Dimensionality-reduced estimation of primaries by sparse inversion

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Motivation

- Data-driven methods

  Estimation of Primaries by Sparse Inversion (EPSI)

- Curse of dimensionality

  In 3D these methods *suffer* from *exponential* growth in computational & storage *demands*

[Herrmann, F. J., 2010, Randomized sampling and sparsity]

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Objective

Reduction in *computational* and *storage* demands:

- *dimensionality*-reduction technique
- *adaptive* low-rank approximation
- *blackbox* model

Outline

- Estimation of Primaries by Sparse Inversion (EPSI)
- Dimensionality reduction via *Randomized* Singular Value Decompositions (r-SVD's)
- Results
- Conclusions

EPSI problem

recorded data   predicted data

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} + \hat{\mathbf{R}}\hat{\mathbf{P}})$$

$\hat{\mathbf{P}}$ total up-going wave-field

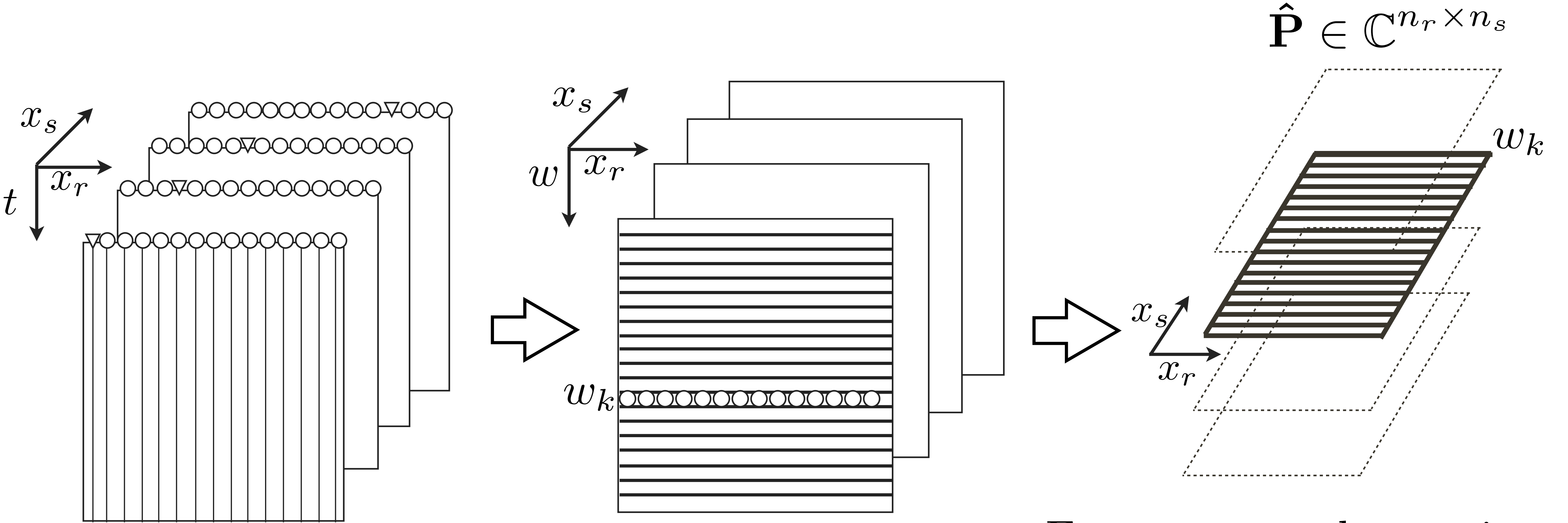
$\hat{\mathbf{Q}}$ down-going source signature

$\hat{\mathbf{R}}$ reflectivity of free surface

$\hat{\mathbf{G}}$ surface-free Green's function

[van Groenestijn and Verschuur, 2009]

Monochromatic “data matrices”



Fourier transform

Extract monochromatic shot records

[Ewoud van Dedem, 2002]

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EPSI problem

recorded data

predicted data

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

$\hat{\mathbf{P}}$ “low-rank” matrix (known)

$\hat{\mathbf{Q}}$ full-rank diagonal matrix (known)

$\hat{\mathbf{R}}$ assume $-\mathbf{I}$

$\hat{\mathbf{G}}$ unknown

EPSI problem

EPSI *linear* algebra format:

$$\mathbf{A}\mathbf{x} \approx \mathbf{b}$$

$$\tilde{\mathbf{x}} = \underbrace{\arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\mathbf{1}}}_{\text{sparsity promoting part}} \quad \text{subject to} \quad \underbrace{||\mathbf{A}\mathbf{x} - \mathbf{b}||_{\mathbf{2}} \leq \sigma}_{\text{data fitting part}}$$

EPSI problem

EPSI *linear* algebra format:

$$\underbrace{\mathbf{F}_t^* \begin{bmatrix} \left((\widehat{\mathbf{Q}} - \widehat{\mathbf{P}})_1^* \otimes \mathbf{I}\right) & & \\ & \ddots & \\ & & \left((\widehat{\mathbf{Q}} - \widehat{\mathbf{P}})_{n_f}^* \otimes \mathbf{I}\right) \end{bmatrix} \mathbf{F}_t}_{\widehat{\mathbf{U}}} \begin{bmatrix} \mathrm{vec}\left(\mathbf{G}_1\right) \\ \vdots \\ \mathrm{vec}\left(\mathbf{G}_{n_t}\right) \end{bmatrix} \approx \begin{bmatrix} \mathrm{vec}\left(\mathbf{P}_1\right) \\ \vdots \\ \mathrm{vec}\left(\mathbf{P}_{n_t}\right) \end{bmatrix}$$

Combination with sparsity promotion:

$\mathbf{A} = \widehat{\mathbf{U}}\mathbf{C}^* \quad \mathbf{C}$ is curvelet transform

$\mathbf{x}$: discrete curvelet representation of $\mathbf{G}$

$\mathbf{b}$: discrete representation of $\mathbf{P}$

EPSI problem

Data matrix $\hat{\mathbf{P}}$

- dense
- low-rank
- extremely large
  - each frequency is a $10^6 \times 10^6$ matrix where $n_r = n_s = 1000$
- expensive to access & store
- high mat-mat multiplication cost $O(N^3)$

EPSI problem

Challenges in solving the optimization problem

- multiple iterations
- multiple evaluations of $\mathbf{A}$, $\mathbf{A}^*$ and $\mathbf{A}^*\mathbf{A}$

Dimensionality-reduction via SVD

Approximate data matrix $\hat{\mathbf{P}}$ with *low-rank* factorization:

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

$$\hat{\mathbf{P}} \approx \mathbf{U\Sigma V}^*$$

$\mathbf{U_{n_r \times k}}$ left singular vectors

$\mathbf{\Sigma_{k \times k}}$ singular values

$\mathbf{V_{n_s \times k}}$ right singular vectors

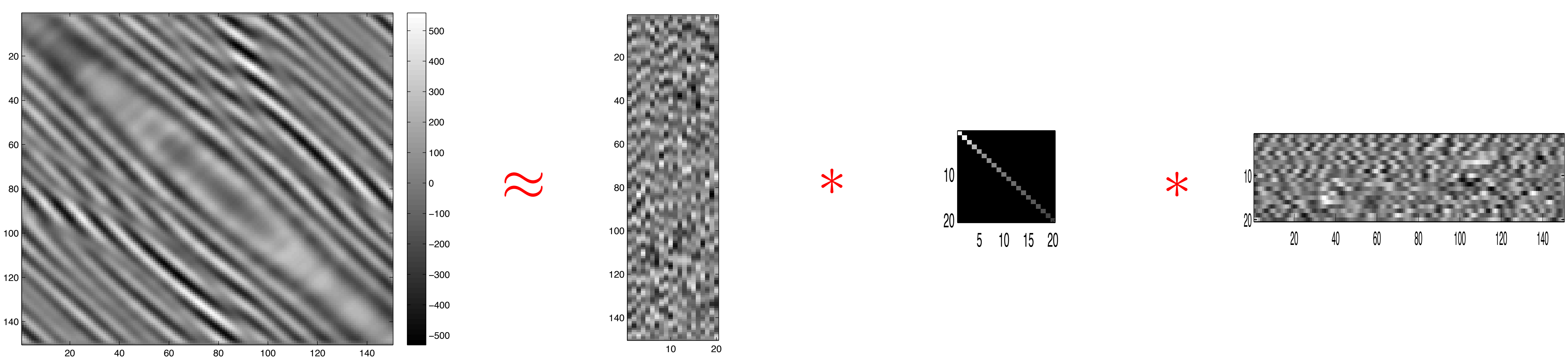
$\mathrm{k} :$ approximate rank

$k << min(n_r, n_s)$

Dimensionality Reduction Via SVD

Approximate data matrix $\hat{\mathbf{P}}$ with *low-rank* factorization:

$$\hat{\mathbf{P}} \approx \mathbf{U} * \mathbf{\Sigma} * \mathbf{V}^*$$



$n_r \times n_s$ $\quad$ $n_r \times k$ $\quad$ $k \times k$ $\quad$ $k \times n_s$

