Limitations of the **deconvolutional** imaging condition for **two-way** propagators

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Muijs et. al. 2007
Whitmore et. al. 2010
Lu et. al., 2011
Poole et. al. 2010

Motivation

When used to image surface-related multiples, the deconvolutional imaging condition *alone* is incapable of eliminating all the coherent artifacts arising from multiples, especially for complex geologies.

Shin et. al. 2001
Gao et. al. 2012

Motivation

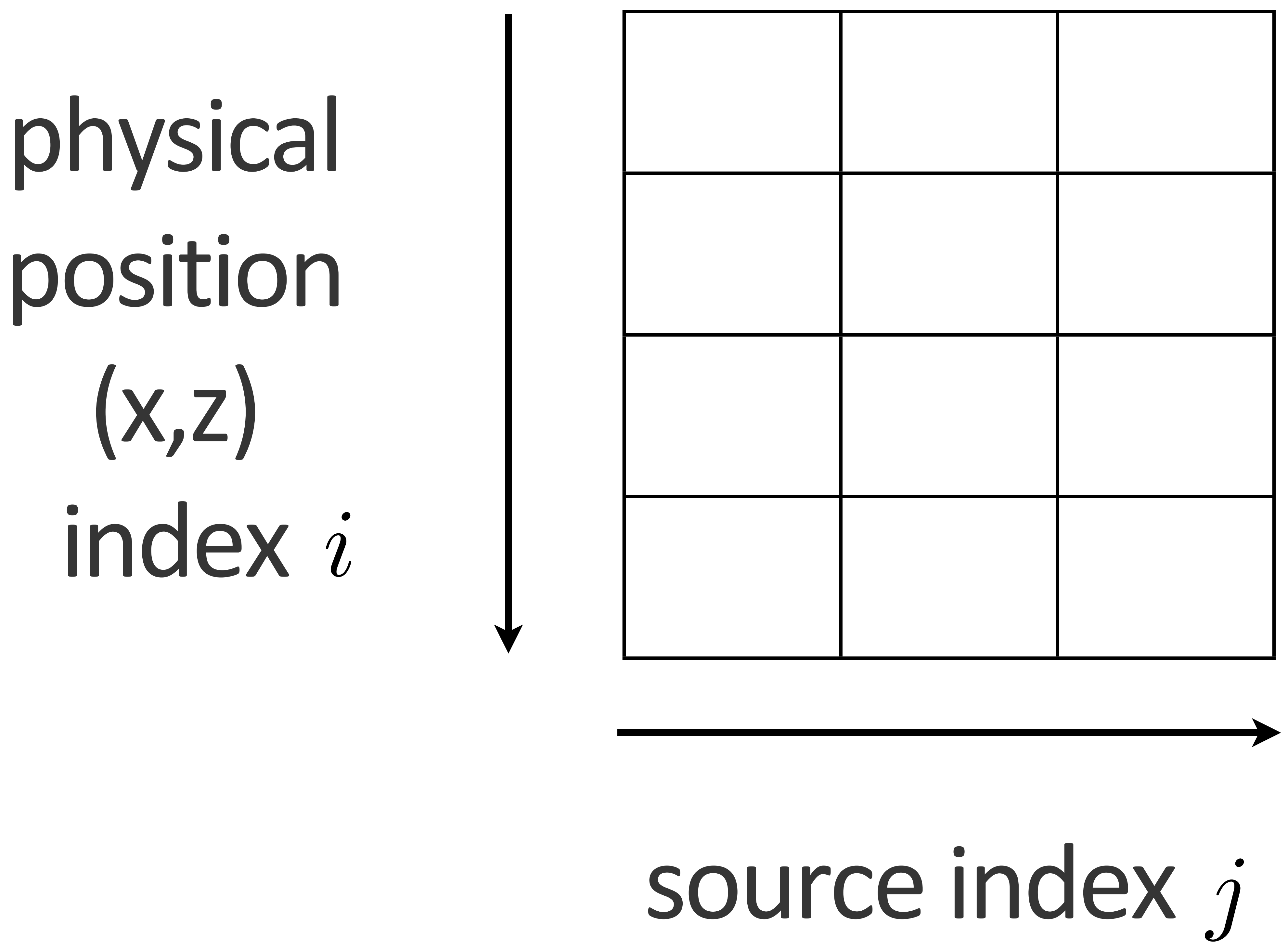
The inverse of the “pseudo-Hessian” is reminiscent of the one-way deconvolutional imaging condition, but with *two-way* propagators.

Two-way vs. one-way: a comparison of *concepts*

	Two-way	One-way
What forms the image	Model perturbation	Reflectivity
Wavefields in the subsurface	**Forward** propagated field	**Downward** propagated field
	Backward propagated field	**Upward** propagated field

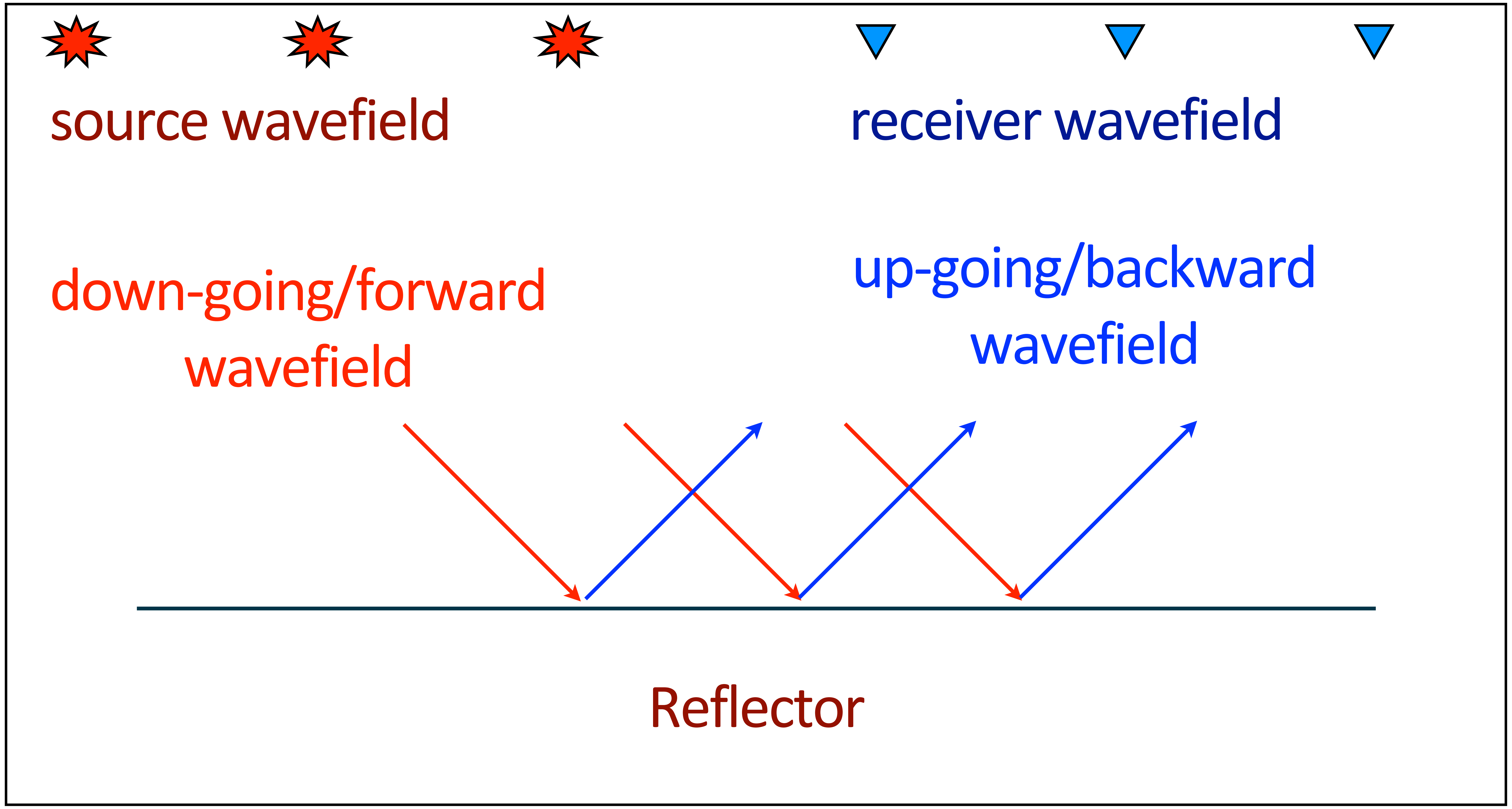
Data matrix notation

Each monochromatic data matrix of angular frequency ω :



Berkhout and Wapenaar 1993
Gray 1997

The deconvolutional imaging condition



Claerbout 1971
de Bruin et. al. 1990
Guitton et. al. 2007
Shin et. al. 2001
Gao et. al. 2012

Two-way vs. one-way: imaging conditions

	Two-way	One-way
Cross-correlation	$\mathbf{I}(\omega) = \mathrm{diag}(\mathbf{V}\mathbf{U}^*) = \sum_j \mathbf{v}_j \odot \overline{\mathbf{u}_j}$	
Deconvolution		$\mathbf{I}(\omega) = \sum_j \mathbf{u}_j \oslash \mathbf{d}_j \approx \sum_j \frac{\mathbf{u}_j \odot \overline{\mathbf{d}_j}}{\mathrm{diag}^{-1}(\mathbf{d}_j \odot \overline{\mathbf{d}_j} + \epsilon)}$

