

Sparsity-promoting migration with multiples

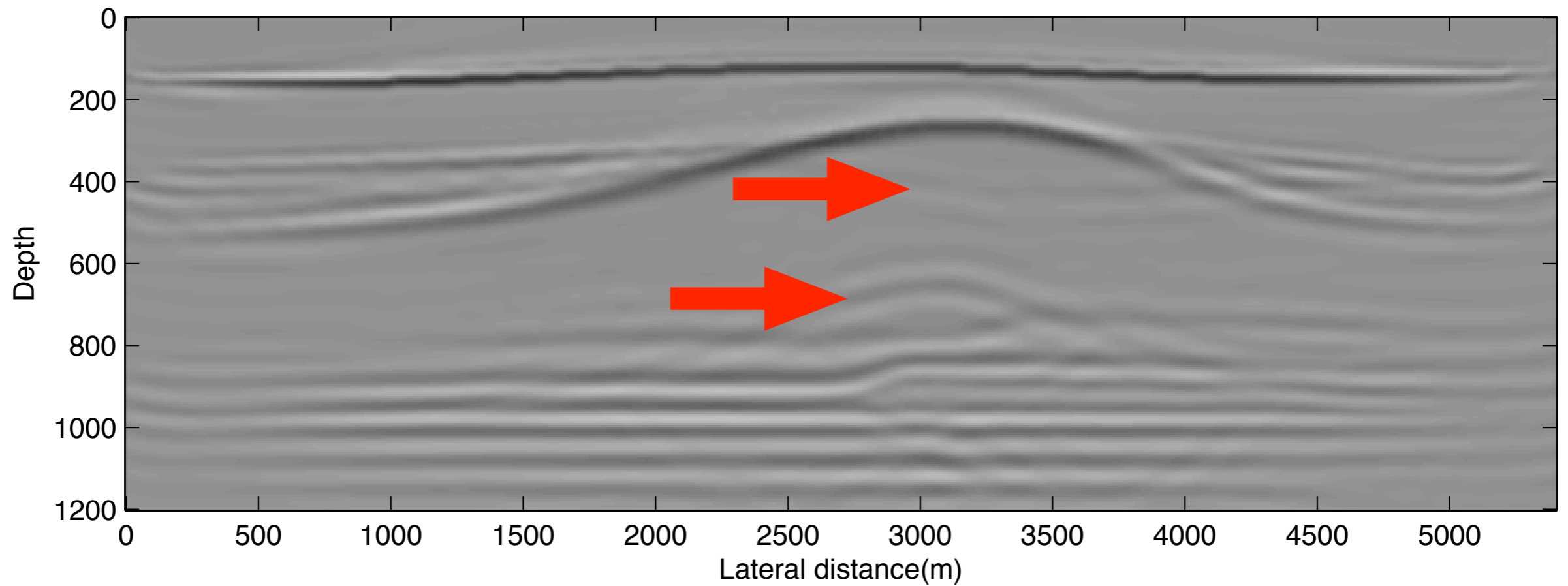
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SLIM 

