Probing the extended image volume for seismic velocity inversion

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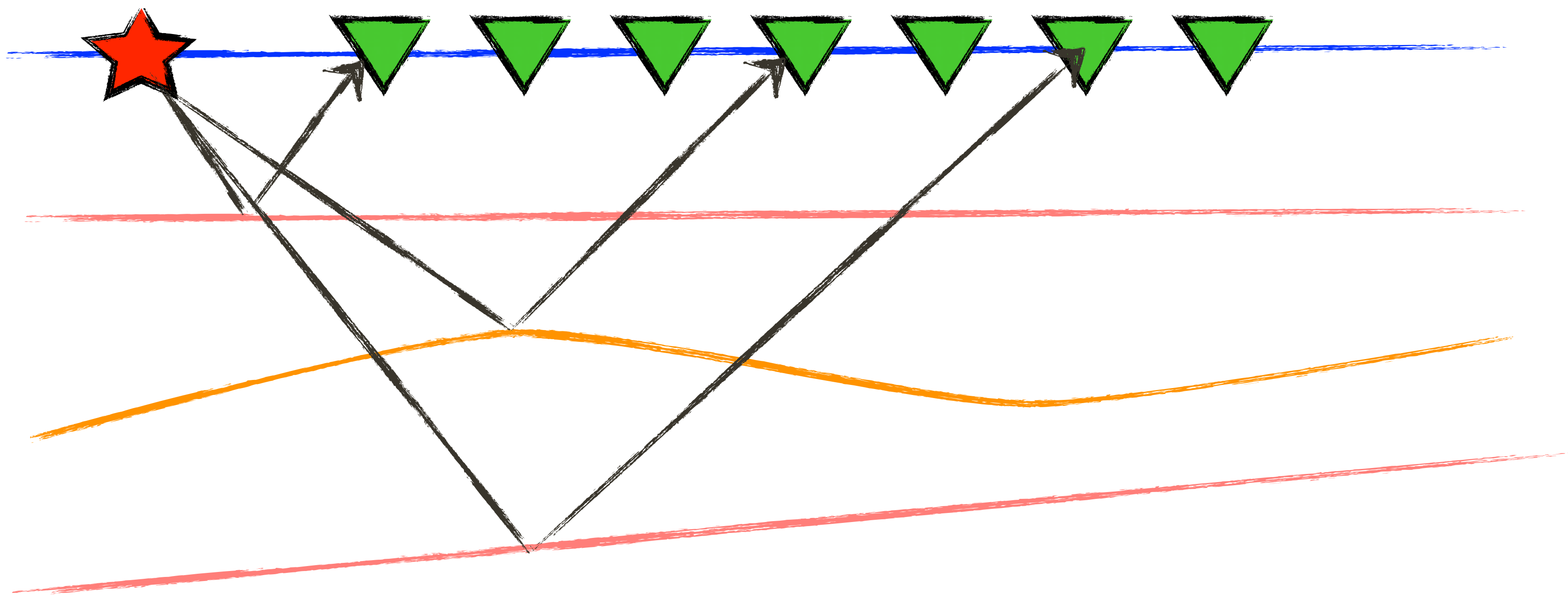
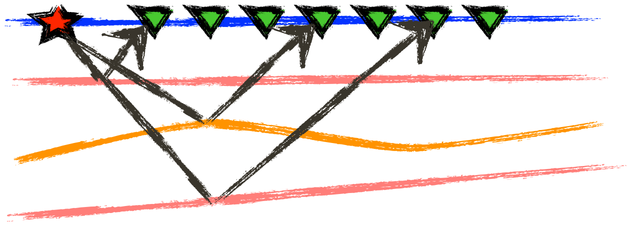
recover subsurface medium parameters from surface data



- *highly non-linear, local minima*
- *~10^9 unknowns*
- *~10^{12} data samples*
- *...*



Overview

- Full waveform inversion
- Imaging & velocity inversion
- Extended images
- Future work
- Conclusions

