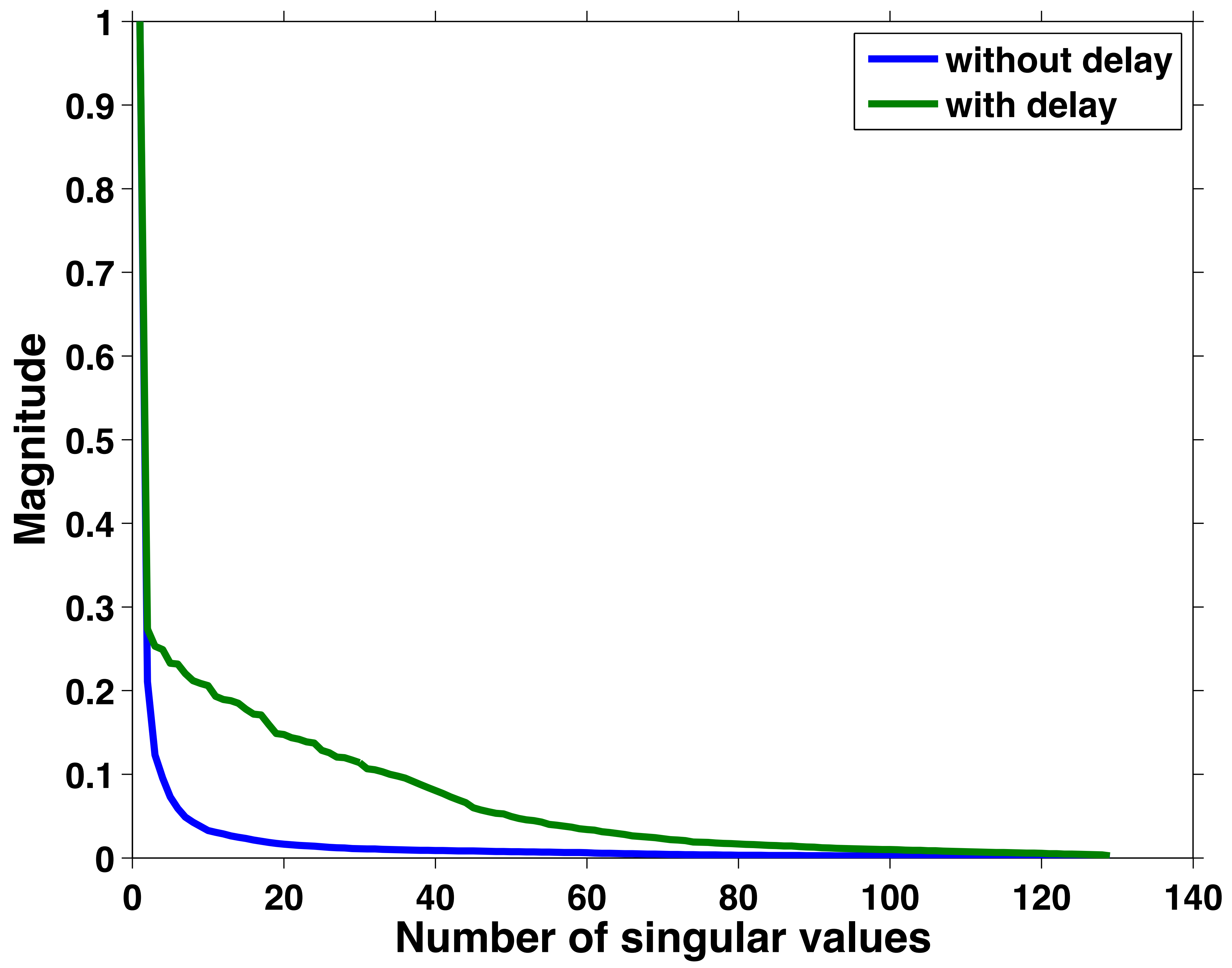


Source separation results

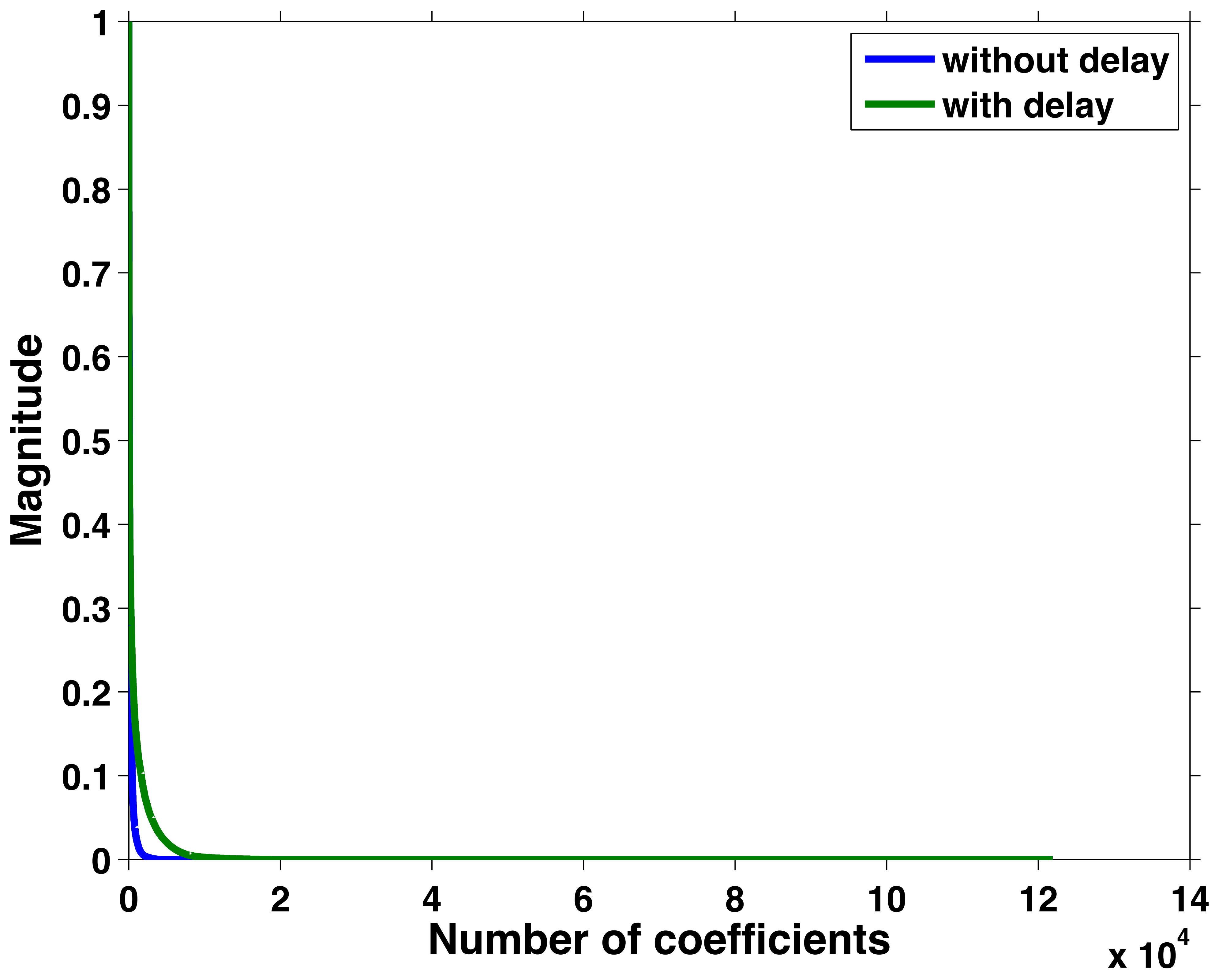
Rank-minimization vs. sparsity-promotion

Rank vs. sparsity

rank-minimization
(midpoint-offset domain)



