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Optimization driven model-space versus data-space approaches to invert elastic data with the acoustic wave equation

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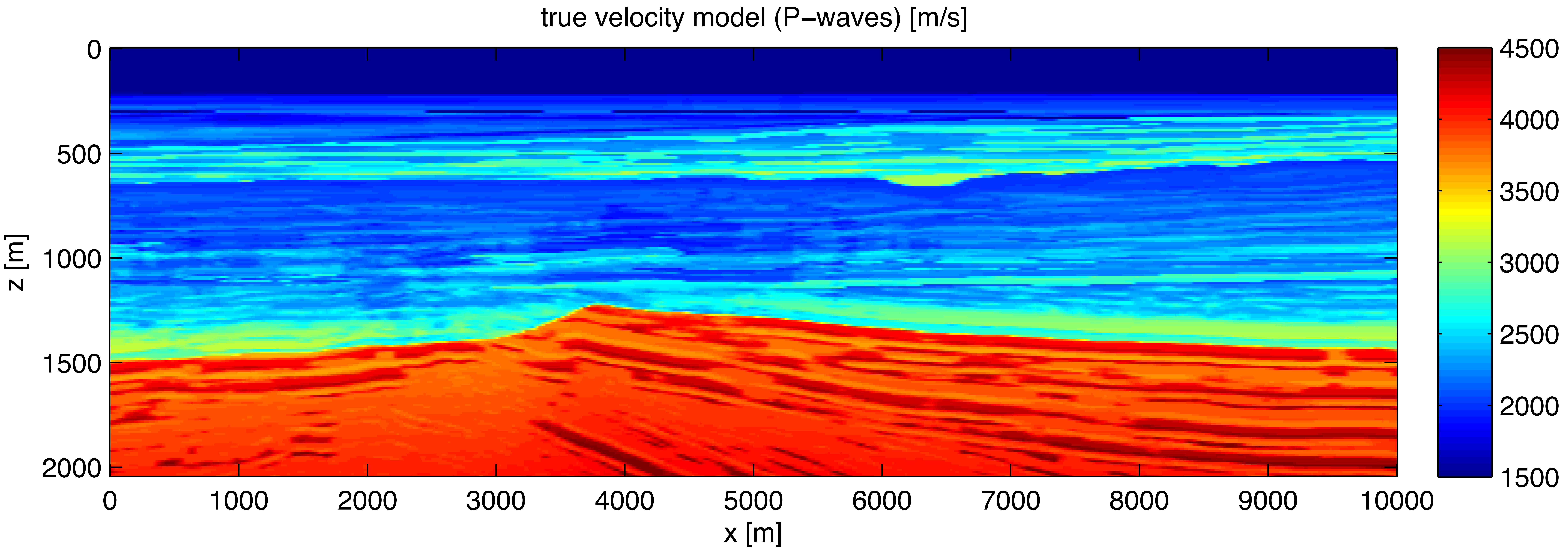
SLIM
University of British Columbia

Motivation

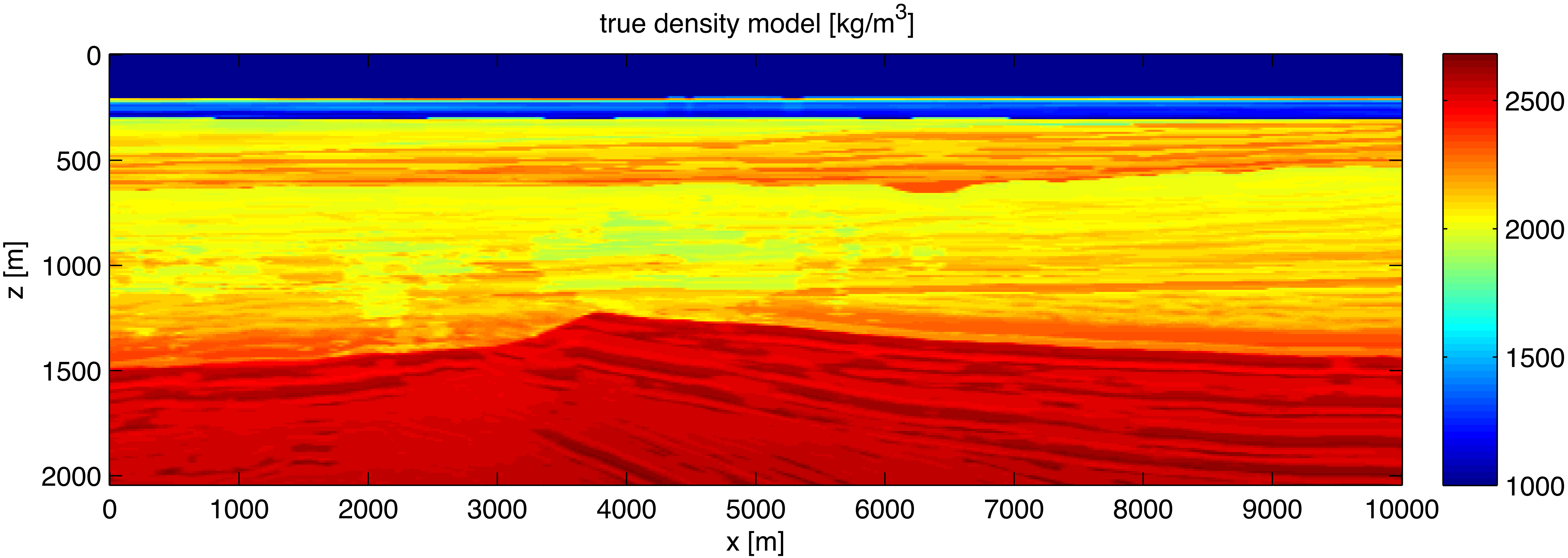
BP group

BG compass model (P-velocity)



Motivation

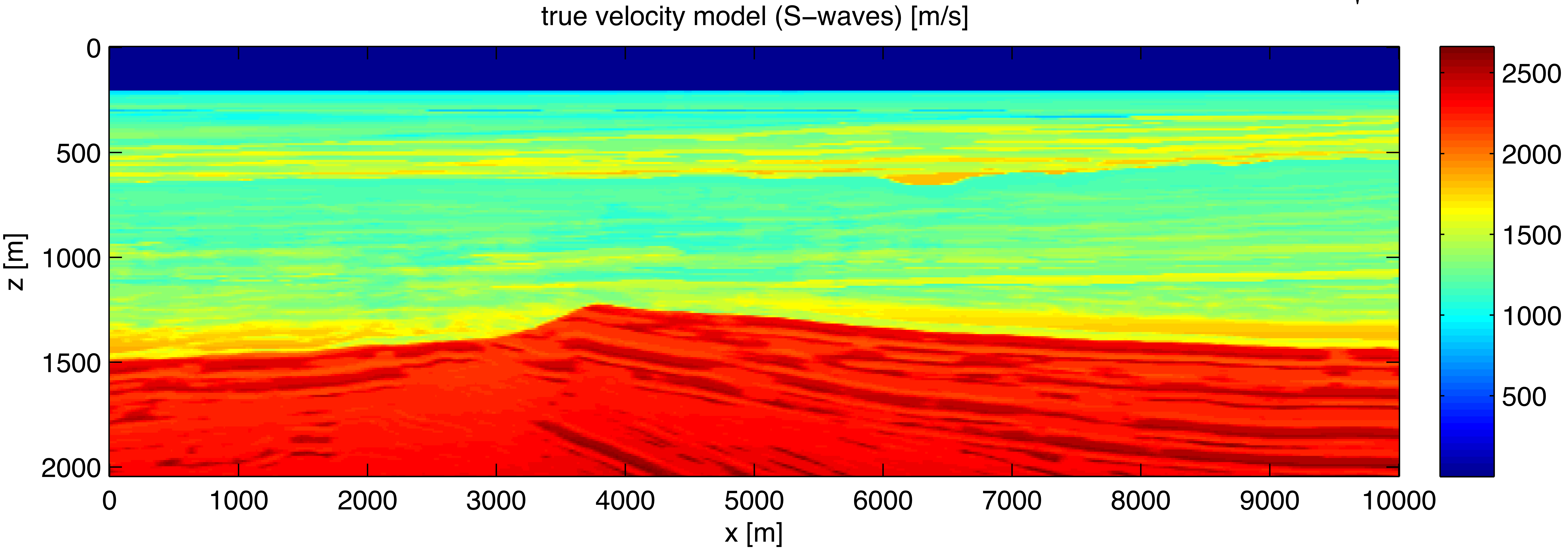
BG compass model (density)



Motivation

BG compass model (S-velocity)

