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SEG 2011
San Antonio

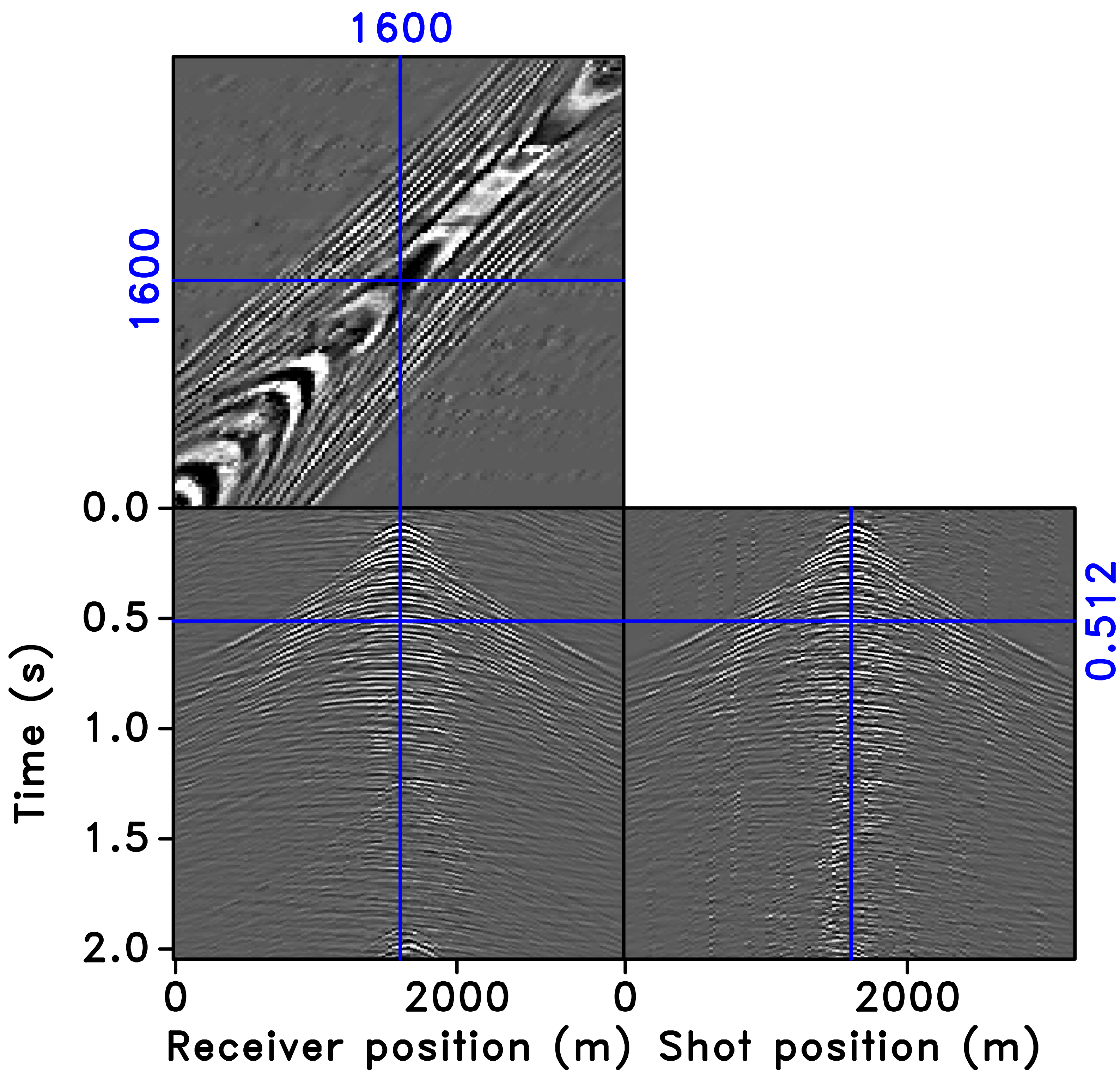
Sparsity-promoting recovery from simultaneous data: a compressive sensing approach

Haneet Wason*, Tim T. Y. Lin, and Felix J. Herrmann

September 19, 2011

SLIM
University of British Columbia

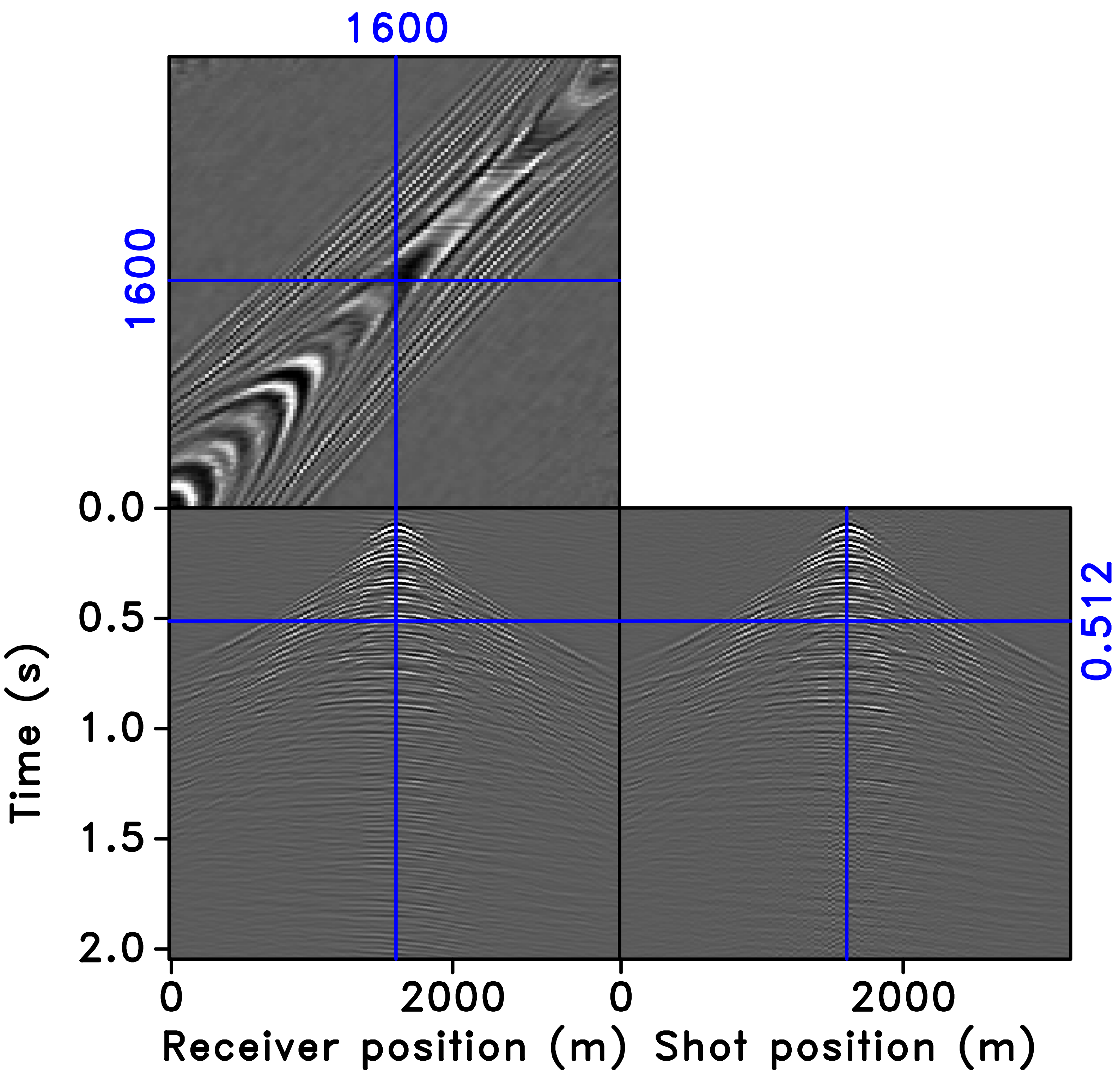
Motivation



Conventional recovery

SNR = 5.04 dB

Motivation



Sparsity-promoting recovery

SNR = 9.52 dB

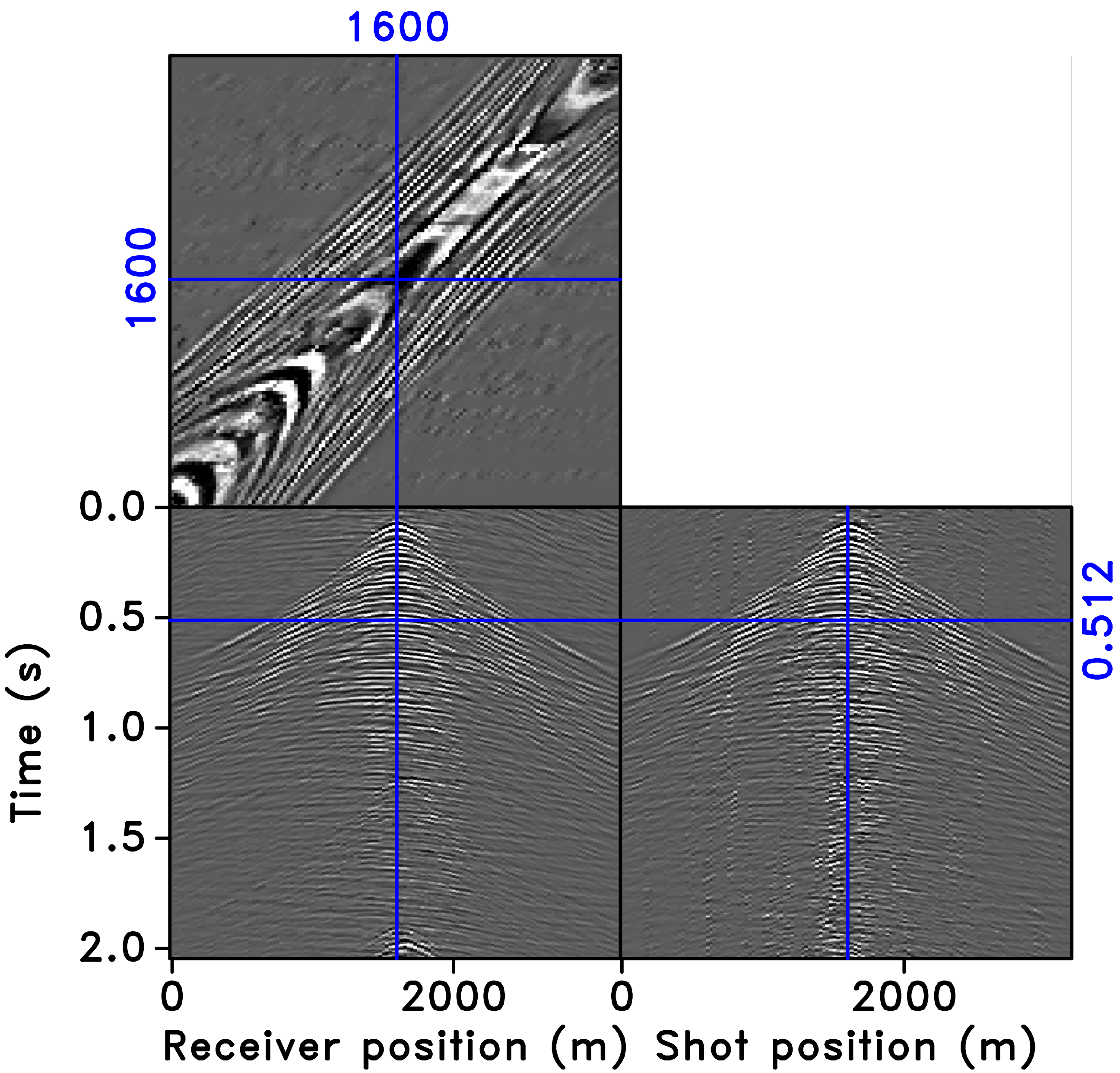
Motivation

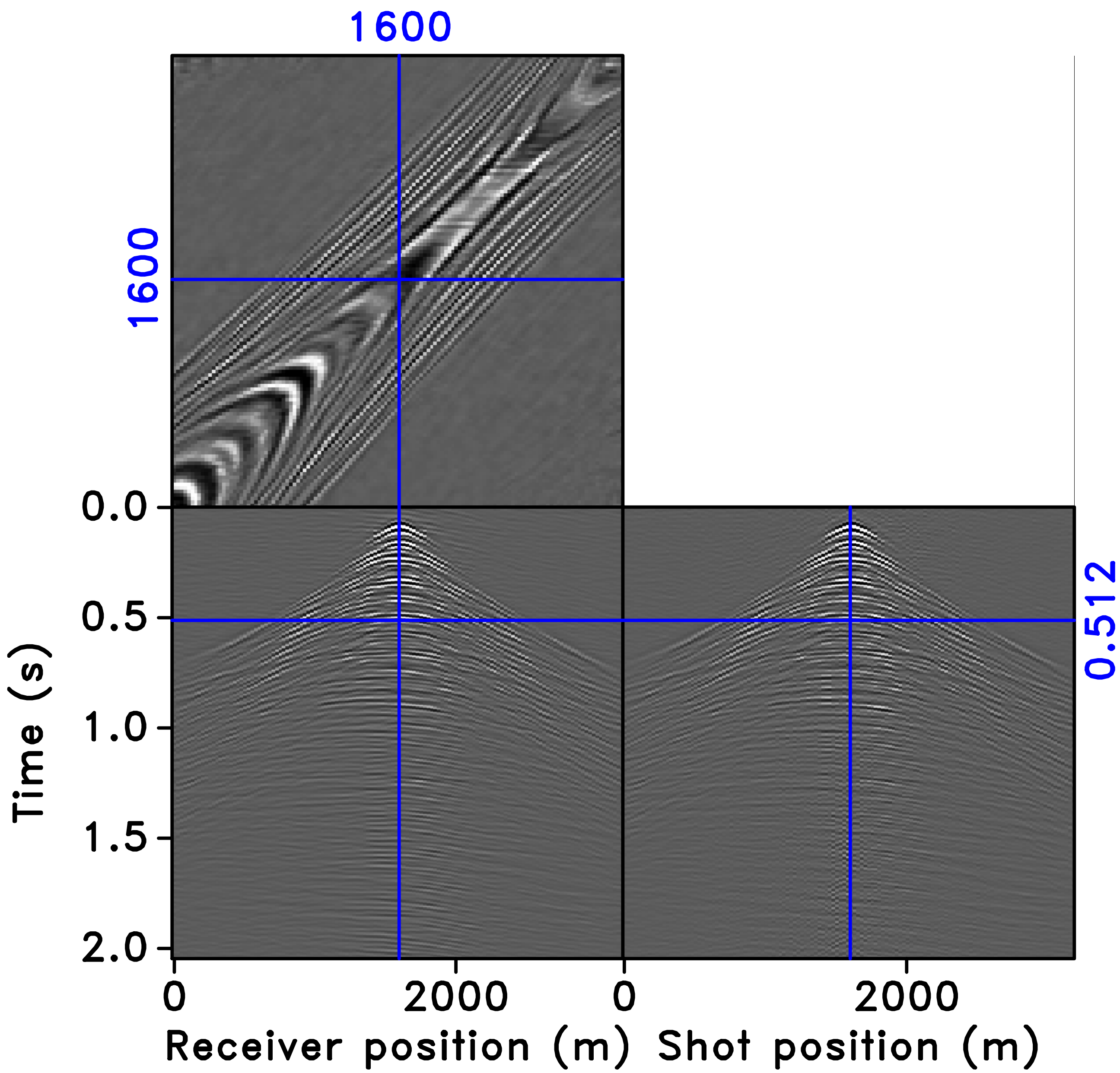
Conventional recovery

SNR = 5.04 dB

Sparsity-promoting recovery

SNR = 9.52 dB





Motivation

- Opportunity to rethink Marine acquisition
- Concentrate on simultaneous sourcing
- Marine acquisition with ocean-bottom nodes

Outline

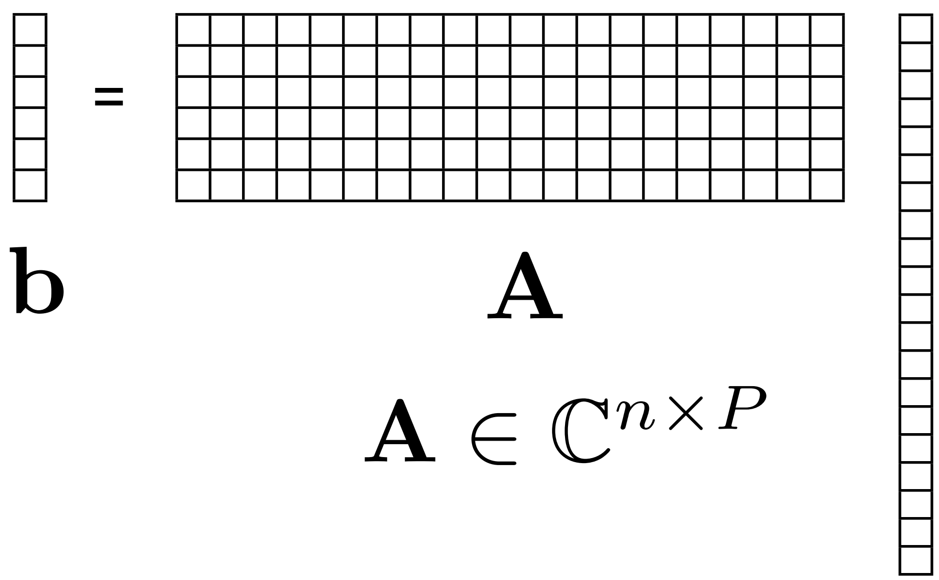
- Compressed sensing (CS) overview
  - design
  - recovery
- *Design* of simultaneous marine acquisition
- Experimental results of *sparsity*-promoting processing

