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Seismic data interpolation via low-rank matrix factorization in the h(ierarchical) s(emi) s(eparable) representation

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Motivation

- acquisition challenges
  - missing data
  - noise

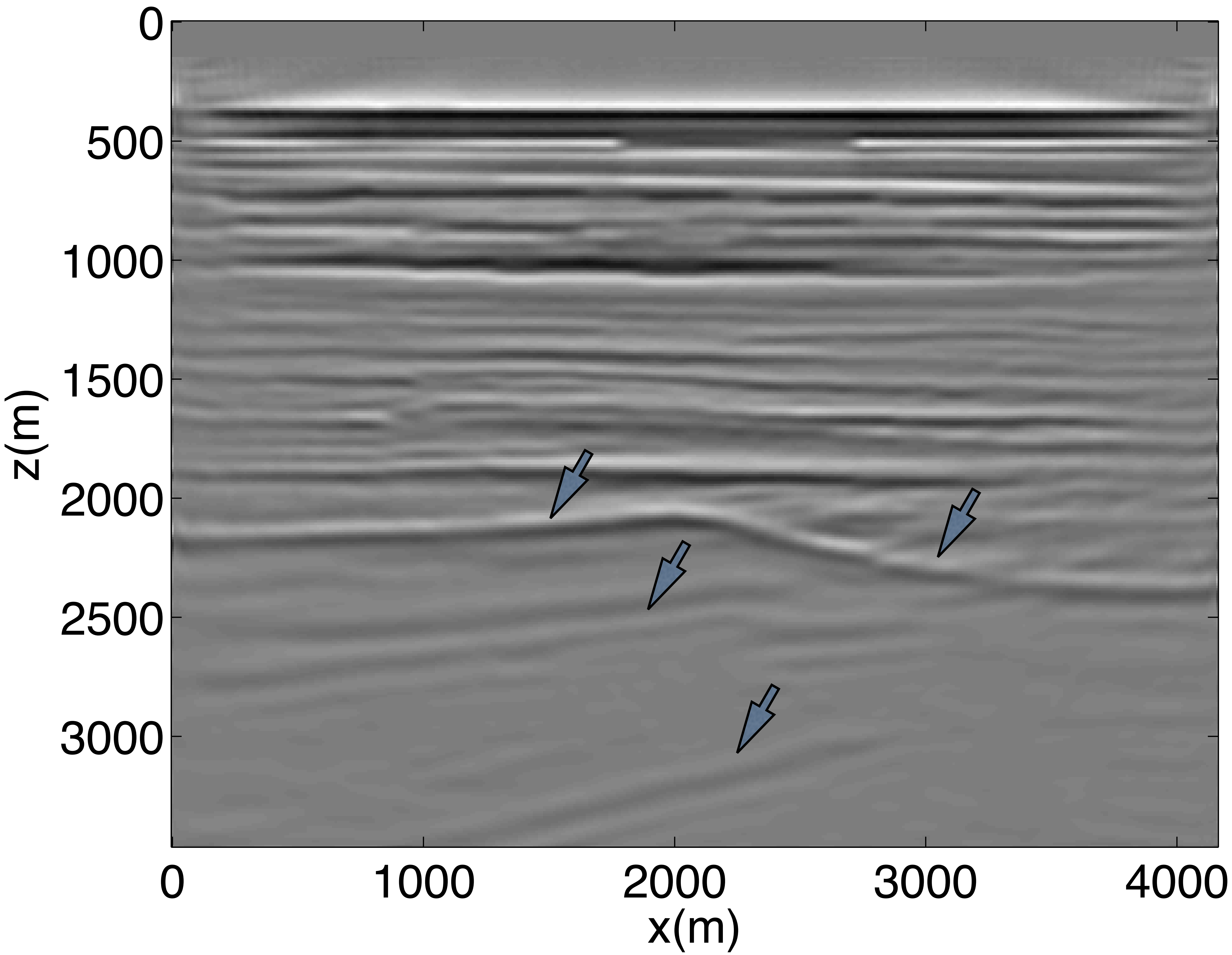
- fully sampled data
  - simultaneous shot based FWI & migration
  - estimation of primaries by sparse inversion & SRME

- exploit *low-rank* structure of seismic data
  - *SVD-free* matrix factorization
  - benefit of HSS

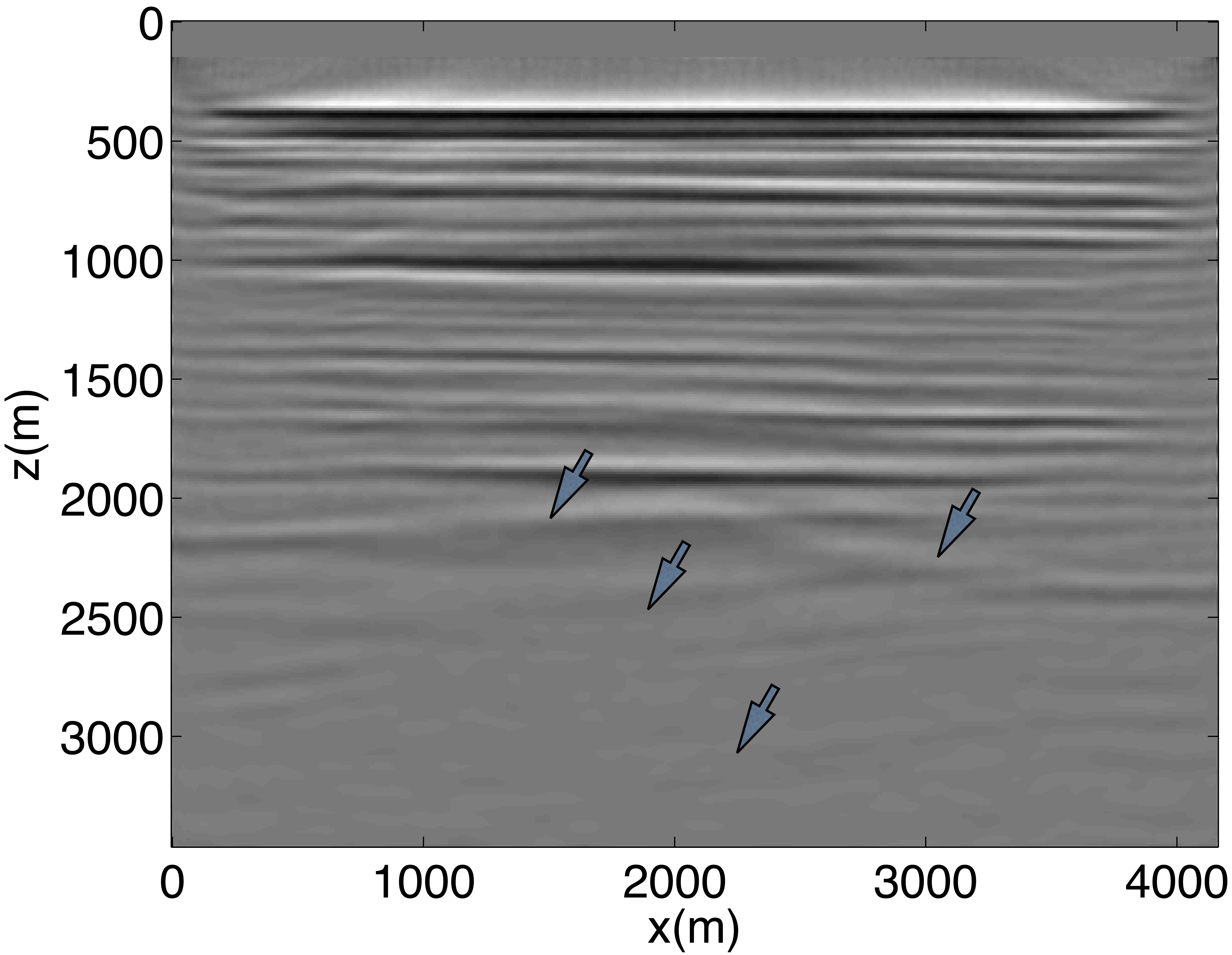
[Tu et. al. 12, li et. al. 12]

Migration

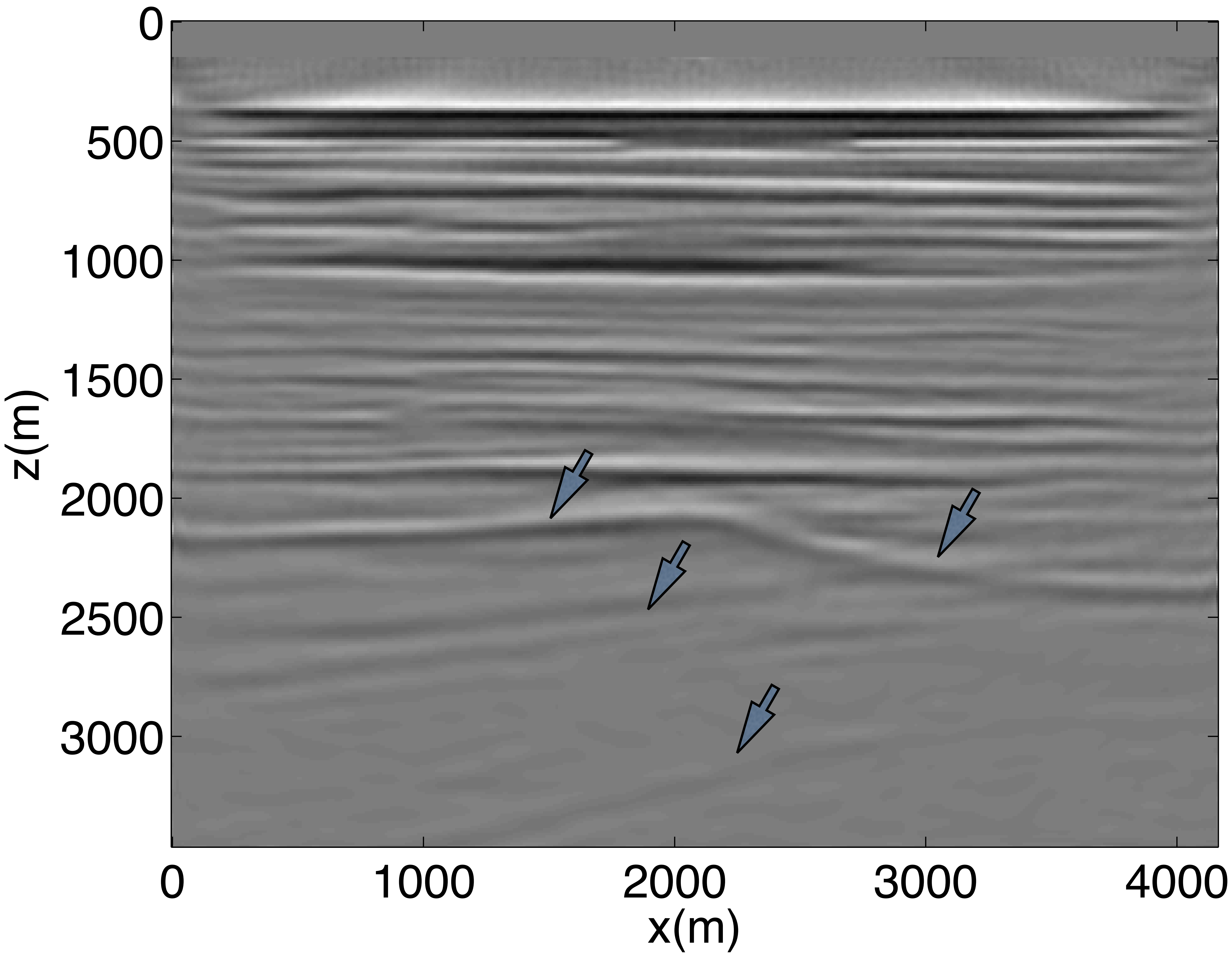
[fully sample data, 30 simultaneous shots, redraw]



Migration
[w/o HSS, 30 simultaneous shots, redraw]



Migration
[w/ HSS, 30 simultaneous shots, redraw]



[1] Oropeza V and M D Sacchi, 2011, Simultaneous seismic data de-noising and reconstruction via Multichannel Singular Spectrum Analysis (MSSA), Geophysics, 76 (3), V25-V32.
[2] Nadia Kreimer, Aaron Stanton and Mauricio D. Sacchi, Tensor completion based on nuclear norm minimization for 5D seismic data reconstruction, Technical report, 2013

Existing work

- SVD computation [1,2]
  - expensive for large scale data
- Overlap windows in space and time [2]
  - heuristic approach

[Candes and Donoho 2000, Herrmann 2008]

Compressive sensing
[transform based]

- signal structure
  - *sparse/compressible*
- sampling scheme
  - random missing traces make signal *less sparse* in transform domain
- recovery using *sparsity promoting* scheme

is *sparsity* the only inherent structure in seismic??

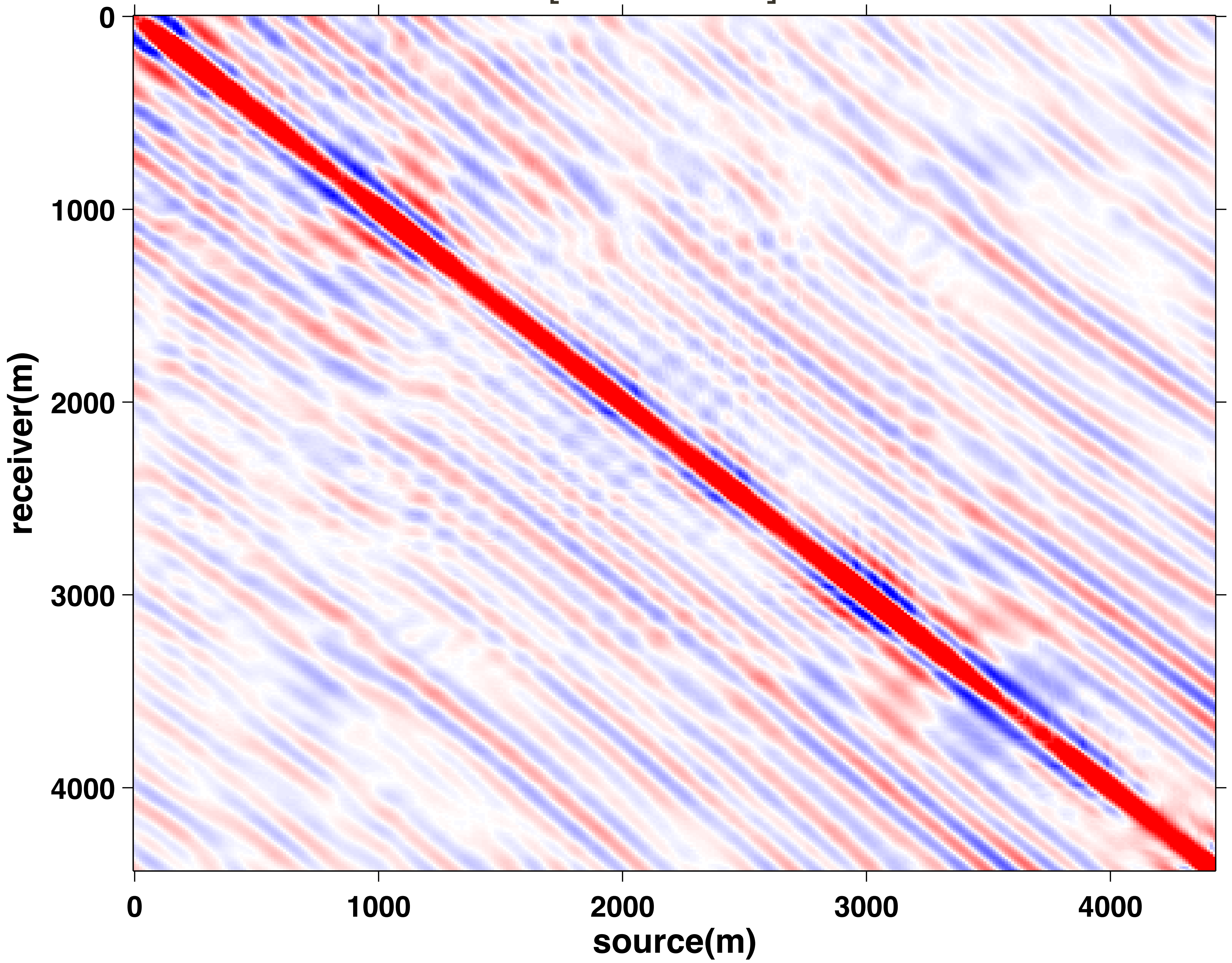
[Candes and Plan 2010, Oropeza and Sacchi 2011]

Matrix completion

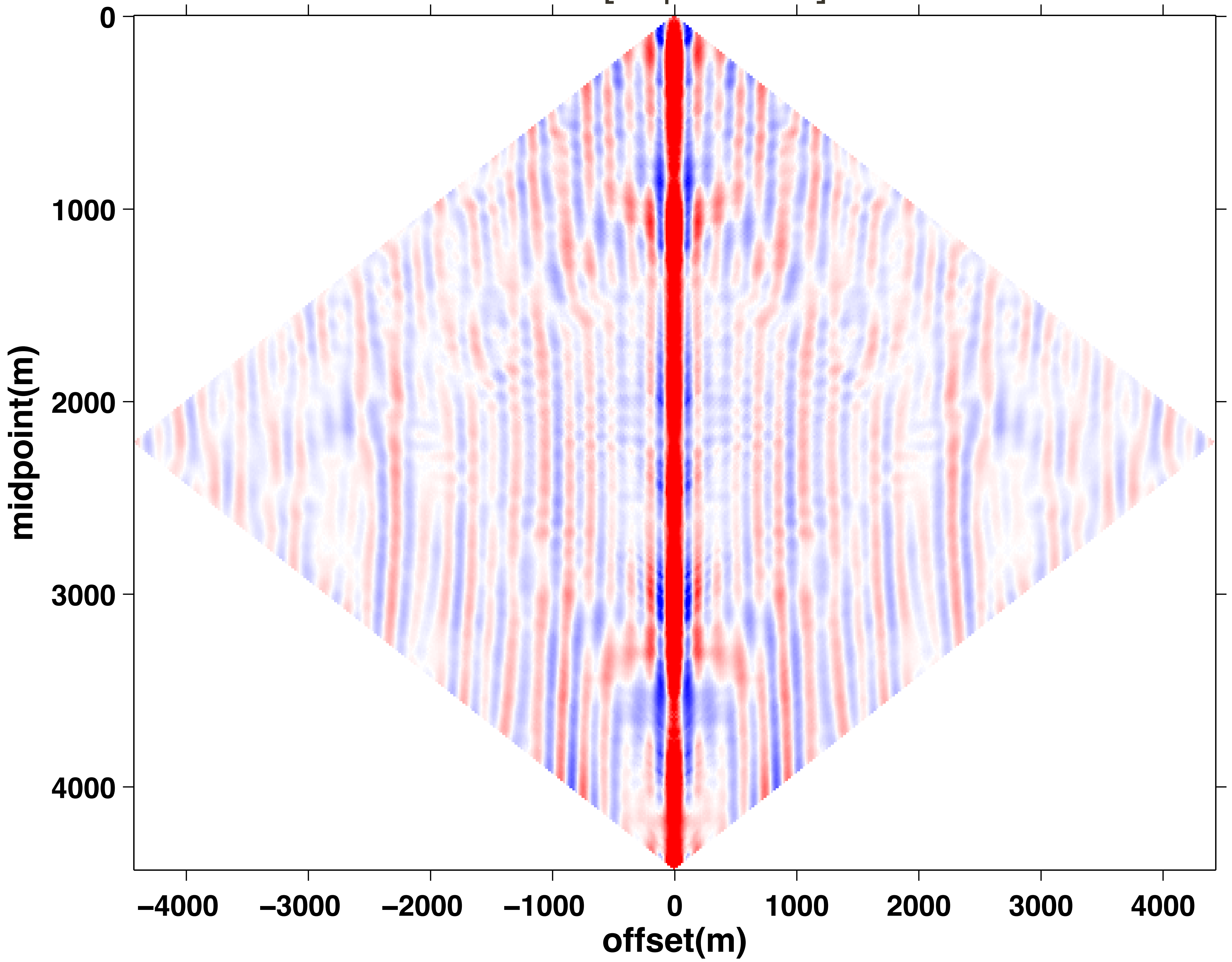
- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Low-rank structure
[2-D acquisition]

