Imaging with multiples accelerated by message passing

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Lin, Tu, and Herrmann, 2010; Verschuur and Berkhout, 2011; Whitmore et.al., 2010; Liu et.al., 2011

Motivation

- making use of primaries and multiples *simultaneously*
- *eliminating artifacts* from multiples
- looking for a computationally *efficient* approach

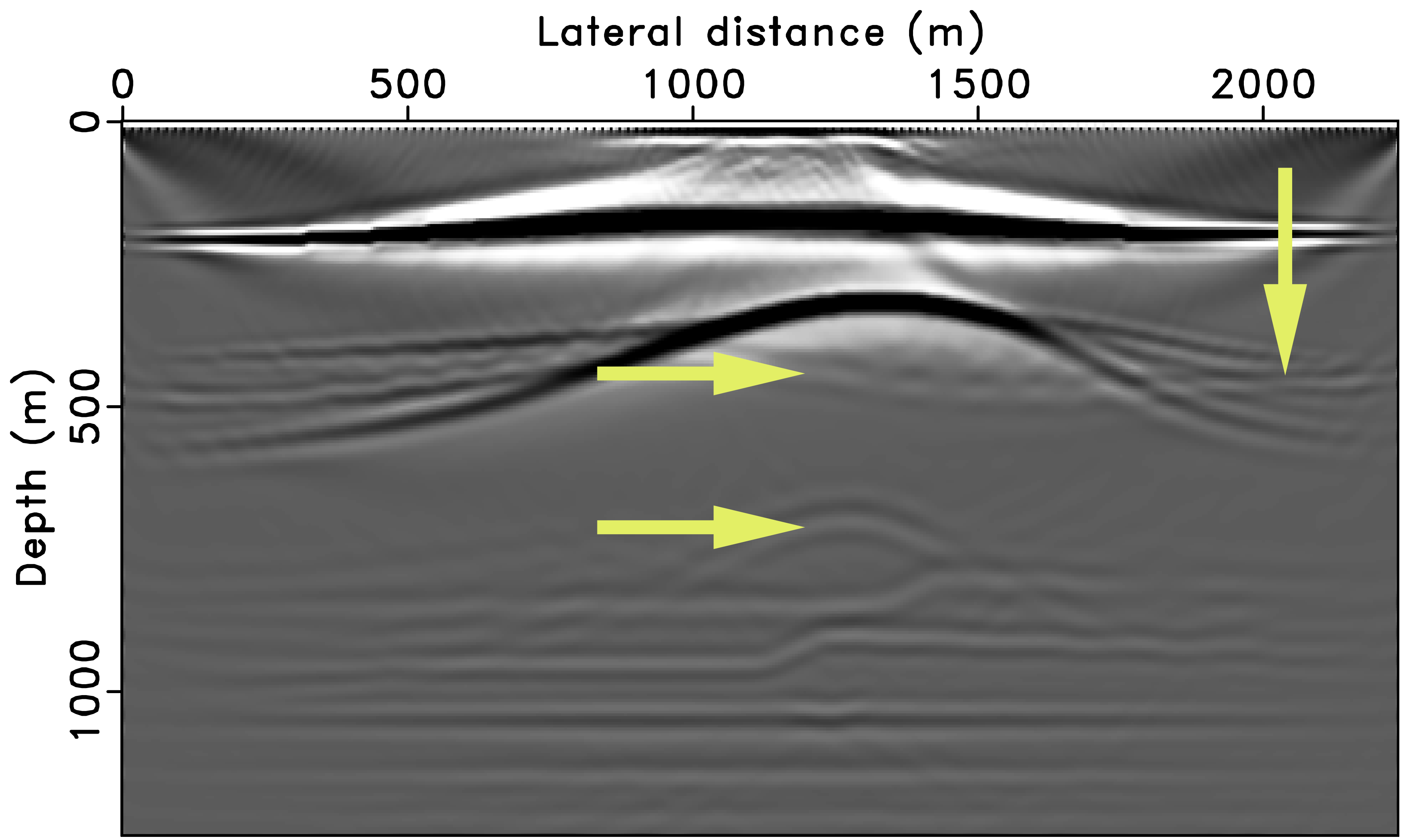
Tu and Herrmann, 2011; Verschuur and Berkhout, 2011; Lu et.al., 2011; Liu et.al., 2011

Primaries & multiples: not ‘or’ but ‘and’

- primaries have a higher signal-to-noise ratio
- multiples can be useful if used correctly
- separating them can be very expensive
- they are not always separable

Conventional RTM image

[use the primary imaging operator]

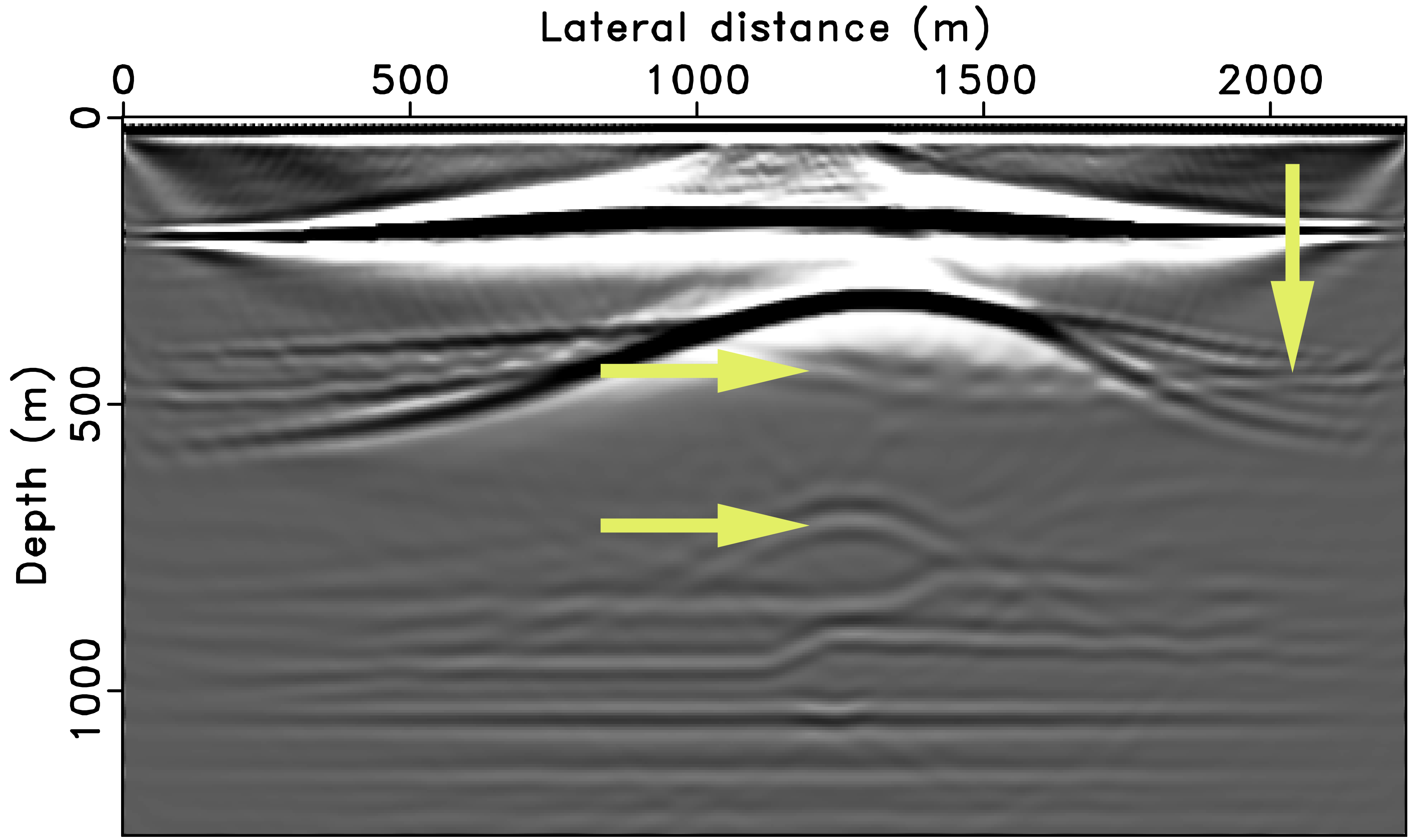


Reverse time migration of *primaries+multiples*, without accounting for multiples

Muijs et al., 2007; Whitmore et.al., 2010; Liu et.al., 2011

RTM of total data

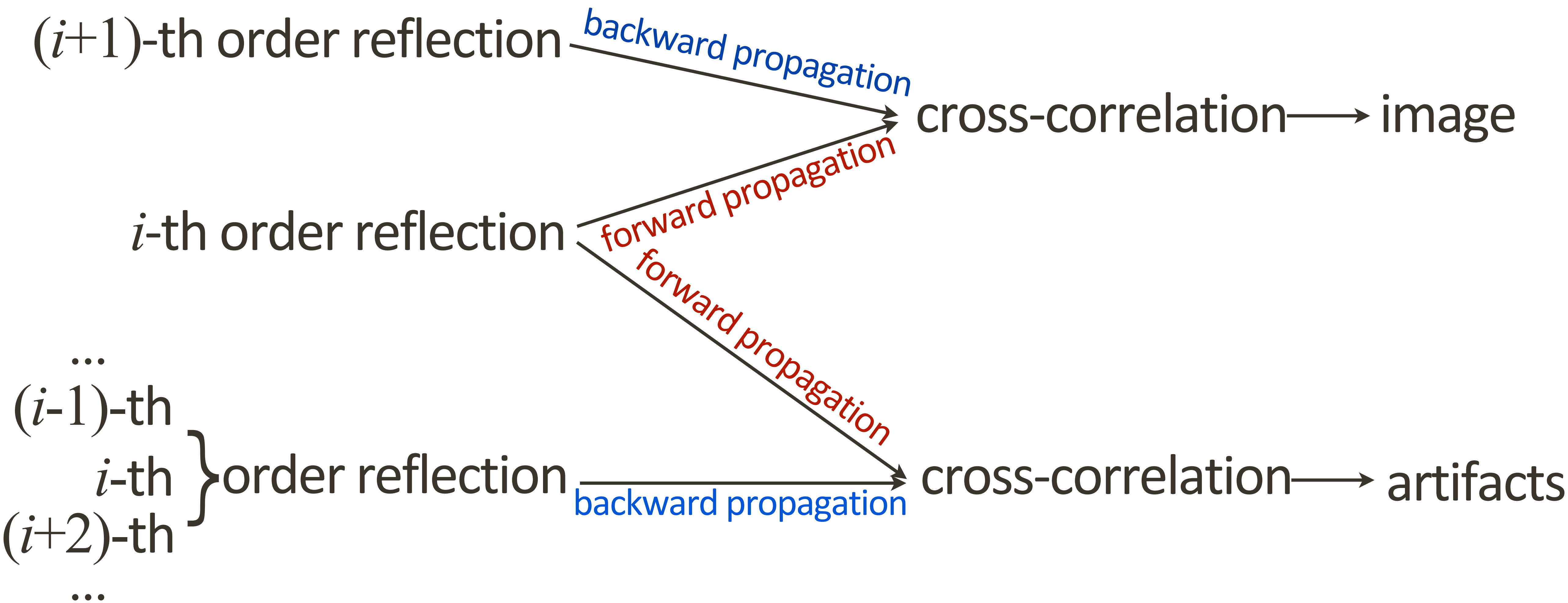
[the imaging operator includes the areal source to account for multiples]