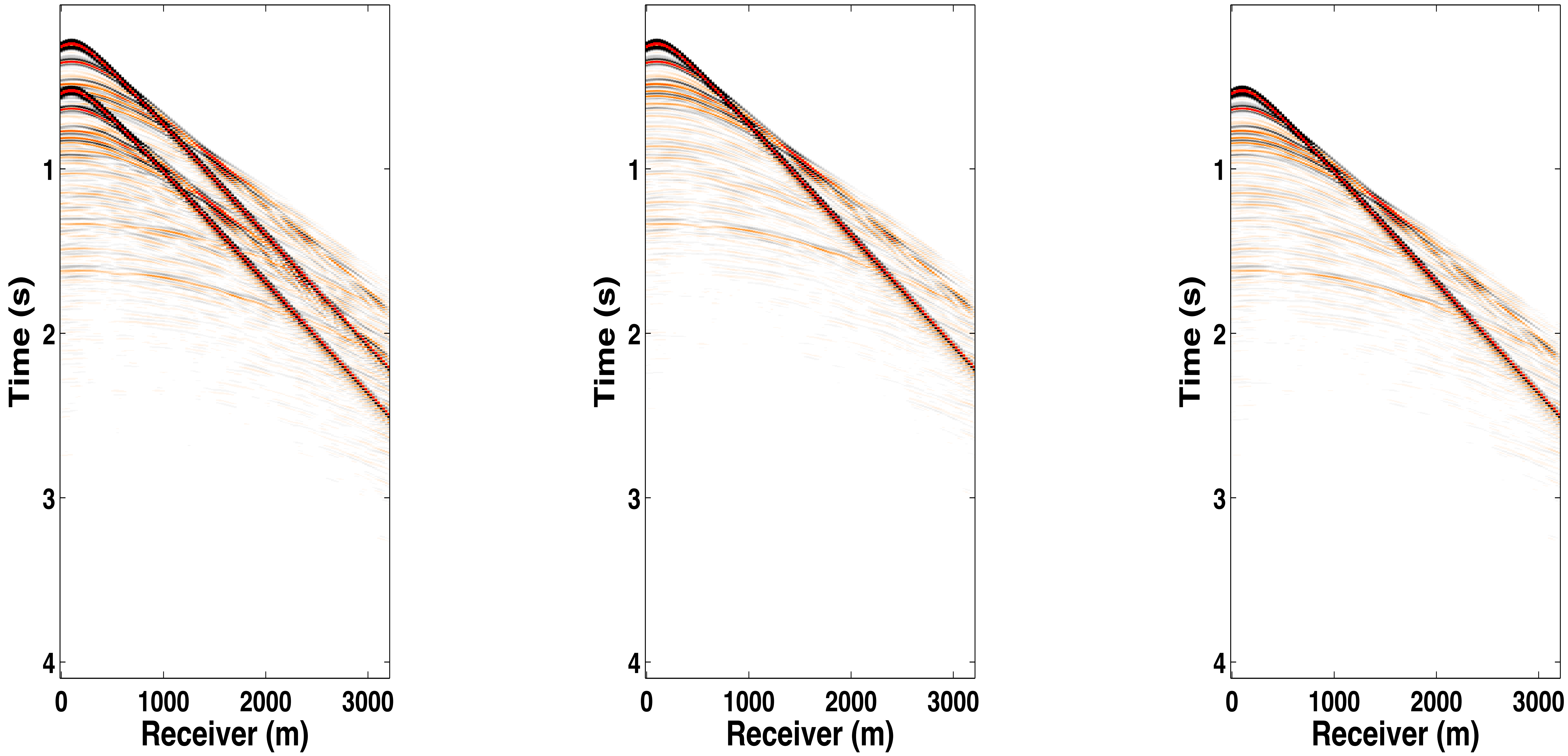
sparsity-promotion
(source-receiver domain)



Blended data (w/ delay)

