Migration with surface-related multiples from incomplete seismic data

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SUMMARY

Seismic acquisition is confined by limited aperture that leads to finite illumination, which, together with other factors, hinders imaging of subsurface objects in complex geological settings such as salt structures. Conventional processing, including surface-related multiple elimination, further reduces the amount of information we can get from seismic data. With the growing consensus that multiples carry valuable information that is missing from primaries, we are motivated to exploit the extra illumination provided by multiples to image the subsurface. In earlier research, we proposed such a method by combining primary estimation and sparsity-promoting migration to invert for model perturbations directly from the total up-going wavefield. In this abstract, we focus on a particular case. By exploiting the extra illumination from surface-related multiples, we mitigate the effects caused by migrating from incomplete data with missing sources and missing near-offsets.

INTRODUCTION

In practice, multiples, especially surface-related multiples, are almost always removed in the early stage of processing, lest they introduce artifacts into seismic images. However, there is a growing consensus that instead of simply being temporally displaced replicas of the primaries, multiples actually provide extra illumination that may not be present in primaries. We removed them in the past because most migration methods are not able to handle multiples effectively. Because multiples travel more than once between reflectors and the surface, they contain higher spatial wavenumber components, especially at far offsets. Therefore, multiples provide wider illumination angles, and they are more sensitive to velocity changes (Verschuur, 2006). This property becomes advantageous when imaging subsalt structures, where far offset information is of prime importance while little knowledge can be obtained from near-offset data since salt bodies almost prohibit the near-offset seismic wave from reaching the structures underneath.

Various methods have emerged in the literature to address the effective use of multiples. Reiter et al. (1991) introduced a free surface to the background-velocity model so the Green's function can explain surface-related multiple reflections. Berkhout and Vershuur (1994) proposed a method that identifies the surface-related multiples as the response when the primary reflections at the surface act as an areal source. They later proposed another method by using focal transform (Berkhout and Verschuur, 2006) to map first-order multiples to primaries. He et al. (2007) used 3D wave-equation interferometric migration for VSP data containing free-surface multiples. These methods either violate the Born-scattering assumptions (Reiter et al., 1991), or are confined to certain migration methods (Berkhout and Vershuur, 1994) or certain types of data (He et al., 2007). However, these limitations can be overcome by inverting the multiple-generating operator to map the total up-going wavefield back to the surface-free Green's function from which migration can be performed.

The recently developed concept, EPSI (Estimation of Primaries via Sparse Inversion) (van Groenestijn and Verschuur, 2009a), has successfully set up a framework to invert the surface-free Green's function from the total up-going wavefield. In earlier research, we proposed to combine EPSI with sparsity-promoting migration to reap benefits from surface-related multiples (Lin et al., 2010; Tu et al., 2011). In this abstract, we concentrate on a particular aspect of this integral approach by addressing the case where the illumination is further limited because of missing sources and missing near-offsets in the data. Again, the information carried by surface-related multiples comes to our rescue enabling us to image regions that are not illuminated because of the missing data.

THEORY

In this section, we will discuss the formulation of robust EPSI, how we combine this approach with sparsity-promoting migration, and how we work with incomplete seismic data.

Robust EPSI

Verschuur et al. (1992) established the following monochromatic relationship between the up-going wavefield and the Green's function:

$$\overbrace{\hat{\mathbf{P}}}^{\text{upgoing}} = \hat{\mathbf{G}}\,\overbrace{(\hat{\mathbf{Q}} - \hat{\mathbf{P}})}^{\text{downgoing}}, \qquad (1)$$

where $\hat{\mathbf{G}}$ represents the Green's function, $\hat{\mathbf{P}}$ is the total up-going wavefield, and $\hat{\mathbf{Q}}$ is the source signature. We assume that the surface reflection operator $\mathbf{R}$ can be approximated by $-\mathbf{I}$ and that the source is stationary, i.e., $\hat{\mathbf{Q}} = \mathrm{q}(\omega)\mathbf{I}$. Hatted quantities represent monochromatic variables. Each bold upper-case variable is a matrix comprised of a single frequency slice of the wavefield. Two sources can be identified in this expression. There is a point source $\mathbf{Q}$ that maps $\mathbf{G}$ to primaries, and there is the down-going surface reflection $-\mathbf{P}$, which can also be treated as an areal source, mapping $\mathbf{G}$ to surface-related multiples. This is what is referred to as "double illumination" (Verschuur, 2011) when both primaries and surface-related multiples are used to image the subsurface. One thing that deserves our attention is that the internal-multiple generating response is contained in the $\mathbf{G}$ of equation (1).

