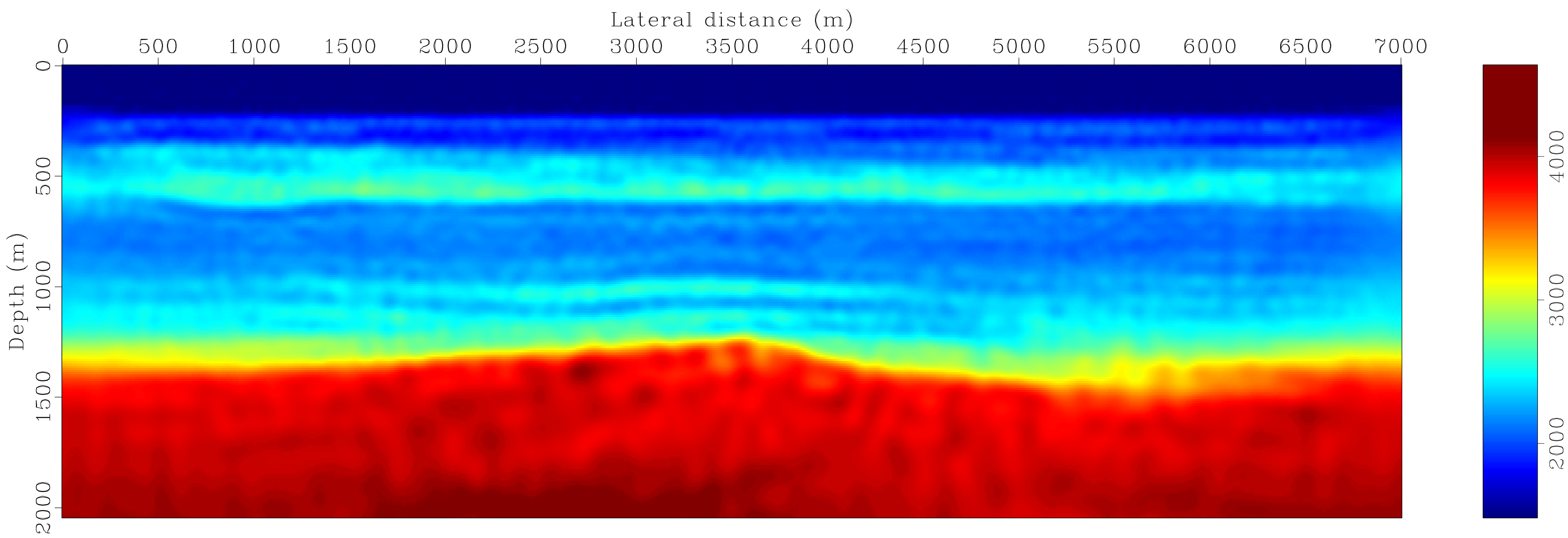
SEG Las Vegas 2012

Sparsity-promoting migration accelerated by message passing

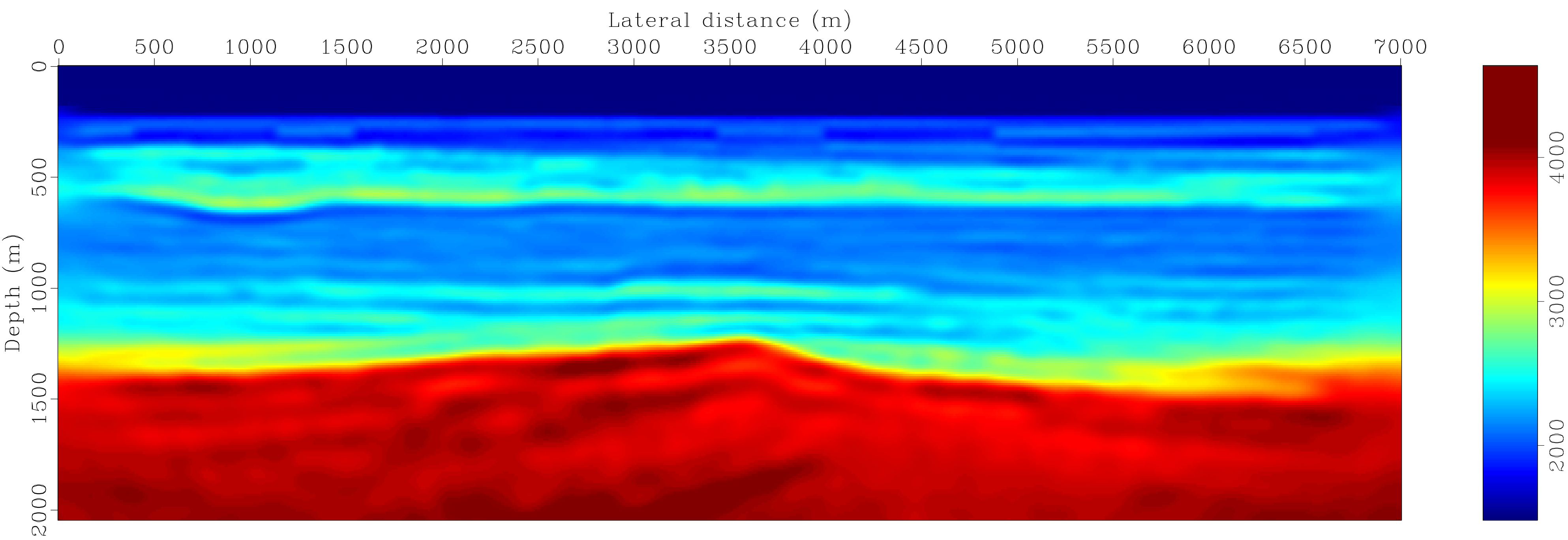
Xiang Li and Felix J. Herrmann

SLIM
University of British Columbia

Motivation

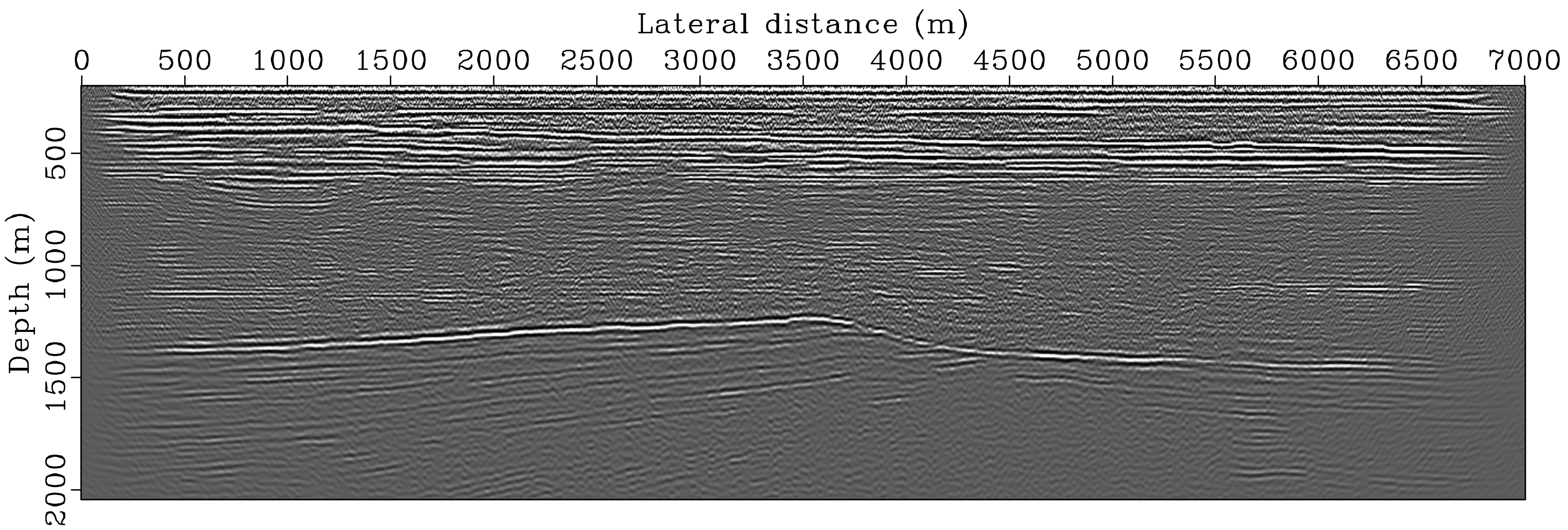


standard GN FWI

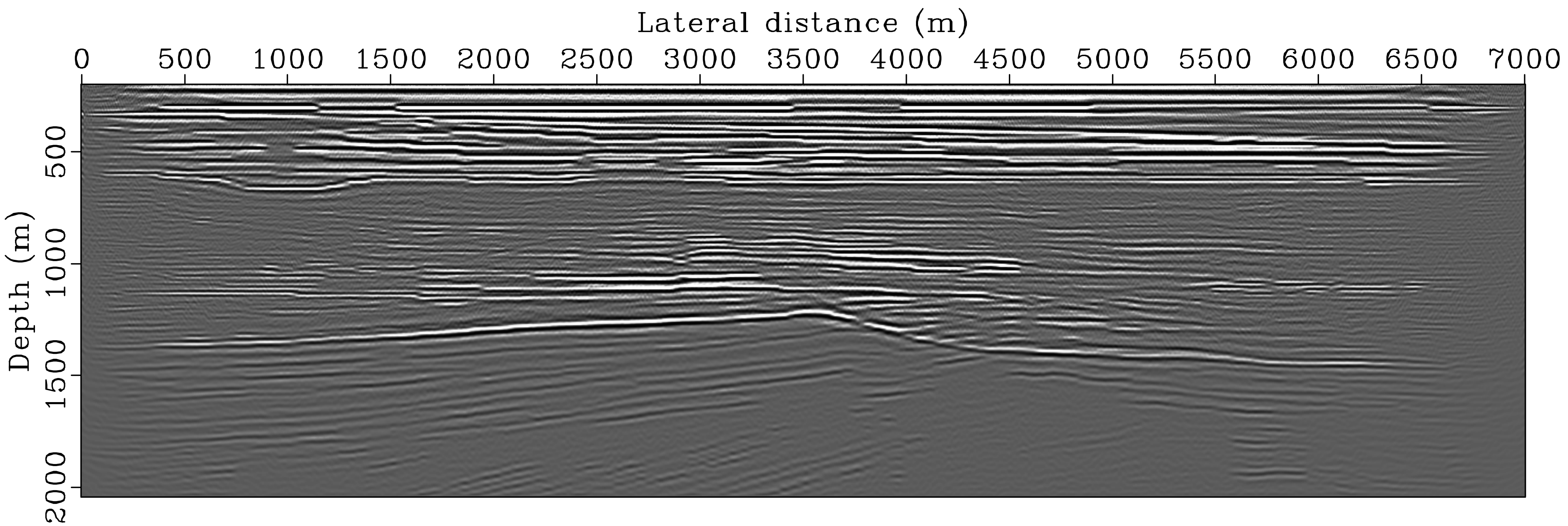


sparsity-promoting GN FWI

Motivation



standard least-squares migration



sparsity promoting least-squares migration

Outline

- sparsity promoting Gauss-Newton FWI to generate initial model
- sparsity-promoting imaging for migration

Making sparsity-promoting imaging computationally feasible...

Full-waveform inversion

Full waveform inversion

$$\underset{\mathbf{m},\alpha}{\text{minimize}} \; \frac{1}{2}\|\mathbf{D} - \alpha\boldsymbol{\mathcal{F}}[\mathbf{m}]\|_F^2$$

$$\begin{aligned}
\mathbf{D} &: \quad \text{observed data} \\
\boldsymbol{\mathcal{F}} &: \quad \text{forward modelling kernel} \\
\alpha &: \quad \text{source wavelet} \\
\mathbf{m} &: \quad \text{model parameters}
\end{aligned}$$

Gauss-Newton

Gauss-Newton subproblem:

$$\underset{\delta\mathbf{m}}{\text{minimize}}\ \frac{1}{2}\|\underbrace{\mathbf{D}-\alpha\boldsymbol{\mathcal{F}}(\mathbf{m})}_{\mathbf{b}}-\underbrace{\alpha\nabla\boldsymbol{\mathcal{F}}(\mathbf{m})}_{\mathbf{A}}\underbrace{\delta\mathbf{m}}_{\mathbf{x}}\|_F^2$$

- least-squares inversion problem
- no explicit Jacobian required

Sparsity-promoting GN

Gauss-Newton subproblem:

$$\underset{\delta\mathbf{m}}{\text{minimize}}\ \frac{1}{2}\|\mathbf{D}-\alpha\boldsymbol{\mathcal{F}}(\mathbf{m})-\alpha\nabla\boldsymbol{\mathcal{F}}(\mathbf{m})\mathbf{C}^T\delta\mathbf{x}\|_F^2 \quad \text{s.t} \quad \|\delta\mathbf{x}\|_1<\tau$$