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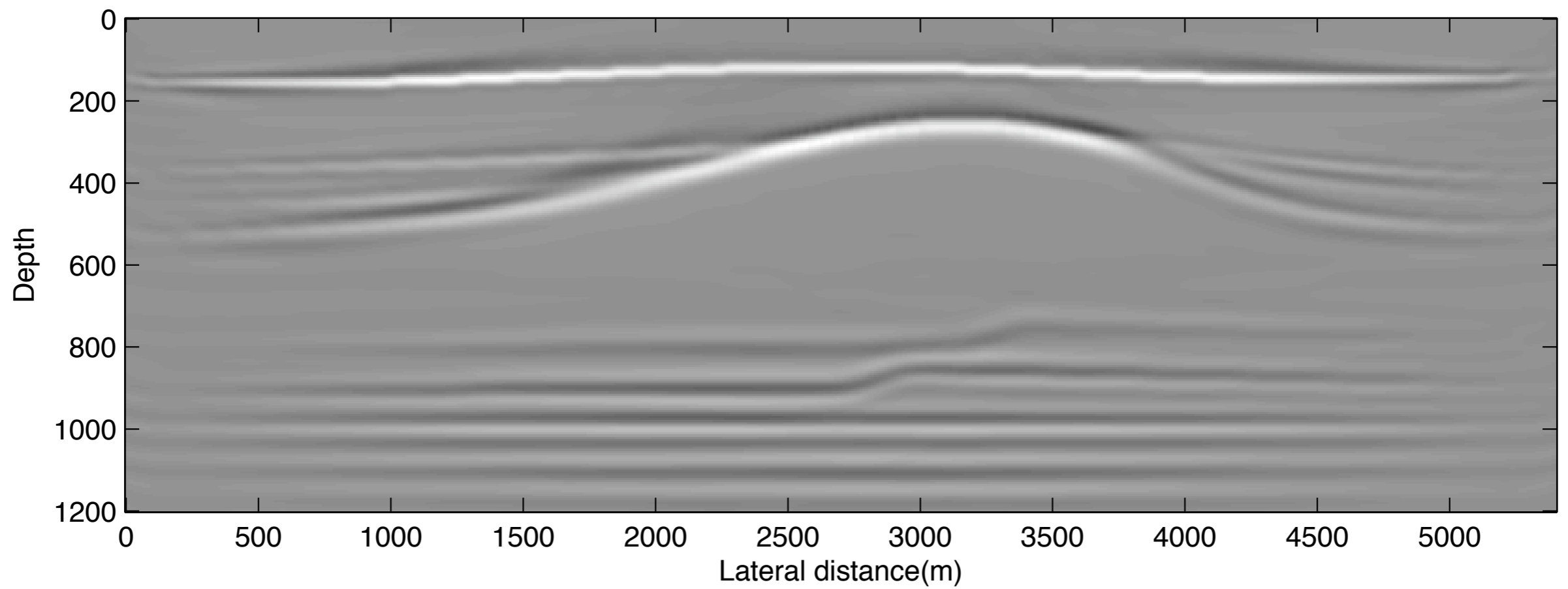
Courtesy of Verschuur, 2009