Problem statement

Solve an *underdetermined* system of *linear* equations:

data (measurements /observations) →

$\mathbf{b} \in \mathbb{C}^n$



$\mathbf{b}$

$\mathbf{A}$

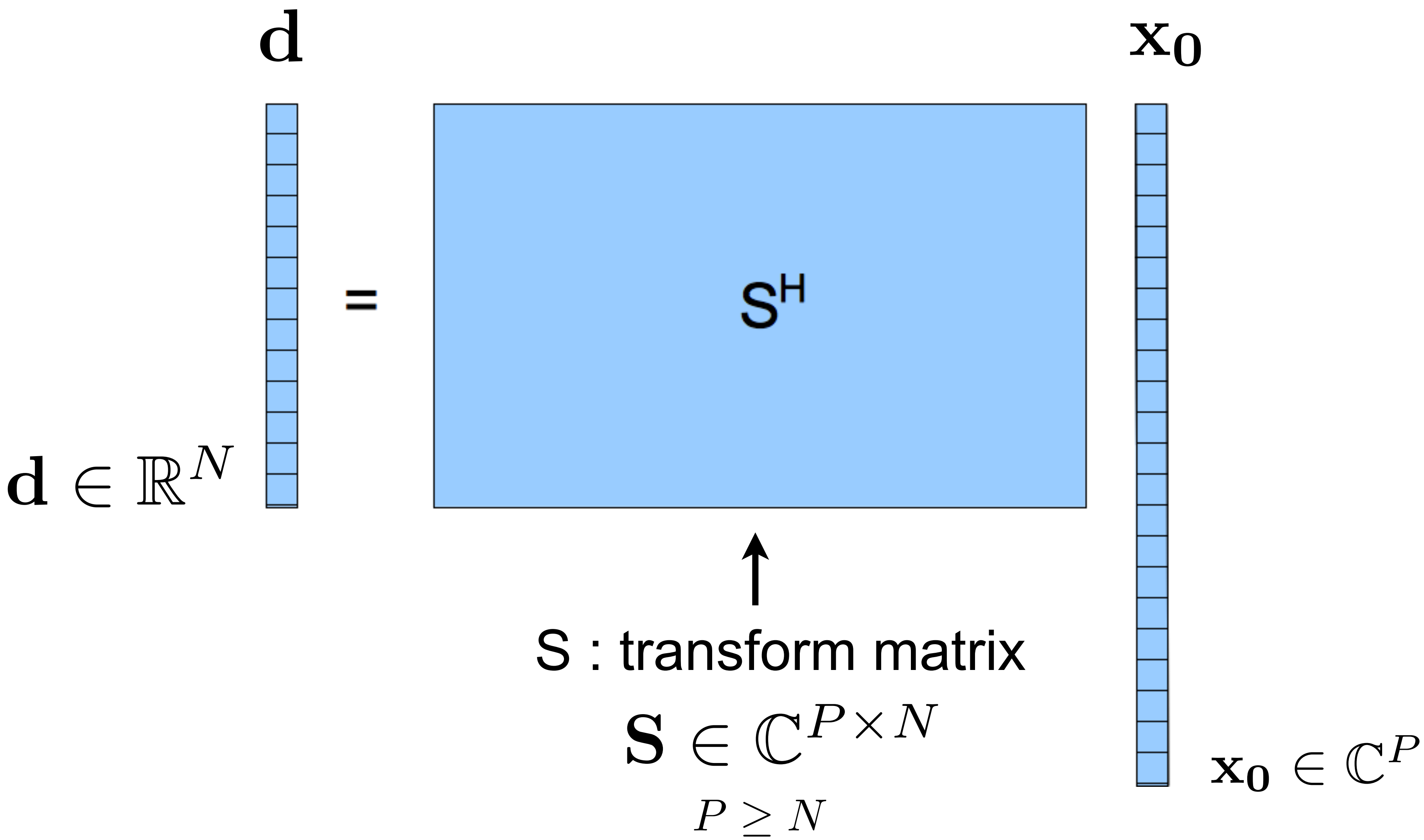
$\mathbf{A} \in \mathbb{C}^{n \times P}$

$\mathbf{x_0}$ ← unknown

$\mathbf{x_0} \in \mathbb{C}^P$

Compressed sensing

- acquisition paradigm for sparse signals
- in some transform domain



Compressed sensing

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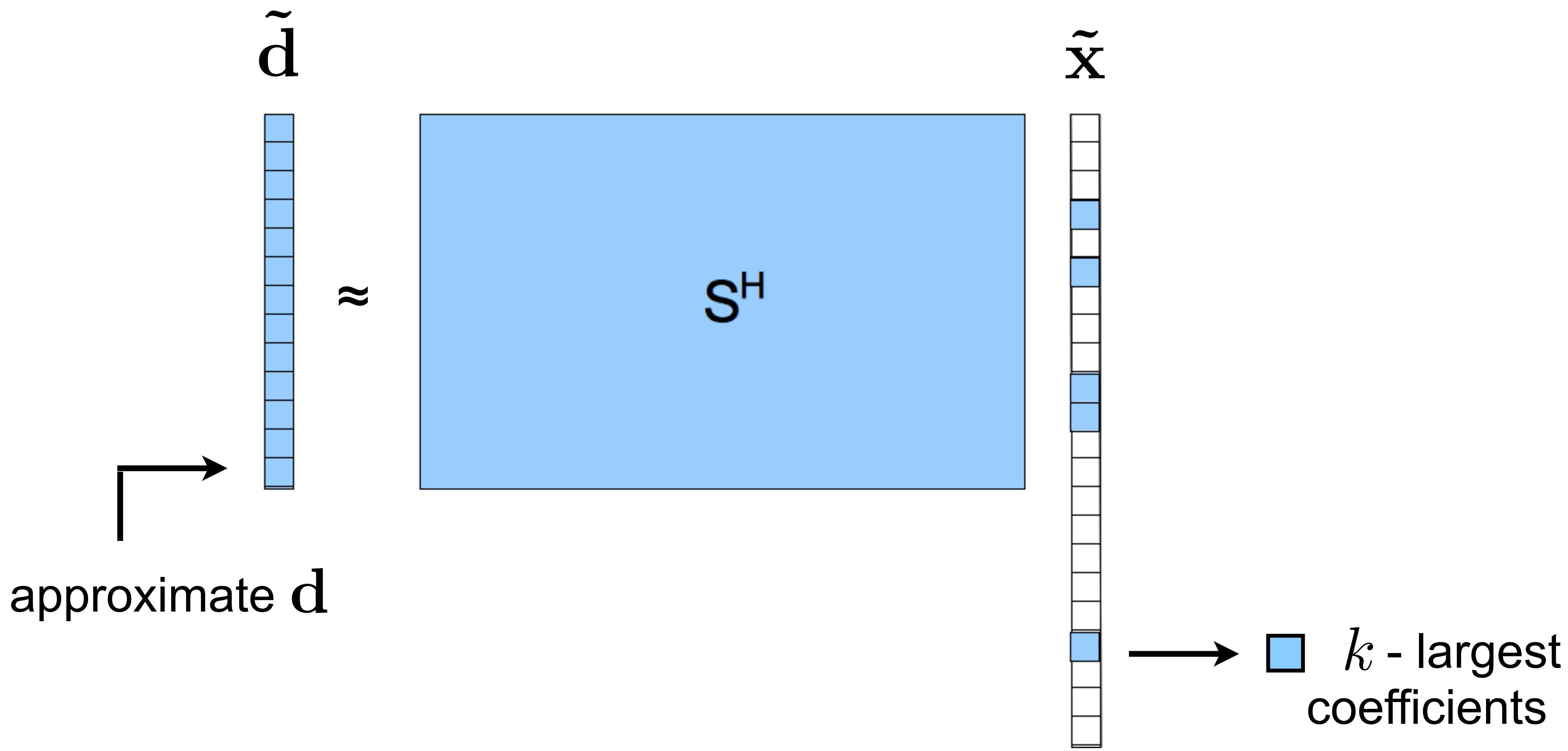
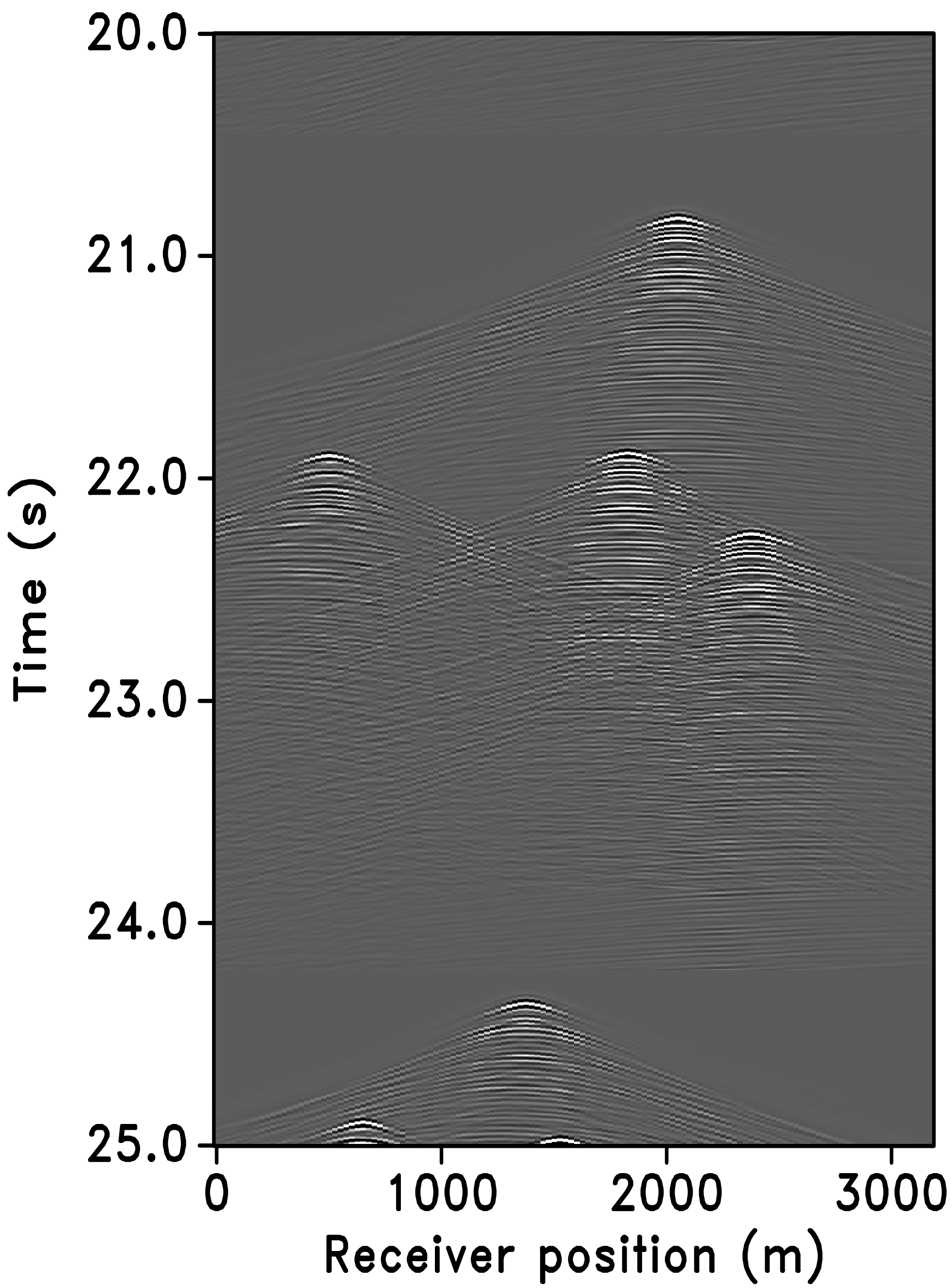
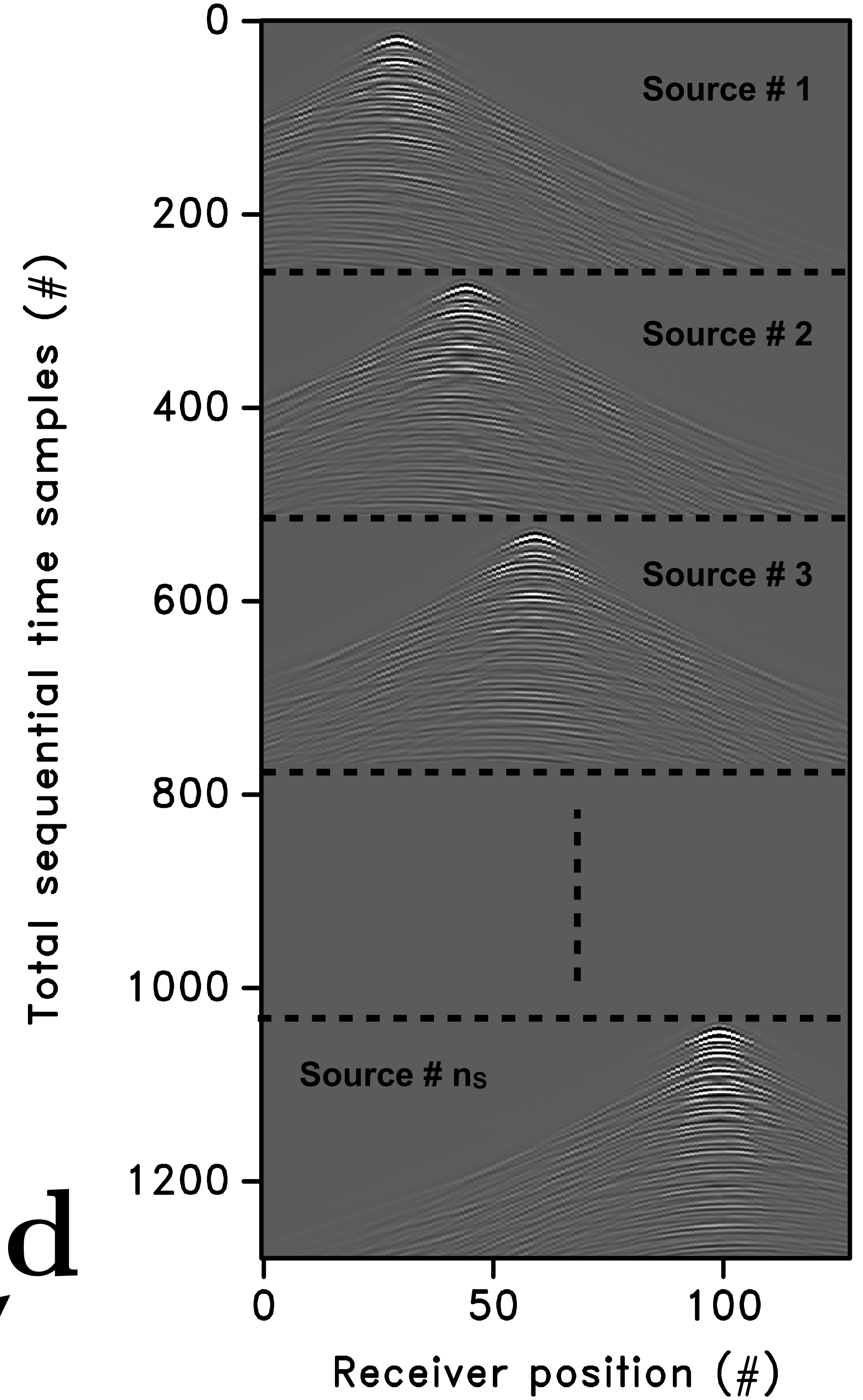


Bigger picture



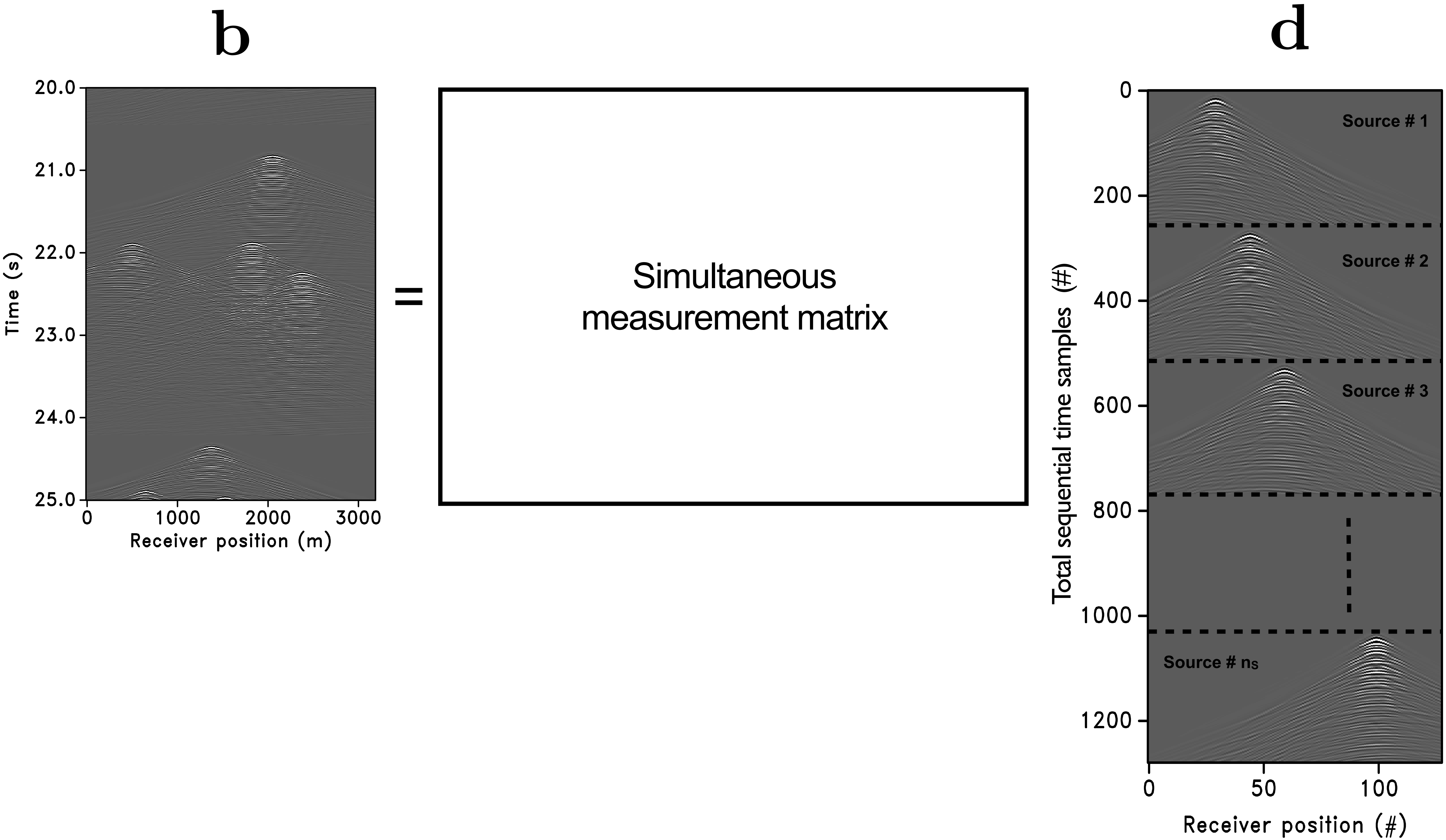
b



d

series of
sequential shots

Bigger picture



Bigger picture

$$\mathbf{b} \quad = \quad \boxed{\text{Simultaneous measurement matrix}} \quad \times \quad \mathbf{d}$$

$$\downarrow$$

$$\tilde{\mathbf{d}} \approx \mathbf{S}^{\mathbf{H}} \tilde{\mathbf{x}}$$

Bigger picture

$$\mathbf{b} = \boxed{\text{Simultaneous measurement matrix}} \times \mathbf{d}$$