k : approximate rank

$k << min(n_r, n_s)$

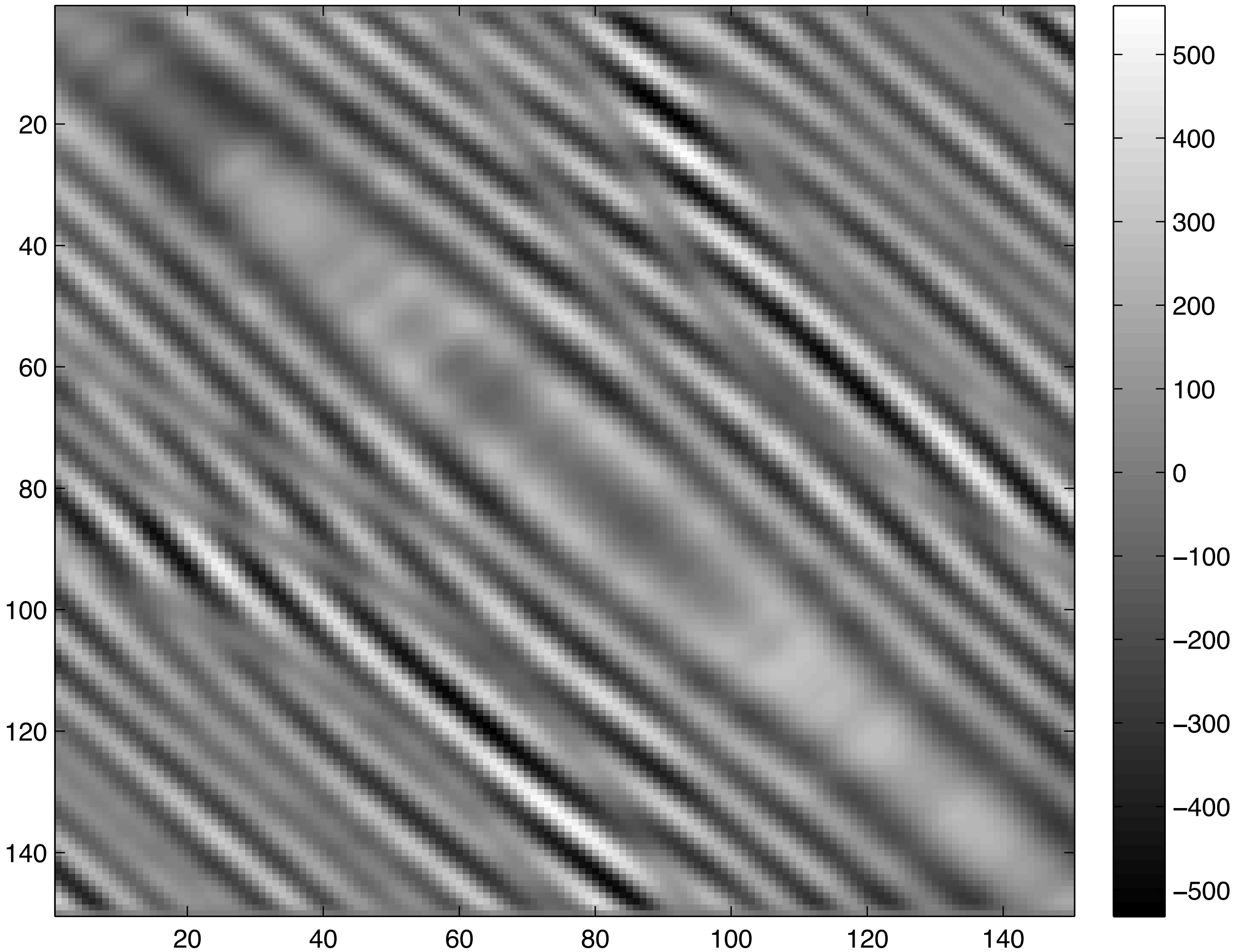
Dimensionality Reduction Via SVD

Advantages of using low rank factorization

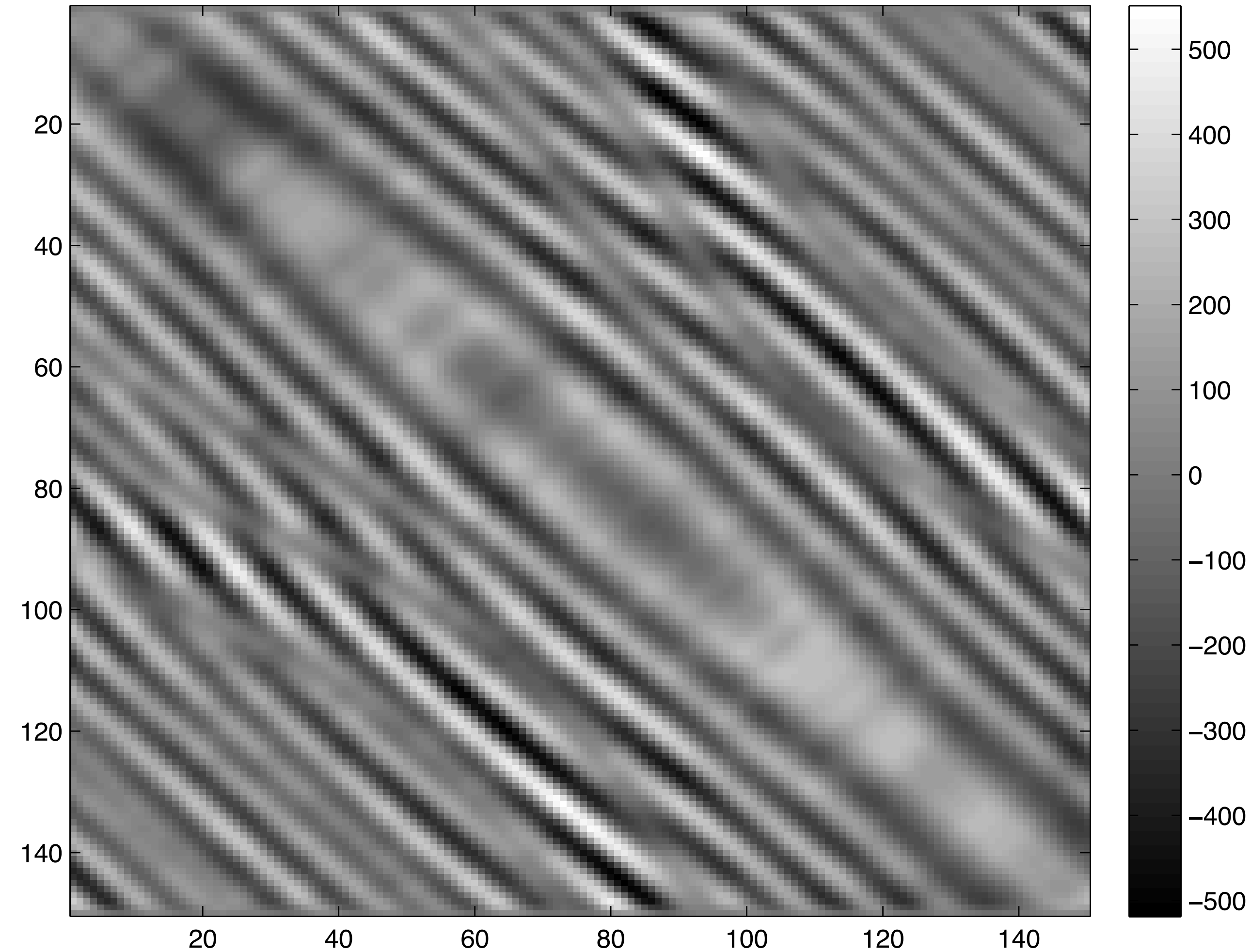
	Regular method	Low-rank approximation
Matrix-Matrix multiplication	$O(N^3)$	$O(kN^2)$
Storage (bytes)	$O(N^2)$	$O(2Nk + k^2)$