Reverse time migration of *primaries+multiples*, accounting for multiples

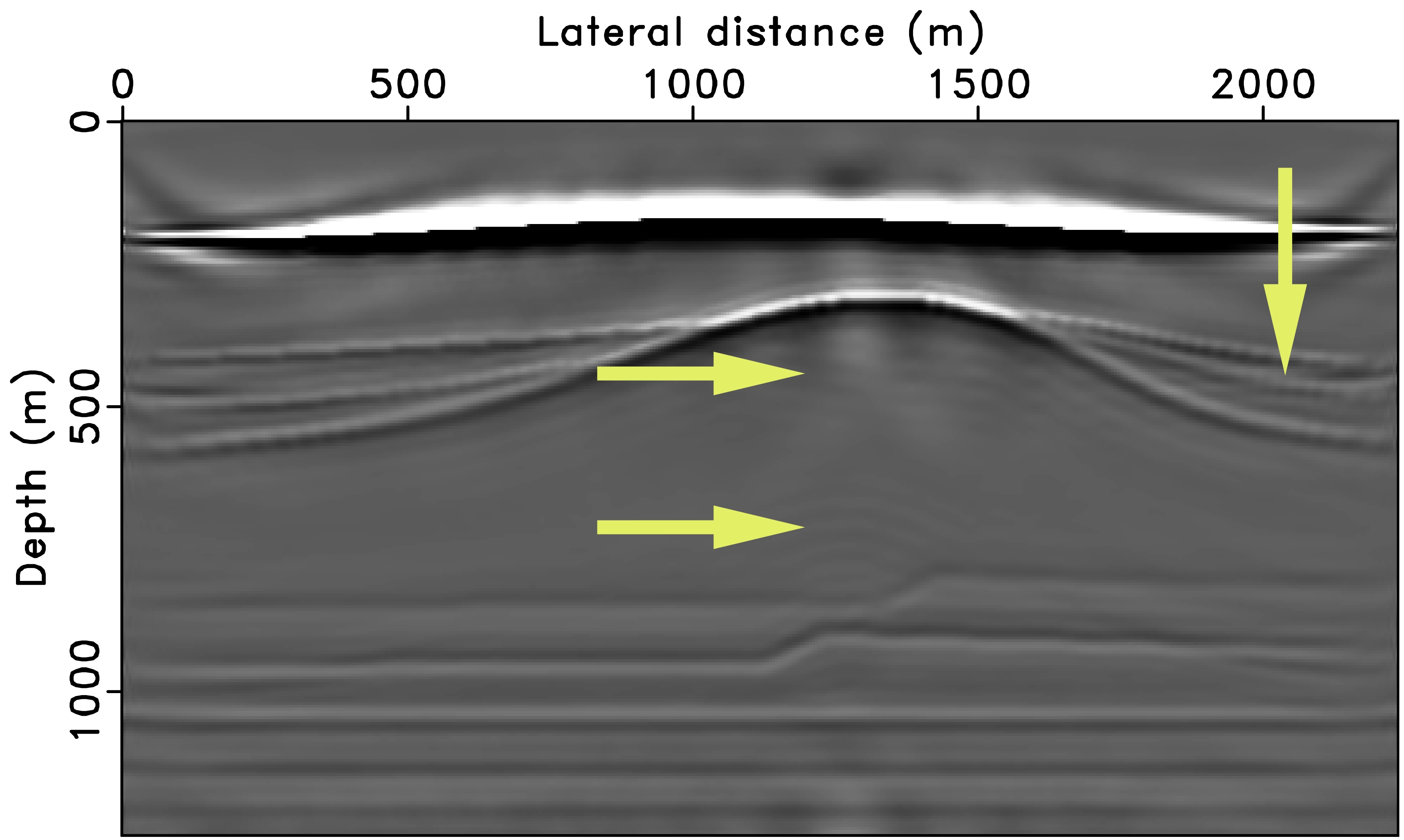
Muijs et al., 2007; Liu et.al., 2011

When a free-surface is present



Lin et. al., 2010; Tu and Herrmann, 2011a

Artifacts-free image by inversion



Imaging of *primaries+multiples* by *inversion*

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Inversion? Sounds expensive...

- repeated evaluations of the Born scattering operator and its adjoint
- each evaluation requires solving 4*(#source)*(#frequencies) PDEs

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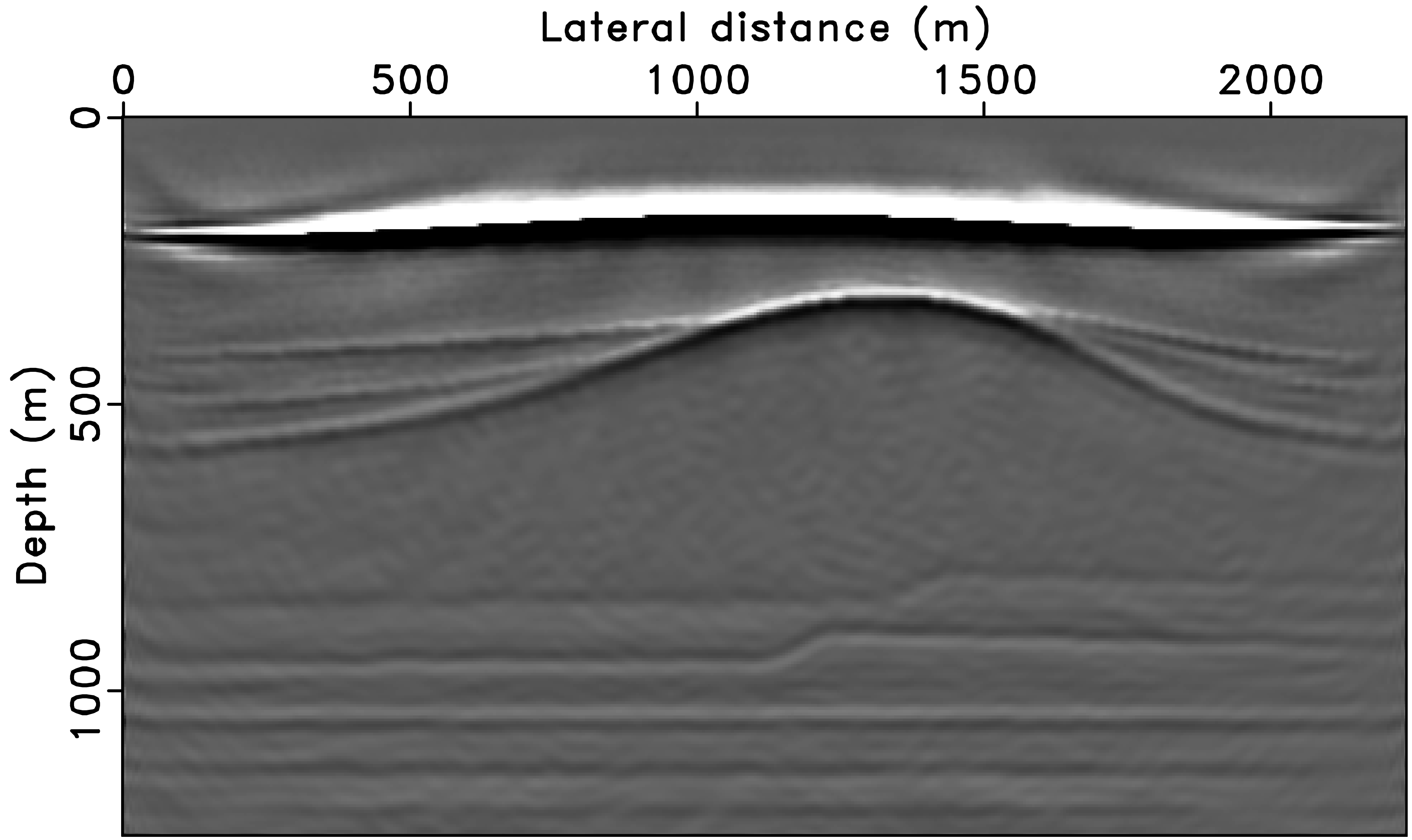
Sneak peek of our result

[with a **120X** speed-up compared to the previous image]



Fast imaging of *primaries+multiples* by sparse inversion

Method

Physics of the free surface

Total data and the surface-free Green's function can be related by the SRME formulation:

$$\mathbf{P}_{\mathrm{i}} = \mathbf{G}_{\mathrm{i}}(\mathbf{Q}_{\mathrm{i}} + \mathbf{R}_{\mathrm{i}}\mathbf{P}_{\mathrm{i}})$$

$\mathbf{P}$: total up-going wavefield
$\mathbf{G}$: surface-free Green's function
$\mathbf{Q}$: source wavelet
$\mathbf{R}$: surface reflectivity

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Expressed in model space

$$
\begin{aligned}
\mathbf{P}_{\mathrm{i}} &= \mathrm{vec}^{-1}(\mathbf{F}_{\mathrm{i}}[\mathbf{m},\mathbf{I}])(\mathbf{Q}_{\mathrm{i}}+\mathbf{R}_{\mathrm{i}}\mathbf{P}_{\mathrm{i}}) \\
&= \mathbf{D}_{\mathrm{r}}\mathbf{H}_{\mathrm{i}}^{-1}\boxed{[\mathbf{m}](\mathbf{D}_{\mathrm{s}}^{*}\mathbf{I})(\mathbf{Q}_{\mathrm{i}}+\mathbf{R}_{\mathrm{i}}\mathbf{P}_{\mathrm{i}})} \\
&= \mathbf{D}_{\mathrm{r}}\mathbf{H}_{\mathrm{i}}^{-1}\boxed{[\mathbf{m}](\mathbf{D}_{\mathrm{s}}^{*}(\mathbf{Q}_{\mathrm{i}}+\mathbf{R}_{\mathrm{i}}\mathbf{P}_{\mathrm{i}}))} \\
&\doteq \mathrm{vec}^{-1}(\mathbf{F}_{\mathrm{i}}[\mathbf{m},\mathbf{Q}_{\mathrm{i}}+\mathbf{R}_{\mathrm{i}}\mathbf{P}_{\mathrm{i}}])
\end{aligned}
$$

$\mathbf{F}$: forward modelling operator

$\mathbf{m}$: true model

$\mathbf{I}$: impulsive source array

$\mathbf{D}_{\mathrm{r}}$, $\mathbf{D}_{\mathrm{s}}$: detection operator at receiver/source locations

$\mathbf{H}$: time-harmonic Helmholtz operator

Linearized forward modelling
[monochromatic]

$$\mathbf{p}_{\mathrm{i}} = \boxed{\nabla\mathbf{F}_{\mathrm{i}}[\mathbf{m}_0, \mathbf{Q}_{\mathrm{i}} + \mathbf{R}_{\mathrm{i}}\mathbf{P}_{\mathrm{i}}]\delta\mathbf{m}}$$ + higher order reflections

$\nabla\mathbf{F}$: Born scattering operator

$\mathbf{m_0}$: background model

$\delta\mathbf{m}$: model perturbation

$\mathbf{p}$: vectorized wavefield

Linearized forward modelling

[*all* frequencies]

$$\mathbf{p} \approx \begin{bmatrix} \nabla \mathbf{F}_1(\mathbf{m}_0, \mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}) \\ \vdots \\ \nabla \mathbf{F}_\mathrm{nf}(\mathbf{m}_0, \mathbf{Q}_\mathrm{i} + \mathbf{R}_\mathrm{i}\mathbf{P}_\mathrm{i}) \end{bmatrix} \delta\mathbf{m}$$

$$\doteq \nabla \mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{R}\mathbf{P}]\delta\mathbf{m}$$

$$\Downarrow$$

$$\delta\mathbf{m} \approx \nabla \mathbf{F}^{-1}[\mathbf{m}_0, \mathbf{Q} + \mathbf{R}\mathbf{P}]\mathbf{p}$$