Full waveform inversion

Large scale non-linear LS problem

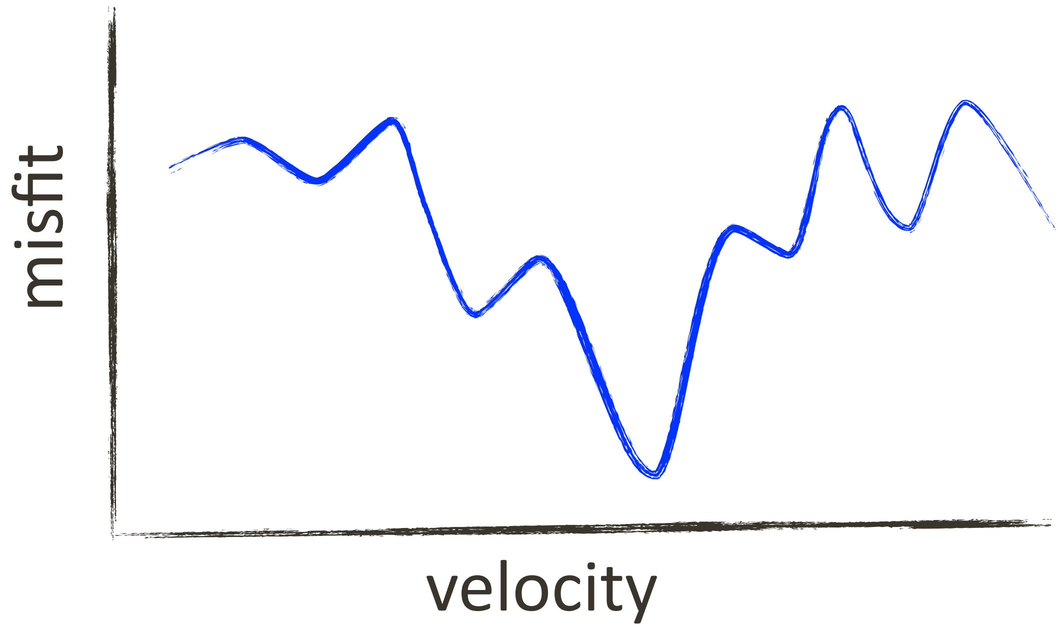
$$\min_{\mathbf{m}} \frac{1}{2} \sum_i ||\mathcal{F}_i[\mathbf{m}] - \mathbf{d}_i||_2^2 = \min_{\mathbf{m}} \frac{1}{2} ||\mathcal{F}[\mathbf{m}] - \mathbf{d}||_2^2$$

$$\mathcal{F}_i[\mathbf{m}] = P_i H_i[\mathbf{m}]^{-1} \mathbf{q}_i$$

$$H_i[\mathbf{m}] = \omega_i^2 \text{diag}(\mathbf{m}) + \nabla^2$$

Full waveform inversion

- Local minima
- Need *very* good initial guess



Full waveform inversion

Traditional approach based on separation of scales:

$$\text{find } \mathbf{m},\, \delta\mathbf{m} \text{ s.t. } \mathcal{F}_i[\mathbf{m}] + \nabla\mathcal{F}_i[\mathbf{m}]\delta\mathbf{m} \simeq \mathbf{d}_i$$

where $\mathbf{m}$ is smooth and $\delta\mathbf{m}$ is oscillatory

Imaging & velocity inversion

Imaging:

$$\delta\mathbf{m} \approx (\nabla\mathcal{F}^*\nabla\mathcal{F})^{-1}\nabla\mathcal{F}[\mathbf{m}]^*\underbrace{(\mathcal{F}[\mathbf{m}]-\mathbf{d})}_{\delta\mathbf{d}}$$

$\nabla\mathcal{F}$ is a GRT, Normal operator is diagonal in phase space.

[Beylkin '84; ten Kroode '98]

Imaging & velocity inversion

Velocity inversion:

$$\text{find } \mathbf{m} \text{ s.t } \delta\mathbf{d} \in \text{range}(\nabla\mathcal{F}[\mathbf{m}])$$

Projection onto the range:

$$\Pi = \nabla\mathcal{F}(\nabla\mathcal{F}^*\nabla\mathcal{F})^{-1}\nabla\mathcal{F}^*$$