Full vs approximated data

$\hat{\mathbf{P}}$

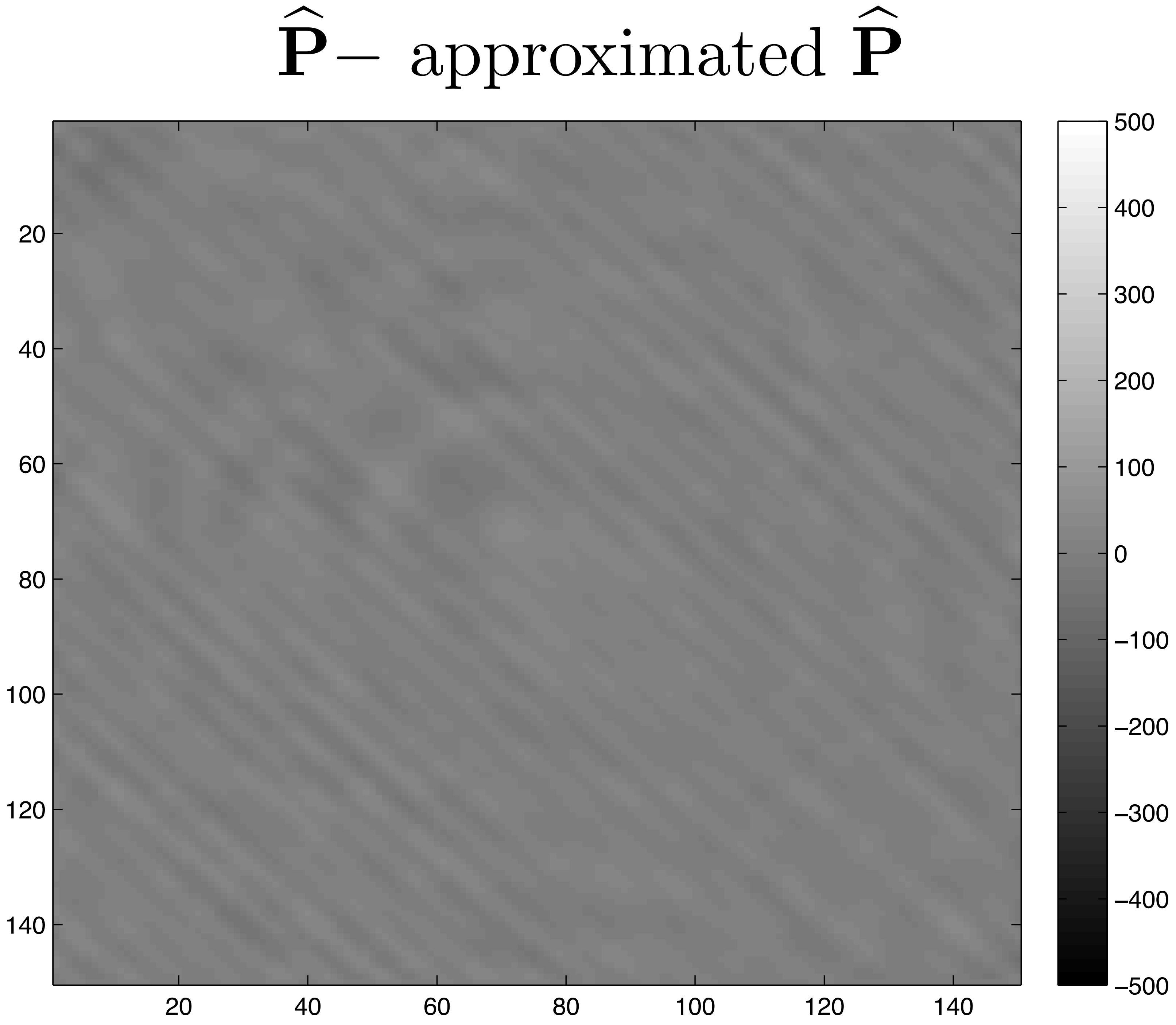


Approximated $\hat{\mathbf{P}}$



$$n_s = n_r = 150$$
$$k = 20 = 14\%$$
$$SNR = 16dB$$

Full vs approximated data



Multiplication speed up

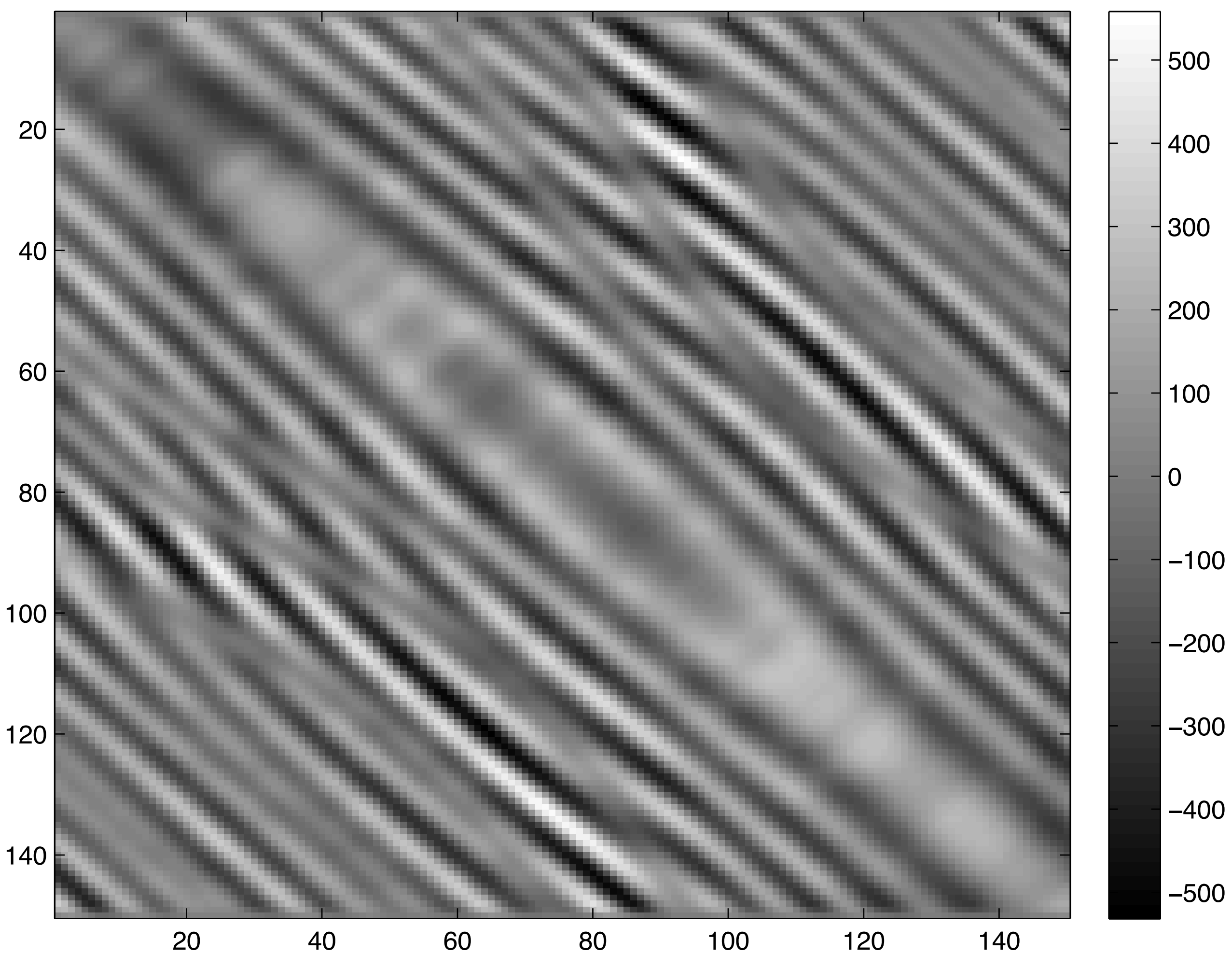
7.5 x

Memory usage

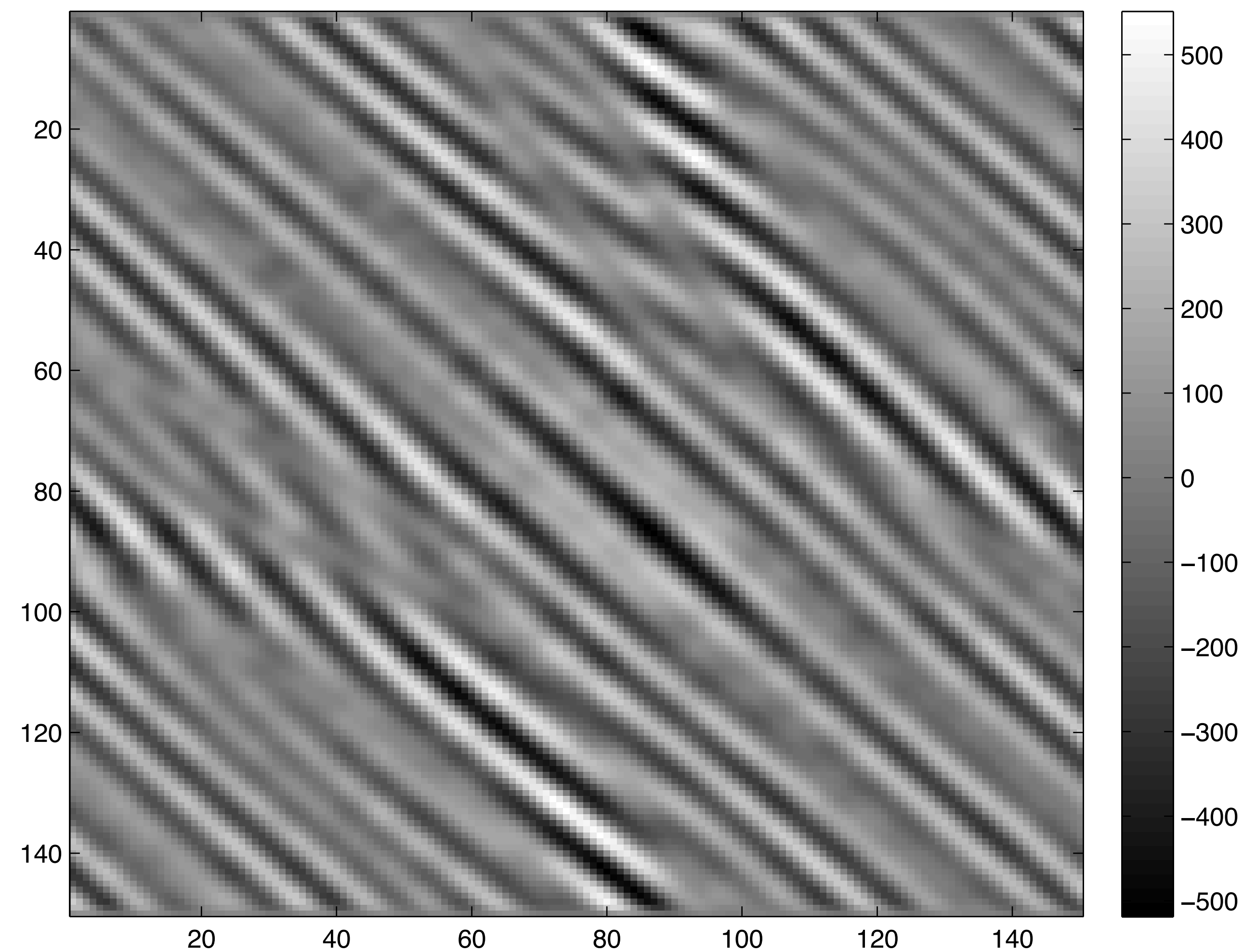
70% less

Full vs approximated data

$\hat{\mathbf{P}}$

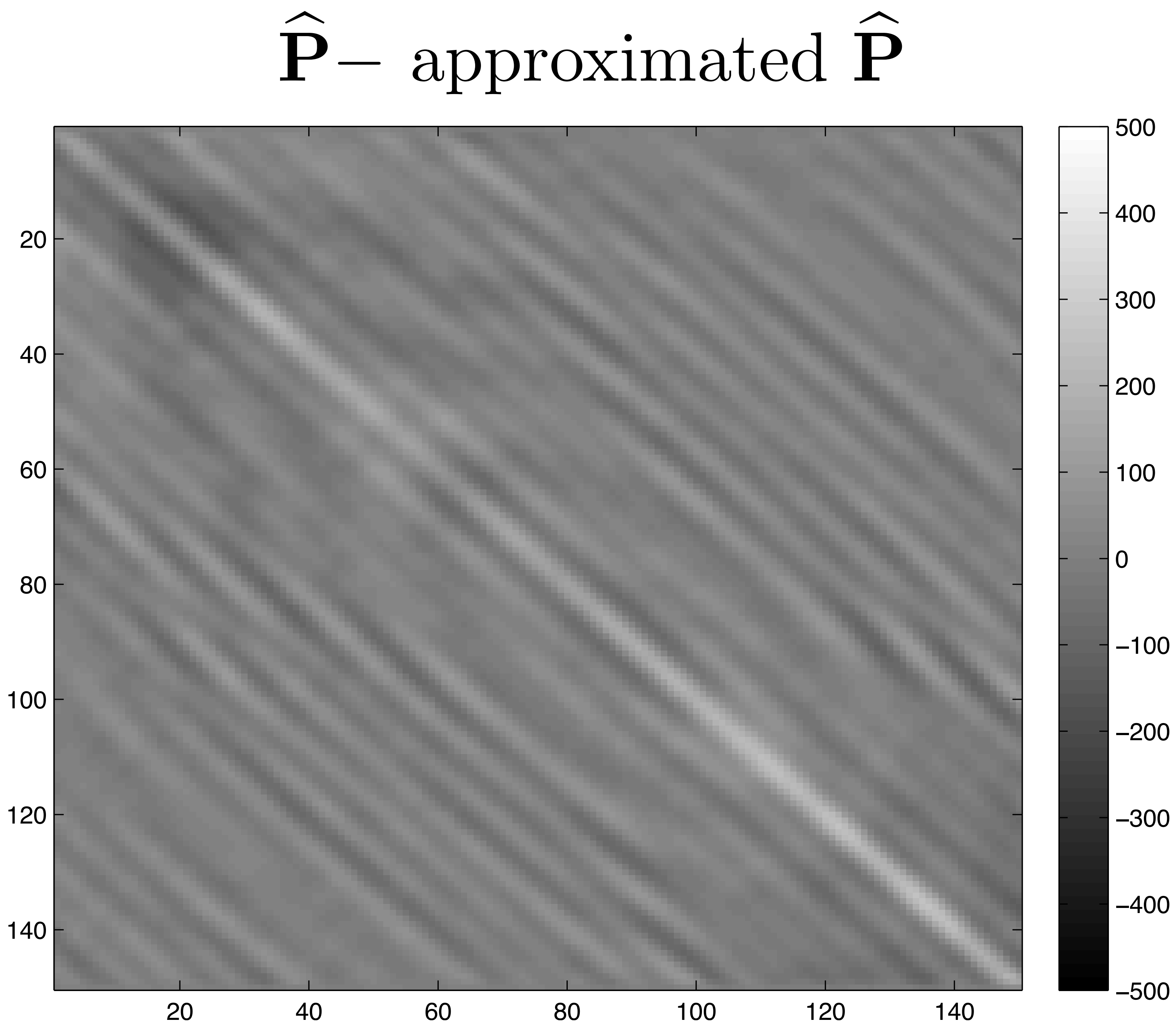


Approximated $\hat{\mathbf{P}}$



$$n_s = n_r = 150$$
$$k = 8 = 5\%$$
$$SNR = 8dB$$

Full vs approximated data



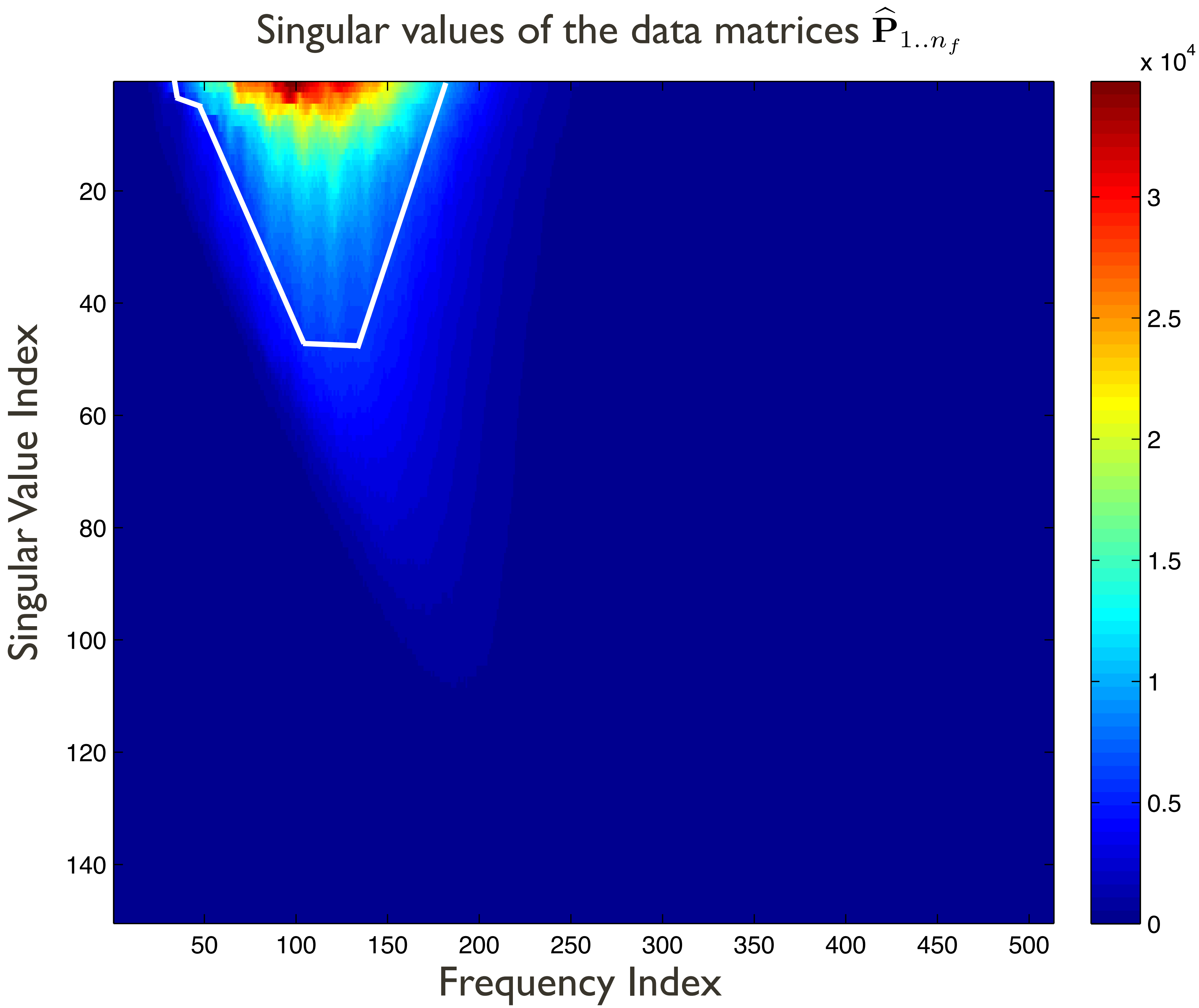
Multiplication speed up

20x

Memory usage

90% less

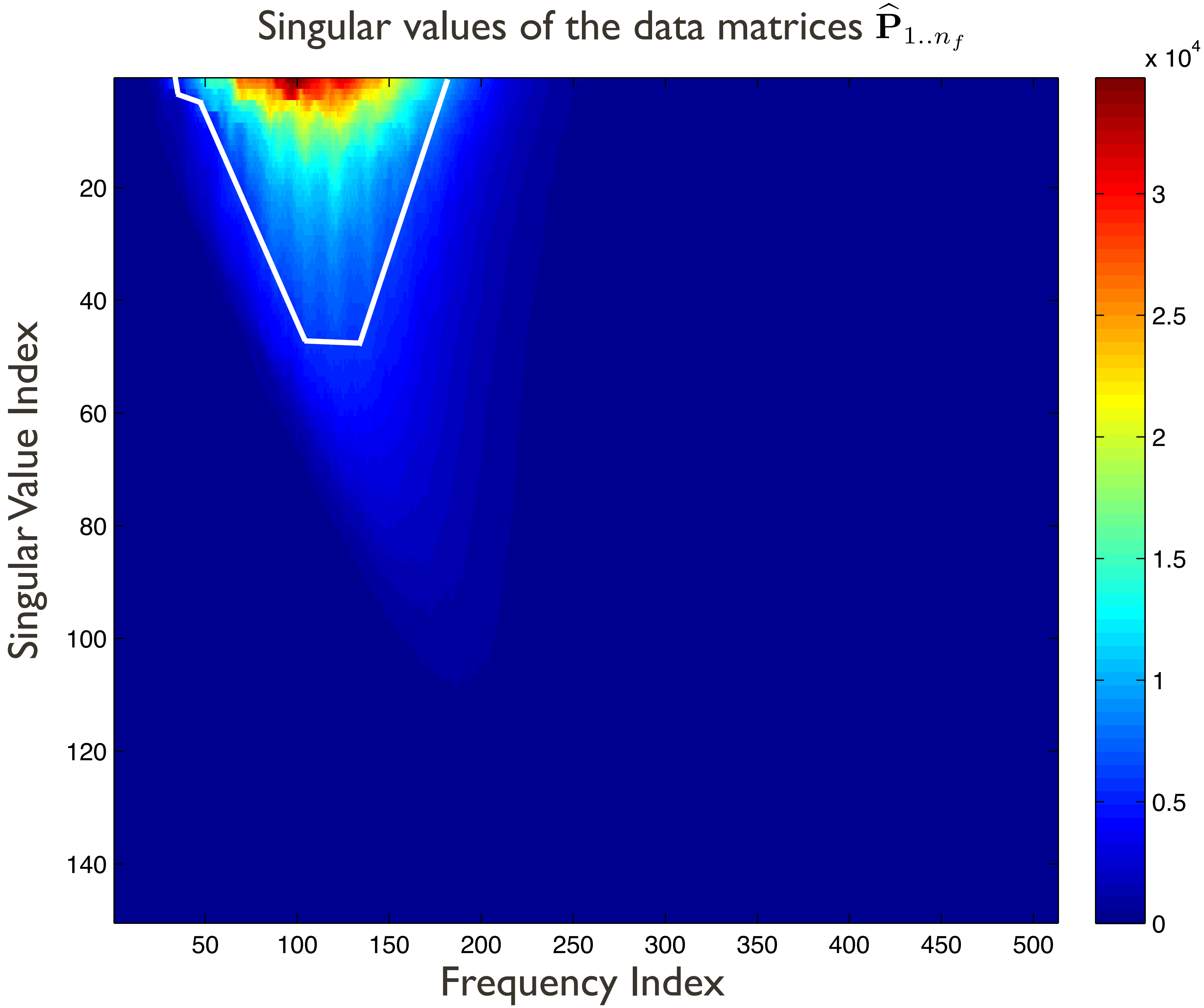
Singular values of the data matrix



Objective:

Approximate *all* frequency slices

Singular values of the data matrix



Challenge:

SVD IS SLOW

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Singular Value Decomposition (SVD)

Randomized SVD:

Requires action of data matrix on small number of *randomized* vectors (*simultaneous* shots)

Fast $O(mnlog(k))$

Classical SVD:

Slow $O(mnk)$

[Halko, N., P. G. Martinsson, and J. A. Tropp, 2011]

Singular Value Decomposition (SVD)

Randomized vs. Classical SVD

Example

# rows = # columns = 10000
k = 20% = 2000

SVD

$$O((10,000)^2 * 2000)$$

R-SVD

$$O((10,000)^2 * log(2000))$$ $200\times$ Faster !