Motivation



a migrated section from data with multiples

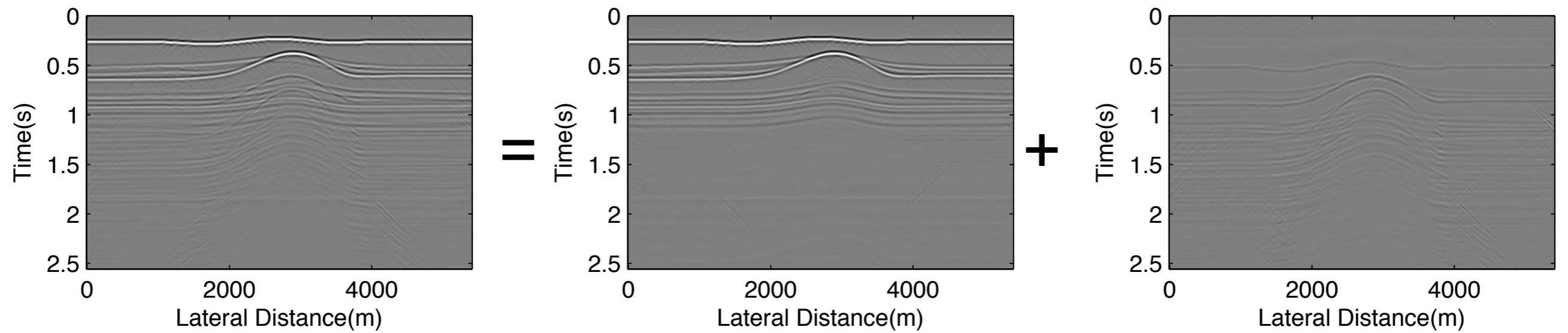
Motivation



a migrated section from multiple free data

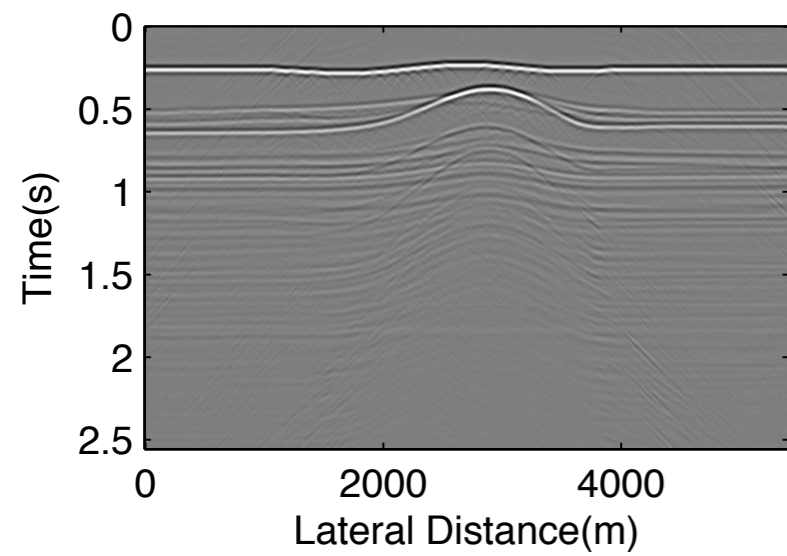
Motivation

So...

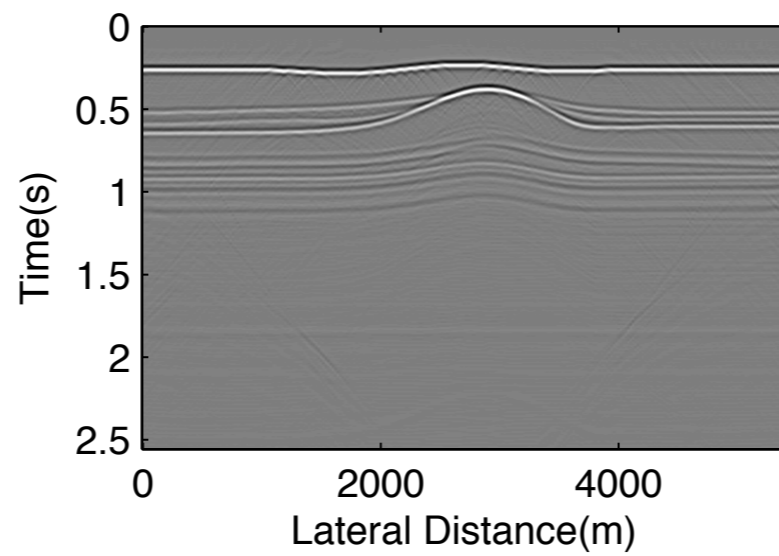


Motivation

So...