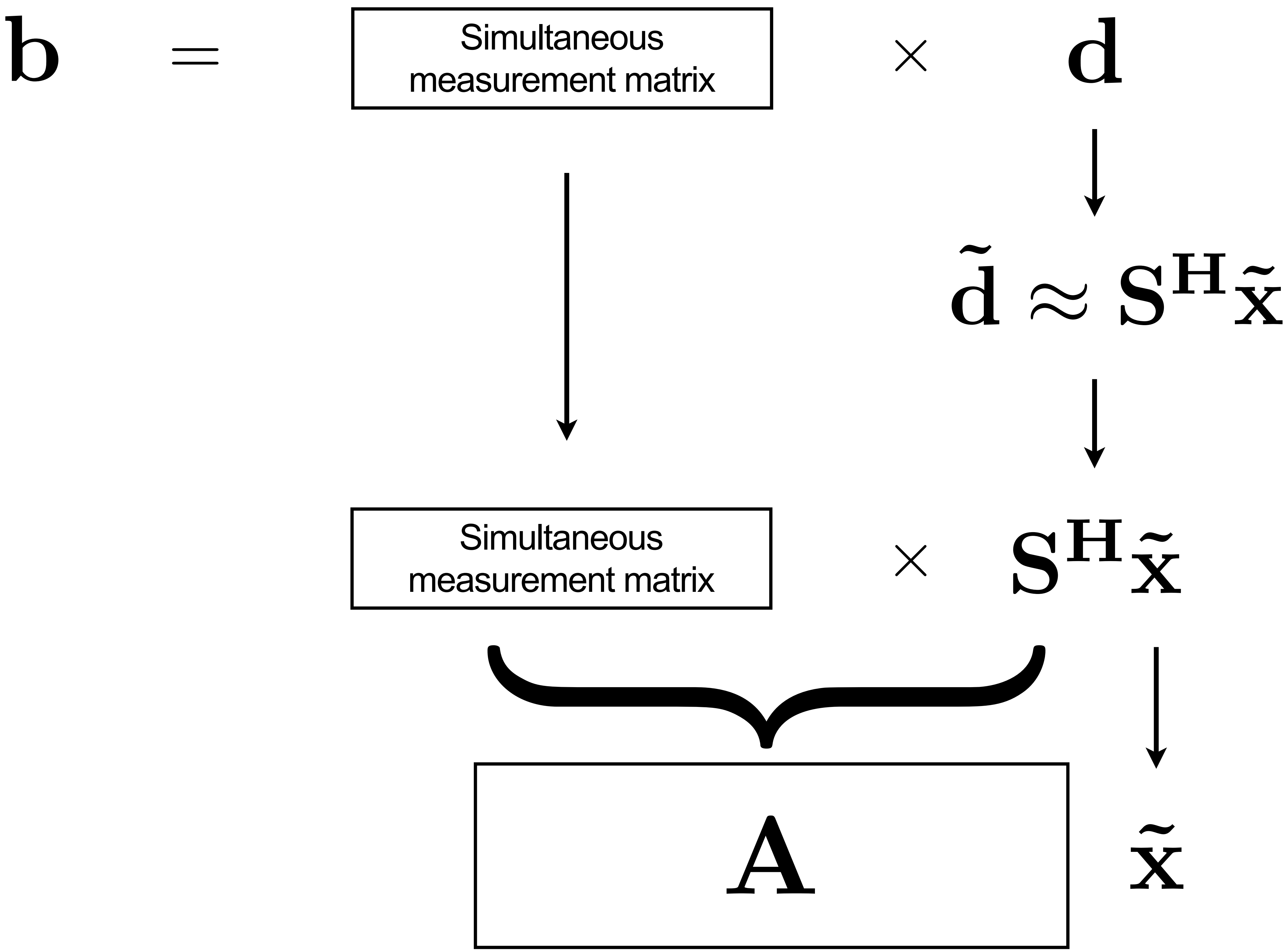
$$\downarrow$$

$$\tilde{\mathbf{d}} \approx \mathbf{S}^{\mathbf{H}}\tilde{\mathbf{x}}$$

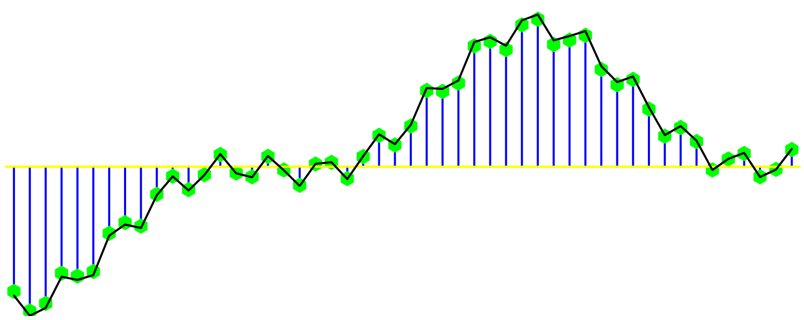
$$\downarrow$$

$$\boxed{\text{Simultaneous measurement matrix}} \times \mathbf{S}^{\mathbf{H}}\tilde{\mathbf{x}}$$

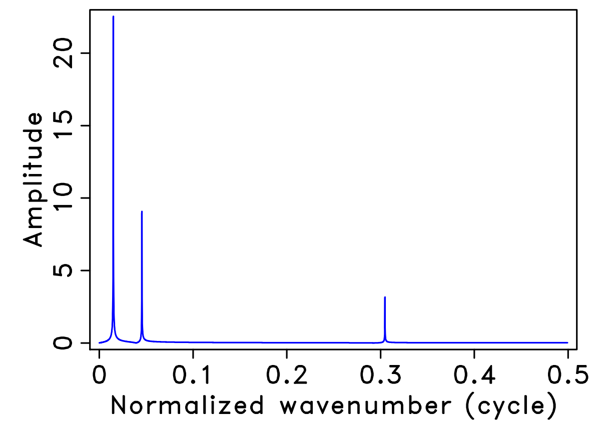
Bigger picture



Coarse sampling schemes

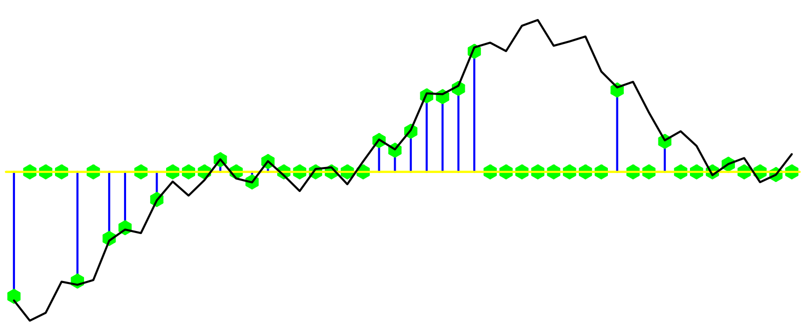


Fourier → transform

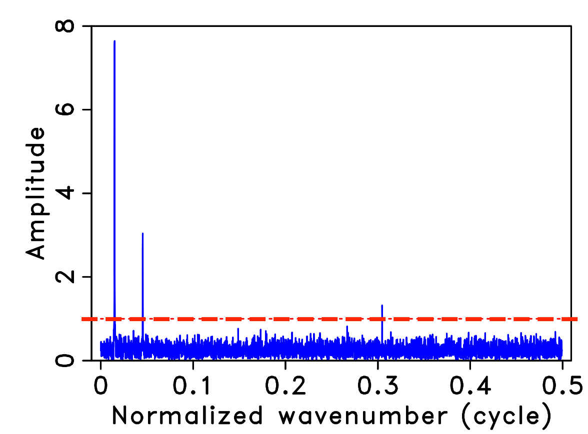


few significant coefficients

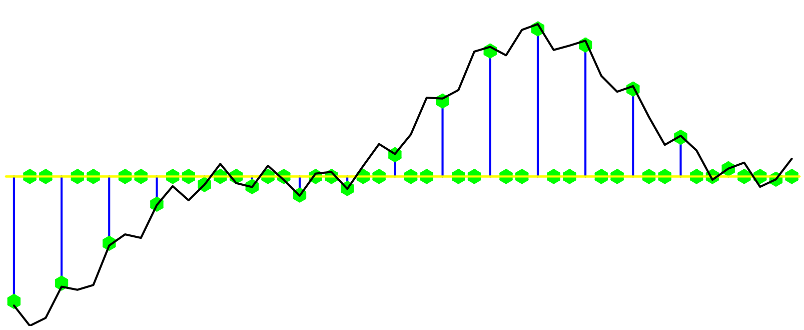
3-fold under-sampling



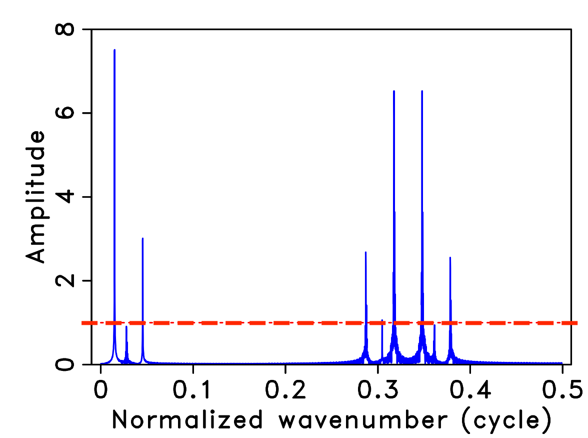
Fourier → transform



✓ significant coefficients detected



Fourier → transform



✗ ambiguity

[Hennenfent & Herrmann, '08]

Mutual coherence

- measures the orthogonality of all columns of $\mathbf{A}$
- equal to the maximum off-diagonal element of the *Gram* matrix
  - ➡ controlled by *compressive sensing* via a combination of
    - *randomization* with
    - *spreading* of *sampling* vectors in the *sparsifying* domain

Restricted isometry property

- indicates whether every group of k columns of $\mathbf{A}$ are nearly orthogonal

- *restricted isometry constant* $0 < \delta_k < 1$ for which

$$(1-\delta_k)\|\mathbf{x}\|_2^2 \leq \|\mathbf{A}\mathbf{x}\|_2^2 \leq (1+\delta_k)\|\mathbf{x}\|_2^2$$

Sparse recovery

Solve the convex optimization problem (one-norm minimization):

$$\tilde{\mathbf{x}} = \underbrace{\arg\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{"sparsity"}} \quad \text{subject to} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{b}}_{\text{data-consistent amplitude recovery}}$$

Sparsity-promoting solver: $\mathbf{SPG}\ell_1$ [van den Berg and Friedlander, '08]

Recover single-source prestack data volume: $\tilde{\mathbf{d}} = \mathbf{S}^{\mathbf{H}}\tilde{\mathbf{x}}$

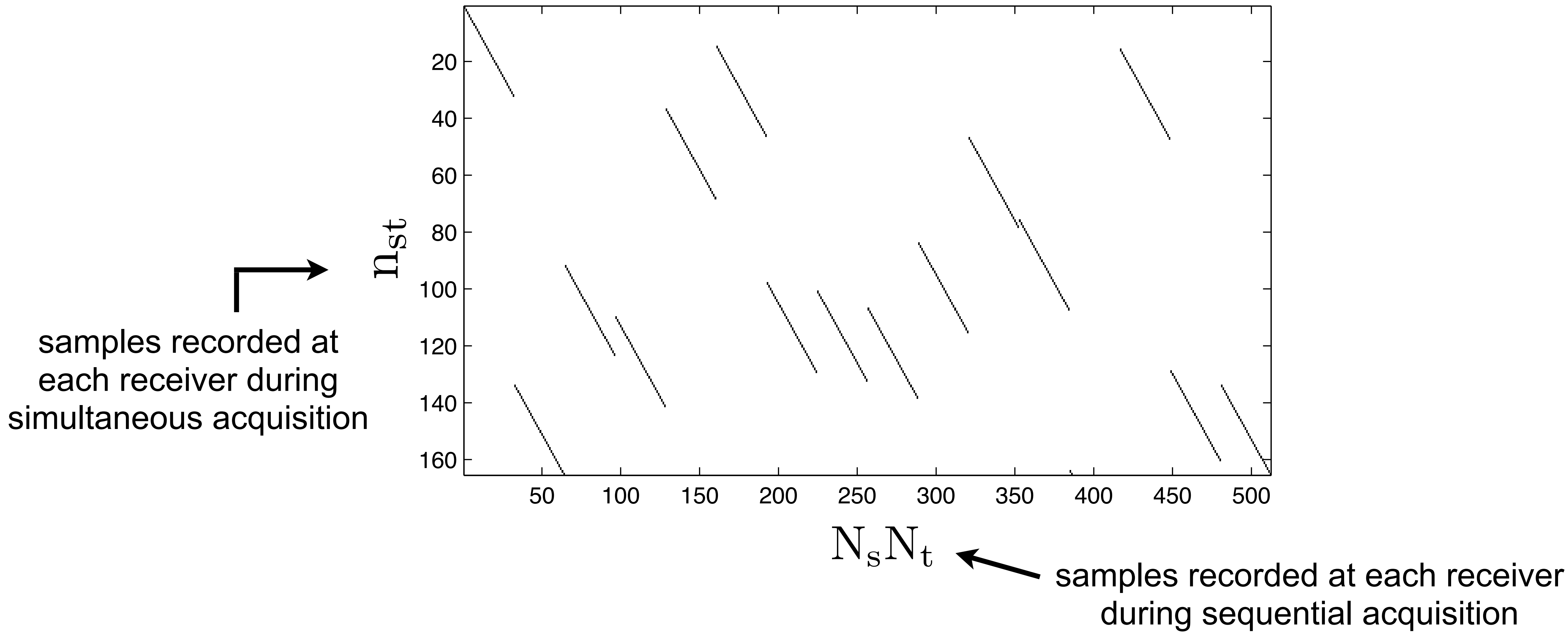
Outline

- Compressed sensing (CS) overview
  - design
  - recovery
- *Design* of simultaneous marine acquisition
- Experimental results of *sparsity*-promoting processing

Simultaneous acquisition matrix

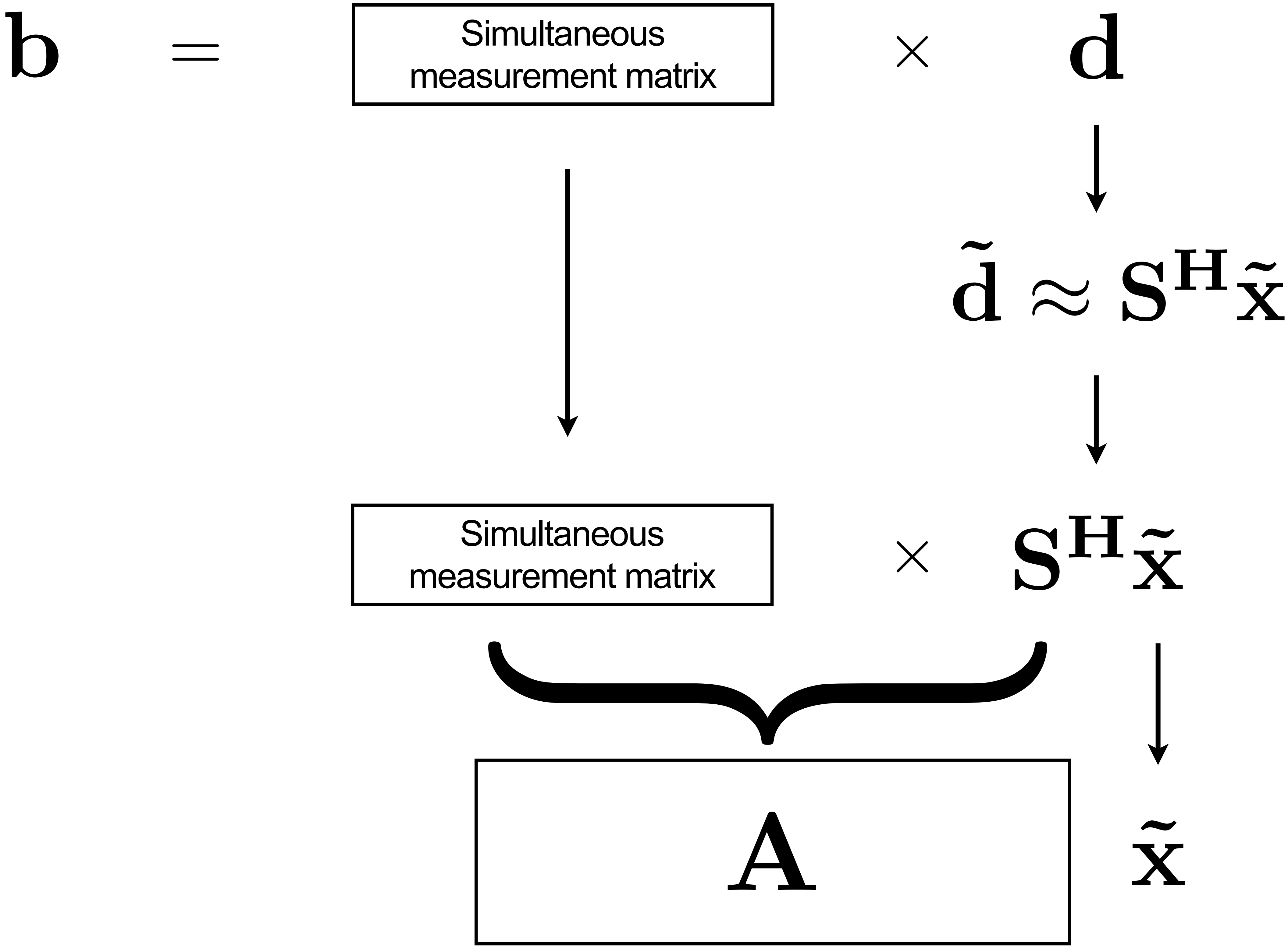
For a seismic line with N_s sources, N_r receivers, and N_t time samples, the sampling matrix is