Dimensionality Reduction Via *RSVD*

Two-stage approach:

1. *capture* action of the data $\hat{\mathbf{P}}$ matrix on $k+l$ *random vectors*

$$\hat{\mathbf{Y}} = \hat{\mathbf{P}}\hat{\mathbf{W}}$$

$\hat{\mathbf{W}}$: Gaussian random matrix

l is a small over sampling parameter (1-8)

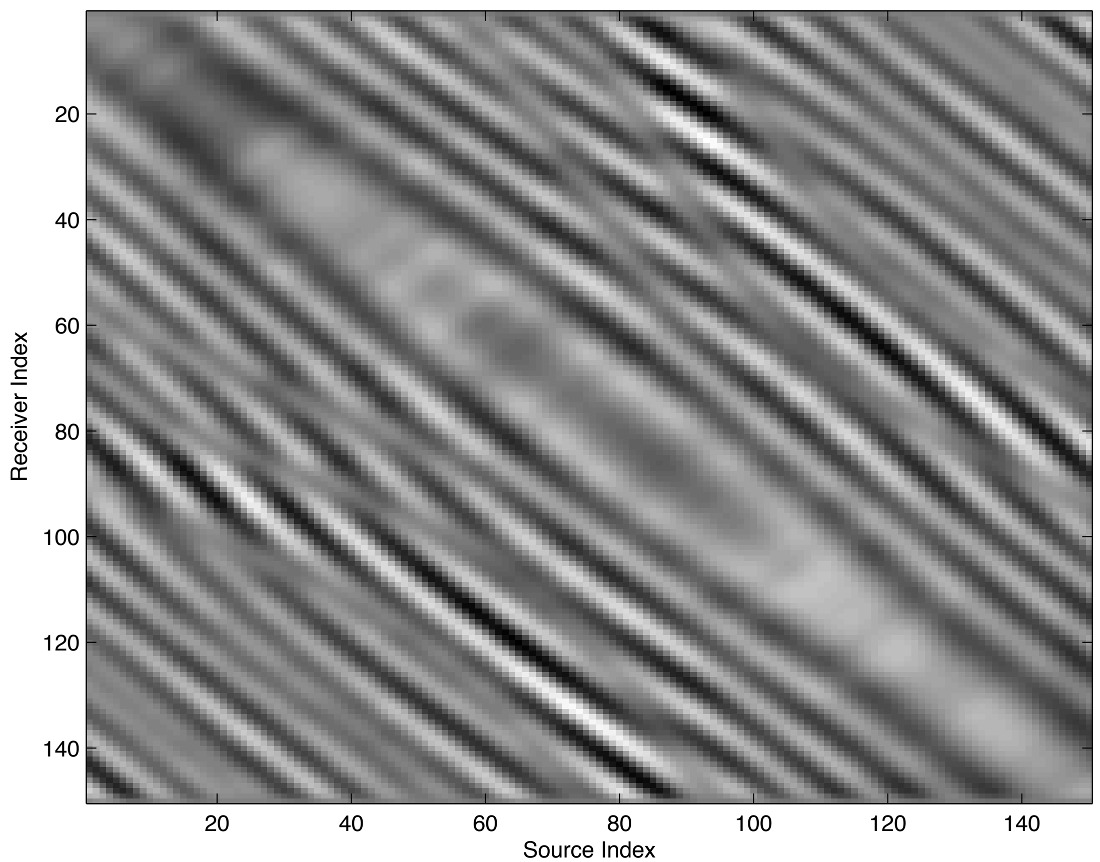
2. *form* a SVD on $\hat{\mathbf{Y}}$

[Halko, N., P. G. Martinsson, and J. A. Tropp, 2011]

Dimensionality Reduction Via *RSVD*

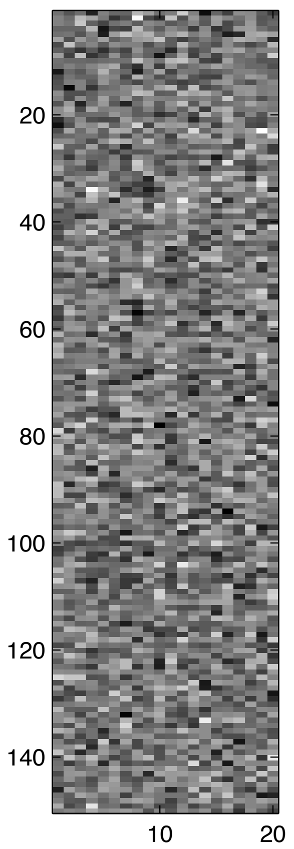
Stage 1: Capturing the action of $\hat{\mathbf{P}}$

$\hat{\mathbf{P}}$



$n_r \times n_s$

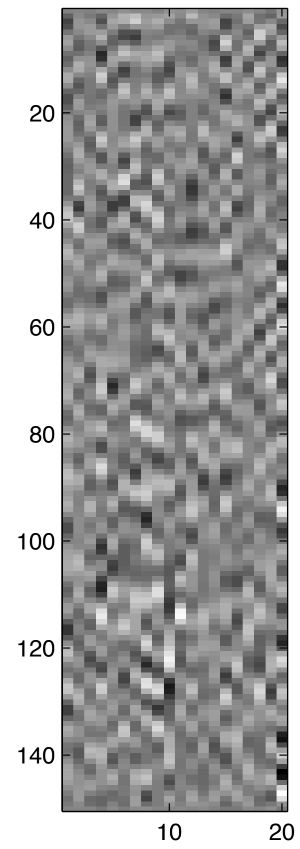
$\hat{\mathbf{W}}$



$n_s \times k$

$=$

$\hat{\mathbf{Y}} = \hat{\mathbf{P}}\hat{\mathbf{W}}$



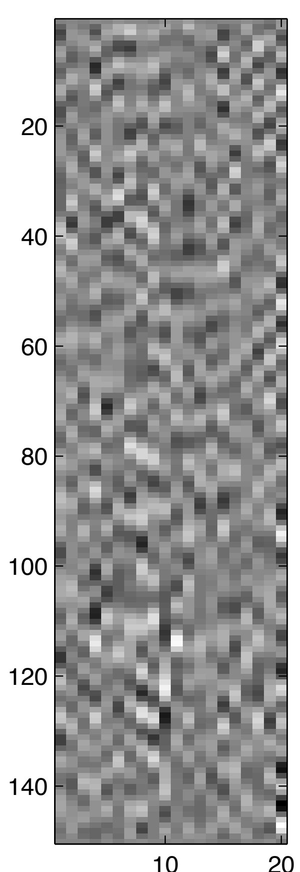
$n_r \times k$

Dimensionality Reduction Via *RSVD*

Stage 1: Capturing the action of $\hat{\mathbf{P}}$

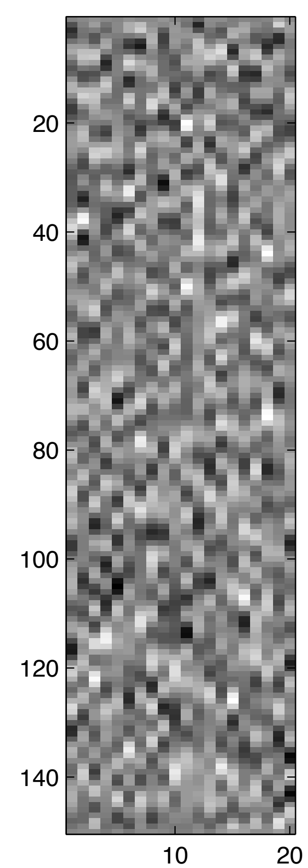
2. Form a low-rank QR factorization $\hat{\mathbf{Y}} \approx \mathbf{QR}$

$\hat{\mathbf{Y}}$ $\quad = \quad$ $\mathbf{Q}$ $\quad$ $\mathbf{R}$

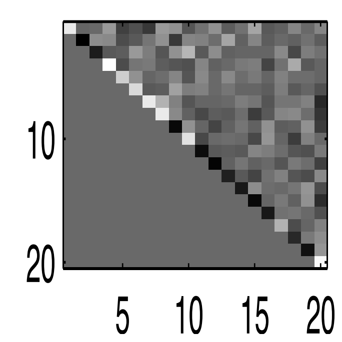


$n_r \times k$

$=$



$n_r \times k$



$k \times k$

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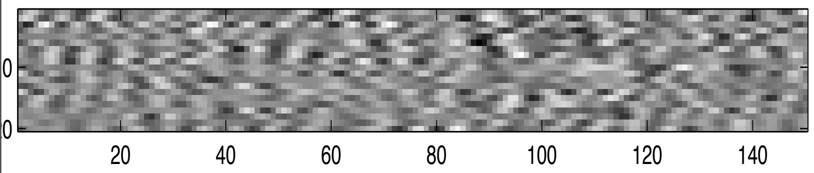
Dimensionality Reduction Via *RSVD*

Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

1. Form $\mathbf{B} = \mathbf{Q}^*\hat{\mathbf{P}}$

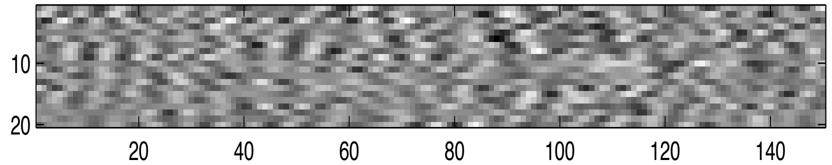
$\mathbf{B}$



$k * n_s$

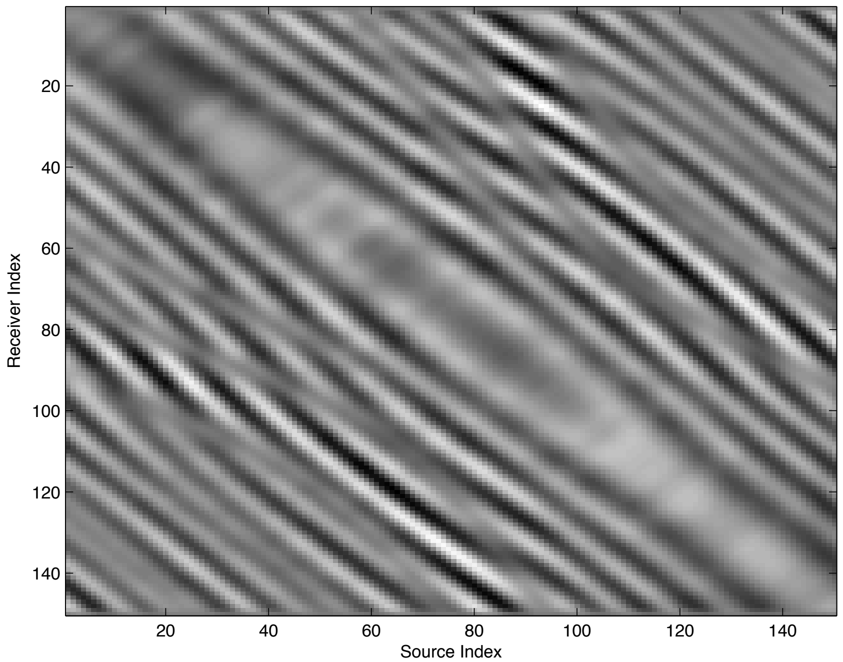
=

$\mathbf{Q}^*$



$k \times n_r$

$\hat{\mathbf{P}}$



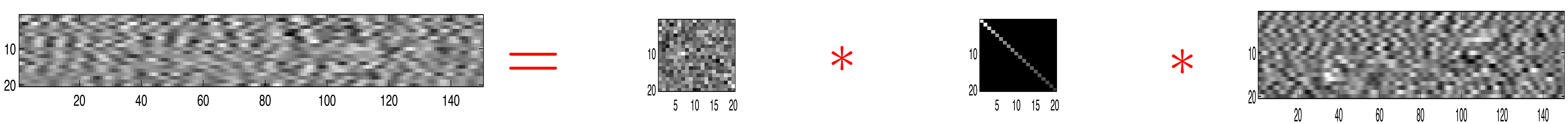
$n_r \times n_s$

Dimensionality Reduction Via *RSVD*

Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

2. Compute **SVD** of the small matrix $\mathbf{B} = \widetilde{\mathbf{U}}\mathbf{\Sigma}\mathbf{V}^*$

$\mathbf{B}$ $\quad$ $\widetilde{\mathbf{U}}$ $\quad$ $\mathbf{\Sigma}$ $\quad$ $\mathbf{V}^*$