Fixed Poisson's ratio 0.25 $$V_s = \sqrt{\frac{1-2\sigma}{2(1-\sigma)}}V_p$$

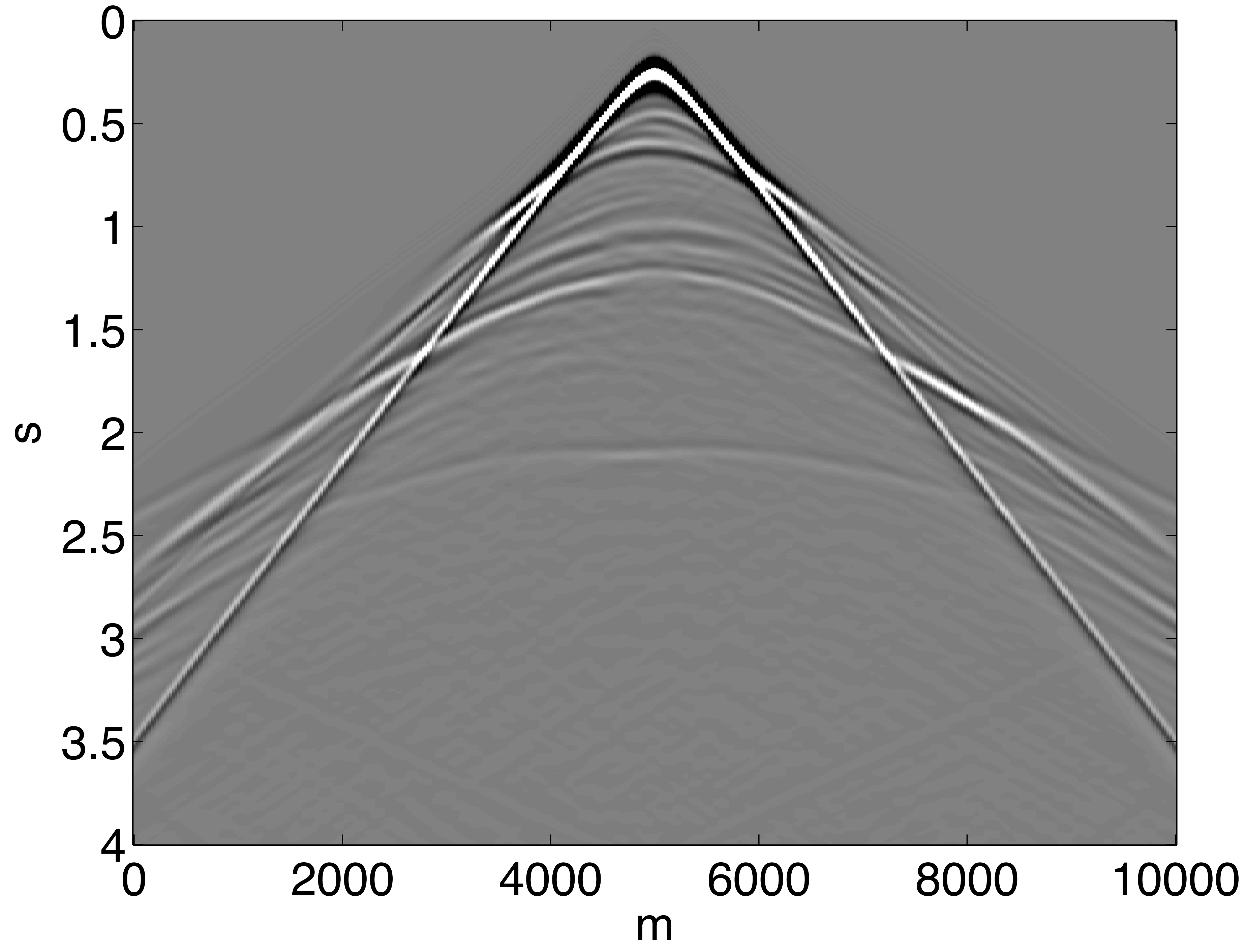


Motivation

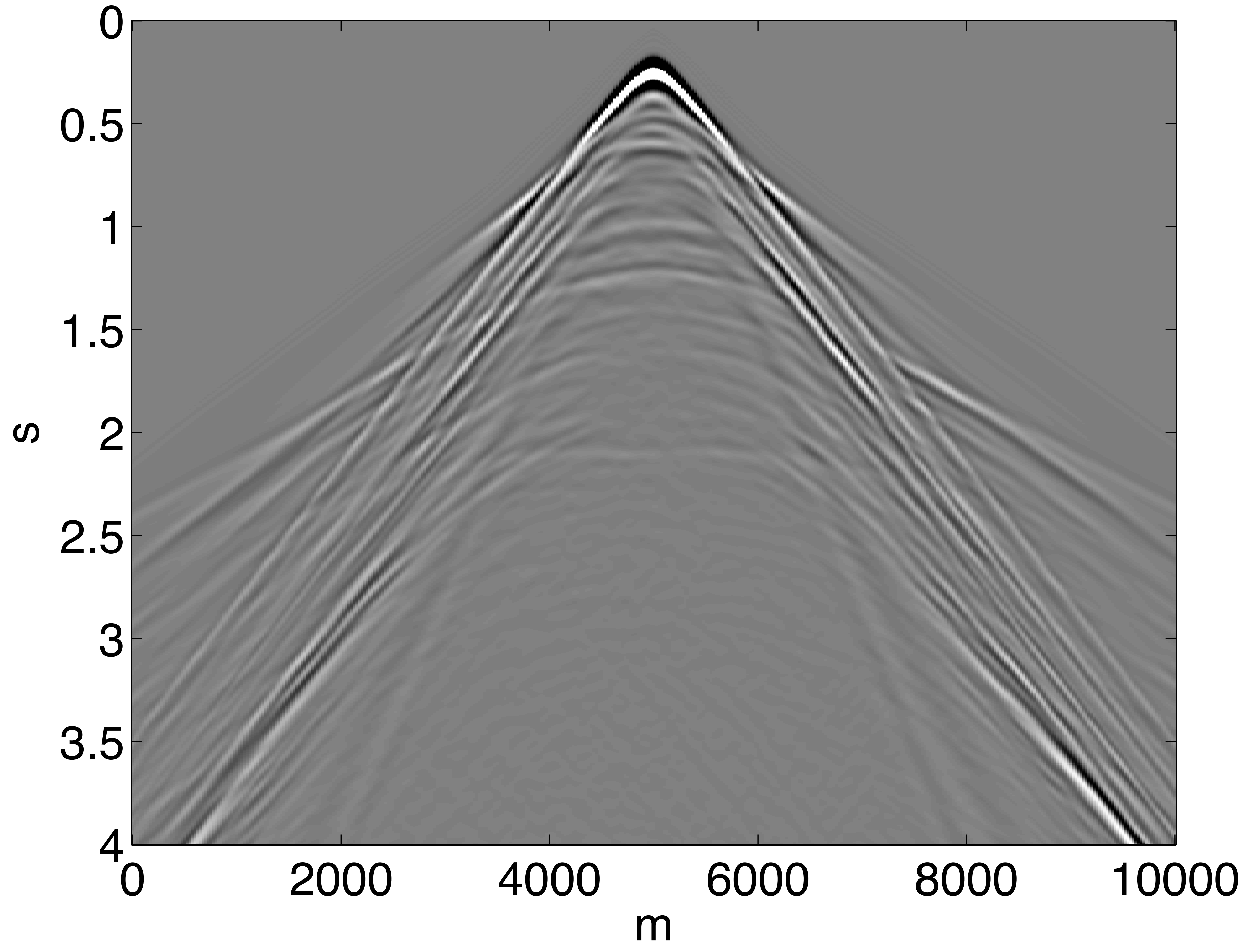
one sample shot in time domain

modeling kernel: time domain
finite difference

Thorbecke, 2013



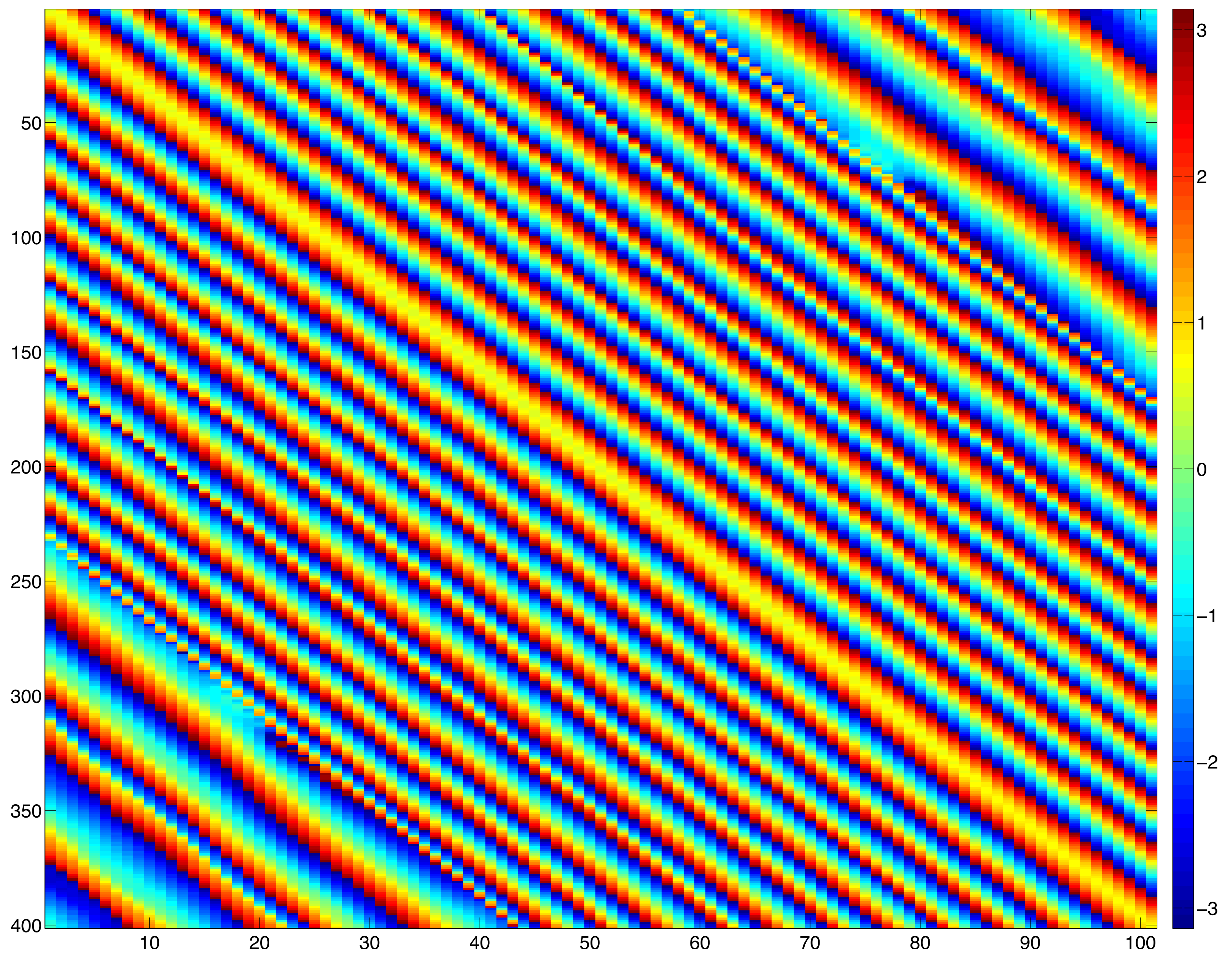
Acoustic



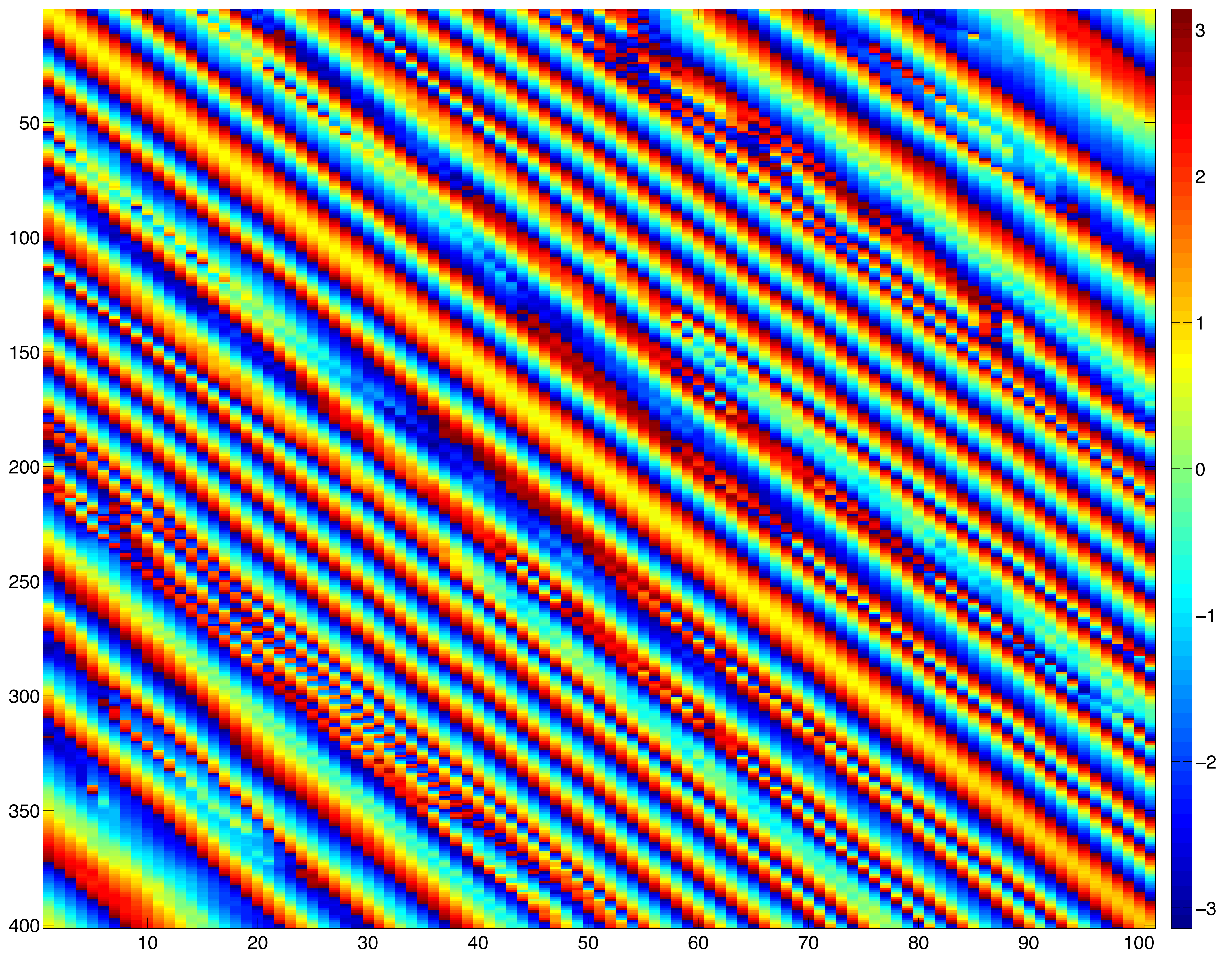
Elastic

Motivation

3Hz frequency slide (phase)