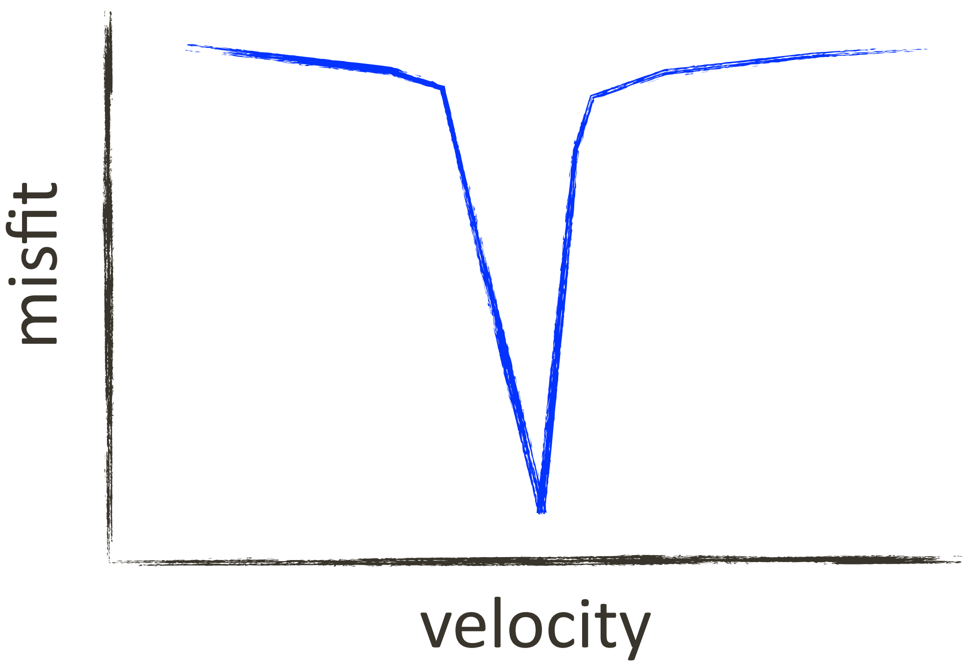
leads to `MBTT'

$$\min_{\mathbf{m}} ||(\Pi[\mathbf{m}] - I)\delta\mathbf{d}||_2^2$$

[Chavent '93; Clement '01]

Imaging & velocity inversion

range changes rapidly when model is perturbed



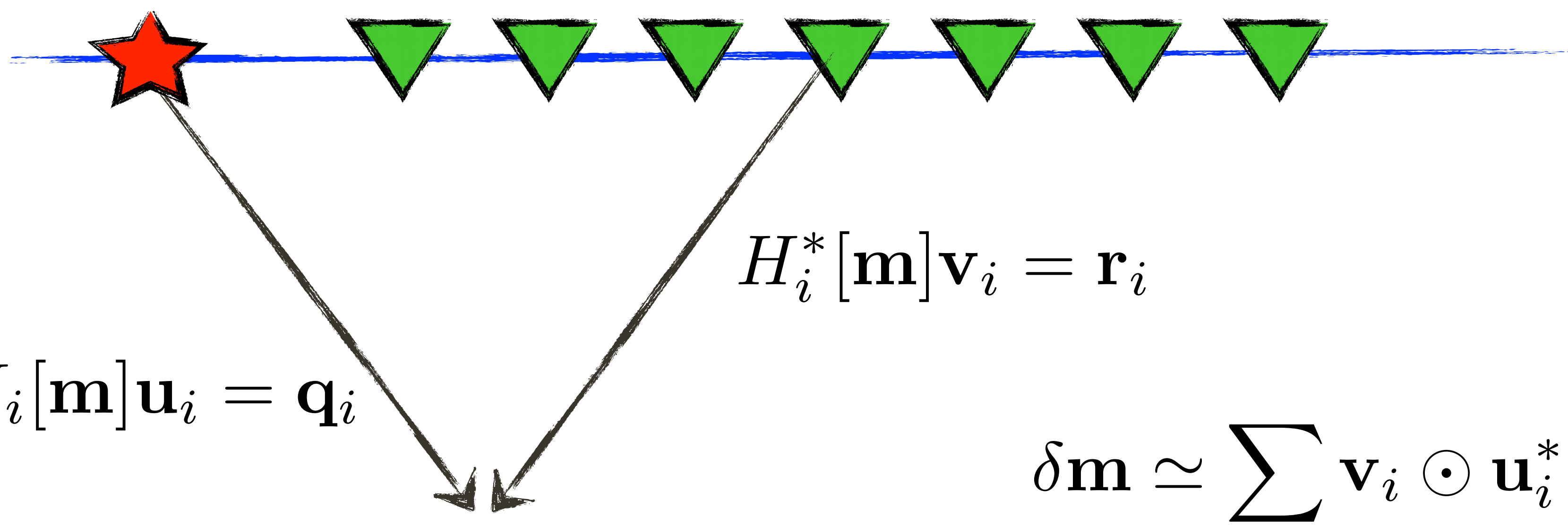
Imaging & velocity inversion

Suppose we have $\nabla\mathcal{F} = \widetilde{\nabla\mathcal{F}}\mathcal{B}$
where $\widetilde{\nabla\mathcal{F}}$ has `full range'.

Then we need to detect whether
$\widetilde{\nabla\mathcal{F}}^{*}\delta\mathbf{d}$ is in the range of $\mathcal{B}$....