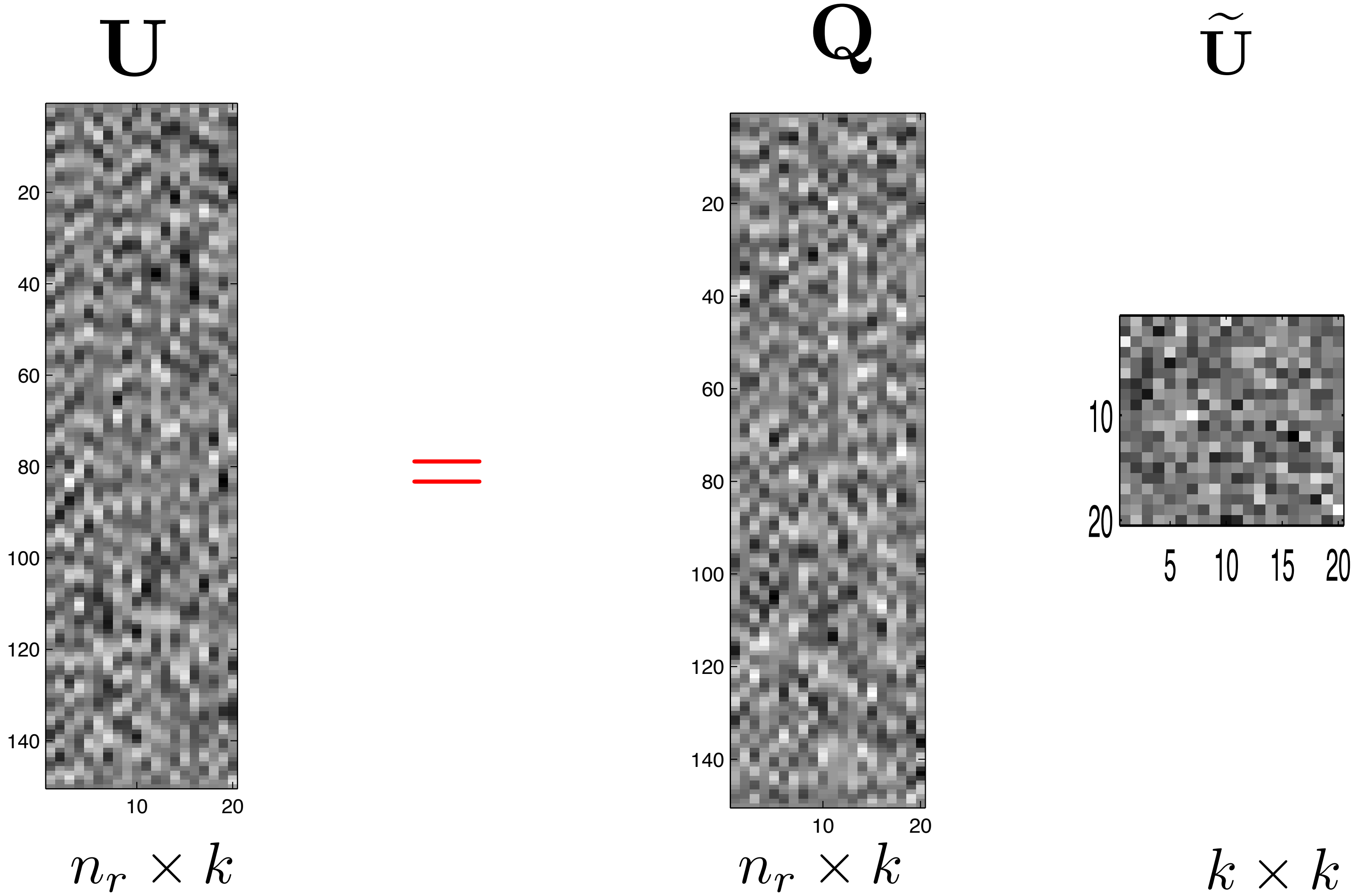


$k * n_s$ $\quad$ $k * k$ $\quad$ $k * k$ $\quad$ $k * n_s$

Dimensionality Reduction Via *RSVD*

Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

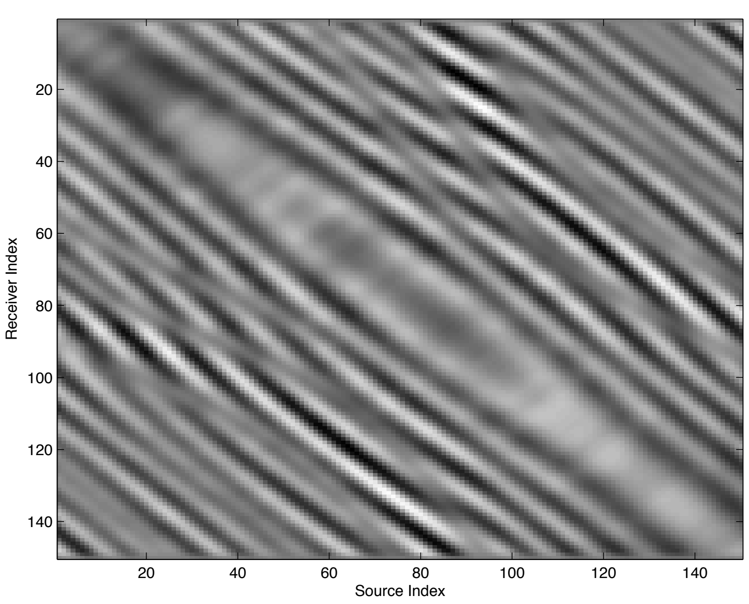
3. Compute $\mathbf{U} = \mathbf{Q}\widetilde{\mathbf{U}}$



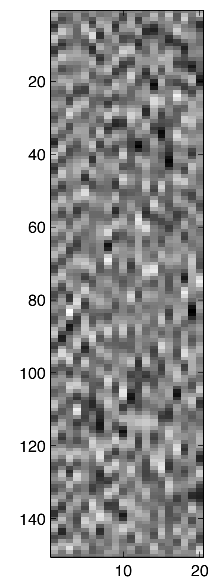
Dimensionality Reduction Via *RSVD*

Stage 2 : Compute an approximate SVD of $\hat{\mathbf{P}}$

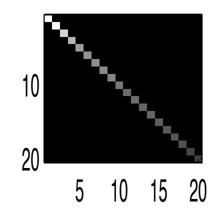
approximated $\hat{\mathbf{P}}$ $\approx$ $\mathbf{U}$ $*$ $\mathbf{\Sigma}$ $*$ $\mathbf{V}^*$



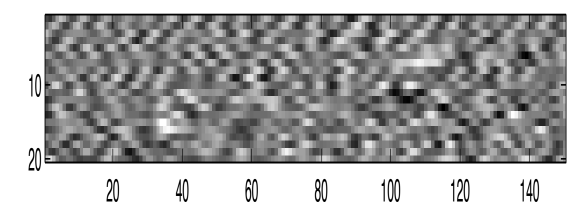
$n_r \times n_s$



$n_r \times k$



$k \times k$



$k \times n_s$

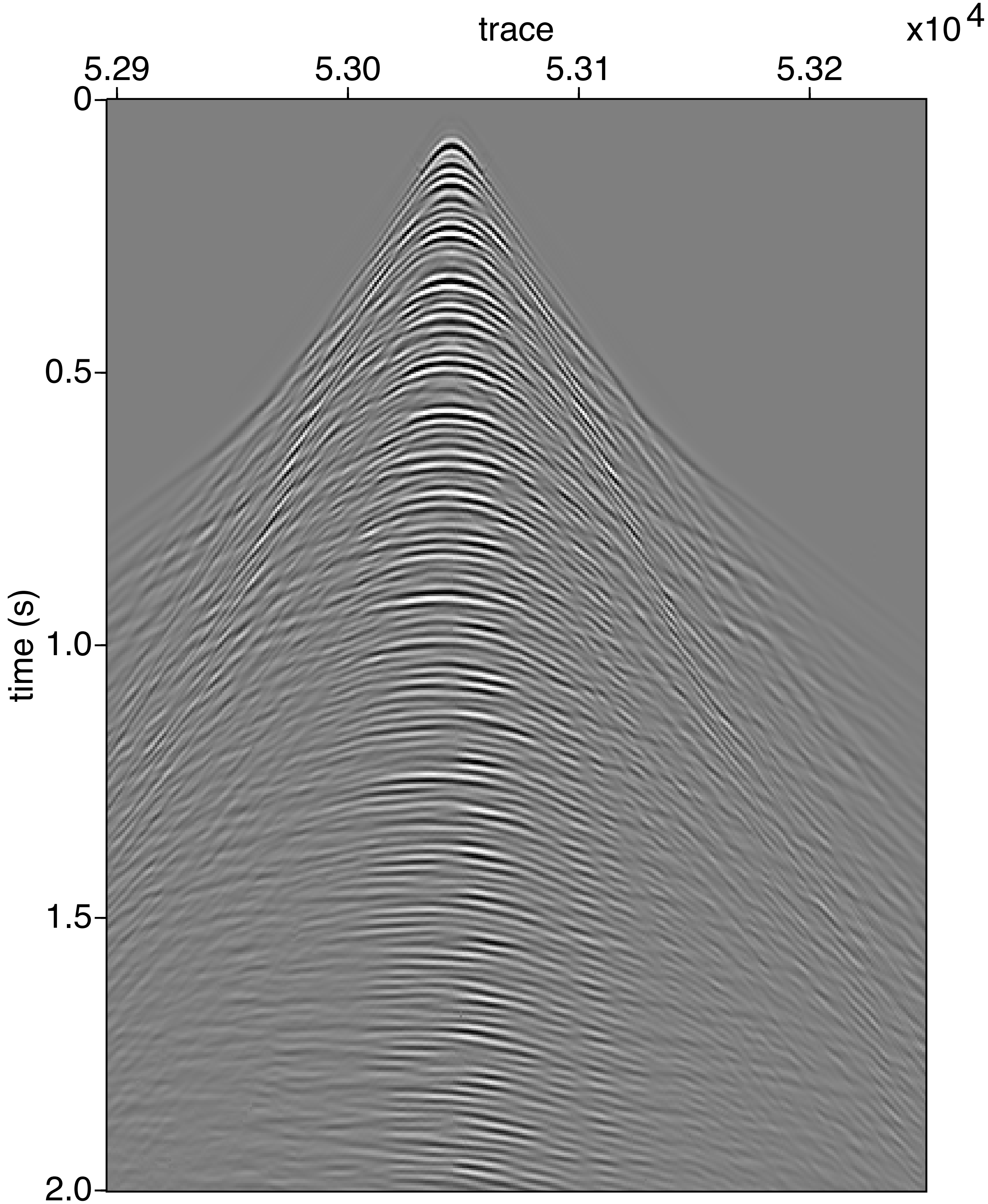
Summery

EPSI Formulation

R-SVD

Results

- 2D seismic line (Gulf of Suez)
  - $n_s = n_r = 355$
  - $n_t = 1024,\ dt = .004s$
- Adaptive approximation
- Compare results from EPSI using full vs. approximated data