$$\delta\mathbf{m}=\mathbf{C}^T\delta\mathbf{x}$$

- suppress incoherent noise/artifacts by one-norm constraint in a transform domain
- randomized subsets of shots to reduce computational costs

[Pratt, '98]

Source estimation

Source estimation at the kth iteration:

$$\alpha_k = \arg\min_{\alpha} \|\mathbf{D}_k - \alpha \boldsymbol{\mathcal{F}}(\mathbf{m}_k)\|_F^2$$

- source estimation handles amplitude problems
- cheap to evaluate

Sparsity-promoting GN FWI

Algorithm 1: Sparsity-promoting Gauss-Newton FWI

Result: Output estimate for the model $\mathbf{m}$

$\mathbf{m} \longleftarrow \mathbf{m}_0$; $k \longleftarrow 0$; `// initial model`

while not converged **do**

  $\alpha_k \longleftarrow \arg\min_{\alpha} \|\mathbf{D}_k - \alpha \boldsymbol{\mathcal{F}}(\mathbf{m}_k)\|_F^2$; `// source estimation`

  $\delta\mathbf{D}_k \longleftarrow \mathbf{D}_k - \alpha_k \boldsymbol{\mathcal{F}}(\mathbf{m}_k)$; `// wavefield residual`

  $\delta\mathbf{x}_k \longleftarrow \arg\min_{\delta\mathbf{x}} \frac{1}{2}\|\delta\mathbf{D}_k - \alpha\nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_k)\mathbf{C}^T\delta\mathbf{x}\|_2^2 \quad \text{s.t.} \quad \|\delta\mathbf{x}\|_1 \leq \tau_k$

  $\mathbf{m}^{k+1} \longleftarrow \mathbf{m}^k + \mathbf{C}^T\delta\mathbf{x}$; `// update model`

  $k \longleftarrow k+1$;

end

Examples

Acquisition topology:

- 350 shots with interval 20 m
- 701 receivers with interval 10 m

Observed data is generated by

- time domain finite difference modeling method with PML boundary
- 12 Hz Ricker wavelet