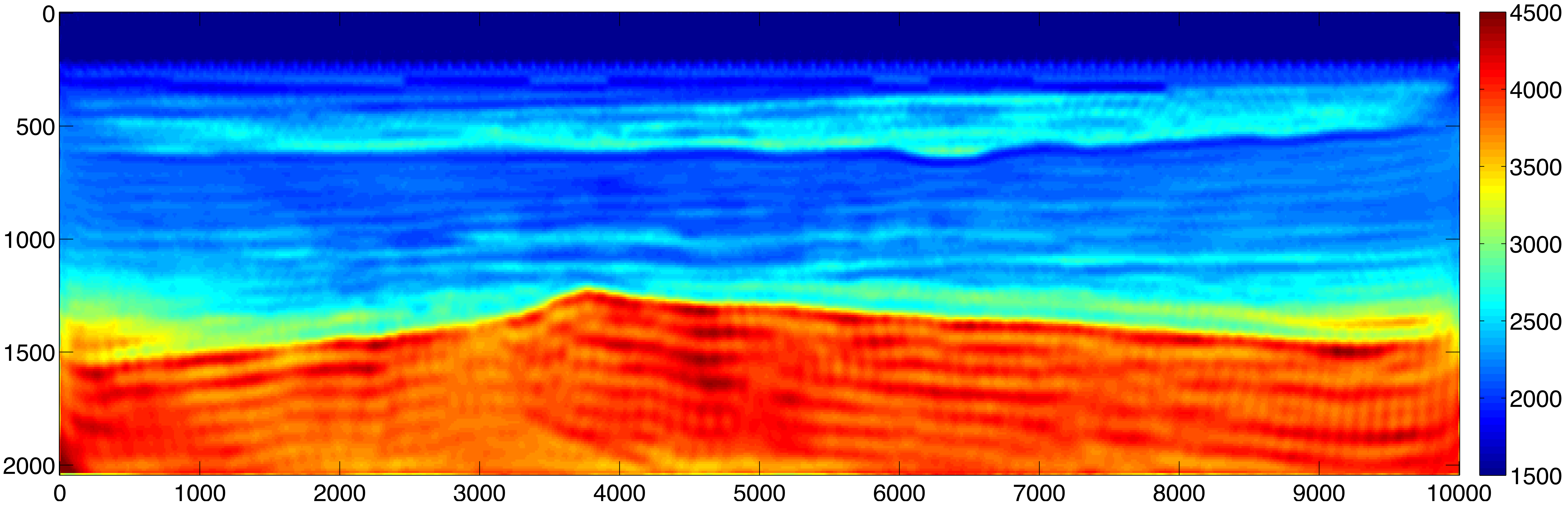
Acoustic



Elastic

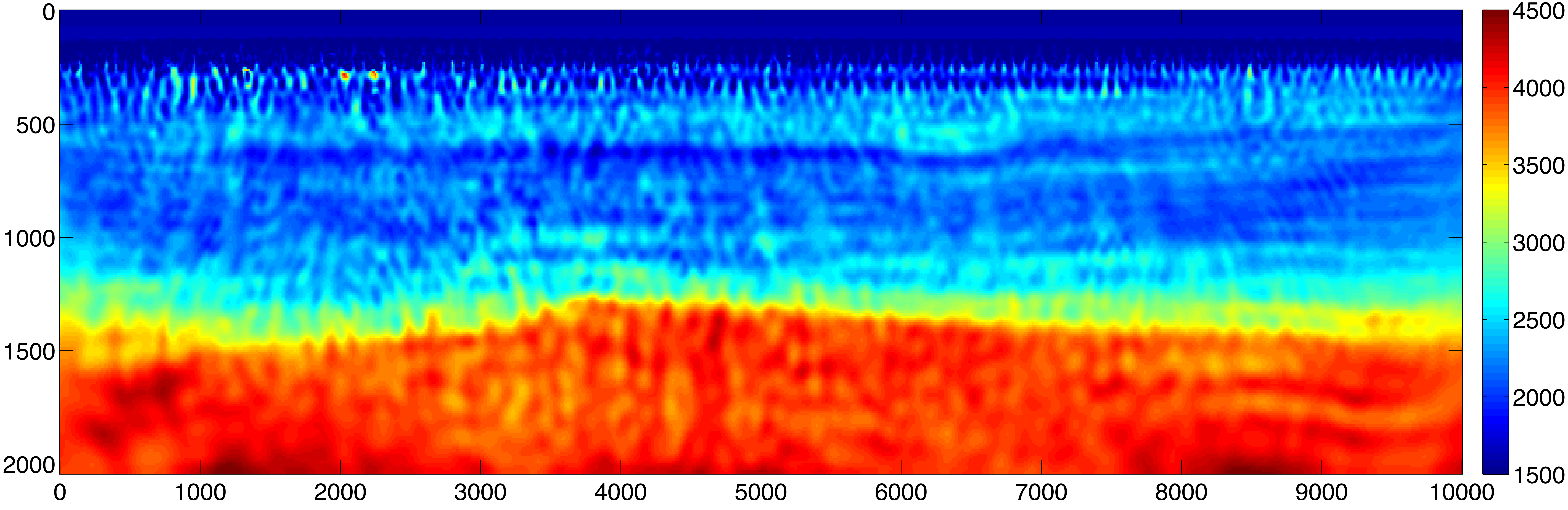
FWI example

standard acoustic Gauss-Newton inversion with *acoustic* data



FWI example

standard acoustic Gauss-Newton inversion with *elastic* data



Question

Can we invert *elastic* data with *acoustic* modeling kernel?

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Can we invert *elastic* data with *acoustic* modeling kernel?

- data-space student’s T
- model-space sparsity promotion

Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m},\mathbf{w}} \phi(\mathbf{m},\mathbf{w}) = \sum_{i=1}^{K} \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - w_i\boldsymbol{\mathcal{F}}_i(\mathbf{m})),$$

$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
$\boldsymbol{\mathcal{B}}_i$	data-processing operator
ρ	penalty function
K	batch size
$\mathbf{m}$	unknown medium parameters

Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m},\mathbf{w}} \phi(\mathbf{m},\mathbf{w}) = \sum_{i=1}^{K} \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - w_i \boldsymbol{\mathcal{F}}_i(\mathbf{m})),$$

penalty function

$\mathbf{d}_i$	observed data for one frequency
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K	batch size
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Penalty functions

Least-squares $$\rho(\mathbf{r}) = \sum_i |r_i|^2$$

Huber

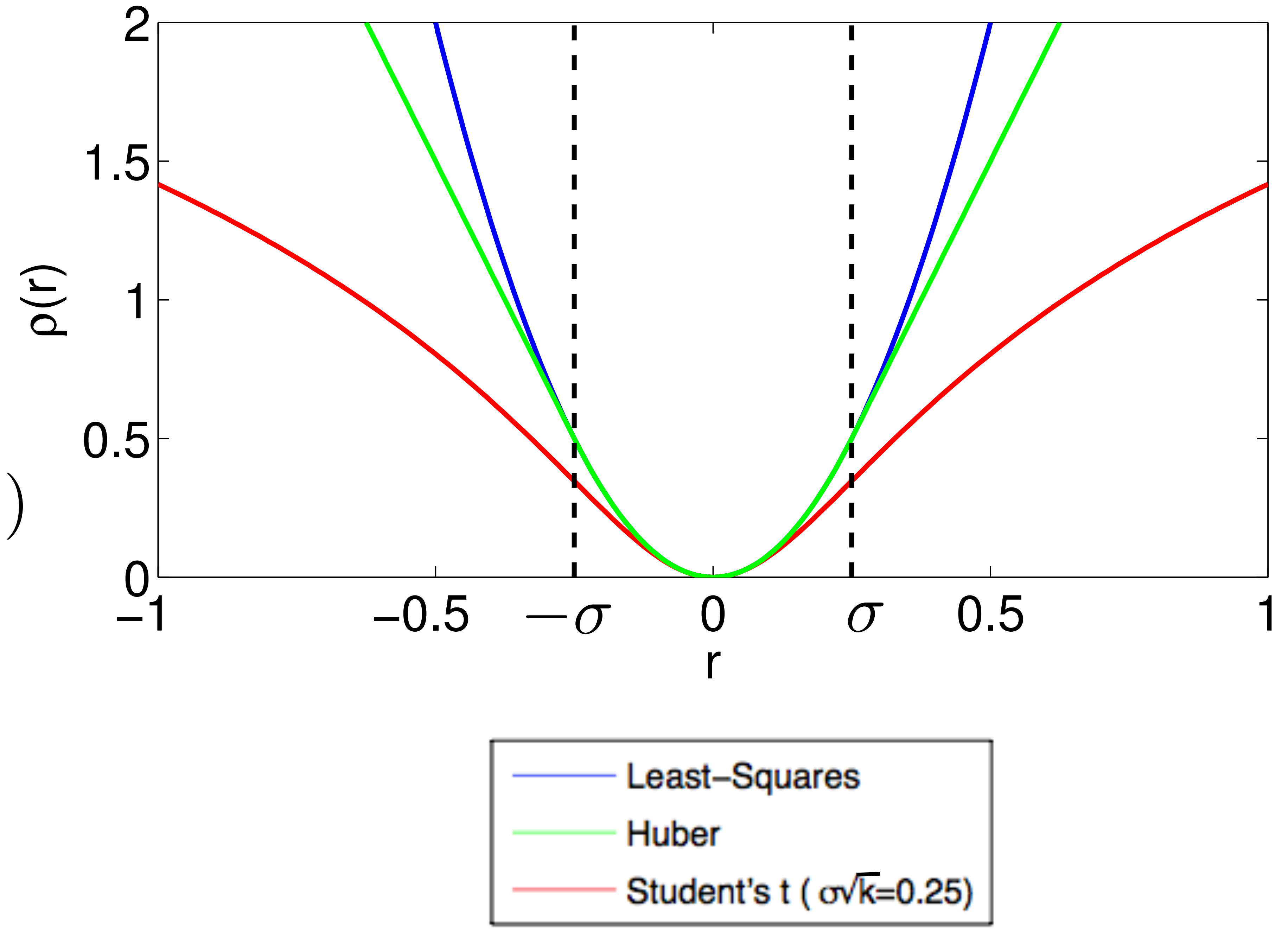
Guitton and Symes, 2003

$$\rho(\mathbf{r}) = \sum_{|r_i|\le\sigma} \frac{1}{2\sigma^2}|r_i|^2 + \sum_{|r_i|>\sigma} (\frac{1}{\sigma}|r_i| - \frac{1}{2})$$

Student's T

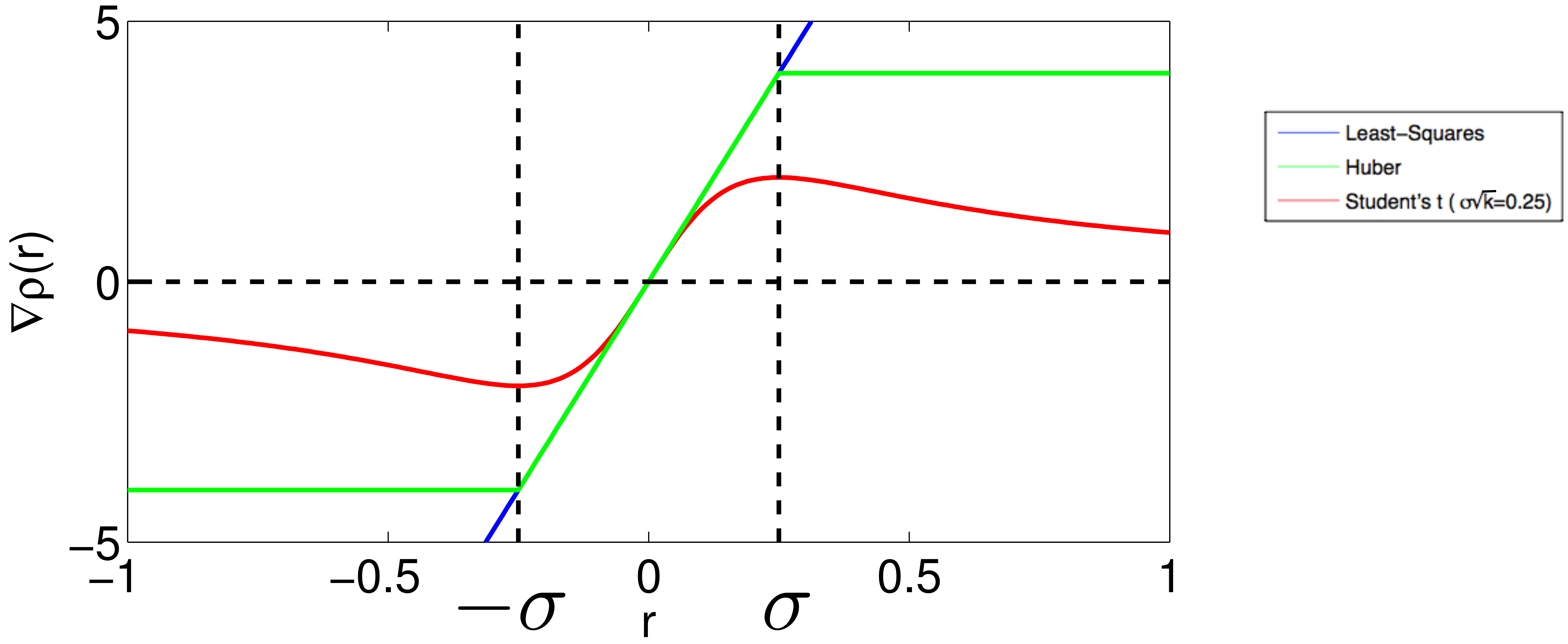
Aravkin et al., 2011

$$\rho(\mathbf{r}) = \sum_i \mathbf{log}(1 + |r_i|^2/\sigma^2 k)$$



- student's T distribution is robust to large outliers and artifacts in the data

Influence functions



Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m},\mathbf{w}} \phi(\mathbf{m},\mathbf{w}) = \sum_{i=1}^{K} \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - w_i\boldsymbol{\mathcal{F}}_i(\mathbf{m})),$$

transforms the residual into a domain where the outliers are localized

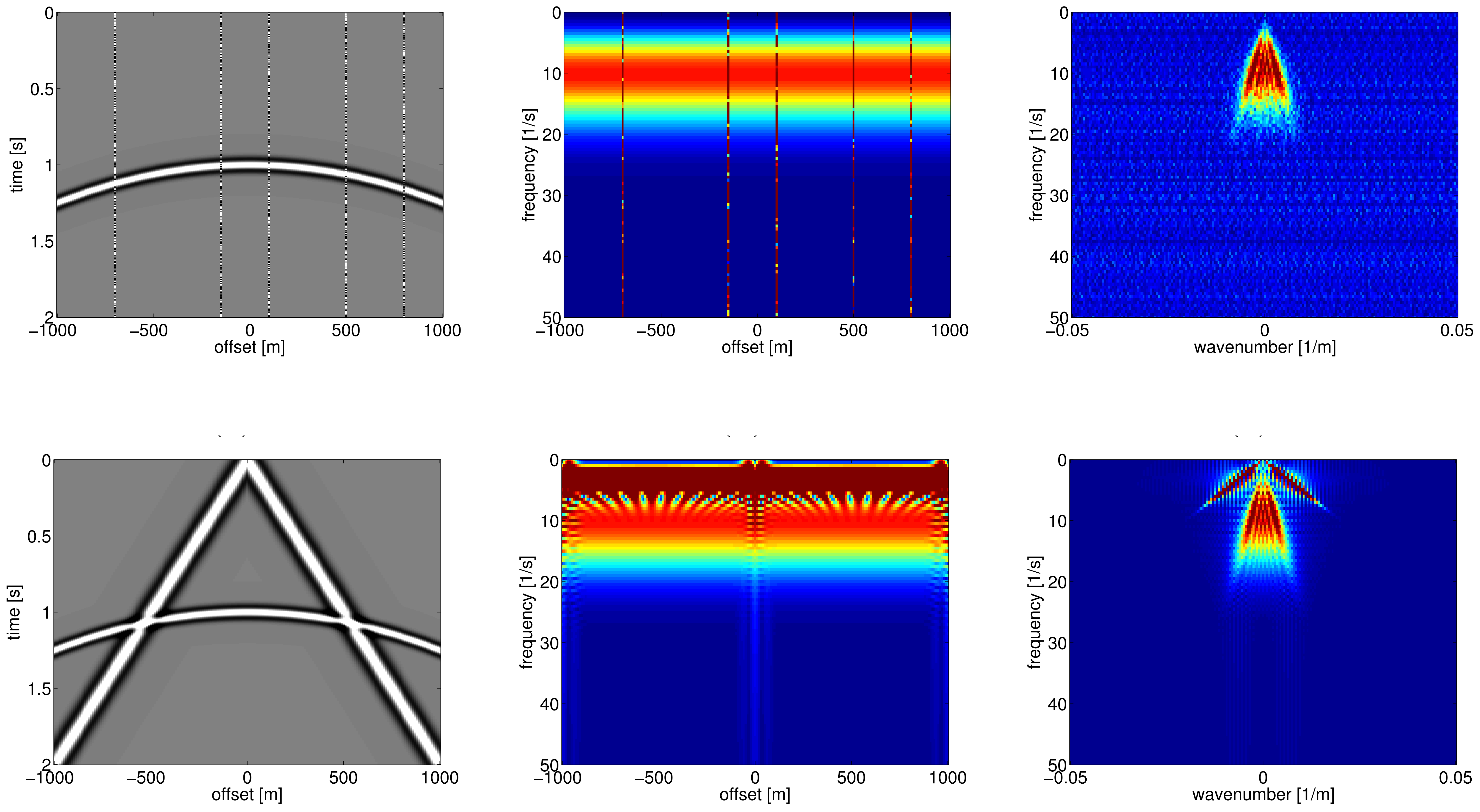
$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
$\boldsymbol{\mathcal{B}}_i$	data-processing operator
ρ	penalty function
K	batch size
$\mathbf{m}$	unknown medium parameters