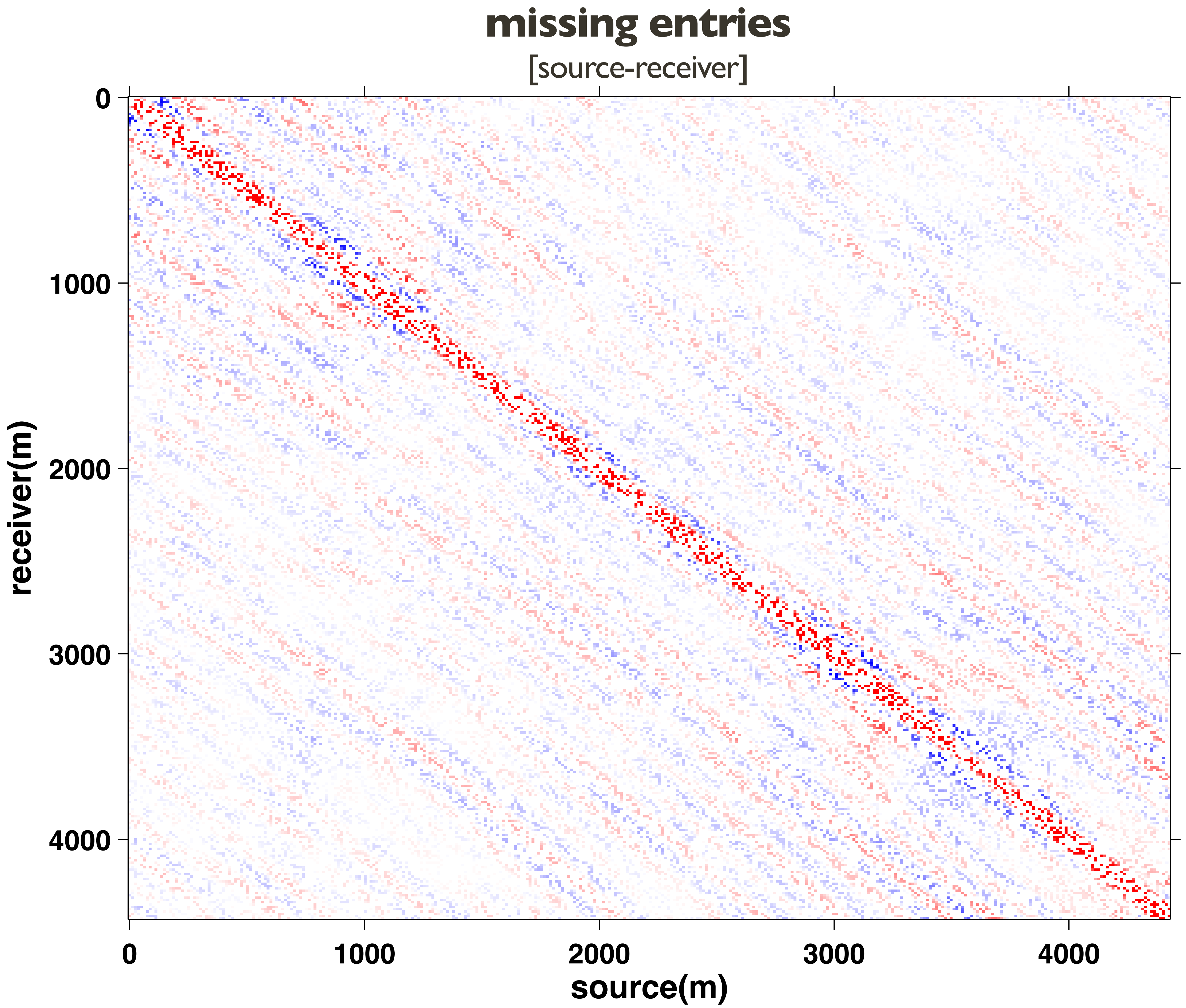
acquisition domain
[source-receiver]

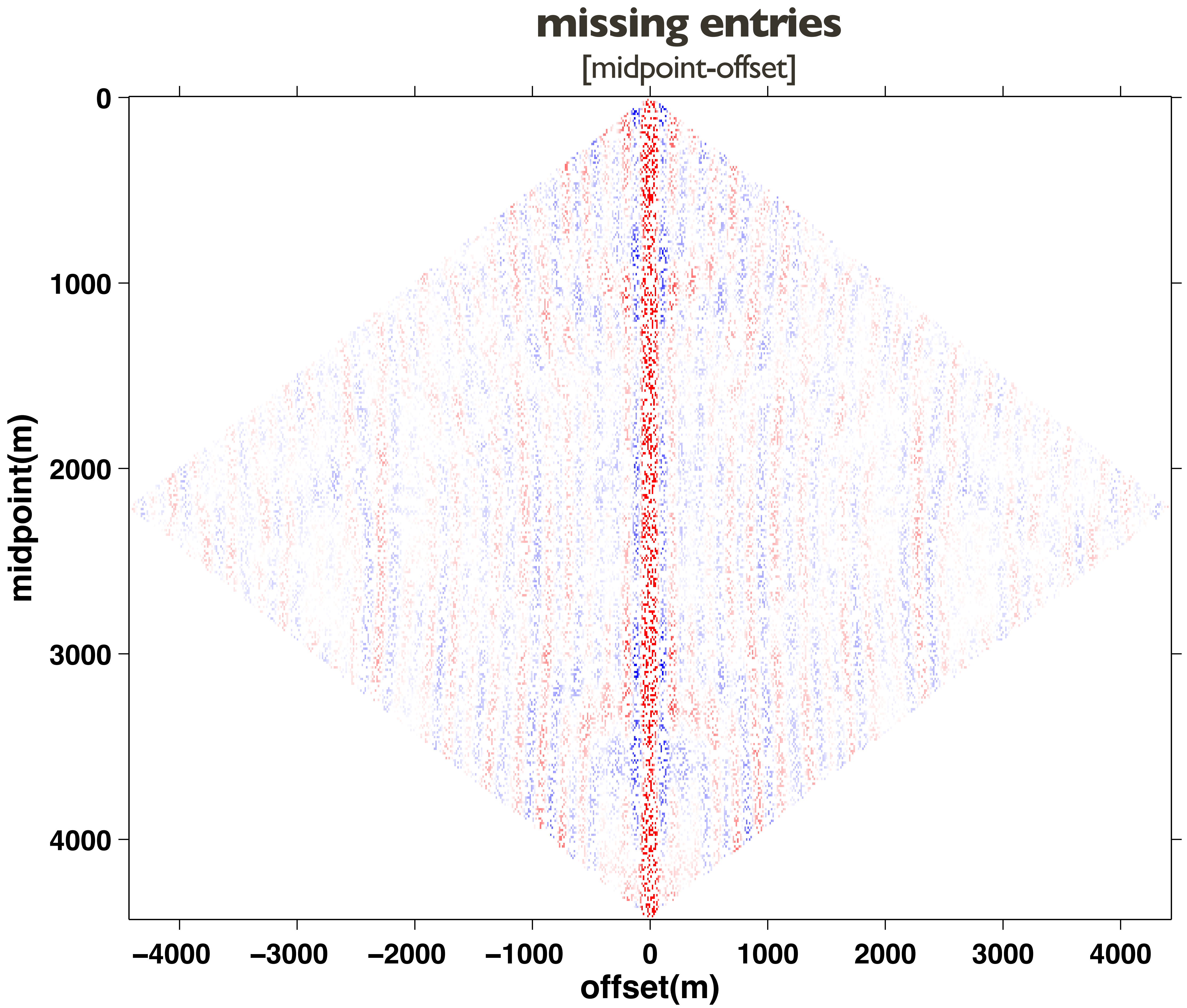


transform domain
[midpoint-offset]

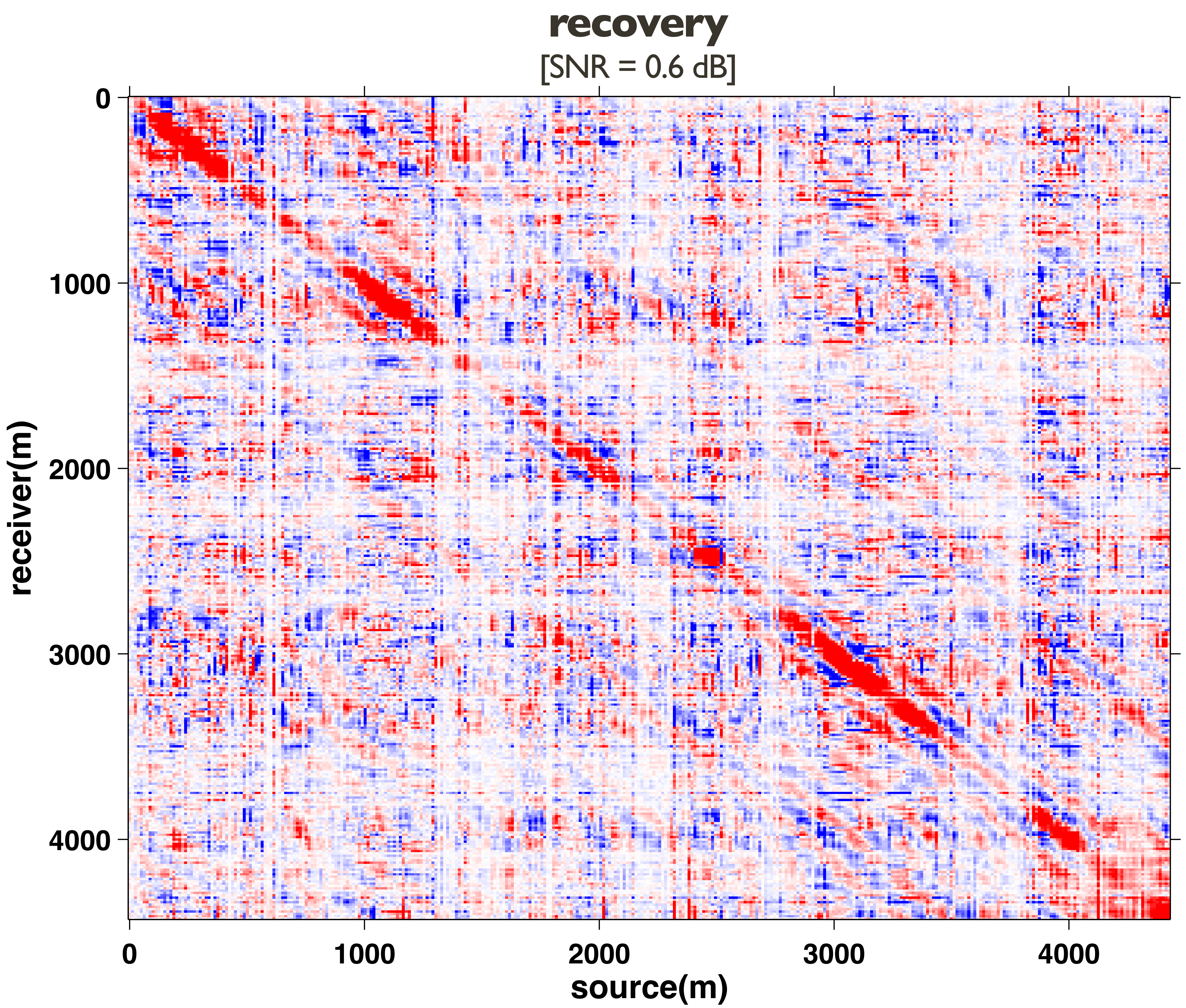


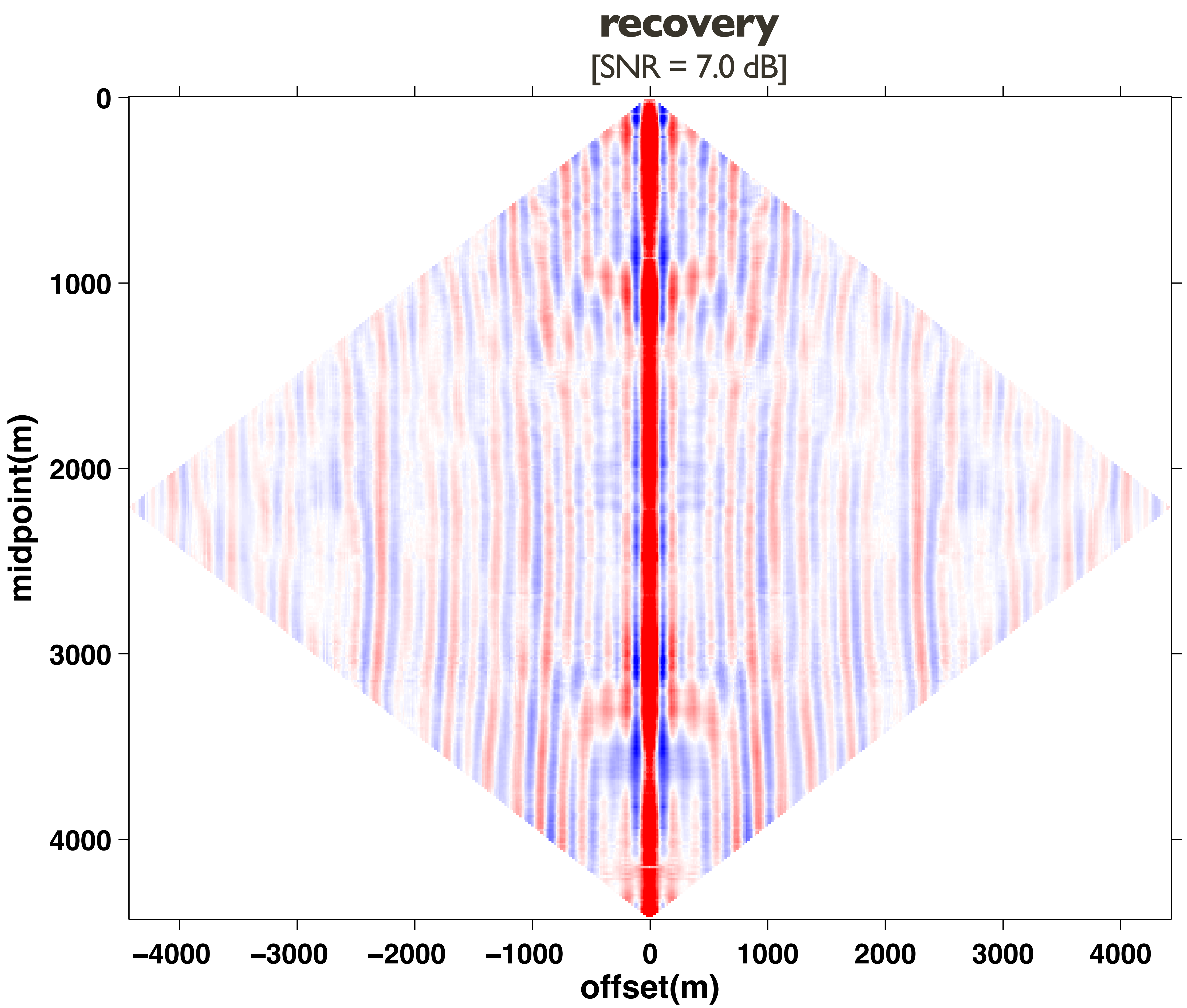
Matrix completion problem



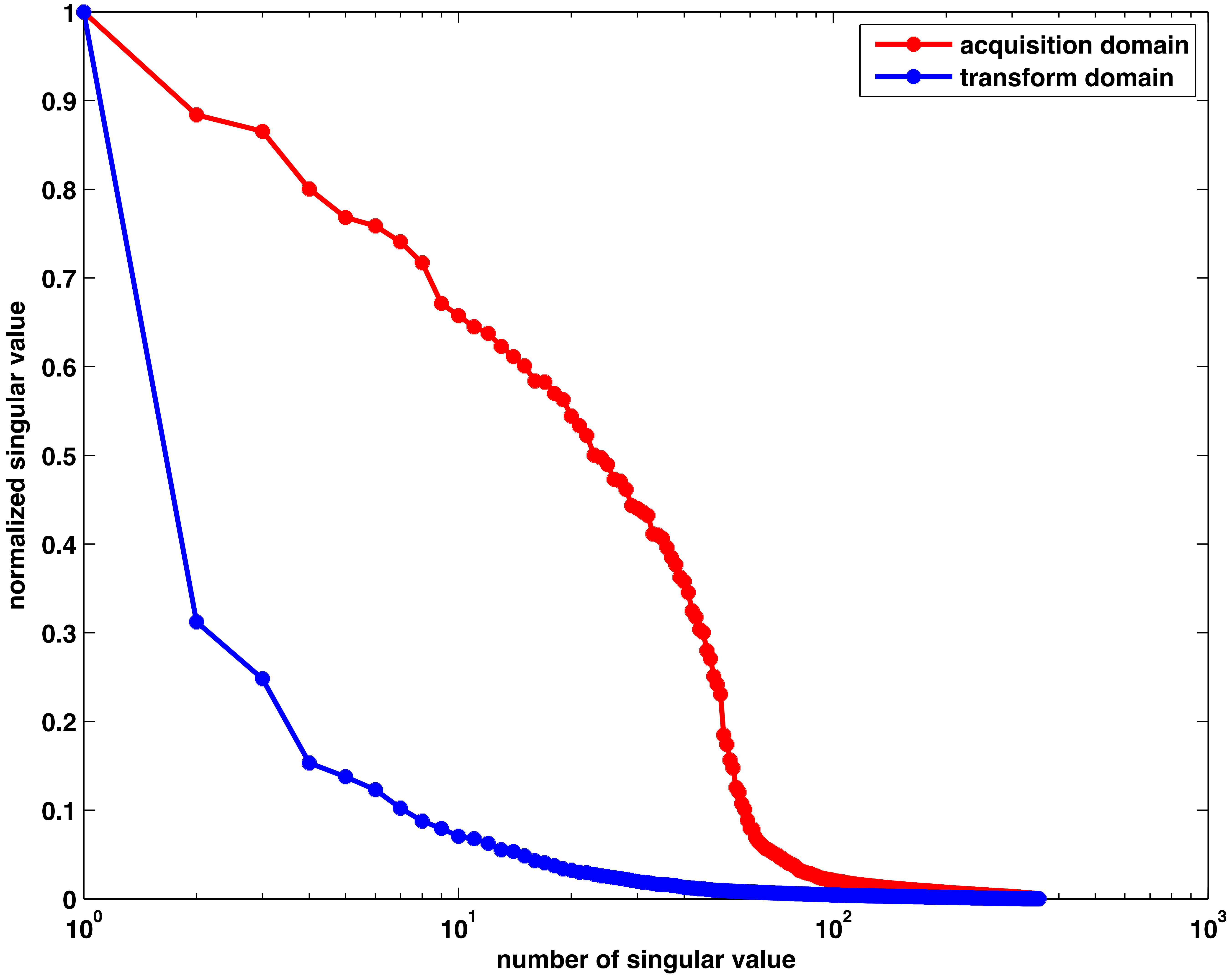


Low-rank interpolation

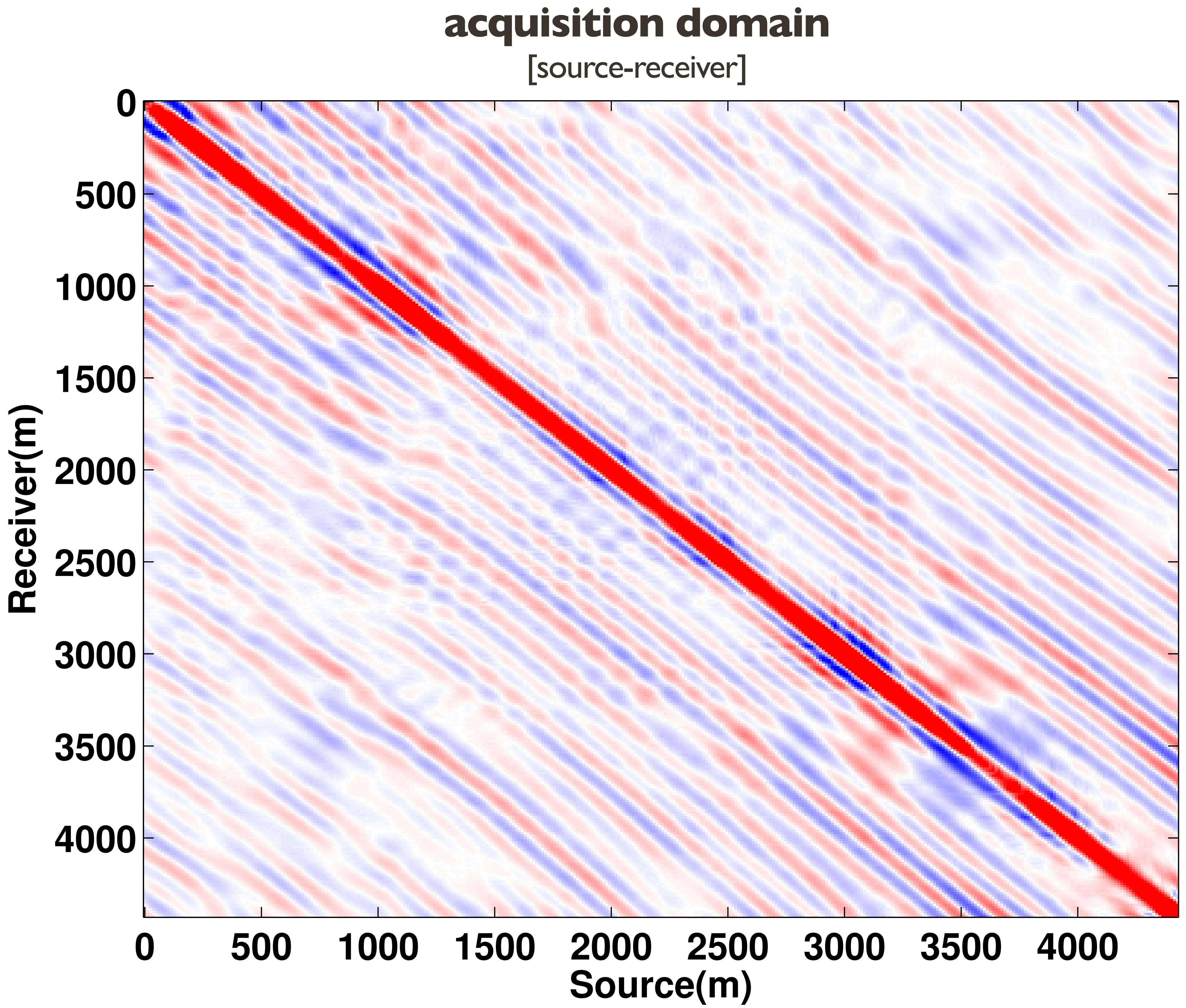


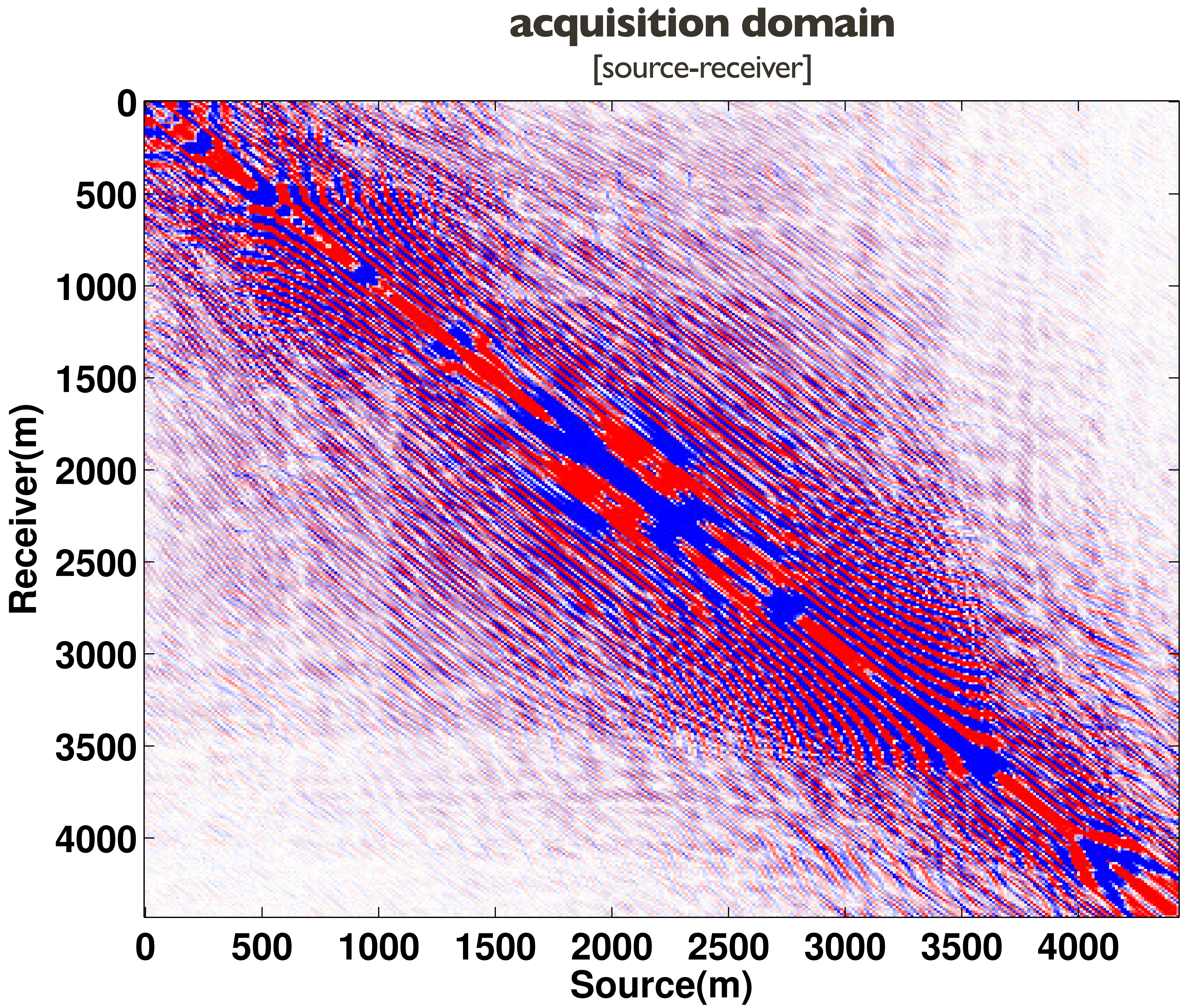


Singular value decay
[2-D acquisition]