Sparse inversion

We use a sparsity-promoting formulation:

$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \operatorname*{argmin}_{\delta\mathbf{x}} ||\delta\mathbf{x}||_1$$

$$\text{subject to } ||\mathbf{p} - \nabla\mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

$\mathbf{C}$: curvelet transform

solver: $\mathrm{SPG}\ell_1$

Example using a simple model

- model grid spacing: 5 meters
- using linearized data including surface related multiples:

$$\nabla \mathbf{F}[\mathbf{m_0}, \mathbf{Q} + \mathbf{RP}]\delta \mathbf{m}$$

- 150 collocated sources/receivers
- 122 frequencies in 0-60Hz range

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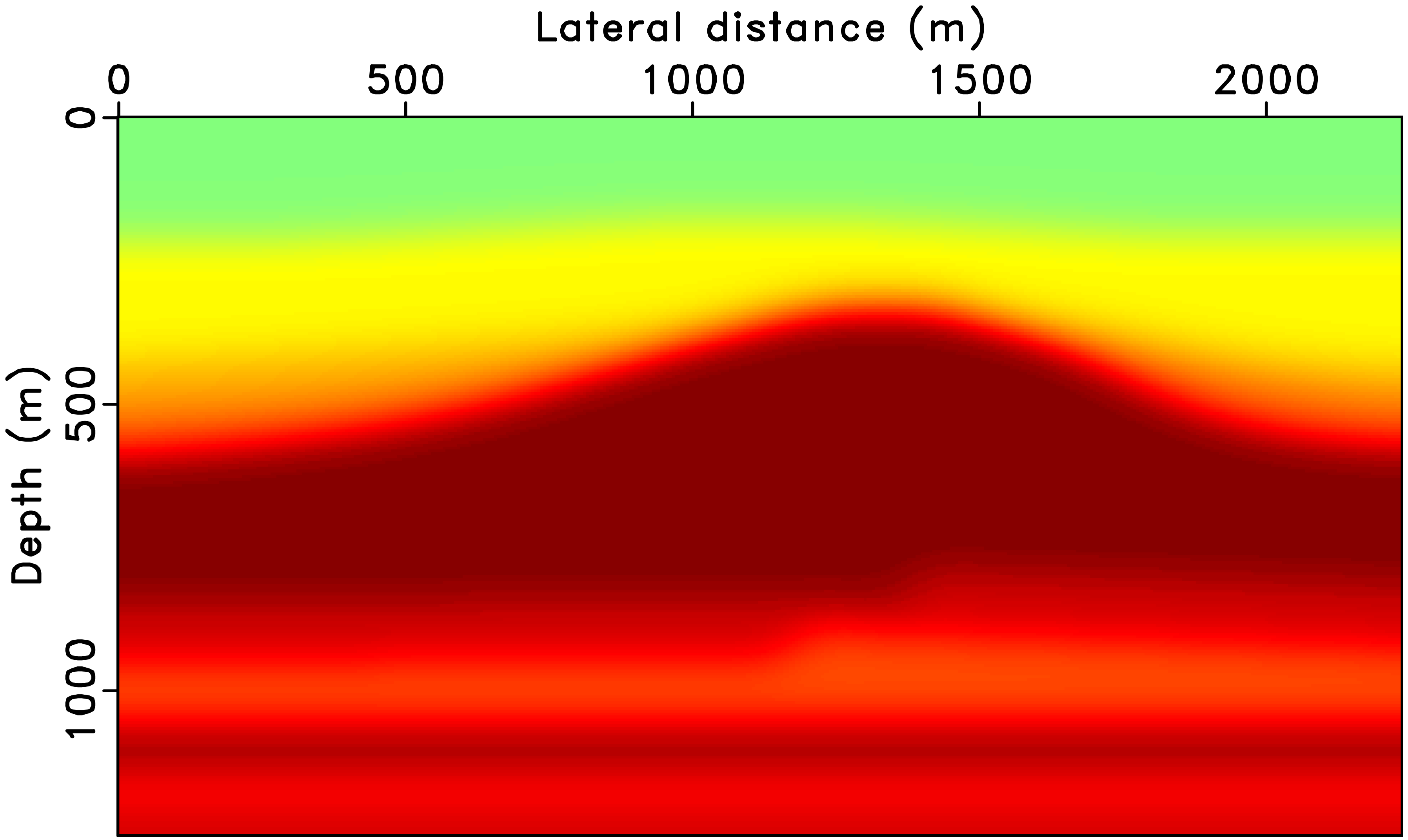
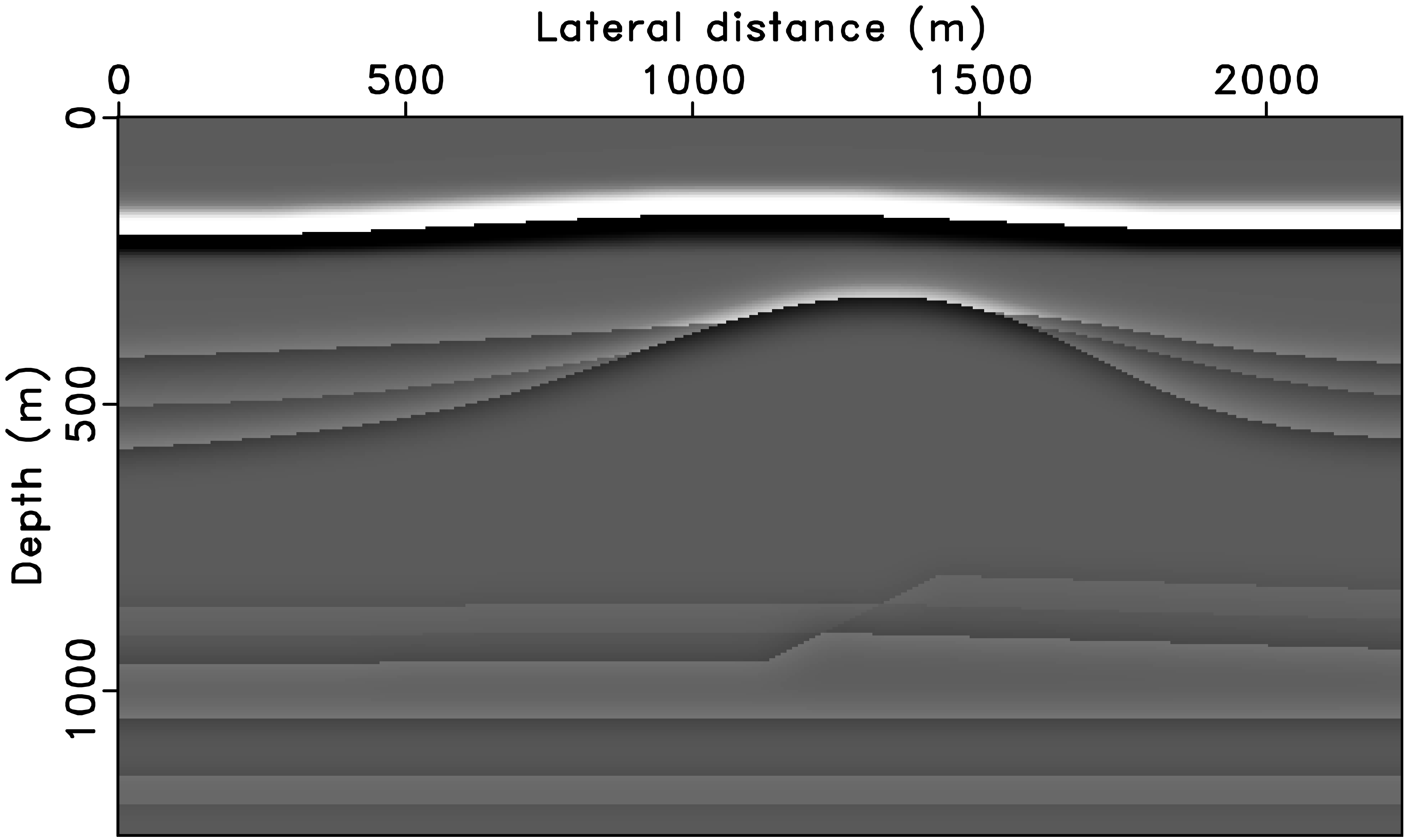
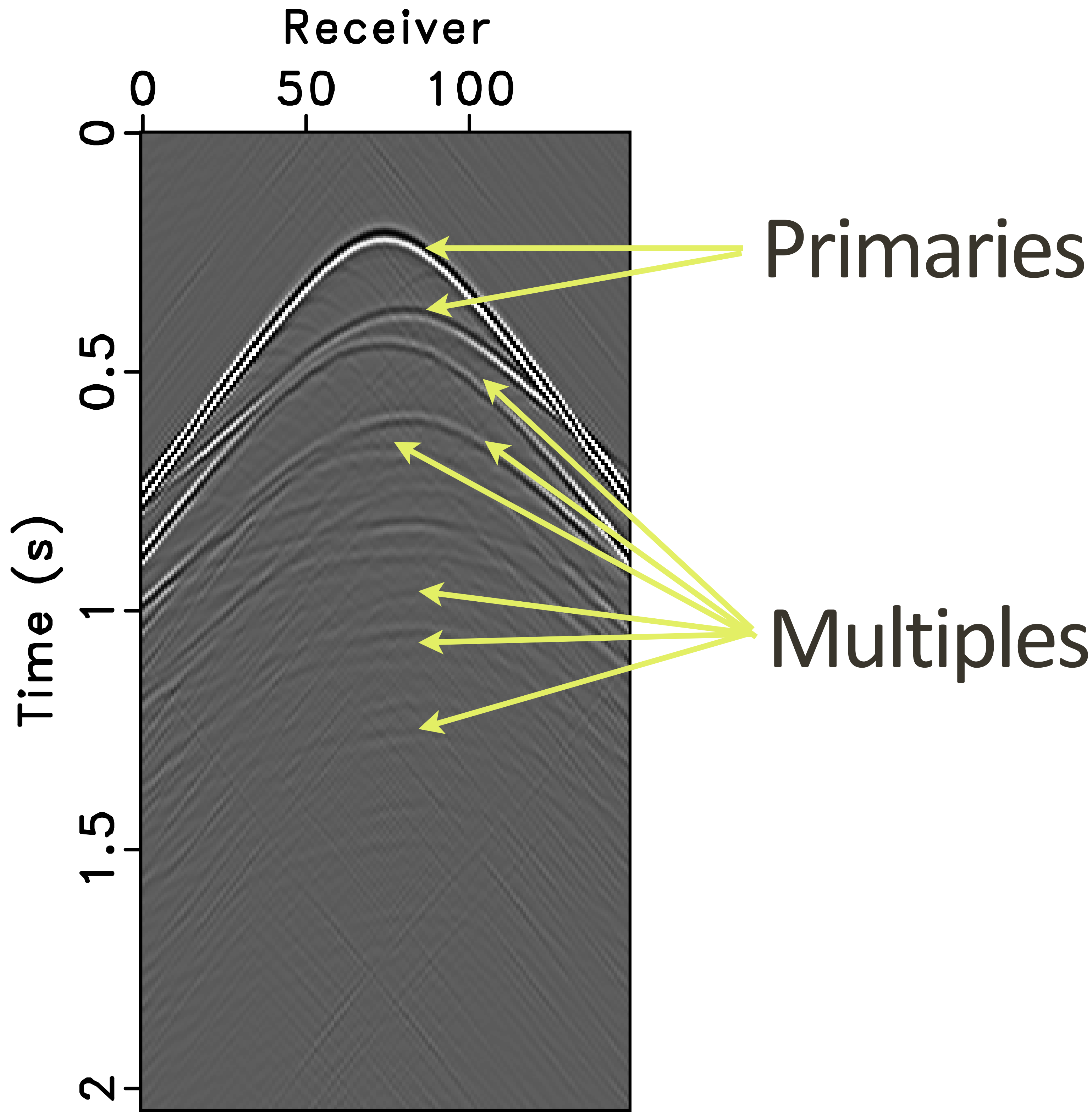
Background velocity model