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Berkhout and Wapenaar 1993
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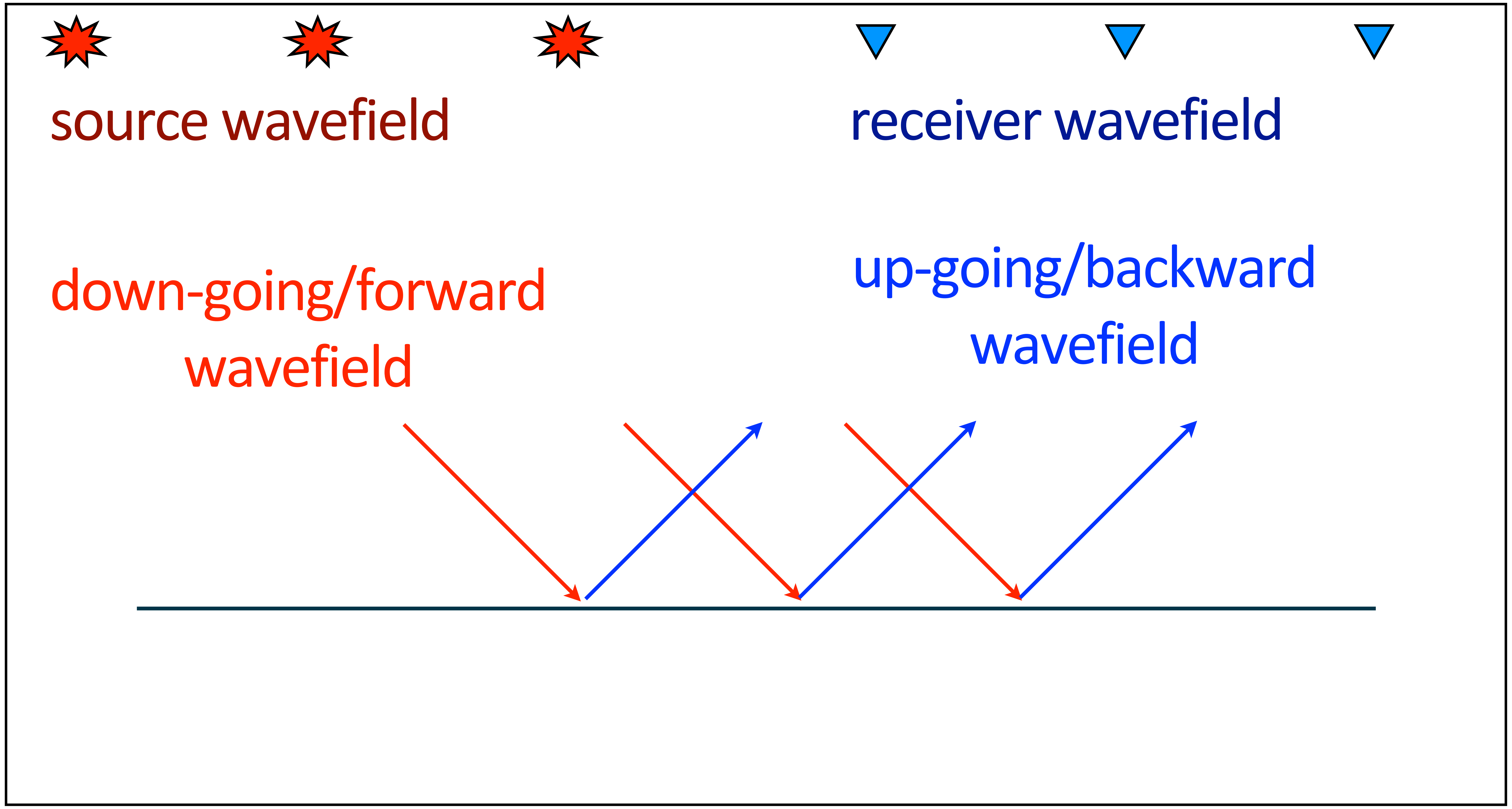
The deconvolutional imaging condition



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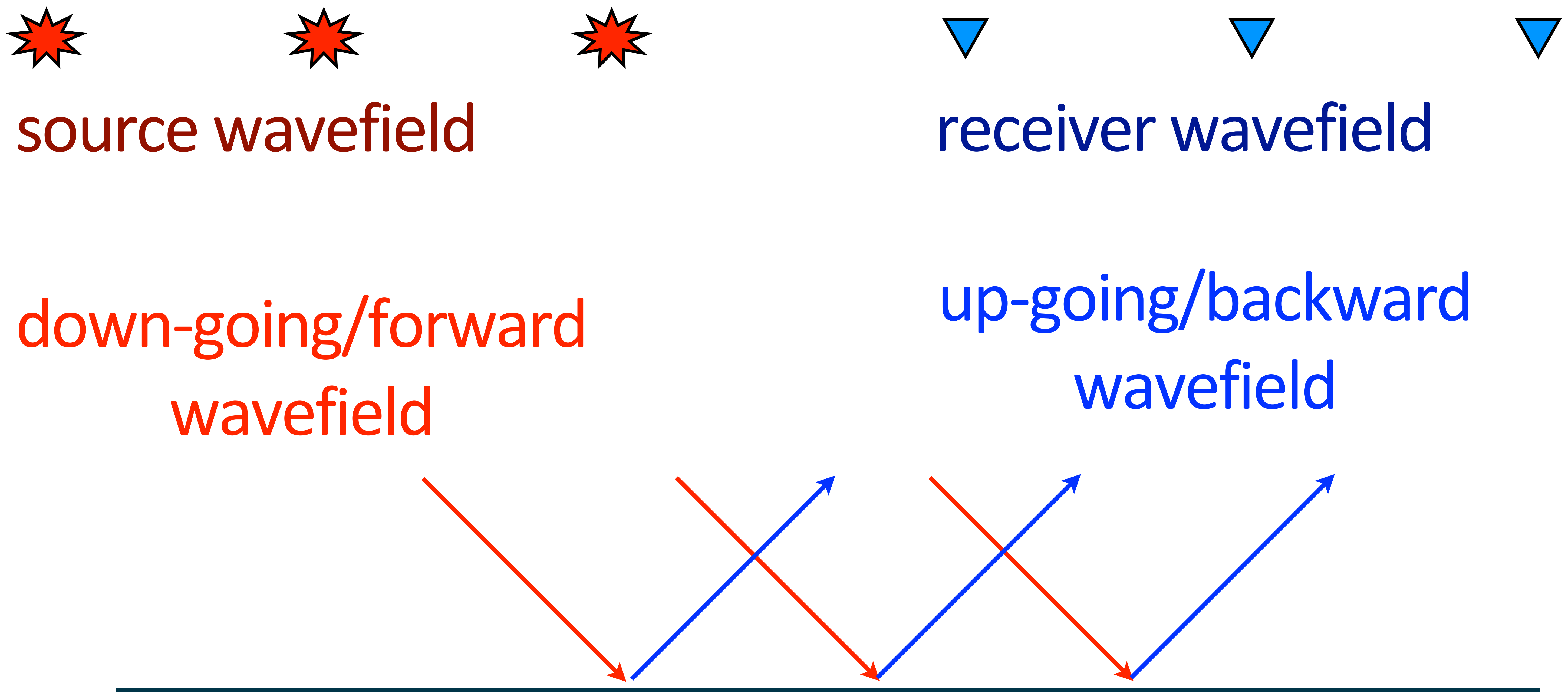
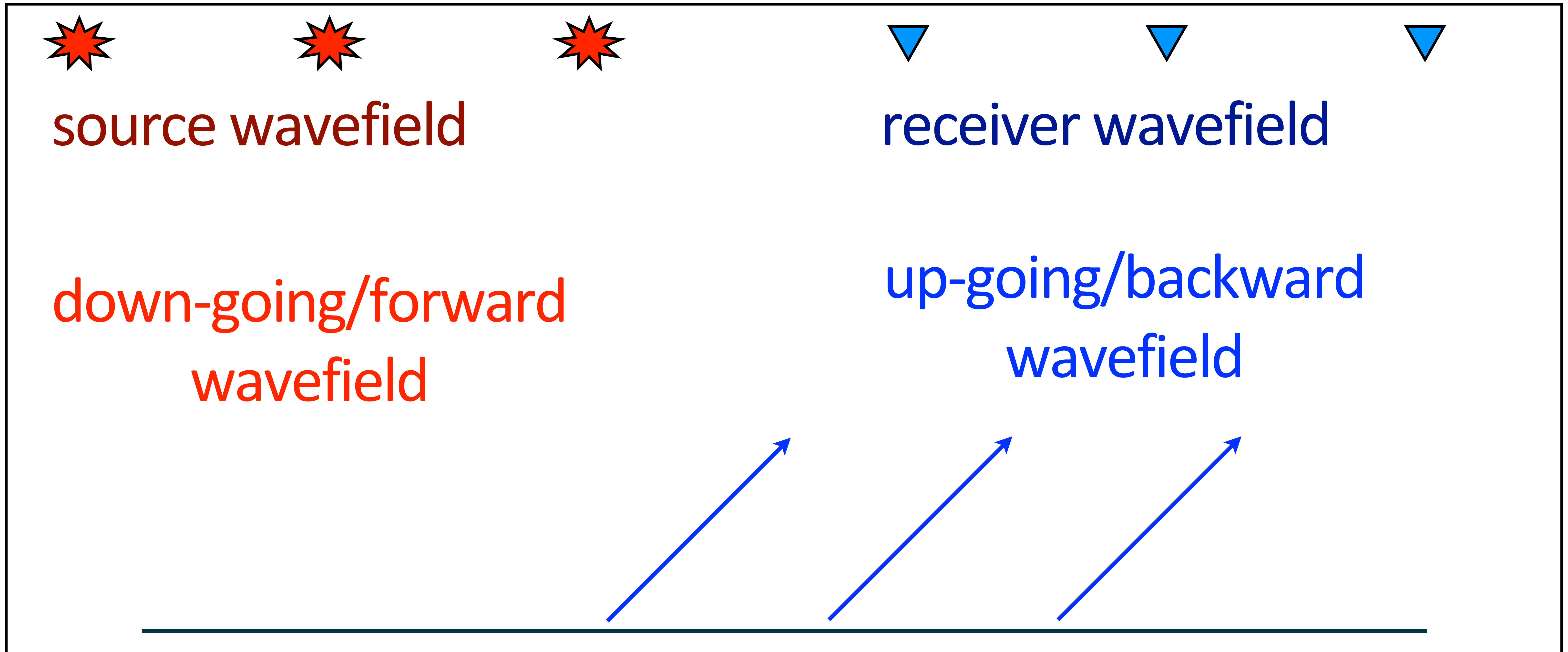
The deconvolutional imaging condition



$$\mathbf{I}(\omega) = \sum_j \mathbf{u}_j \oslash \mathbf{d}_j$$

Berkhout and Wapenaar 1993
Gray 1997

The deconvolutional imaging condition



$$\mathbf{I}(\omega) = \sum_j \mathbf{u}_j \oslash \mathbf{d}_j$$

Linearized inversion

$$\mathbf{LS}: \quad \underset{\delta \mathbf{m}}{\text{minimize}} \quad \|\nabla \mathbf{F}[\mathbf{m}_0, \mathbf{q}]\delta \mathbf{m} - \mathbf{d}\|_2,$$