Imaging & velocity inversion

"As I go forward, you go backward and somewhere we will meet" -Thom York, Radiohead.



$$H_i^*[\mathbf{m}]\mathbf{v}_i = \mathbf{r}_i$$

$$H_i[\mathbf{m}]\mathbf{u}_i = \mathbf{q}_i$$

$$\delta\mathbf{m} \simeq \sum_i \mathbf{v}_i \odot \mathbf{u}_i^*$$

Extended images

Extended image:

$$E[\mathbf{m}] = \sum_i \mathbf{v}_i \mathbf{u}_i^*$$

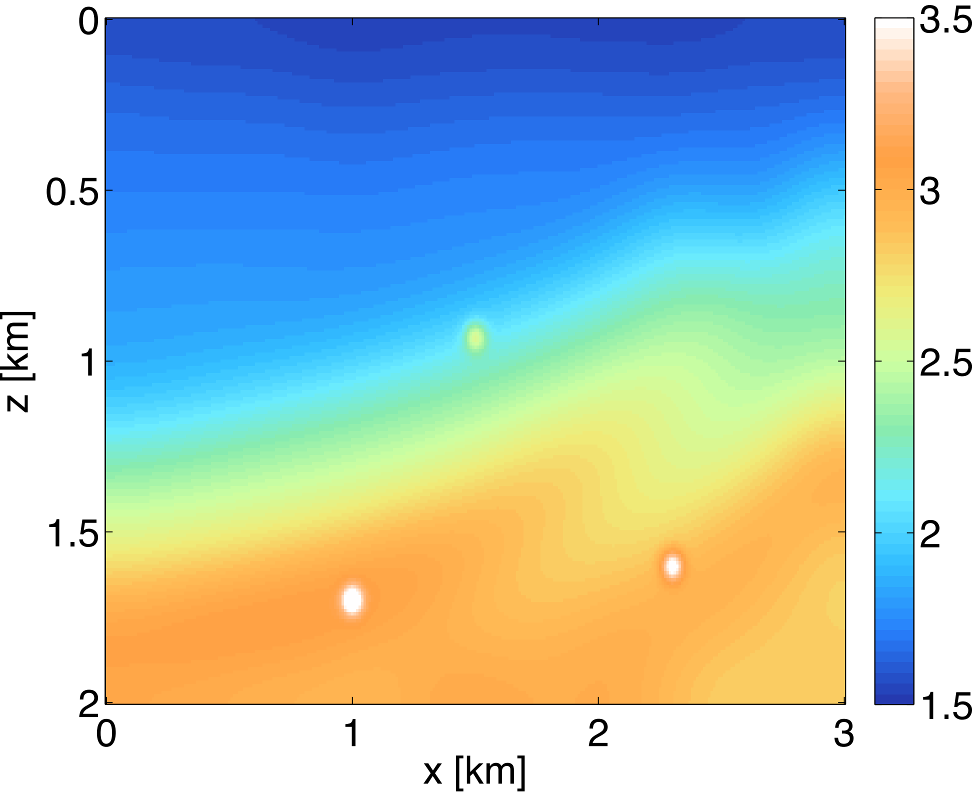
For the `correct' $\mathbf{m}$

$$E[\mathbf{m}] \simeq \mathrm{diag}(\delta \mathbf{m})$$

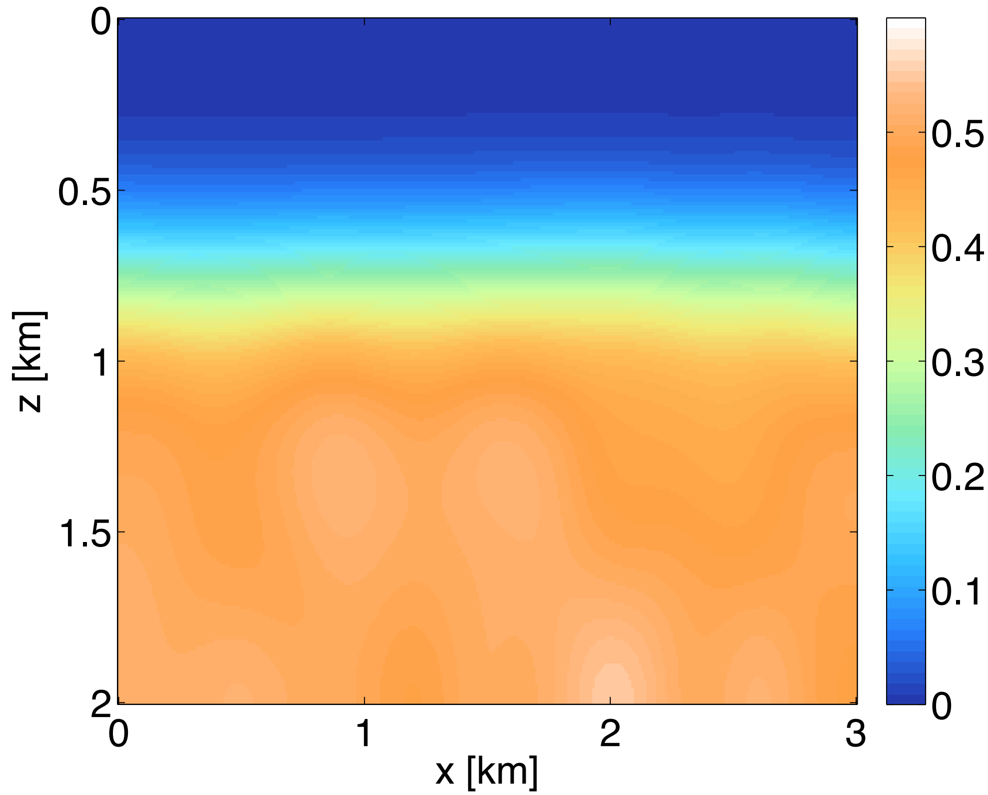
This defines $\widetilde{\nabla \mathcal{F}}$ and $\mathcal{B}$