True perturbation

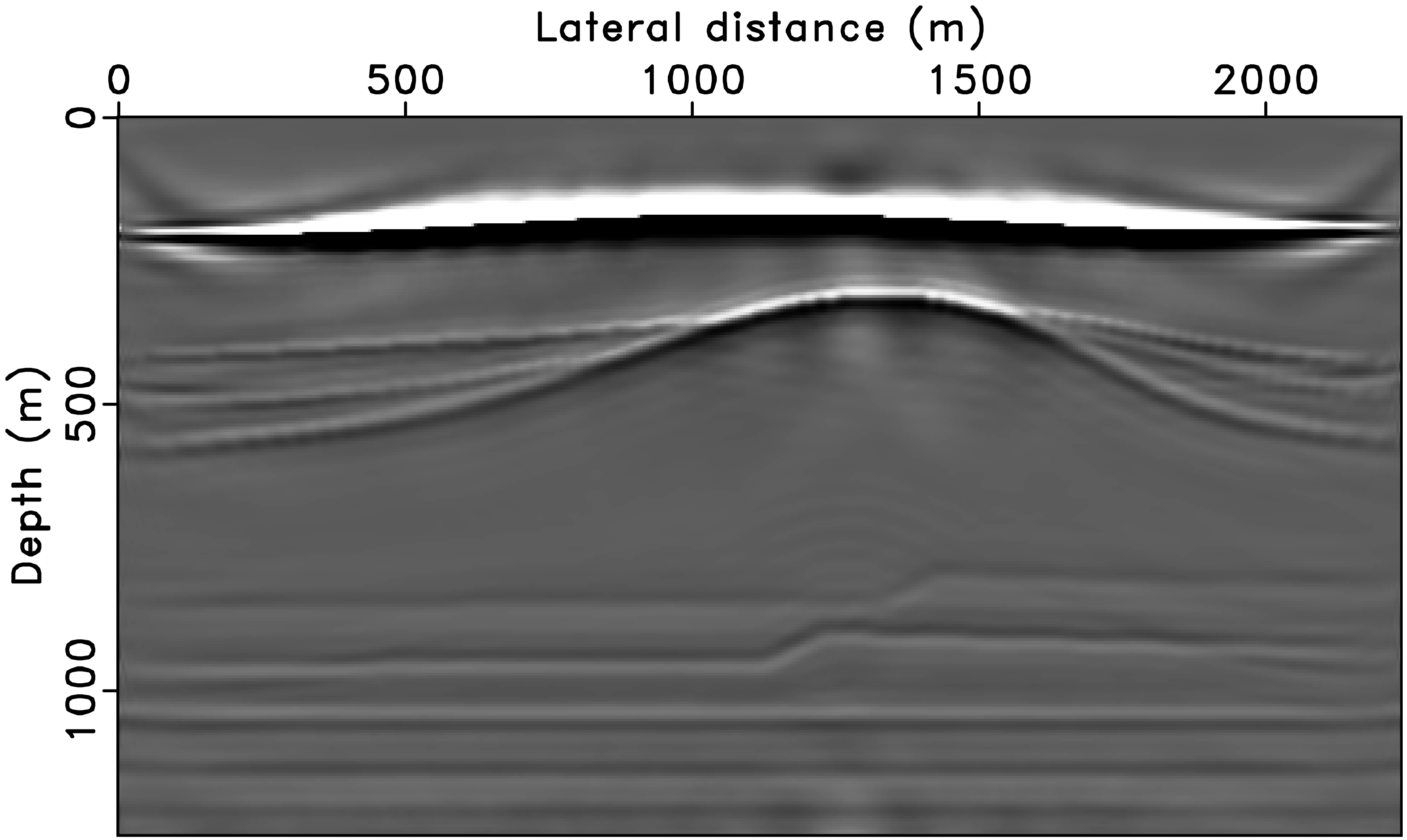


Linearized total data



Inversion of total data

[by computing the inverse of the Born scattering operator]



Inversion of the total up-going wavefield using all sequential sources and all frequencies
number of PDE solves: ~4.4 million (by calculation)

Speed up inversion

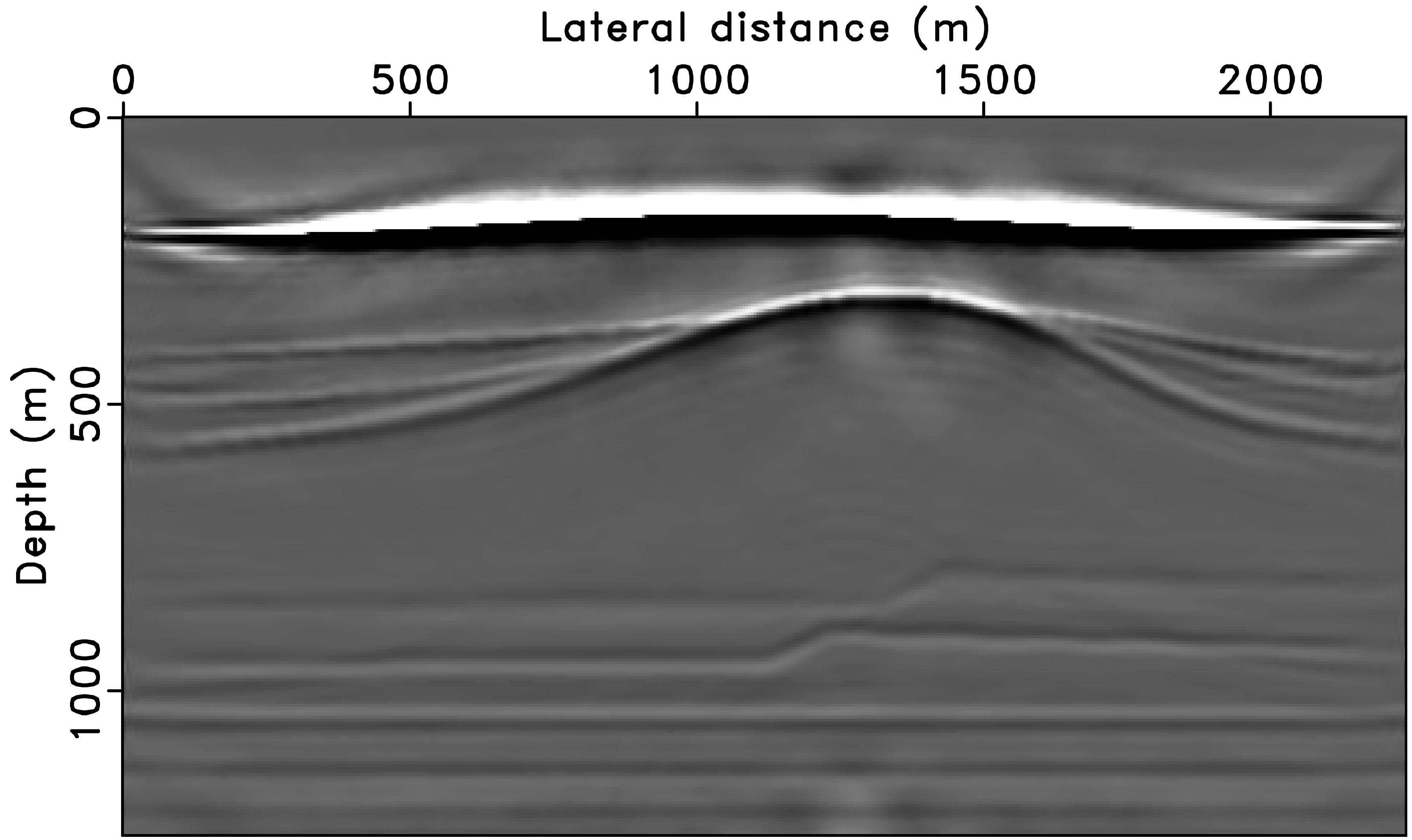
$$\delta\tilde{\mathbf{m}} = \mathbf{C}^H \underset{\delta\mathbf{x}}{\operatorname{argmin}} ||\delta\mathbf{x}||_1$$

$$\text{subject to } ||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2 \leq \sigma$$

source: combine all sequential sources into a few simultaneous sources

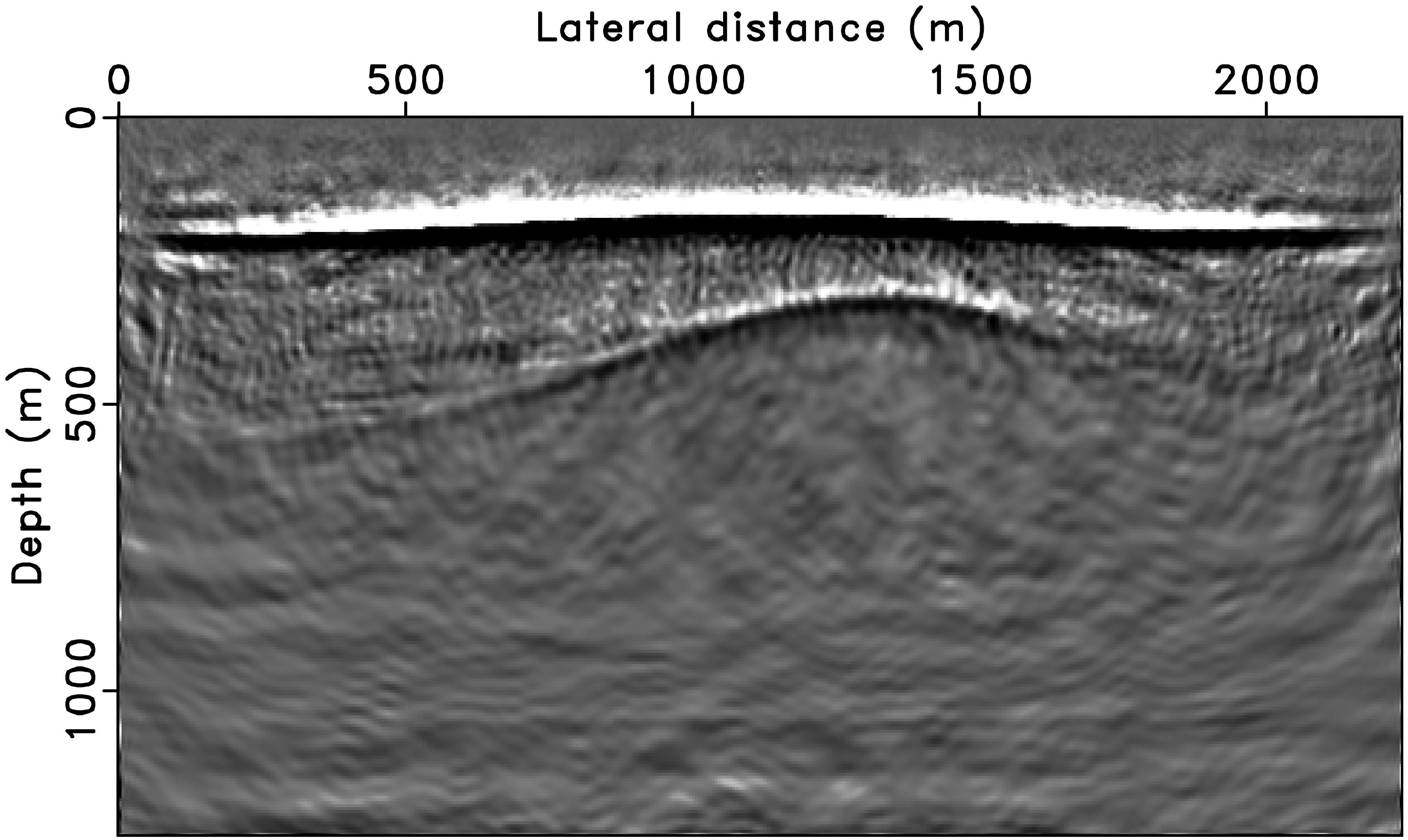
frequency: randomly choose a subset from all of them