Contrary to predicting multiples and then removing them in conventional methods like SRME (Verschuur et al., 1992), the EPSI approach (van Groenestijn and Verschuur, 2009a) inverts $\mathbf{G}$ directly from $\mathbf{P}$ by solving a ℓ_0-norm constrained optimization problem. Mathematically, the ℓ_0-norm of a vector is the number of non-zero entries. Unfortunately, this ℓ_0-norm minimization is non-convex and computationally prohibitive. In addition, the ℓ_0-norm is by definition sensitive to noise. To overcome these shortcomings, Lin and Herrmann (2010) proposed robust EPSI by replacing the ℓ_0-norm with the ℓ_1-norm. The ℓ_1-norm of a vector is the sum of the absolute value of each entry. A ℓ_1-norm constrained optimization problem can be solved with improved computational efficiency and robustness to noise. To facilitate further discussion, the EPSI operator $\mathbf{E}$ can be formulated mathematically in the canonical form of a linear operator acting on a vector:

$$\underbrace{\mathscr{F}_t^* \mathrm{Blockdiag}_{1\cdots \mathrm{n_f}}[(\hat{\mathbf{Q}}-\hat{\mathbf{P}})^* \otimes \mathbf{I}]\mathscr{F}_\mathbf{t}}_{\mathbf{E}}\,\mathbf{g} = \mathbf{p}, \qquad (2)$$

where lower case quantities $\mathbf{g}$ and $\mathbf{p}$ represent vectorized wavefields; $\mathscr{F}_t$ is the Fourier transform that operates along the time axis of the vectorized wavefield $\mathbf{g}$, and its adjoint operator $\mathscr{F}_t^*$ brings the wavefield back to the time domain; n_f is the number of frequencies. The block diagonal term varies over frequencies. The symbol $\otimes$ refers to the Kronecker product which turns matrix multiplications into matrix-vector multiplications. Equation (2) is conducive to a solution with sparsity promotion (Lin and Herrmann, 2010):

$$\tilde{\mathbf{g}} = \underset{\mathbf{g}}{\mathrm{argmin}}\,||\mathbf{g}||_1 \quad \text{subject to } ||\mathbf{p}-\mathbf{E}\mathbf{g}||_2 \leq \sigma. \qquad (3)$$

In this expression, we assume the seismic wavelet $\mathbf{Q}$ to be known. In practice, we also estimate $\mathbf{Q}$ before the EPSI operator is made. The effectiveness of EPSI has been repeatedly demonstrated (van Groenestijn and Verschuur, 2009b; Lin and Herrmann, 2010; Baardman et al., 2010) for the purpose of primary estimation and near-offset data reconstruction on real data. In this abstract, we take this success a step further by integrating EPSI into migration.

Combined EPSI and Sparsity-promoting migration

The success of EPSI compared to transform-based techniques is that it properly incorporates the physical information of the free surface, and consequently allows us to estimate the surface-free Green's function. By ignoring internal multiples, this surface-free Green's function can be related to model perturbations directly via the Born-scattering operator. Incorporating this scattering operator allows us to leverage curvelet-domain sparsity in the image space, which is by definition sparser because the image lives in a lower dimensional space. To leverage this image space sparsity, we solve the following sparsity promoting program:

$$\delta\tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta\mathbf{x}}{\mathrm{argmin}}\,||\delta\mathbf{x}||_1 \quad \text{subject to } ||\mathbf{RM}(\mathbf{g}-\mathbf{KS}^*\delta\mathbf{x})||_2 \leq \sigma \qquad (4)$$

where $\mathbf{S}^*$ denotes the curvelet synthesis operator, $\mathbf{RM}$ is a possible restriction operator that simulates data with missing sources or receivers in real acquisition, which is an identity matrix for complete data in the ideal case.