[Claerbout '85; Rickett '02; Sava '04; Stolk '04]

Extended images



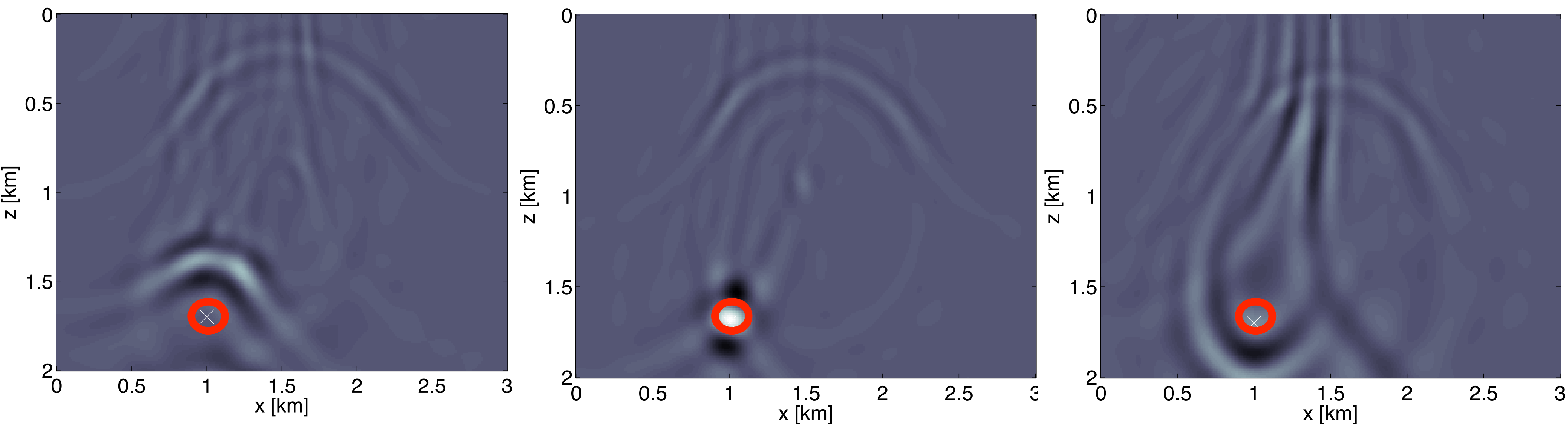
velocity [km/s]



perturbation [km/s]