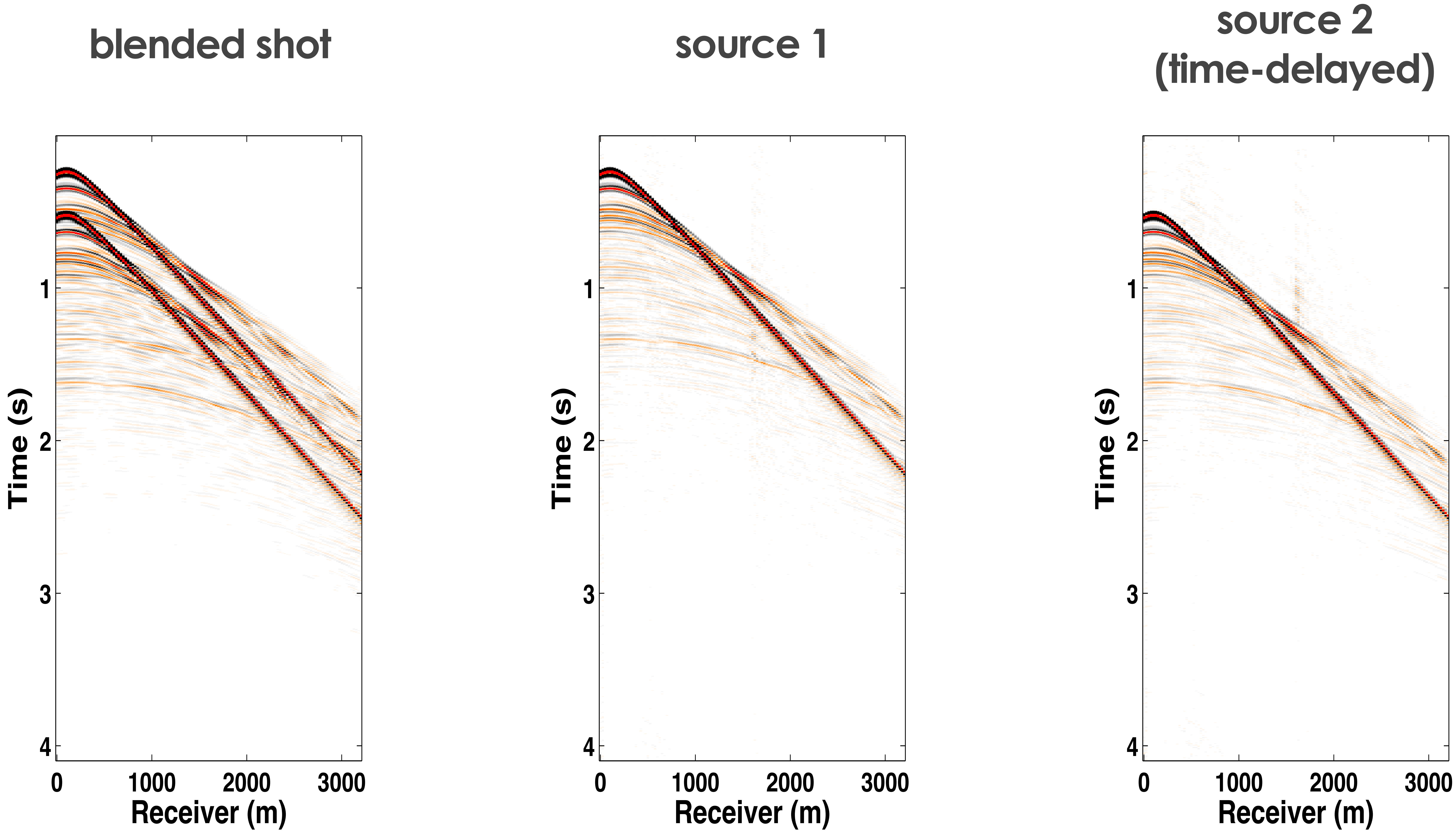
[random time delays applied to source 2]

blended shot = source 1 + source 2 (time-delayed)