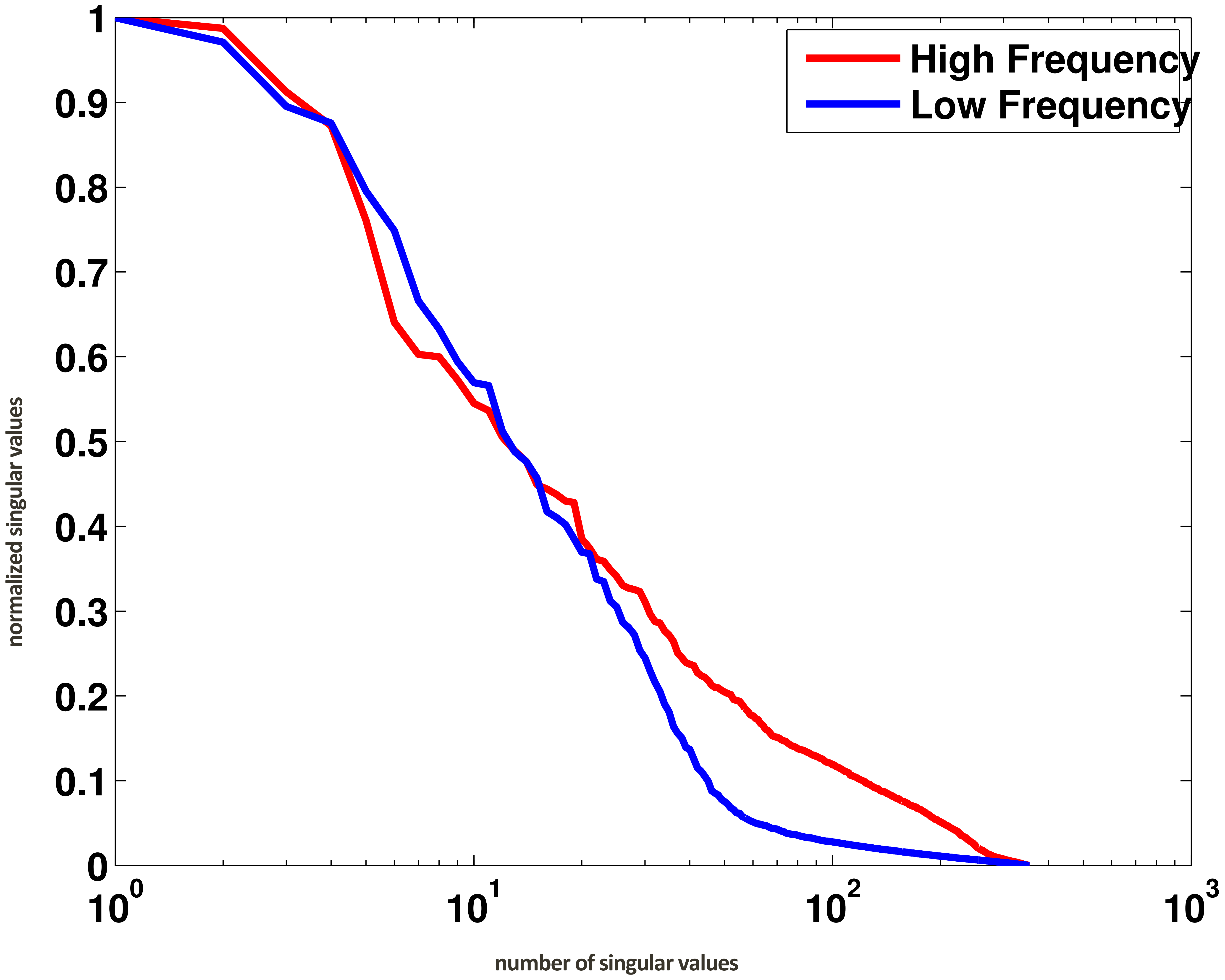


Is high frequency low-rank ?



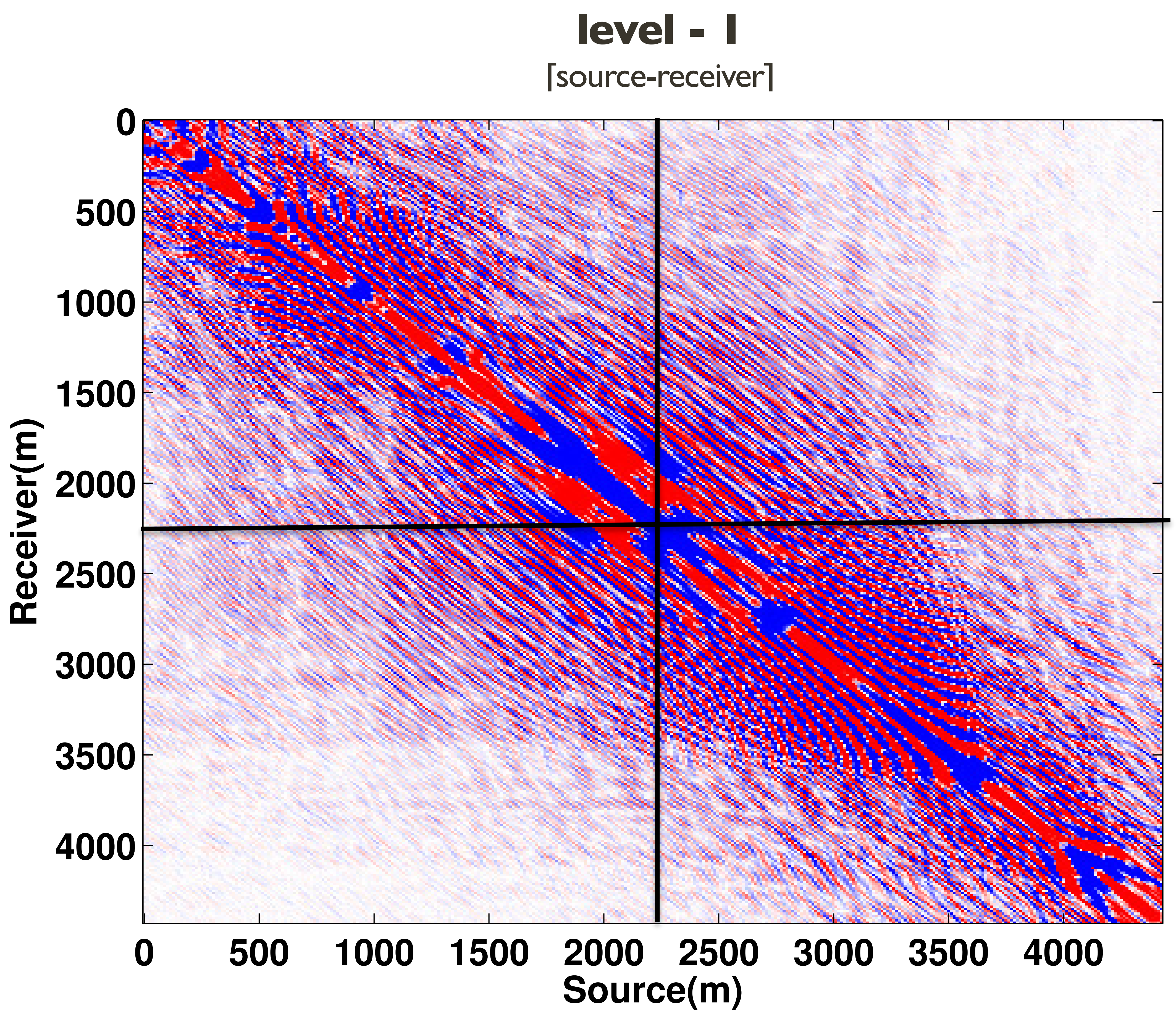


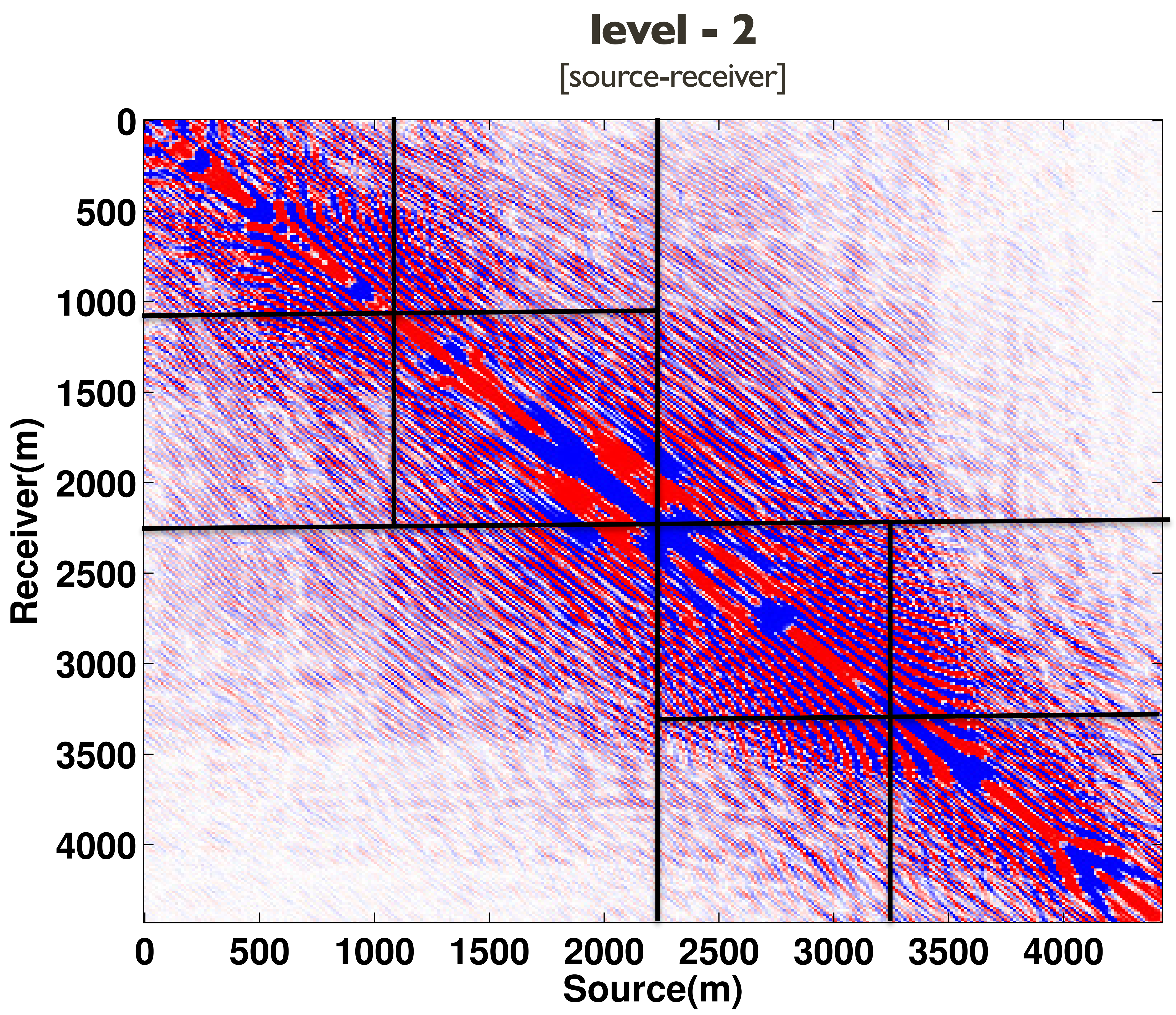
Singular value decay



[Chandrasekaran et al. (2006)]

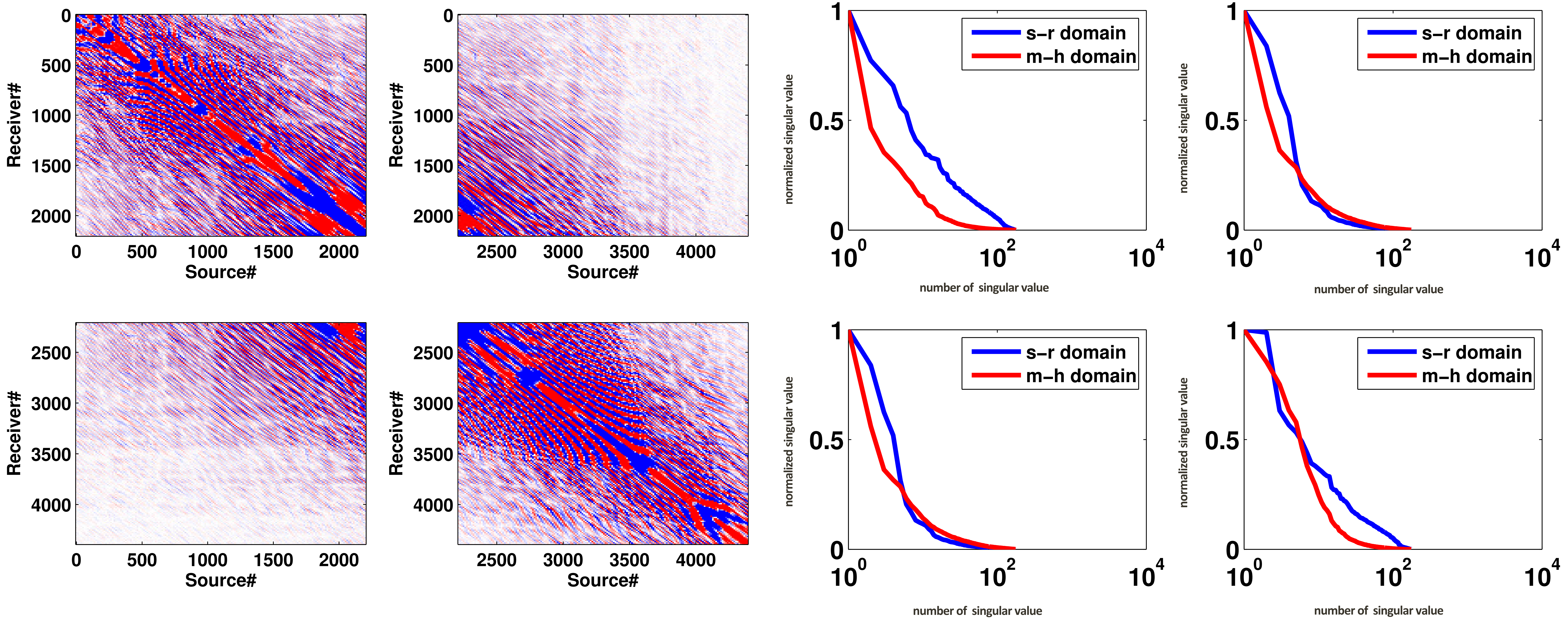
HSS representation





HSS representation

[level-1]



Matrix completion

- signal structure
  - *low rank/fast decay* of singular values

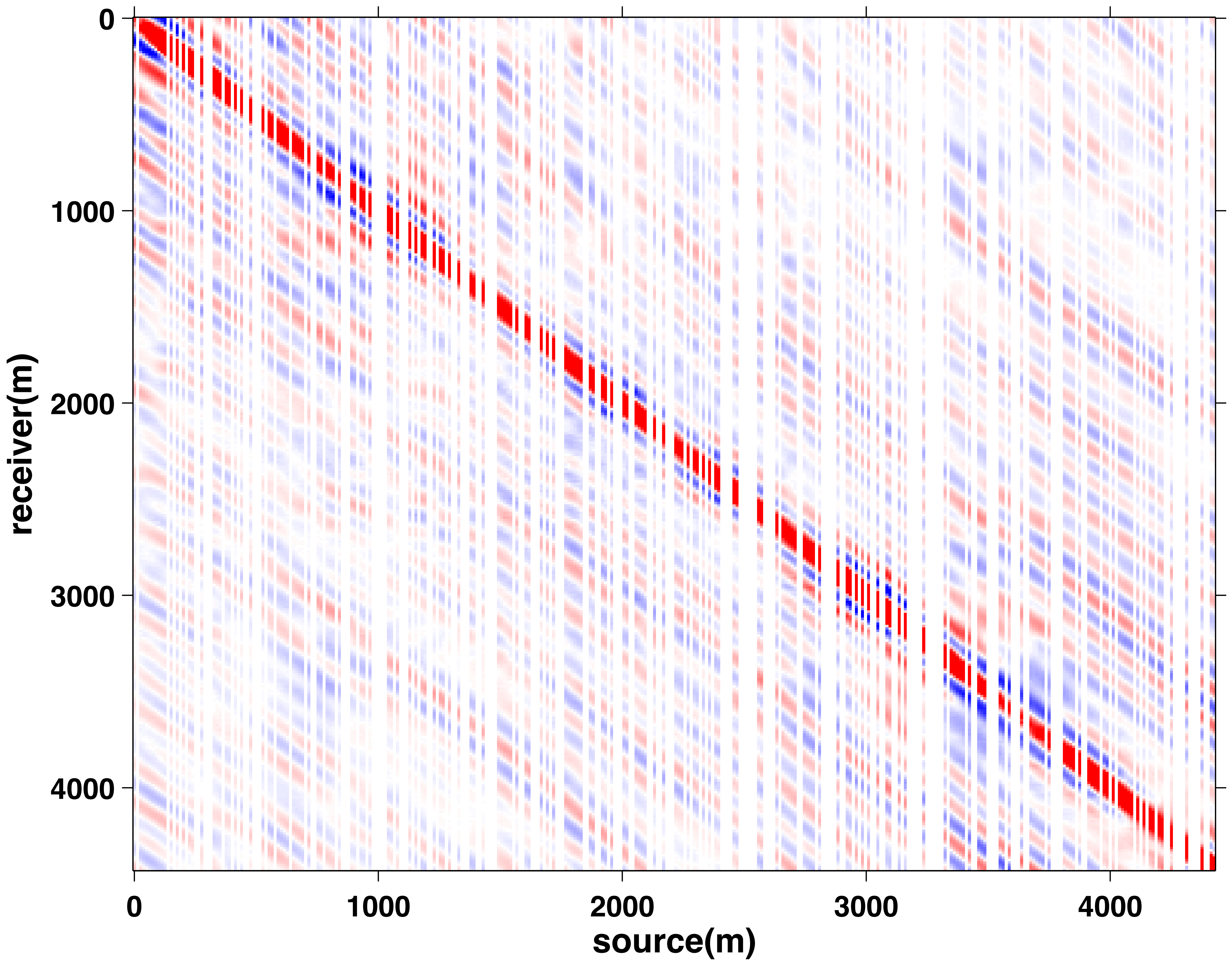
- sampling scheme
  - missing data *increase* rank in "transform domain"

- recovery using *rank penalization* scheme

2-D acquisition
[randomized sampling]