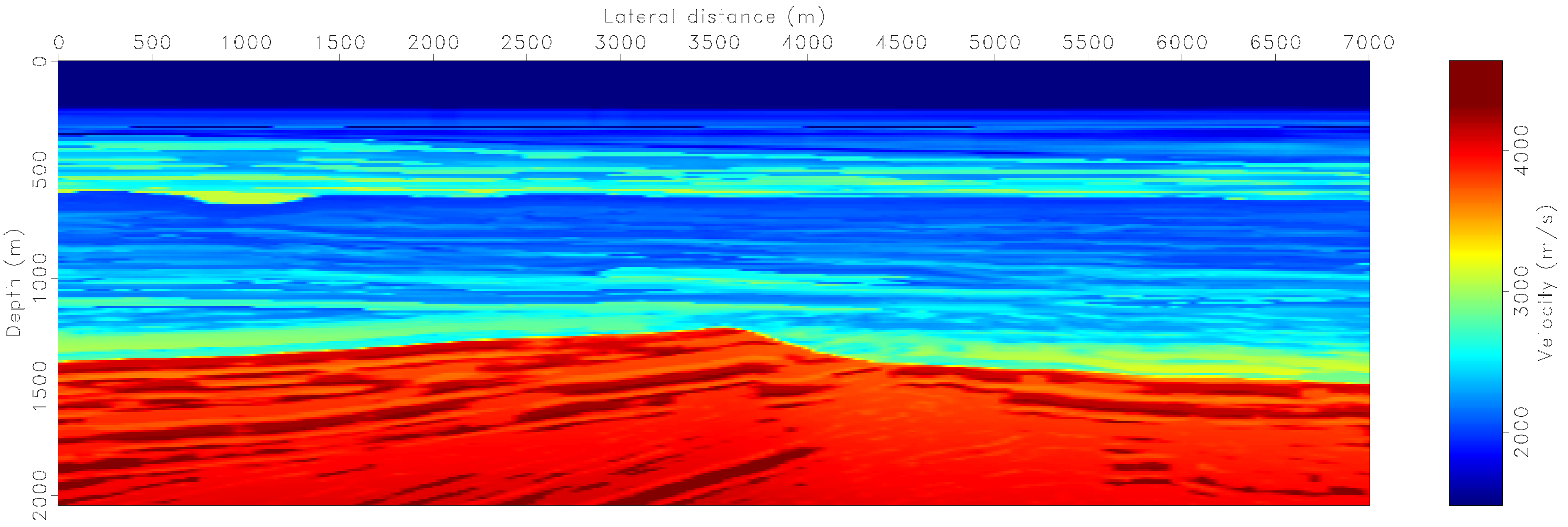
Examples

Inversion parameters:

- 10 frequency bands for 3-12 Hz.
- each band contain 4 frequencies
- inversion using frequency modeling kernel (Helmholtz)
- grid size is determined by minimal wavelength
- solve 5-10 GN subproblems for each frequency band
- < 20 iterations for each GN subproblem