Result with 15x speed-up



Inversion of the total up-going wavefield using 10 simultaneous sources and all frequencies
number of PDE solves: ~0.3 million[15X speed-up]

Too much subsampling brings artifacts [120X speed-up]



Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand [120X speed-up]

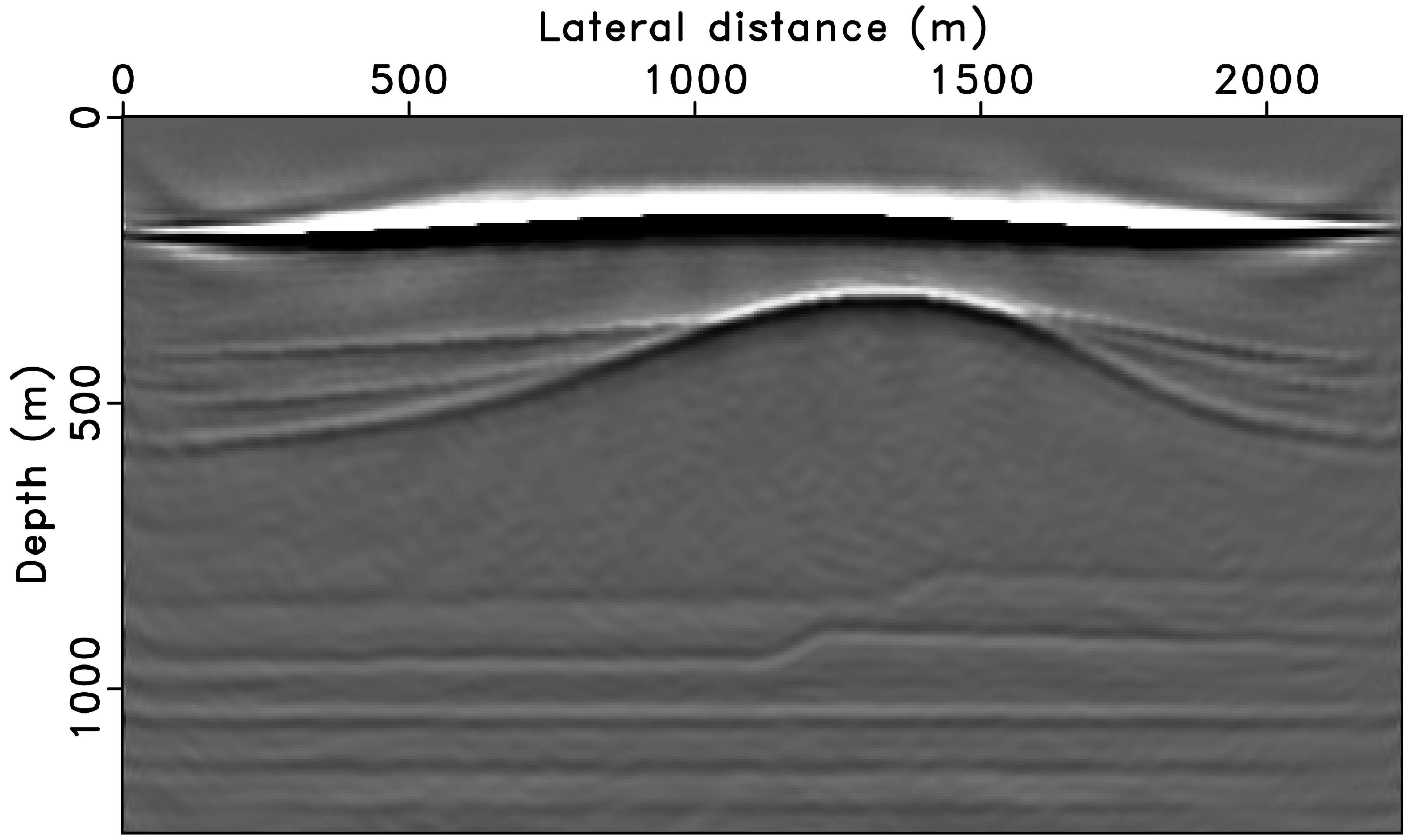
Rerandomization

- $\mathrm{SPG}\ell_1$ solves a series of subproblems:

$$\begin{aligned}&\underset{\delta\mathbf{x}}{\operatorname{argmin}}\,||\underline{\mathbf{p}} - \nabla\mathbf{F}[\mathbf{m_0}, \underline{\mathbf{Q}} + \underline{\mathbf{RP}}]\mathbf{C}^H\delta\mathbf{x}||_2\\ &\text{subject to } ||\delta\mathbf{x}||_1 \le \tau\end{aligned}$$

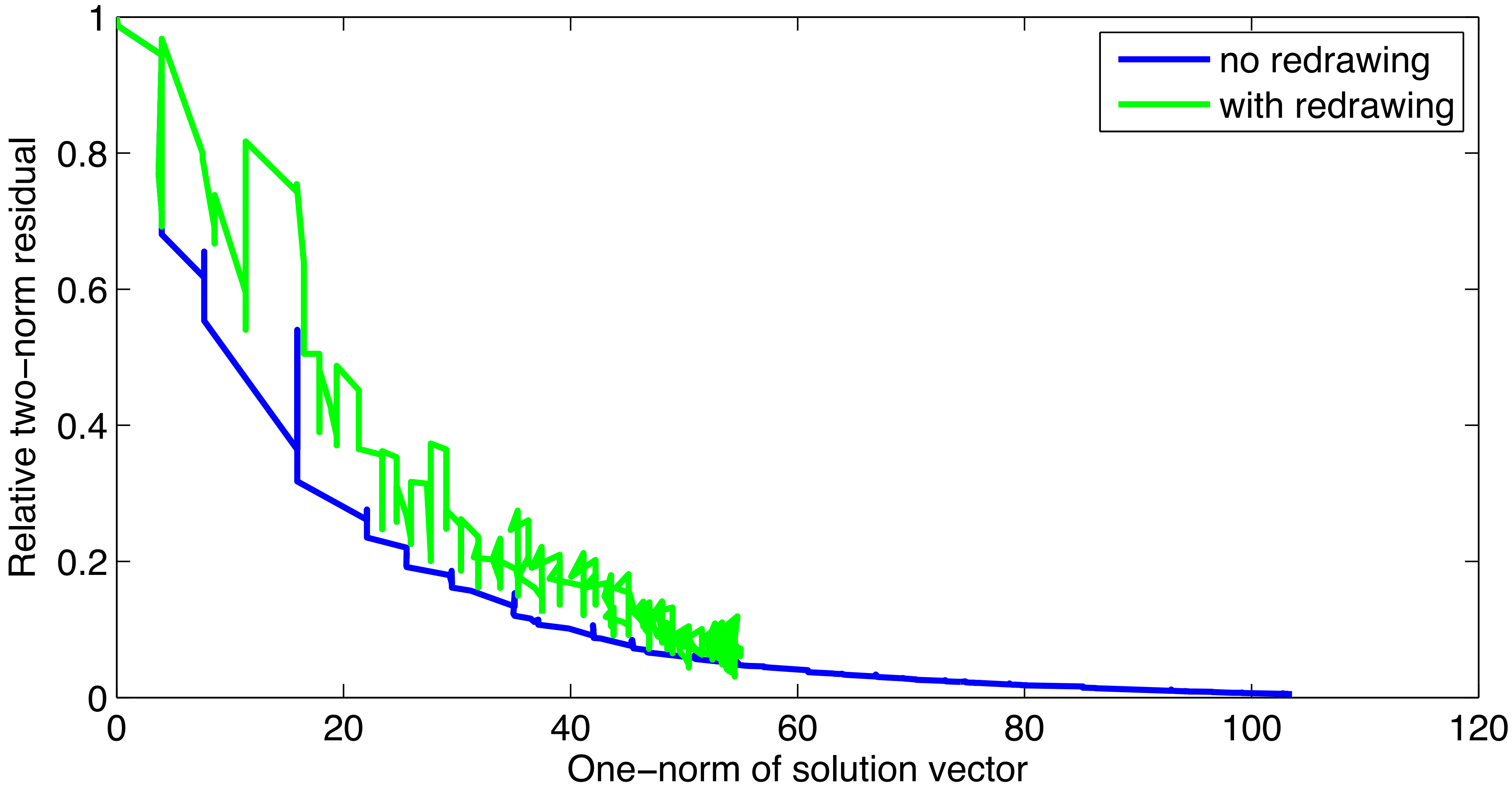
- redraw subsampling operator for each new subproblem
- motivated by insights from approximate message passing

Redraw sim. sources and frequencies [120X speed-up]