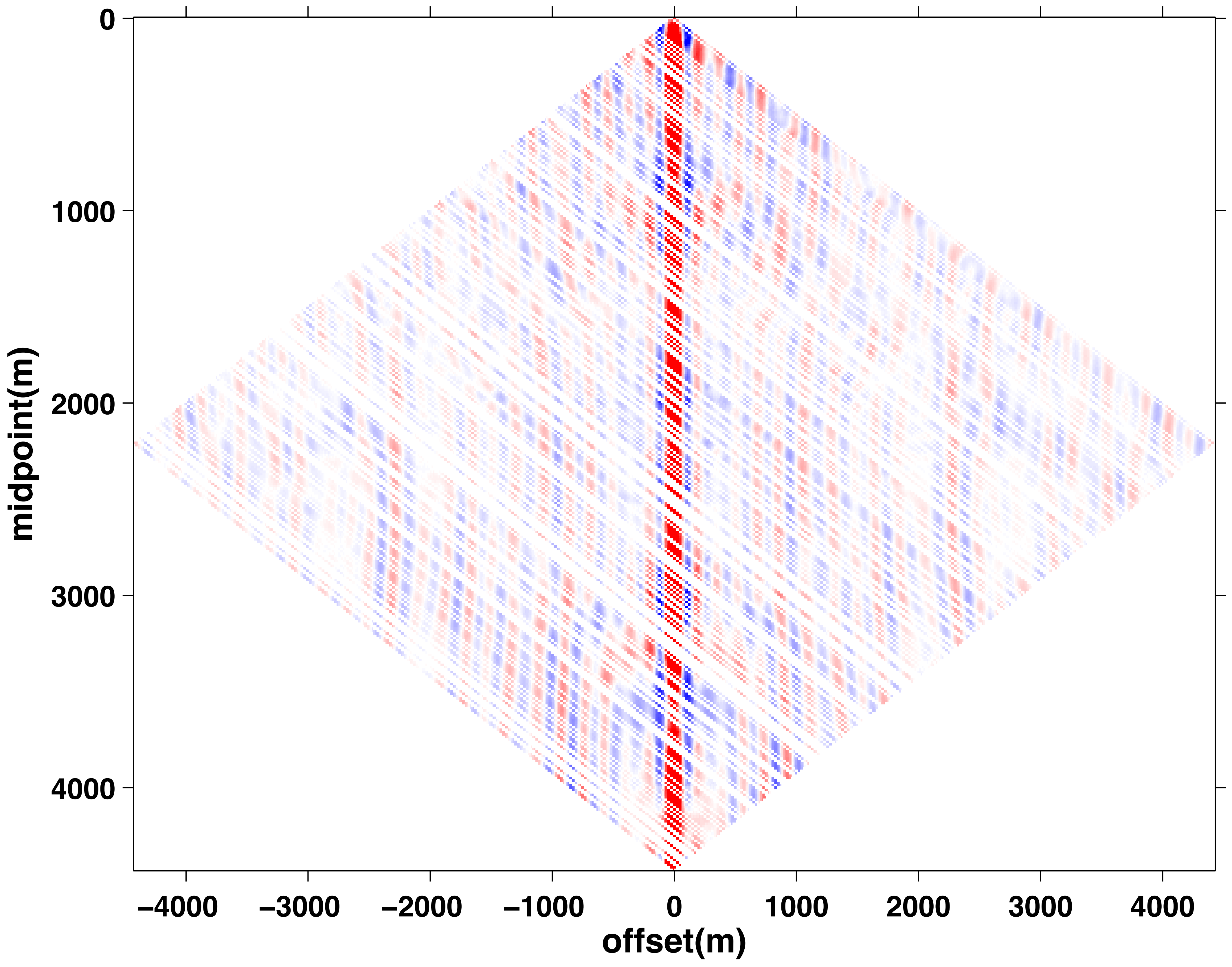
acquisition domain

missing columns *do not* increase rank



transform domain

missing columns *do* increase rank



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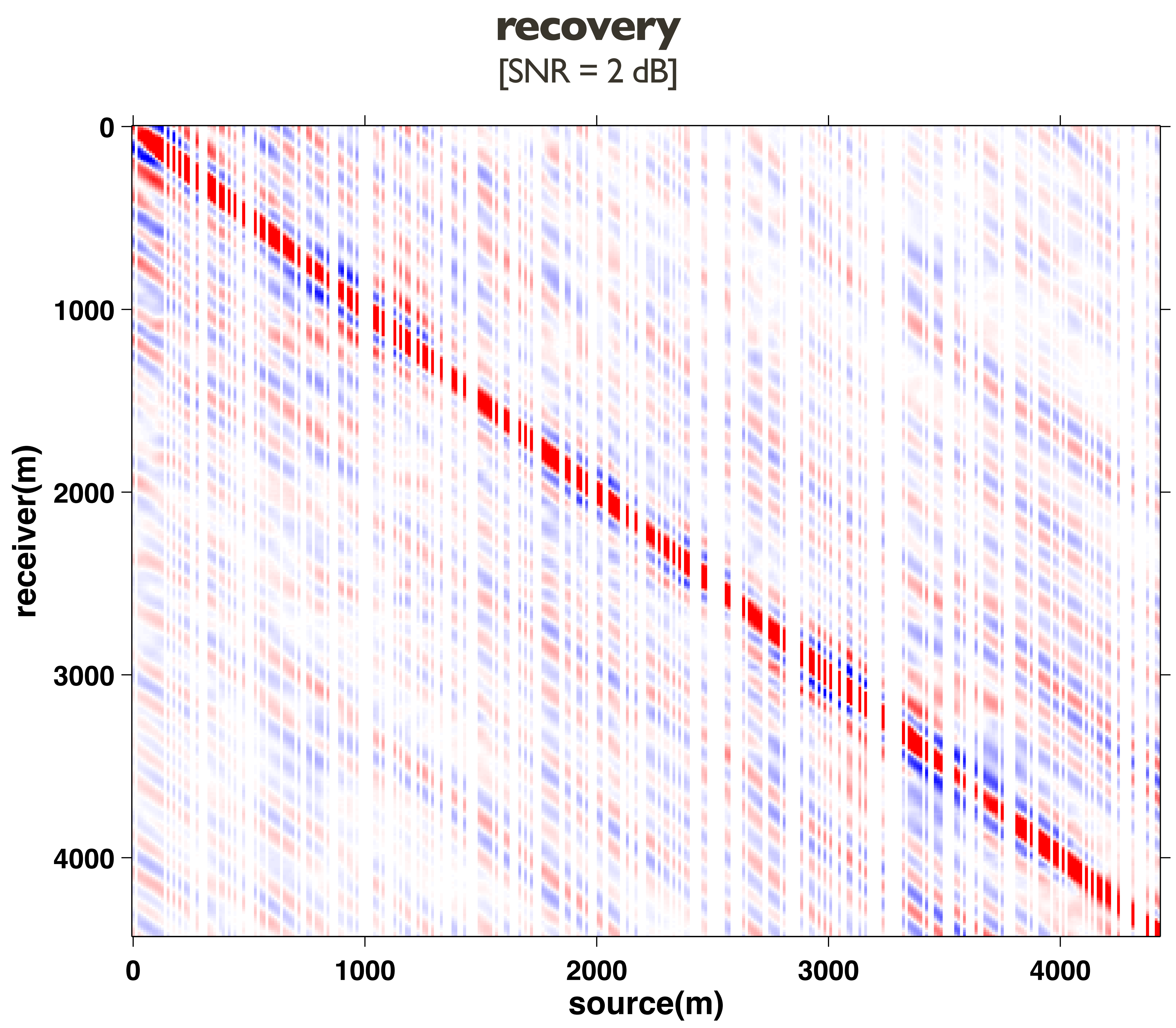
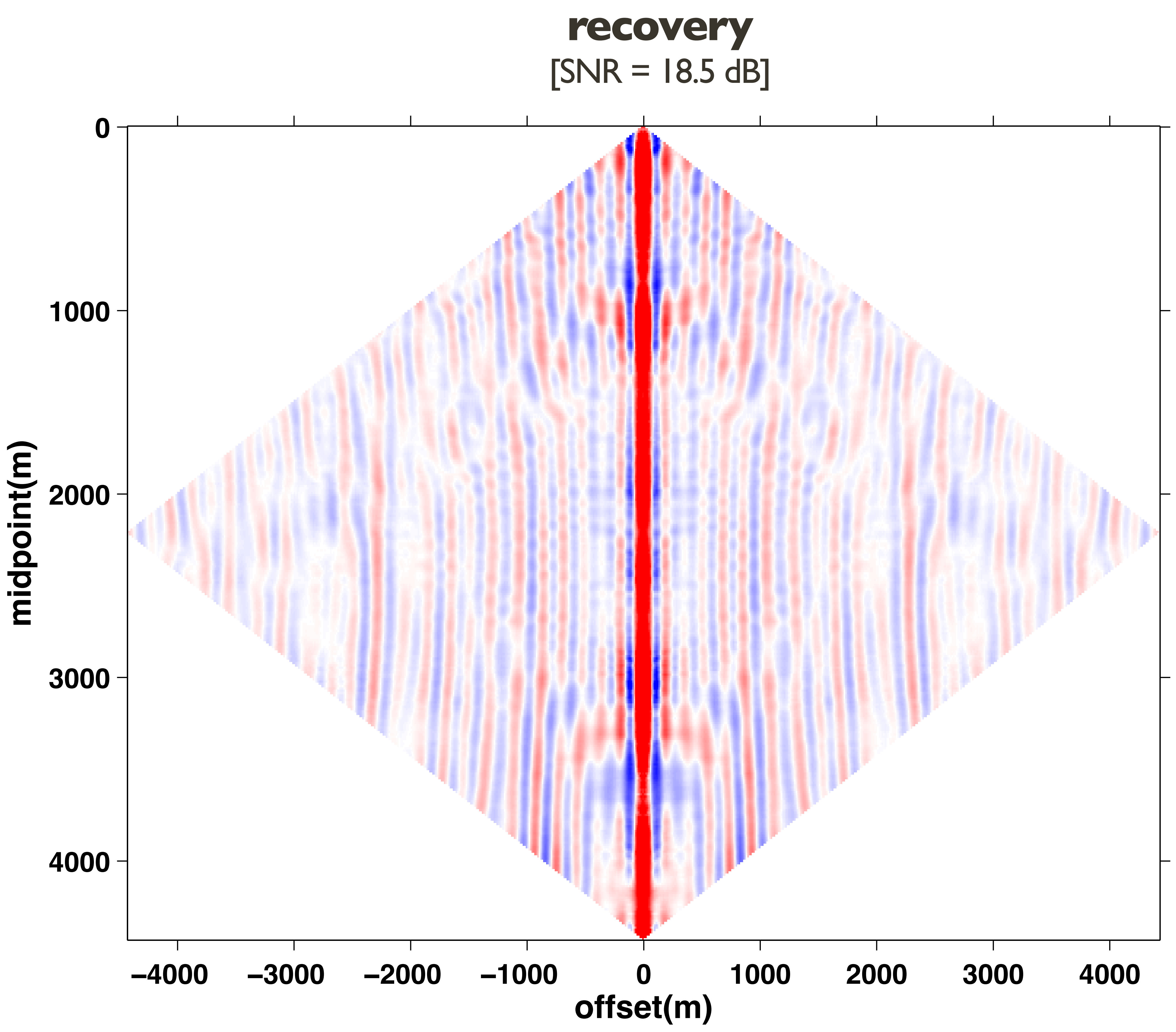
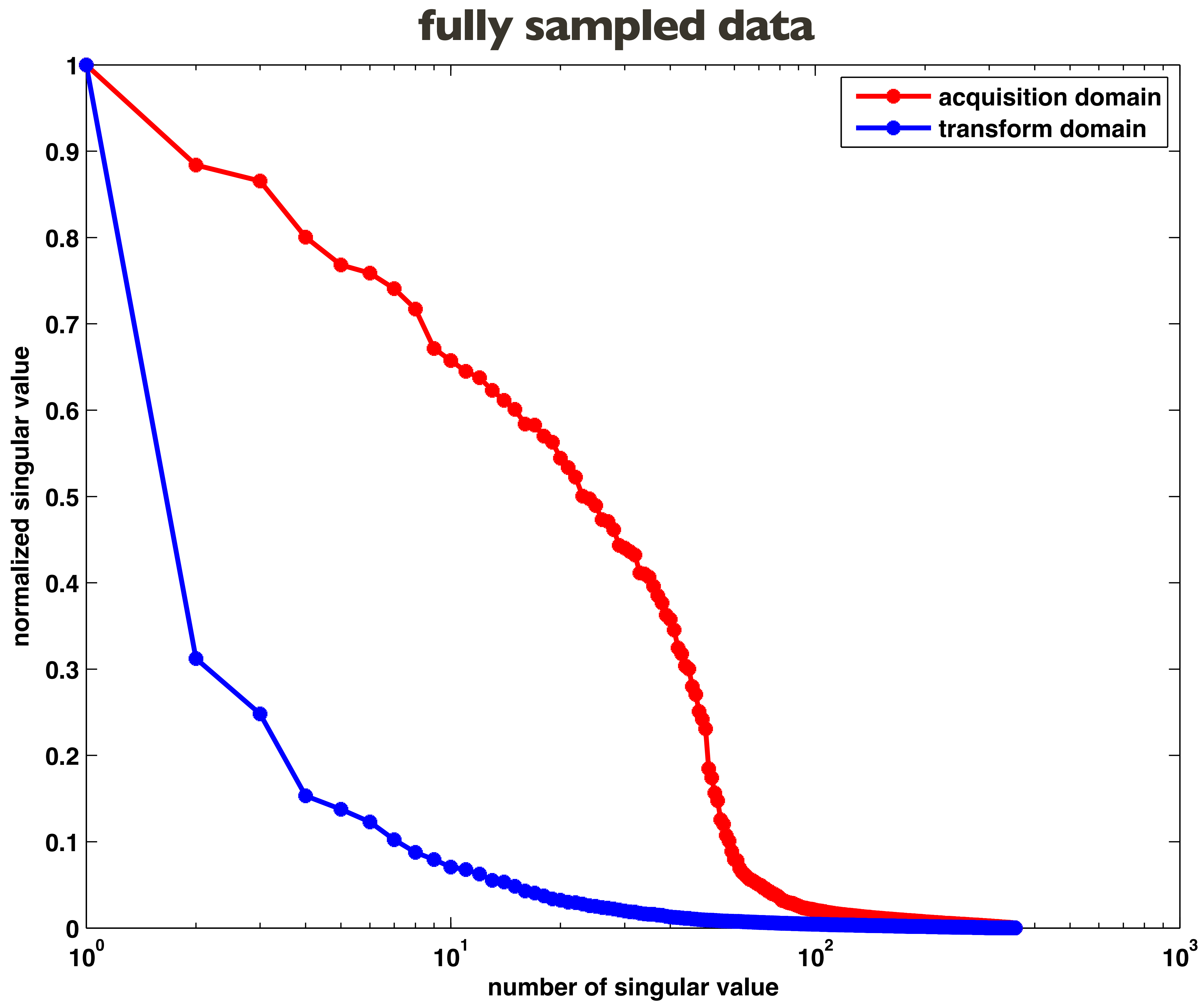
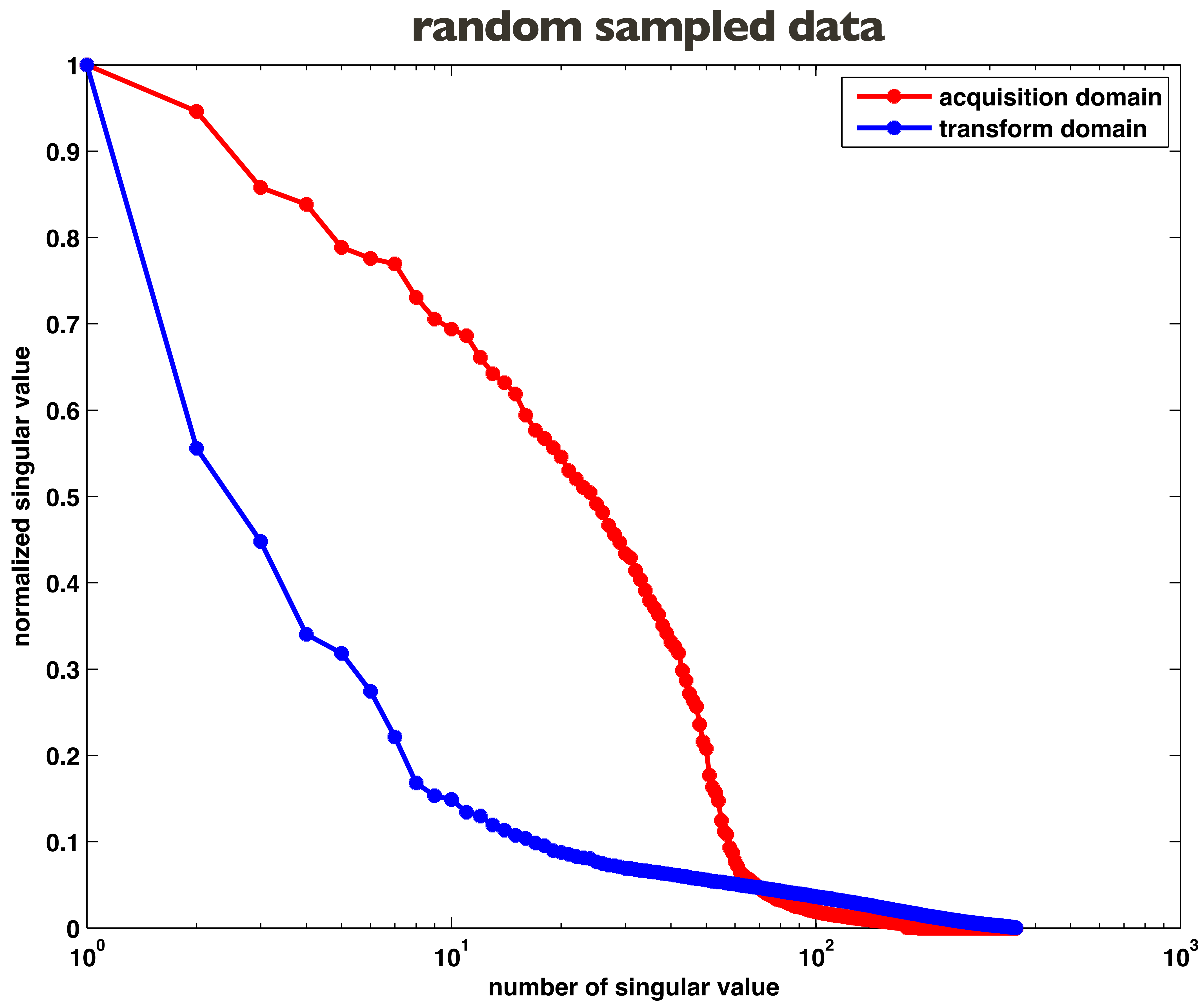
Low-rank interpolation





Randomized sampling
[singular value decay]

