Sparsity-promoting recovery from simultaneous data: a compressive sensing approach

Haneet Wason*, Tim T. Y. Lin, and Felix J. Herrmann

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Motivation

Conventional recovery

SNR = 5.04 dB
Motivation

Sparsity-promoting recovery

SNR = 9.52 dB
Motivation

**Conventional recovery**

SNR = 5.04 dB

✗

**Sparsity-promoting recovery**

SNR = 9.52 dB

✓
Motivation

- Opportunity to rethink Marine acquisition
- Concentrate on simultaneous sourcing
- Marine acquisition with ocean-bottom nodes
Outline

- Compressed sensing (CS) overview
  - design
  - recovery
- Design of simultaneous marine acquisition
- Experimental results of sparsity-promoting processing
Problem statement

Solve an *underdetermined* system of *linear* equations:

\[
\begin{align*}
\text{data} & \quad \xrightarrow{\text{measurements/observations}} \\
\mathbf{b} \in \mathbb{C}^n & \quad = \\
\mathbf{b} & \quad \begin{pmatrix} \mathbf{A} \\ \mathbf{x}_0 \end{pmatrix} \\
\mathbf{A} & \in \mathbb{C}^{n \times P} \\
\mathbf{x}_0 & \leftarrow \text{unknown}
\end{align*}
\]
Compressed sensing

- acquisition paradigm for sparse signals
- in some transform domain

\[ \mathbf{d} \in \mathbb{R}^N \]
\[ \mathbf{S} : \text{transform matrix} \]
\[ \mathbf{S} \in \mathbb{C}^{P \times N} \]
\[ P \geq N \]
\[ \mathbf{x}_0 \in \mathbb{C}^P \]
Compressed sensing

- acquisition paradigm for sparse signals
- in some transform domain

\[ \tilde{d} \approx \tilde{x} \]

\[ S^H \]

approximate \( d \)

\( k \)-largest coefficients
Bigger picture

Source # 1
Source # 2
Source # 3
Source # n
Source # ns

series of sequential shots

Receiver position (m)
Time (s)

Total sequential time samples (#)

Receiver position (＃)
Simultaneous measurement matrix
Bigger picture

\[ b = \text{Simultaneous measurement matrix} \times d \]

\[ \tilde{d} \approx S^H \tilde{x} \]
\[ b = \text{Simultaneous measurement matrix} \times d \]

\[ \tilde{d} \approx S^H \tilde{x} \]

\[ b = \text{Simultaneous measurement matrix} \times \tilde{d} \approx S^H \tilde{x} \]
Bigger picture

\[ b = \text{Simultaneous measurement matrix} \times d \]

\[ \tilde{d} \approx S^H \tilde{x} \]

\[ \text{Simultaneous measurement matrix} \times S^H \tilde{x} \]

\[ \underbrace{A}_{\text{Simultaneous measurement matrix}} \rightarrow \tilde{x} \]
Coarse sampling schemes

3-fold under-sampling

- Fourier transform
  - Few significant coefficients detected
  - Ambiguity

[Hennenfent & Herrmann, '08]
Mutual coherence

- measures the orthogonality of all columns of $\mathbf{A}$
- equal to the maximum off-diagonal element of the Gram matrix
  - controlled by compressive sensing via a combination of
    - randomization with
    - spreading of sampling vectors in the sparsifying domain
Restricted isometry property

- indicates whether every group of $k$ columns of $A$ are nearly orthogonal

- restricted isometry constant $0 < \delta_k < 1$ for which

\[
(1 - \delta_k) \|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta_k) \|x\|_2^2
\]
Sparse recovery

Solve the convex optimization problem (one-norm minimization):

$$\tilde{x} = \arg \min_x \|x\|_1 \quad \text{subject to} \quad Ax = b$$

“sparsity”

data-consistent amplitude recovery

Sparsity-promoting solver: $\text{SPG}_1$

[van den Berg and Friedlander, ’08]

Recover single-source prestack data volume: $\tilde{d} = S^H \tilde{x}$
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Simultaneous acquisition matrix