While sparsity-promoting migration by itself only admits multiple-free input, the formulation in equation (5) overcomes this shortcoming with the EPSI modeling operator that maps the surface-free Green's function to surface-related multiples. Since our background velocity model is smooth, we can identify the total up-going wavefield with the model perturbations. As a result, we can estimate $\delta\mathbf{m}$ from $\mathbf{p}$ with a σ adjusted to allow for misfit from internal multiples:

$$\delta\tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta\mathbf{x}}{\mathrm{argmin}}\,||\delta\mathbf{x}||_1 \quad \text{subject to } ||\mathbf{RM}(\mathbf{p}-\mathbf{EKS}^*\delta\mathbf{x})||_2 \leq \sigma. \qquad (5)$$

In this way, we use surface-related multiples in the hope of obtaining wider illumination angles. To demonstrate how surface-related multiples can contribute to the formation of the image, we migrate solely from surface-related multiples:

$$\delta\tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta\mathbf{x}}{\mathrm{argmin}}\,||\delta\mathbf{x}||_1 \quad \text{subject to } ||\mathbf{RM}(\mathbf{u}-\mathbf{UKS}^*\delta\mathbf{x})||_2 \leq \sigma, \qquad (6)$$

where $\mathbf{u}$ is the vectorized surface-related multiples, and U corresponds to the $-\mathbf{P}$ term in the EPSI operator that maps the Green's function to surface-related multiples:

$$\underbrace{\mathscr{F}_t^* \mathrm{BlockDiag}_\mathrm{f}[(-\hat{\mathbf{P}})^* \otimes \mathbf{I}]\mathscr{F}_\mathbf{t}}_{\mathbf{U}}\,\mathbf{g} = \mathbf{u}. \qquad (7)$$

Migration from incomplete data

Marine seismic surveys are confined by limited in-line and cross-line apertures due to finite number of hydrophones and streamers, as well as missing near-offsets and far-offsets. To illustrate how the extra illumination from surface-related multiples can be exploited by combining EPSI with migration, we migrate from incomplete data with missing sources and missing near-offsets. Incomplete data is made by applying the corresponding restriction operator to complete data. For now, however, this is a fictitious example since we still need the complete data to make the EPSI operator in equation (5) and (6). Overcoming this shortcoming is the subject of future research.

SYNTHETIC DATA EXAMPLES

A full synthetic data is made using a time domain finite-difference method from a synthetic salt-dome model (Verschuur, 2011). A free-surface together with a monopole source is used to generate surface-related multiples. Grid spacing in the model is 5m. A Ricker wavelet with a central frequency of 30Hz is used as the source. There are 128 sources and receivers placed at 5m below the free surface. Source and receiver spacings are both 20m. The physical size of the model is 1250m in depth by 5400m horizontally, while the sources and receivers only cover the horizontal range from 1420m to 3960m.

After the full data is simulated, an alternating optimization approach, namely robust EPSI, (Lin and Herrmann, 2010, 2011) is used to estimate both the Green's function and the source wavelet, as well as to separate primaries from surface-related multiples. Incomplete data for equation (4) to (6) are then made by applying a restriction operator to the Green's function, the full data, and the surface-related multiples respectively. However, we still assume that the full data is available to make the EPSI operator.

The restriction operator we use here is a continuous recording operator (a framework can be found in Berkhout (2008)) combined with a missing near-offset operator. The continuous recording operator randomly selects 16 shot-gathers from a total number of 128 sequential shot-gathers, randomly permutes their order, and puts each shot-gather randomly along a time axis that is 8X the length of a single shot-gather. The missing near-offset operator removes the data within 120 meters from the zero-offset. The incomplete data **P** of equation (5) is shown in Figure (1). Although this particular design is not directly applicable to towed-streamer marine seismic surveys, one can imagine a potential relation to continuous-recording ocean-bottom cable survey scenarios in shallow water, where near-offset data cannot be trusted.

We then migrate from the incomplete data according to equation (4) to (6). A Born-scattering operator is made with a time-harmonic (frequency domain) Helmholtz operator with absorbing boundary conditions. Frequency domain method is used for easier parallelization (Herrmann et al., 2009). A smooth background velocity is made by applying a 5-point moving-average filter to the true model for 10 times along the depth dimension. The parameters are chosen to be the same as what is used in the finite difference forward modelling. After migration, we combine the EPSI operator with the Born-scattering operator, and apply it to the recovered $\delta\mathbf{m}$ from equation (5) to recover the total up-going wavefield. The 64th shot gather of the recovered data is shown in Figure (1(b)).