$$\mathbf{RM}$$

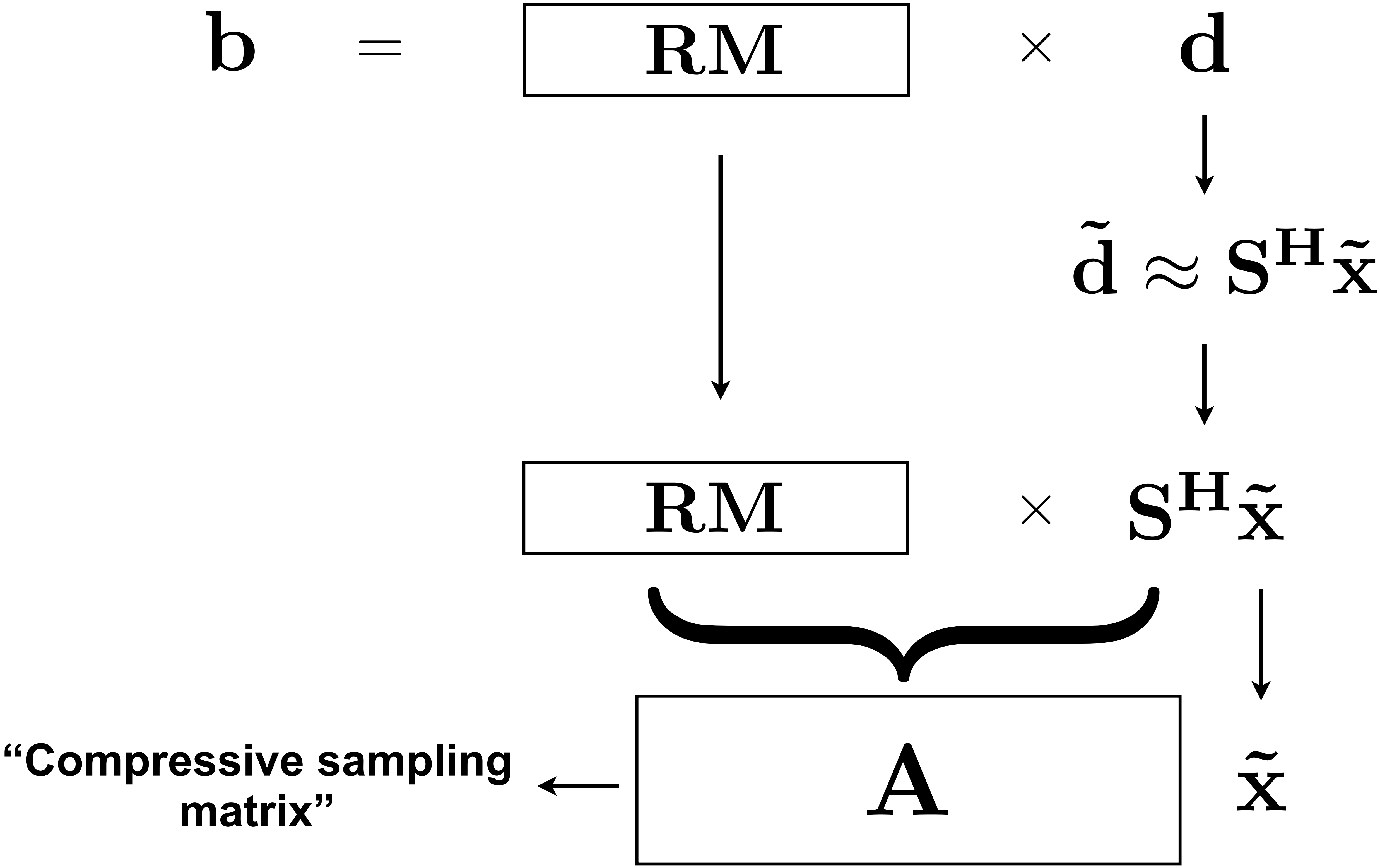


[Mansour et.al., '11]

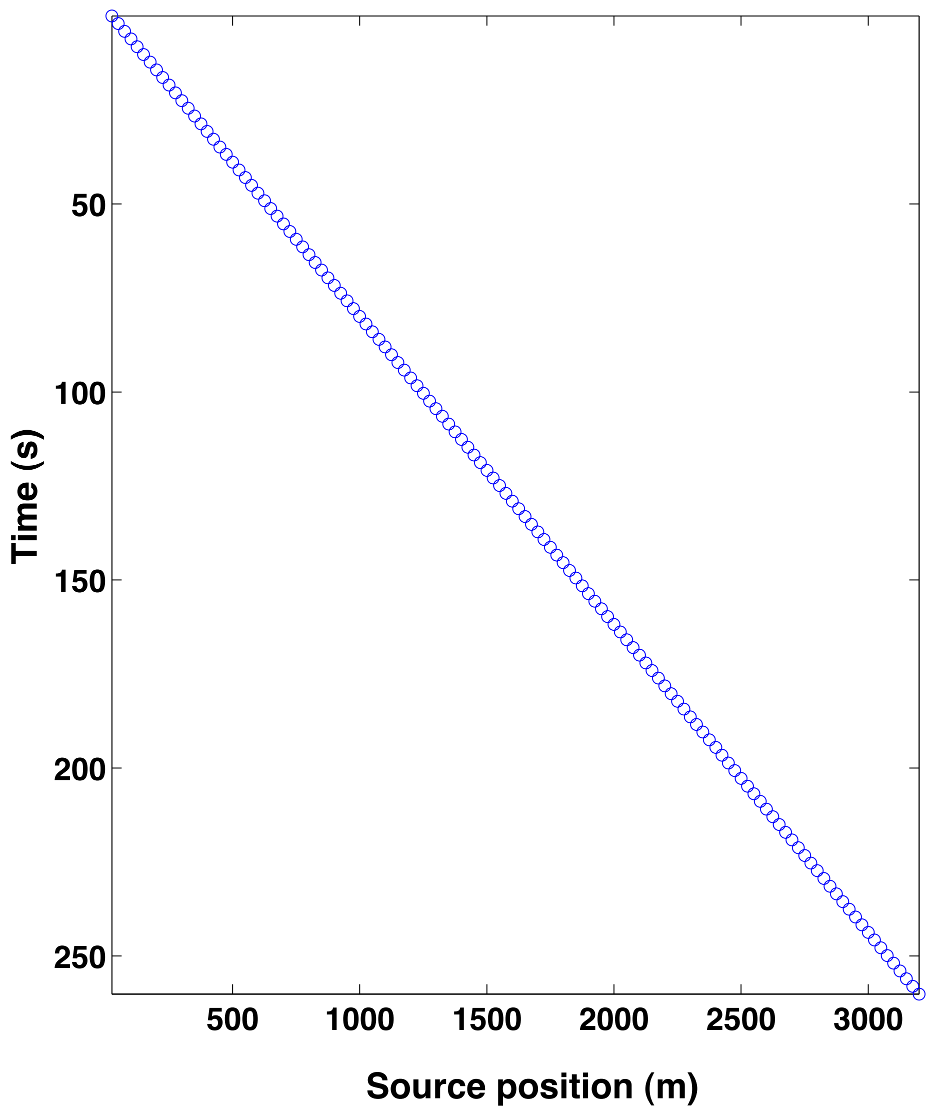
Bigger picture



Bigger picture

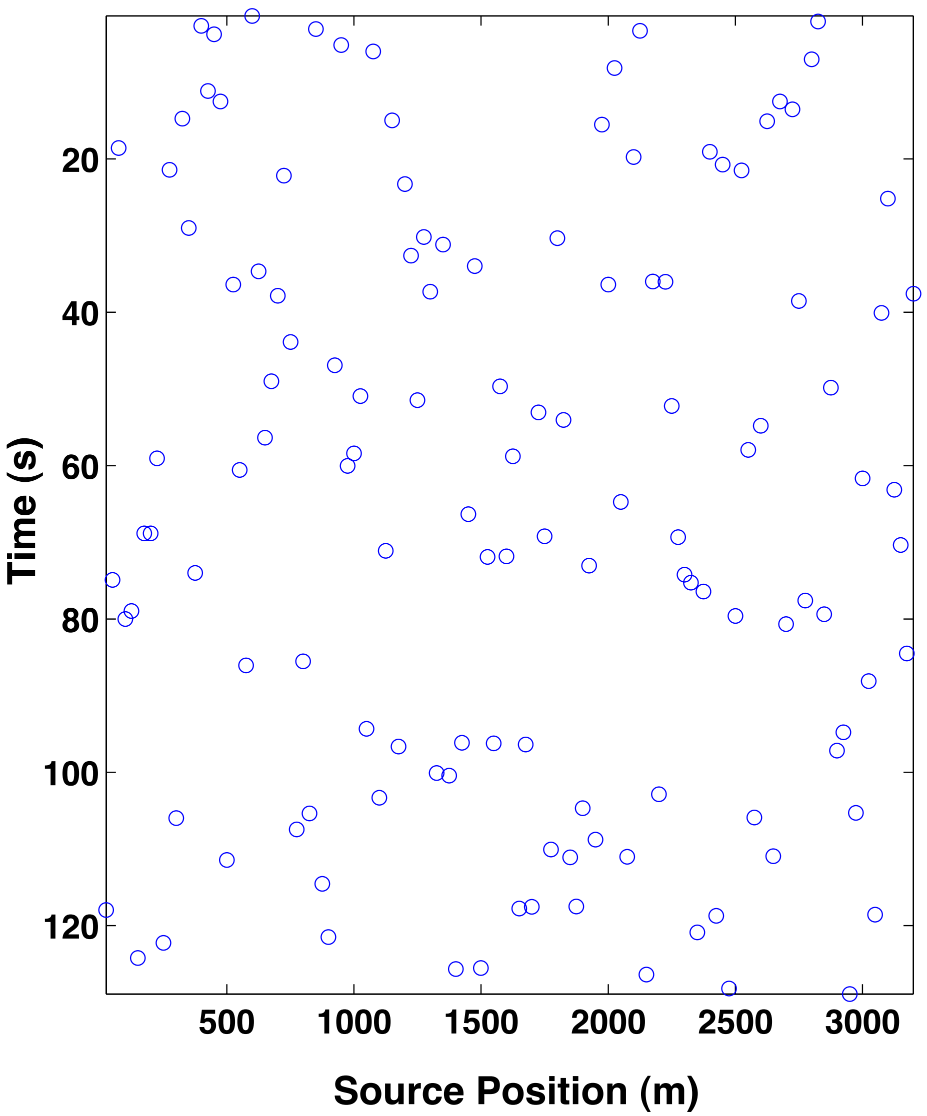


Sequential vs. simultaneous sources



Sequential acquisition

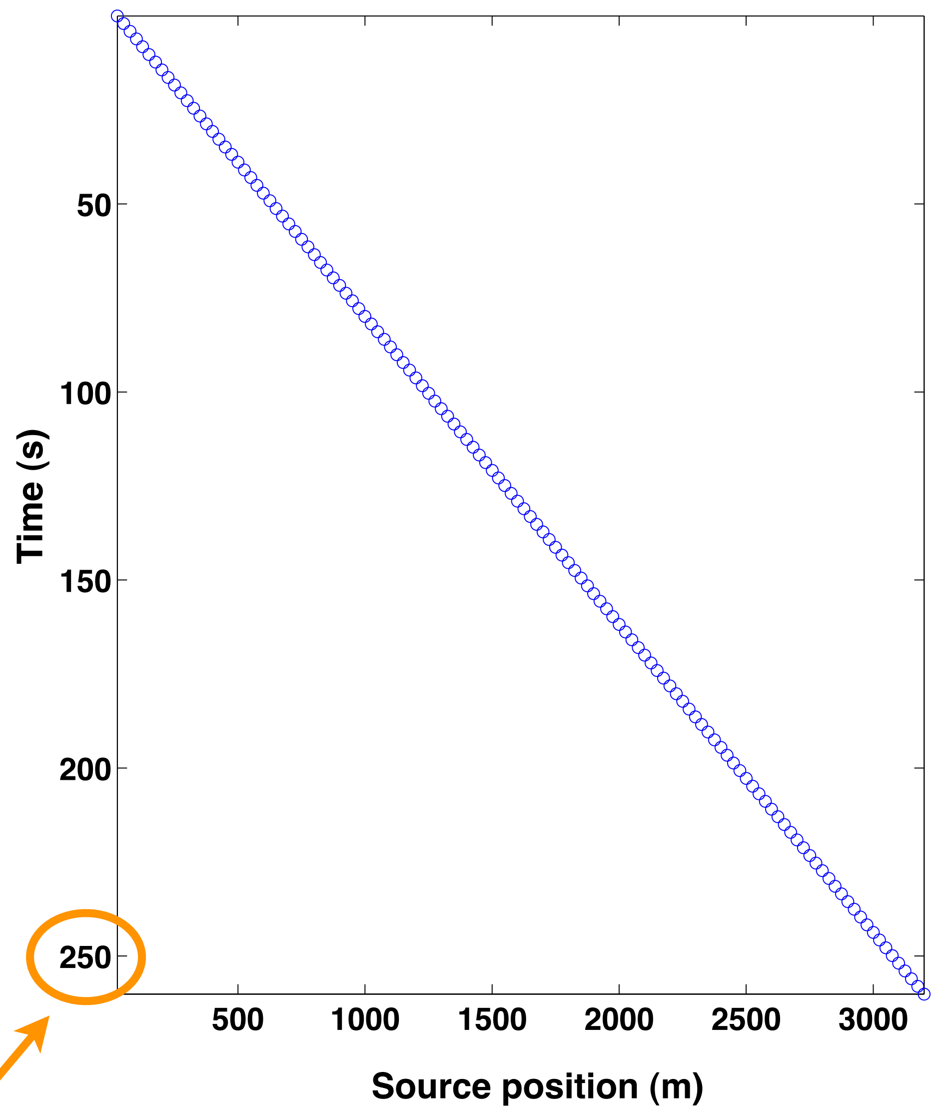
Sampling scheme: Random dithering



Simultaneous acquisition

Sequential vs. simultaneous sources

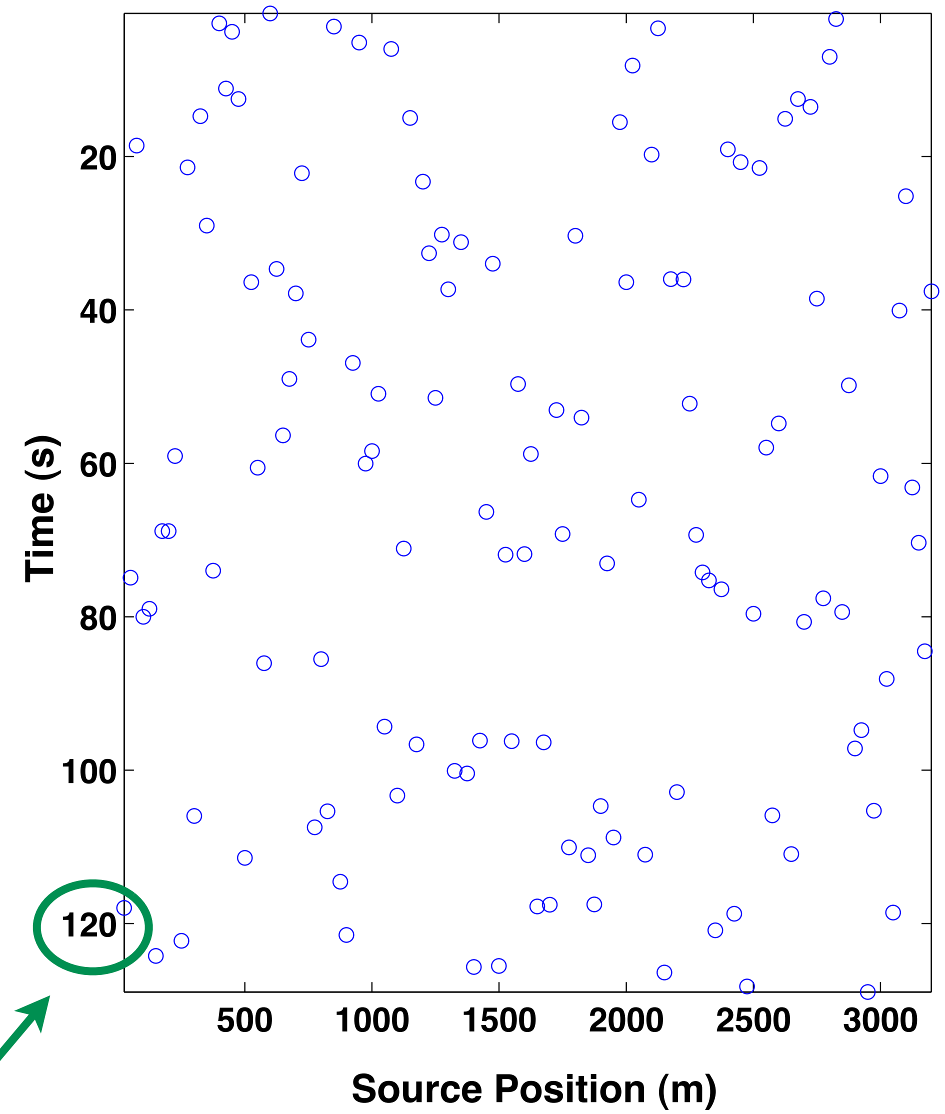
Sampling scheme: Random dithering



Conventional survey time:

$$t = \mathbf{N_s} \times \mathbf{N_t}$$

Sequential acquisition



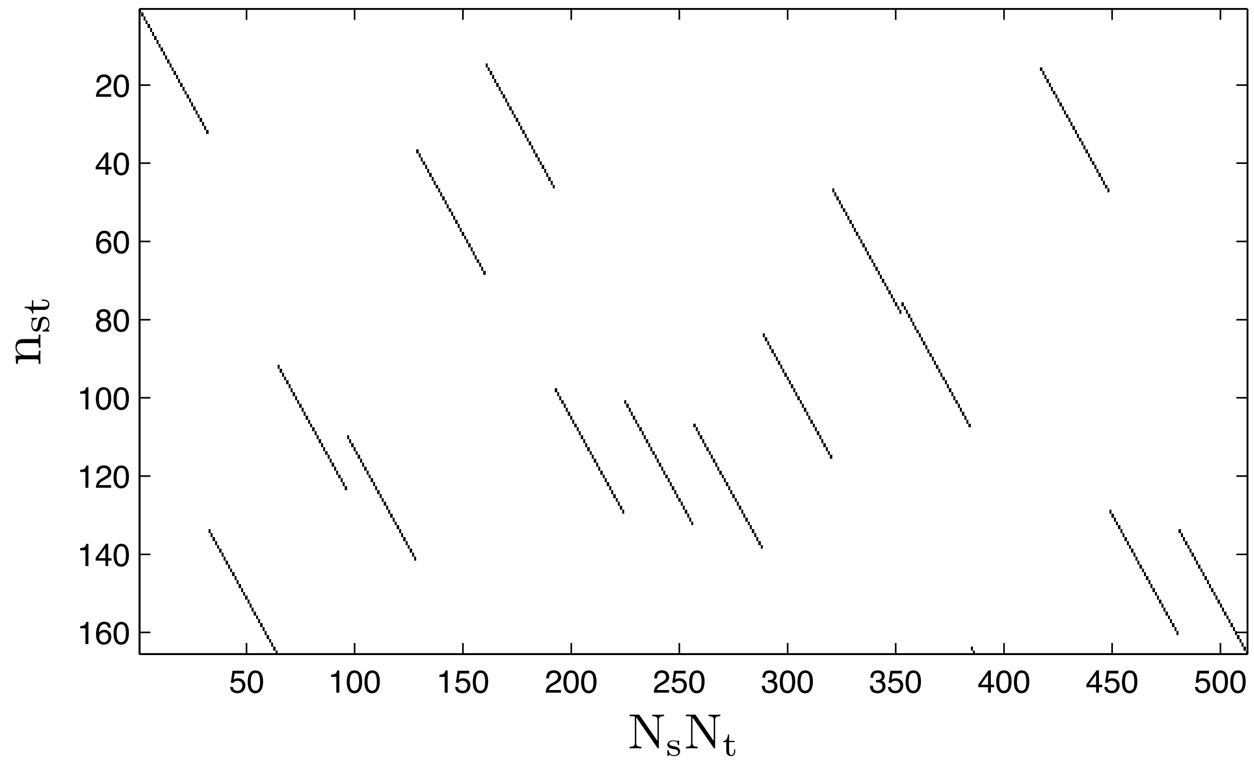
Theoretical survey time:

$$t = n_{st} \ll n_s \times \mathbf{N_t}$$

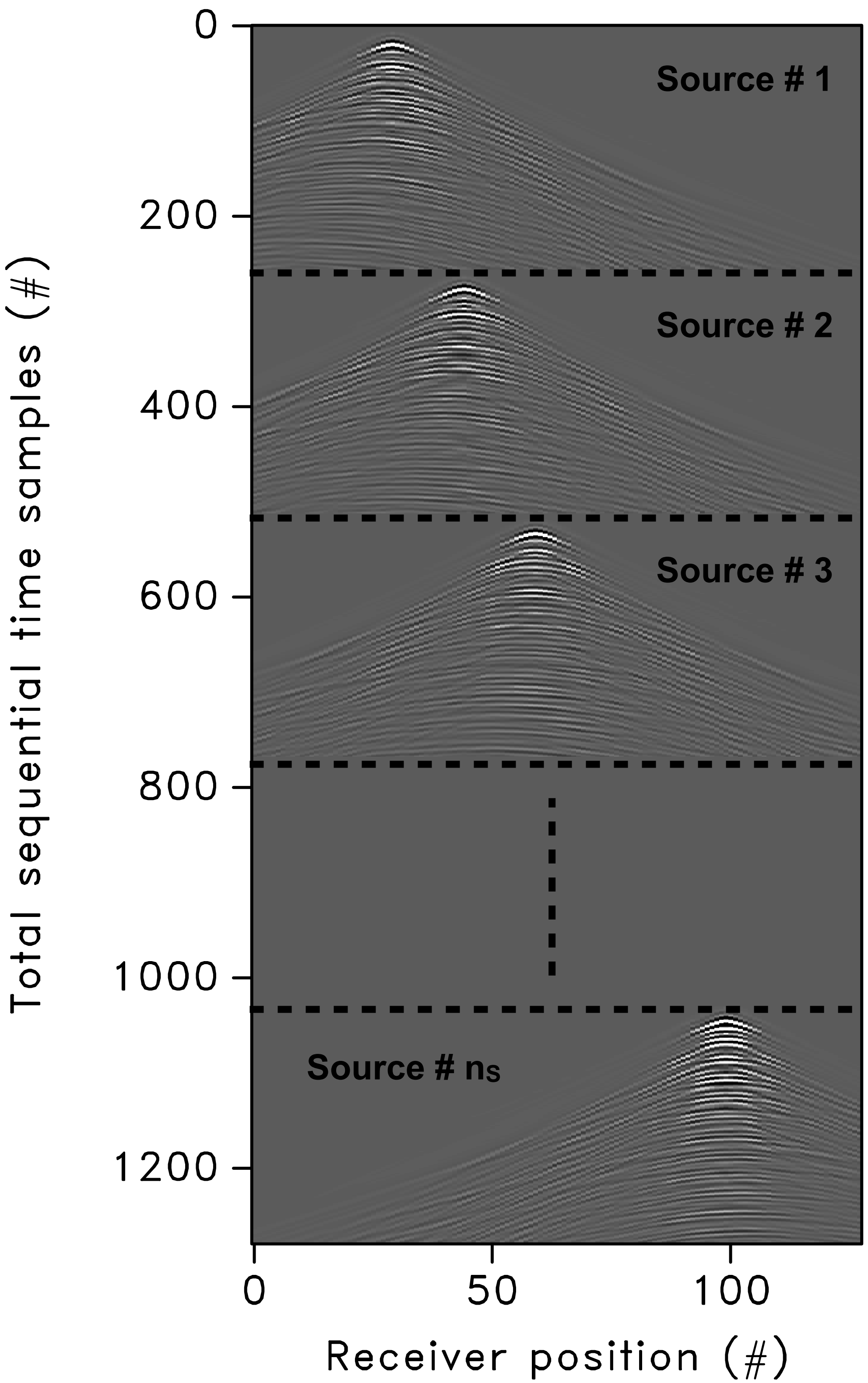
Simultaneous acquisition

Sampling scheme: Random dithering

$\mathbf{RM}$

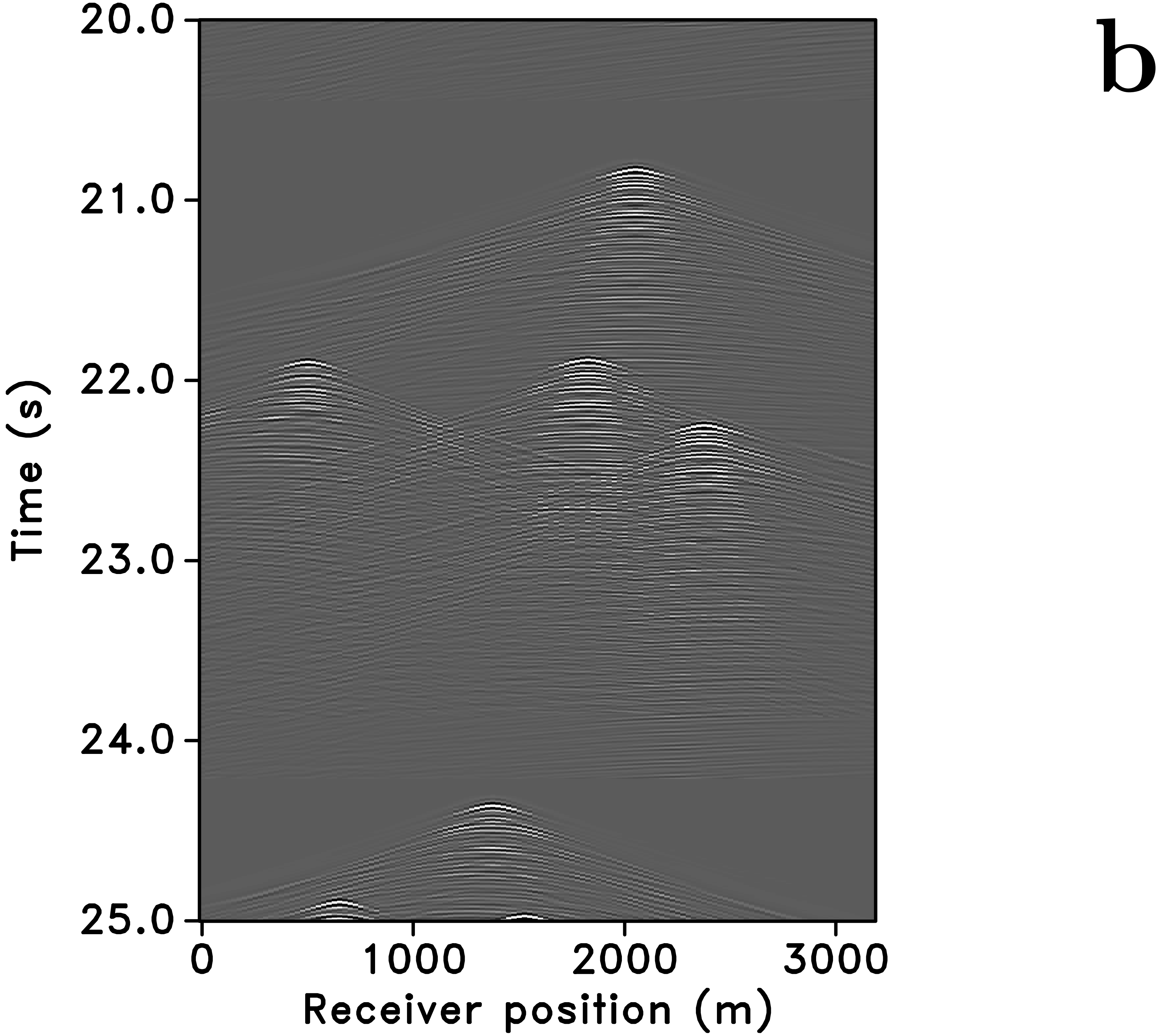


$\mathbf{d}$

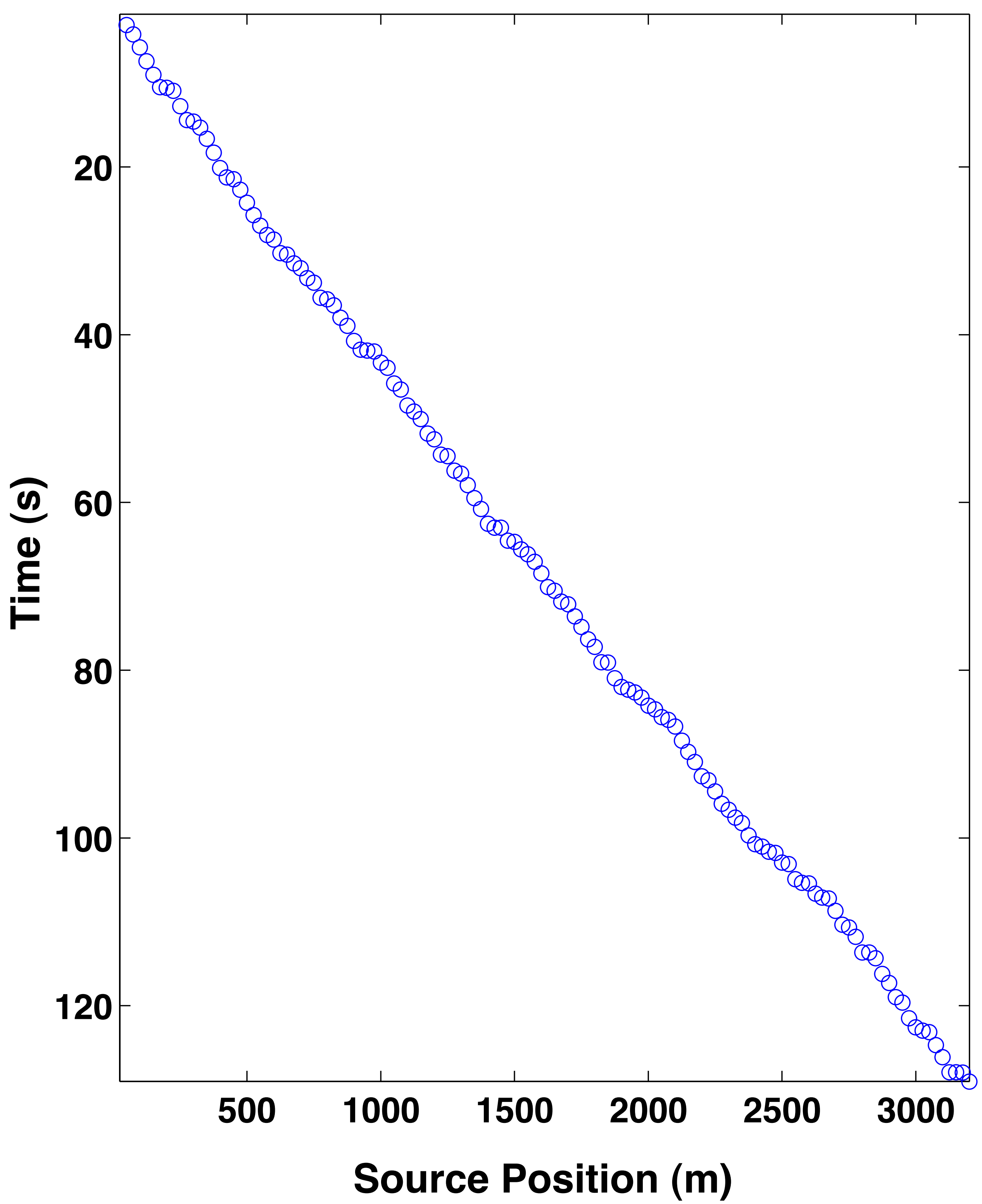


series of
sequential shots

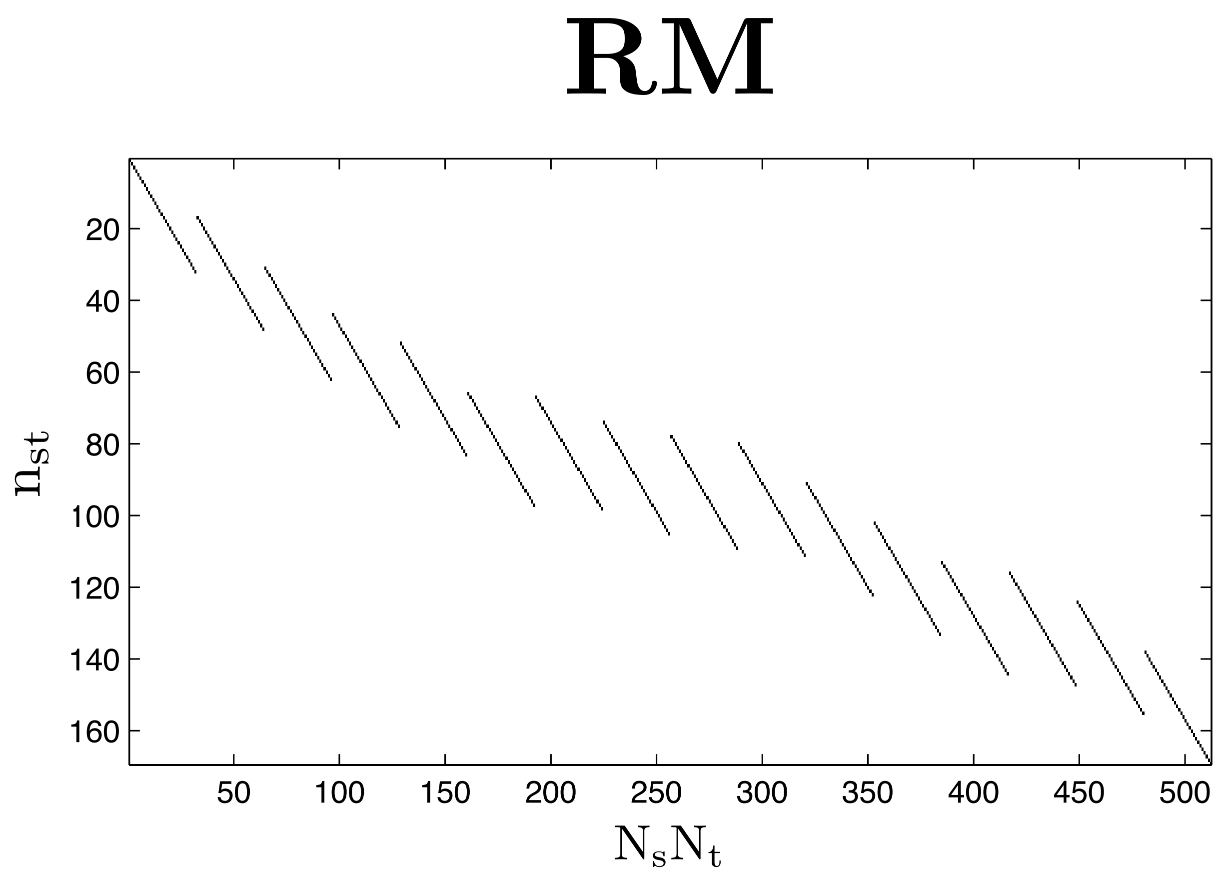
Sampling scheme: Random dithering

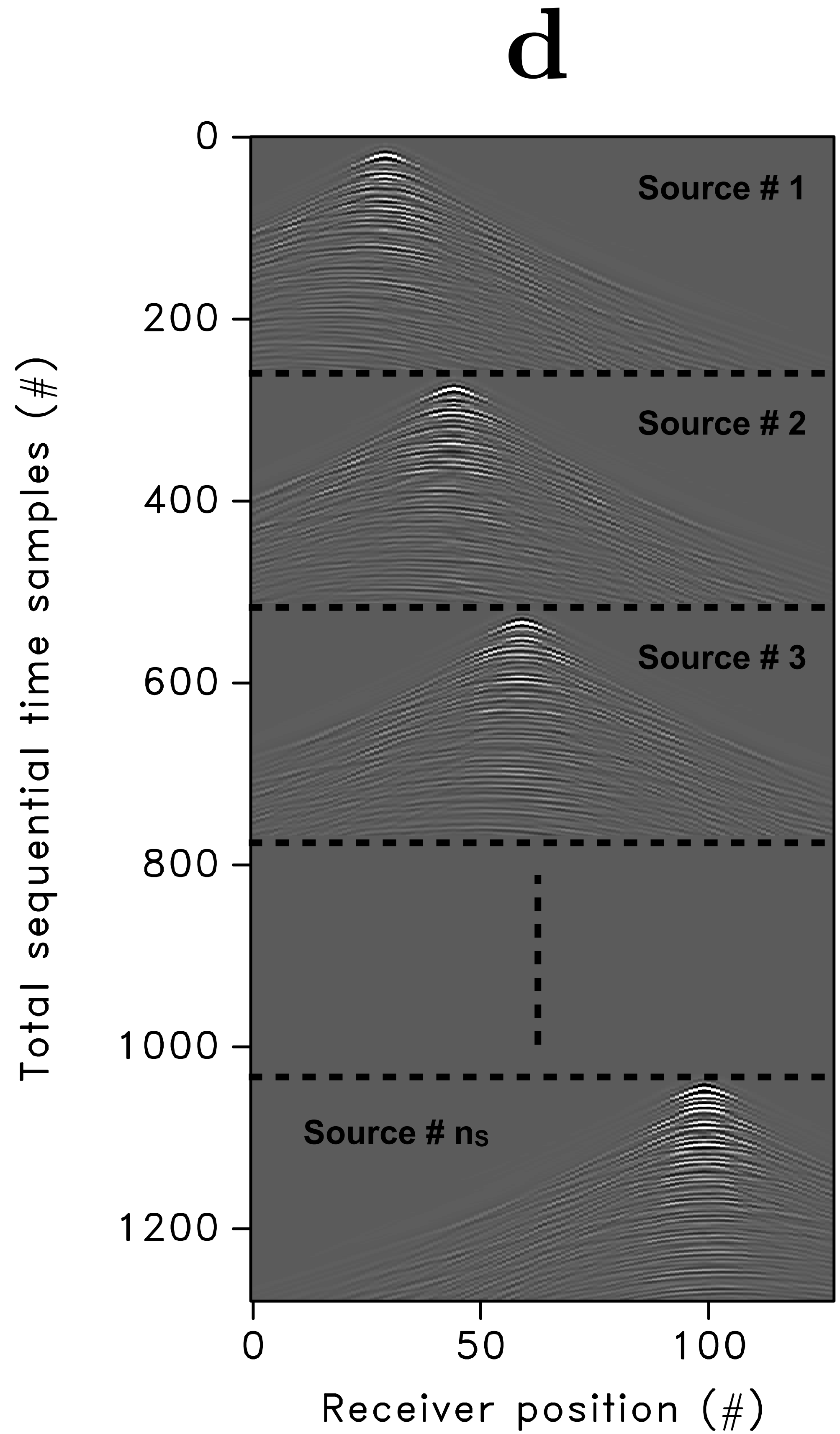


Sampling scheme: Random time-shifting



Sampling scheme: Random time-shifting





series of
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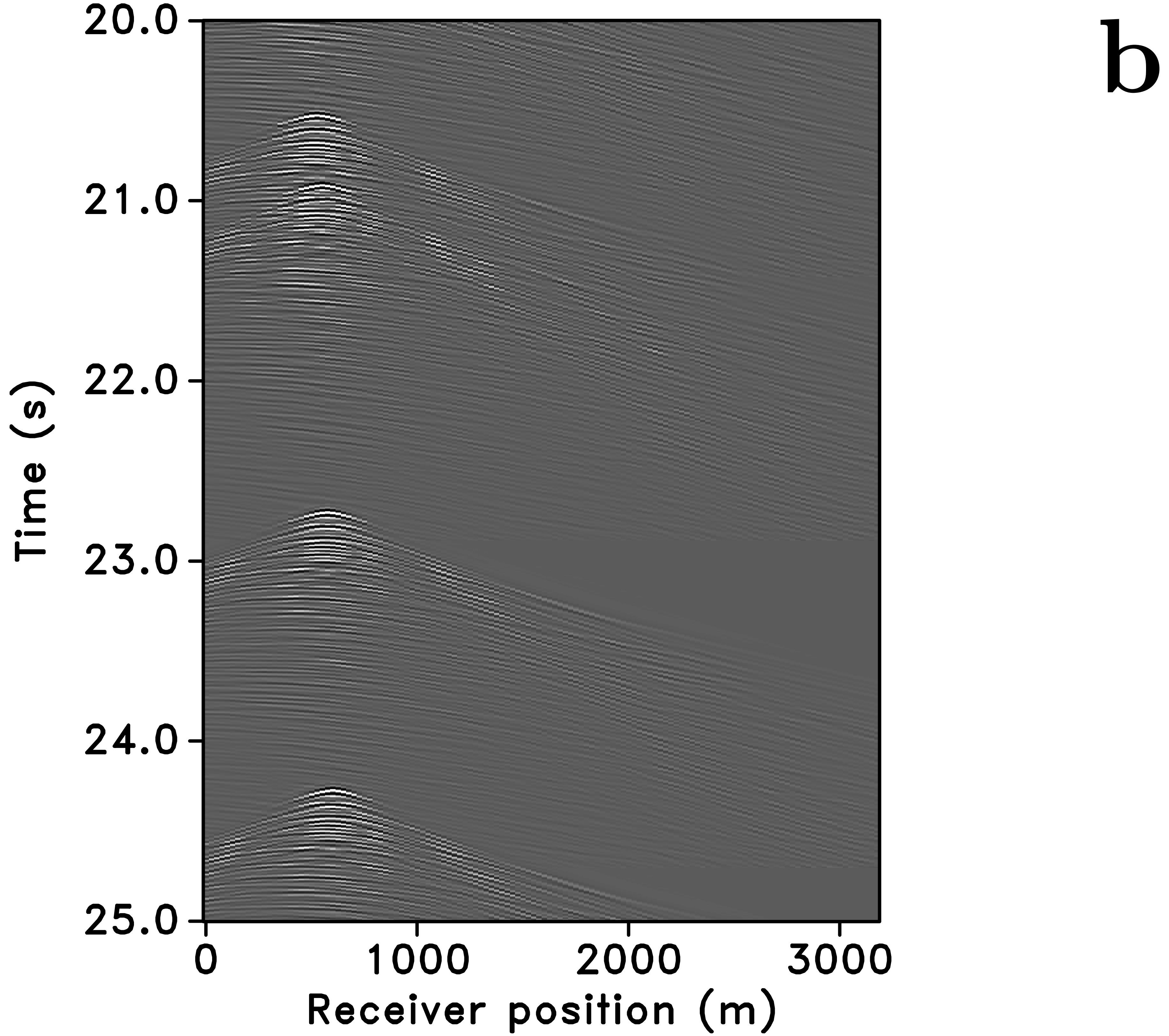
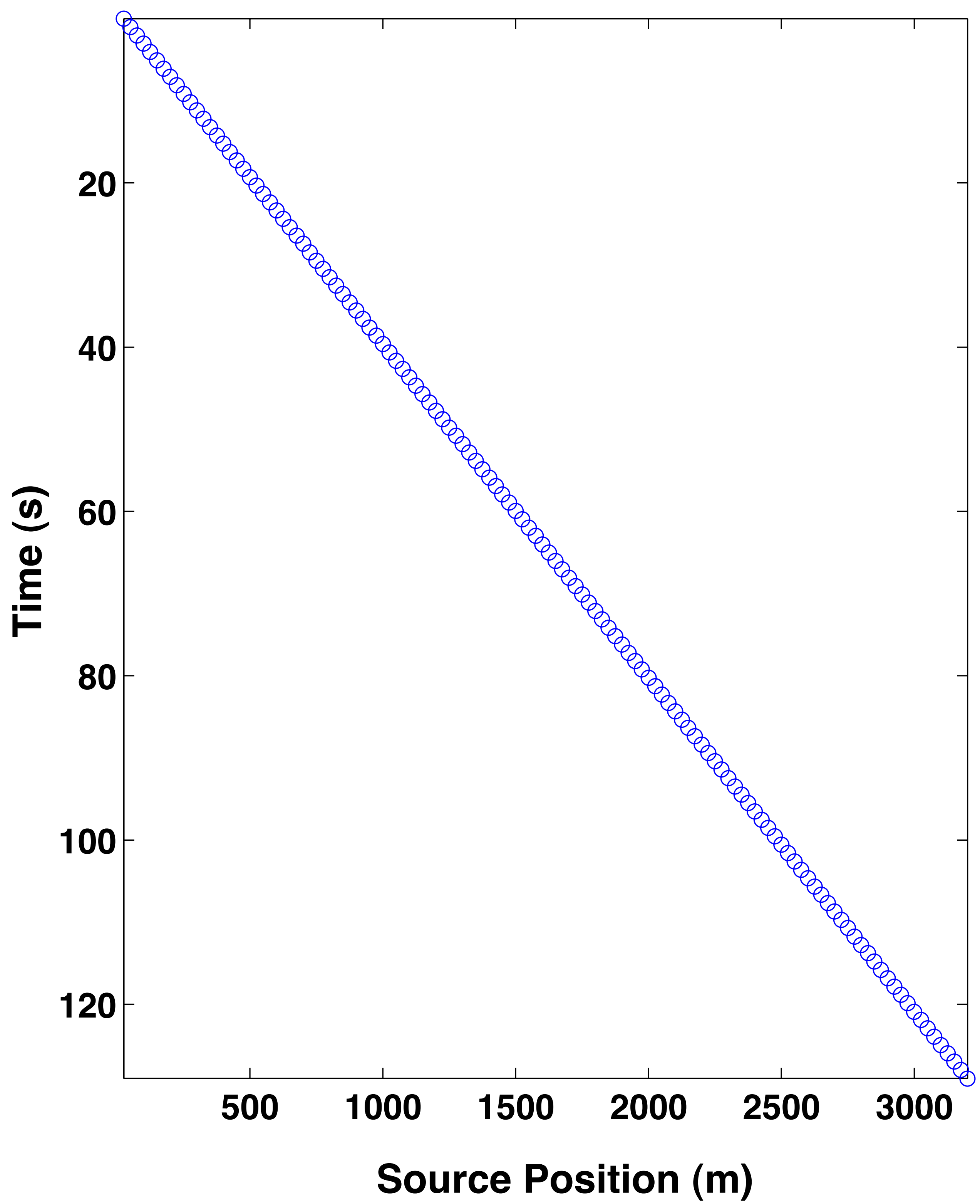
Sampling scheme: Random time-shifting

