Imaging with multiples accelerated by message passing
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Motivation

• making use of primaries and multiples *simultaneously*
• *eliminating artifacts* from multiples
• looking for a computationally *efficient* approach

Lin, Tu, and Herrmann, 2010; Verschuur and Berkhout, 2011; Whitmore et.al., 2010; Liu et.al., 2011
Primaries & multiples: not ‘or’ but ‘and’

- primaries have a higher signal-to-noise ratio
- multiples can be useful if used correctly
- separating them can be very expensive
- they are not always separable
Conventional RTM image
[use the primary imaging operator]

Reverse time migration of *primaries+multiples*, without accounting for multiples
RTM of total data

[the imaging operator includes the areal source to account for multiples]

Reverse time migration of *primaries+multiples*, accounting for multiples

Muijs et al., 2007; Whitmore et al., 2010; Liu et al., 2011
When a free-surface is present

Muijs et al., 2007; Liu et al., 2011

(i+1)-th order reflection → backward propagation → cross-correlation → image

i-th order reflection → forward propagation

(i-1)-th order reflection → forward propagation

(i-th, i+1-th, i+2-th) order reflection → backward propagation → cross-correlation → artifacts

...
Artifacts-free image by inversion

Lin et. al., 2010; Tu and Herrmann, 2011a

Imaging of *primaries+multiples* by *inversion*
Inversion? Sounds expensive...

- repeated evaluations of the Born scattering operator and its adjoint
- each evaluation requires solving $4 \times (#\text{source}) \times (#\text{frequencies})$ PDEs
Sneak peek of our result
(with a 120X speed-up compared to the previous image)

Fast imaging of *primaries*+*multiples* by sparse inversion
Method
Physics of the free surface

Total data and the surface-free Green’s function can be related by the SRME formulation:

\[ P_i = G_i(Q_i + R_i P_i) \]

- **P** : total up-going wavefield
- **G** : surface-free Green’s function
- **Q** : source wavelet
- **R** : surface reflectivity
Expressed in model space

\[ P_i = \text{vec}^{-1}(F_i[m, I])(Q_i + R_i P_i) \]
\[ = D_r H_i^{-1} [m](D_s^* I)(Q_i + R_i P_i) \]
\[ = D_r H_i^{-1} [m](D_s^*(Q_i + R_i P_i)) \]
\[ = \text{vec}^{-1}(F_i[m, Q_i + R_i P_i]) \]

\textbf{F} : forward modelling operator
\textbf{m} : true model
\textbf{I} : impulsive source array
\textbf{D}_r, \textbf{D}_s : detection operator at receiver/source locations
\textbf{H} : time-harmonic Helmholtz operator
Linearized forward modelling
[monochromatic]

\[ p_i = \nabla F_i [m_0, Q_i + R_i P_i] \delta m + \text{higher order reflections} \]

\( \nabla F \): Born scattering operator
\( m_0 \): background model
\( \delta m \): model perturbation
\( P \): vectorized wavefield
Linearized forward modelling

(all frequencies)

\[
\mathbf{p} \approx \begin{bmatrix}
\nabla F_1(m_0, Q_i + R_i P_i) \\
\vdots \\
\nabla F_{nf}(m_0, Q_i + R_i P_i)
\end{bmatrix} \delta m \\
\hat{=} \nabla F[m_0, Q + RP] \delta m
\]

\[
\delta m \approx \nabla F^{-1}[m_0, Q + RP] \mathbf{p}
\]
Sparse inversion

We use a sparsity-promoting formulation:

$$\delta \tilde{m} = C^H \arg\min_{\delta x} ||\delta x||_1$$

subject to $$||p - \nabla F[m_0, Q + RP]C^H \delta x||_2 \leq \sigma$$

$C$: curvelet transform
solver: SPG$\ell_1$
Example using a simple model

- model grid spacing: 5 meters
- using linearized data including surface related multiples:
  \[ \nabla F[m_0, Q + RP] \delta m \]
- 150 collocated sources/receivers
- 122 frequencies in 0-60Hz range
Background velocity model
True perturbation
Linearized total data

- Primaries
- Multiples
Inversion of total data

[by computing the inverse of the Born scattering operator]

Inversion of the total up-going wavefield using all sequential sources and all frequencies

number of PDE solves: ~4.4 million (by calculation)
Speed up inversion

\[ \delta \tilde{m} = C^H \argmin_{\delta x} ||\delta x||_1 \]

subject to \[ ||p - \nabla F[m_0, Q + RP]C^H \delta x||_2 \leq \sigma \]

**source**: combine all sequential sources into a few simultaneous sources

**frequency**: randomly choose a subset from all of them
Result with **15x speed-up**

Inversion of the total up-going wavefield using 10 simultaneous sources and all frequencies number of PDE solves: ~0.3 million[15X speed-up]
Too much subsampling brings artifacts [120X speed-up]

Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies number of PDE solves: 36.6 thousand [120X speed-up]
Rerandomization

- \text{SPG}\ell_1 \text{solves a series of subproblems:}

  \[ \underset{\delta x}{\arg\min} \| p - \nabla F[m_0, Q + RP]C^H \delta x \|_2 \]

  subject to \( \| \delta x \|_1 \leq \tau \)

- redraw subsampling operator for each new subproblem
- motivated by insights from approximate message passing
Redraw sim. sources and frequencies [120X speed-up]

Inversion of the total up-going wavefield using 2 simultaneous sources and 15 frequencies

number of PDE solves: 36.6 thousand (by calculation)
Solution path

One-norm of solution vector vs. Relative two-norm residual.

- Blue line: no redrawing
- Green line: with redrawing
Model error decrease

Note: outliers are intermediate line-search results, not a concern; number of PDE solves in practice has ~50% overhead due to line search, etc.
Case study
Using a complex model
[cropped from the Sigsbee 2B model]

- model grid spacing: 7.62m
- using linearized data
- 261 sequential sources
- ~8s recording time, 278 frequencies in 0-34Hz range
- using 8 simultaneous sources and 15 frequencies with rerandomization
The true velocity model
Background velocity model
True perturbation
Preview:

total data

Receiver

0 200

Time (s)

0 2 4 6 8

primaries

multiples
Fast inversion of total data

(with the same computational budget as a single RTM with all data)
Example with coarse source sampling

- suppose only 21 shots regularly sampled from all 261 shots are available
- an analogue of limited number of ocean bottom nodes by source-receiver reciprocity
- SRME and EPSI have difficulty to predict or invert multiples
- we directly image from the total data using the proposed method
Inverted primaries by EPSI

Data

Multiple events

Inverted

Multiple events

True

Not there
Image from EPSI inverted data
Our result
Conclusions

• It is plausible to image with multiples without the artifacts from them.
• Non-causal cross correlations caused by multiples are eliminated by inversion.
• Multi-dimensional convolution in multiple prediction can be implicitly carried out by the wave-equation solver.
Conclusions [cont.]

- We gain significant speed-up in sparsity-promoting RTM by subsampling over sources/frequencies and rerandomization.
- Our method can handle data with very large source (or receiver by reciprocity) gaps by optimizing in the image space instead of data space (e.g., EPSI).
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