Outliers in different domains

Van Leeuwen, 2013

bad traces

Spurious events



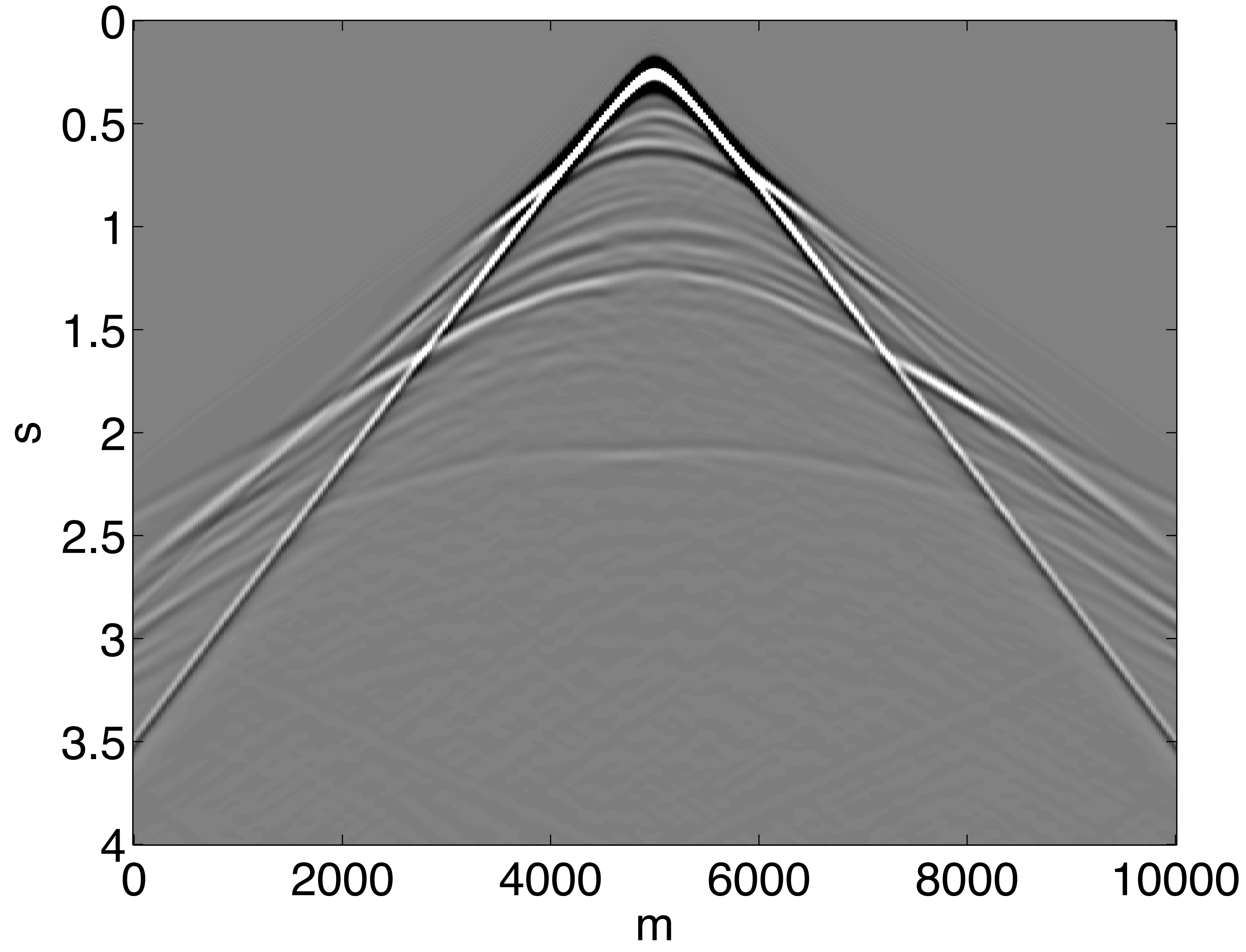
t-x

F-x

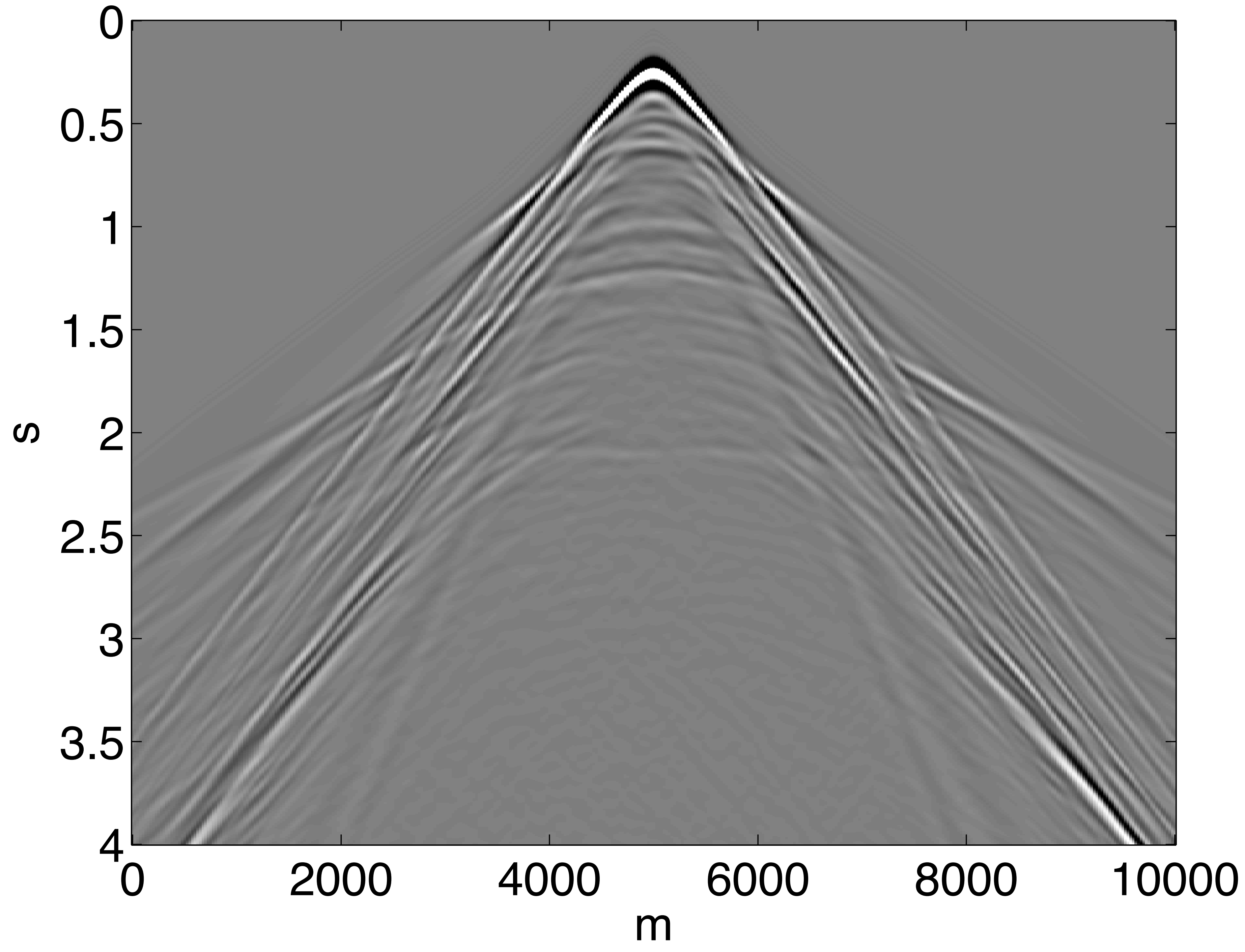
F-K

shot records

one sample shot in time domain



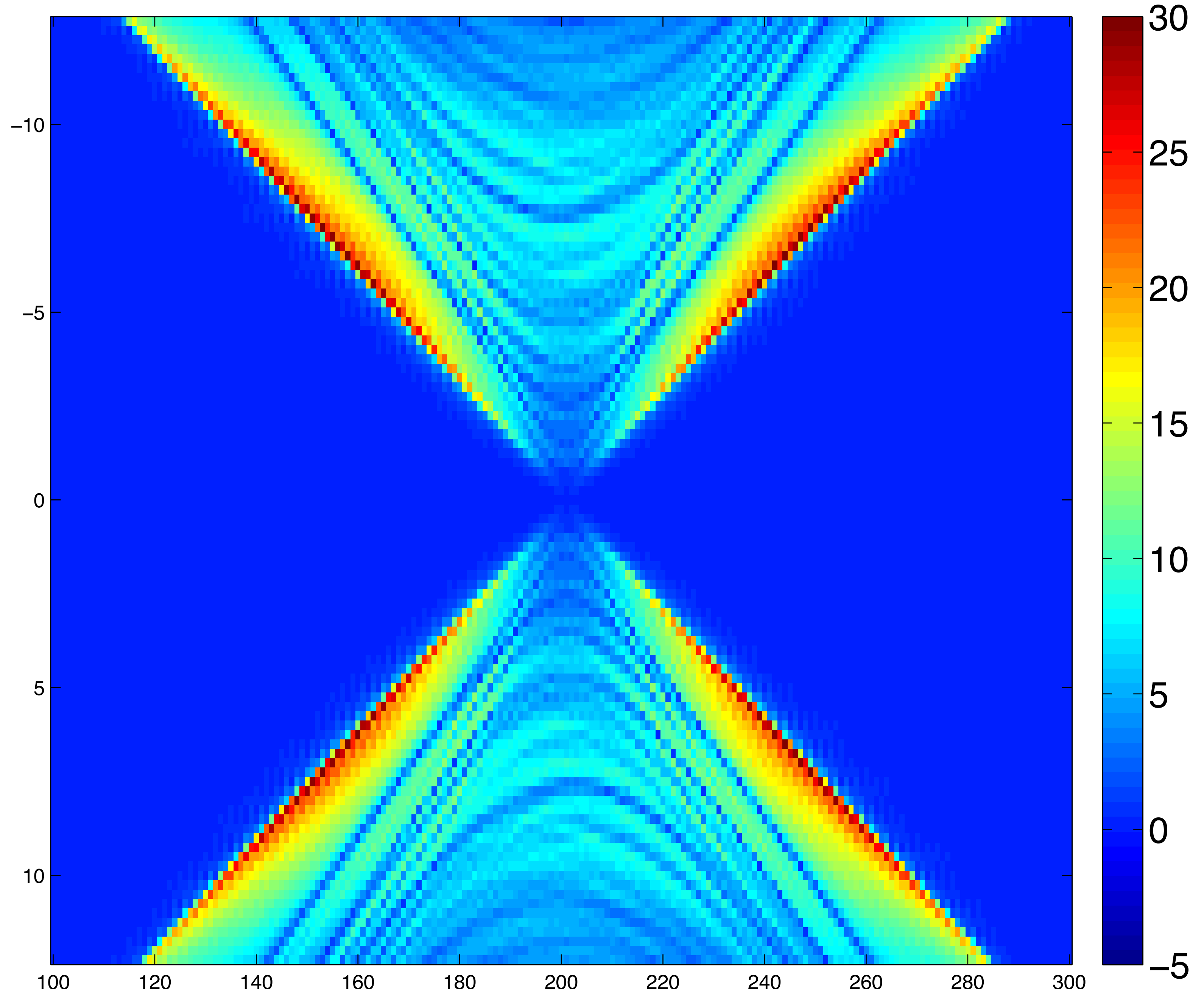
Acoustic



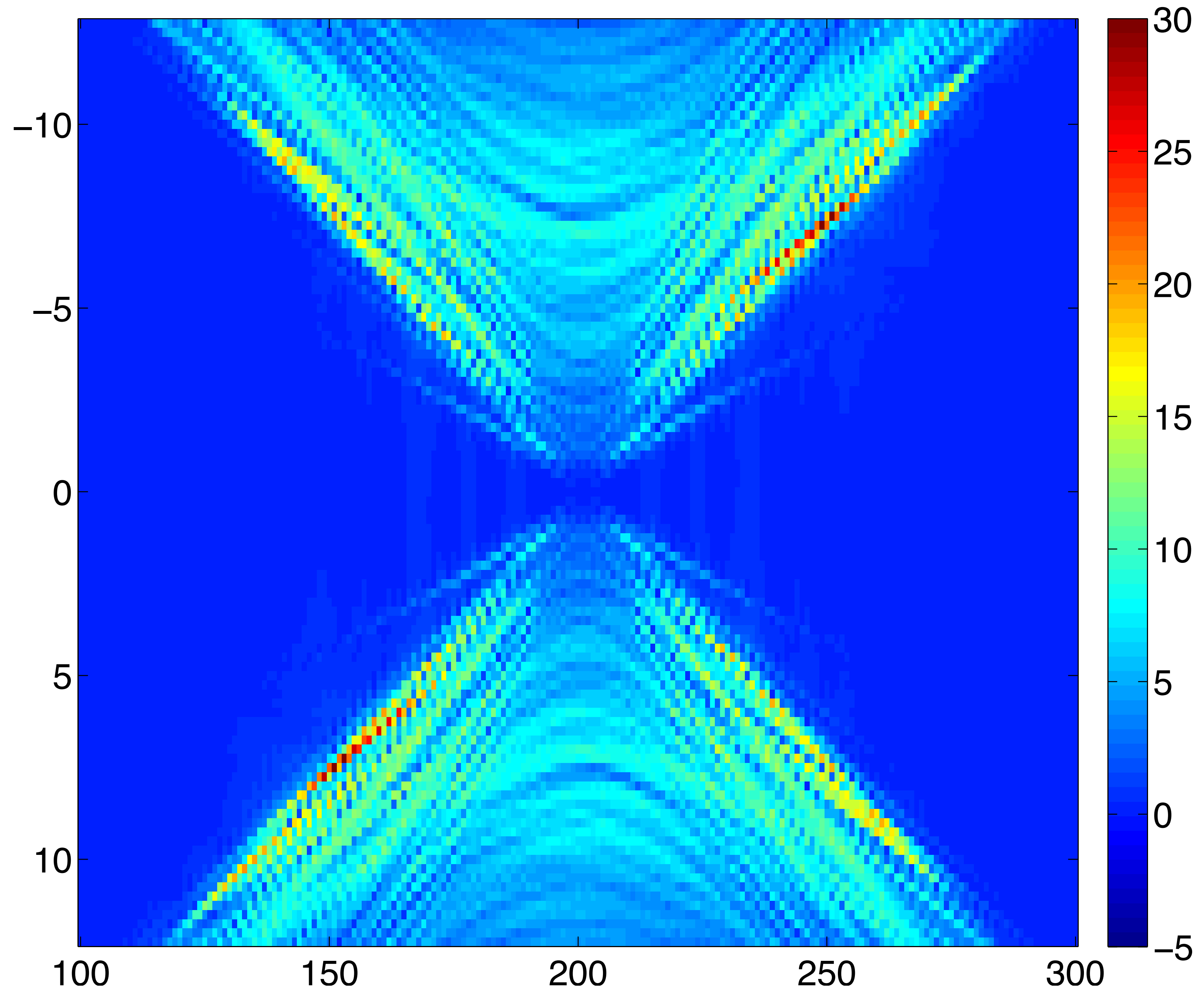
Elastic

FK spectrum

one shot in FK domain



Acoustic



Elastic

Full-waveform inversion

problem formulation in frequency domain

$$\min_{\mathbf{m},\mathbf{w}} \phi(\mathbf{m},\mathbf{w}) = \sum_{i=1}^{K} \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - w_i \boldsymbol{\mathcal{F}}_i(\mathbf{m}))),$$

source estimation

$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
$\boldsymbol{\mathcal{B}}_i$	data-processing operator
ρ	penalty function
K	batch size
$\mathbf{m}$	unknown medium parameters

Source estimation

scaler optimization problem

$$\bar{w}_i = \arg\min_w \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - w\boldsymbol{\mathcal{F}}_i(\mathbf{m})))$$

- Newton-like algorithm

$$w_i^{k+1} = w_i^k - \frac{\mathbf{G}(w_i^k)}{\mathbf{H}(w_i^k)}$$

$\mathbf{G}$ and $\mathbf{H}$ are the first and second derivatives of the objective

  - Least-Squares solution