The results of equation (4) to (6) are shown in Figure (2), from top to bottom, plotted on the same color scale. Comparing Figure (2(a)) with (2(b)), we can clearly see the improvement in the continuity in the reflectors, especially in the ocean-bottom reflector, brought by using surface-related multiples. Since the areal source that gives rise to the surface-related multiples is the total up-going wavefield, which provides more illumination than a point source, migration solely from the surface-related multiples also yields a image (Figure (2(c))) of higher fidelity than Figure (2(a)). In field seismic data, however, surface-related multiples have lower signal to noise ratio than primaries, therefore it is still usually beneficial to migrate from total up-going wavefields.

CONCLUSIONS AND DISCUSSION

By combining EPSI with sparsity-promoting migration in a joint-inversion process, we are able to reap extra illumination from surface-related multiples. In this abstract, we have demonstrated how this extra illumination can help to image the subsurface when there is missing sources and missing near-offsets in the data. This will lead to a conceptual transition from multiple removal to using multiples in future.

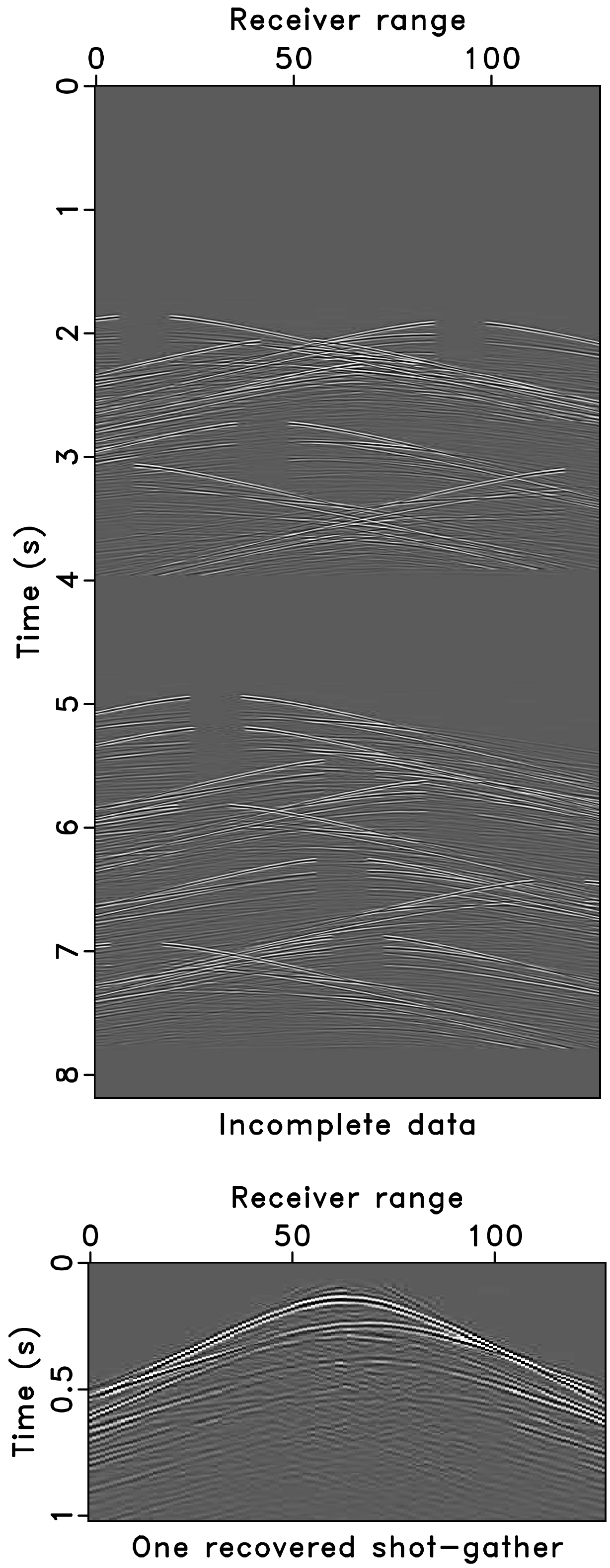

Figure 1: Upper: incomplete up-going wavefield; lower: the 64th shot-gather of the recovered data