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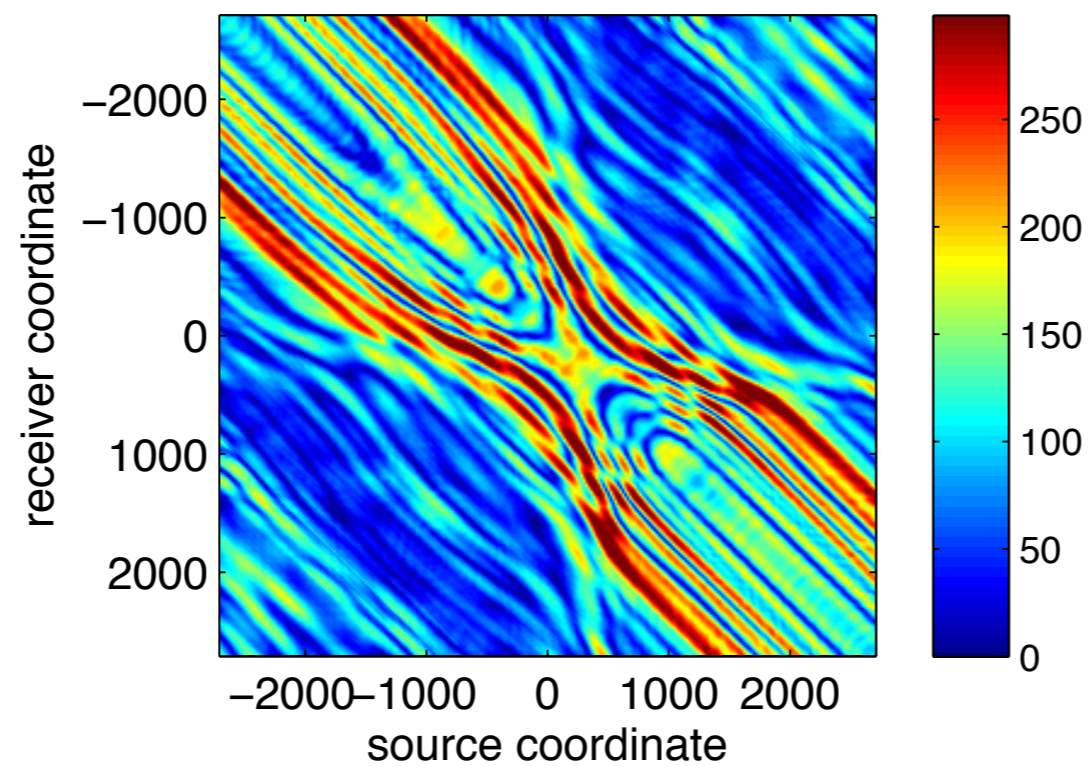


+



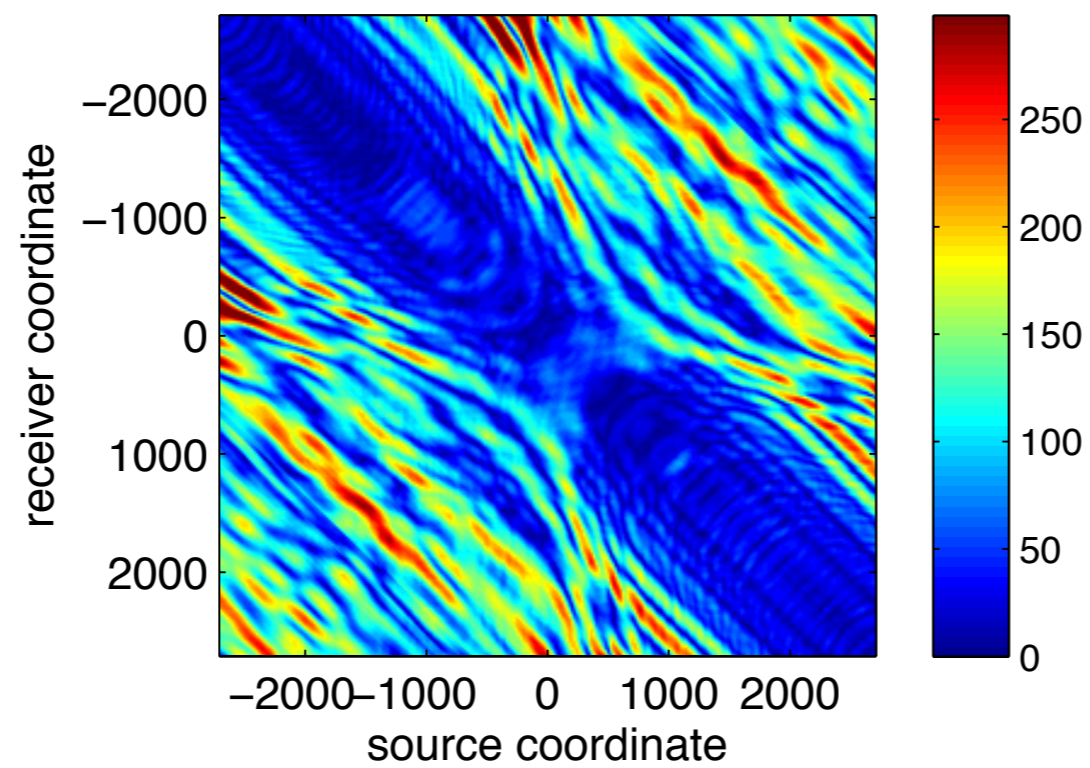
Rethink multiples

But wait a minute, are they really garbage?