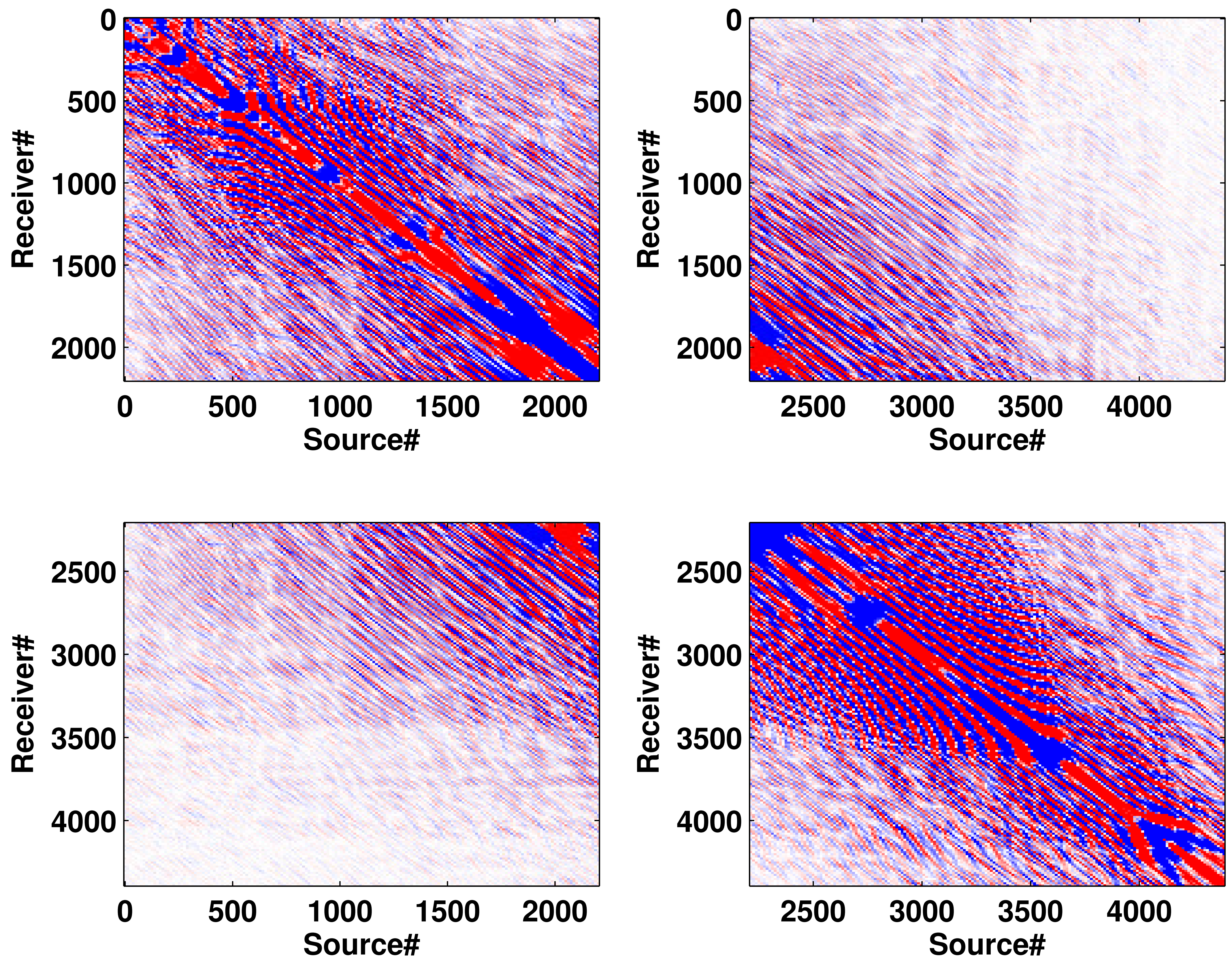




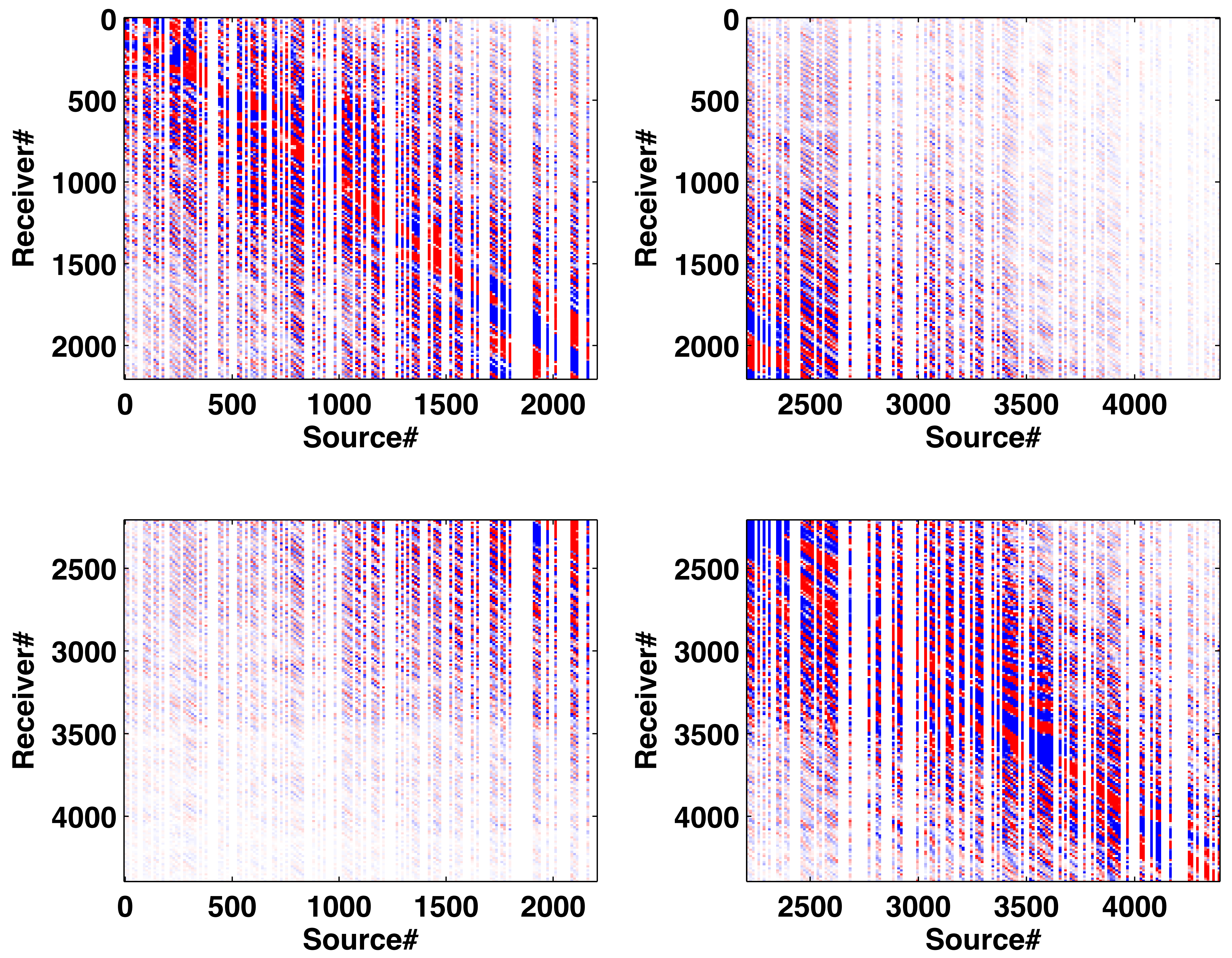
HSS representation

[level-1]

fully sampled data

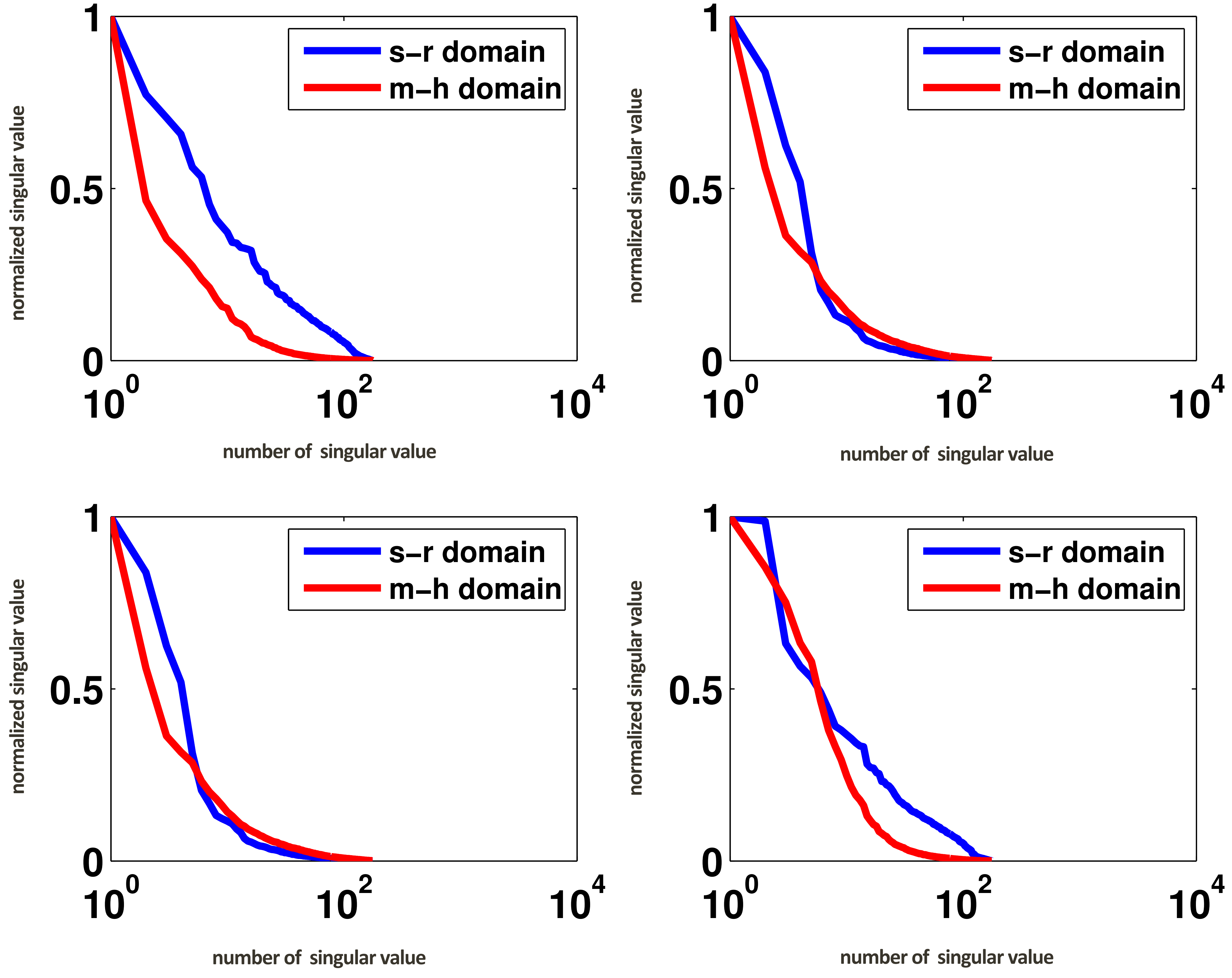


random sampled data

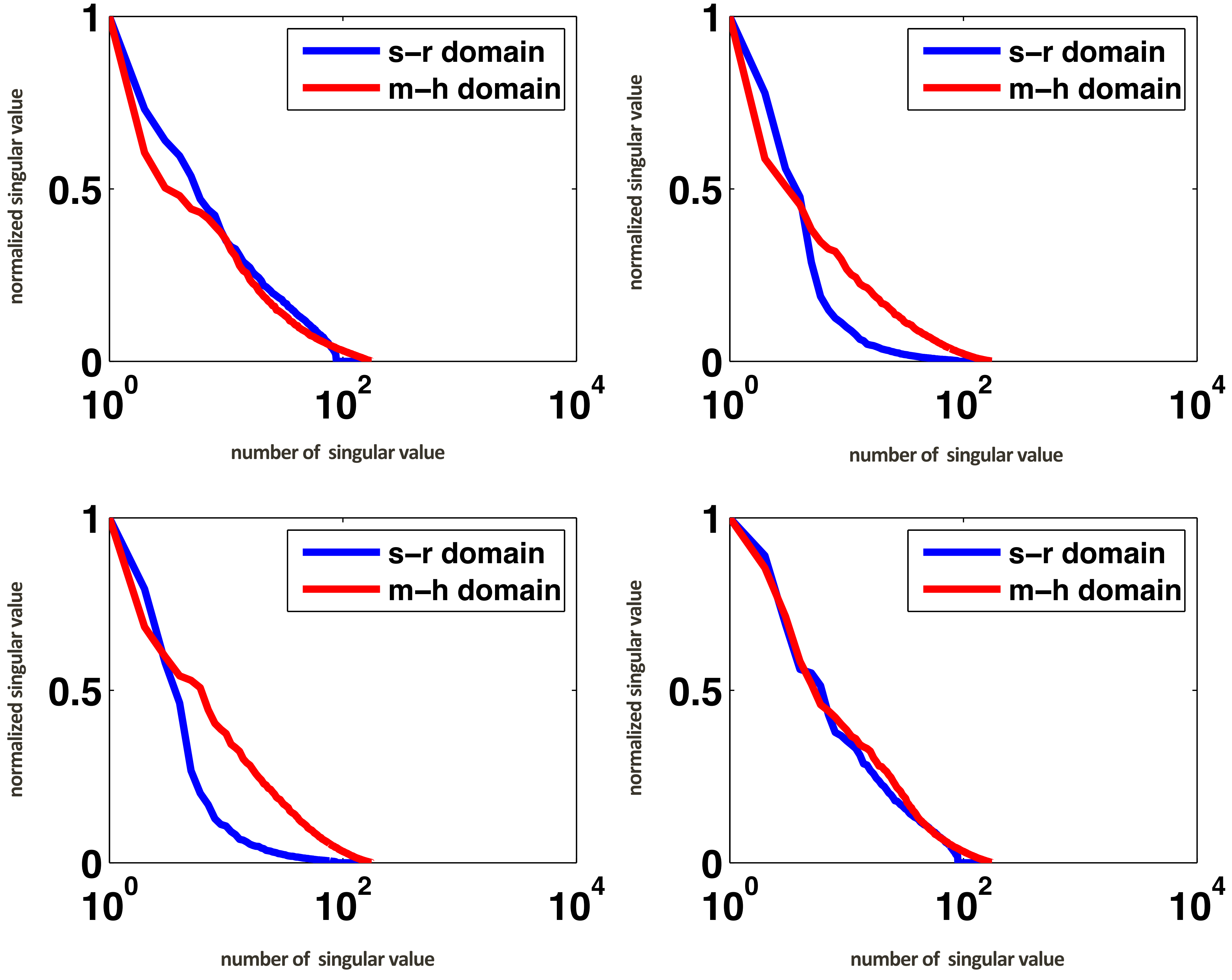


Singular value decay
[HSS, level-1]

fully sampled data



random sampled data



Matrix completion

- signal structure
  - *low rank/fast decay* of singular values
- sampling scheme
  - missing data *increase* rank in "transform domain"
- recovery using *rank penalization* scheme

Rank minimization

- given a set of measurements $\mathbf{b}$, aim is to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} \quad \mathrm{rank}(\mathbf{X}) \quad \text{s.t.} \ ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where $\mathrm{rank}(\mathbf{X}) =$ number of singular values of $\mathbf{X}$

- $\mathcal{A}$ is the transform-sampling operator defined as

$$\mathcal{A} = \mathbf{RM}\mathcal{S}^H$$

where

$\mathbf{R}$: restriction operator
$\mathbf{M}$: measurement operator
$\mathcal{S}^H$: transform operator

Rank minimization

- prohibitively *expensive*
  - do not know rank value in advance
  - search over all possible values of rank
- instead solve nuclear-norm minimization
  - convex relaxation of rank-minimization [Recht et. al. 2010]

[Recht et. al. 2010]

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Nuclear-norm minimization

- we want to solve

$$(BPDN_\sigma) \qquad \min_{\mathbf{X}} ||\mathbf{X}||_* \quad \text{s.t. } ||\mathcal{A}(\mathbf{X}) - \mathbf{b}||_2^2 \leq \sigma$$

where

$$\|\mathbf{X}\|_* = \sum_{i=1}^{m} \lambda_i = \|\lambda\|_1$$

where λ_i are the *singular* values

Challenges

- requires repeated application of *SVD* for projections

- expensive to compute for large system
  - curse of dimensionality

- can we exploit rank structure "*SVD* free"

[Rennie and Srebro 2005, Lee et. al. 2010, Recht and Re 2011]

Factorized formulation

$$\mathbf{X} \in \mathbb{R}^{n \times m} = \mathbf{L} \in \mathbb{R}^{n \times k} \quad \mathbf{R}^H \in \mathbb{R}^{k \times m}$$

$$\mathbf{X} = \mathbf{L}\mathbf{R}^H$$

[Berg and Friedlander 2008, Aravkin et al. 2012b]

Factorized formulation

- reformulate $(BPDN_\sigma)$ formulation

$$\min_{\mathbf{L},\mathbf{R}} ||\mathbf{L}\mathbf{R}^H||_* \quad \text{s.t.} \quad ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \leq \sigma$$

- approximately solve a series of $LASSO_\tau$ formulation

$$v(\tau) = \min_{\mathbf{L},\mathbf{R}} ||\mathcal{A}(\mathbf{L}\mathbf{R}^H) - \mathbf{b}||_2^2 \quad \text{s.t.} \quad ||\mathbf{L}\mathbf{R}^H||_* \leq \tau$$

where τ is a rank regularization parameter

[Rennie and Srebro 2005]

Factorized formulation

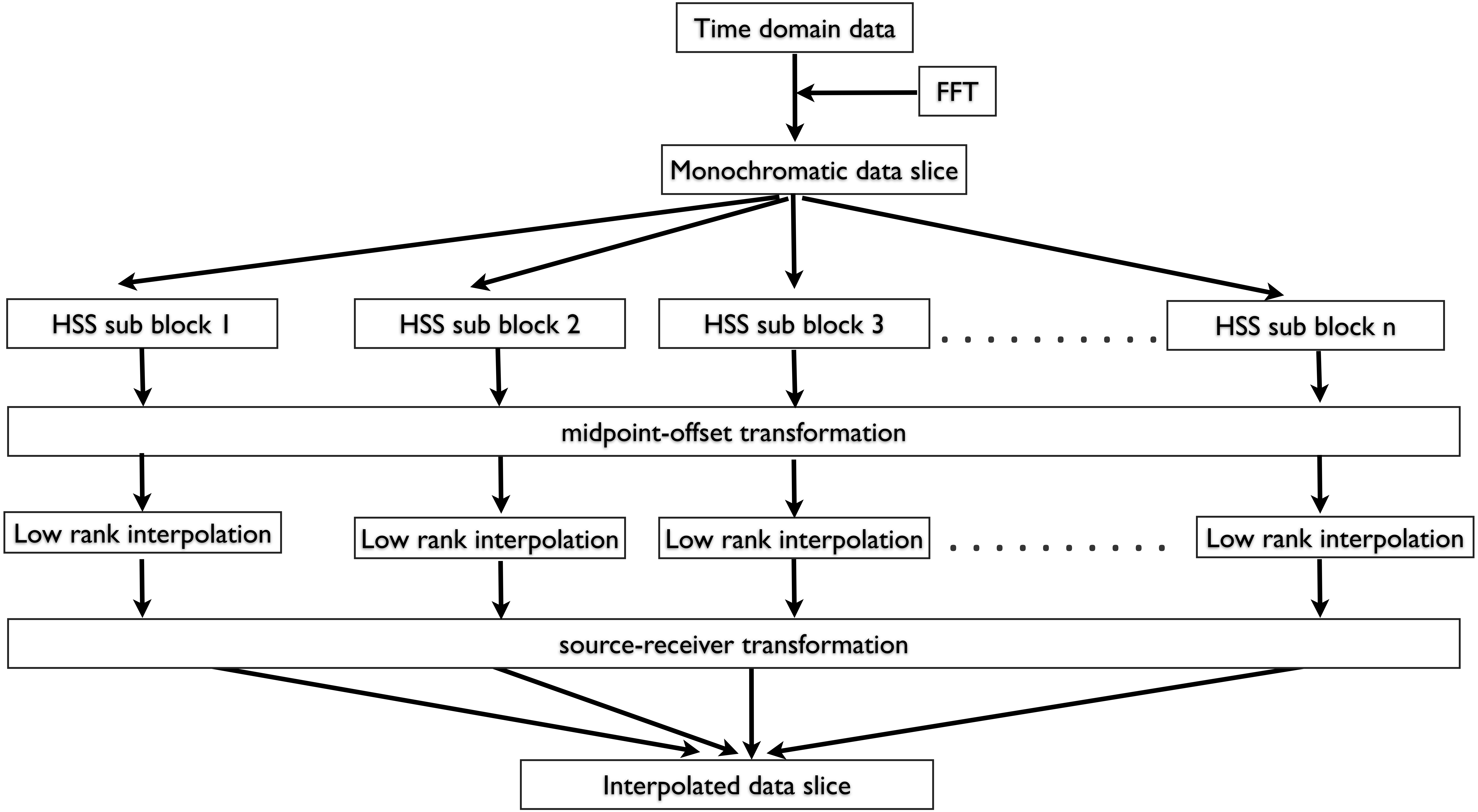
- Upper-bound on nuclear norm is defined as

$$\|\mathbf{L}\mathbf{R}^H\|_* \leq \frac{1}{2}\left\|\begin{bmatrix}\mathbf{L}\\ \mathbf{R}\end{bmatrix}\right\|_F^2$$