$\delta \mathbf{m}$: model perturbation

$\nabla \mathbf{F}$: linearized two-way modelling operator

$\mathbf{m}_0$: background model

$\mathbf{q}$: *vectorized* source wavefields

$\mathbf{d}$: *vectorized* residual wavefields

Herrmann and Li, 2012
Candes et. al., 2006

Fast inversion w. sparsity promotion

$$\mathbf{BPDN}: \quad \text{minimize} \quad \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{q}}]\mathbf{C}^*\mathbf{x} - \underline{\mathbf{d}}\|_2 \leq \sigma$$

$\mathbf{C}$: Curvelet transform

σ : tolerance for noise/modelling error

Tu and Herrmann, 2012
Tu et. al., 2013

Alternative formulation

$$
\textbf{LASSO}: \quad \text{minimize} \quad \|\nabla \mathbf{F}[\mathbf{m}_0, \underline{\mathbf{q}}]\mathbf{C}^* \mathbf{x} - \underline{\mathbf{d}}\|_2 \\
\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau
$$

τ : sparsity level

Rerandomization:

For each LASSO subproblem, we draw:

- new randomized source aggregates
- new frequency subsets

Tu and Herrmann, 2012

Imaging (with) surface-related multiples

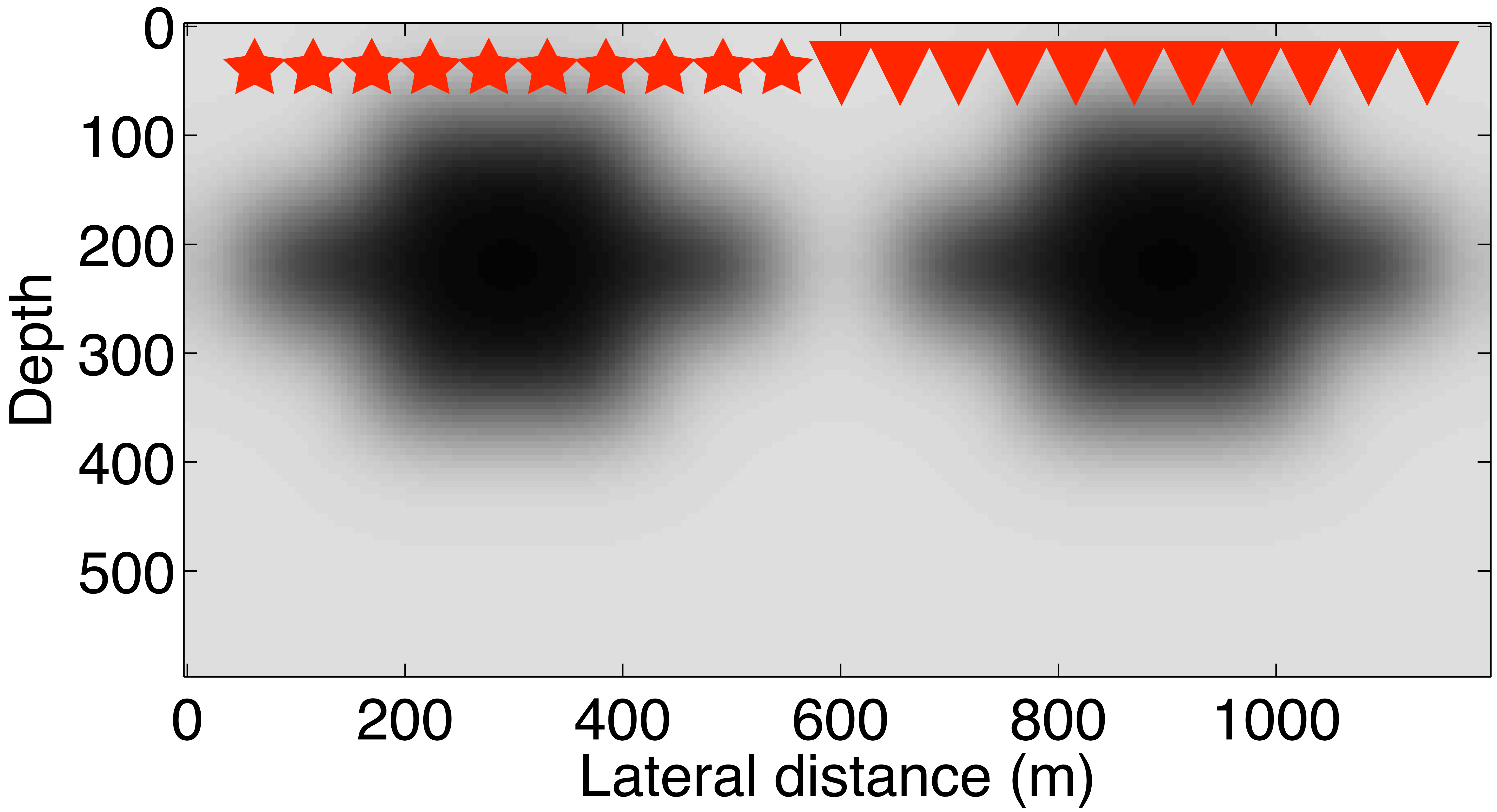
$$\textbf{LASSO}: \quad \text{minimize} \quad \|\nabla\mathbf{F}[\mathbf{m}_0, \underline{\mathbf{q}}]\mathbf{C}^*\mathbf{x} - \underline{\mathbf{d}}\|_2$$
$$\text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau$$

DATA	PRIMARY	MULTIPLE	TOTAL DATA
SOURCE WAVEFIELD	$q(\omega)\mathbf{I}$	$-\mathbf{D}(\omega)$	$q(\omega)\mathbf{I} - \mathbf{D}(\omega)$

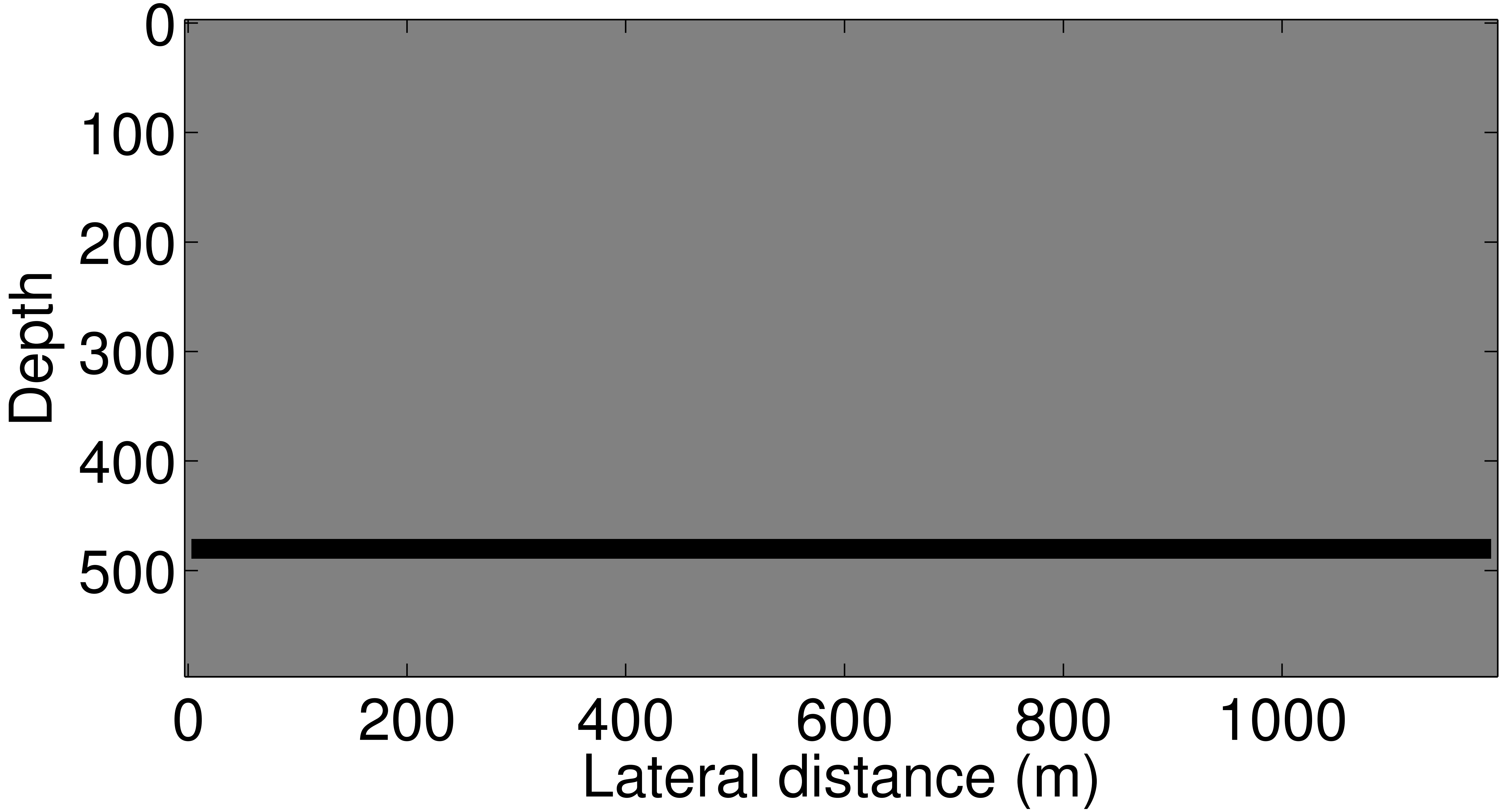
Stylized example

- background model has smooth velocity *anomalies symmetrically* on both the source and receiver side
- geometrically *symmetric* source (left) and receiver (right) distribution, for a total number of 50 each, with 12m spacing
- 2s recording time, 100 frequency samples between 0-50Hz
- linearized modelling, primary wavefield only