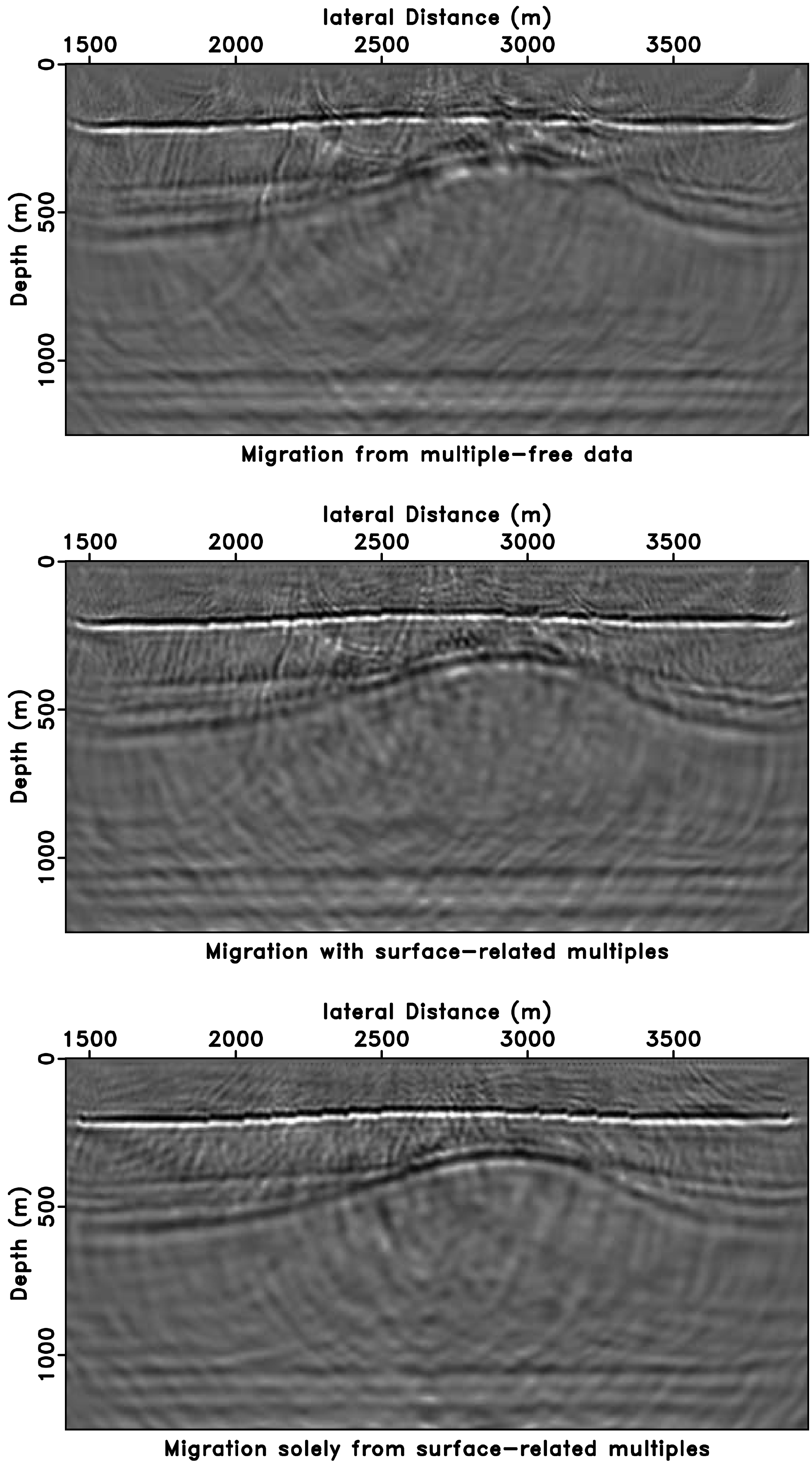

Figure 2: Migration from incomplete data. upper:migration from multiple-free data; middle: migration from data with surface-related multiples; lower: migration solely from surface-related multiples

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