Extended images

Slice through extended image

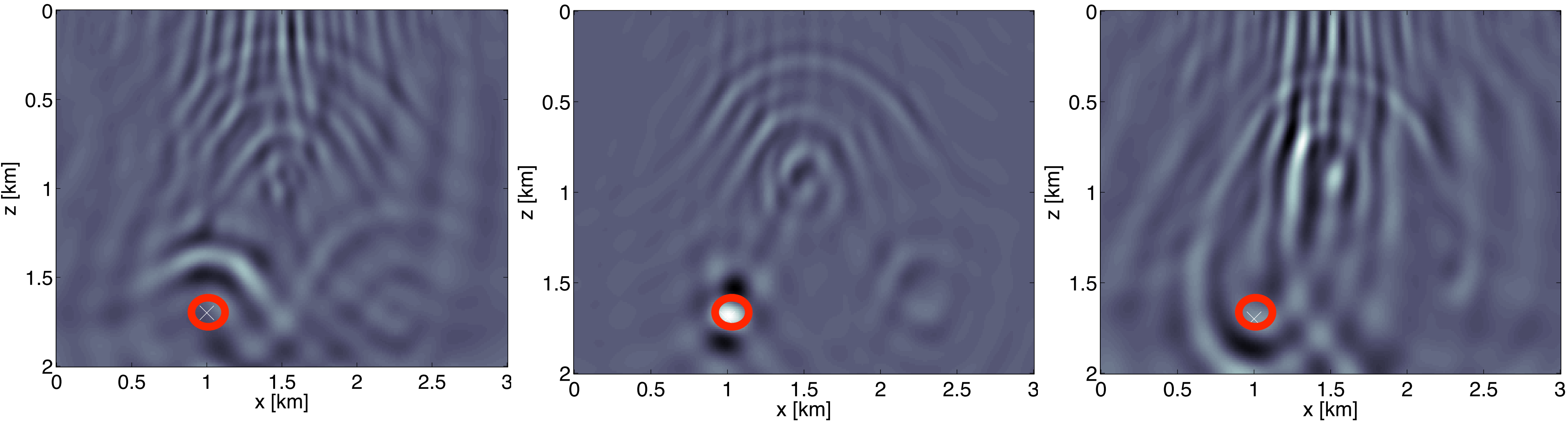


low

high

Extended images

Images for 10 simultaneous sources (less computation)



low

high

Extended images

Misfit criterion: weighted norm

$$||W \odot E||_?$$

- minimize off-diagonal energy:

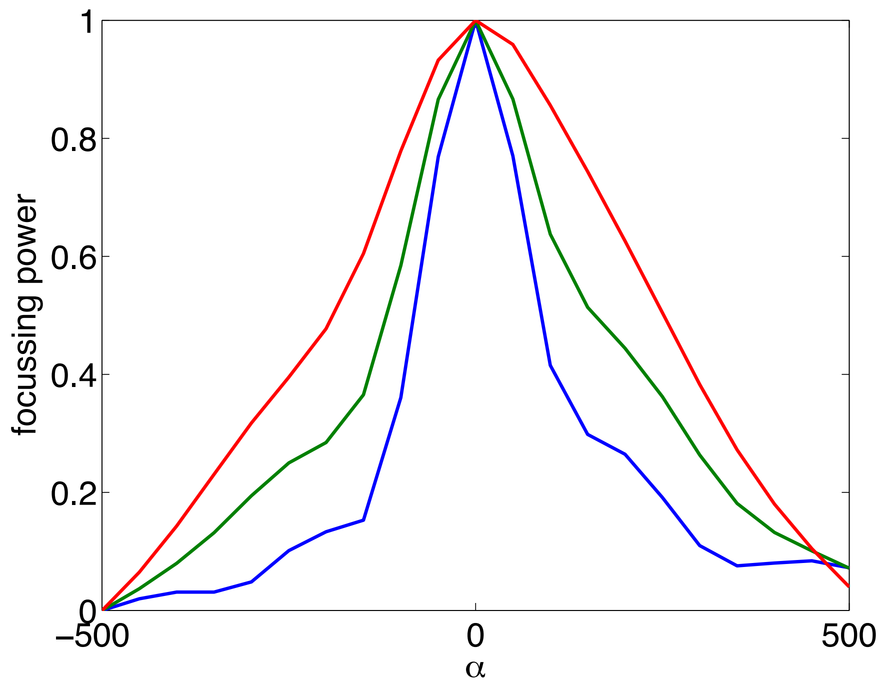
$$w_{ij} \propto (i - j)$$ [Stolk '04; Shen '08]

- maximize near-diagonal energy:

$$w_{ij} \propto e^{-\alpha(i-j)^2}$$ [TvL '08]

Extended images

focussing power for small, medium and large scale



Extended images

Computation of the image.

- Organize quantities as

$$U = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n]$$

then: $E = VU^*$

- matvec's via PDE solve:

$$E\mathbf{x} = VU^*\mathbf{x} = H^{-1}(R\mathbf{x})$$

[TvL '11]

Extended FWI

Introduce `extended' wave-equation: $\left[\omega^2 M + \nabla^2\right] \mathbf{u} = \mathbf{q}$

Inverse problem:

$$\min_{M} \rho(M) \quad \text{s.t.} \quad ||\widetilde{\mathcal{F}}[M] - \mathbf{d}||_2 \leq \sigma$$

[Symes '08]

