Sparsity-promoting recovery from simultaneous data: a compressive sensing approach

Haneet Wason*, Tim T. Y. Lin, and Felix J. Herrmann
September 19, 2011
Motivation

Conventional recovery

SNR = 5.04 dB
Motivation

Sparsity-promoting recovery

SNR = 9.52 dB
Motivation

[X] Conventional recovery
SNR = 5.04 dB

✓ Sparsity-promoting recovery
SNR = 9.52 dB
Motivation

- Opportunity to rethink Marine acquisition
- Concentrate on simultaneous sourcing
- Marine acquisition with ocean-bottom nodes
Outline

- Compressed sensing (CS) overview
  - design
  - recovery

- Design of simultaneous marine acquisition

- Experimental results of sparsity-promoting processing
Problem statement

Solve an underdetermined system of linear equations:

\[Ax_0 = b\]

where:
- **data** (measurements/observations): \(b \in \mathbb{C}^n\)
- **unknown**: \(x_0 \in \mathbb{C}^P\)
- **linear system**: \(A \in \mathbb{C}^{n \times P}\)
Compressed sensing

- acquisition paradigm for sparse signals
- in some transform domain

\[ d \in \mathbb{R}^N \]

\[ \mathbf{S} : \text{transform matrix} \]

\[ \mathbf{S} \in \mathbb{C}^{P \times N} \]

\[ P \geq N \]

\[ \mathbf{x}_0 \in \mathbb{C}^P \]
Compressed sensing

- acquisition paradigm for sparse signals
- in some transform domain

\[ \tilde{d} \approx S^H \tilde{x} \]

approximate \( d \)

\( k \) - largest coefficients
Bigger picture

b
d

series of sequential shots
Bigger picture

\[ b = \text{Simultaneous measurement matrix} \]

Source # 1

Source # 2

Source # 3

Source # n

Total sequential time samples (#)

Receiver position (#)

Source # ns

Receiver position (m)
Bigger picture

\[ b = \text{Simultaneous measurement matrix} \times d \]

\[ \tilde{d} \approx S^H \tilde{x} \]
Bigger picture

\[ \mathbf{b} = (\text{Simultaneous measurement matrix}) \times \mathbf{d} \]
\[ \tilde{\mathbf{d}} \approx \mathbf{S}^H \tilde{\mathbf{x}} \]
\[ \times (\text{Simultaneous measurement matrix}) \]
\[ \times \mathbf{S}^H \tilde{\mathbf{x}} \]
\[ \bm{b} = \text{Simultaneous measurement matrix} \times \bm{d} \]

\[ \tilde{\bm{d}} \approx \bm{S}^H \tilde{\bm{x}} \]

\[ \tilde{\bm{d}} \approx \text{Simultaneous measurement matrix} \times \bm{S}^H \tilde{\bm{x}} \]

\[ \boxed{\bm{A}} \]

\[ \tilde{\bm{x}} \]
Coarse sampling schemes

3-fold under-sampling

Fourier transform

few significant coefficients
detected

significant coefficients detected

[ Hennenfent & Herrmann, '08 ]
Mutual coherence

- measures the orthogonality of all columns of $\mathbf{A}$

- equal to the maximum off-diagonal element of the Gram matrix
  - controlled by compressive sensing via a combination of
    - randomization with
    - spreading of sampling vectors in the sparsifying domain
Restricted isometry property

- indicates whether every group of $k$ columns of $A$ are nearly orthogonal

- restricted isometry constant $0 < \delta_k < 1$ for which

\[
(1 - \delta_k)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta_k)\|x\|_2^2
\]
Sparse recovery

Solve the convex optimization problem (one-norm minimization):

\[ \tilde{x} = \text{arg min}_{x} \|x\|_1 \quad \text{subject to} \quad Ax = b \]

“sparsity”

data-consistent amplitude recovery

|Sparsity-promoting solver: \(\text{SPG}_{\ell_1}\)|

Recover single-source prestack data volume: \(\tilde{d} = S^H\tilde{x}\)

[van den Berg and Friedlander, '08]
Outline

- Compressed sensing (CS) overview
  - design
  - recovery

- Design of simultaneous marine acquisition

- Experimental results of sparsity-promoting processing
Simultaneous acquisition matrix