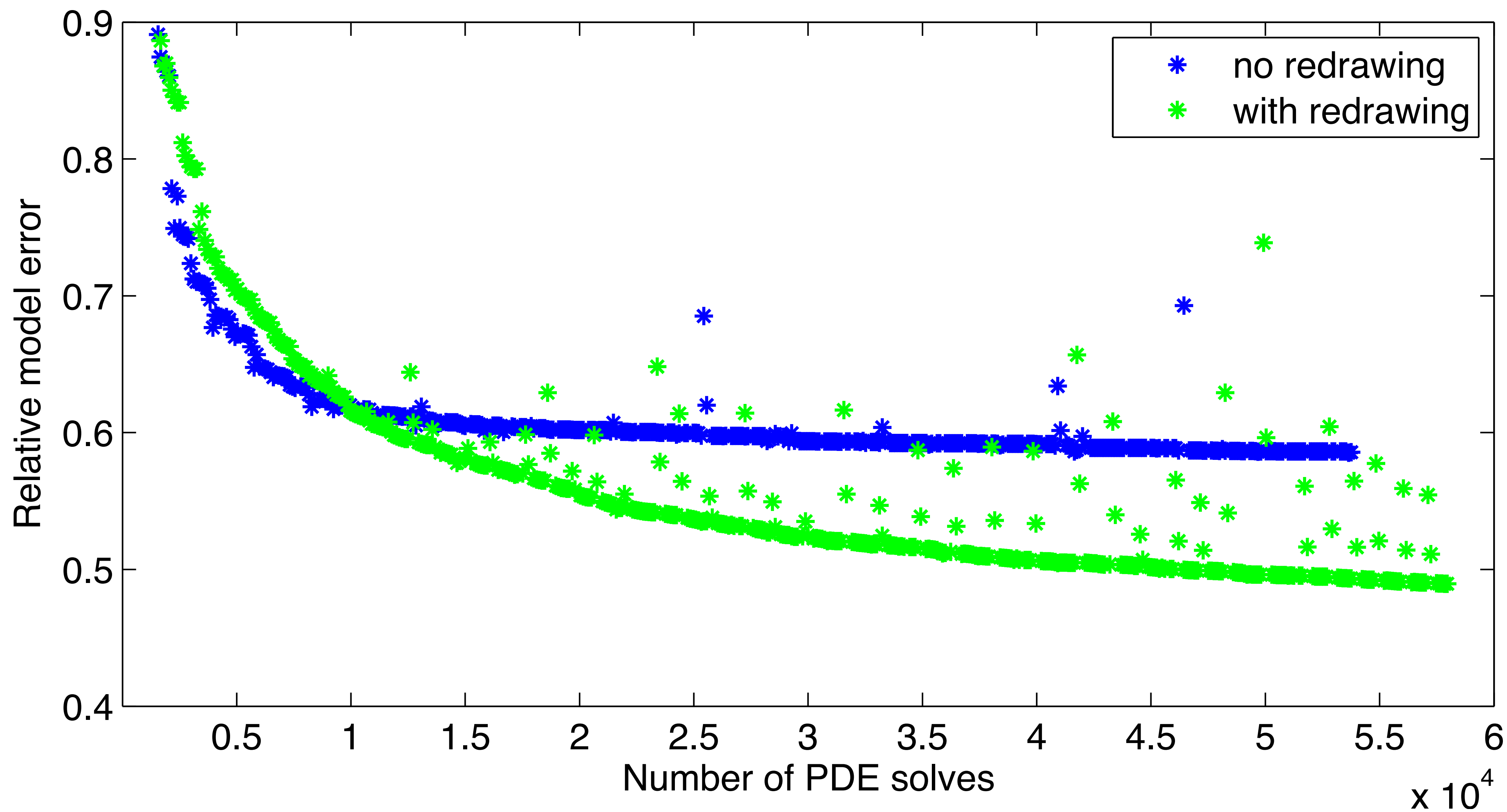


Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies
number of PDE solves: 36.6 thousand (by calculation)

Solution path



Model error decrease



Note: outliers are intermediate line-search results, not a concern; number of PDE solves in practice has ~50% overhead due to line search, etc.

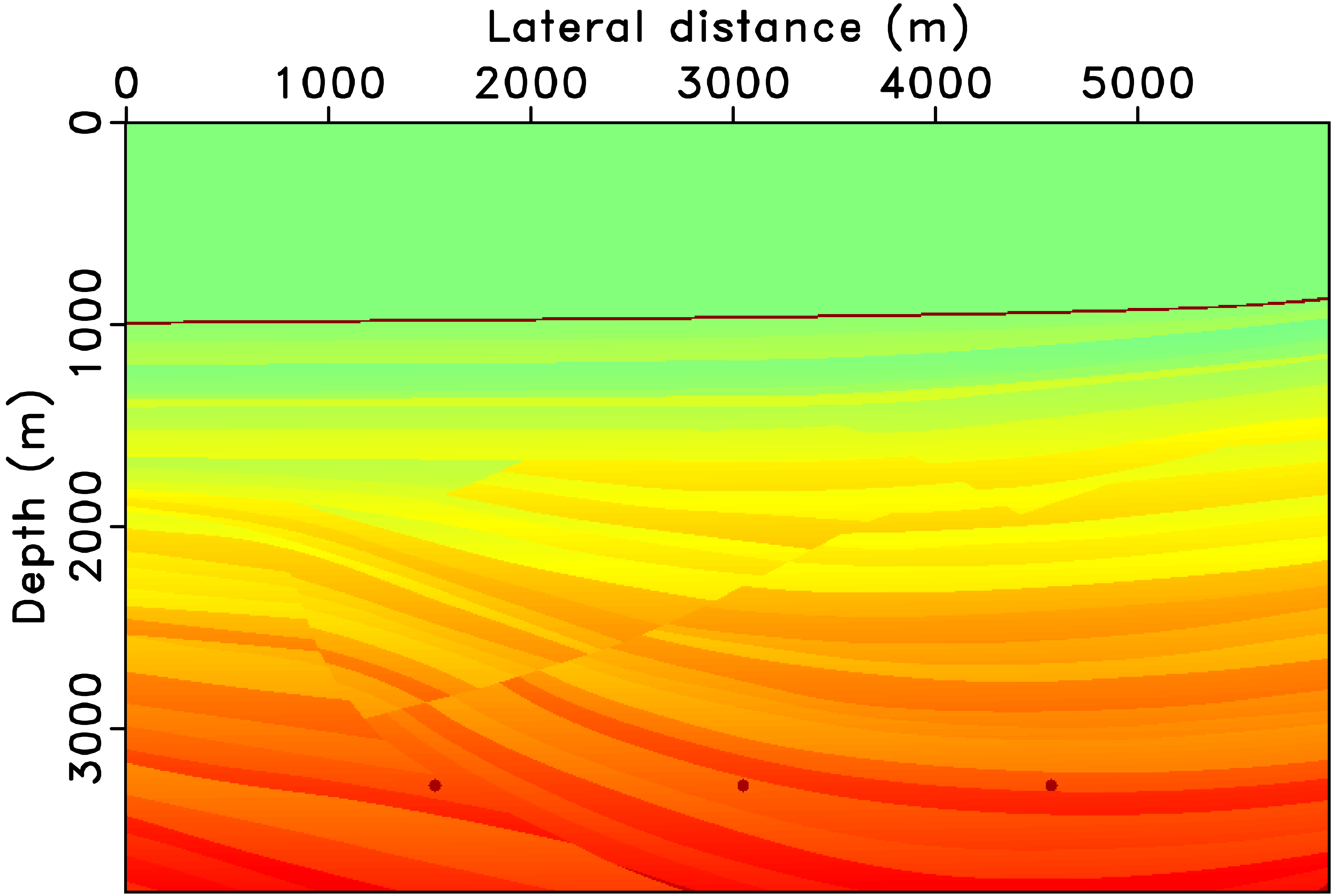
Case study

Using a complex model

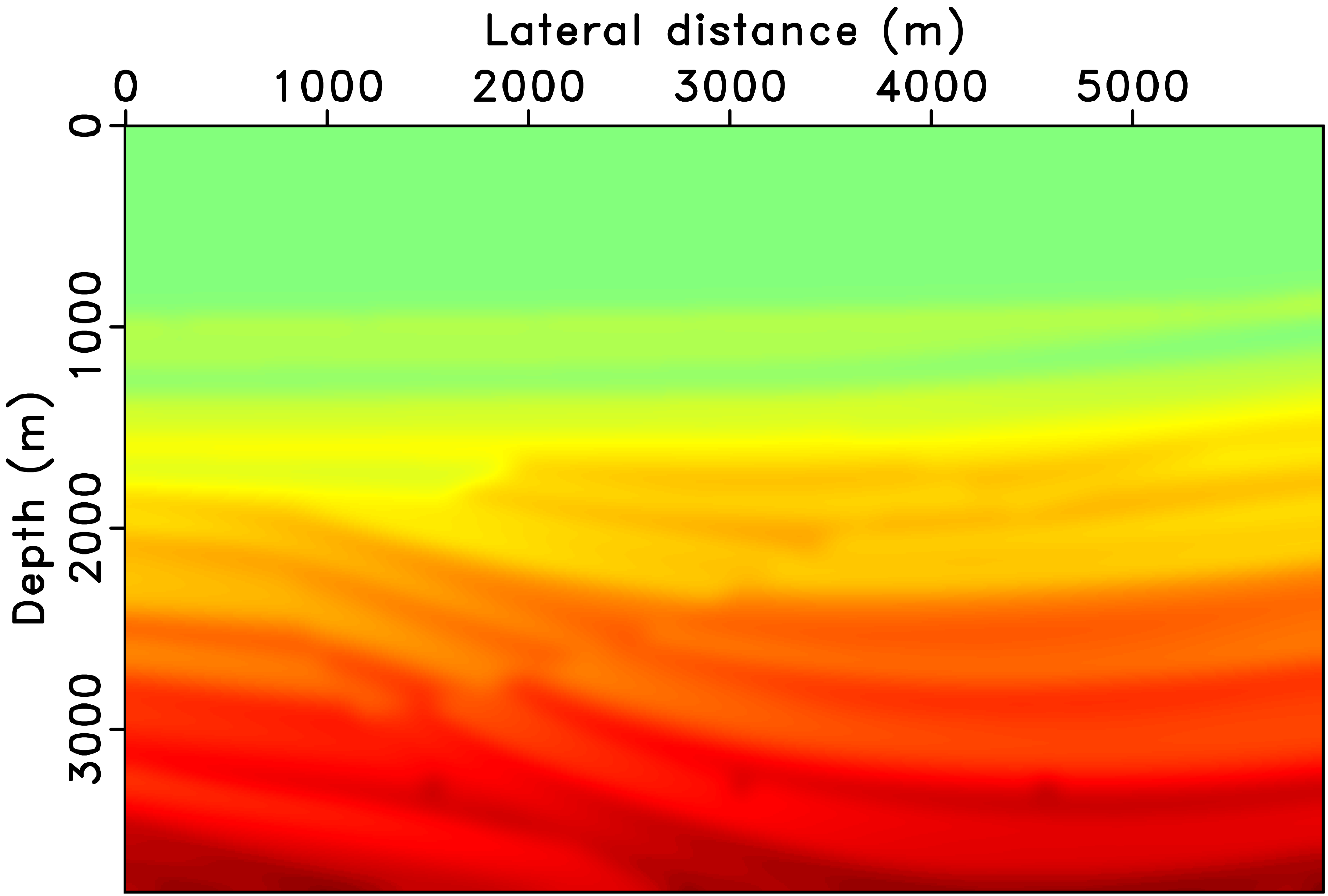
[cropped from the Sigsbee 2B model]

- model grid spacing: 7.62m
- using linearized data
- 261 sequential sources
- ~8s recording time, 278 frequencies in 0-34Hz range
- using 8 simultaneous sources and 15 frequencies with rerandomization