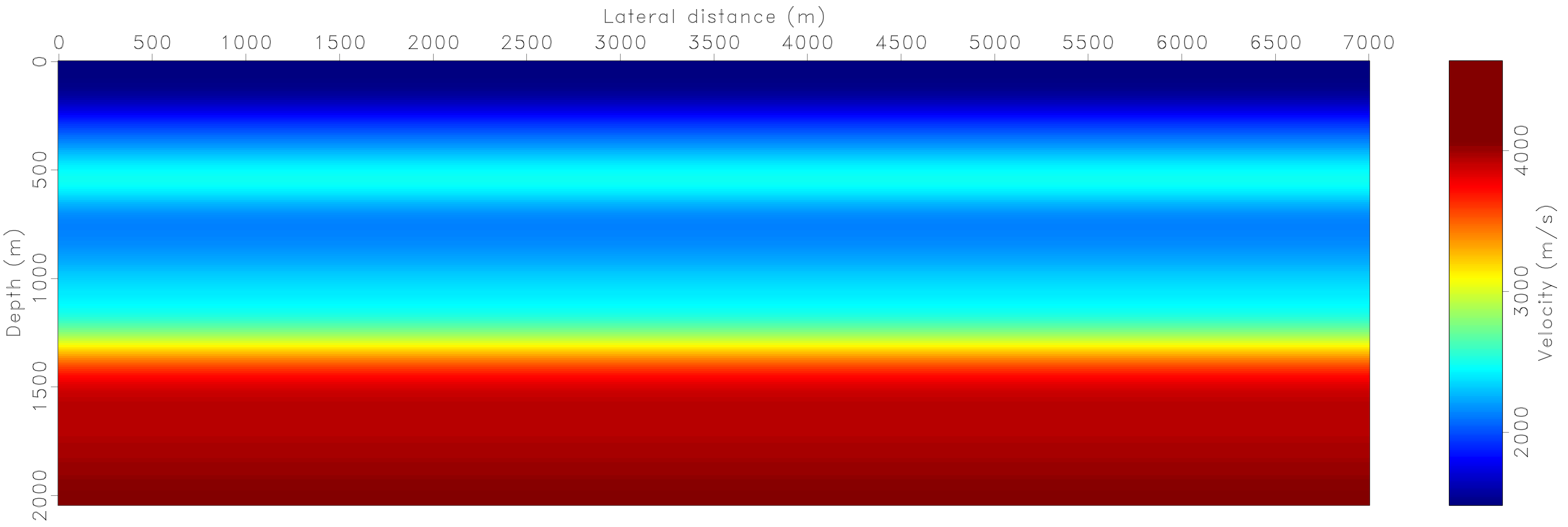
True model

BG compass



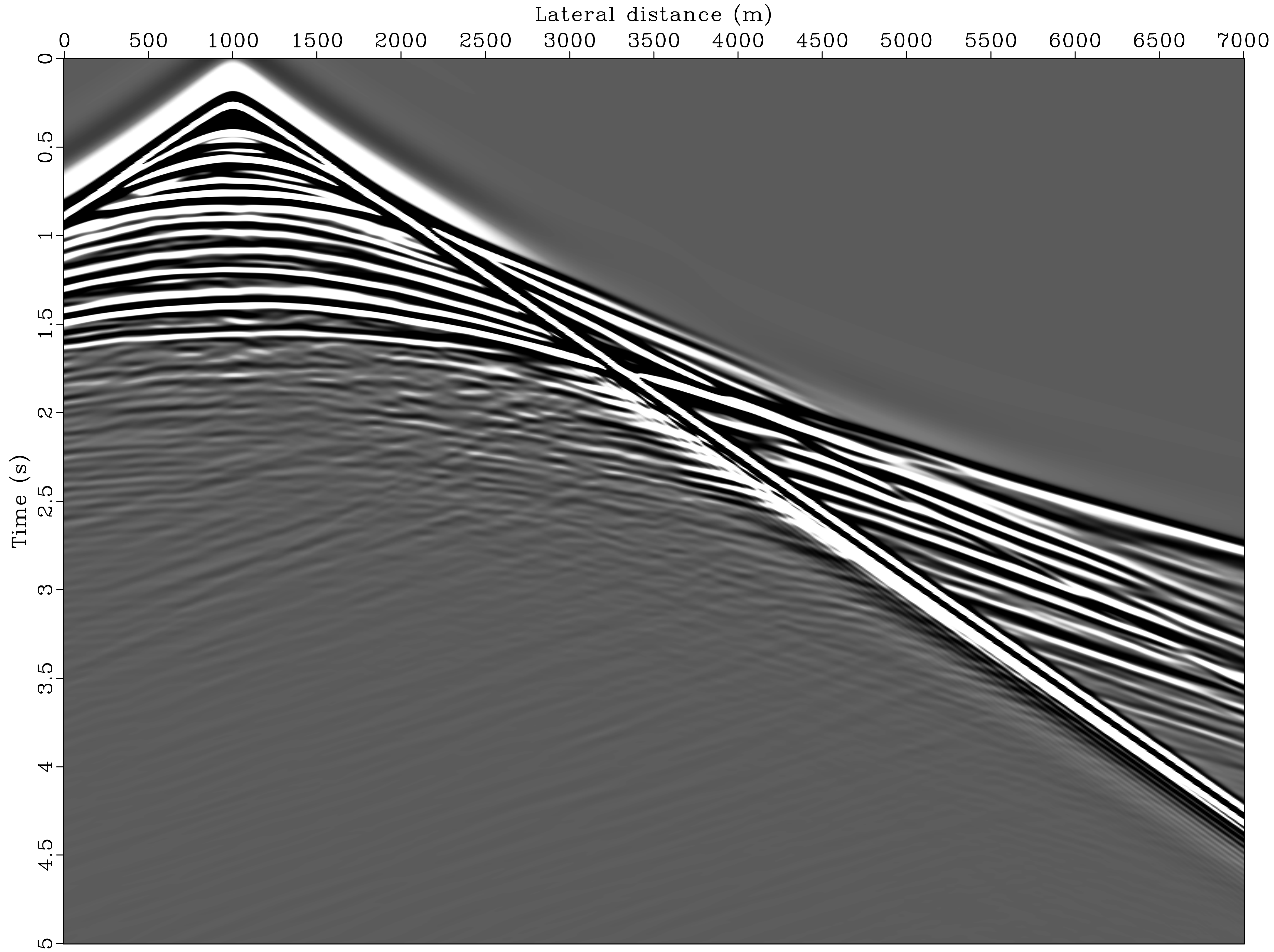
Initial model

Initial model



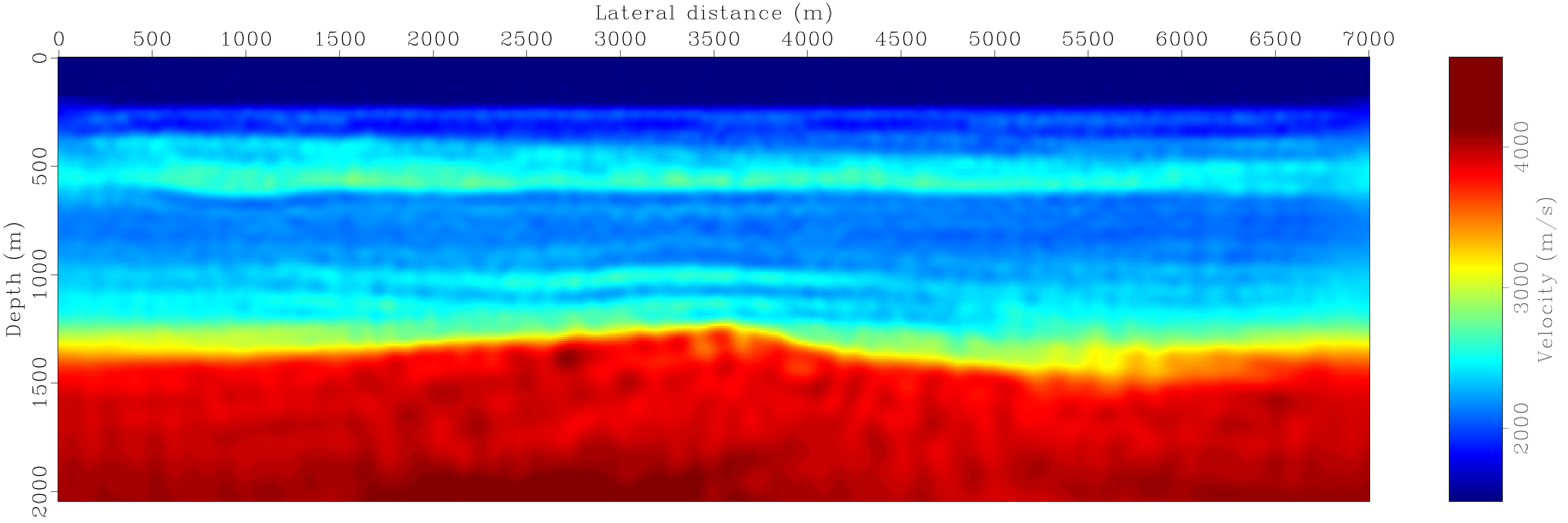
One shot record

Time-domain finite-differences (PML boundary)