For a seismic line with $N_s$ sources, $N_r$ receivers, and $N_t$ time samples, the sampling matrix is

$$RM$$

samples recorded at each receiver during simultaneous acquisition

samples recorded at each receiver during sequential acquisition

[Mansour et.al., ‘11]
Bigger picture

\[ b = \text{Simultaneous measurement matrix} \times d \]

\[ \tilde{d} \approx S^H \tilde{x} \]

\[ \tilde{x} \]
Bigger picture

\[ b = \begin{array}{c}
RM \\
\end{array} \times d \\
\]

\[ \tilde{d} \approx S^H \tilde{x} \]

\[ \begin{array}{c}
RM \\
\end{array} \times S^H \tilde{x} \]

"Compressive sampling matrix"
Sequential vs. simultaneous sources

Sampling scheme:
Random dithering

Sequential acquisition

Simultaneous acquisition
Sequential vs. simultaneous sources

Conventional survey time:
\[ t = N_s \times N_t \]

Sequential acquisition

Theoretical survey time:
\[ t = n_{st} \ll n_s \times N_t \]

Simultaneous acquisition

Sampling scheme:
Random dithering
Sampling scheme: Random dithering

RM

Source # 1
Source # 2
Source # 3
Source # ns

series of sequential shots

Total sequential time samples (#)

Receiver position (#)
Sampling scheme: Random dithering
Sampling scheme: Random time-shifting

![Graph showing the relationship between source position (m) and time (s).](image-url)
Sampling scheme: Random time-shifting

RM

Source # 1
Source # 2
Source # 3
Source # ns

Series of sequential shots

Total sequential time samples (#)

Receiver position (#)
Sampling scheme: Random time-shifting
Sampling scheme: Constant time-shifting

![Graph showing the relationship between source position (m) and time (s). The graph is a straight line, indicating a linear relationship.](image)
Sampling scheme: Constant time-shifting

RM

Source # 1
Source # 2
Source # 3
Source # ns

Series of sequential shots

Total sequential time samples (#)

Receiver position (#)
Sampling scheme: Constant time-shifting
Outline

- Compressed sensing (CS) overview
  - design
  - recovery
- Design of simultaneous marine acquisition
- Experimental results of sparsity-promoting processing
Experimental setup

- Three sampling schemes:
  - Random dithering
  - Random time-shifting
  - Constant time-shifting

- Fully sampled sequential data (a seismic line from the Gulf of Suez) with $N_s = 128$ sources, $N_r = 128$ receivers, and $N_t = 512$ time samples

- Subsampling ratio, $\gamma = 0.5$

- Recover prestack data from simultaneous data
  - $\ell_1$ minimization
  - sparsifying transform: 3-D curvelets

- All sources see the same receivers
  - marine acquisition with ocean-bottom nodes
Algorithm

1. Fully sampled sequential data
2. Restricted simultaneous-acquisition sampling matrix
3. Sparsifying transform: Curvelet
4. Compressive sampling matrix
5. Compressively sampled measurements
6. Recover sparsest set of curvelet coefficients
7. Sequential data recovery

\[ \tilde{x} = \arg \min_{x} \|x\|_1 \quad \text{s.t.} \quad Ax = b \]
\[ \tilde{d} = C^H \tilde{x} \]

\[ d \quad \text{RM} \quad C \quad A = RMC^H \quad b = RMd \]
Curvelets
Detect the wavefronts
Original data
(Sequential acquisition)
Sparsity-promoting recovery: Random dithering
SNR = 10.5 dB
Conventional recovery: Random time-shifting

SNR = 5.04 dB
Sparsity-promoting recovery: Random time-shifting

SNR = 9.52 dB
Sparsity-promoting recovery: Constant time-shifting

$\text{SNR} = 4.80 \text{ dB}$
Simultaneous acquisition is a linear subsampling system.

Critical for reconstruction quality:

- design of source subsampling schemes (i.e., acquisition scenarios)
- appropriate sparsifying transform
- sparsity-promoting solver
Future plans

- Extensions to simultaneous acquisition frameworks for *towed streamer surveys*

- Use different transforms for *sparsity*-promoting processing


References


Acknowledgements


E. van den Berg and M. Friedlander for SPGL1 (www.cs.ubc.ca/labs/scl/spgl1)

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08). This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BP, Chevron, ConocoPhillips, Petrobras, PGP, PGS, Total SA, and WesternGeco.
Thank you!

slim.eos.ubc.ca