    - closed form solution $\bar{w}_i = \frac{\boldsymbol{\mathcal{F}}_i(\mathbf{m})^T\mathbf{d}_i}{||\boldsymbol{\mathcal{F}}_i(\mathbf{m})||_2^2} \quad (\boldsymbol{\mathcal{B}}_i = 1)$ Pratt, 1999

  - student's T solution Aravkin, 2012

    - Newton iteration $w_i^{k+1} = w_i^k - \frac{\sum_i \mathbf{r}_i(w_i^k)\boldsymbol{\mathcal{F}}_i/\sqrt{1+\mathbf{r}_i(w_i^k)}}{\sigma\sum_i \boldsymbol{\mathcal{F}}_i^2/\sqrt{1+\mathbf{r}_i(w_i^k)}} \quad (\boldsymbol{\mathcal{B}}_i = 1)$

    - strictly approximation to the second derivative

Example

observed data simulation

- DELPHI Package (delmodc) Thorbecke, 2013

ocean Bottom acquisition

Forward modeling (Elastic)		
101 shots	interval = 100m	depth = 200m
401 receivers	interval = 25 m	depth = 10m
Sampling time	interval = 4ms	total = 4 seconds

Inversion setting

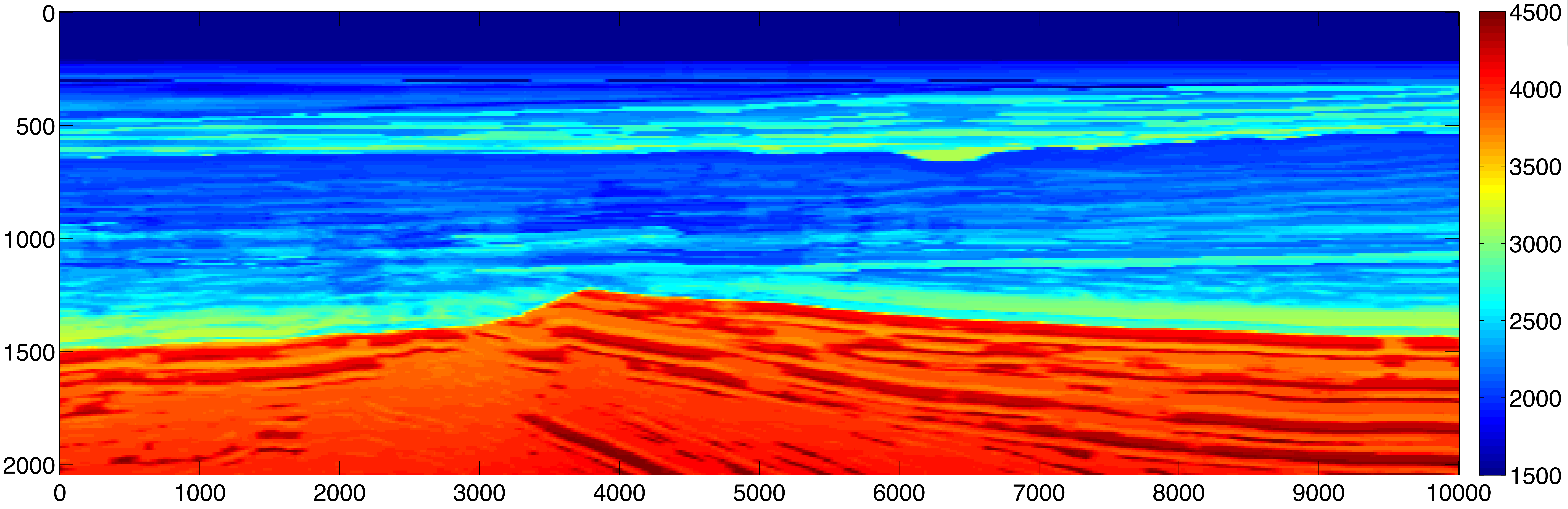
modeling kernel

- Frequency domain Helmholtz (acoustic)

acoustic inversion of elastic data

- 11 frequency band (3-12Hz), each has 3 frequcencies
- compute 10 *quasi-Newton (L-BFGS) updates* for each frequency band
- each L-BFGS update uses *all* shots

True model



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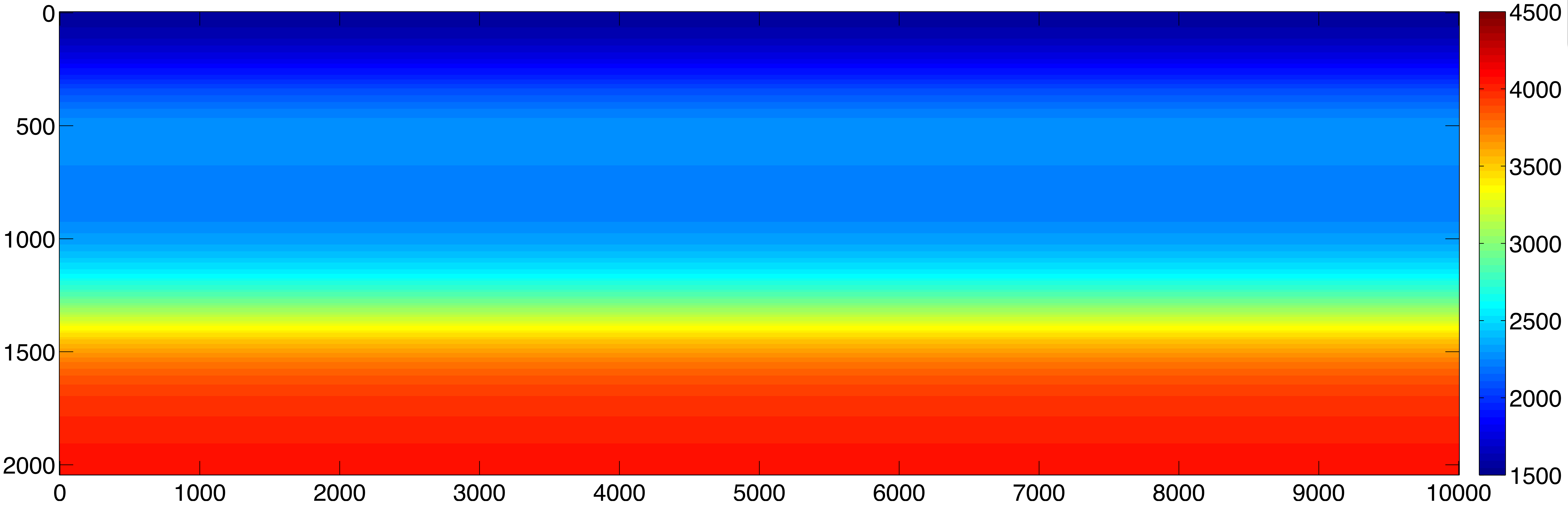
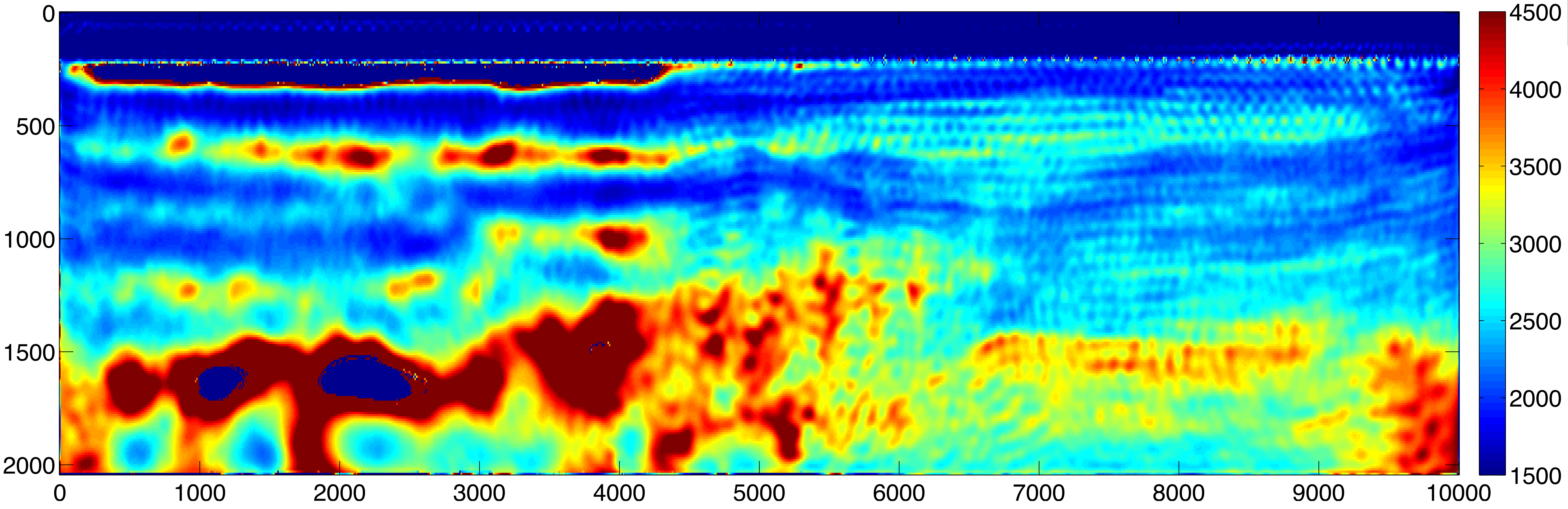
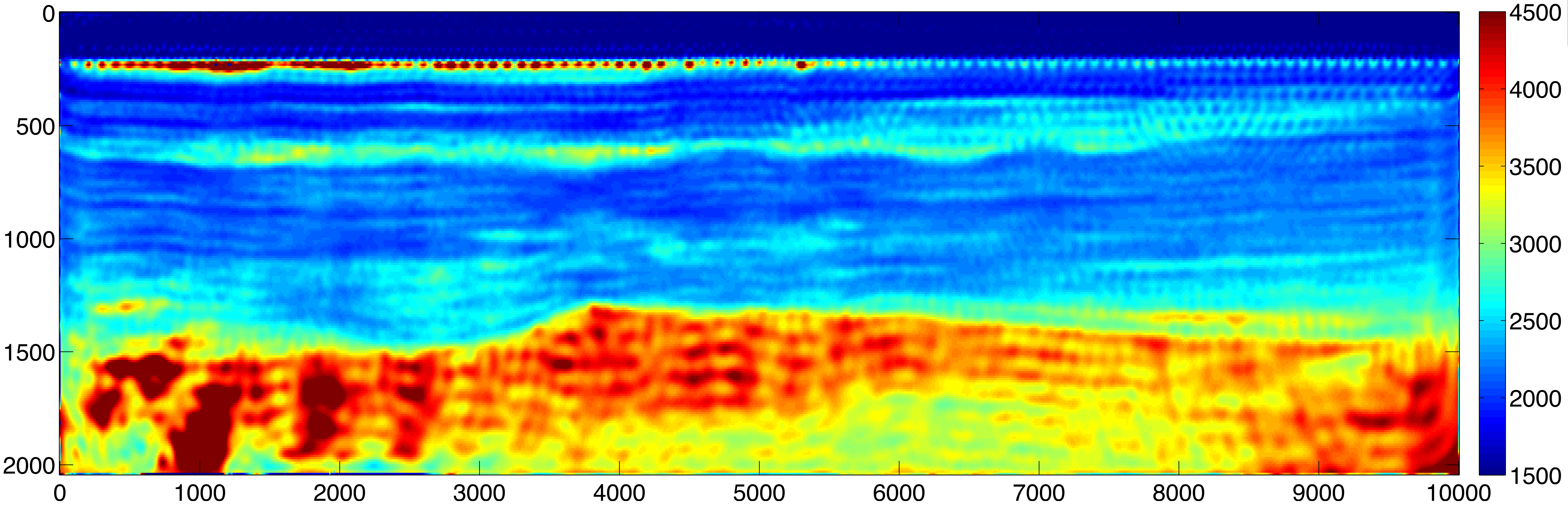
Initial model