Normal Gauss-Newton

Each GN subproblem uses 20 randomly selected sequential shots

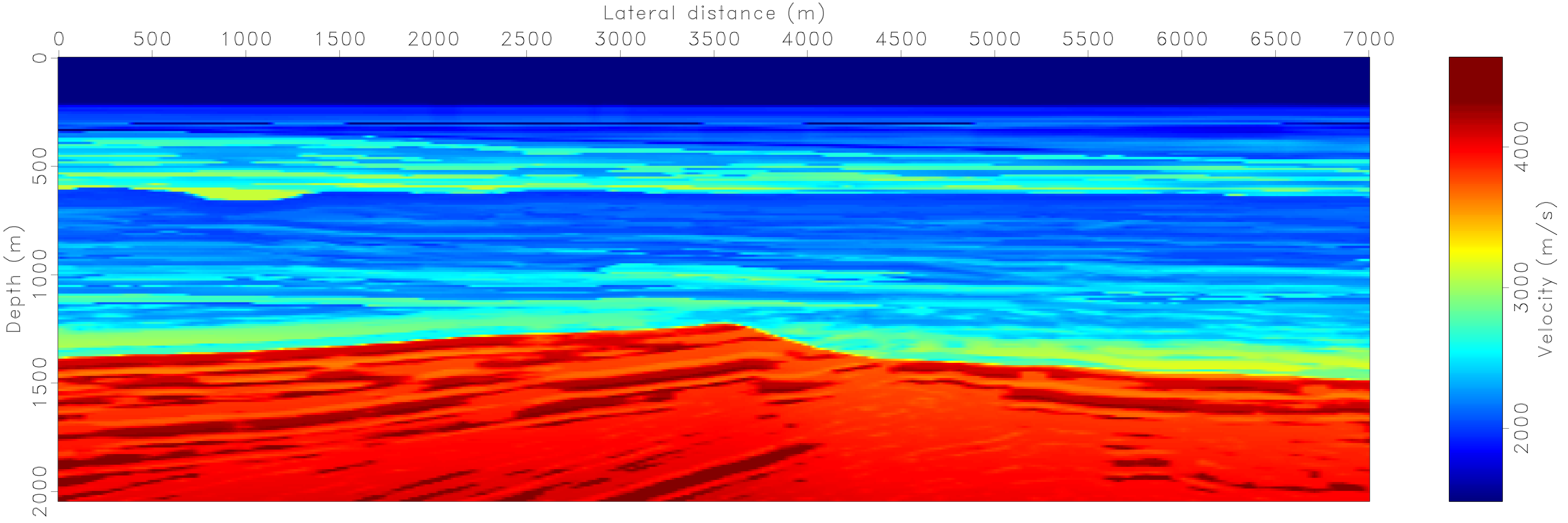


Each GN subproblem using 5 lsqr iterations

1x node, 4 x cpus, an hour

True model

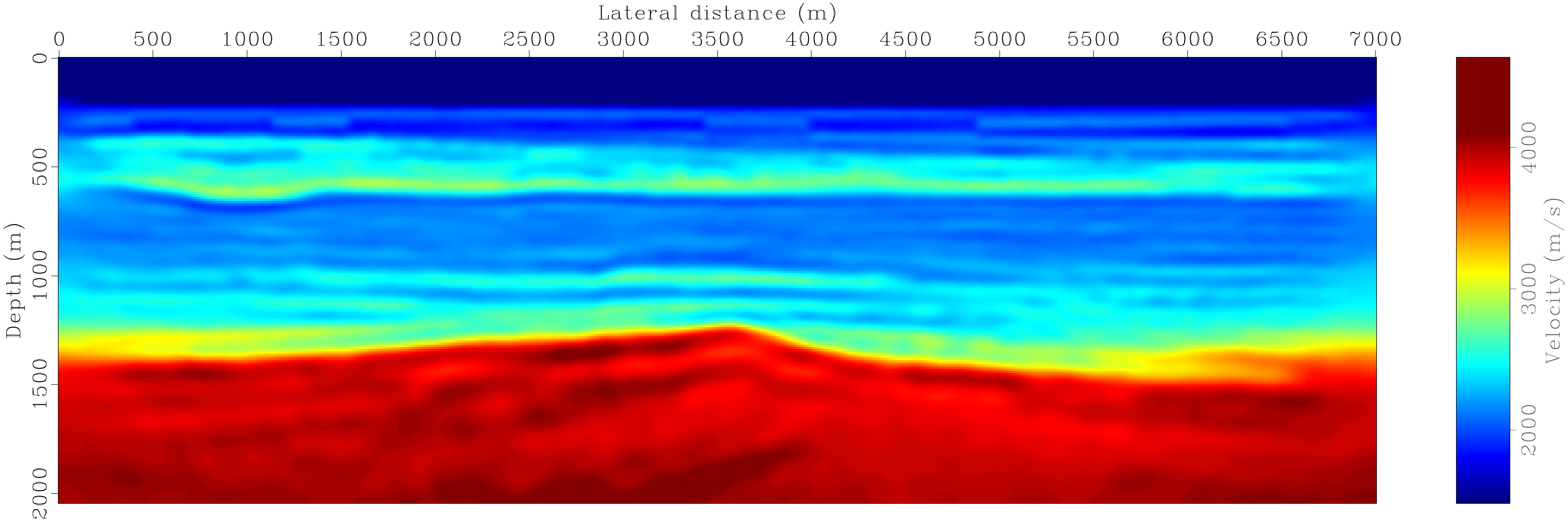
BG compass



Sparsity promoting Gauss-Newton

SLIM

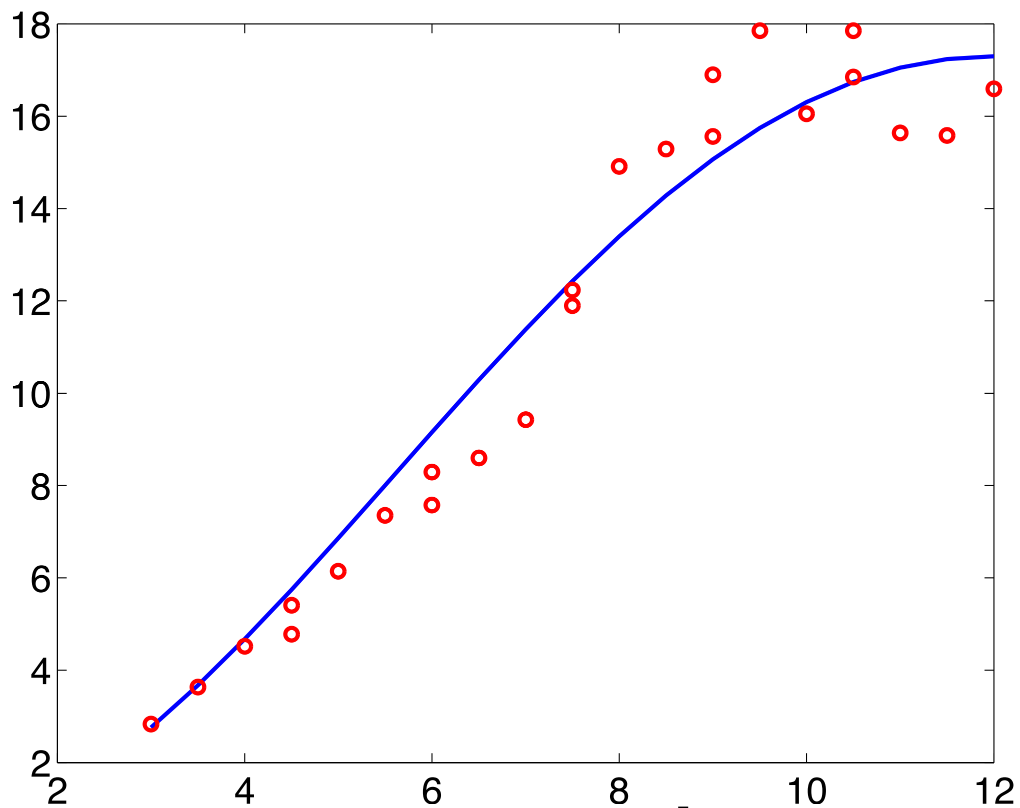
Each GN subproblem using 20 randomly selected shots



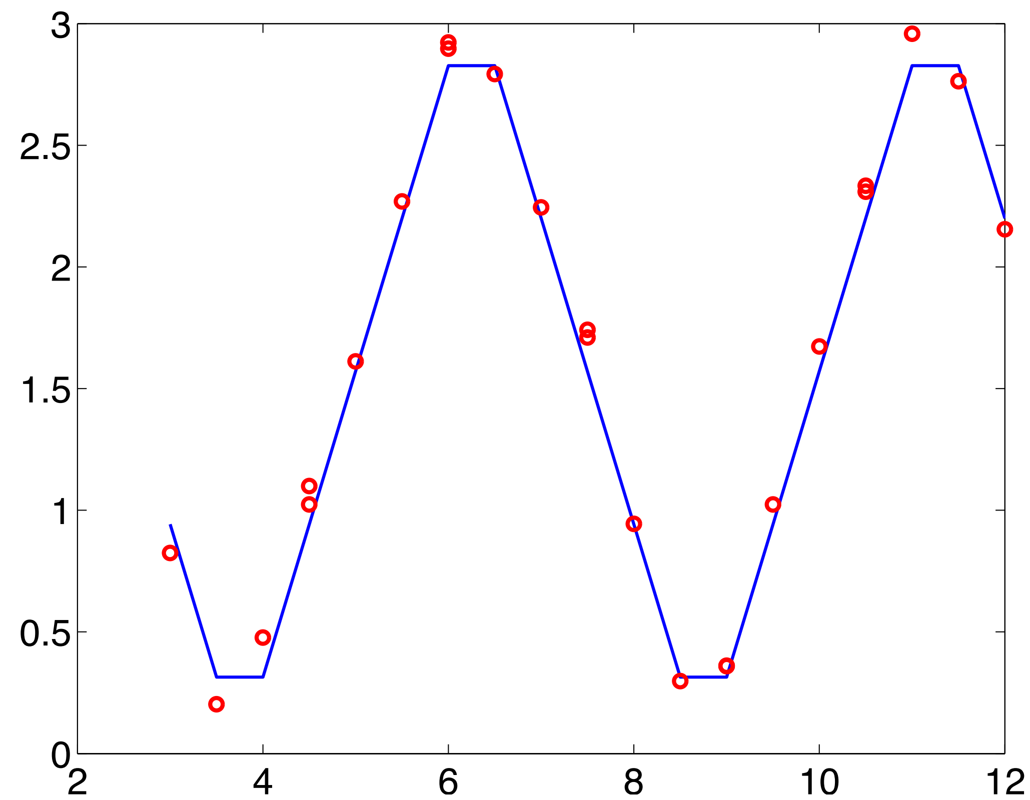
Uses <10 spectral-projected gradient iterations