For a seismic line with $N_s$ sources, $N_r$ receivers, and $N_t$ time samples, the sampling matrix is

$\begin{bmatrix} RM \end{bmatrix}$

samples recorded at each receiver during simultaneous acquisition

samples recorded at each receiver during sequential acquisition

[Mansour et al., '11]
$$\mathbf{b} = \text{Simultaneous measurement matrix} \times \mathbf{d}$$

$$\tilde{\mathbf{d}} \approx \mathbf{S}^H \tilde{\mathbf{x}}$$

$$\tilde{\mathbf{d}} \approx \mathbf{S}^H \tilde{\mathbf{x}} \times \mathbf{S}^H \tilde{\mathbf{x}}$$

$$\mathbf{A} \tilde{\mathbf{x}}$$
Bigger picture

\[ b = \text{RM} \times d \]

\[ \tilde{d} \approx S^H \tilde{x} \]

\[ \text{RM} \times S^H \tilde{x} \]

“Compressive sampling matrix”

\[ A \]

\[ \tilde{x} \]
Sequential vs. simultaneous sources

Sampling scheme:
Random dithering

Sequential acquisition  Simultaneous acquisition
Sequential vs. simultaneous sources

Sampling scheme: Random dithering

Conventional survey time: \( t = N_s \times N_t \)

Theoretical survey time: \( t = n_{st} \ll n_s \times N_t \)
Sampling scheme: Random dithering

RM

Source # 1
Source # 2
Source # 3
Source # ns

Source # 1
Source # 2
Source # 3
Source # ns

Series of sequential shots

Receiver position (#)

Total sequential time samples (#)
Sampling scheme: Random dithering
Sampling scheme: Random time-shifting
Sampling scheme: Random time-shifting

RM

Series of sequential shots

Source # 1

Source # 2

Source # 3

Source # ns

Total sequential time samples (#)

Receiver position (#)
Sampling scheme: Random time-shifting
Sampling scheme: Constant time-shifting
Sampling scheme: Constant time-shifting

Series of sequential shots
Sampling scheme: Constant time-shifting
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Experimental setup

- Three sampling schemes:
  - Random dithering
  - Random time-shifting
  - Constant time-shifting

- Fully sampled sequential data (a seismic line from the Gulf of Suez) with $N_s = 128$ sources, $N_r = 128$ receivers, and $N_t = 512$ time samples

- Subsampling ratio, $\gamma = 0.5$

- Recover prestack data from simultaneous data
  - $\ell_1$ minimization
  - sparsifying transform: 3-D curvelets

- All sources see the same receivers
  - marine acquisition with ocean-bottom nodes
**Algorithm**

- Fully sampled sequential data
- Restricted simultaneous-acquisition sampling matrix
- Sparsifying transform: Curvelet
- Compressive sampling matrix
- Compressively sampled measurements
- Recover sparsest set of curvelet coefficients
- Sequential data recovery

\[
d \quad \rightarrow \quad RM \quad \rightarrow \quad C \quad \rightarrow \quad A = RMC^H \quad \rightarrow \quad b = RMd
\]

\[
\tilde{x} = \arg \min_x \|x\|_1 \quad \text{s.t.} \quad Ax = b
\]

\[
\tilde{d} = C^H \tilde{x}
\]
Curvelets
Detect the wavefronts
Original data
(Sequential acquisition)
Sparsity-promoting recovery: Random dithering

SNR = 10.5 dB
Conventional recovery: Random time-shifting

SNR = 5.04 dB
Sparsity-promoting recovery: Random time-shifting
SNR = 9.52 dB
Sparsity-promoting recovery: Constant time-shifting

SNR = 4.80 dB
Simultaneous acquisition is a linear subsampling system

Critical for reconstruction quality:

- design of source subsampling schemes (i.e., acquisition scenarios)
- appropriate sparsifying transform
- sparsity-promoting solver

Conclusions
Future plans

- Extensions to simultaneous acquisition frameworks for *towed streamer surveys*

- Use different transforms for *sparsity*-promoting processing


References


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Thank you!

slim.eos.ubc.ca