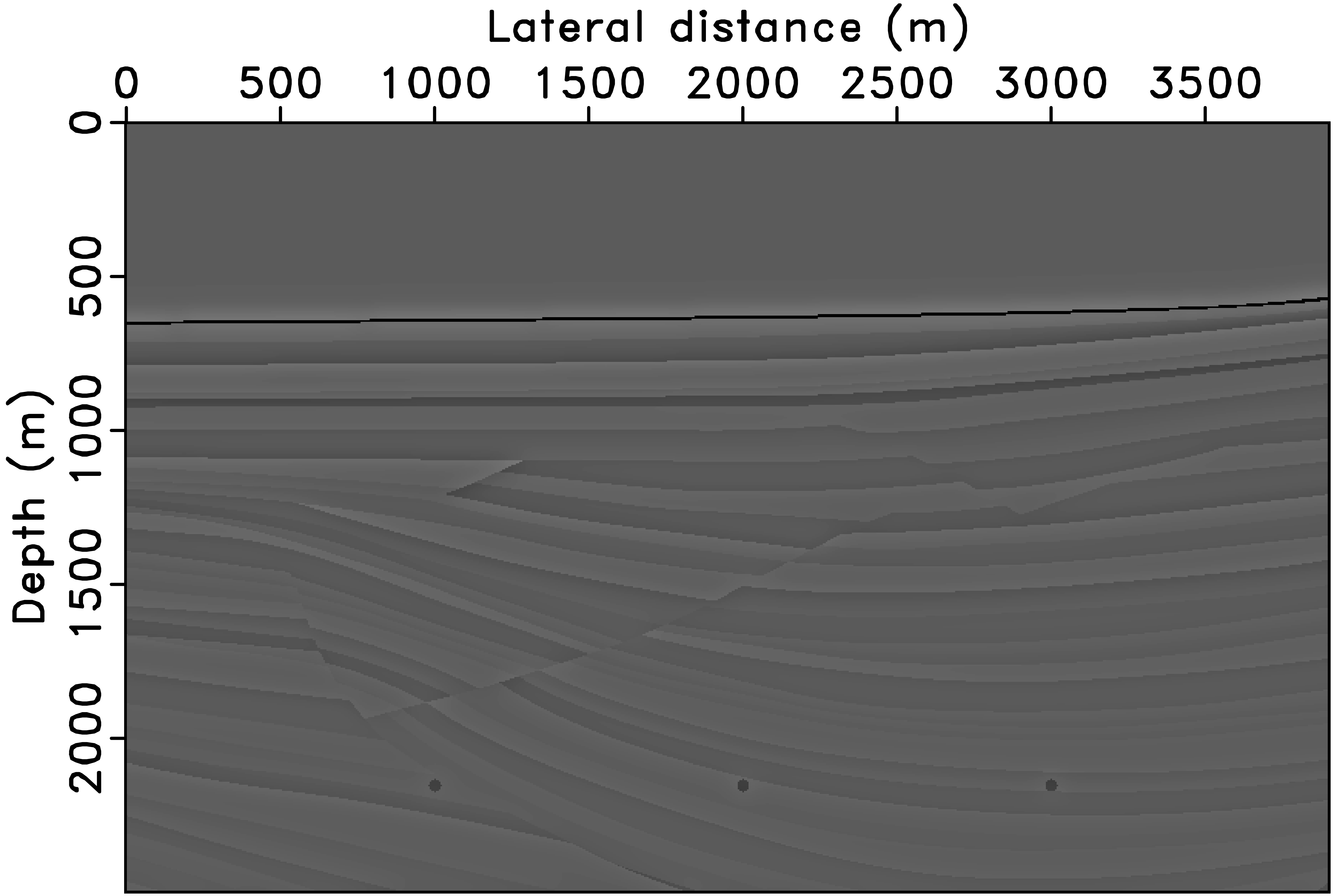
The true velocity model



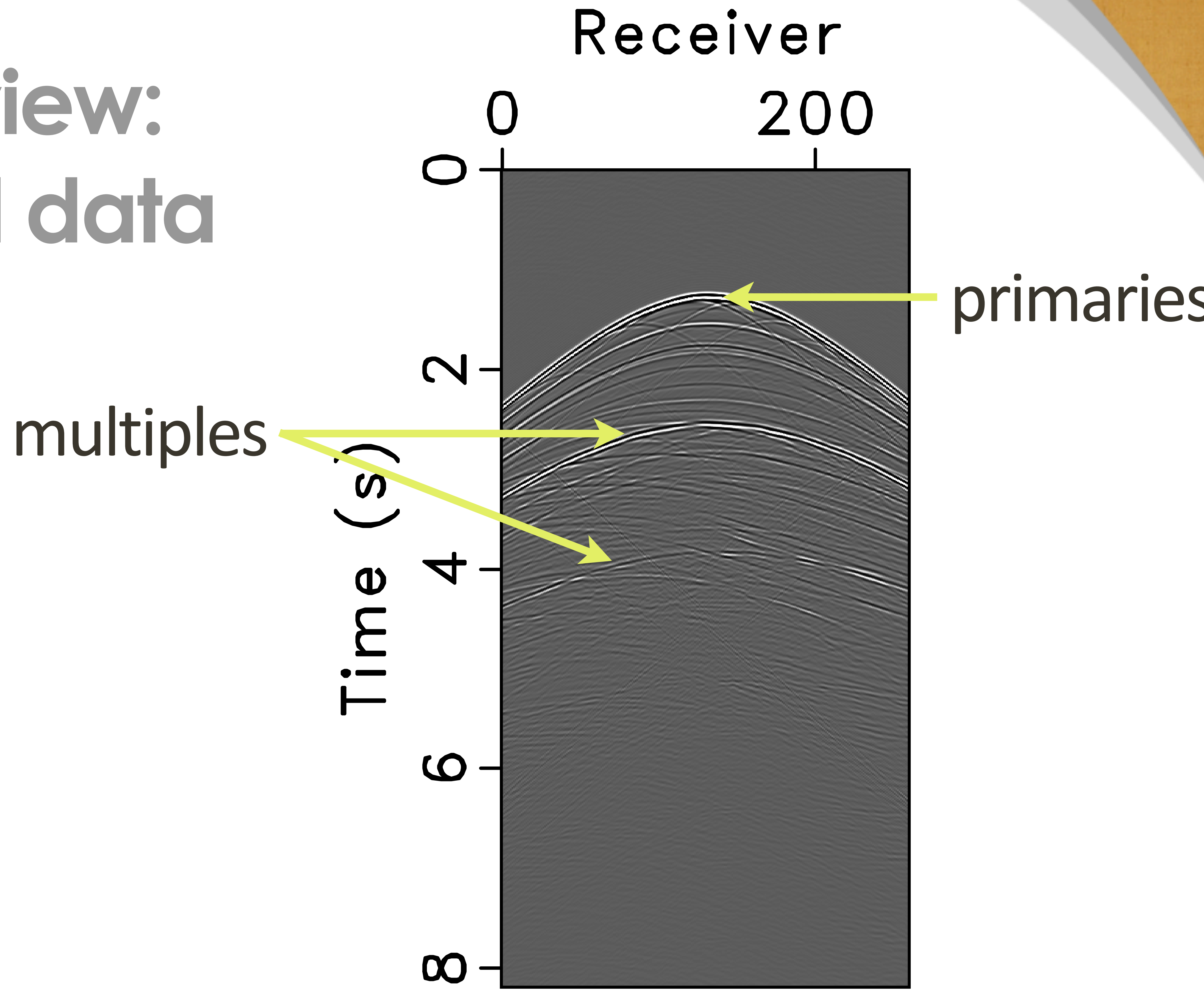
Background velocity model



True perturbation

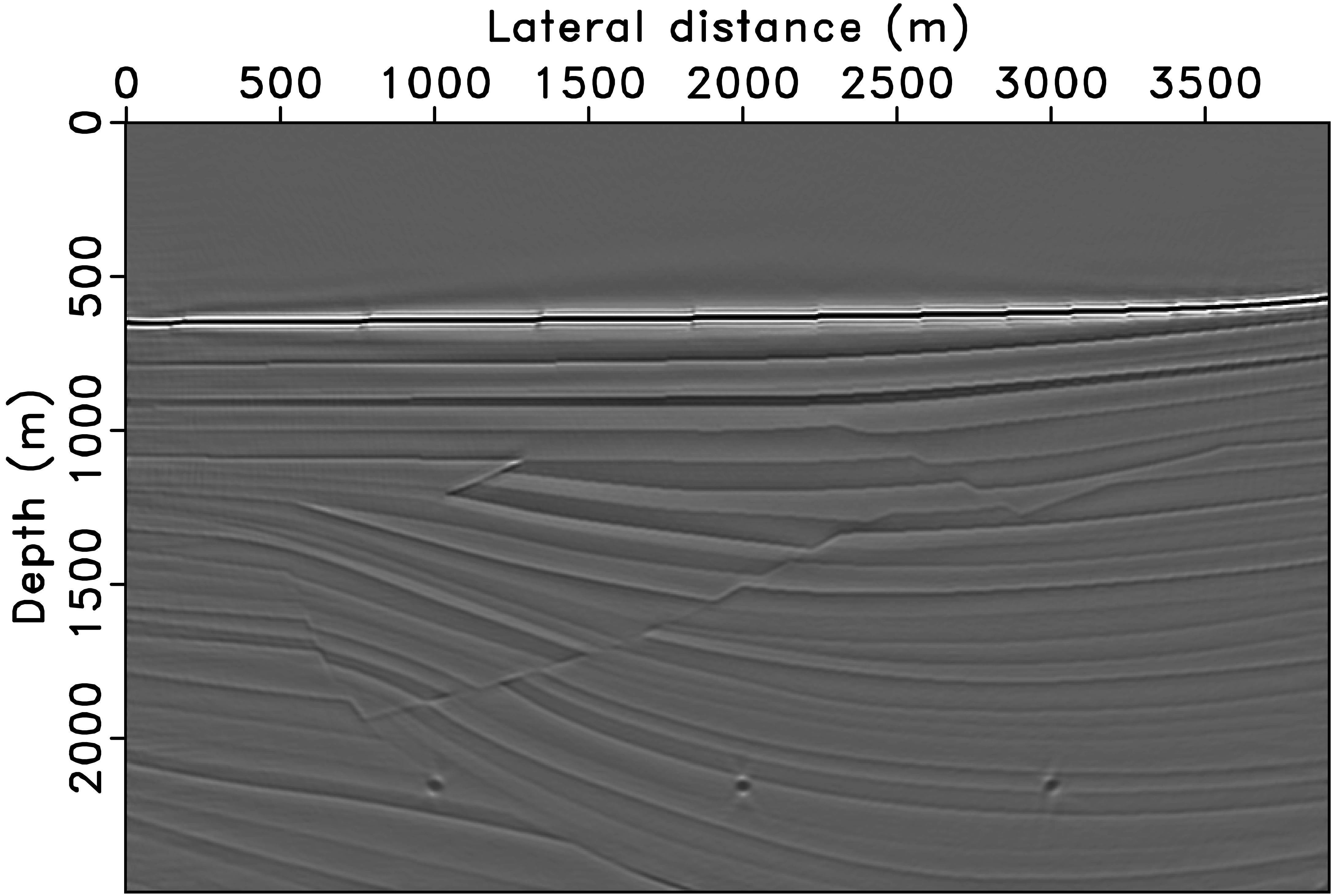


Preview: total data



Fast inversion of total data

[with the **same** computational budget **as a single RTM with all data**]