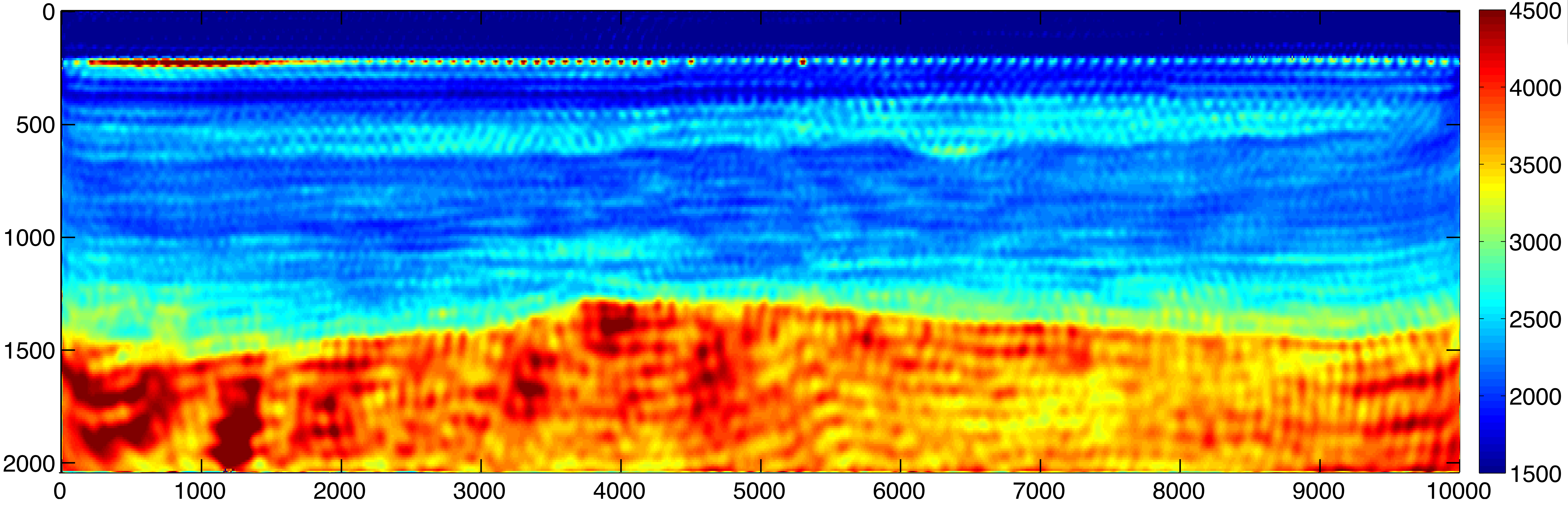
Least-Squares penalty result



Student's T penalty result



Student's T penalty in FK domain



Observation

- It is possible to invert elastic data with acoustic modeling kernel, although the result is not perfect yet

- Student's T penalty function is insensitive to large outliers

- Appropriate transforms will help localize the outliers, improving the resolution

Full-waveform inversion

problem formulation in frequency domain

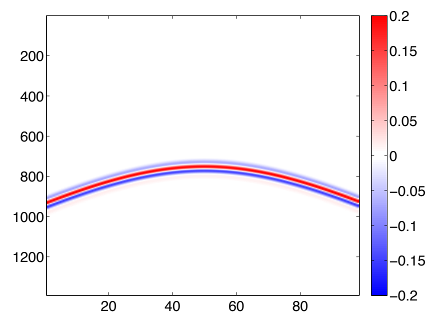
$$\min_{\mathbf{m},\mathbf{w}} \phi(\mathbf{m},\mathbf{w}) = \sum_{i=1}^{K} \rho(\boldsymbol{\mathcal{B}}_i(\mathbf{d}_i - w_i \boldsymbol{\mathcal{F}}_i(\mathbf{m})),$$

$\mathbf{d}_i$	observed data for one frequency
$\boldsymbol{\mathcal{F}}_i(\mathbf{m})$	modelling operator
$\boldsymbol{\mathcal{B}}_i$	data-processing operator
ρ	penalty function
K	batch size
$\mathbf{m}$	unknown medium parameters

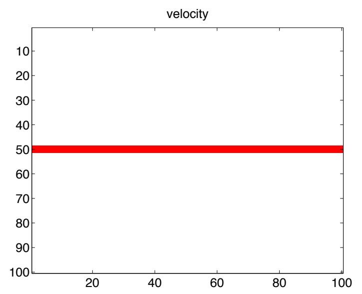
Gauss-Newton update

Gauss-Newton subproblem (Least-Squares penalty):

$$\delta\mathbf{m} = \underset{\delta\mathbf{m}}{\arg\min}\ \frac{1}{2}\|\underline{\delta\mathbf{D}} - \mathbf{diag}(\underline{\mathbf{w}})\nabla\boldsymbol{\mathcal{F}}[\mathbf{m_0}]\quad\delta\mathbf{m}\|_F^2$$



Jacobian operator (born modeling operator)

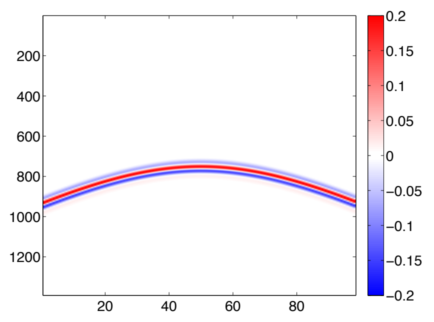


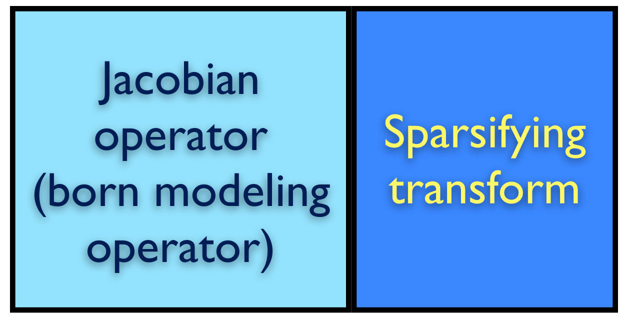
$$\underline{\delta\mathbf{D}} = \underline{\mathbf{D}} - \mathbf{diag}(\underline{\mathbf{w}})\boldsymbol{\mathcal{F}}(\mathbf{m_0}),\ \text{with}\ \underline{\mathbf{D}} \subset [\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3, \cdots, \mathbf{d}_K]$$

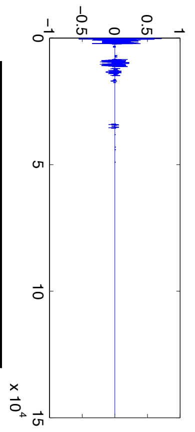
Sparsity promoting Gauss-Newton update

modified Gauss-Newton subproblem:

$$\delta\mathbf{m} = \mathbf{C^H} \arg\min_{\mathbf{x}} \frac{1}{2} \|\underline{\delta\mathbf{D}} - \mathbf{diag}(\underline{\mathbf{w}})\nabla\boldsymbol{\mathcal{F}}[\mathbf{m_0}]\mathbf{C^H}\mathbf{x}\|_F^2 \text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau,$$





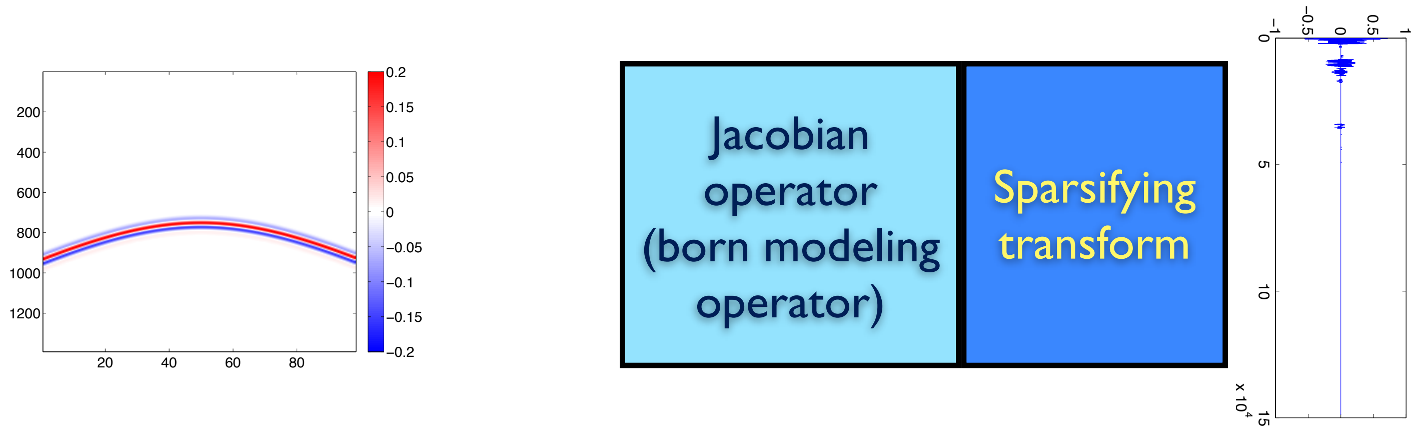


- $\mathbf{C^H}$ inverse curvelet transfrom

Sparsity promoting Gauss-Newton update

modified Gauss-Newton subproblem:

$$\delta\mathbf{m} = \mathbf{C^H} \arg\min_{\mathbf{x}} \frac{1}{2}\|\underline{\delta\mathbf{D}} - \mathbf{diag}(\underline{\mathbf{w}})\nabla\boldsymbol{\mathcal{F}}[\mathbf{m_0}]\mathbf{C^H}\mathbf{x}\|_F^2 \text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau,$$



- $\mathbf{C^H}$ inverse curvelet transfrom

- $\ell 1$ norm constraint is given by

$$\tau = \frac{\|\underline{\delta\mathbf{D}}\|_2}{\|\mathbf{C}\nabla\boldsymbol{\mathcal{F}}^T(\mathbf{m_0})\mathbf{diag}(\underline{\mathbf{w}})\underline{\delta\mathbf{D}}\|_\infty}$$