Source separation via *rank*-minimization

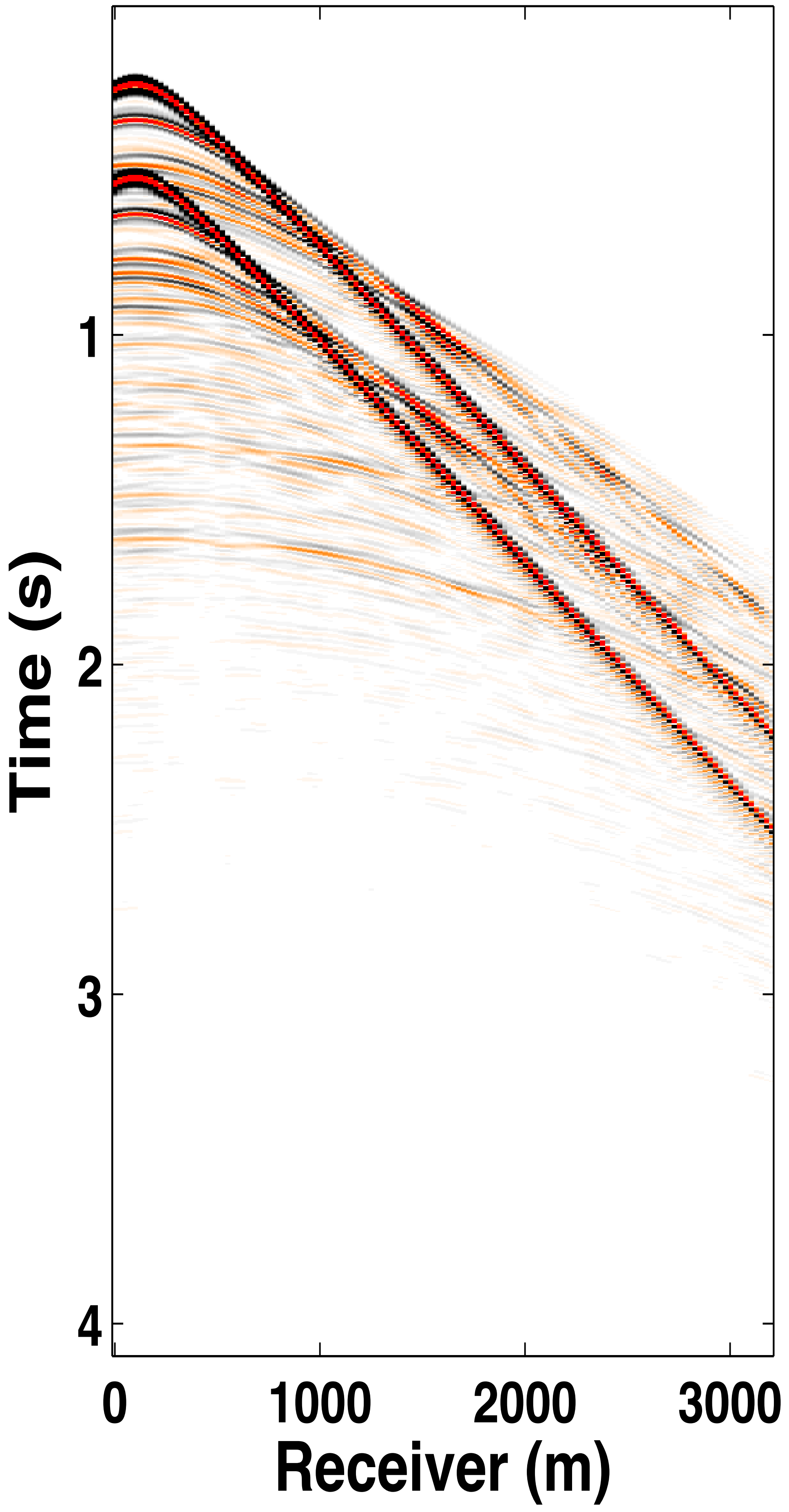
[computation time = 4 x 10 hours]