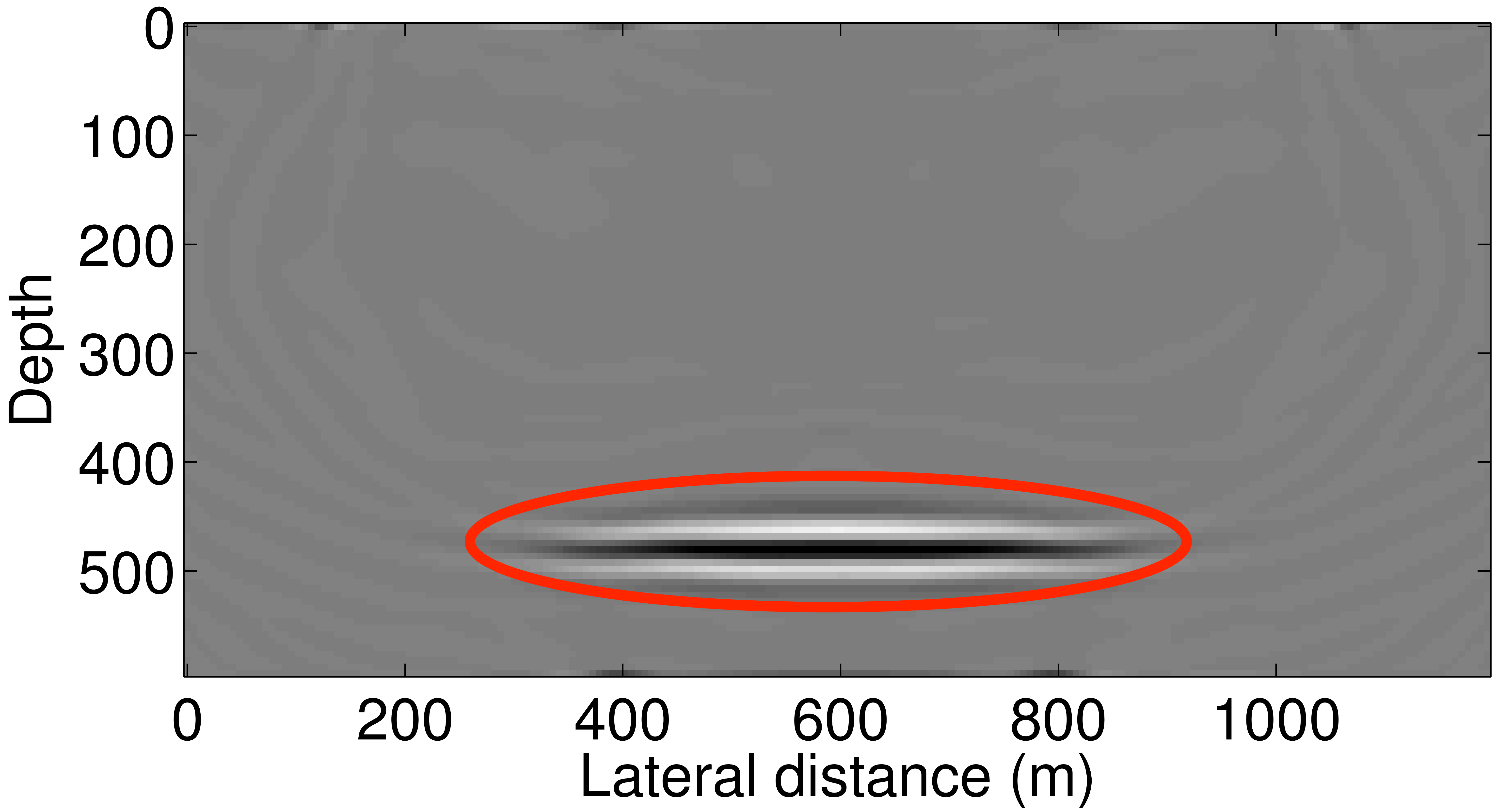
Background model



Model perturbation

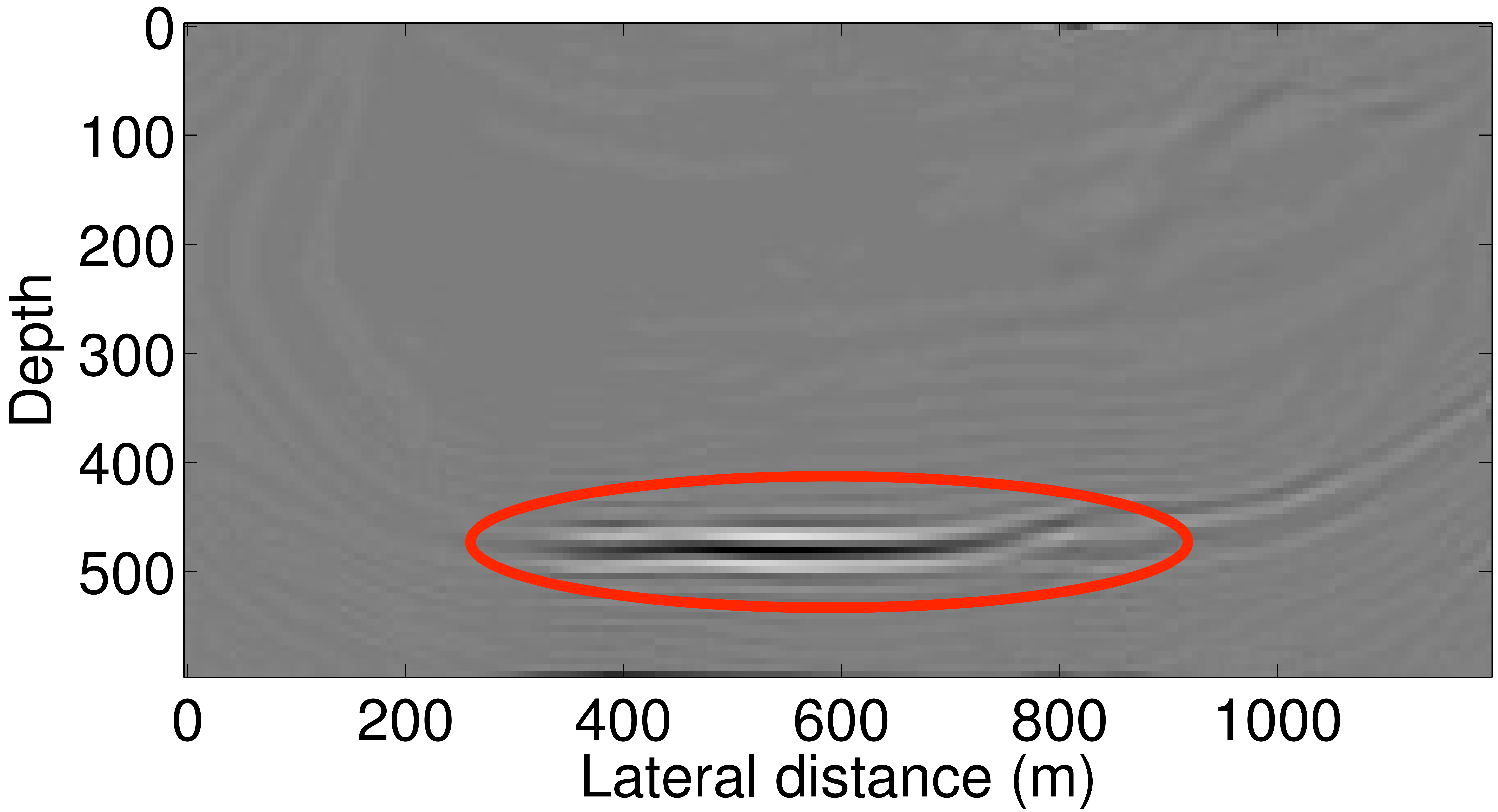


RTM image
[with all sources and frequencies]

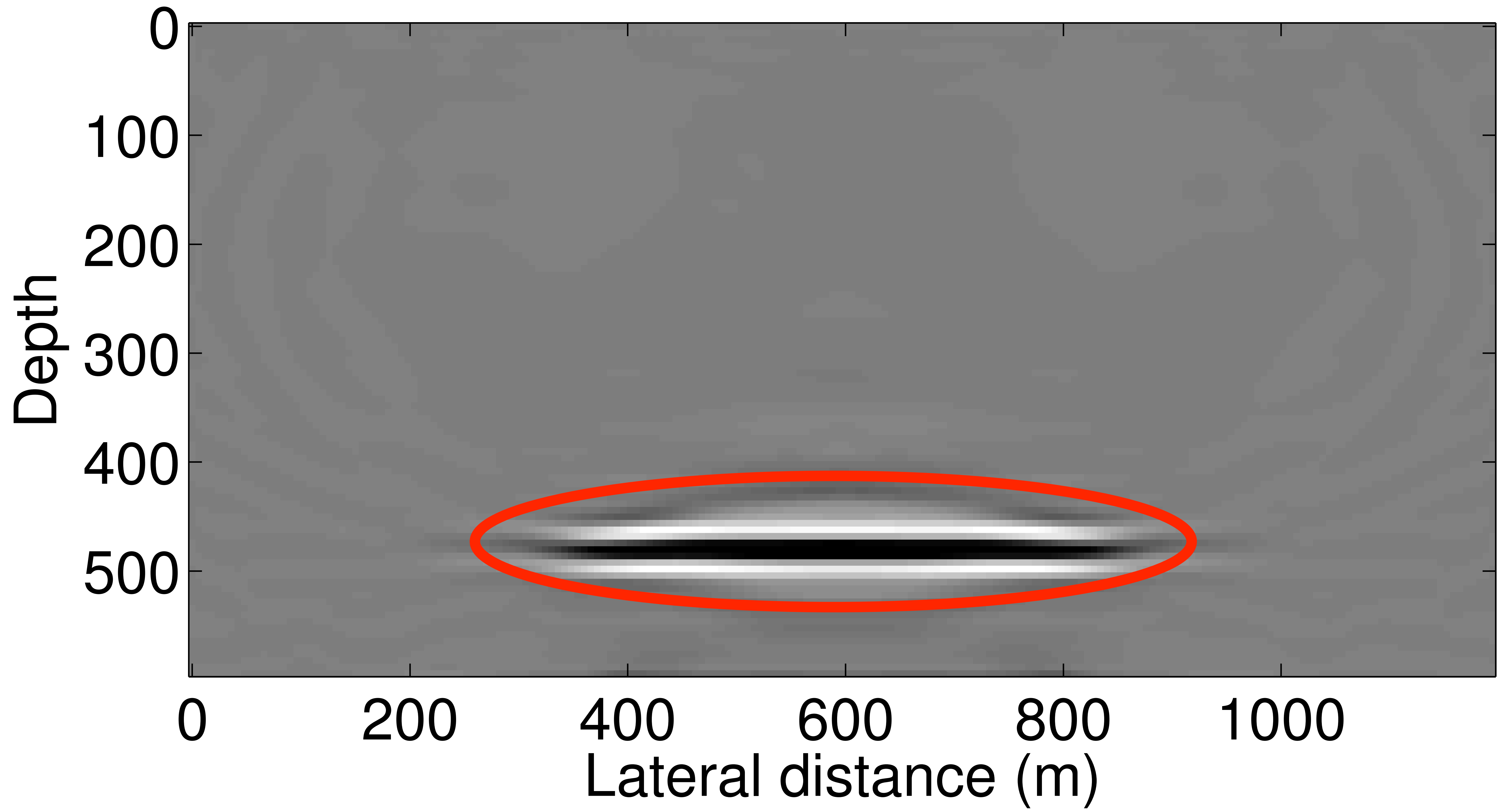


Weighted by inverse of *pseudo-Hessian*

[two-way deconvolutional imaging, w. damping stabilization]

