Fast linearized inversion with surface-related multiples

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1 Summary

In the well-known SRME relation, multiples are expressed as the multi-dimensional convolution between the subsurface Green's function and the downgoing receiver wavefield. Therefore these multiples can be considered as the response of the subsurface to an "areal" source term given by the same downgoing receiver wavefield. This relation can be used to treat these multiples as signals instead of considering them as noise that must removed before seismic imaging. However, when we use conventional reverse-time migration to image these multiple events, it results in acausal imaging artifacts caused by cross-correlations between wrong pairs of up- and downgoing wavefields. We find that these artifacts can be removed by adopting a sparsity-promoting least-squares inversion approach. However, iterative inversions go at the expense of excessive computational costs. By combining the SRME relation and wave-equation based linearized modelling, we are able to significantly reduce the costs by avoiding the dense matrix-matrix multiplications of SRME and the number of wave-equation solves via source subsampling. As a result, we arrive at a method with a cost comparable to that of a single reverse-time migration with all shots.

2 Method

After combining the SRME relation and the linearized wave-equation based forward modelling [Verschuur et al., 1992], we arrive the following expression in the canonical form of a linear system of equation $\mathbf{b} = \mathbf{A}\mathbf{x}$, i.e.,

$$\mathbf{p} \approx \nabla\mathbf{F}[\mathbf{m}_0, \mathbf{q}]\delta\mathbf{m}, \tag{1}$$

where $\mathbf{p}$ is the vectorized upgoing wavefield at the surface, and $\mathbf{q}$ is the *corresponding* downgoing wavefield at the surface. By "corresponding", we mean when $\mathbf{p}$ is the *total* upgoing wavefield, $\mathbf{q}$ is the *total* downgoing wavefield consisting of the point sources and the downgoing receiver wavefield; or when $\mathbf{p}$ is the upgoing *multiple* wavefield, $\mathbf{q}$ is the downgoing *receiver* wavefield. One can choose whether the total upgoing data or only the multiples are to be imaged (the former needs the knowledge of the source wavelet while ther latter needs separated multiple and primary wavefields). The linear operator $\nabla\mathbf{F}$ represents the linearized demigration (i.e., Born modelling) operator for a given background velocity model $\mathbf{m}_0$, and $\delta\mathbf{m}$ is the model perturbations in the squared slowness. We use the $\approx$ symbol to indicate that we ignore the higher-order scattering events such as internal multiples.

The above system of equations (cf. Equation (1)) is tall, i.e., multiple source experiments yield more number of equations than unknowns. This redundancy in the data allows us to subsample the shots and therefore reduce the computational cost that scales linearly with the number of sources. To "subsample", we form fewer randomized source aggregates from all sequential sources, and randomly select a subset of frequencies [see Herrmann and Li, 2012, for more details]. As a result, to effectively solve Equation (1), we solve a series of the following LASSO subproblems:

$$\begin{aligned} &\underset{\mathbf{x}}{\text{minimize}} \; \|\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{q}}]\mathbf{C}^{\mathrm{H}}\mathbf{x}\| \\ &\text{subject to.} \quad \|\mathbf{x}\|_1 \leq \tau, \end{aligned} \tag{2}$$

for a gradually relaxed sparsity parameter τ. We also draw new subsets of randomized source experiments and randomized frequencies after each LASSO subproblem is solved, to speed up the convergence of the inversion algorithm. In this expression, the underlined variables indicate randomly *subsampled* quantities. The matrix $\mathbf{C}^{\mathrm{H}}$ represents the curvelet synthesis operator. Following van den Berg and Friedlander [2008], we update the ℓ_1 constraint τ using the Newton's method on the Pareto trade-off curve between the ℓ_2-norm of the data residual and the ℓ_1-norm of the solution vector. We can also include source estimation into the above inversion process by using variable projection [Tu et al., 2013].

3 Results

We show some synthetic examples using a sedimentary part of the Sigsbee 2B model. We simulate "observed" data using iWave with a free-surface boundary condition, and invert the data set using our in-house frequency-domain modelling engine. Data preprocessing, such as receiver deghosting and data extrapolation from receivers to the surface, has been applied. The results are shown in Figure (1).