Source wavelet

Estimates source function from 3 to 12 Hz



magnitude



phase

[Wang & Sacchi, '07]

Seismic imaging

Least-squares migration:

$$\delta\widetilde{\mathbf{m}} = \arg\min_{\delta\mathbf{m}} \frac{1}{2}\|\delta\mathbf{d} - \alpha\nabla\boldsymbol{\mathcal{F}}(\mathbf{m}_0)\delta\mathbf{m}\|_2^2$$

$$\begin{aligned}
\delta\mathbf{d} &= \text{Multi-source multi-frequency data residue}\\
\boldsymbol{\nabla}\boldsymbol{\mathcal{F}}(\mathbf{m}_0) &= \text{Linearized Born-scattering operator}\\
\mathbf{m}_0 &= \text{Background velocity model}\\
\mathbf{Q} &= \text{Sources}\\
\delta\tilde{\mathbf{m}} &= \text{image}
\end{aligned}$$

[Donoho Chen, '06; Li&Herrmann, '10]

Sparsity-promoting imaging

Basis pursuit denoising problem:

$$\delta\widetilde{\mathbf{m}} = \mathbf{C}^T \arg\min_{\delta\mathbf{x}} \|\delta\mathbf{x}\|_{\ell_1} \quad \text{subject to} \quad \|\delta\underline{\mathbf{d}} - \alpha\nabla\boldsymbol{\mathcal{F}}[\mathbf{m}_0;\underline{\mathbf{Q}}]\mathbf{C}^T\delta\mathbf{x}\|_2 \leq \sigma$$

$$\begin{aligned} \delta\mathbf{x} &= \text{Sparse curvelet-coefficient vector} \\ \mathbf{C}^T &= \text{Curvelet synthesis} \end{aligned}$$

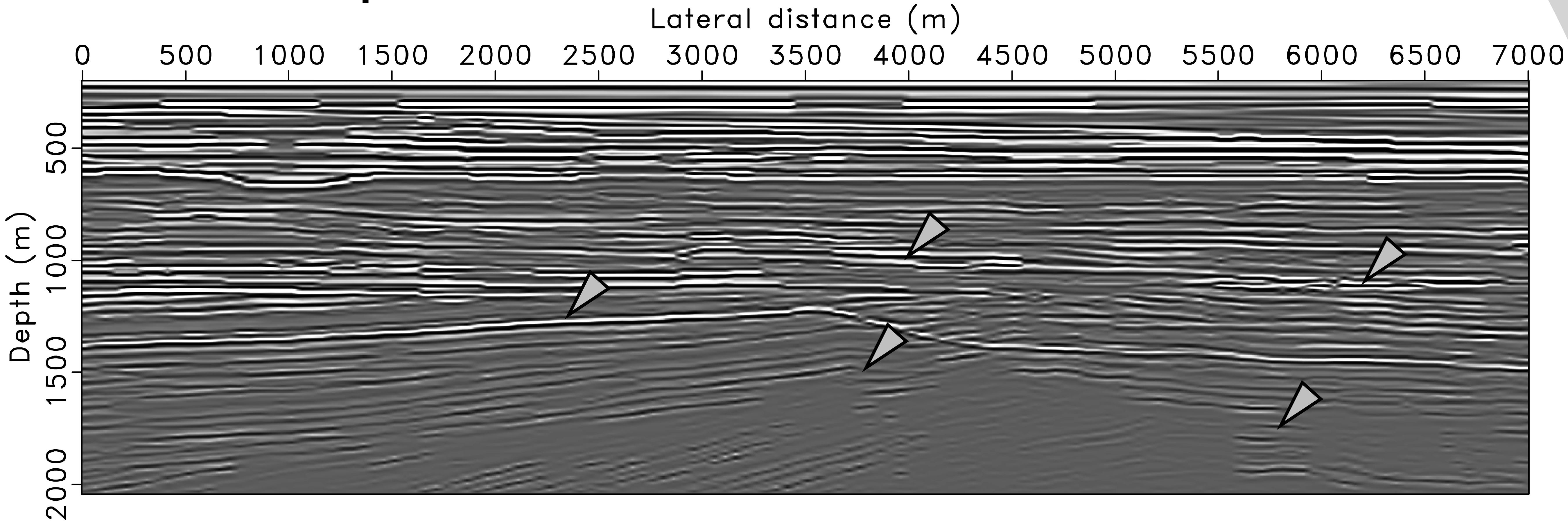
Remarkable speedup of convergence can be obtained by *Message passing*

Examples

- 409x1401 with mesh size of 5m
- 10 randomly selected frequencies (30-50Hz)
- 3 randomly combined simultaneous shots / 17 randomly selected sequential shots
- 60 iterations with 10 redraws