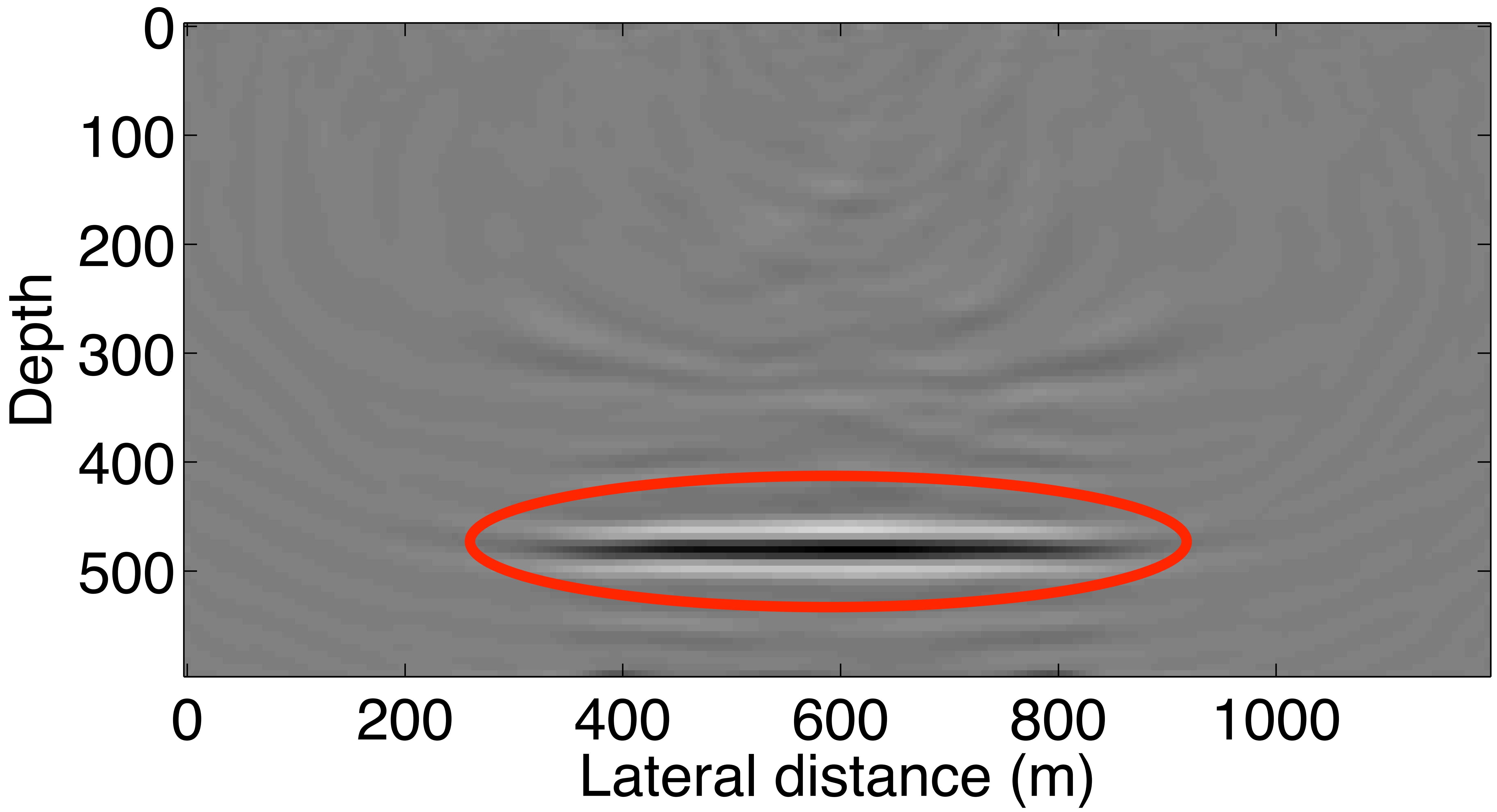


Inversion with sparsity-promotion
[with all 50 sources, 100 frequencies and 100 iterations]



Fast inversion with sparsity-promotion

[with 10 sim. sources, 10 frequencies, and 100 iterations, 50X speed-up]