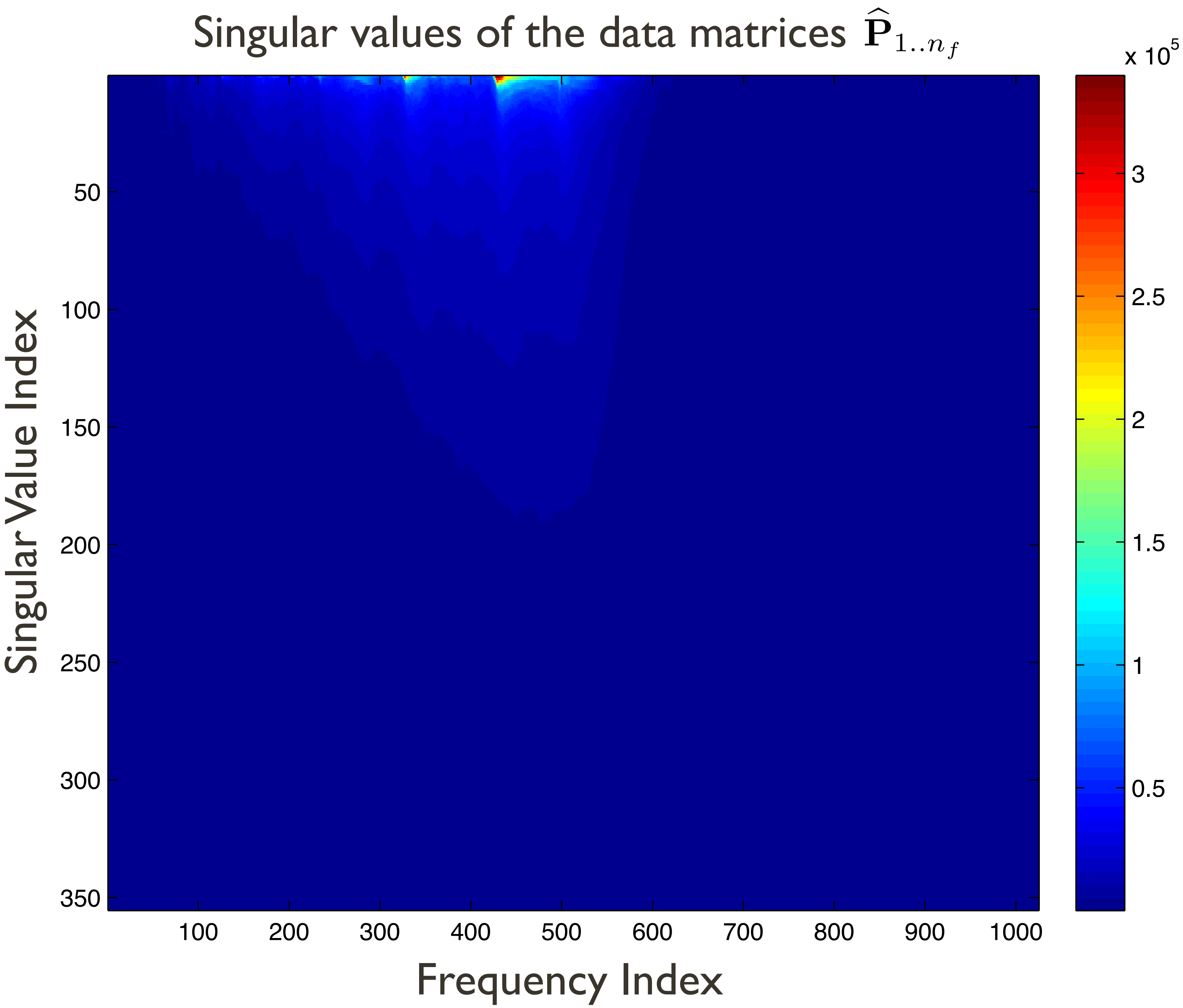
Gulf of Suez

Total Data

shot gather

$$n_r = 355$$
$$n_s = 355$$
$$n_t = 1024$$
$$dt = .004s$$

Singular values of the data matrix



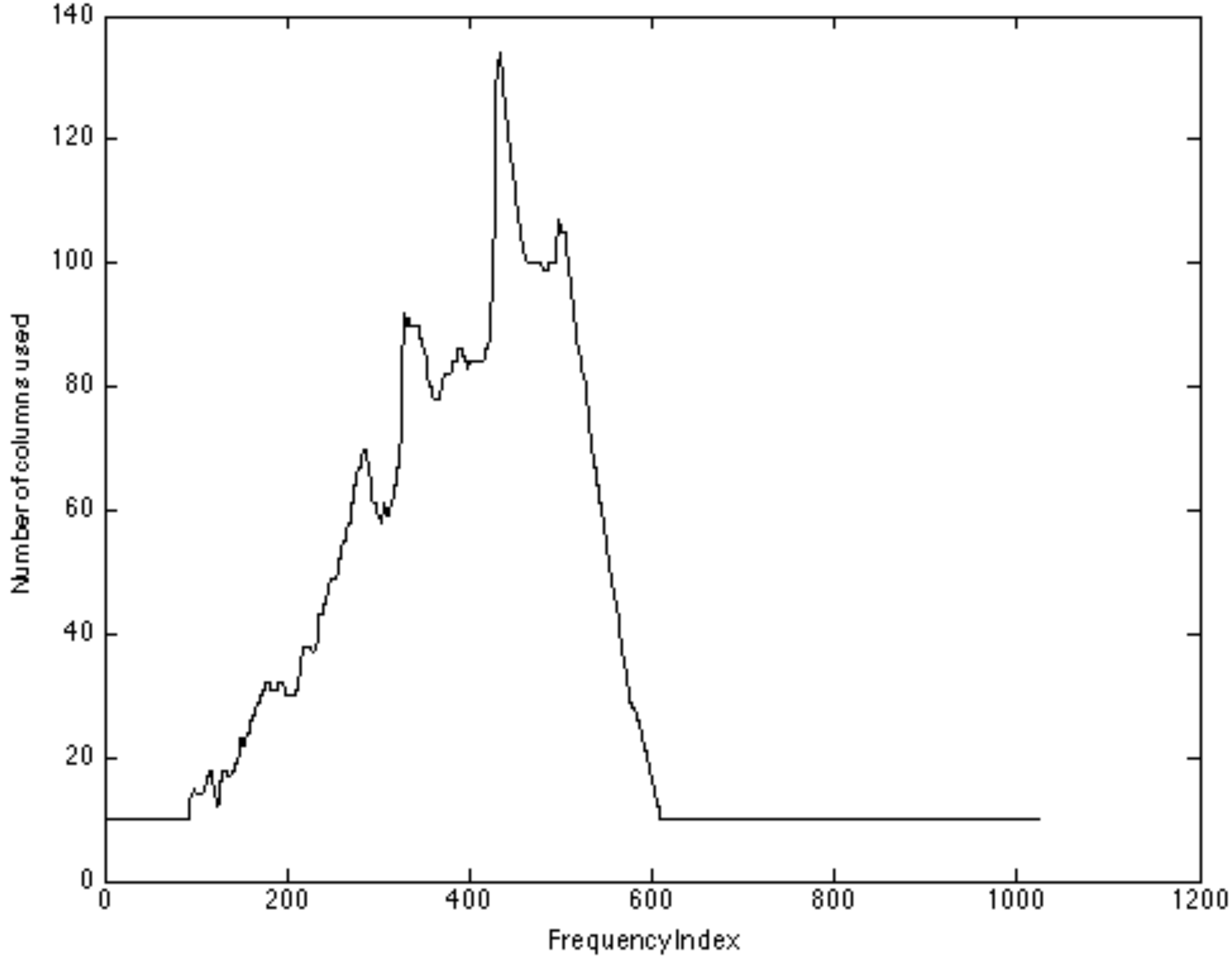
Adaptive approximation

For each frequency find rank k such that

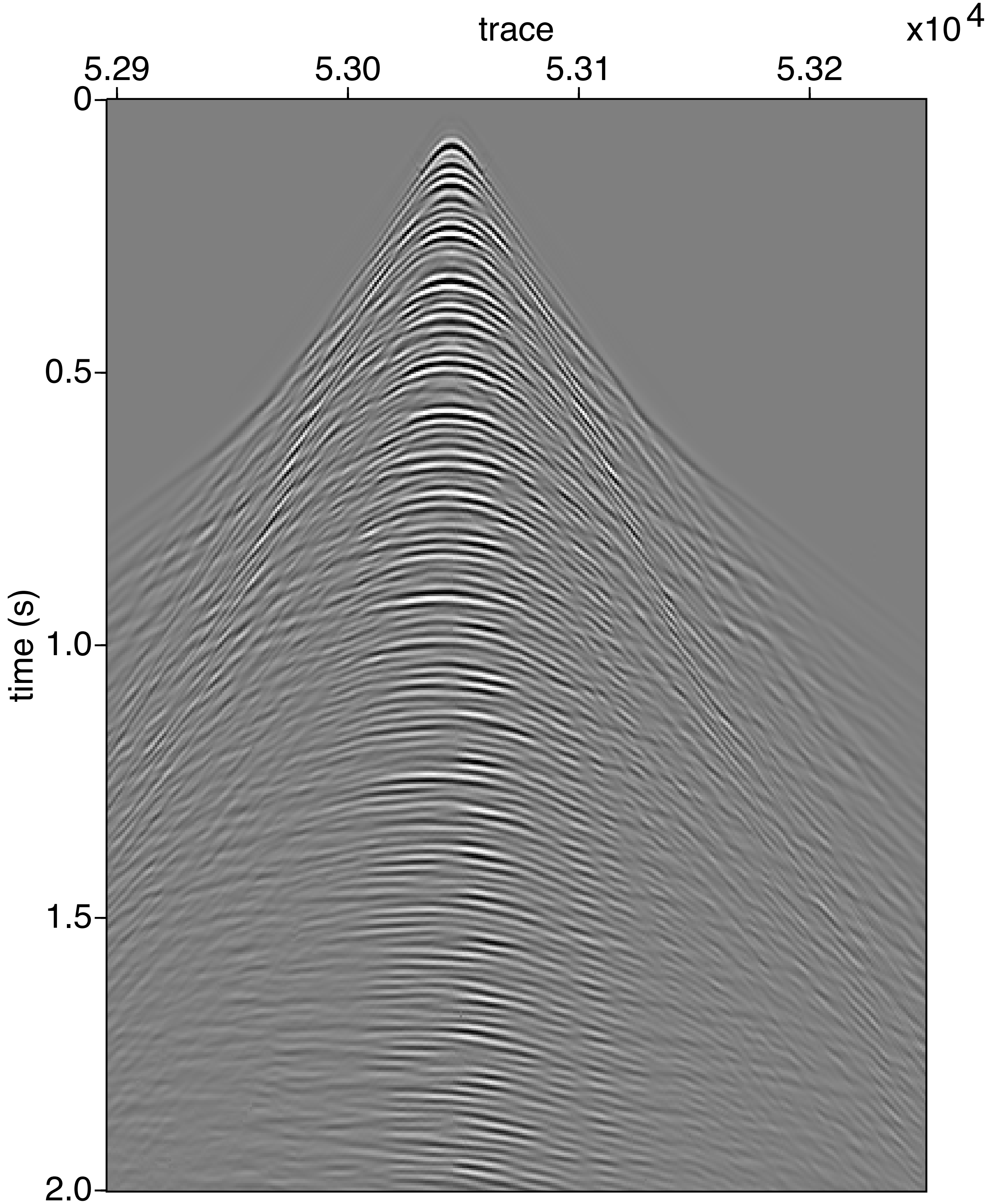
$$\|\mathbf{P} - \mathbf{USV}^*\|_{\mathbf{s}} \leq \epsilon$$

Rank budget = total number of columns

Adaptive rank selection



sum of all k ranks used is 9% of total number of columns

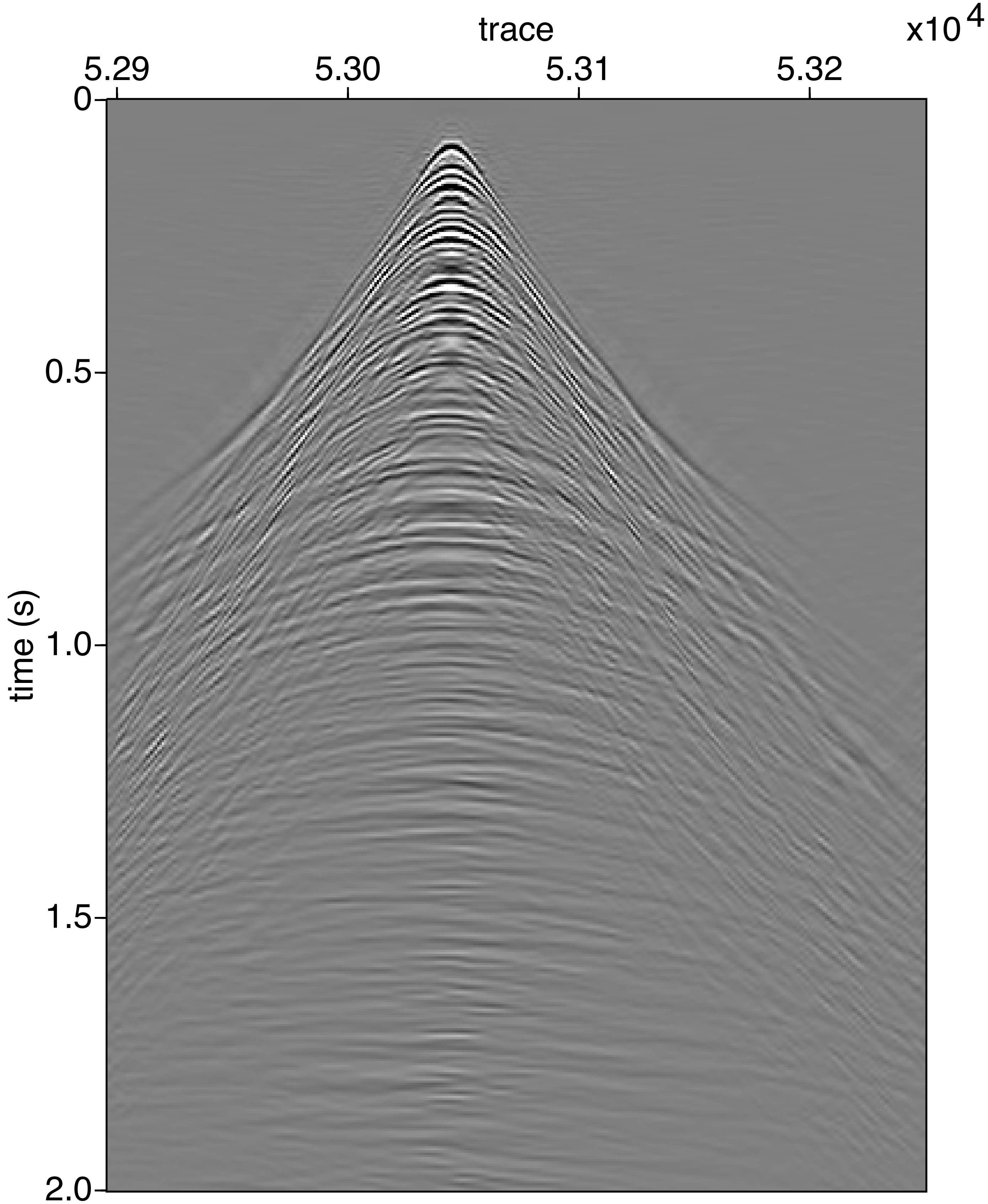


Gulf of Suez

Total Data

shot gather

$$n_r = 355$$
$$n_s = 355$$
$$n_t = 1024$$
$$dt = .004s$$



Gulf of Suez

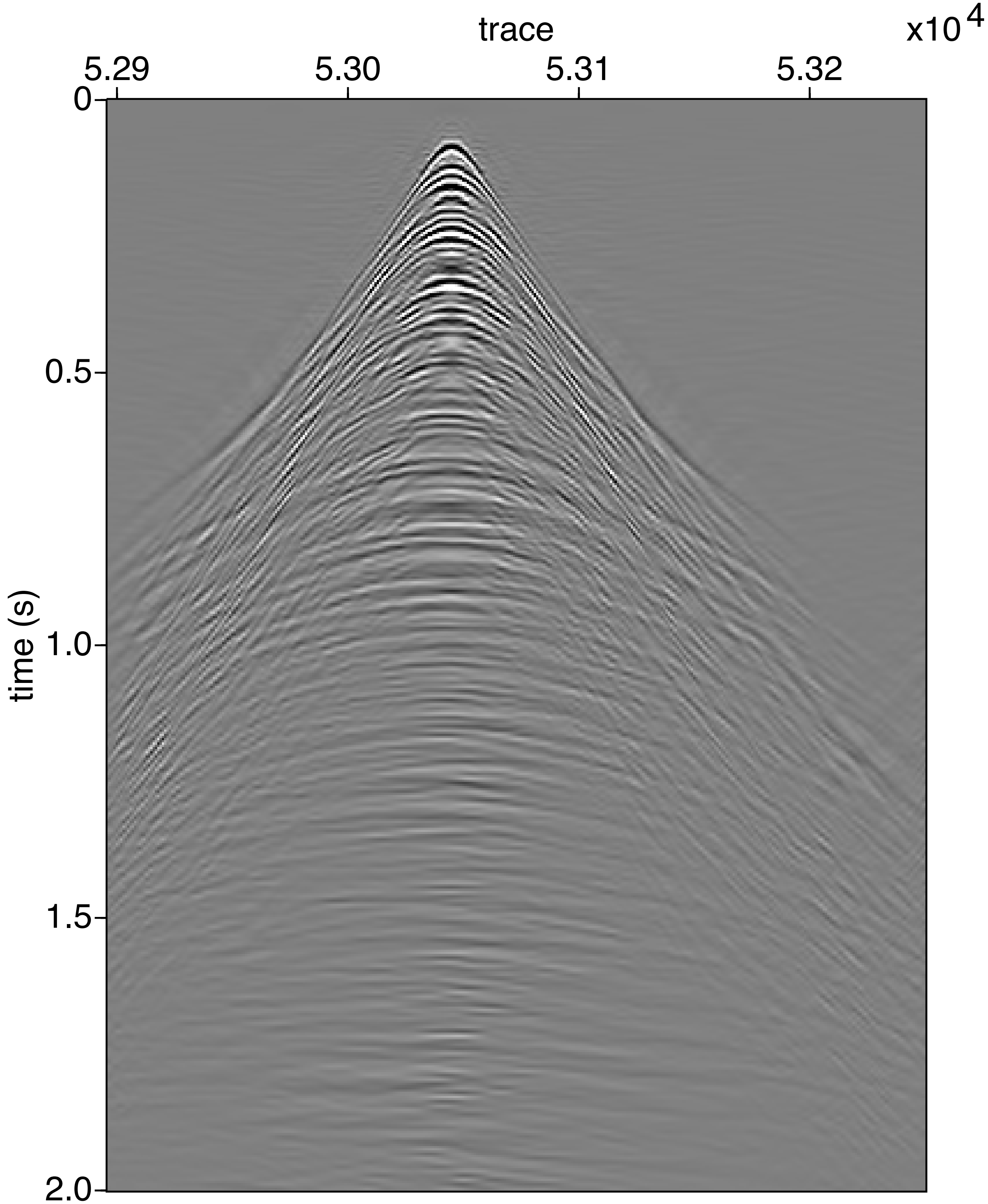
Full Data

Primary IR (G)

shot gather

2D Curvelet (Src-Rcv)