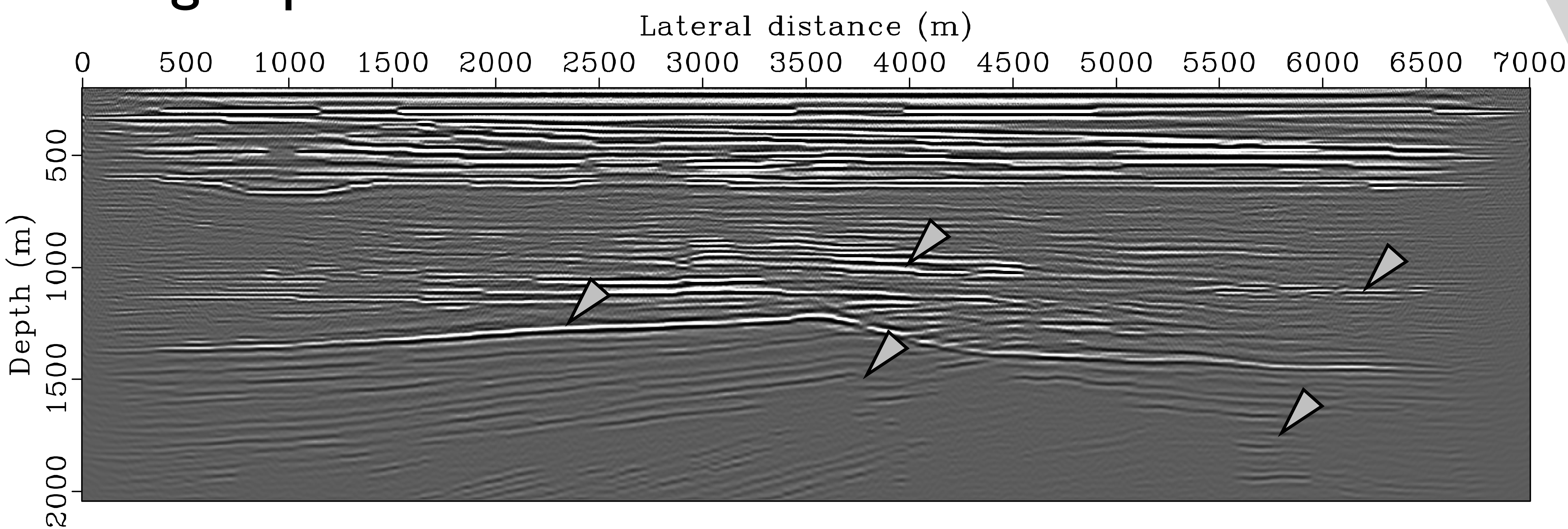
Migration results

True perturbation



Migration results
underdetermined

imaged perturbation with L1 with AMP

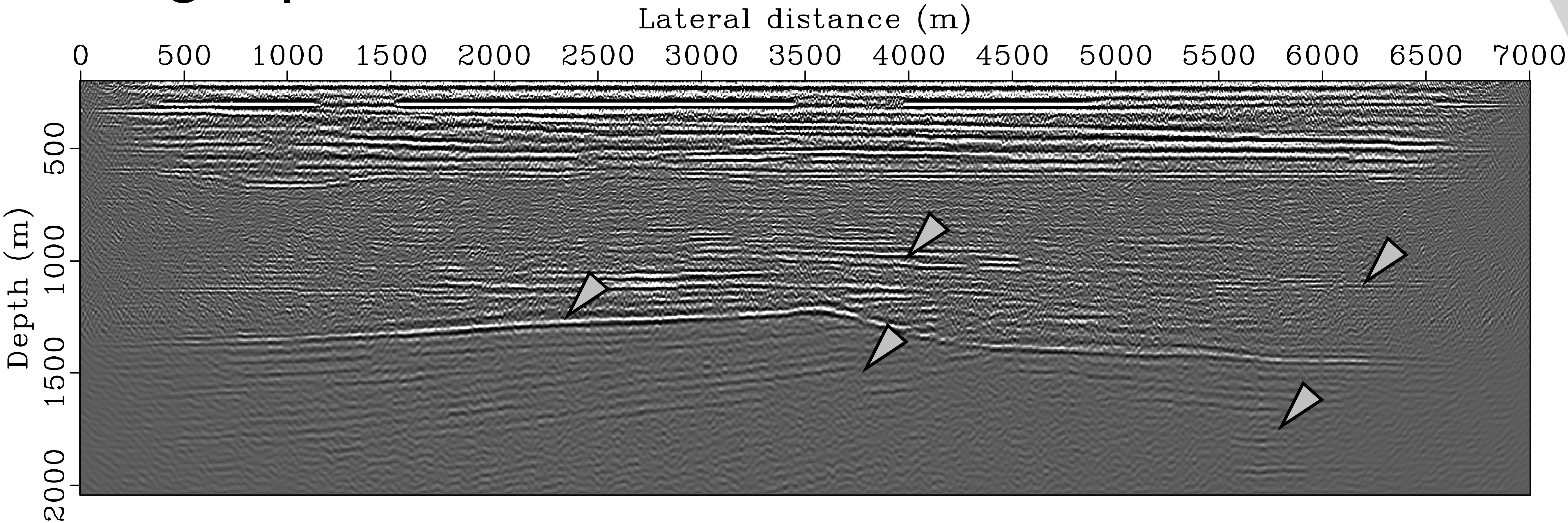


3 *simultaneous* shots

time: ~= 24 hours

Migration results
underdetermined

imaged perturbation with L2 with AMP

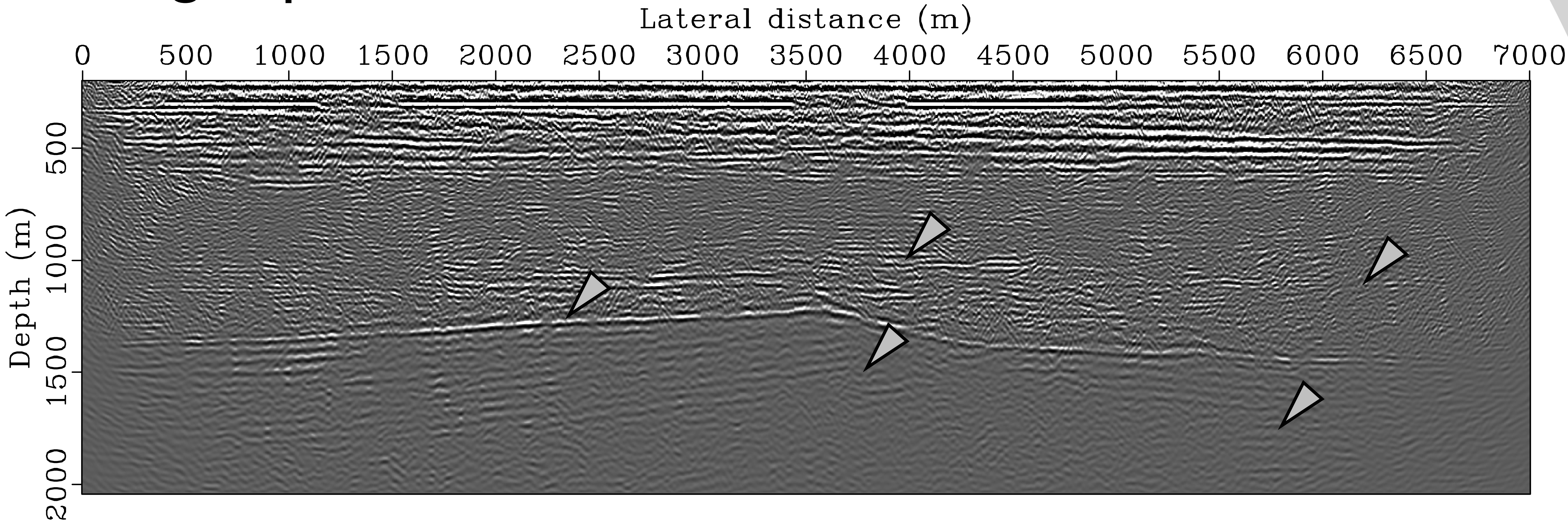


3 *simultaneous* shots

time: ~= 24 hours

Migration results
underdetermined

imaged perturbation with L1 without AMP

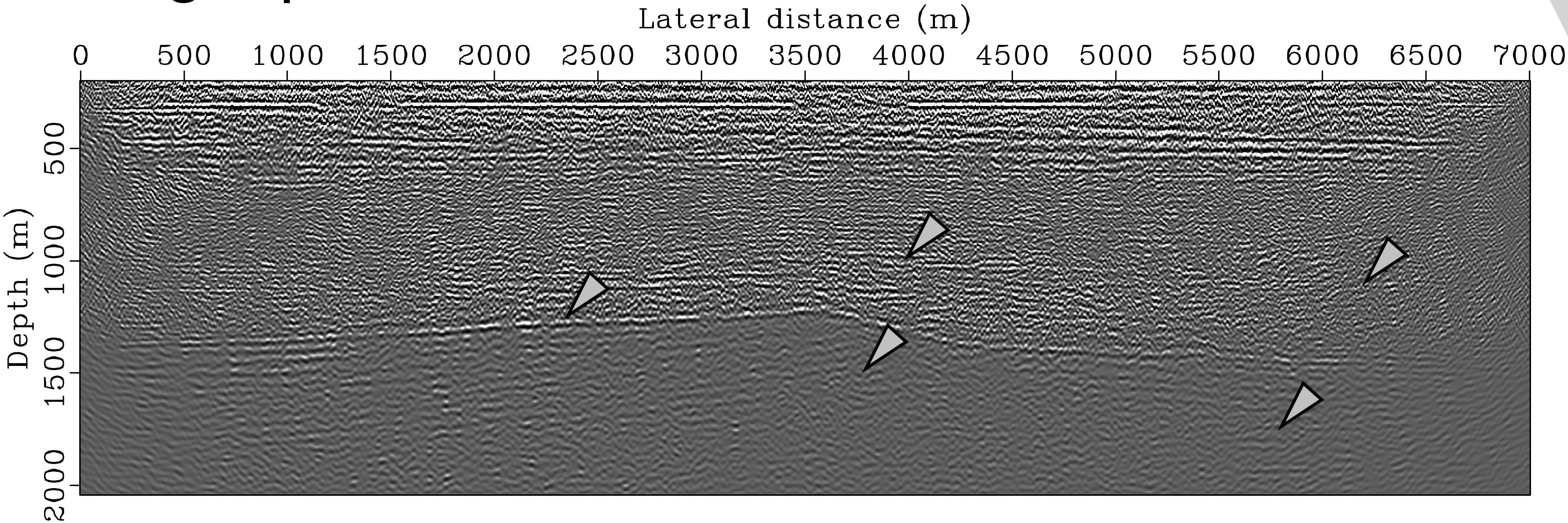


3 *simultaneous* shots

time: ~= 24 hours

Migration results
underdetermined

imaged perturbation with L2 without AMP



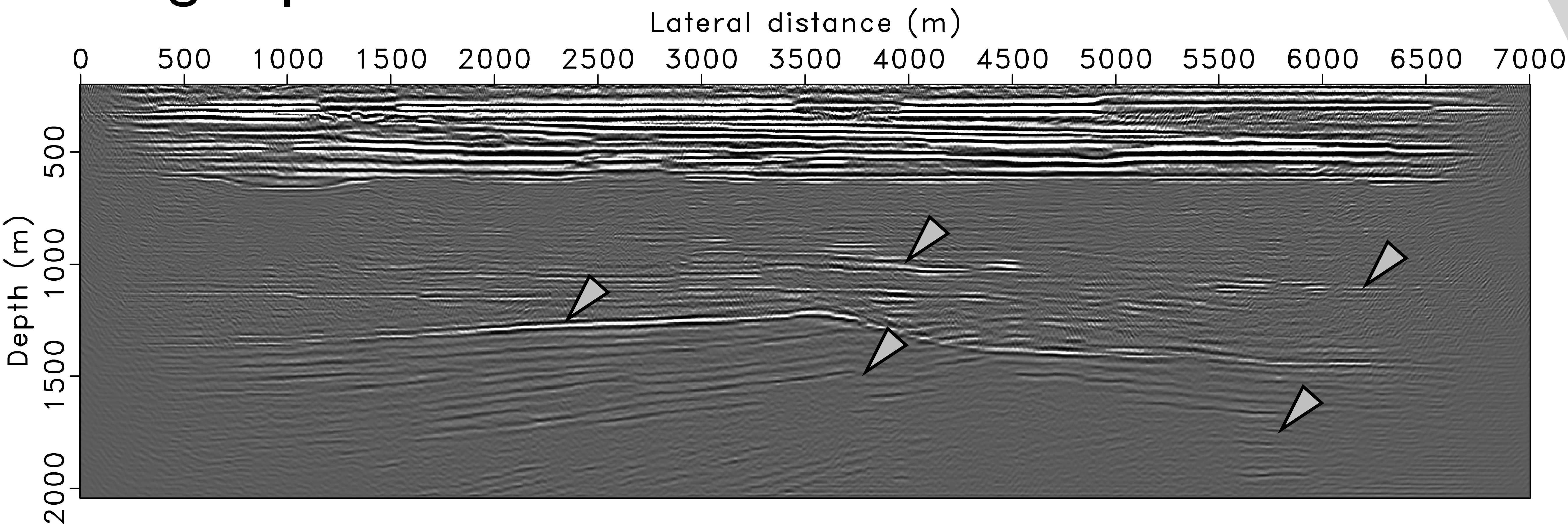
3 *simultaneous* shots

time: ~= 24 hours

SLIM

Migration results
underdetermined

imaged perturbation with L1 with AMP

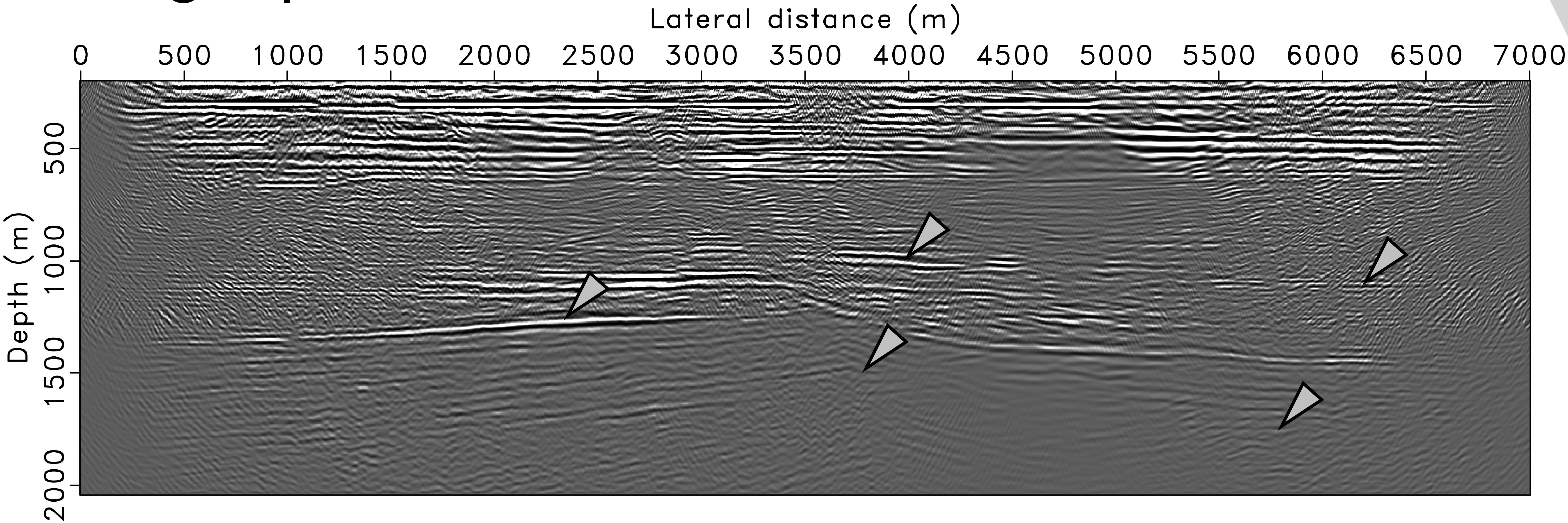


17 *sequential* shots with marine acquisition

time: ~= 40 hours

Migration results
underdetermined

imaged perturbation with L1 without AMP



17 *sequential* shots with marine acquisition

time: ~= 40 hours

Conclusion

- *Curvelet Transform* is efficiently in representing geological models
- *Sparsity regularization in Curvelet domain* can significantly suppress the model space artifacts
- High-resolution FWI and Imaging results is attainable through Sparsity promotion
- Ideas from message passing leads to a remarkable speedup of convergence

SLIM

Acknowledgements

- Authors of CurveLab, SPGL1 and Spot
- My colleagues

SINBAD

NSERC CRSNG

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BGP, BP, Chevron, ConocoPhillips, Petrobras, PGS, Total SA, and WesternGeco.

SLIM

Thank you

https://slimweb.eos.ubc.ca/

Friday, Ballroom C. Workshop: Gulf of Mexico Imaging Challenges: What can FWI achieve?