where $\|\cdot\|_F^2$ is sum of squares of all entries

- choose k *explicitly* & avoid costly *SVD's*

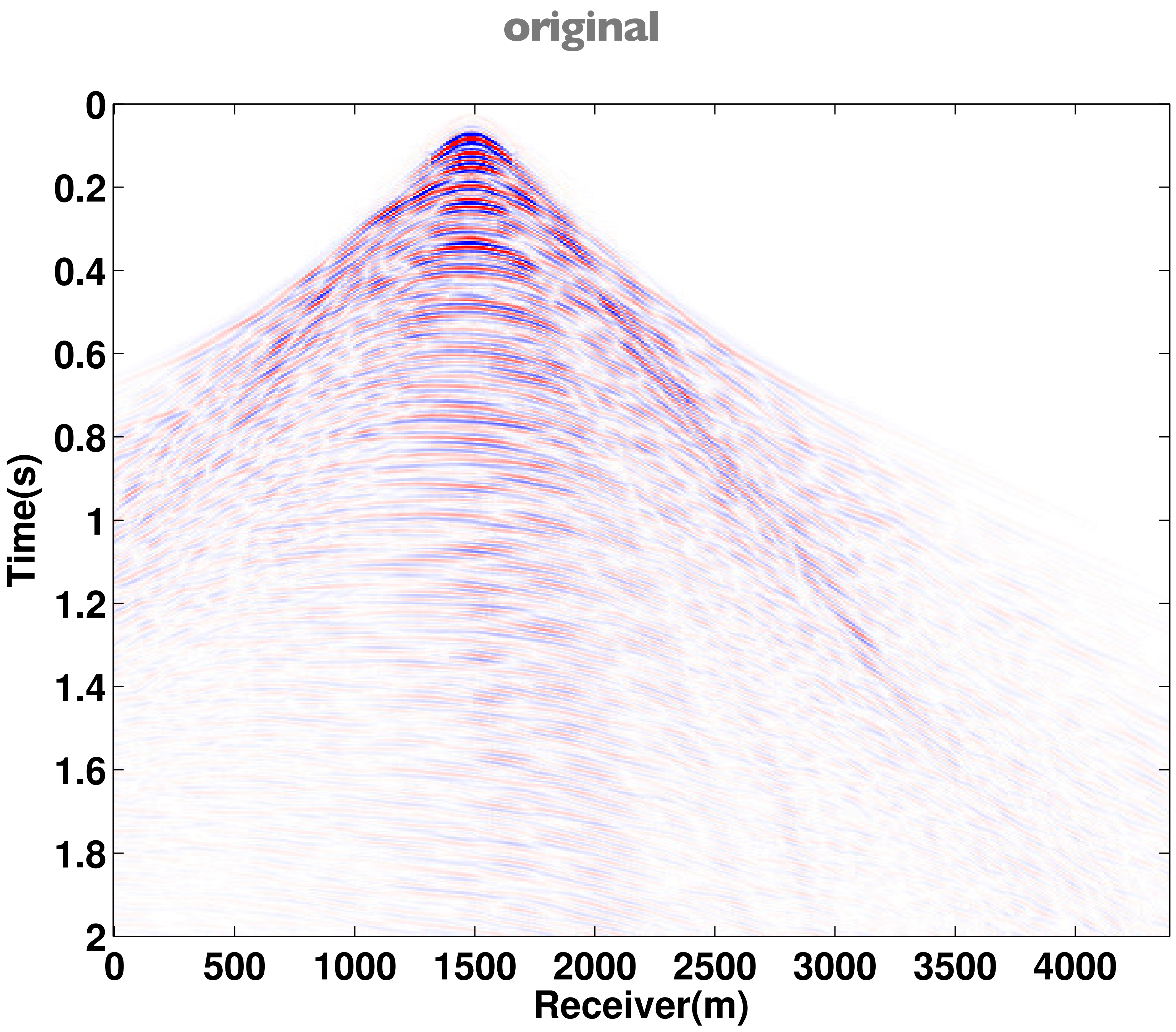
Interpolation flow chart

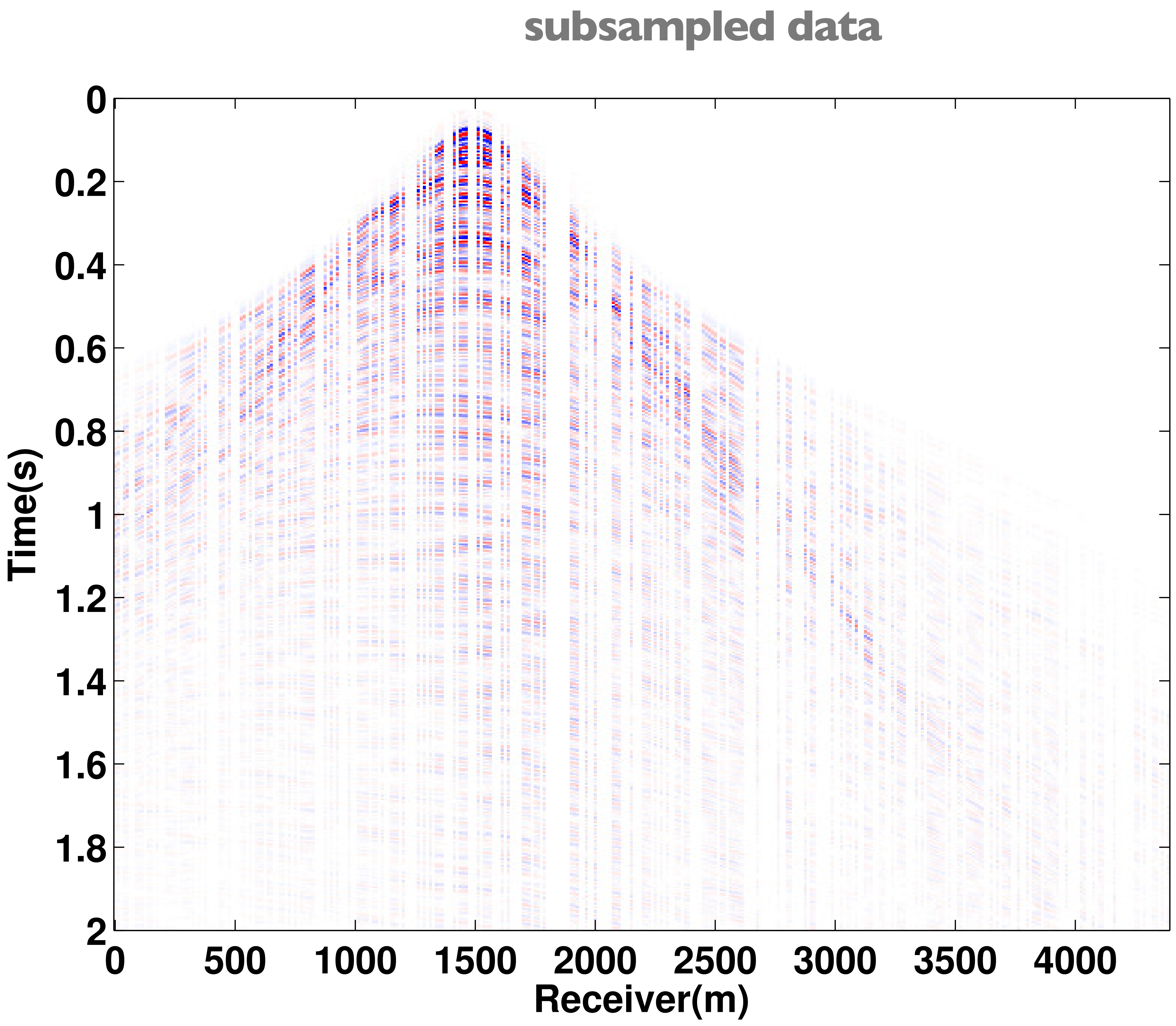


Experiments and Results

- Gulf of Suez
  - 2-D seismic line
  - 50 % missing traces
  - interpolation with HSS level 1,2,3
  - rank = 5
  - 300 iterations

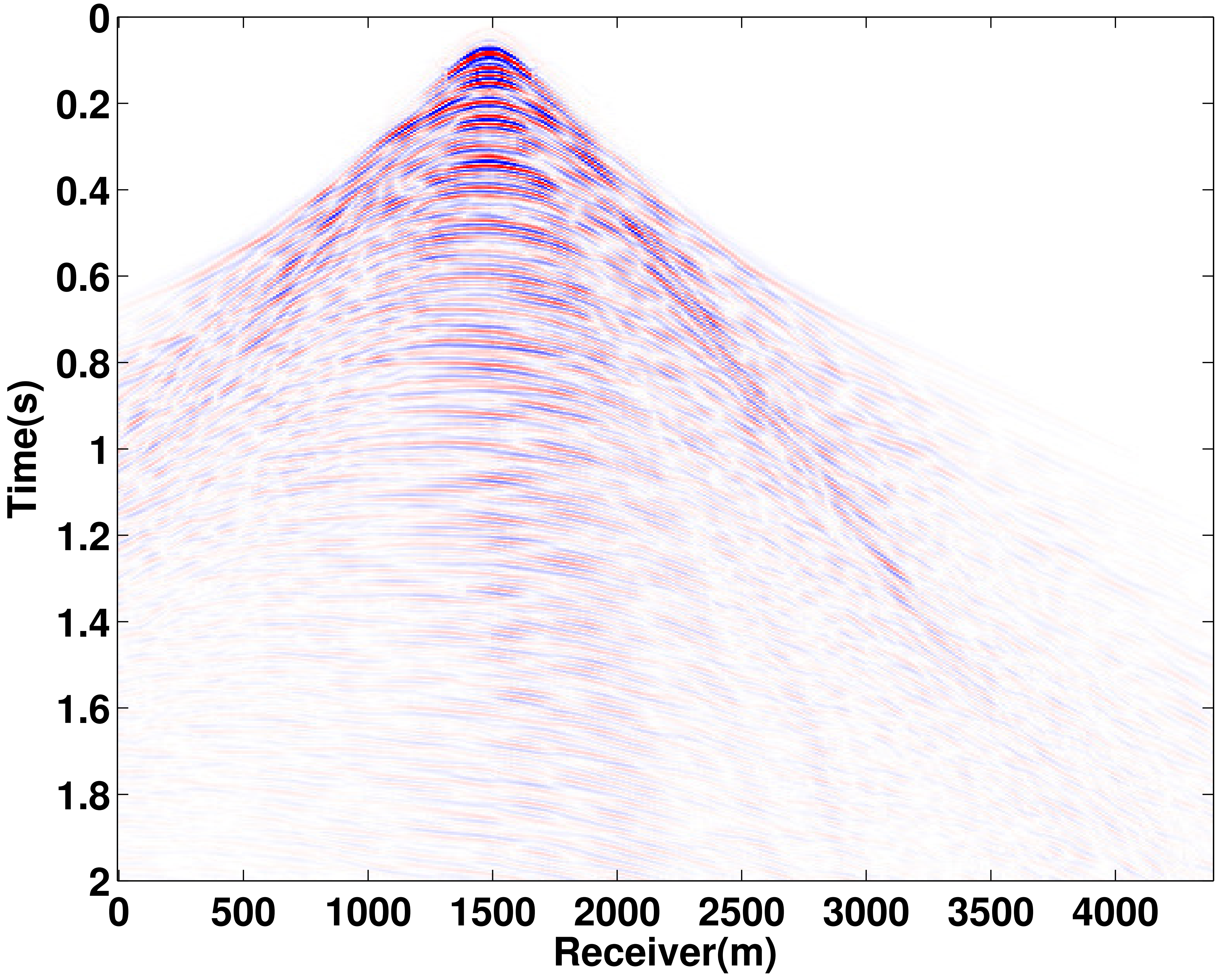
Gulf of Suez


original
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time(s)
0
500
1000
1500
2000
2500
3000
3500
4000
Receiver(m)

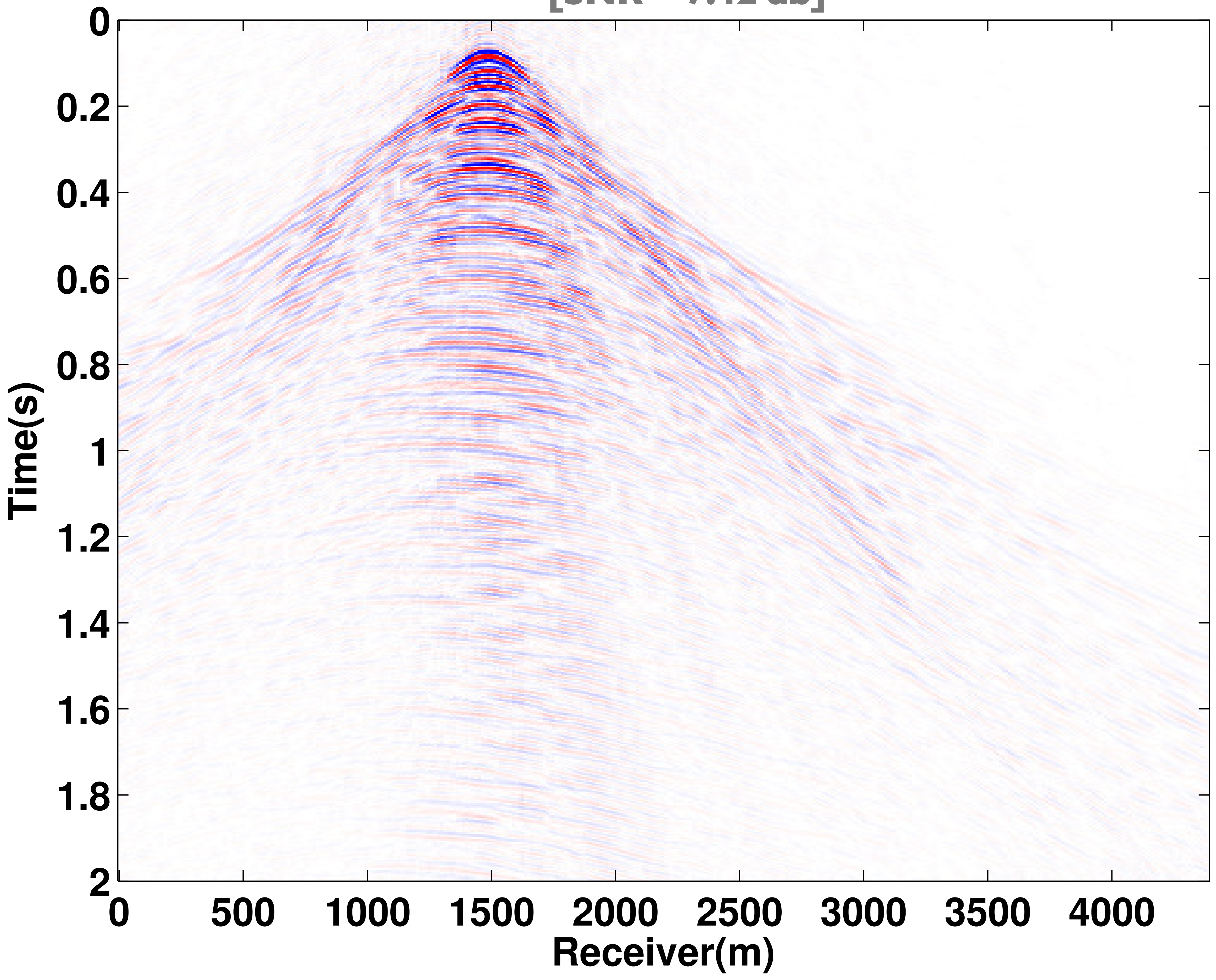

subsampled data
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time(s)
0
500
1000
1500
2000
2500
3000
3500
4000
Receiver(m)

Gulf of Suez
[w/o HSS]

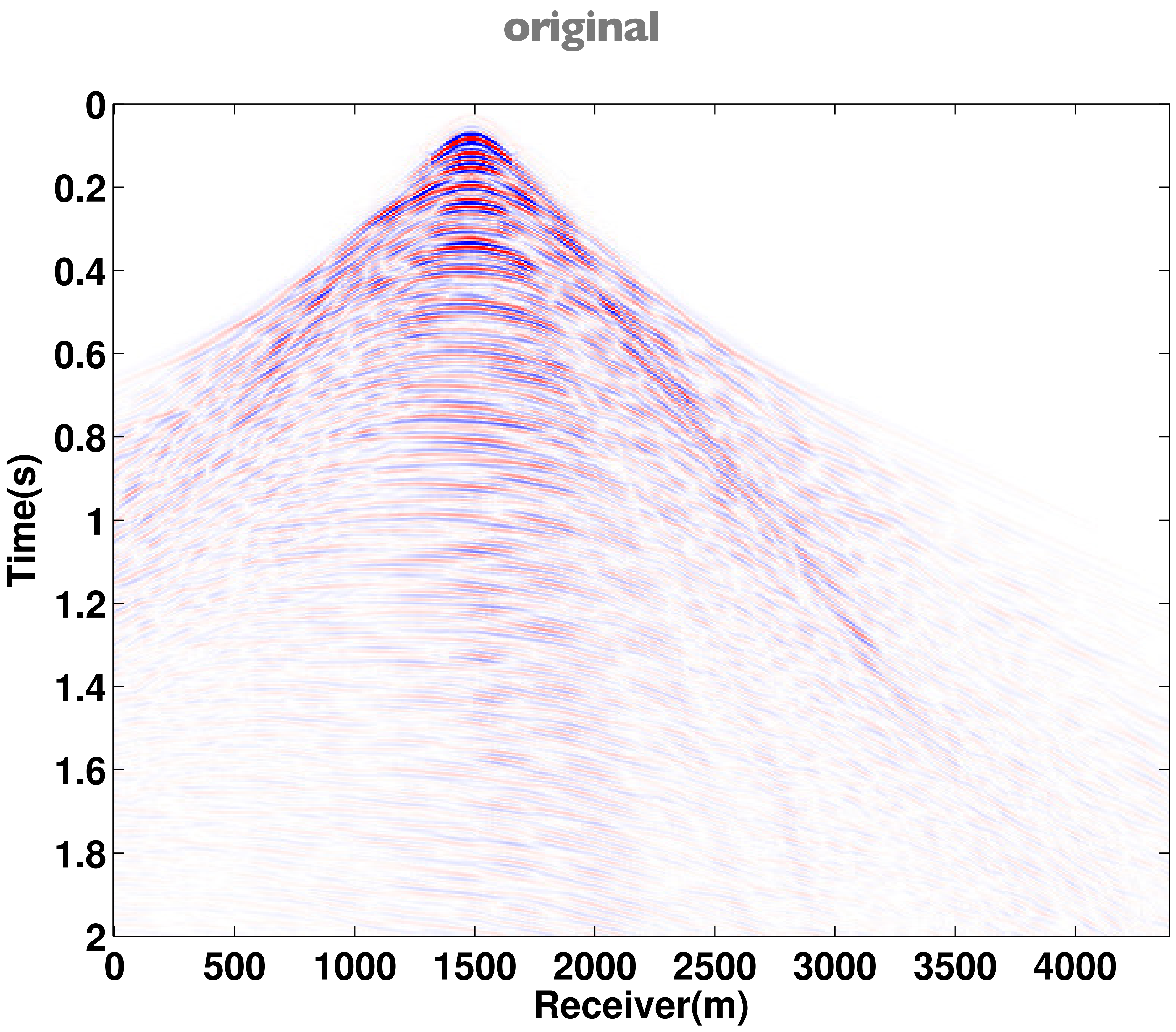
original

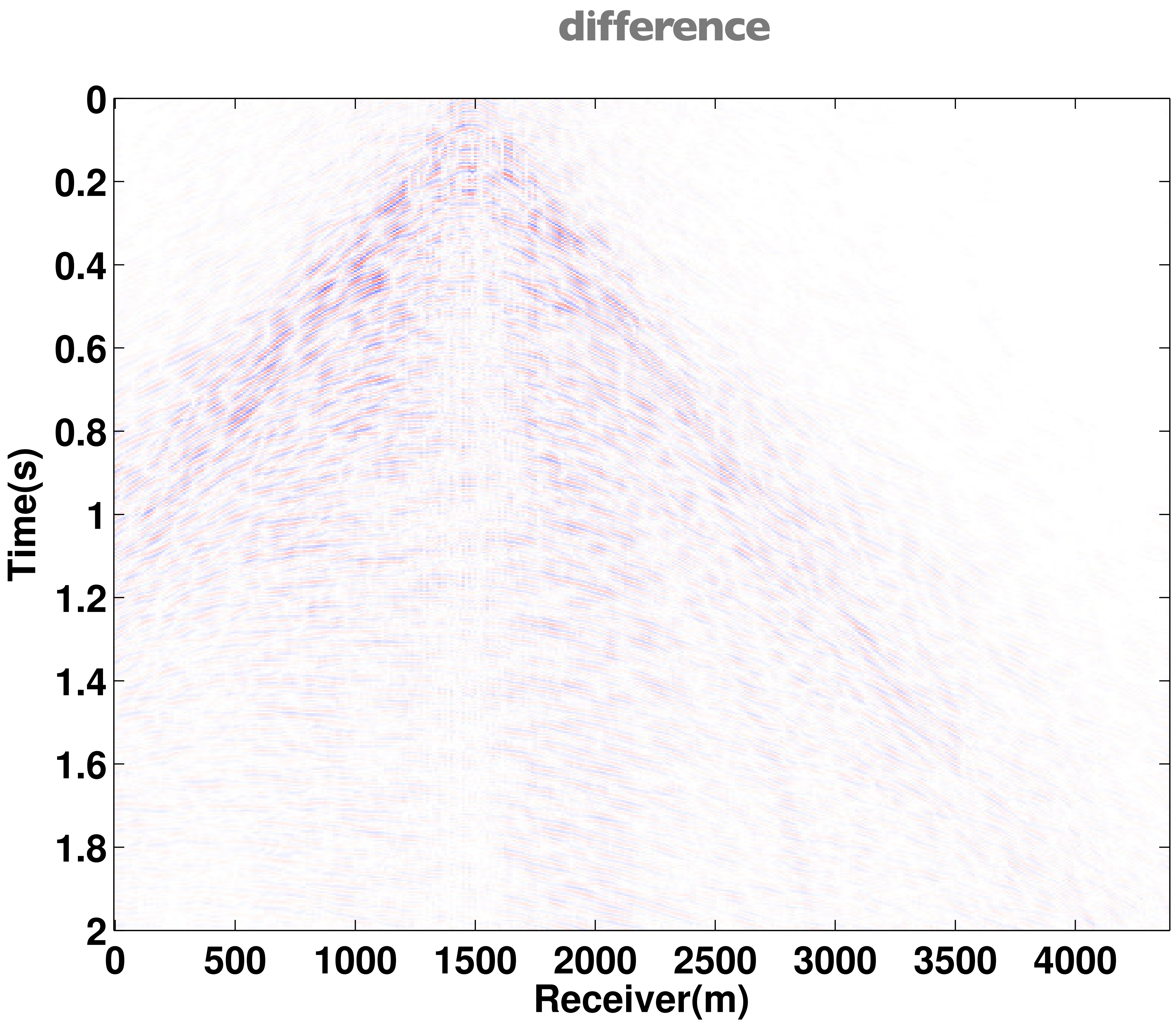


interpolation
[SNR = 7.42 db]



Gulf of Suez
[w/o HSS]