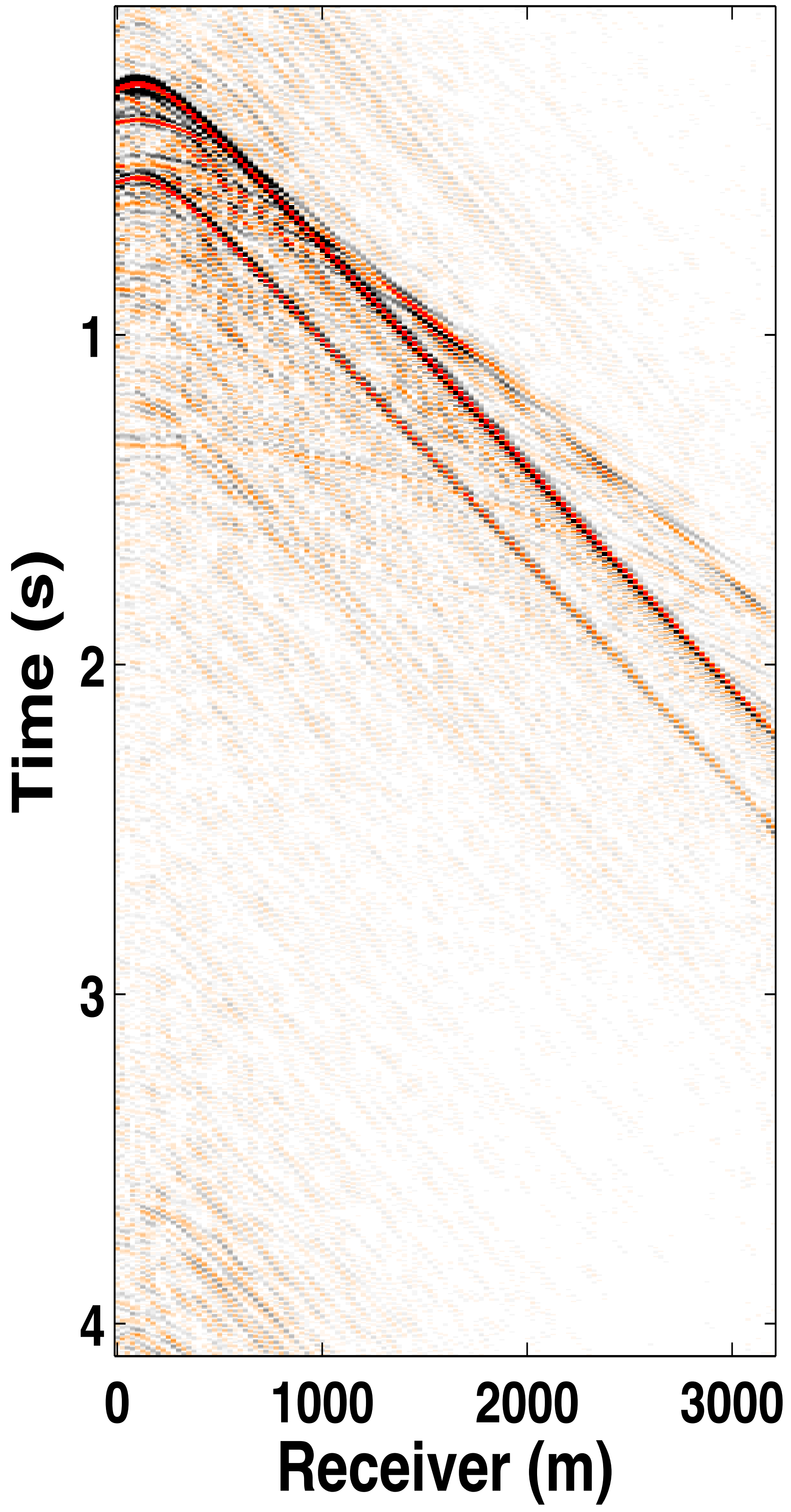
Source separation via *sparsity*-promotion

[computation time = 4 x 25 hours]