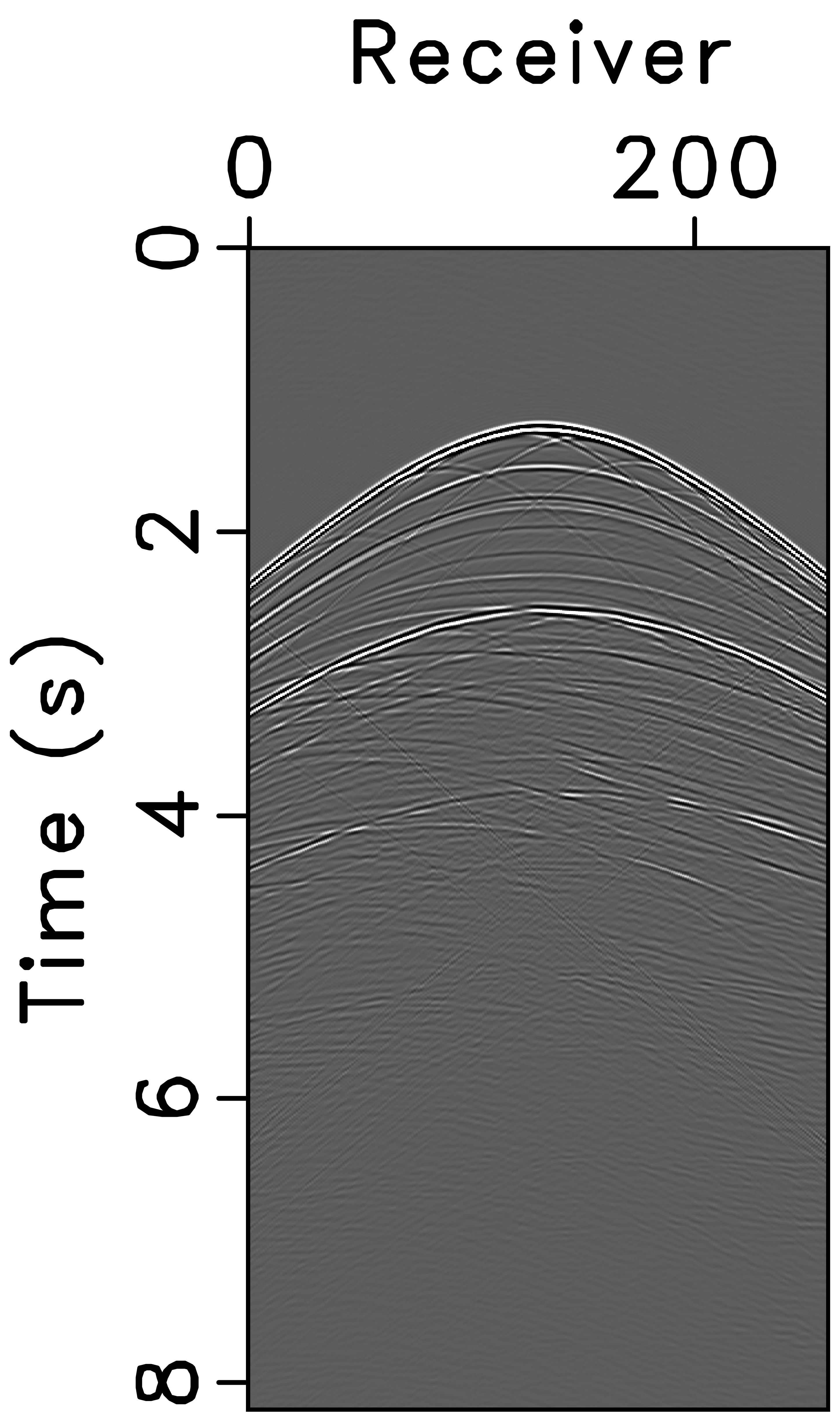


Imaging of multiples

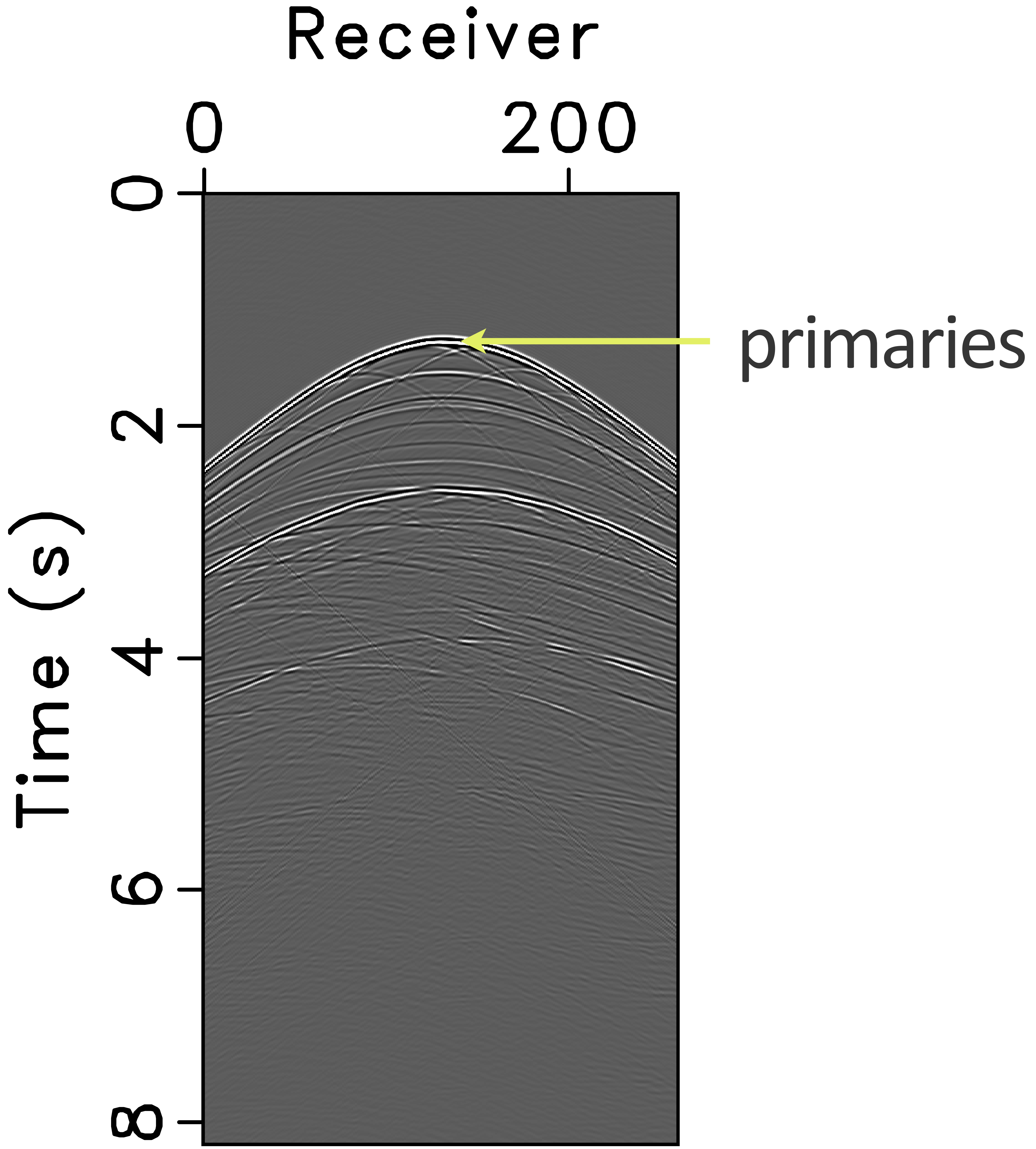
Example setup

- model cropped from the sedimentary part of the Sigsbee 2B model
- model grid spacing: 7.62m
- linearized modelling
- 261 sequential sources w. co-located receivers, fixed spread, 22.86m spacing
- ~8s recording time, 311 frequencies in 0-38Hz range
- imaging *multiples* only