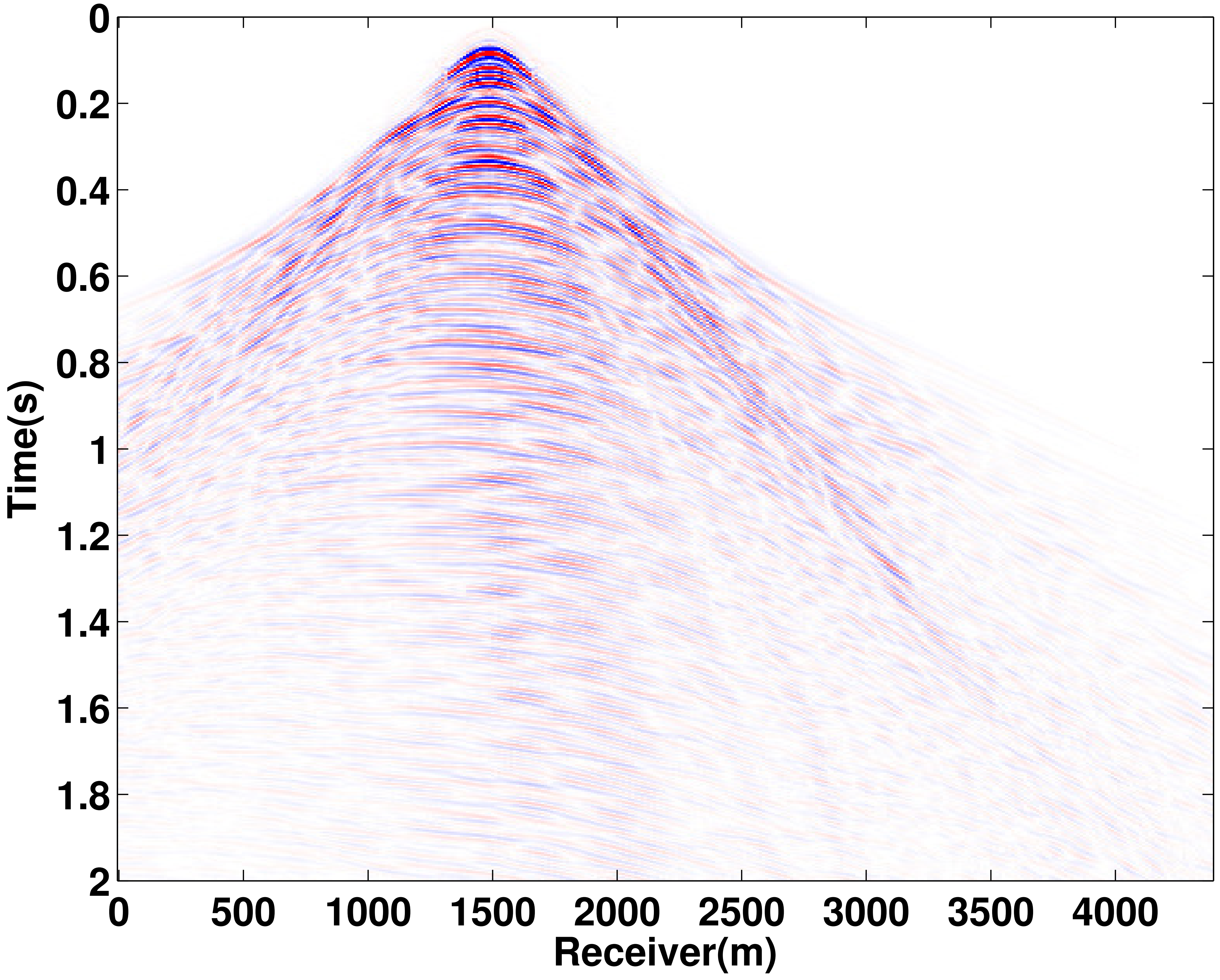




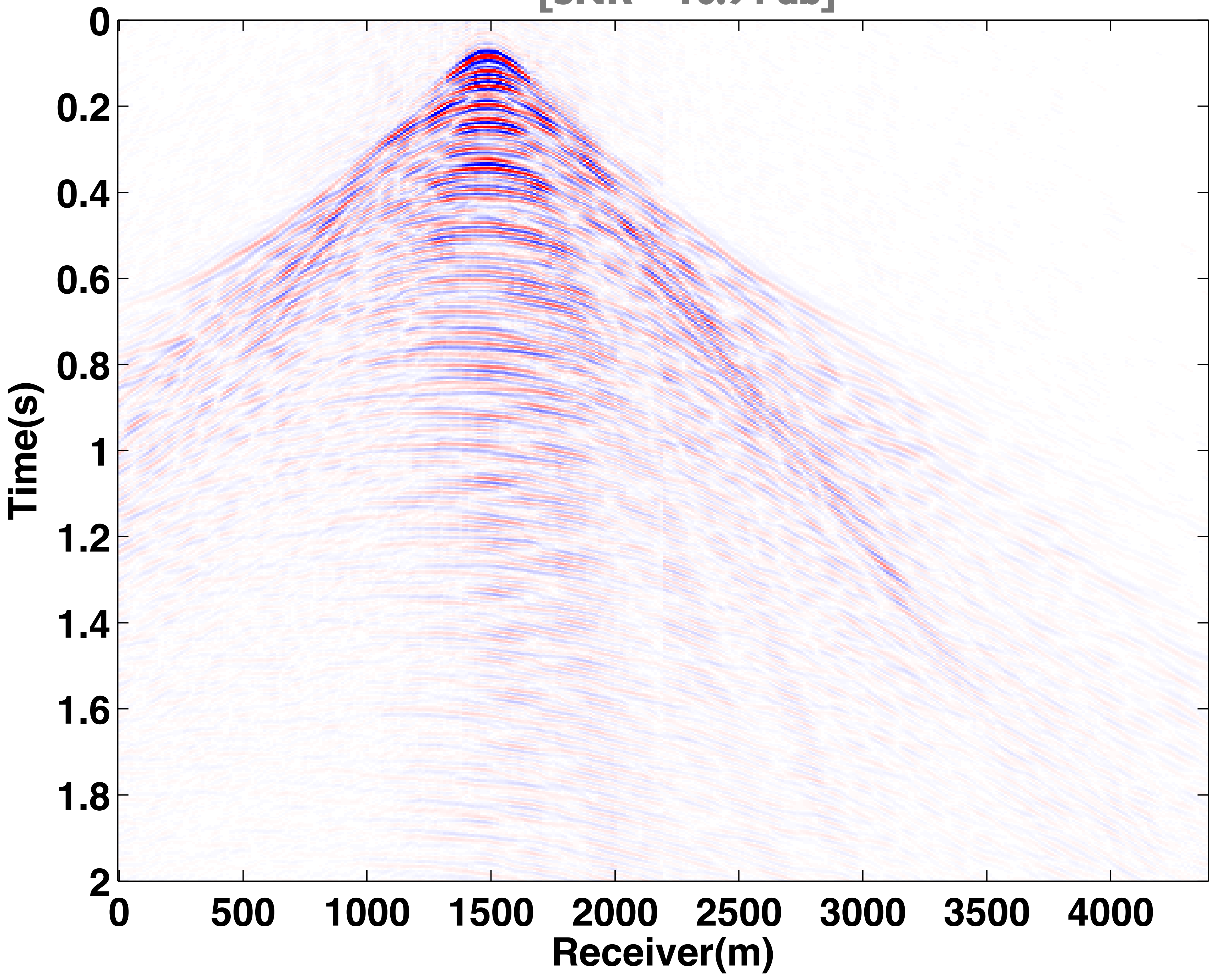
Gulf of Suez

[w/ HSS, level-1]

original

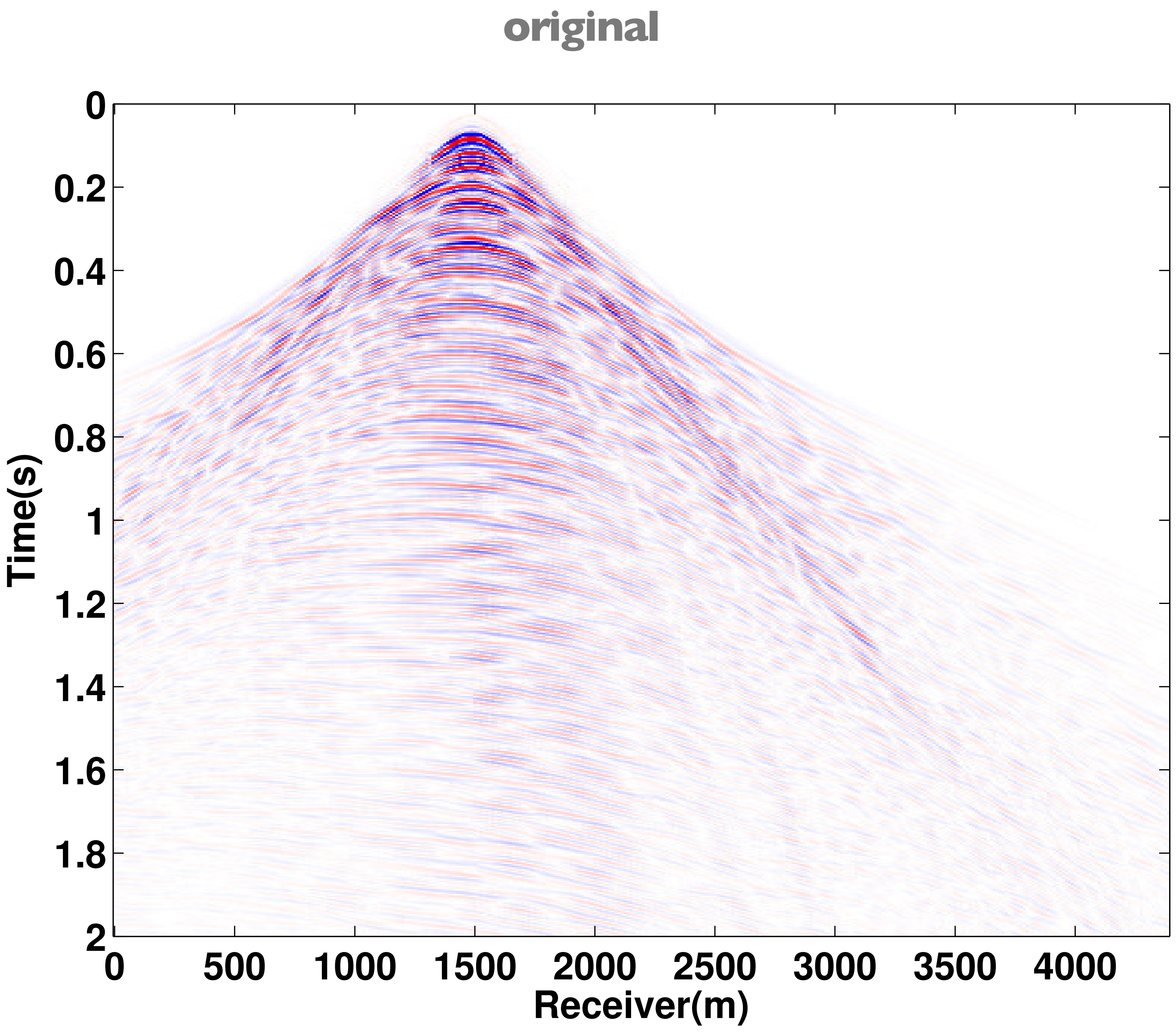


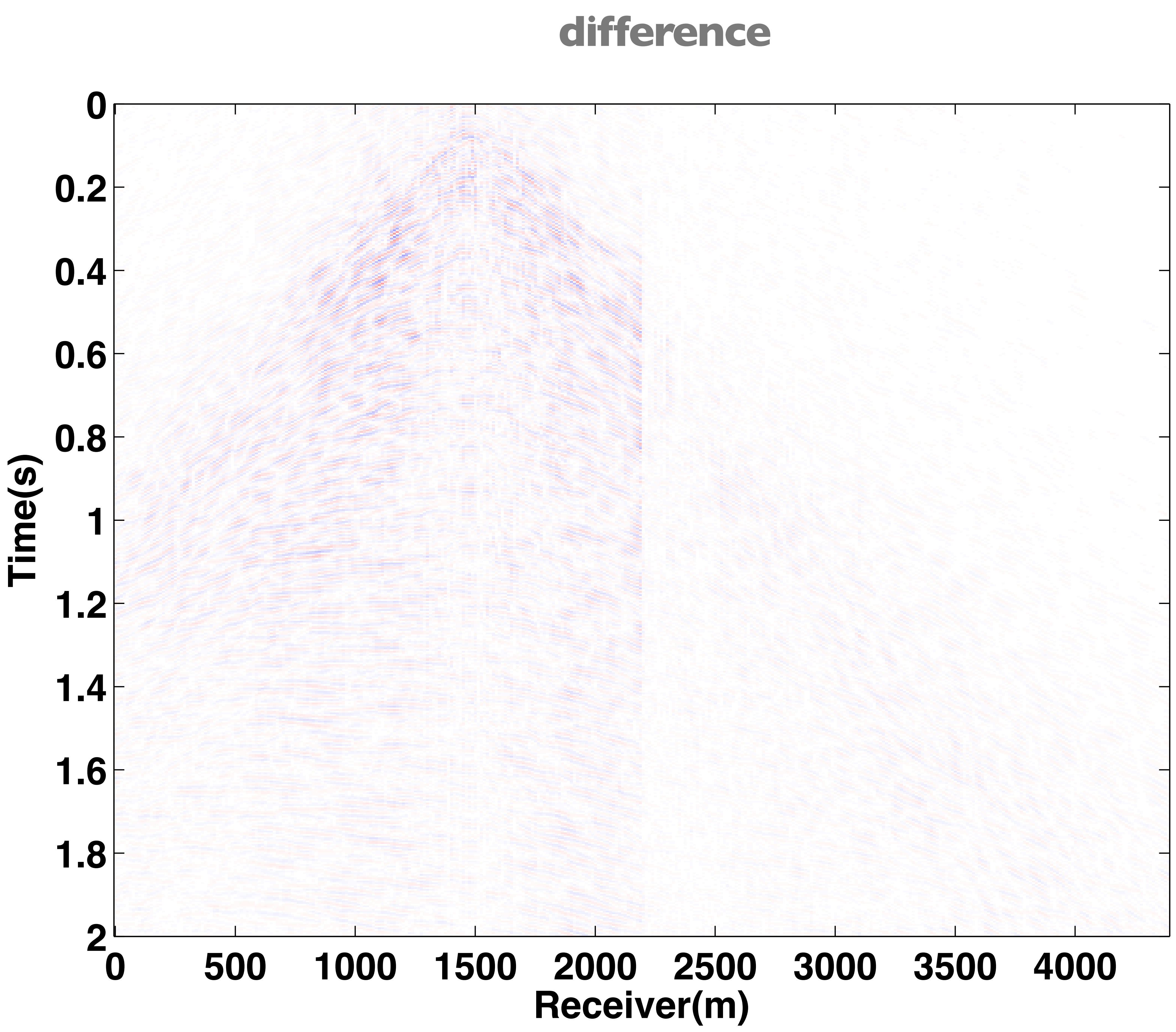
interpolation
[SNR = 10.94 db]



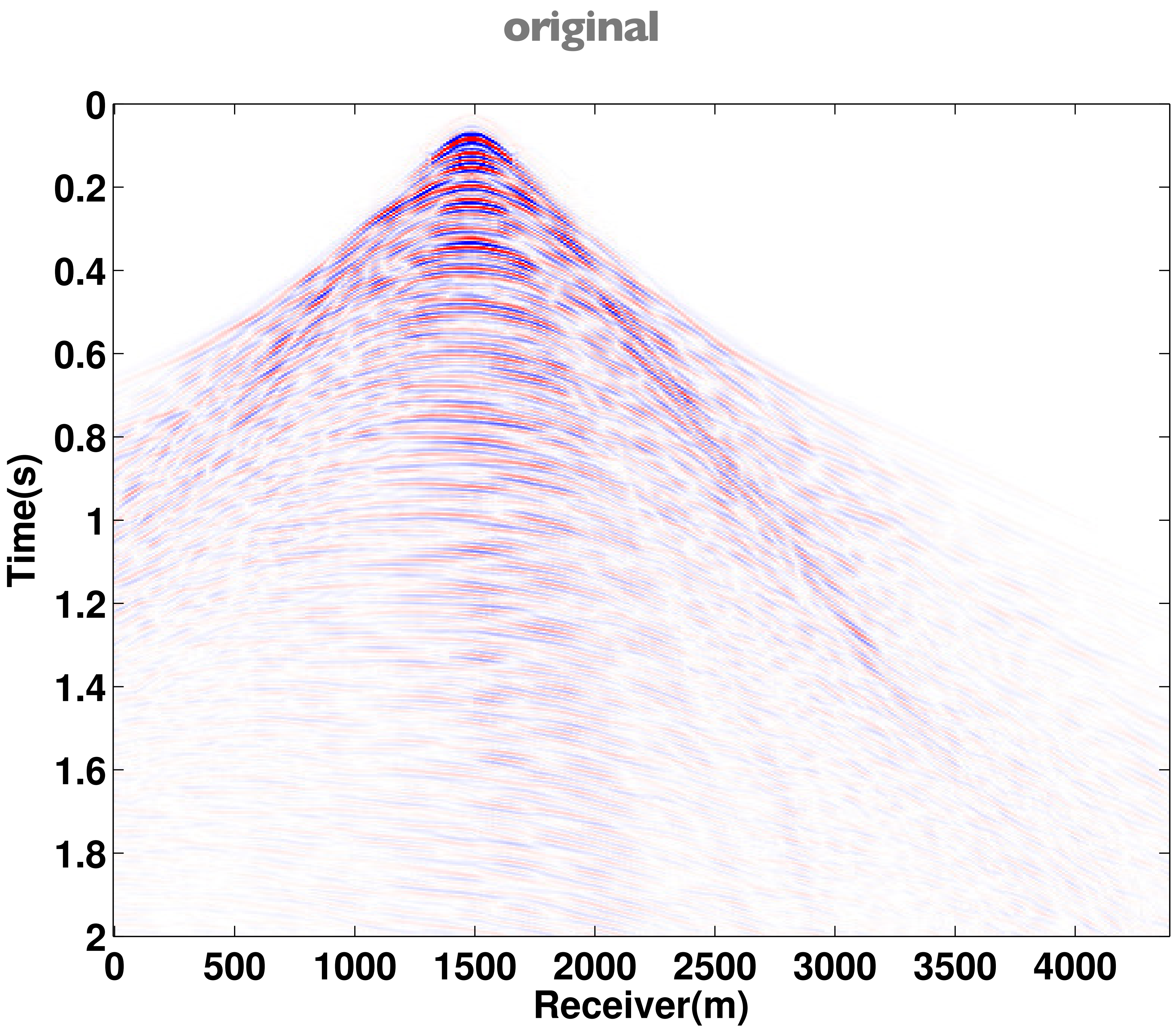
Gulf of Suez

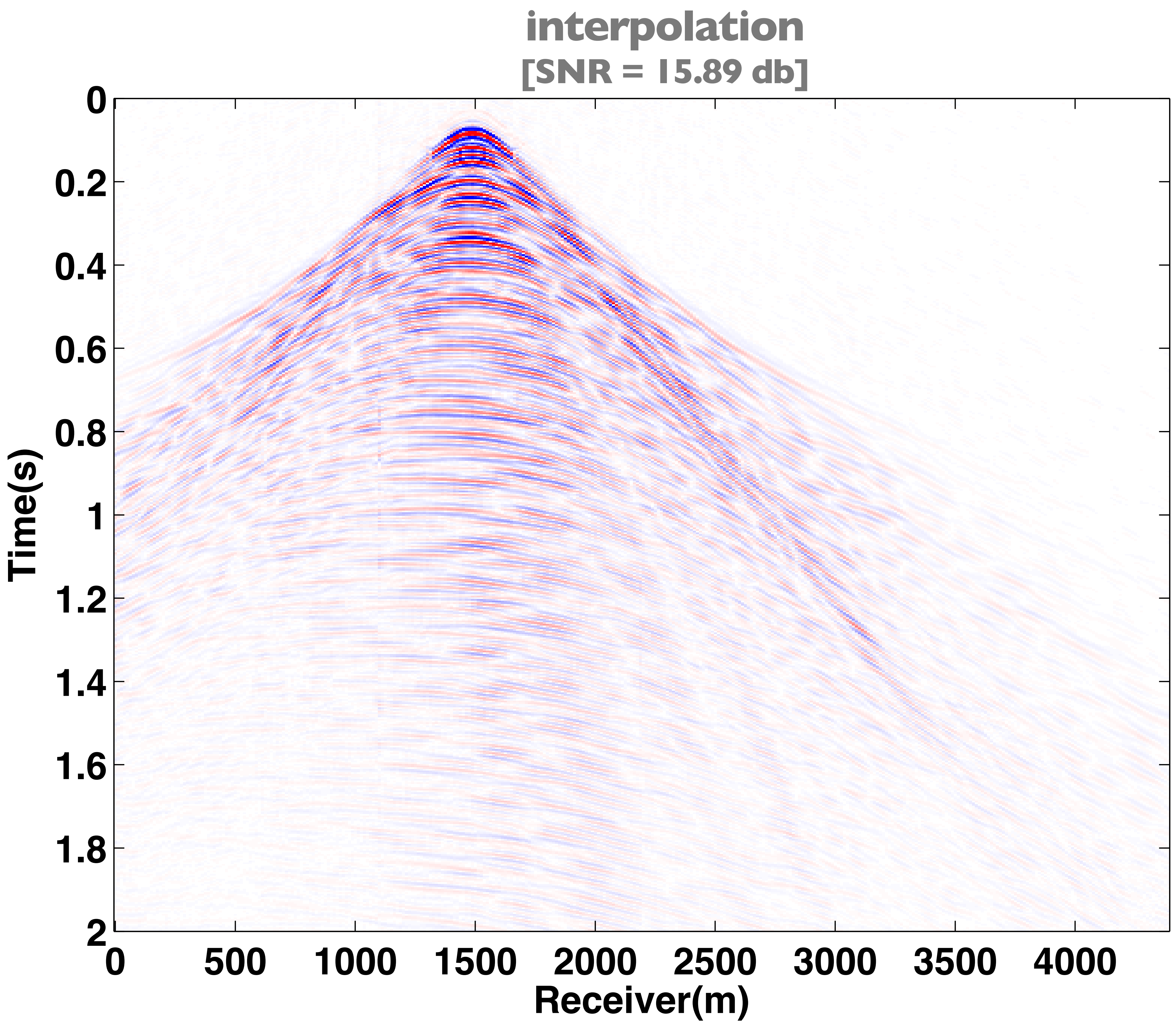
[w/ HSS, level-1]