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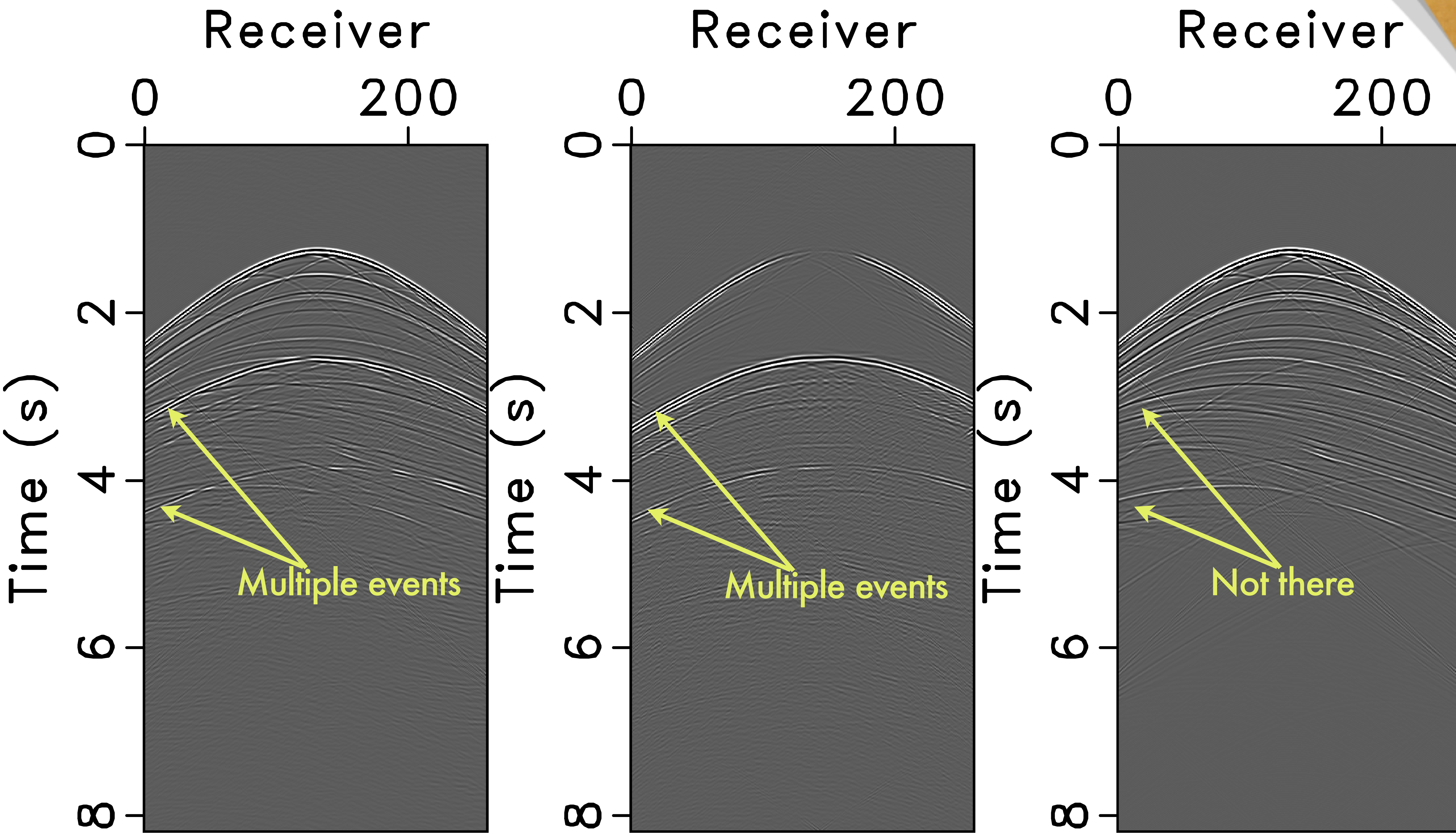
Example with coarse source sampling

- suppose only 21 shots regularly sampled from all 261 shots are available
- an analogue of limited number of ocean bottom nodes by source-receiver reciprocity
- SRME and EPSI have difficulty to predict or invert multiples
- we directly image from the total data using the proposed method

Inverted primaries by EPSI

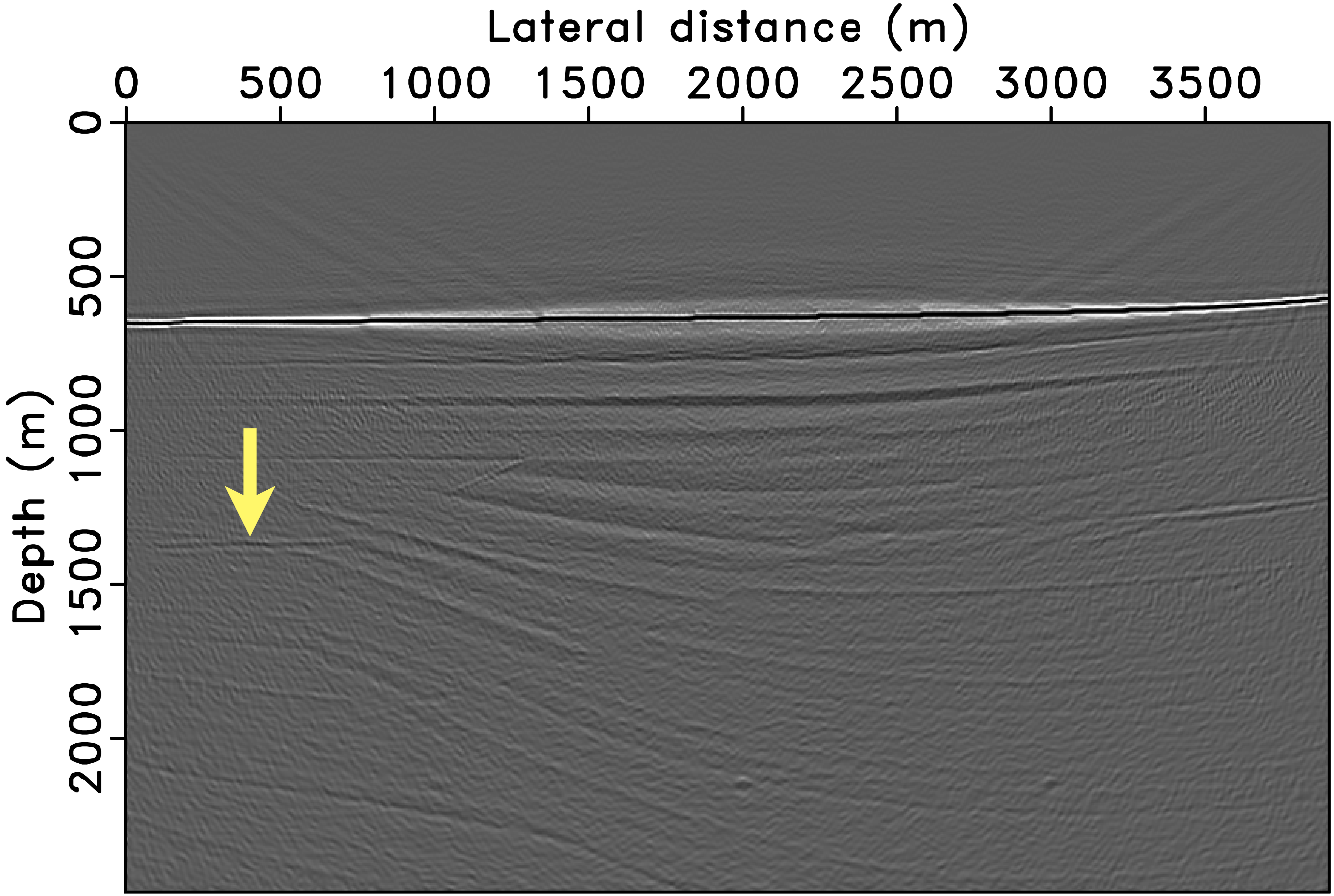


Data

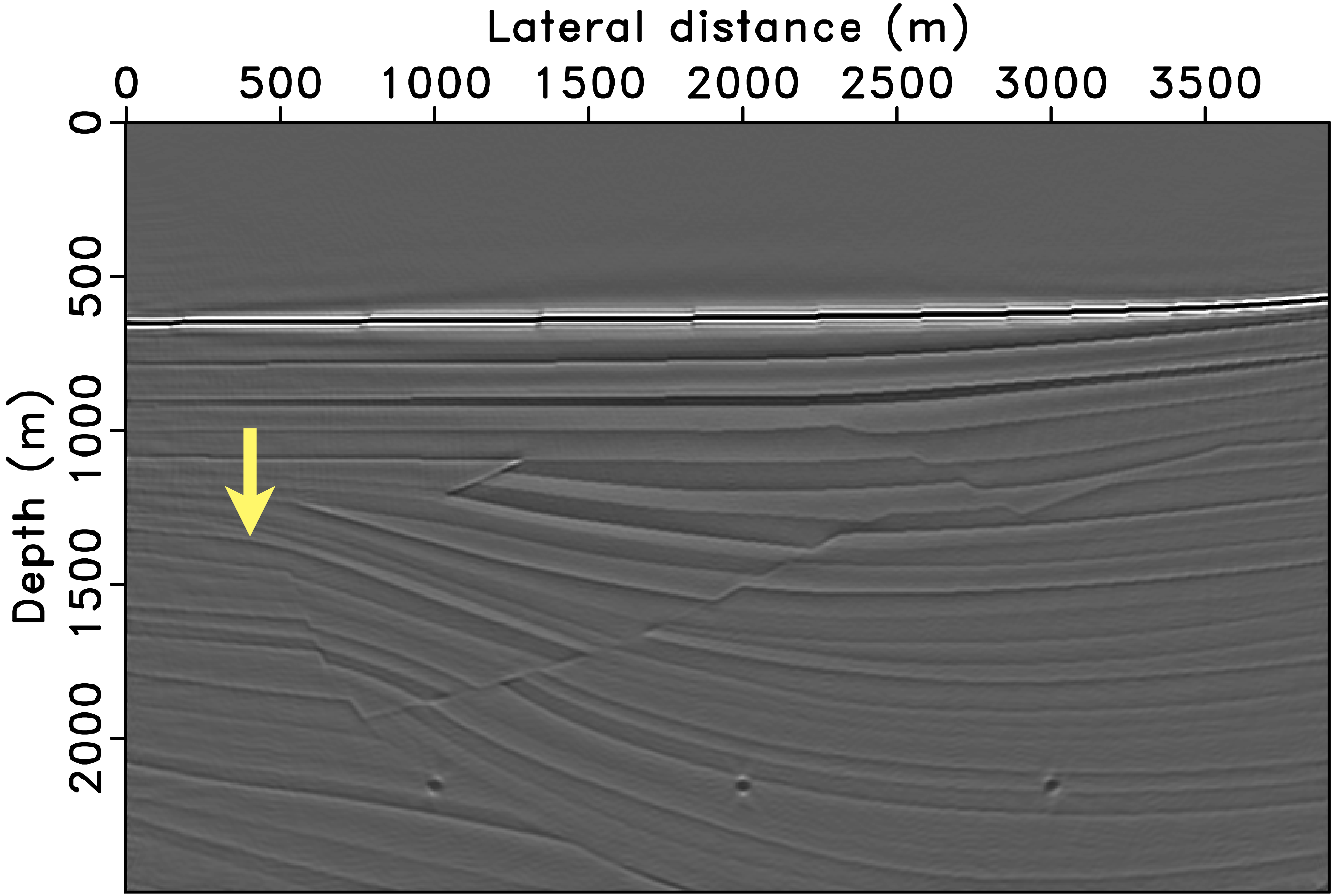
Inverted

True

Image from EPSI inverted data



Our result



Lateral distance (m)
0
500
1000
1500
2000
2500
3000
3500
Depth (m)
0
500
1000
1500
2000

Conclusions

- It is plausible to image with multiples without the artifacts from them.
- Non-causal cross correlations caused by multiples are eliminated by inversion.
- Multi-dimensional convolution in multiple prediction can be implicitly carried out by the wave-equation solver.

Conclusions *[cont.]*

- We gain significant speed-up in sparsity-promoting RTM by subsampling over sources/frequencies and rerandomization.
- Our method can handle data with very large source (or receiver by reciprocity) gaps by optimizing in the image space instead of data space (e.g., EPSI).

Acknowledgements

SMAART JV for sharing the Sigsbee 2B model.

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