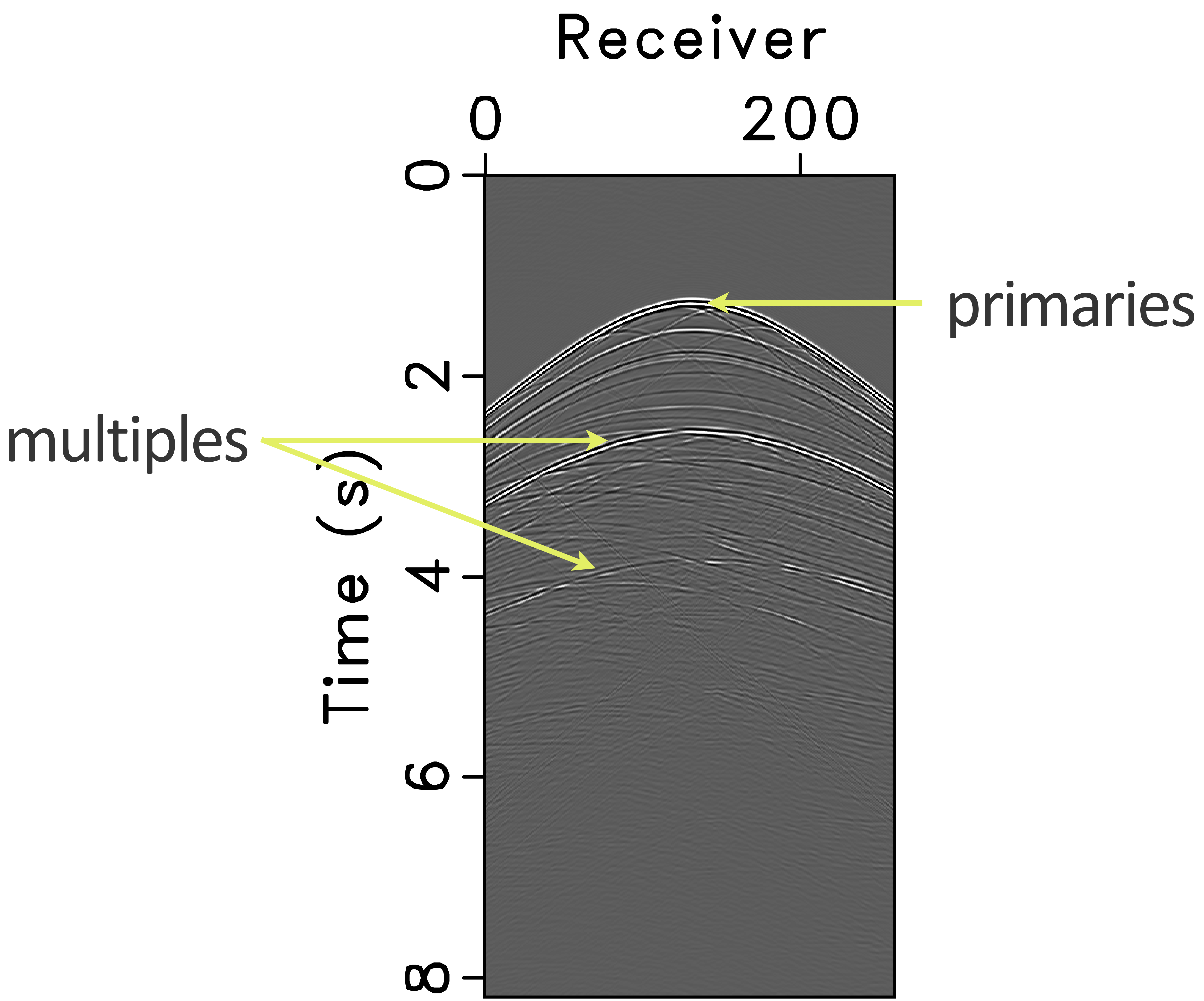
A shot-gather



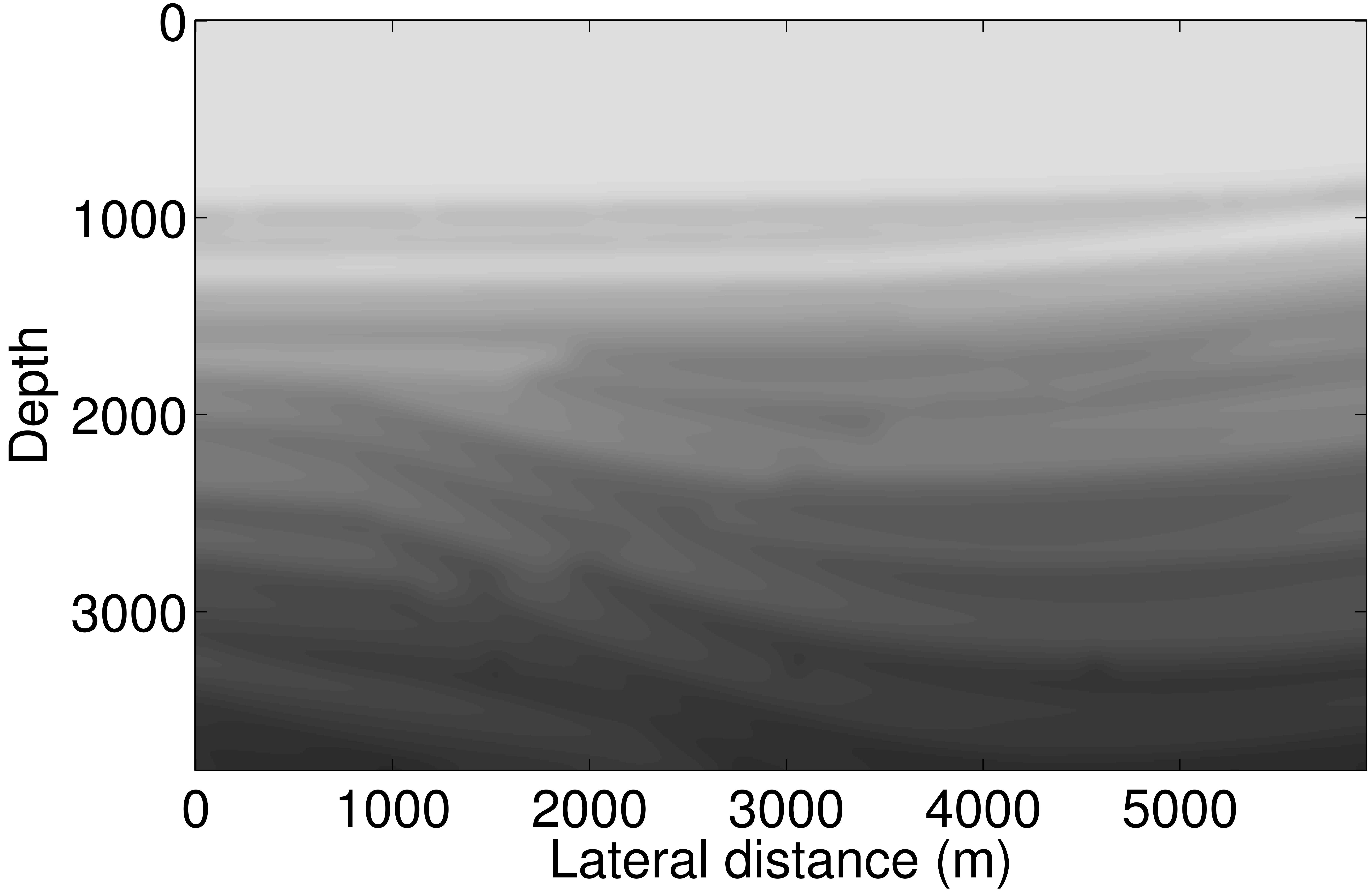
A shot-gather



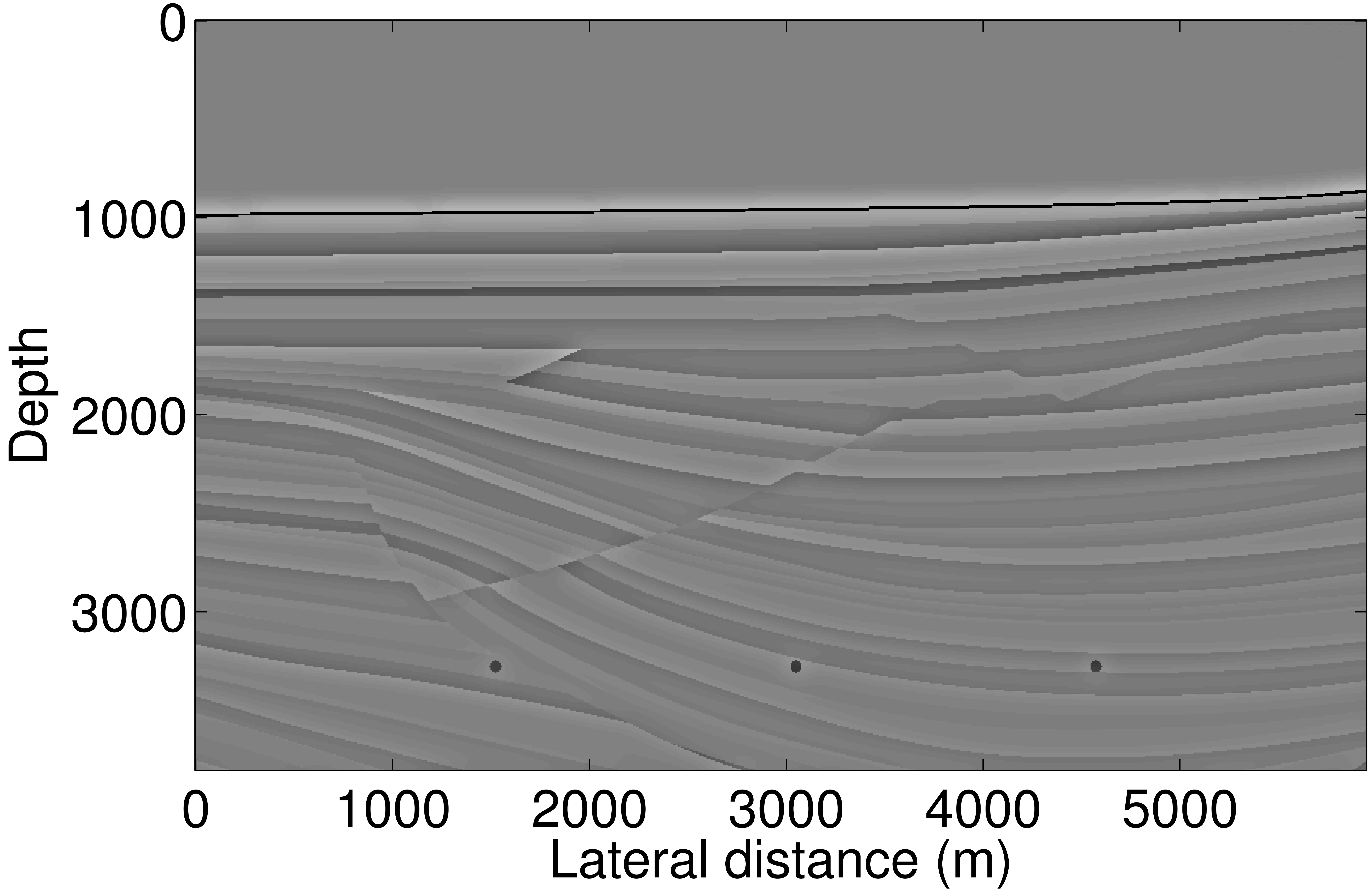
A shot-gather



Background model

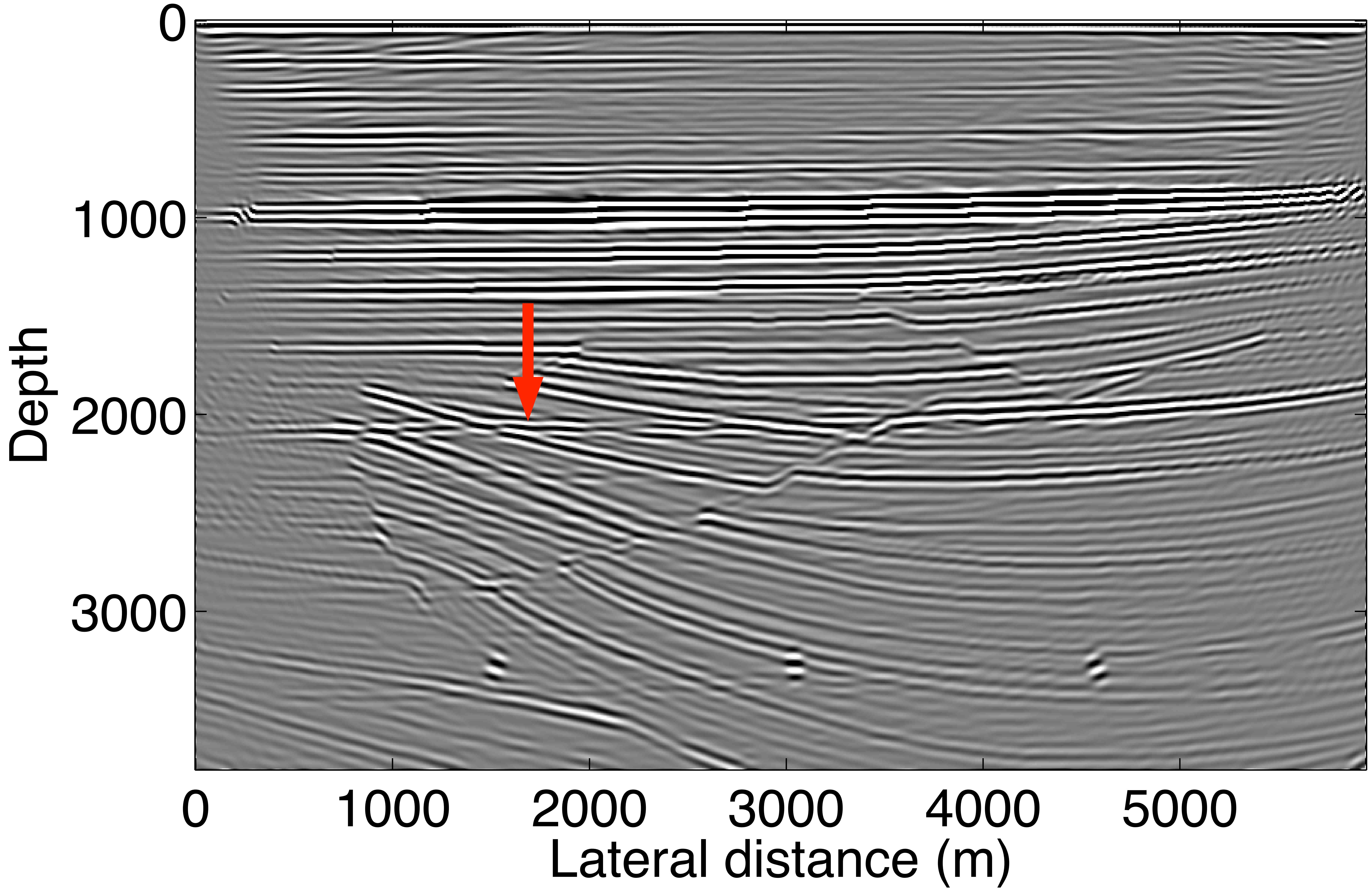


Model perturbation



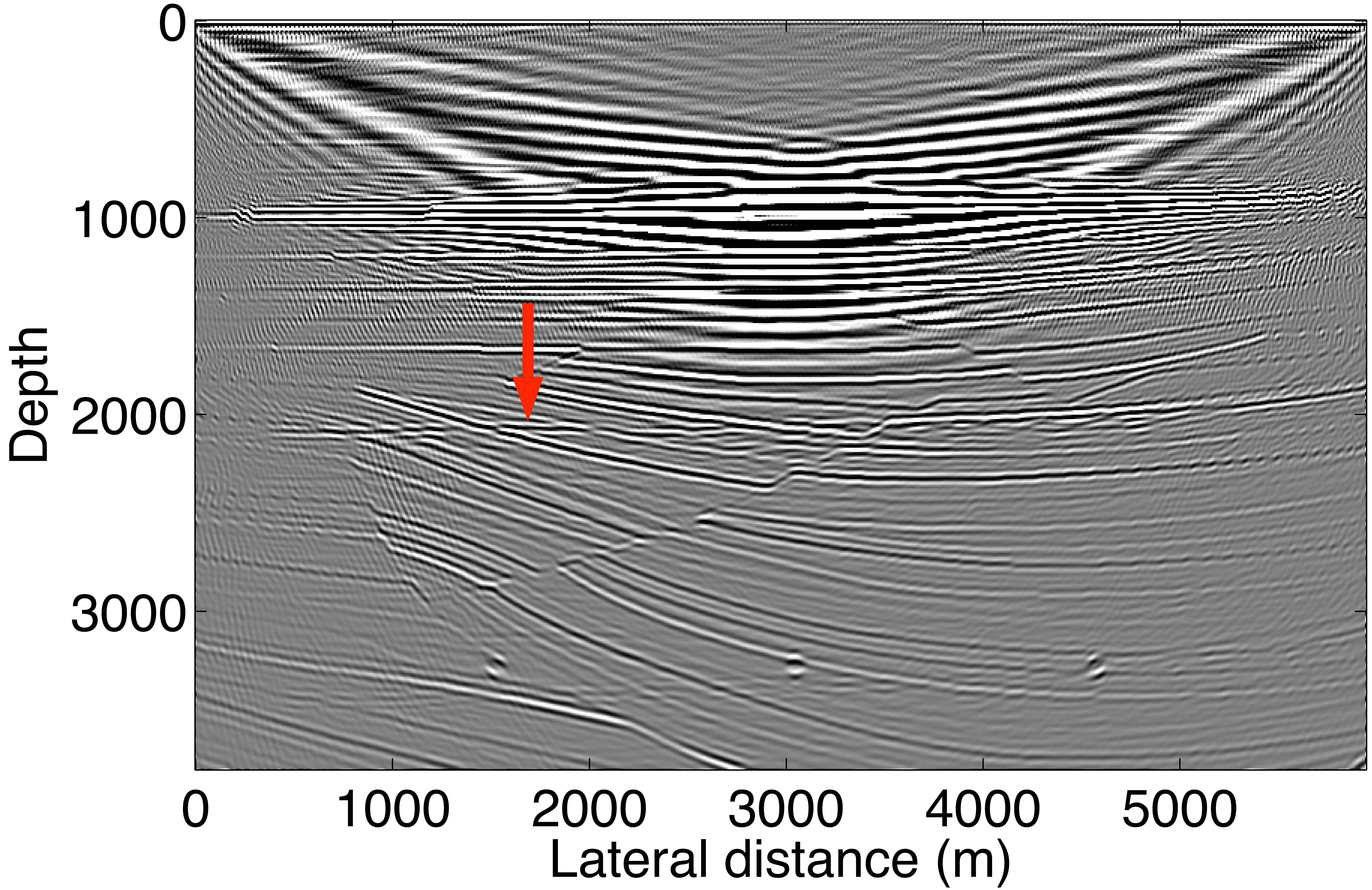
RTM of multiples

[with all 261 sources and 311 frequencies, vertical derivative]



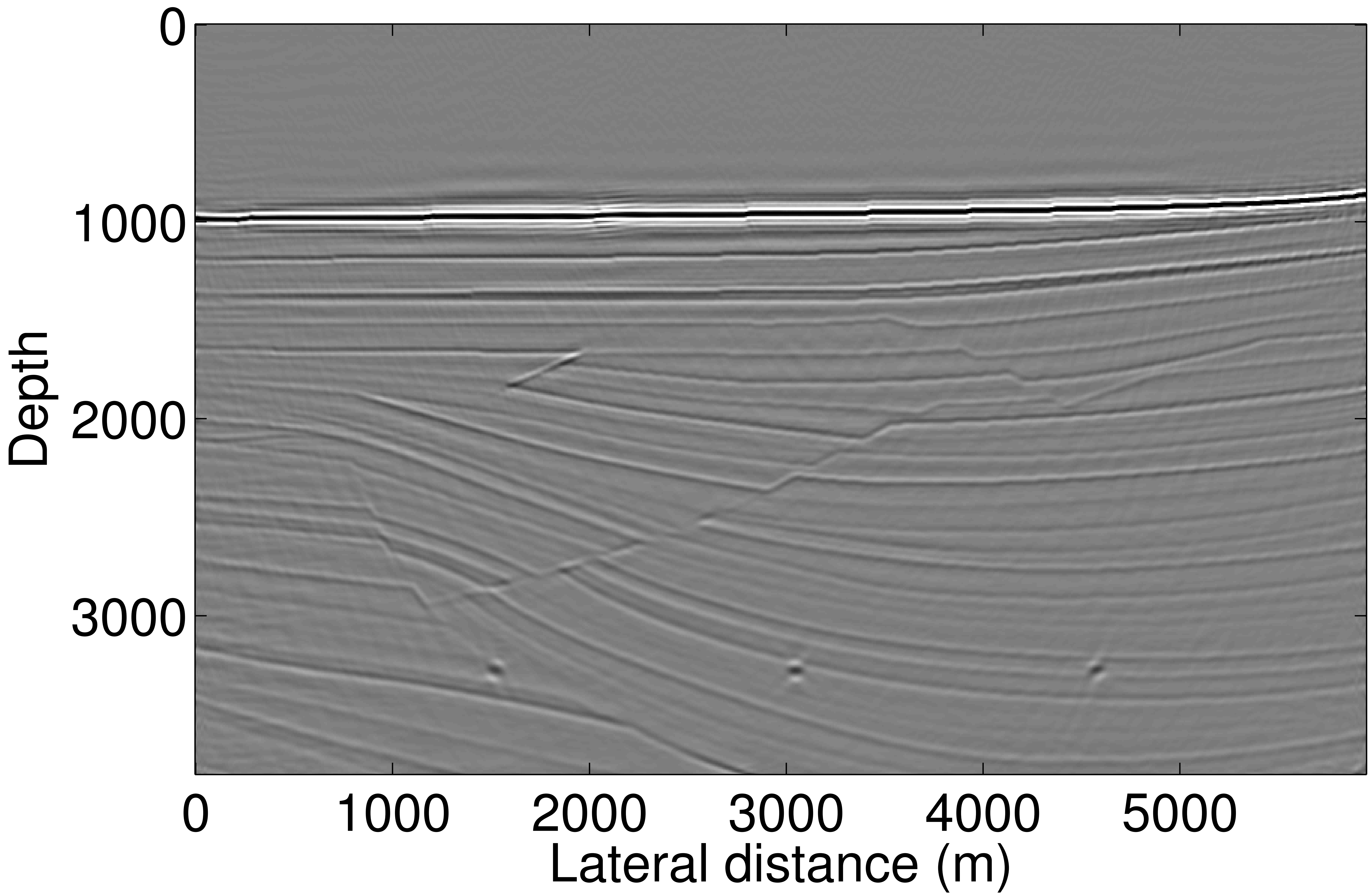
Weighted by inverse of *pseudo-Hessian*

[two-way deconv. imaging, w. damping stabilization, vertical derivative]



Fast inversion of multiples

[15 freq., 8 sim. src., ~300 iter., simulation cost ~1 RTM w. all data]