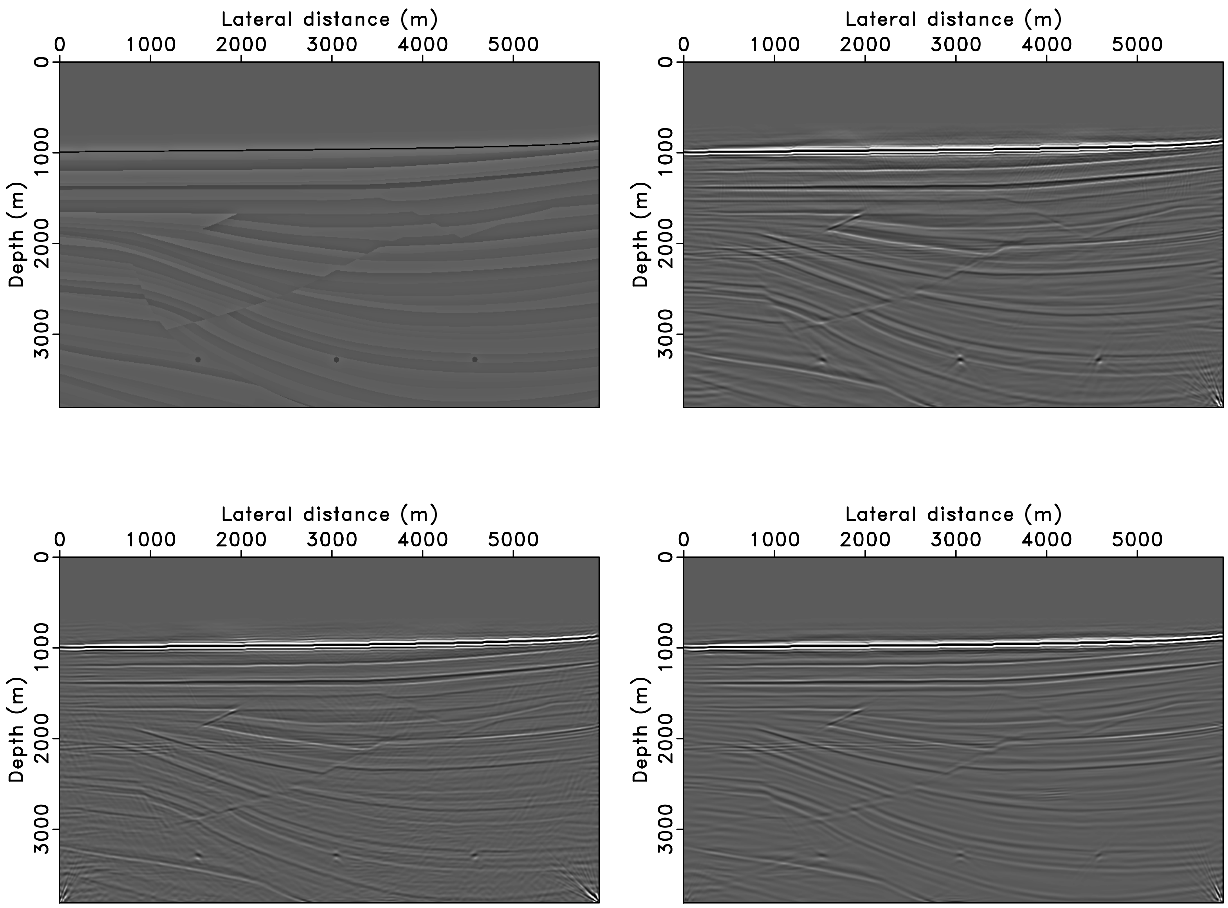

Figure 1: (a) True perturbation. (b) and (c) Imaging of both primaries and multiples simultaneously, with true source wavelet (b) and with source estimation (c). (d) Imaging of multiples only.

References

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