150 iterations



Gulf of Suez

20% of rank budget

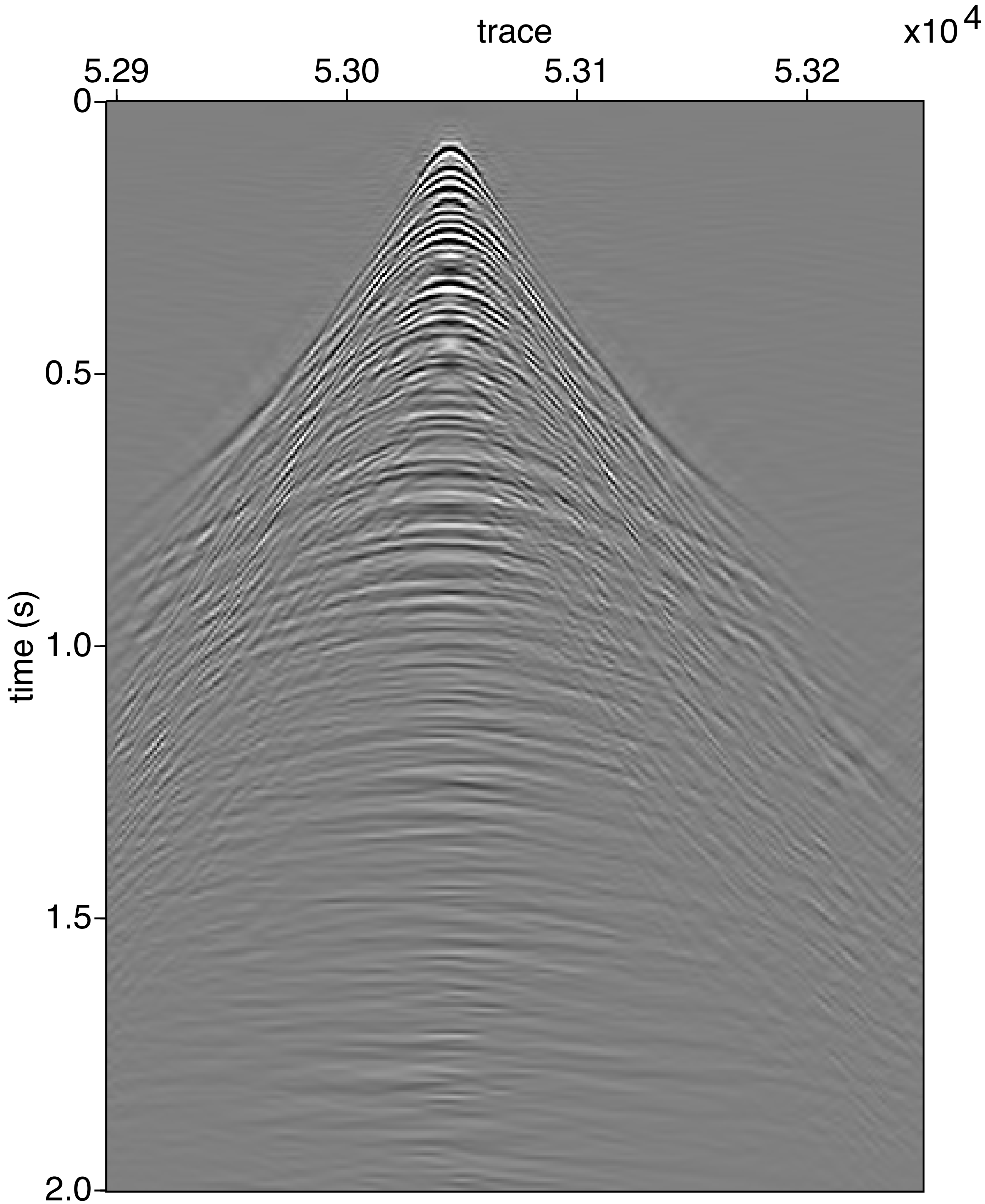
Primary IR (G)

SNR = 27dB

shot gather

2D Curvelet (Src-Rcv)

150 iterations



Gulf of Suez

12% of rank budget

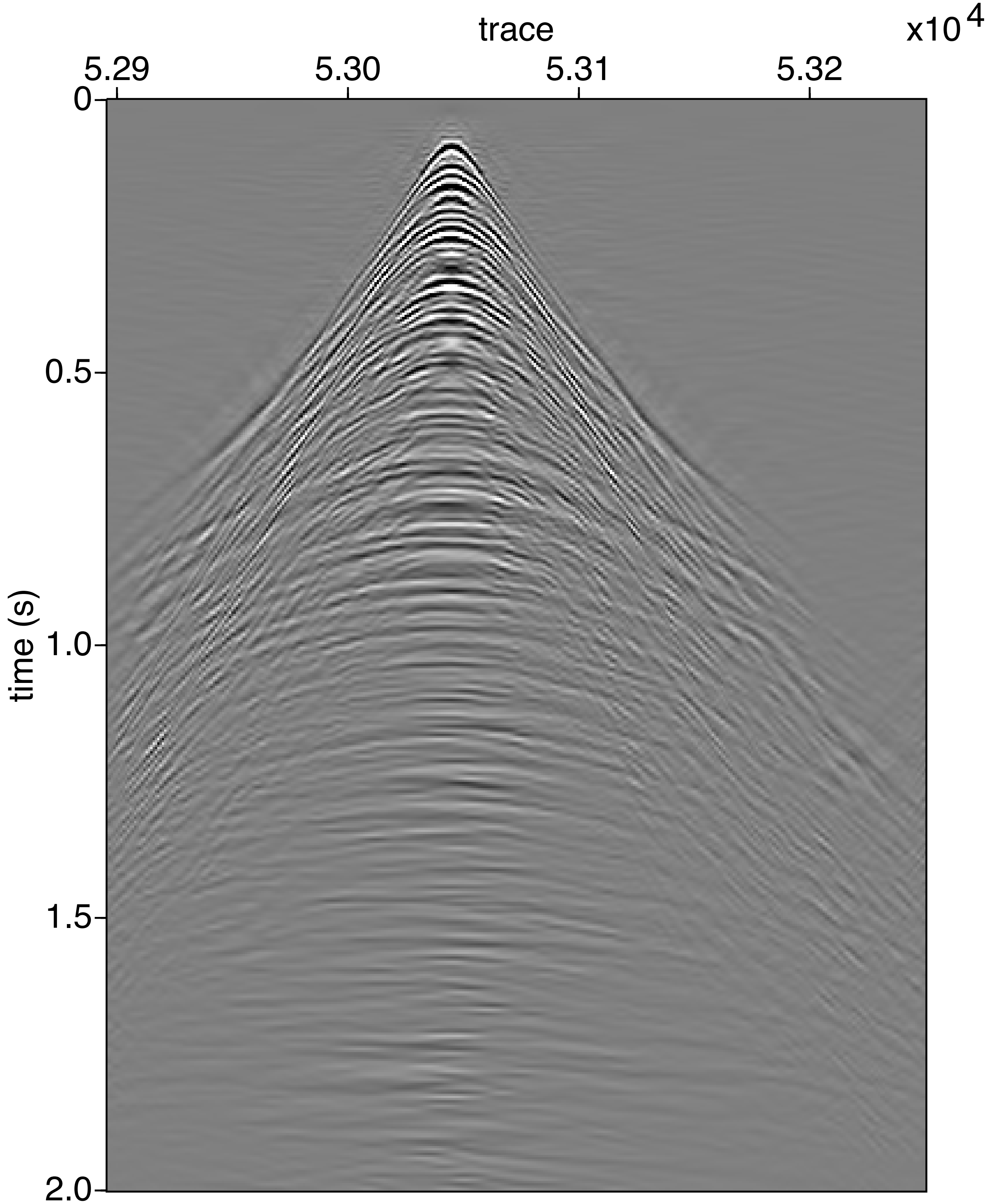
Primary IR (G)

SNR = 17dB

shot gather

2D Curvelet (Src-Rcv)

150 iterations



Gulf of Suez

8% of rank budget

Primary IR (G)

SNR = 12dB

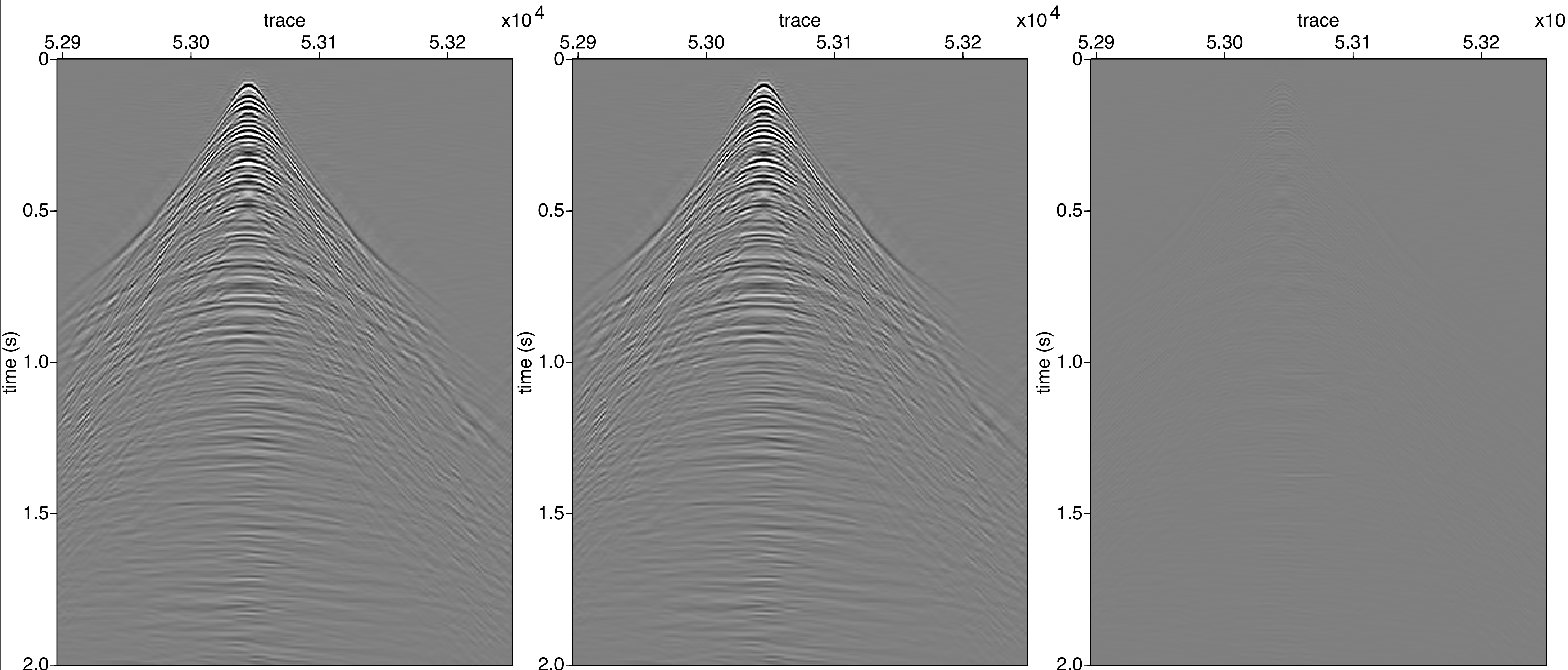
shot gather

2D Curvelet (Src-Rcv)