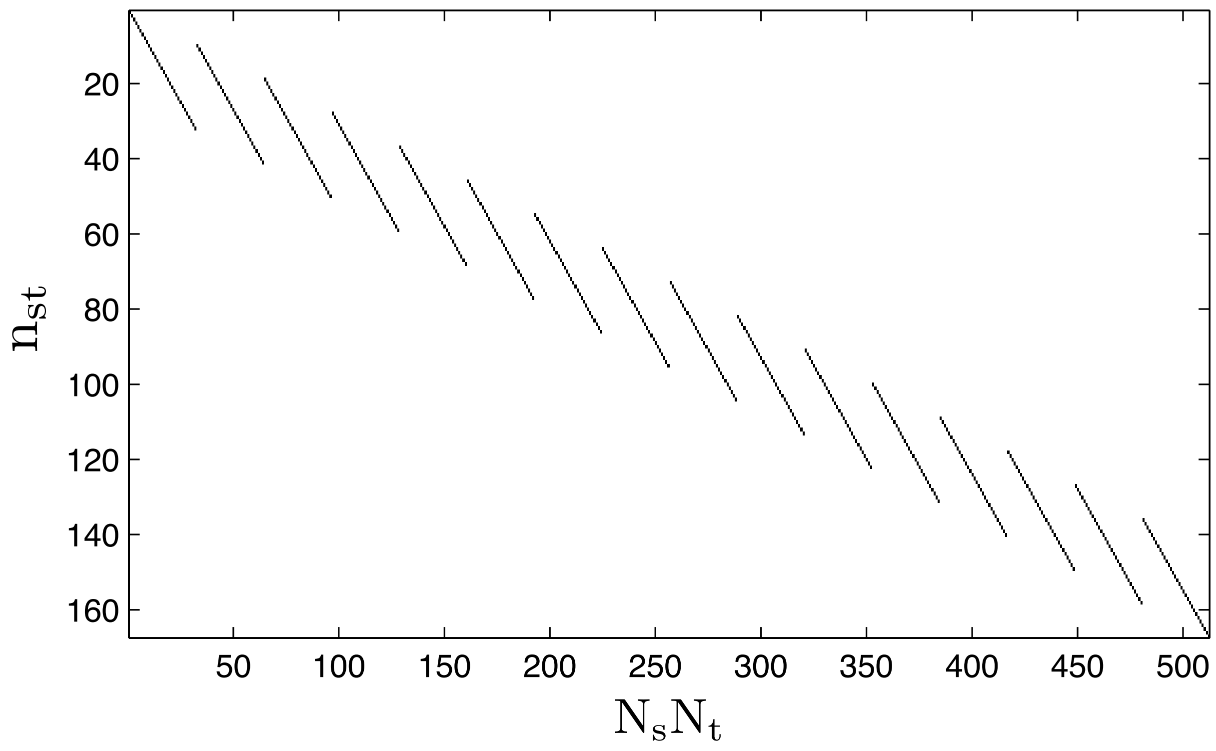


Sampling scheme: Constant time-shifting

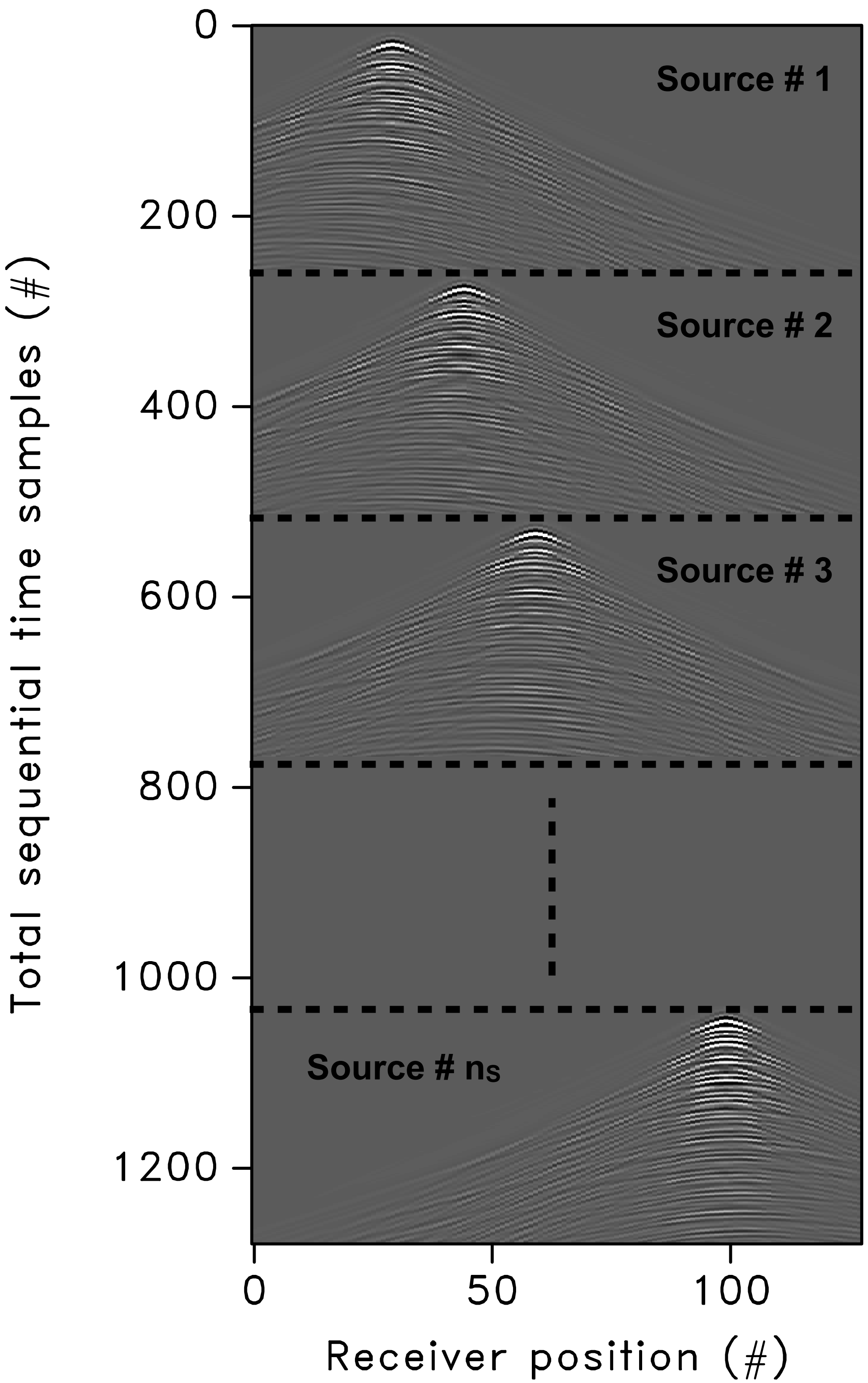


Sampling scheme: Constant time-shifting

$\mathbf{RM}$

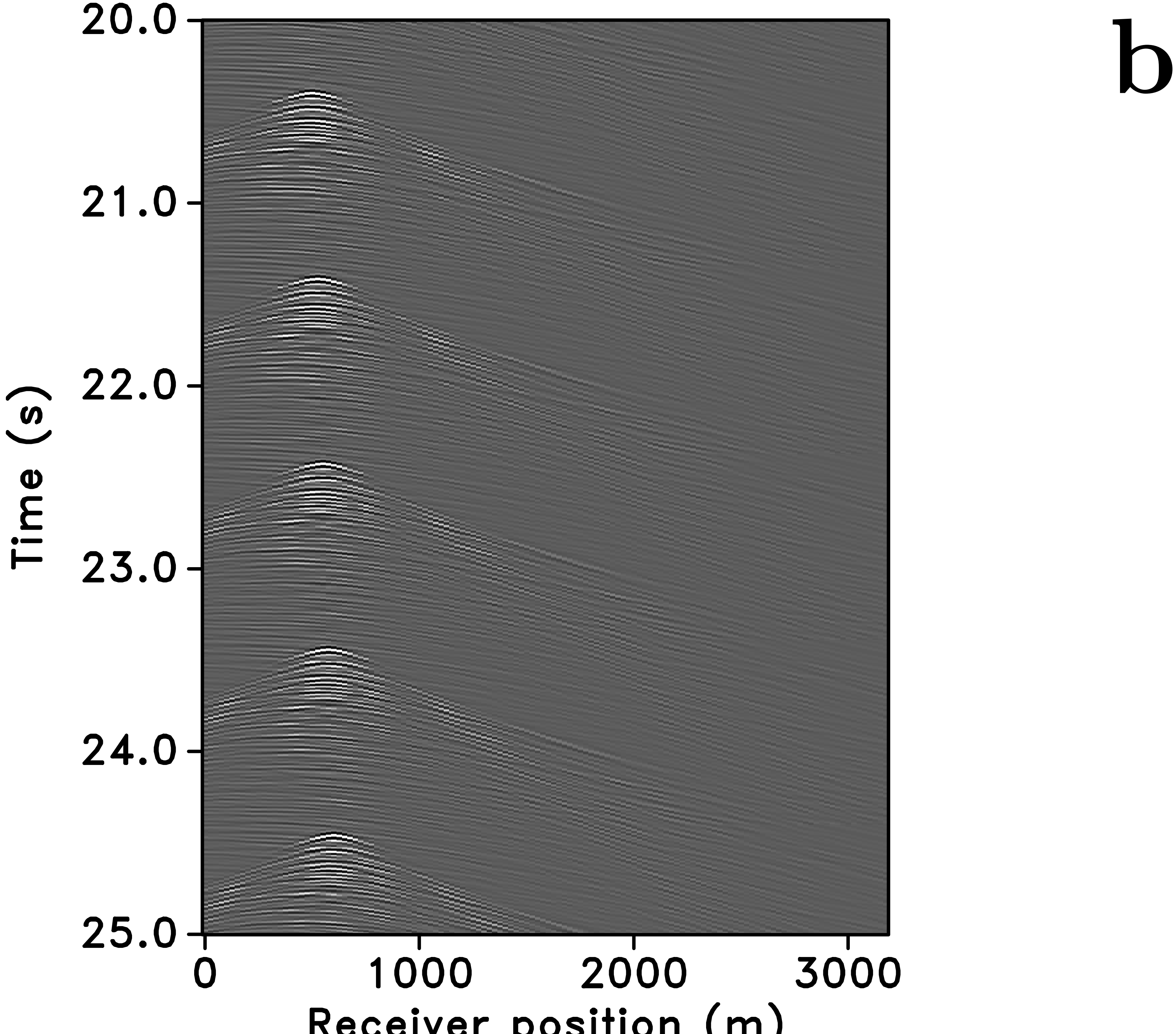


$\mathbf{d}$



series of sequential shots

Sampling scheme: Constant time-shifting



Outline

- Compressed sensing (CS) overview
  - design
  - recovery
- *Design* of simultaneous marine acquisition
- Experimental results of *sparsity*-promoting processing

Experimental setup

SLIM

- Three sampling schemes:
  - Random dithering
  - Random time-shifting
  - Constant time-shifting
- Fully sampled sequential data (a seismic line from the Gulf of Suez) with $\mathbf{N_s} = 128$ sources, $\mathbf{N_r} = 128$ receivers, and $\mathbf{N_t} = 512$ time samples
- Subsampling ratio, $\gamma = 0.5$
- Recover prestack data from simultaneous data
  - ℓ_1 minimization
  - sparsifying transform: 3-D curvelets
- All sources see the same receivers
  - marine acquisition with ocean-bottom nodes

Algorithm

Fully sampled sequential data ← $\mathbf{d}$

↓

Restricted simultaneous-acquisition sampling matrix ← $\mathbf{RM}$

↓

Sparsifying transform : Curvelet ← $\mathbf{C}$

↓

Compressive sampling matrix ← $\mathbf{A} = \mathbf{RMC^H}$

↓

Compressively sampled measurements ← $\mathbf{b} = \mathbf{RMd}$

↓

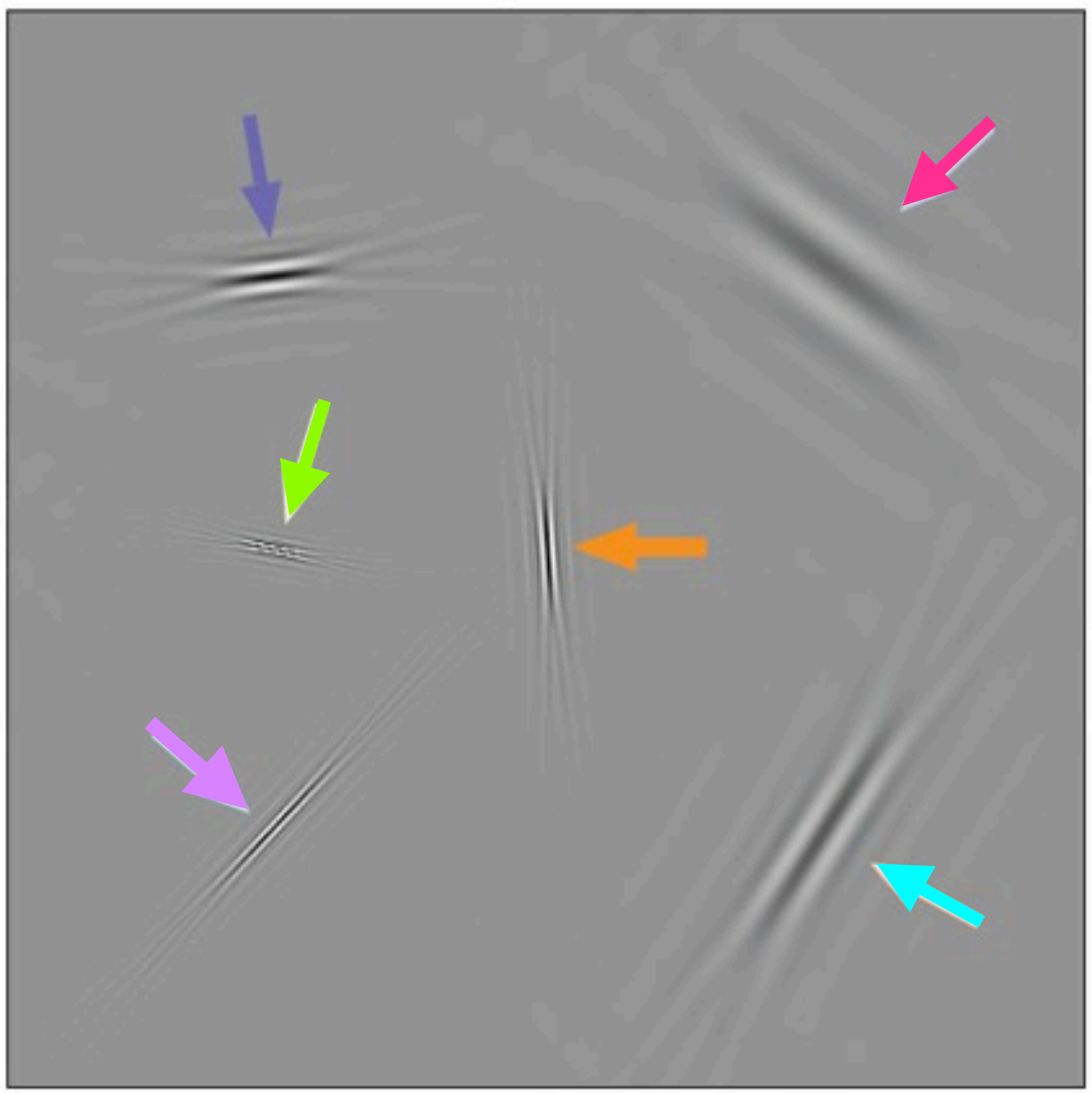
Recover sparsest set of curvelet coefficients

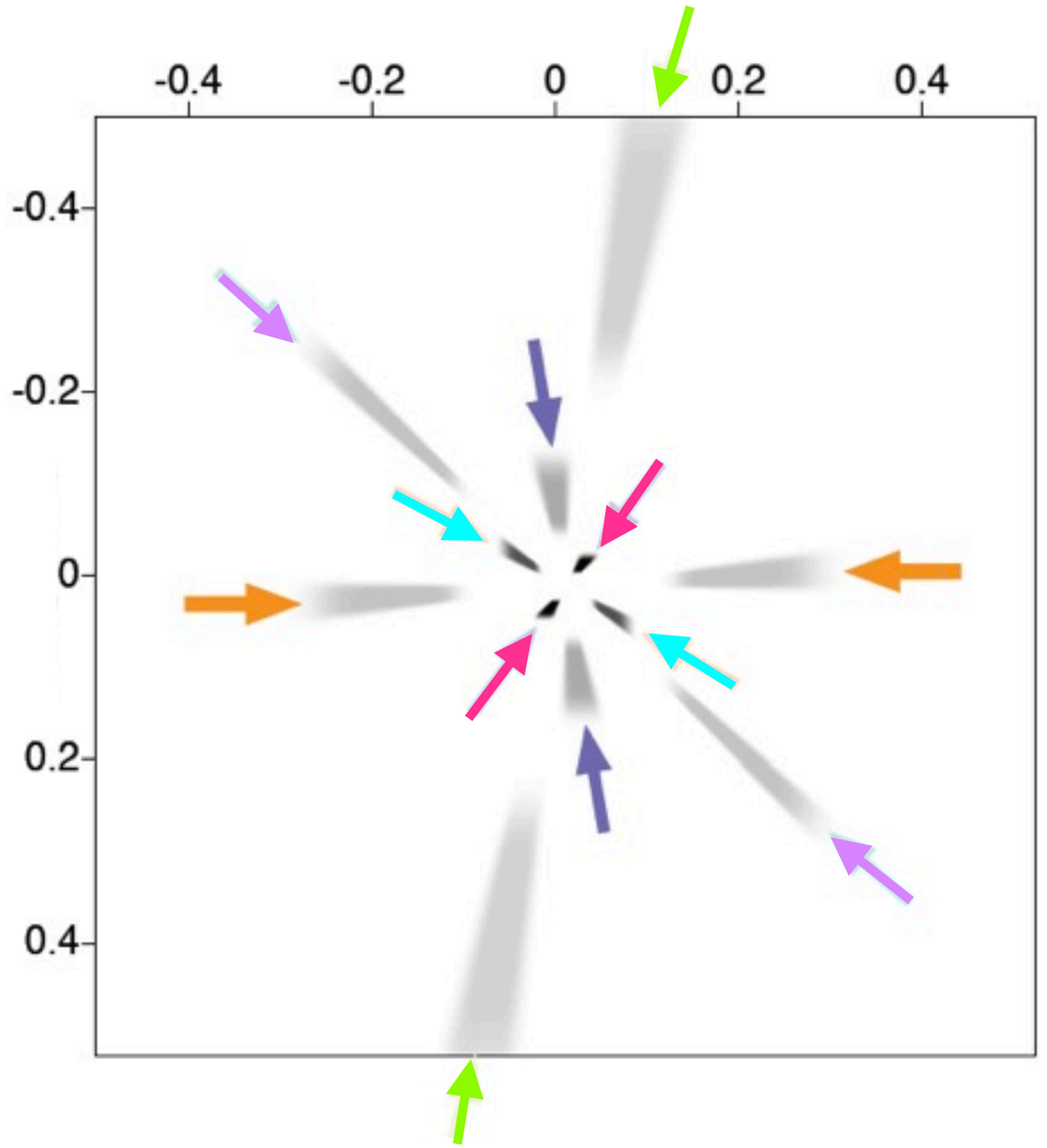
$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \mathbf{Ax} = \mathbf{b}$$

↓

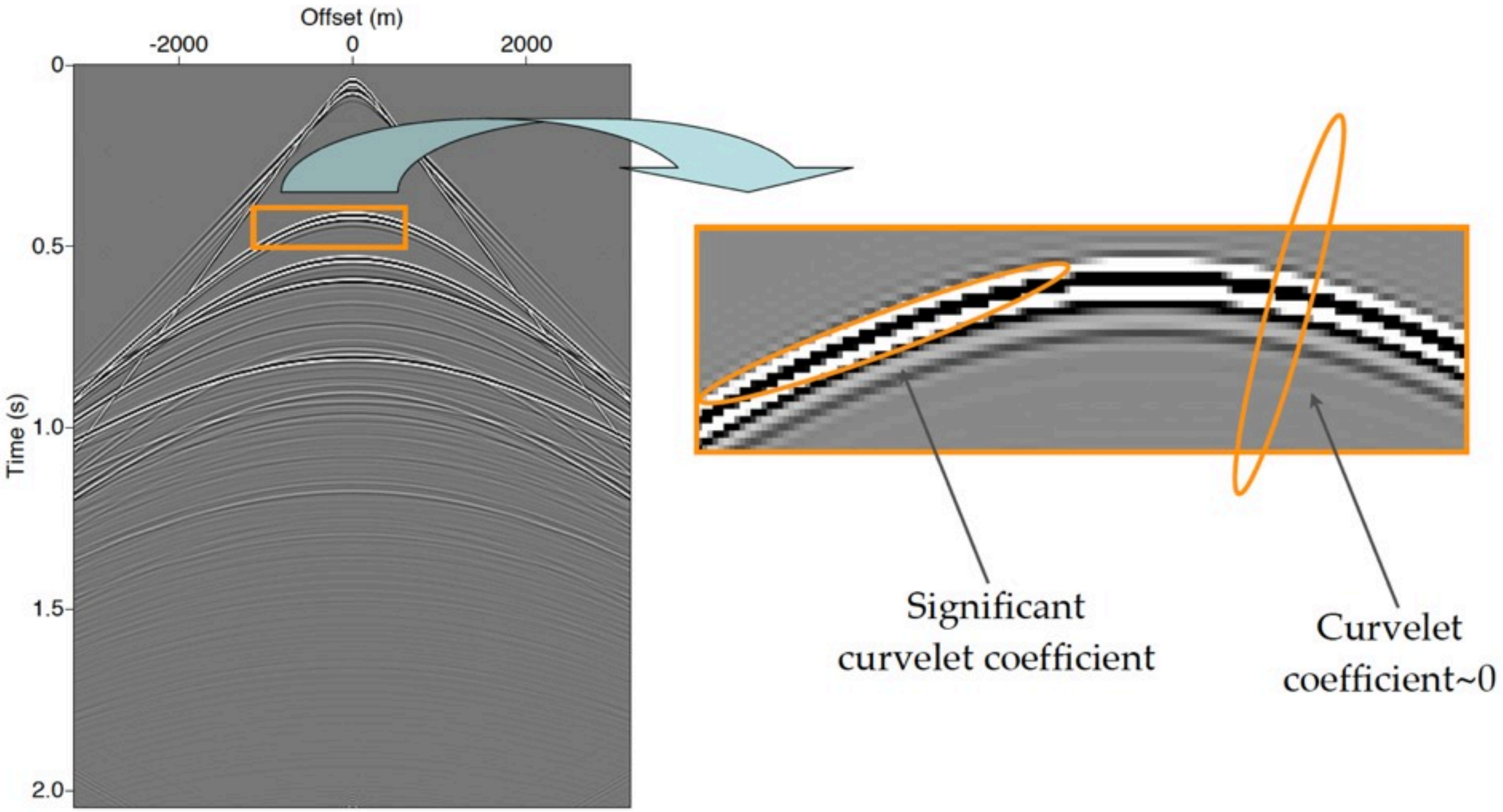
Sequential data recovery ← $\tilde{\mathbf{d}} = \mathbf{C^H}\tilde{\mathbf{x}}$