Extended FWI

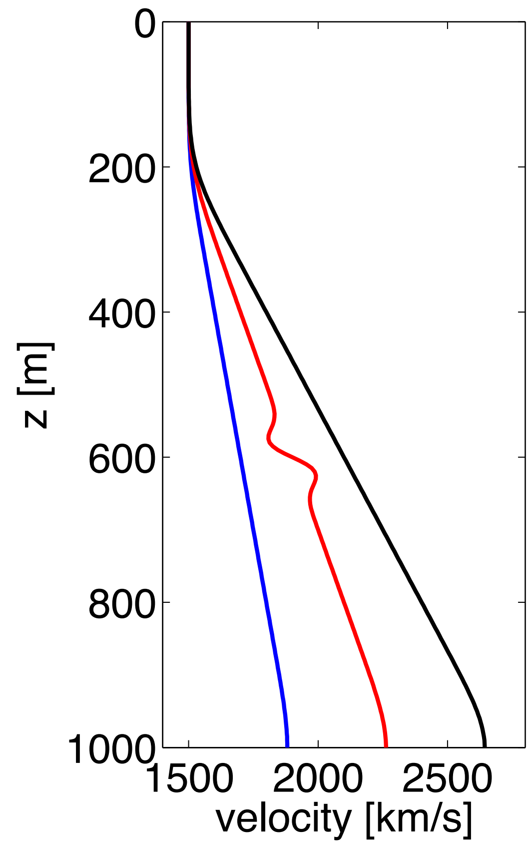
For layered models, M can be taken to be `Toeplitz'.

$$\left[\omega^2 \text{diag}(\hat{\mathbf{m}}) + \partial_z^2 - k_x^2\right] \hat{\mathbf{u}} = \hat{\mathbf{q}}$$

The model is now a function of depth and horizontal wavenumber

[Symes '08]

Extended FWI



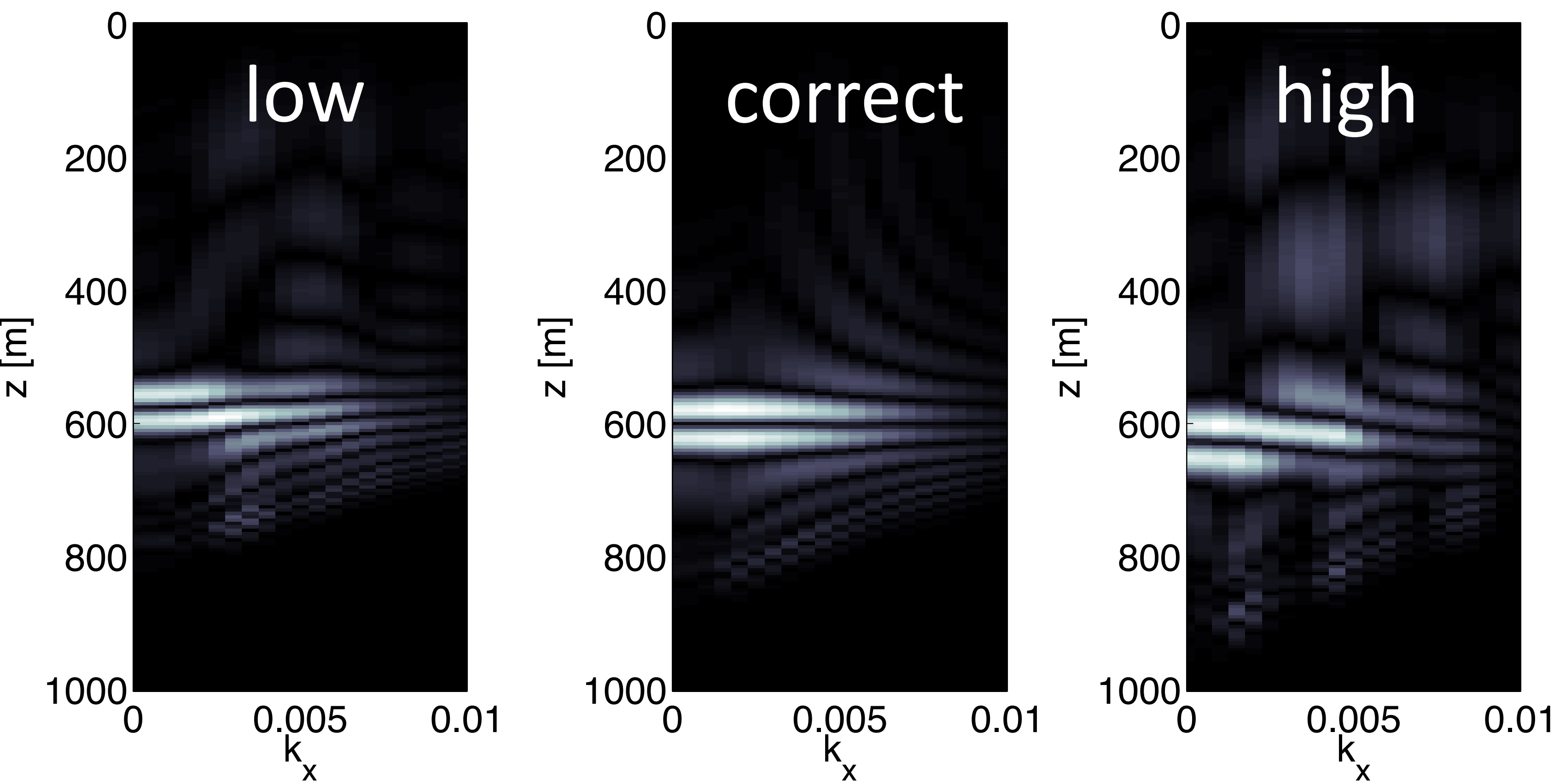
solve:

$$\min_{\widehat{\mathbf{m}}} ||\widetilde{\mathcal{F}}[\mathbf{m}_0 + \widehat{\mathbf{m}}] - \mathbf{d}||_2^2$$

for various reference models

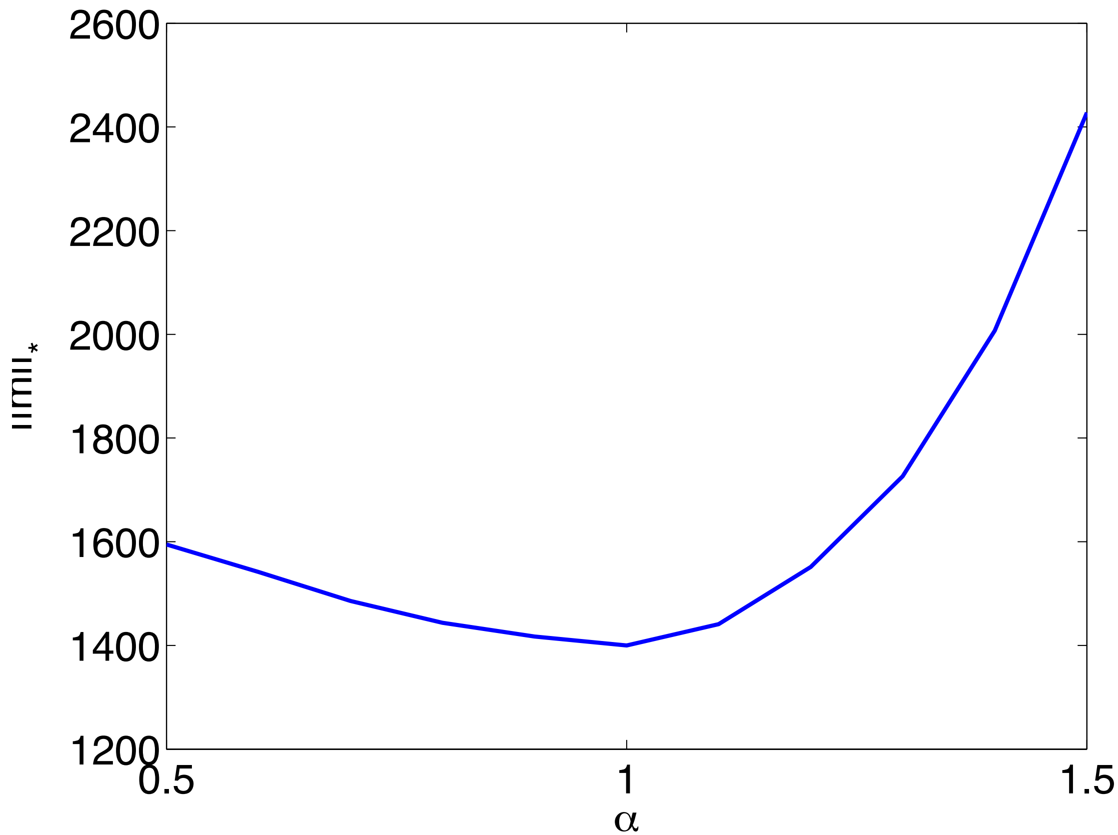
Extended FWI

reconstructed perturbations



Extended FWI

penalize variation in $\mathbf{k}_x$



Conclusions

- Extended modelling concept useful for solving the seismic inverse problem
- Efficient calculation of `image volumes’
- Can be incorporated in non-linear FWI formulation

Future work

- How to parametrize M and guarantee positivity
- How to best exploit `factored' form of updates
- How to define and compute penalty using only matvec's