150 iterations

Difference in EPSI Result

20% rank budget



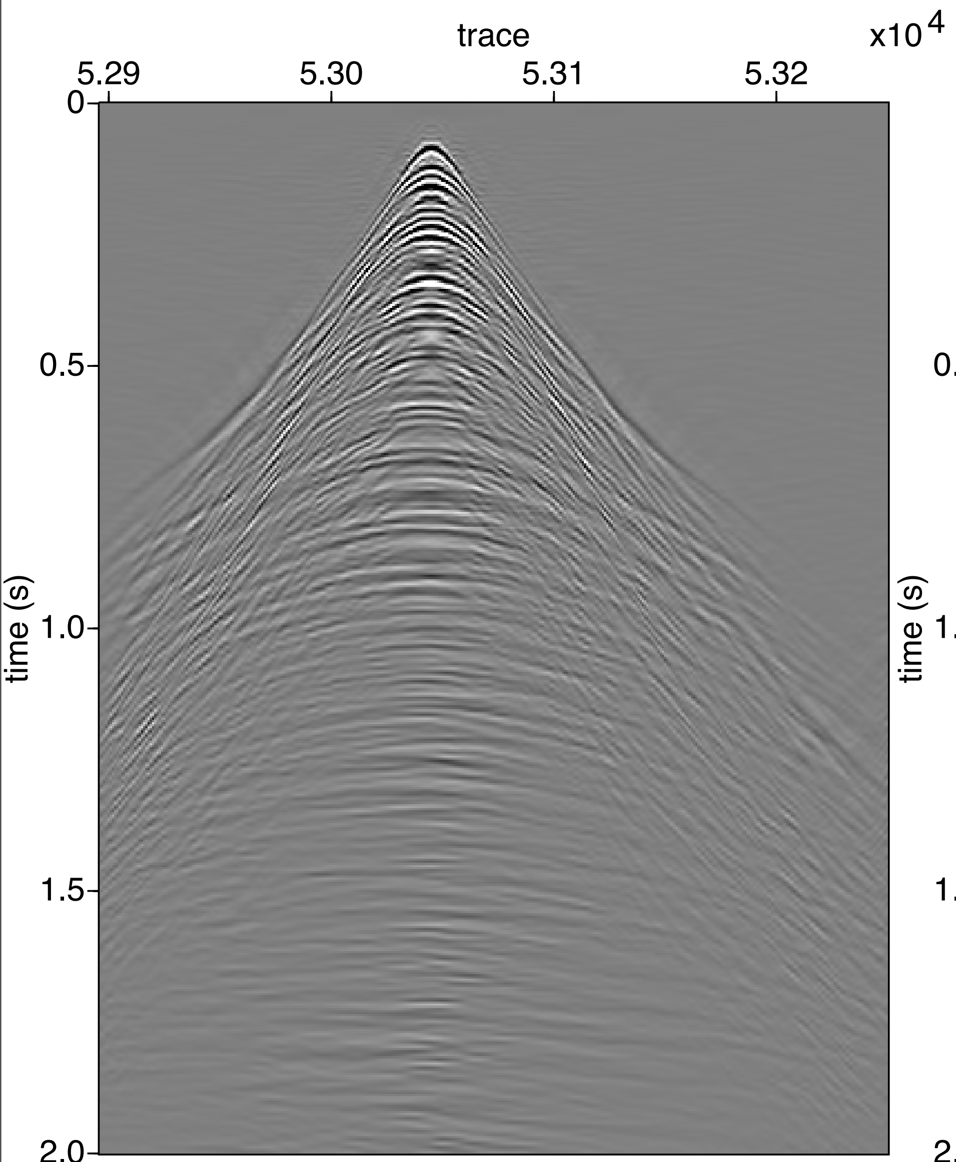
Primary IR full data

Primary IR approximated Data

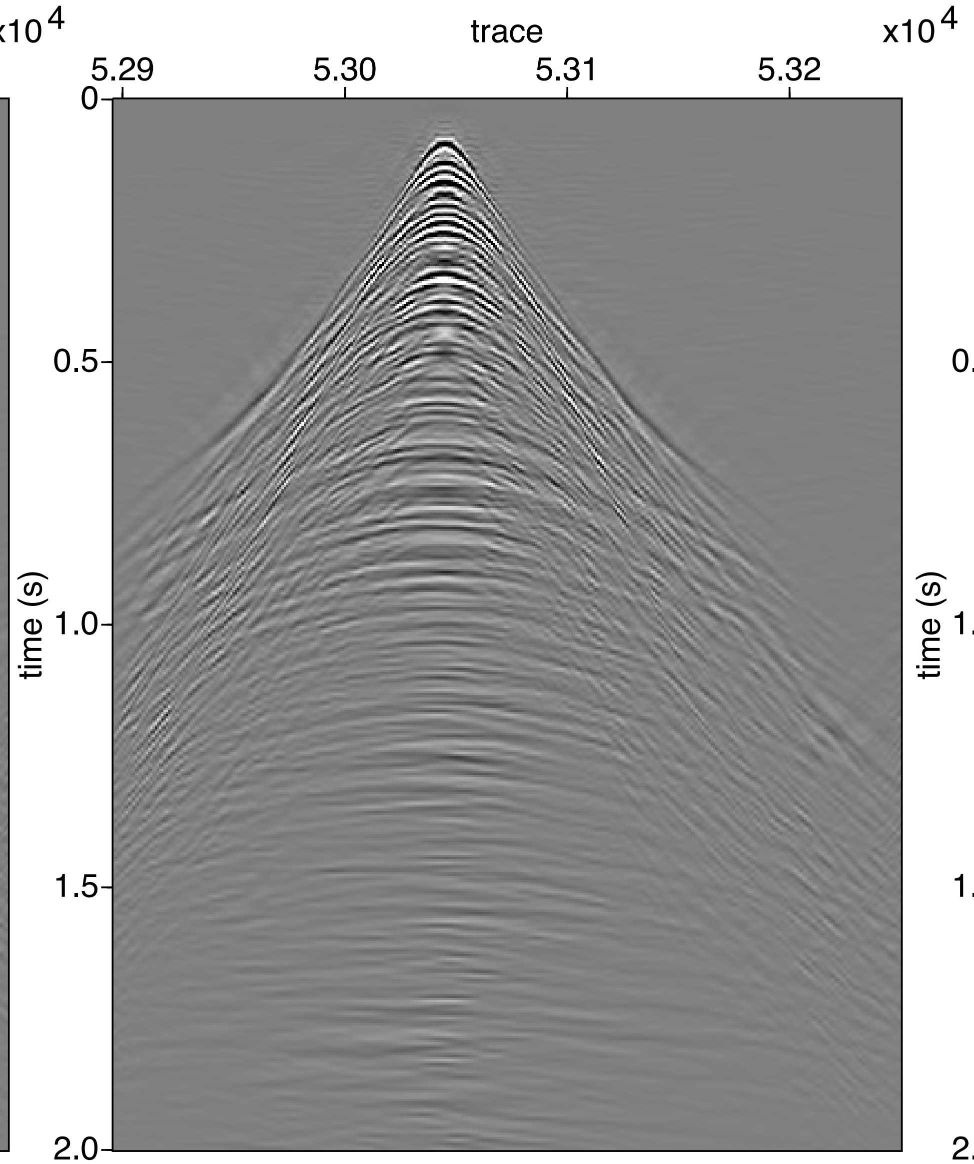
Difference

Difference in EPSI Result

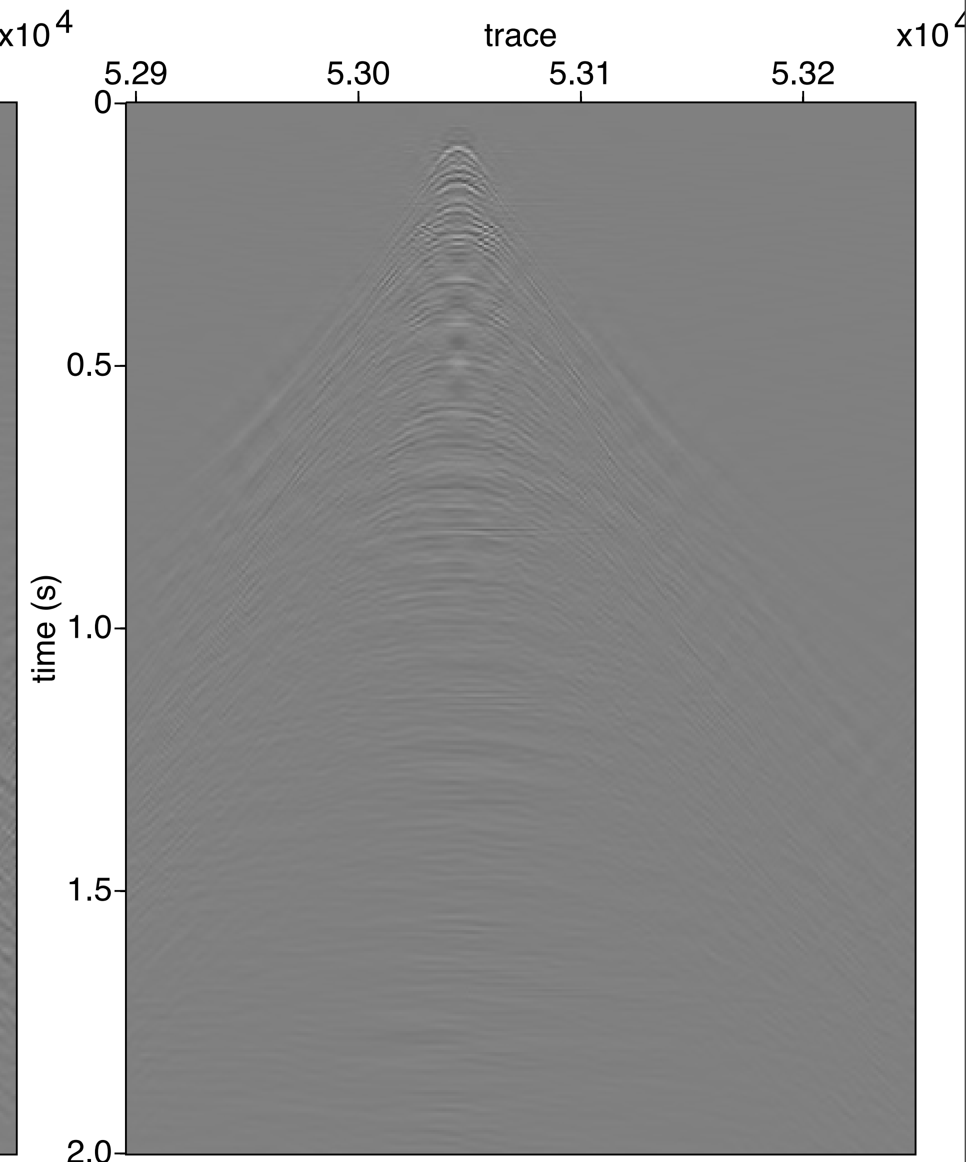
12% rank budget



Primary IR
full data



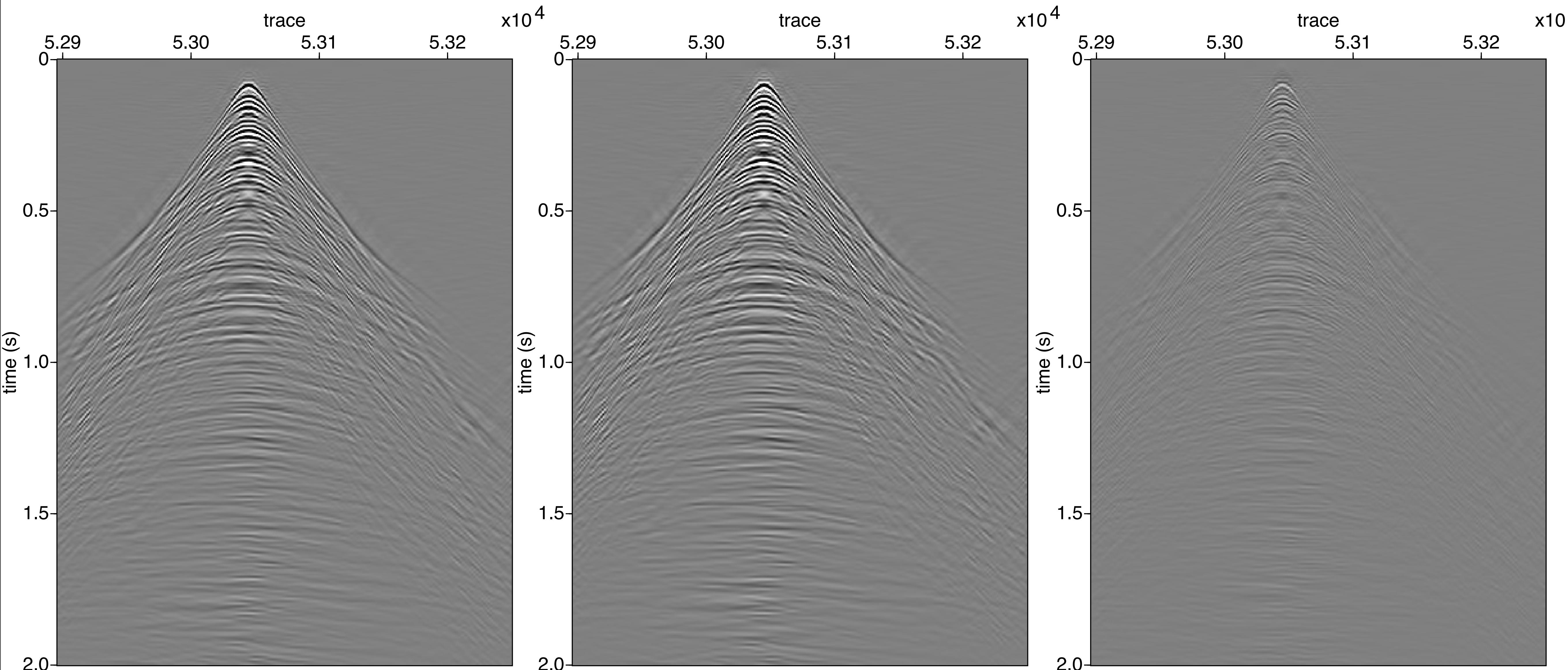
Primary IR
approximated Data



Difference

Difference in EPSI Result

8% rank budget



Primary IR full data

Primary IR approximated Data

Difference

Performance Summary

Rank Percentage	50%	20%	12%	8%
SNR (dB)	30	27	17	12
Multiplication Speedup	1.6x	2x	3.5x	5.7x
Memory savings	40%	50%	71%	82%

Conclusions

- Data driven methods - e.g. EPSI - suffers from the 'curse of dimensionality' when moving to 3D
- We utilize insights from random matrix theory to approximate action of the data matrix
- Reductions in multiplication and storage costs
- Up-Front cost of RSVD is O(mnlog(k))

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Thank you!

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