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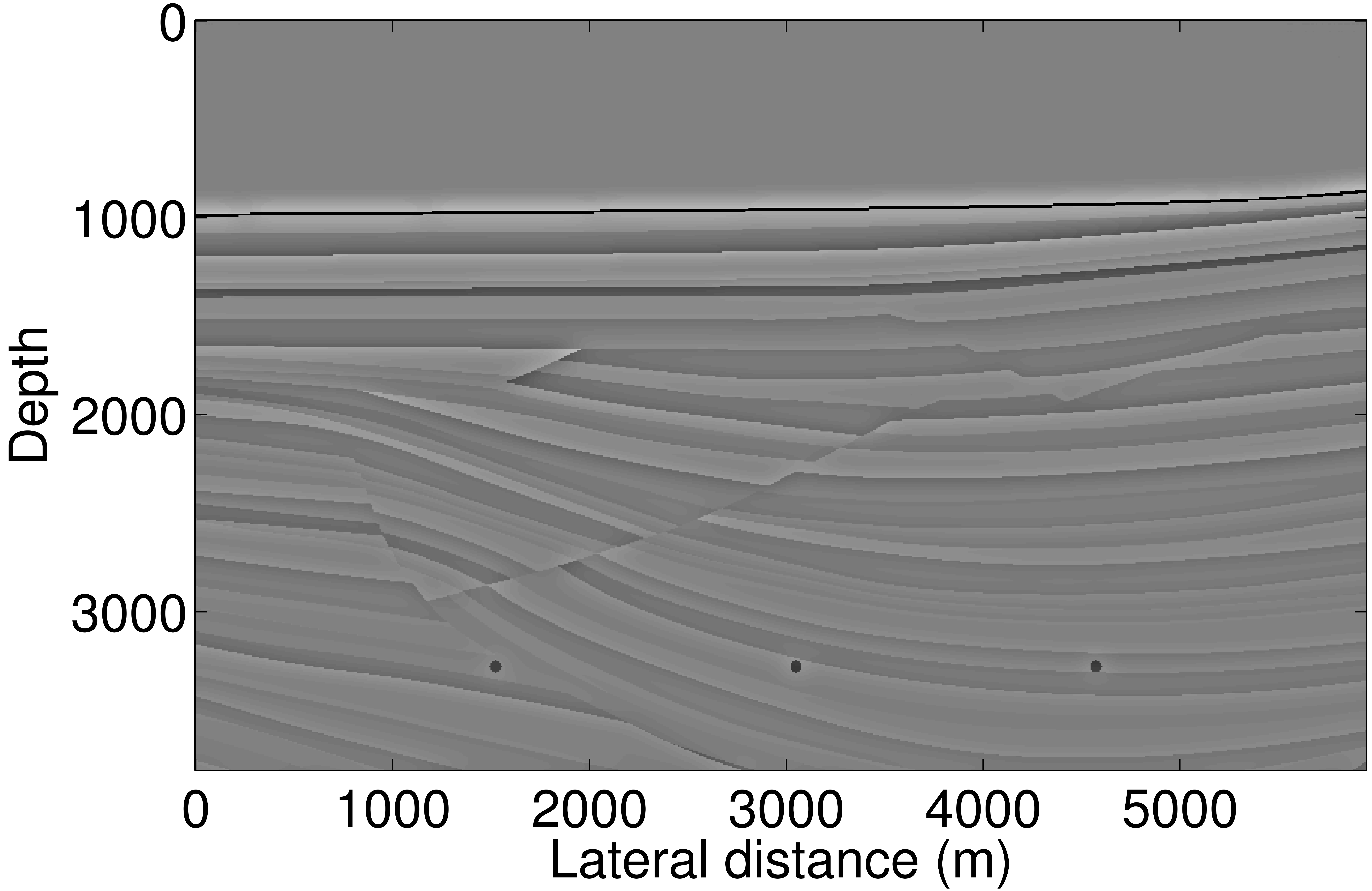

Conclusions

- The two-way analogue of the deconvolutional imaging condition:
  - can distort the image by not accounting for receiver-side propagation effects;
  - is incapable of eliminating all coherent artifacts when used alone to image surface multiples.
- These issues can be addressed by properly inverting the linearized demigration operator, which can be done in a computationally efficient way.

Acknowledgements

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Thank you for your attention!

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