SPGl1 by Friedlander, 2008

Inversion setting

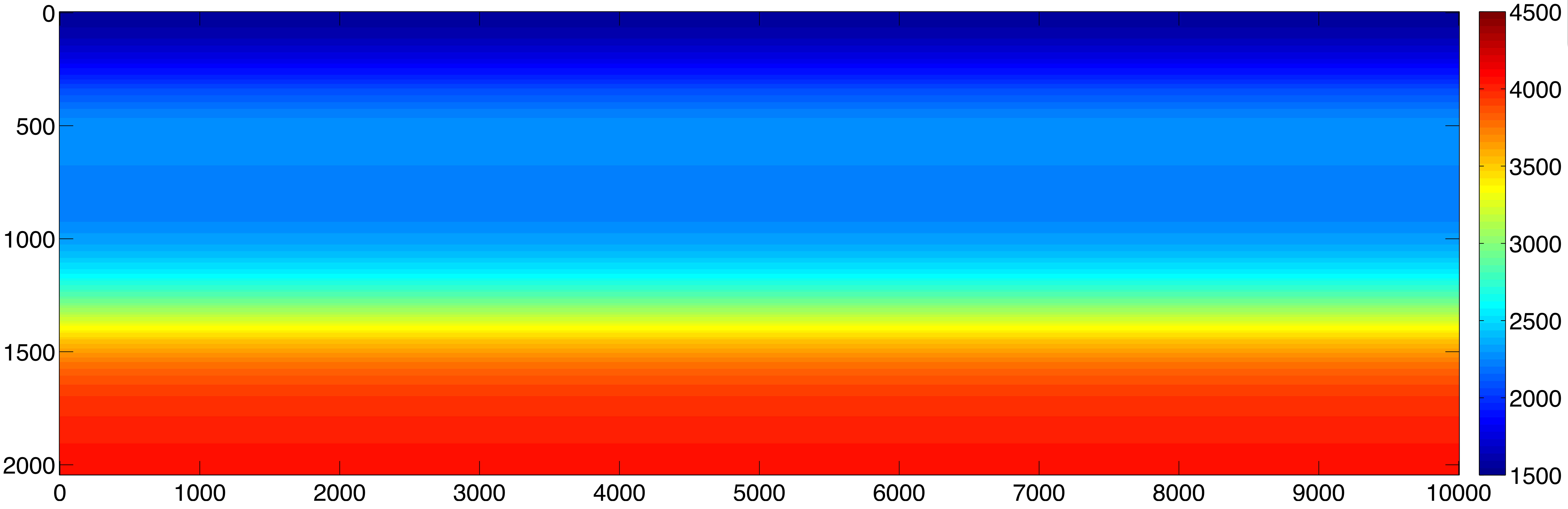
modeling kernel

- Frequency domain Helmholtz (acoustic)

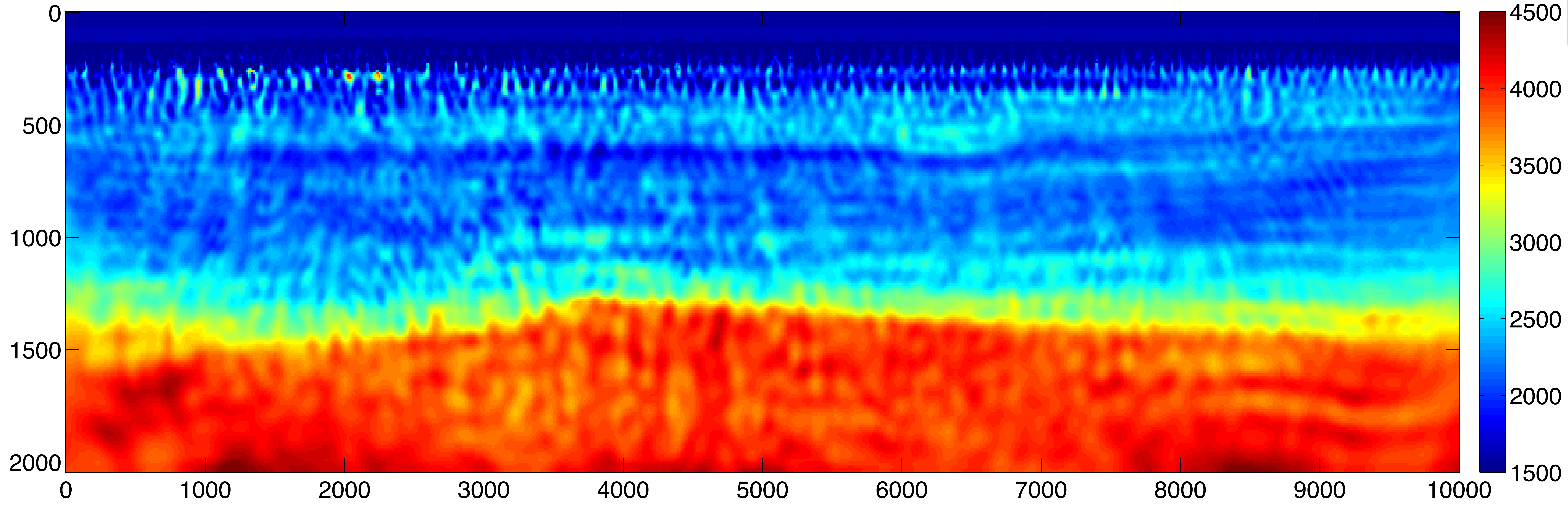
acoustic inversion of elastic data

- 11 frequency band (3-12Hz), each has 3 frequcencies
- compute 5 sparsity-promoting GN *updates* for each frequency band
- each GN update uses *50 randomly selected* shots

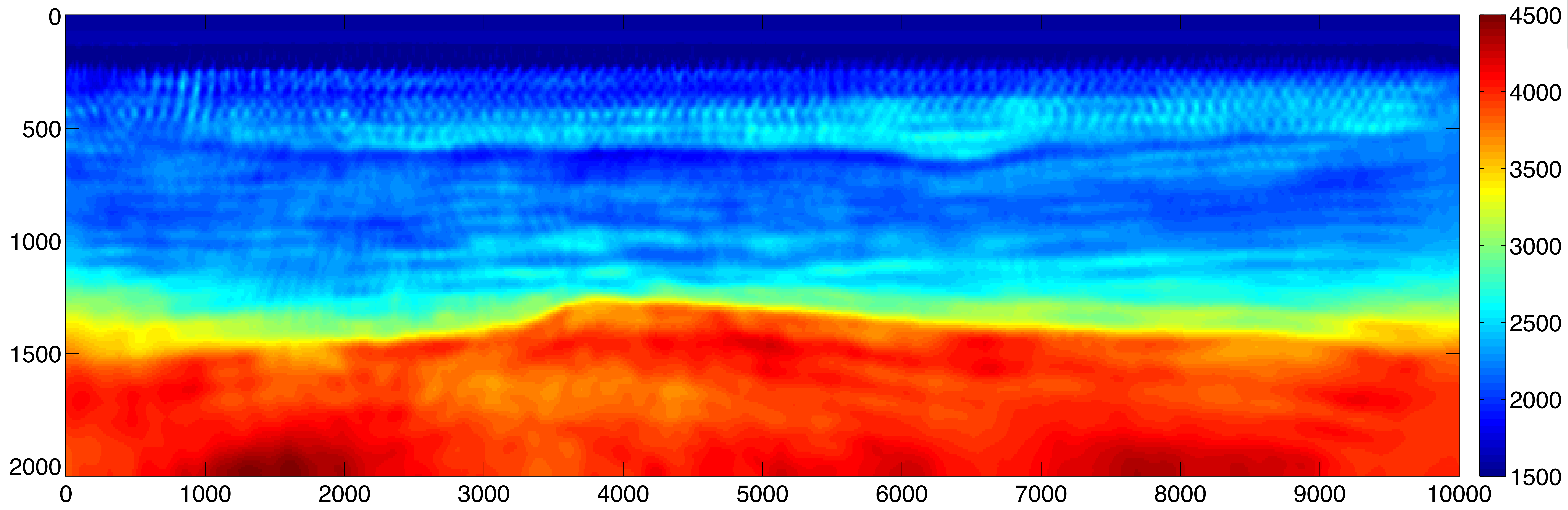
Initial model



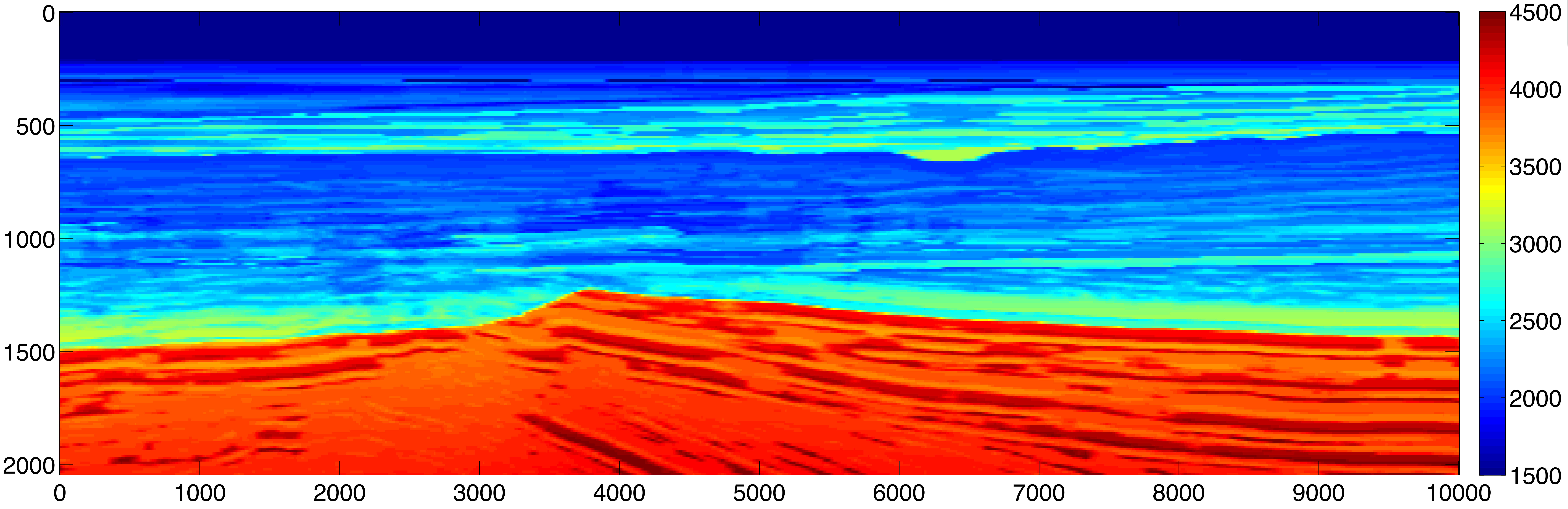
GN inversion without sparsity promoting



GN inversion with sparsity promoting



True model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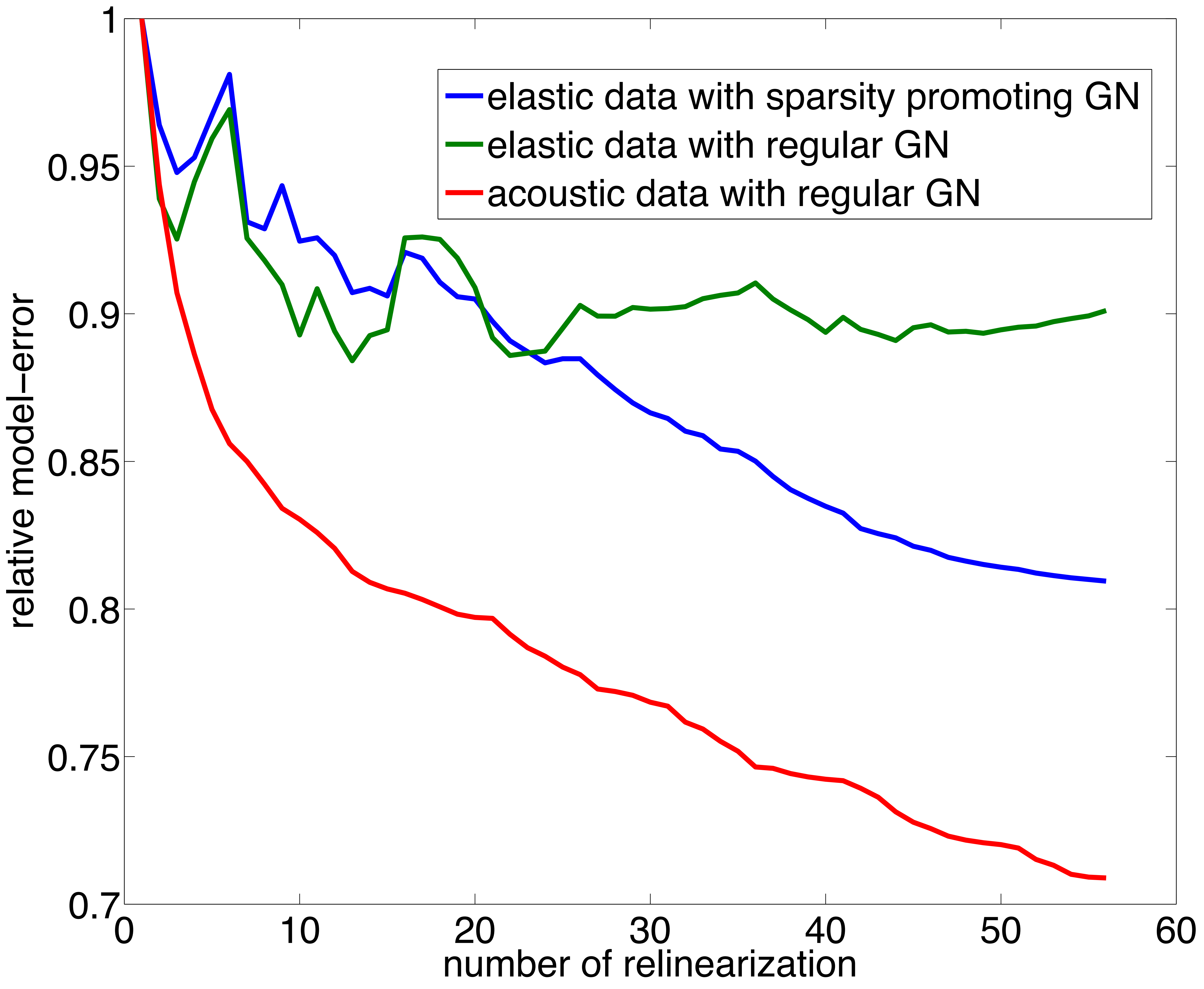
Relative model-error