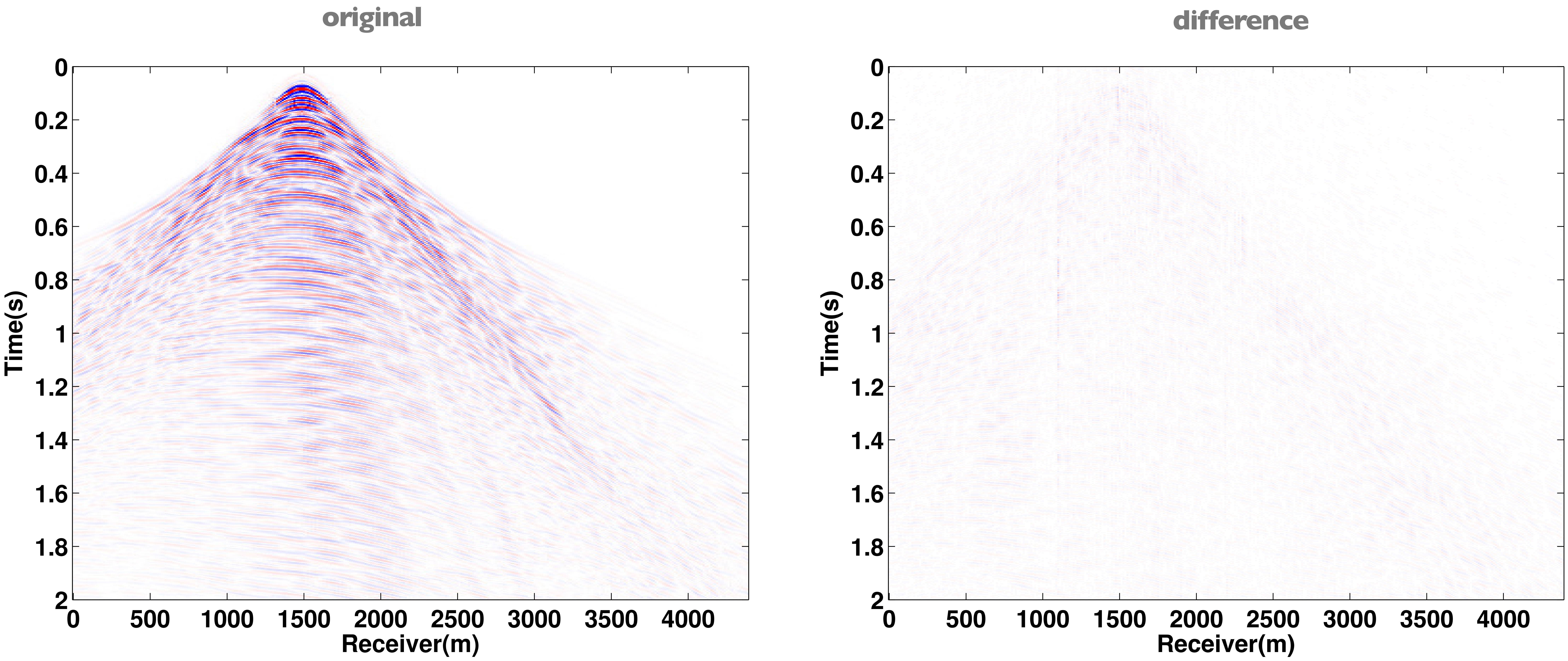


Gulf of Suez
[w/ HSS, level-2]



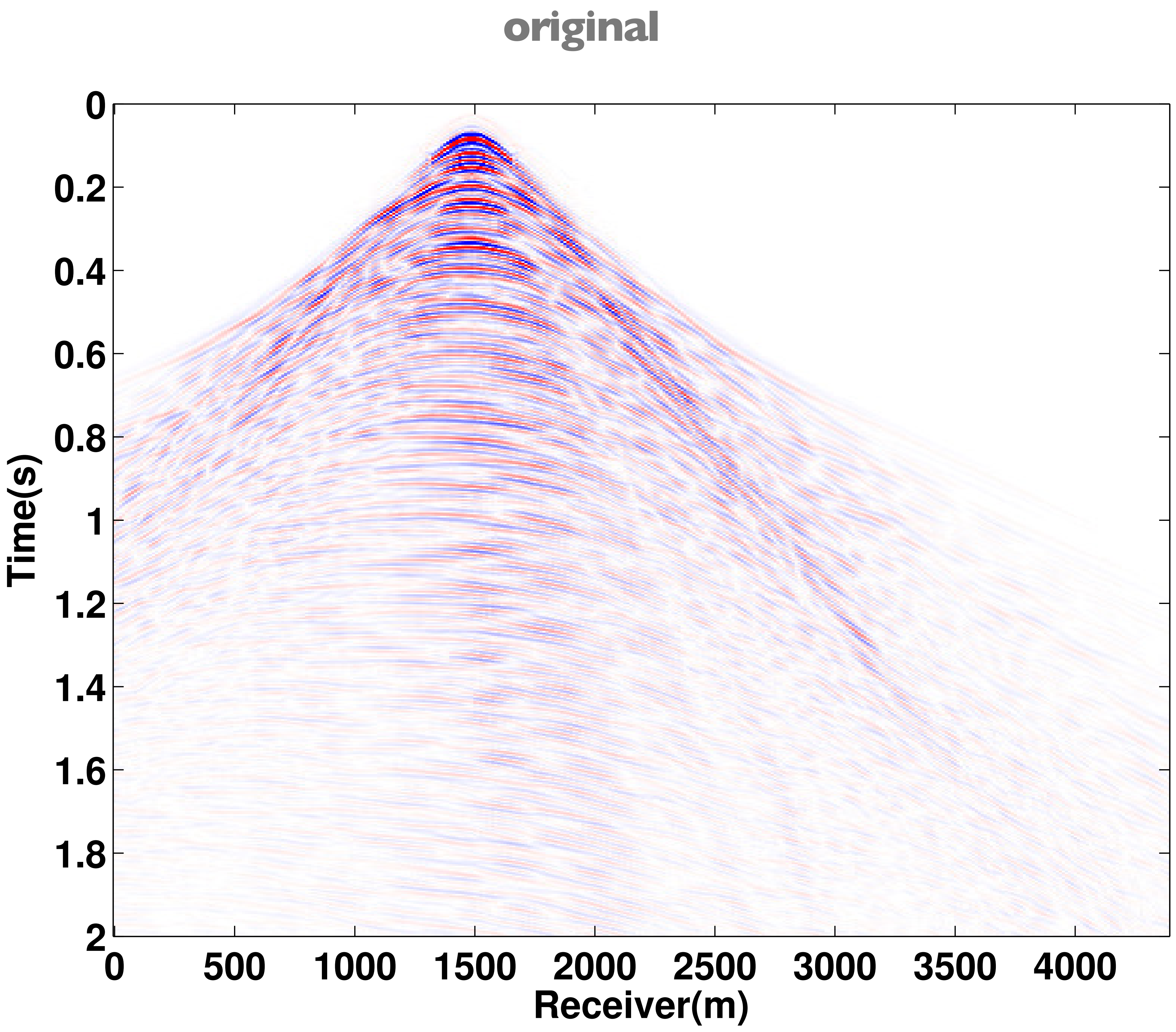


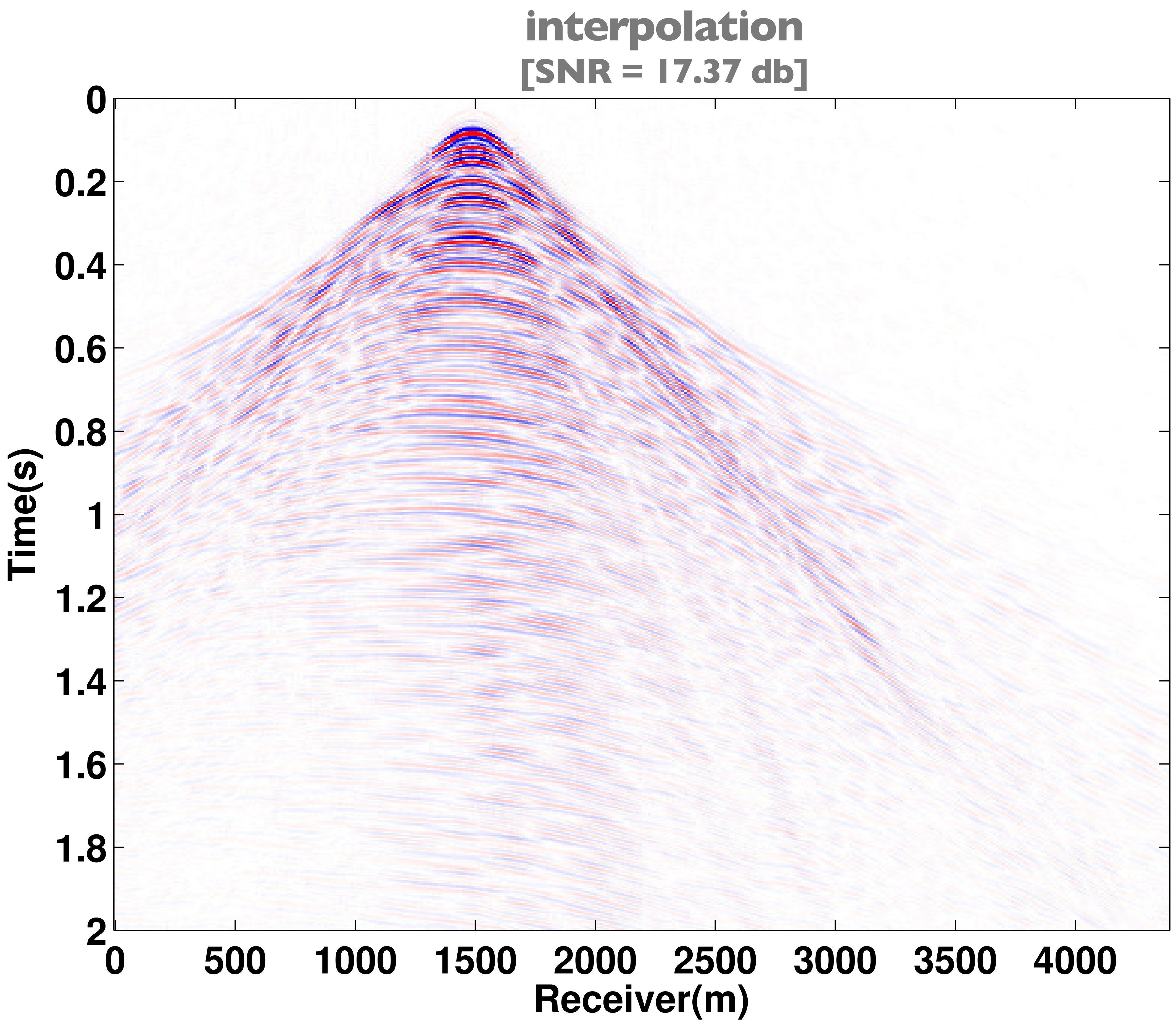
Gulf of Suez
[w/ HSS, level-2]



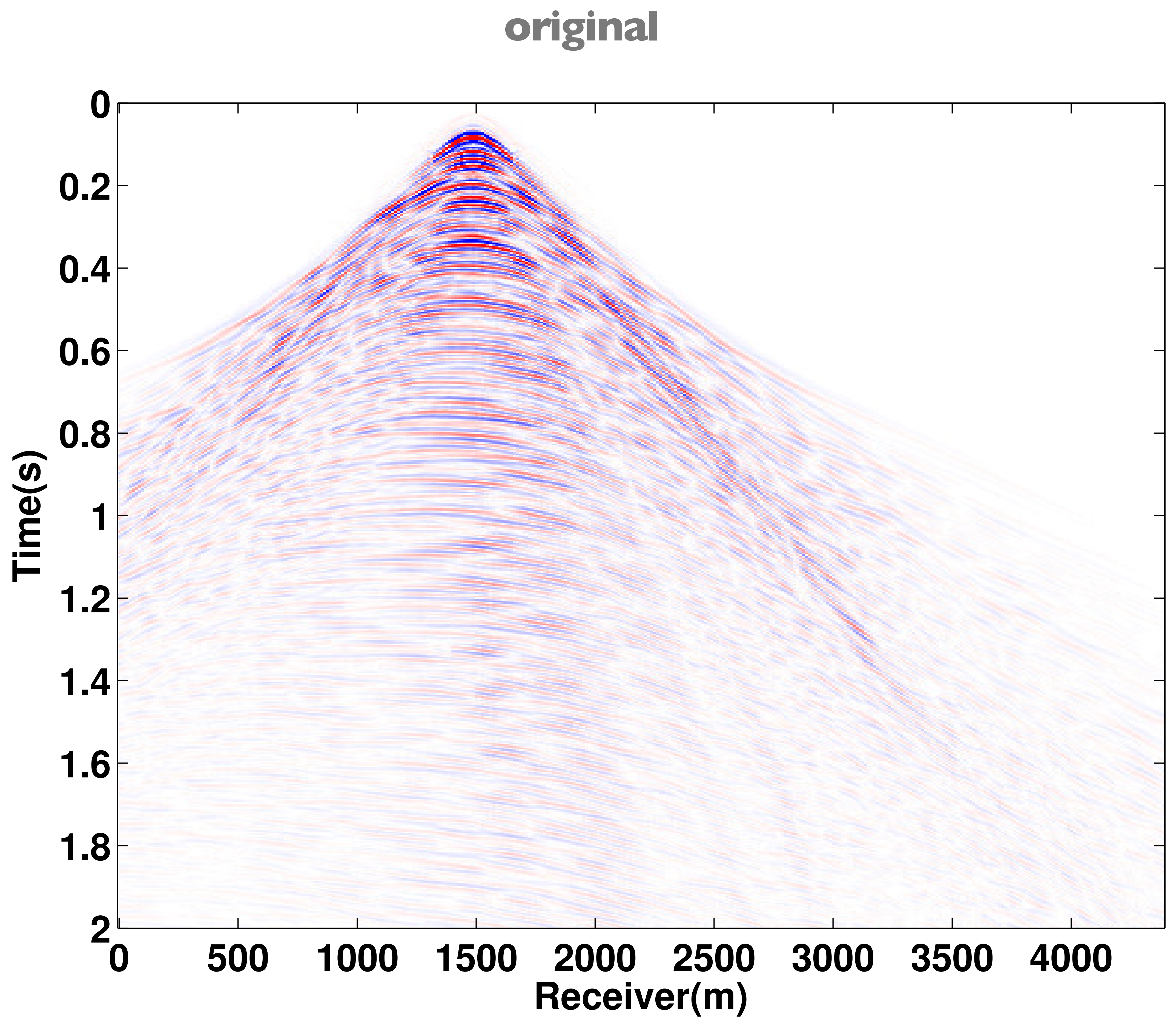
Gulf of Suez

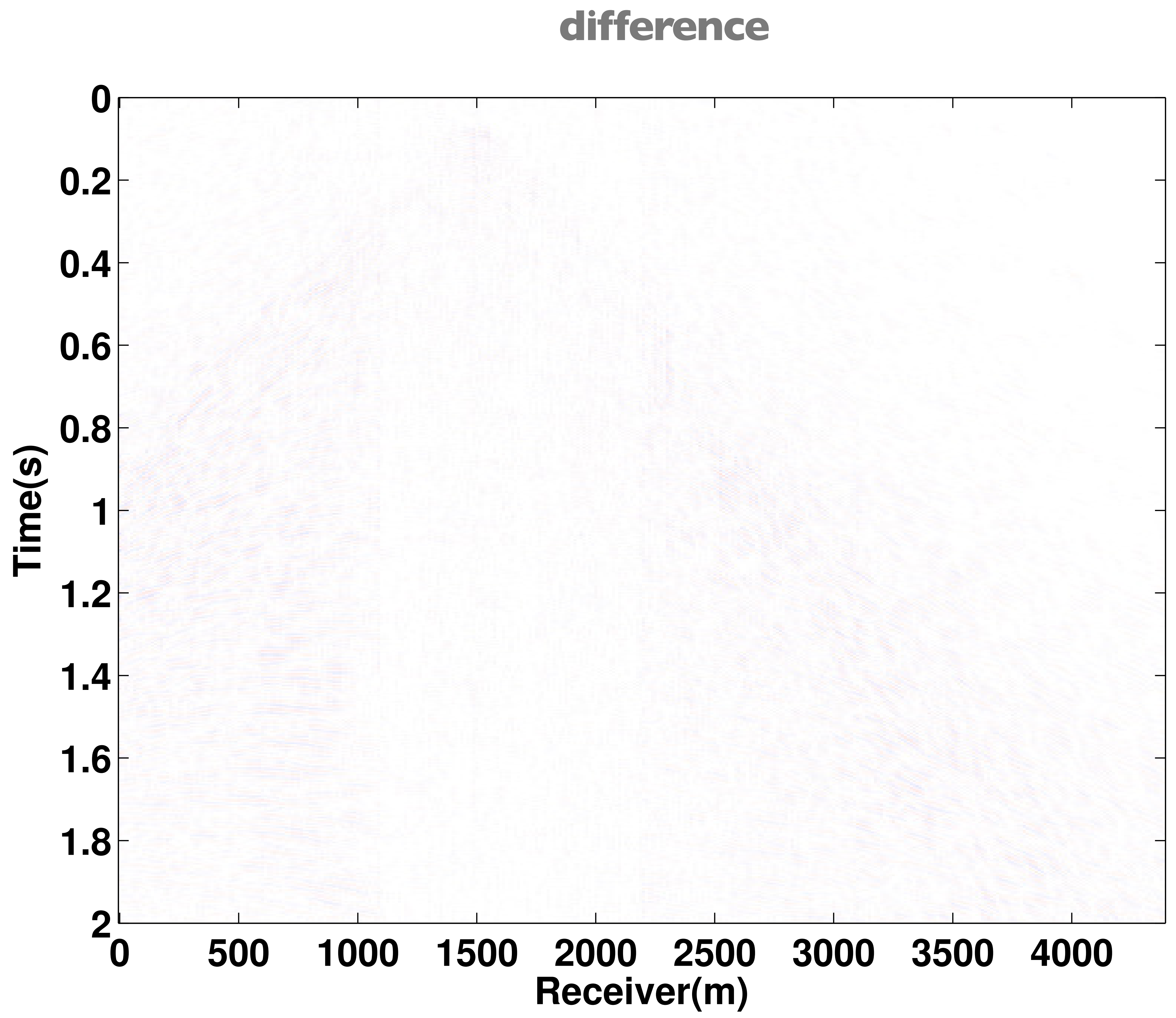
[w/ HSS, level-3]



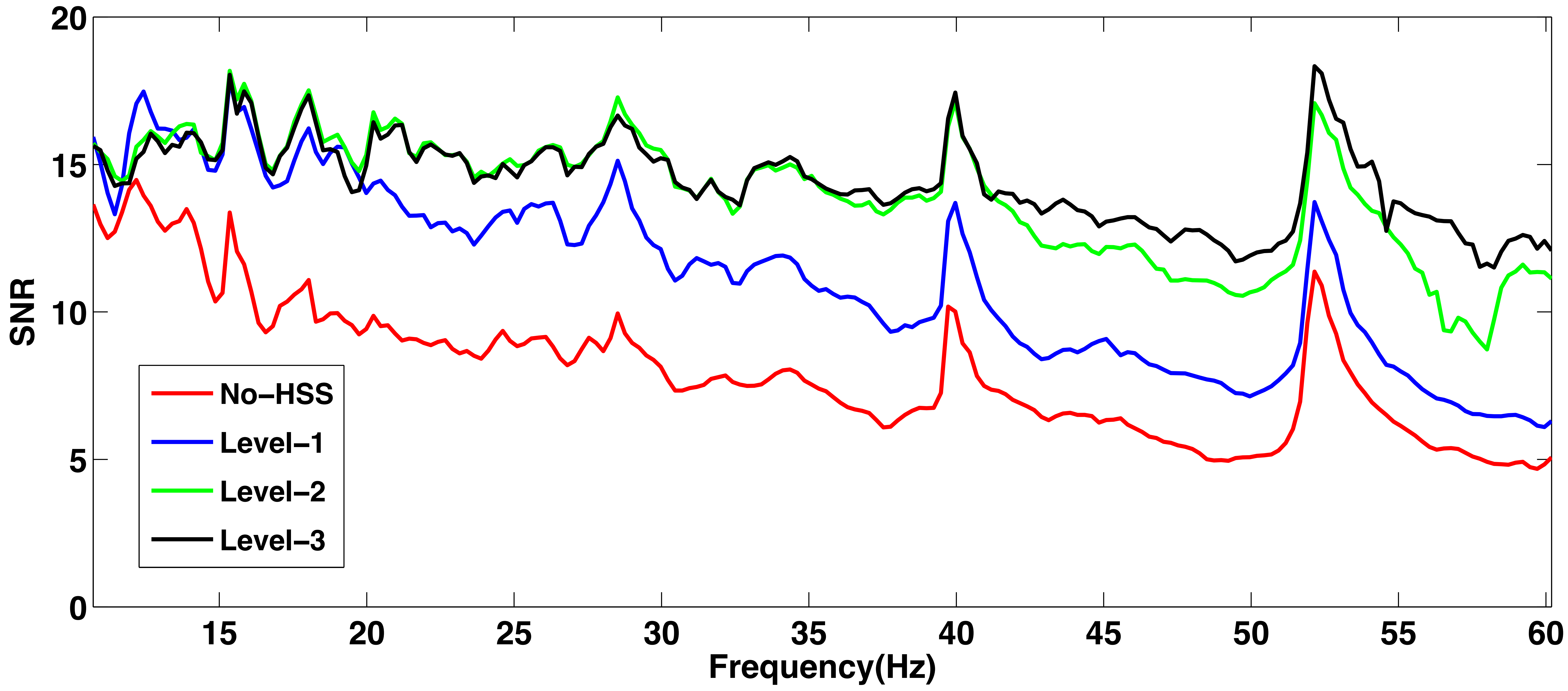


Gulf of Suez
[w/ HSS, level-3]





Comparison
[w/o and w/ HSS]



Conclusion

- firmly established a connection to compressive sensing
  - low-rank structure
  - rank revealing transform domain
  - proper optimization framework

- HSS gives advantage to attack high-rank structures using the partitioning

Conclusion

- matrix factorization allows *SVD-free* low-rank methods that work fast on large data
- low-rank structure holds promise for data recovery and more compact representation

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Acknowledgements

Thank you for your attention !

https://www.slim.eos.ubc.ca/

SINBAD

NSERC CRSNG

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