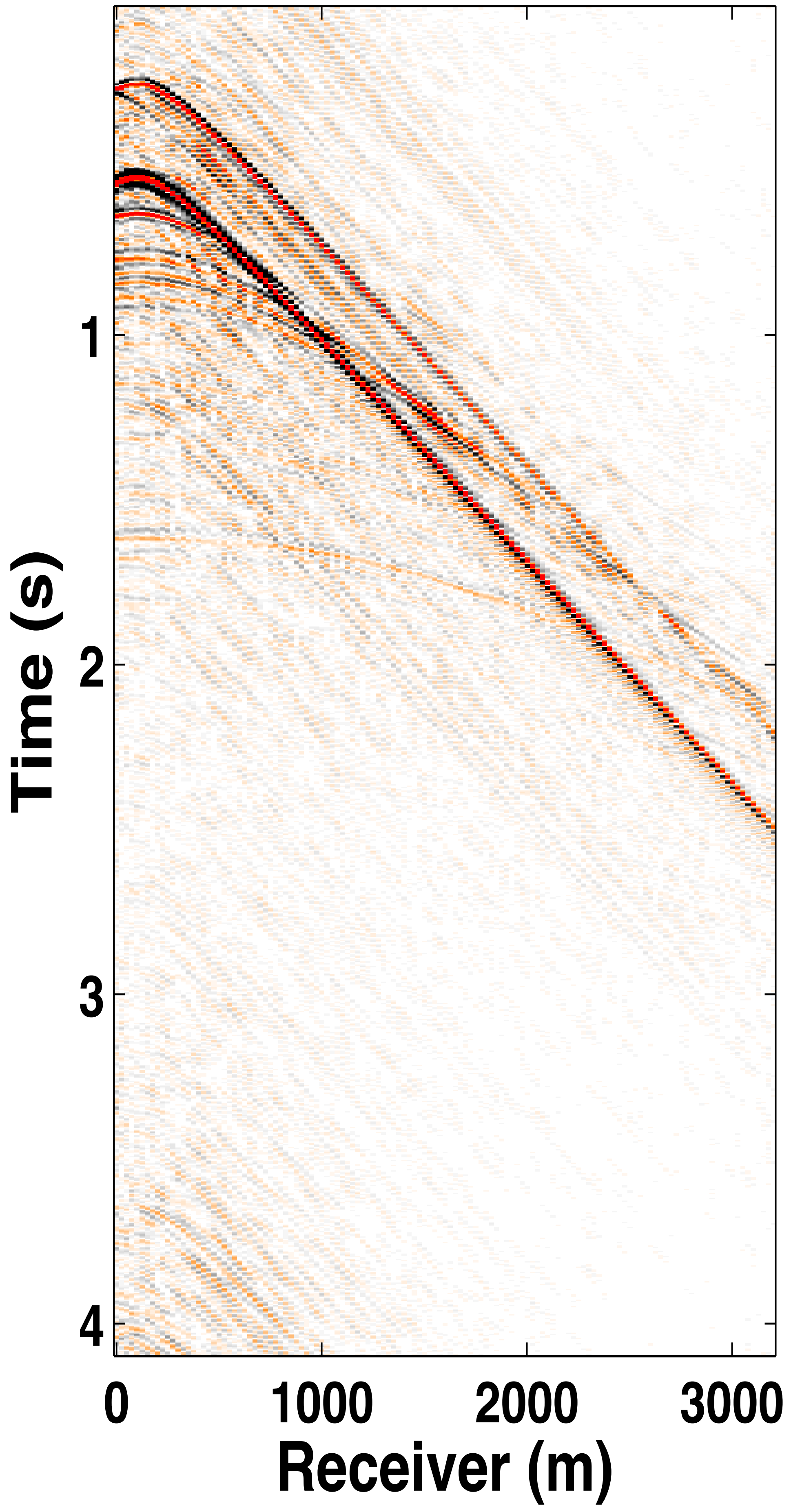
blended shot



source 1



source 2
(time-delayed)



Observations

- Source separation for *low variability* acquisition scenarios can be treated as a *rank-minimization* problem
  - towed-array (streamer) acquisition
- Small variability in shot-times does *not* seem *desirable* for source separation via *sparsity-promotion*

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