Fi

Observation

- *Curvelet transform* efficiently represents geological models
- *sparsity regularization in Curvelet domain* can significantly suppress model space artifacts

Gulf of Mexico data

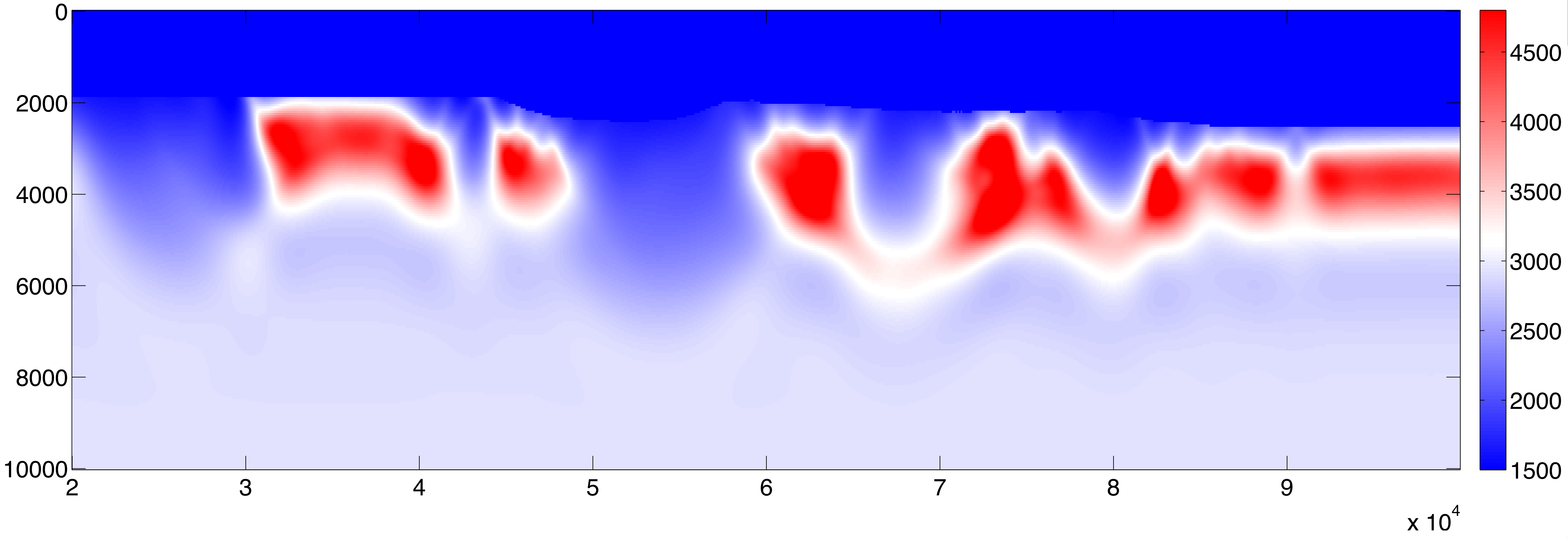
Chevron blind test

- 3201 shots with interval 25 m
- 801 receivers with interval 25 m, yielding 20km offset
- record time 14s, sample rate 4ms
- free surface
- isotropic elastic

W-4: Gulf of Mexico Imaging Challenge Part II: FWI, WEMVA, Workflows &...

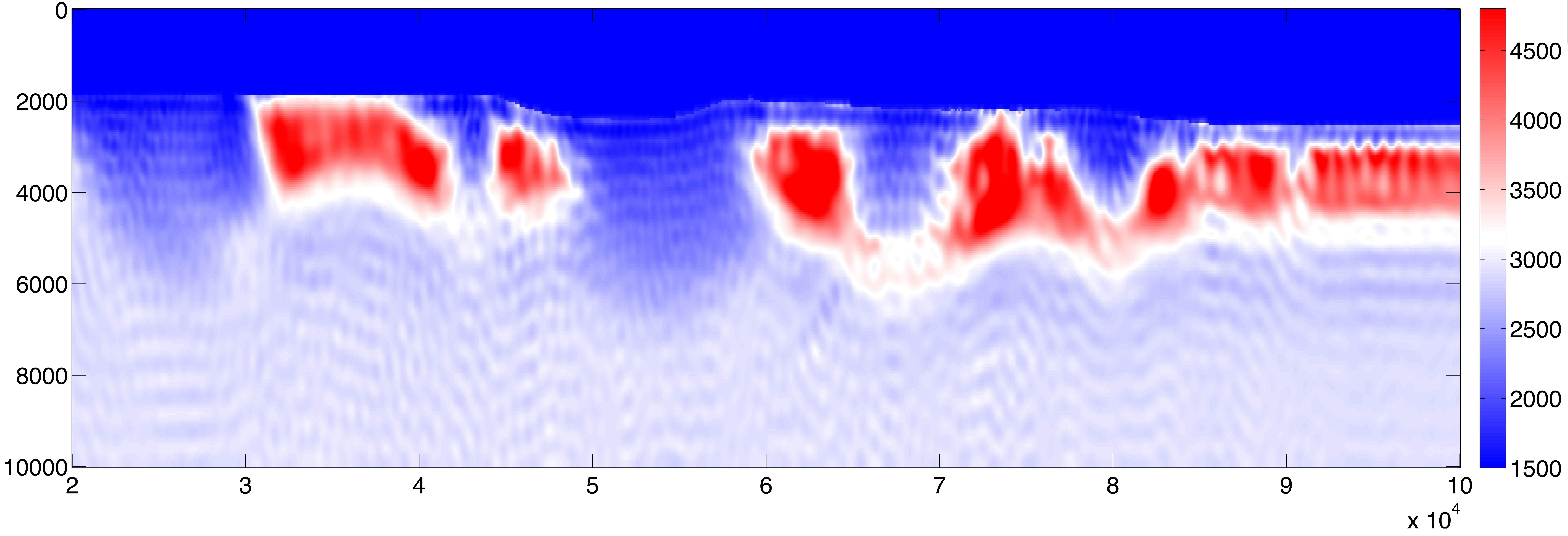
Thursday, 26 September, 1:30-5:00pm

Initial model [ray-based tomography]

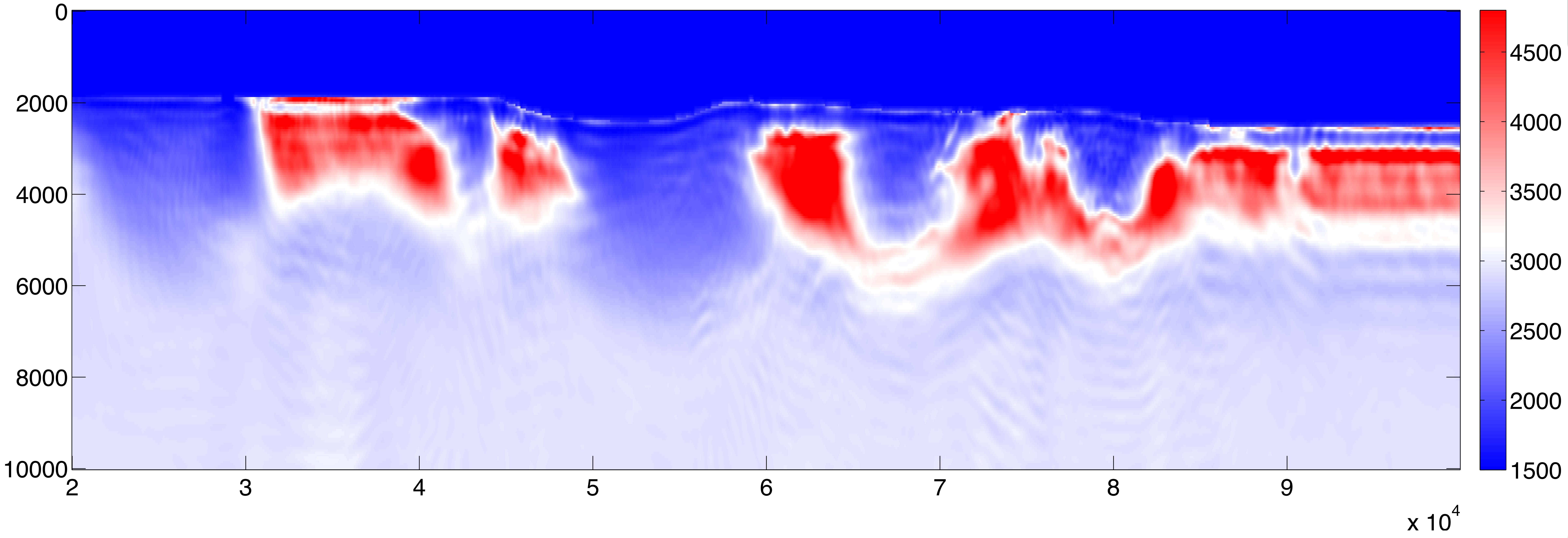


Andrew J. Calvert

Standard GN FWI without sparsity promoting

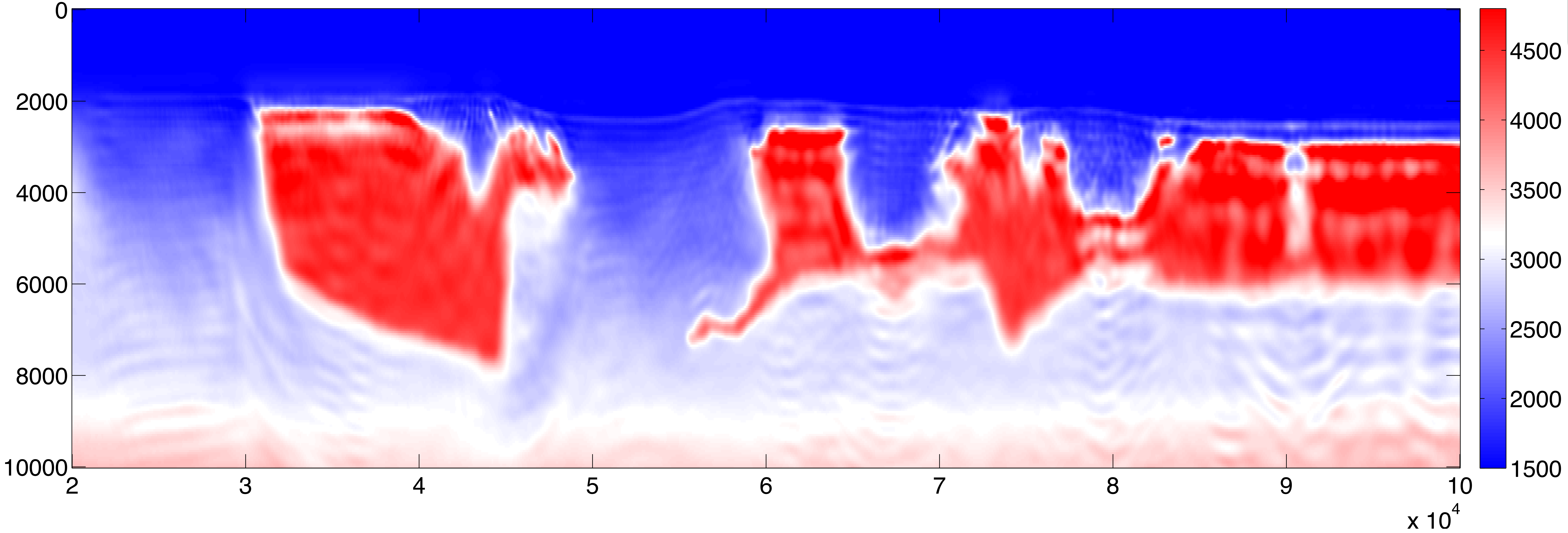


Sparsity promoting GN FWI



This is not our “latest” result

"Latest" result



W-4: Gulf of Mexico Imaging Challenge Part II: FWI, WEMVA, Workflows &...

Thursday, 26 September, 1:30-5:00pm

Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

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