Curvelets

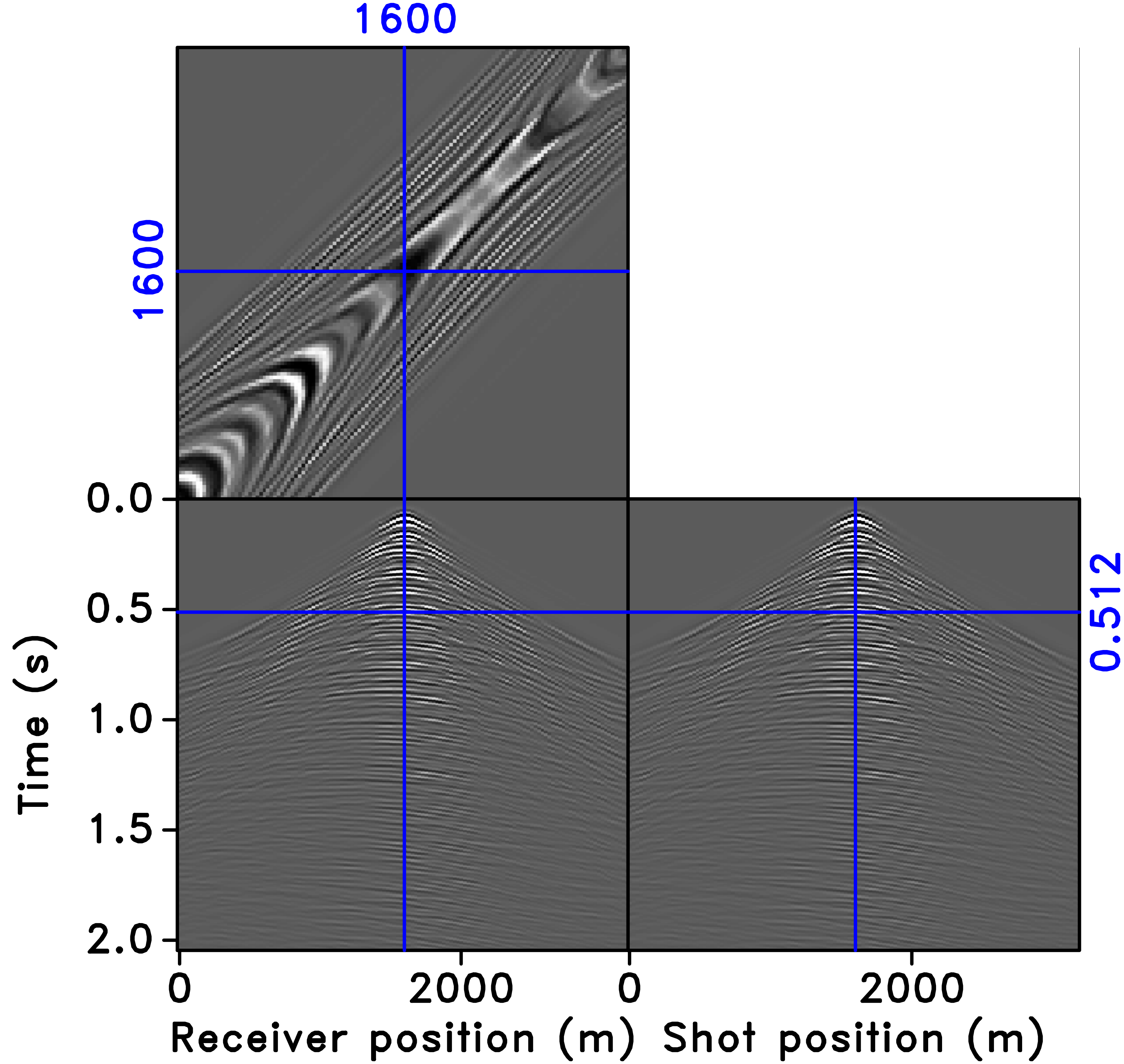




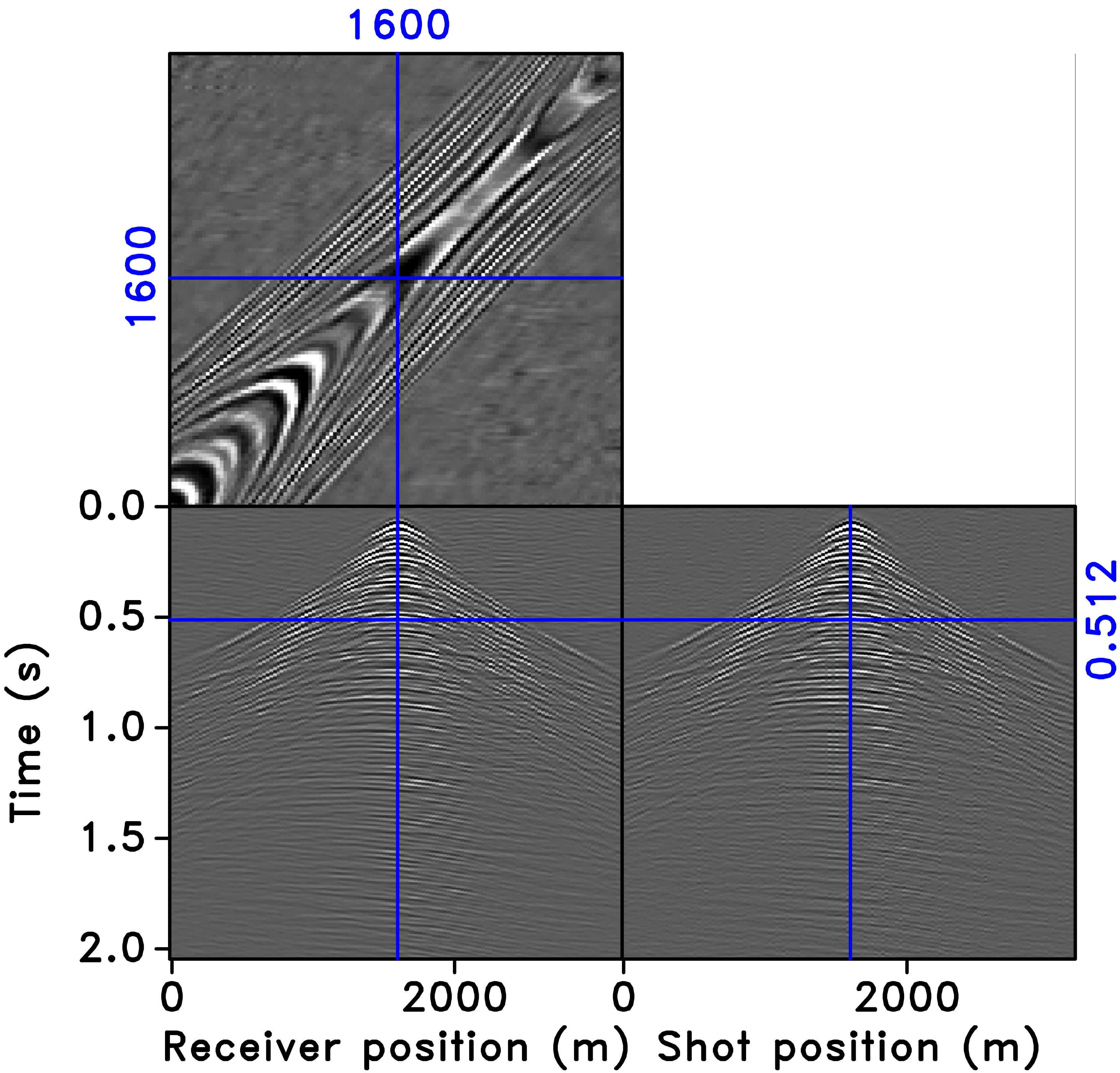
Detect the wavefronts



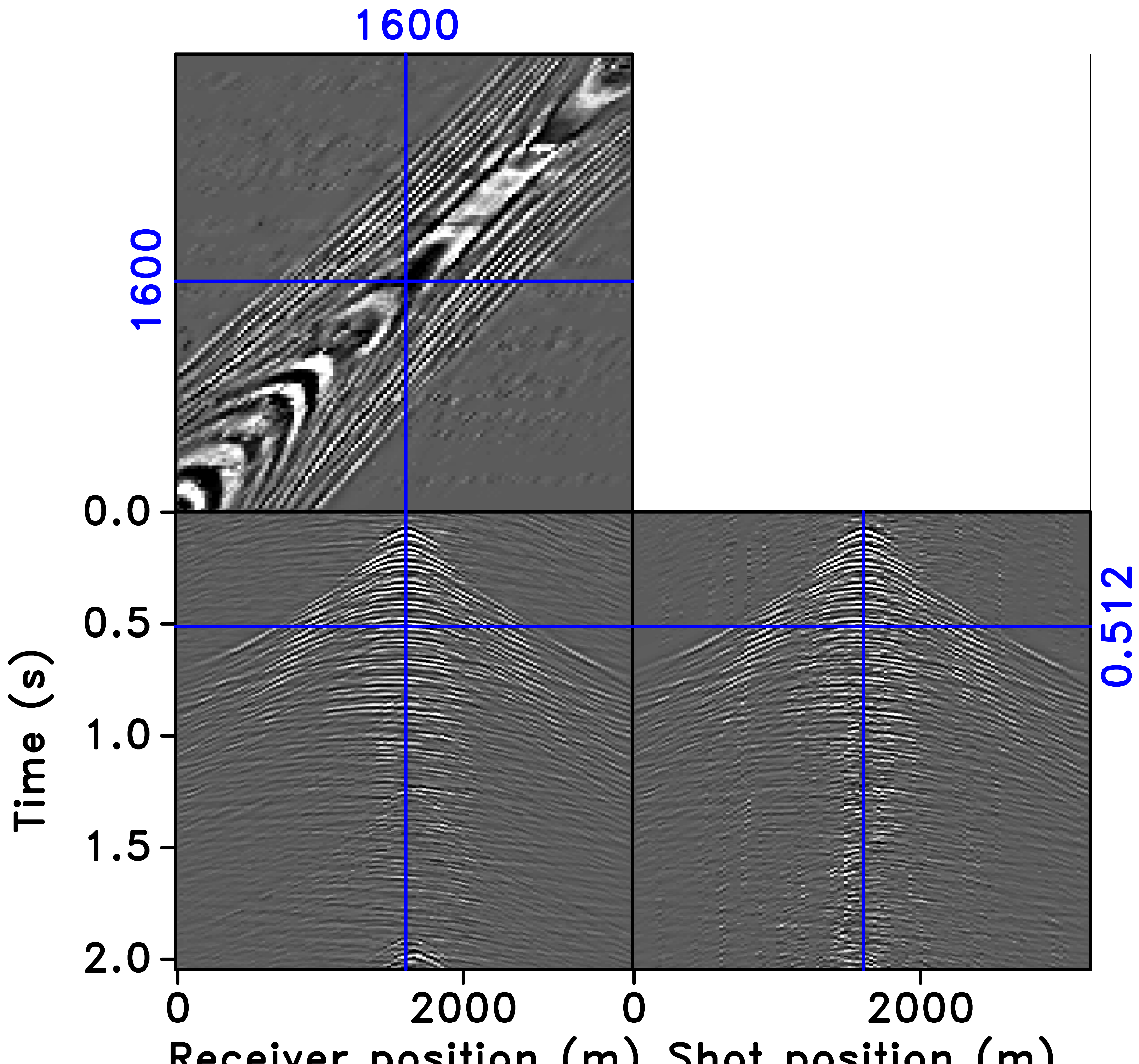
Original data
(Sequential acquisition)



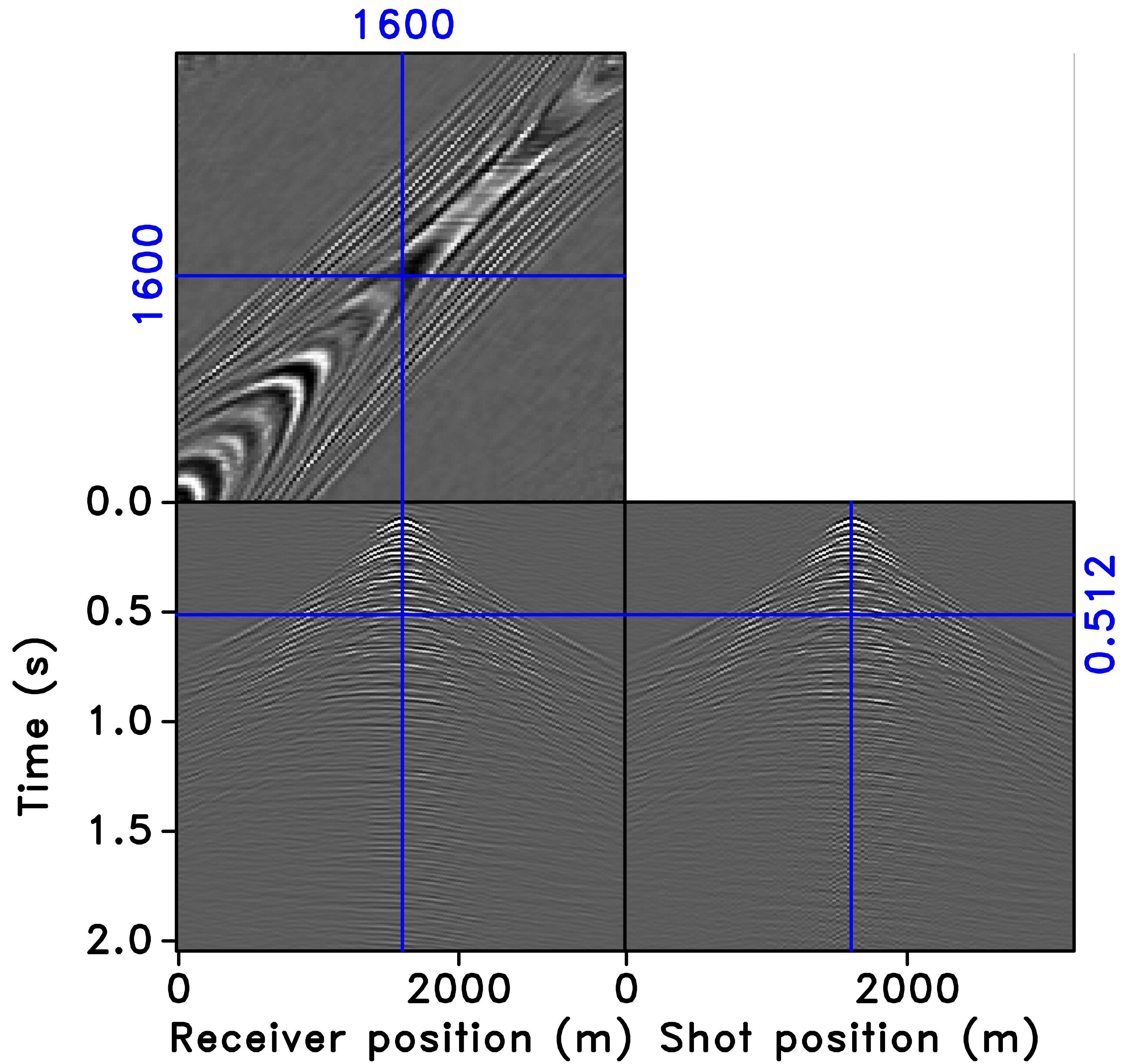
Sparsity-promoting recovery: Random dithering
SNR = 10.5 dB



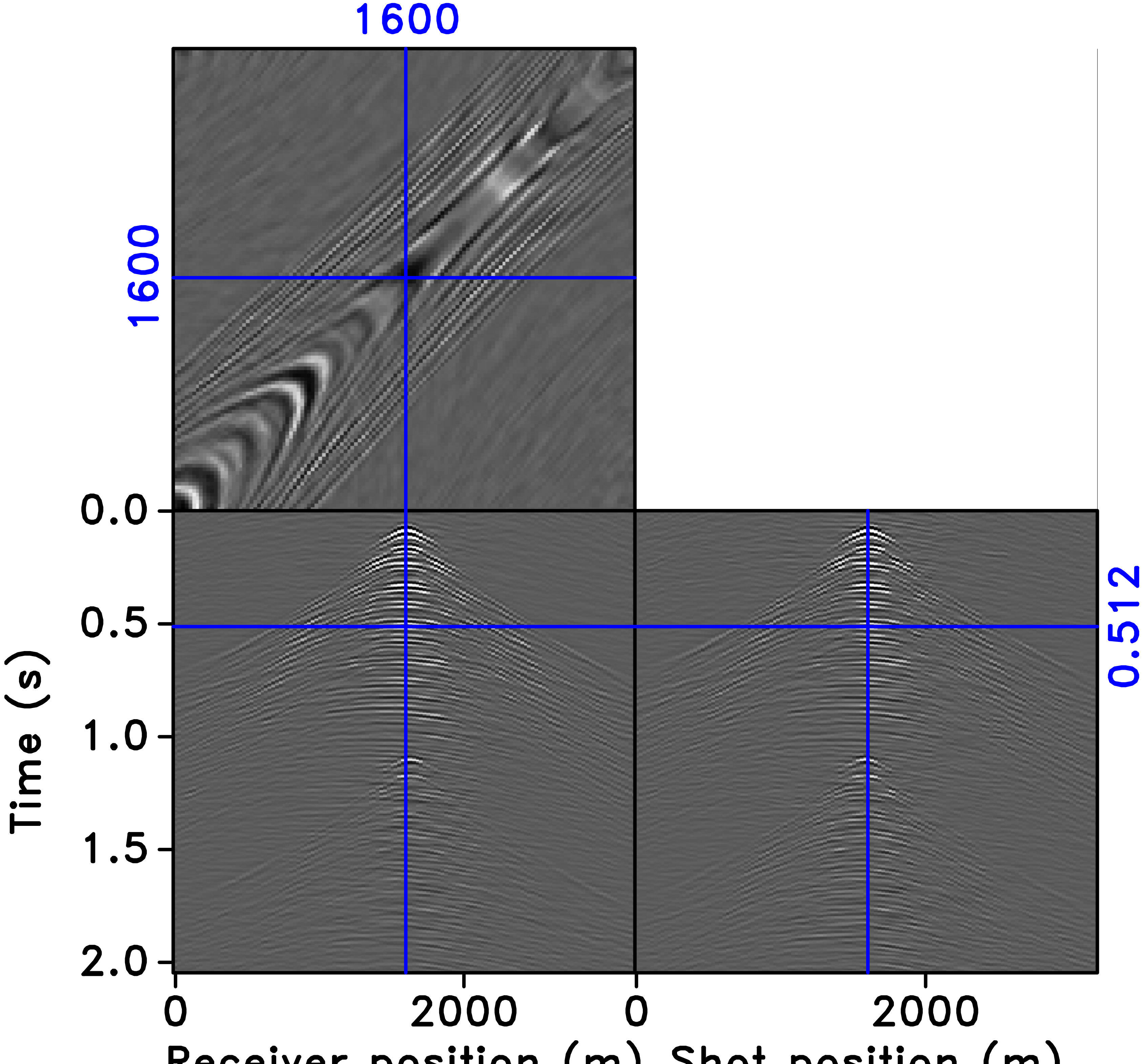
Conventional recovery: Random time-shifting
SNR = 5.04 dB



Sparsity-promoting recovery: Random time-shifting
SNR = 9.52 dB



Sparsity-promoting recovery: Constant time-shifting
SNR = 4.80 dB



Conclusions

Simultaneous acquisition is a *linear subsampling* system

Critical for reconstruction quality:

- *design* of source subsampling schemes (i.e., acquisition scenarios)
- appropriate *sparsifying* transform
- *sparsity*-promoting solver

Future plans

- Extensions to simultaneous acquisition frameworks for *towed streamer surveys*
- Use different transforms for *sparsity*-promoting processing

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SINBAD

NSERC CRSNG

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Thank you!

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