amplitude spectrum: primaries @15Hz

Rethink multiples



amplitude spectrum: multiples @15Hz

Rethink multiples

Surface-related multiples:

- provide wider illumination angles
- contain more higher spatial wave number contents
- more sensitive to velocity changes

Rethink multiples

They may help to deduce subsurface structure...but how?

Motivation

EPSI (Estimation of Primaries via Sparse Inversion) **exploits the sparsity of the up-going Green's function**

- **EPSI tries to derive the up-going Green's function**
- **velocity model is a lot sparser than Green's function**

Motivation

There seems to be some interaction between EPSI and imaging...what about let them get married, and how?

Multiples in imaging

Introduce free surface to the ‘smooth’ background velocity model

- violates the Born approximation assumptions
- more requirements on the exactness of the velocity model

Multiples in imaging

Full-waveform inversion

- “de-multiple” before inversion
- consists of several migration based updates

Multiples in imaging

Focal transform

- first multiples mapped to primaries
- needs the estimate of the primaries as the operator
- de-multiple followed by migration

Our approach

We combine EPSI with migration

- EPSI models primaries as well as multiples
- combine EPSI with sparsity promoting migration

EPSI Formulation

EPSI reveals the relationship:

$$\hat{\mathbf{P}} = \hat{\mathbf{G}}(\hat{\mathbf{Q}} - \hat{\mathbf{P}})$$

Formulating the EPSI operator:

$$\underbrace{\mathcal{F}_t^* \text{BlockDiag}_f [(\hat{\mathbf{Q}} - \hat{\mathbf{P}})^* \otimes \mathbf{I}] \mathcal{F}_t}_{\mathbf{M}} \mathbf{g} = \mathbf{p}$$

EPSI Formulation

$$\tilde{\mathbf{g}} = \underset{\mathbf{g}}{\operatorname{argmin}} \|\mathbf{p} - \mathbf{M}\mathbf{g}\|_2 \text{ s.t. } \|\mathbf{g}\|_0 \leq \mathbf{k}\tau$$

$\tilde{\mathbf{g}}$: estimate of the up-going Green's function

\mathbf{p} : the up-going wavefield

Robust EPSI

Replace the computationally prohibitive l_0 norm with l_1 norm.

Robust EPSI:

$$\tilde{\mathbf{g}} = \underset{\mathbf{g}}{\operatorname{argmin}} \|\mathbf{g}\|_1 \text{ s.t. } \|\mathbf{p} - \mathbf{M}\mathbf{g}\|_2 \leq \sigma$$

Nemeth, 1999

Wang and Sacchi, 2007

Regularized least-squares migration

Regularized least-squares migration:

$$\delta\tilde{\mathbf{m}} = \operatorname{argmin}_{\delta\mathbf{m}} \frac{1}{2} \|\mathbf{g} - \mathbf{K}\delta\mathbf{m}\|_2^2 + \lambda \|\delta\mathbf{m}\|_2^2$$

Sparsity promoting migration

Sparsity-promoting migration:

$$\delta \tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta \mathbf{x}}{\operatorname{argmin}} \|\delta \mathbf{x}\|_1 \text{ s.t. } \|\mathbf{g} - \mathbf{KS}^* \delta \mathbf{x}\|_2 \leq \sigma$$

Combine EPSI with migration

We formulate this linearized inversion process as

$$\delta\tilde{\mathbf{m}} = \mathbf{S}^* \operatorname{argmin}_{\delta\mathbf{x}} \|\delta\mathbf{x}\|_1 \text{ s.t. } \|\mathbf{p} - \mathbf{MKS}^* \delta\mathbf{x}\|_2 \leq \sigma$$

Numerical experiments

Make linearized data:

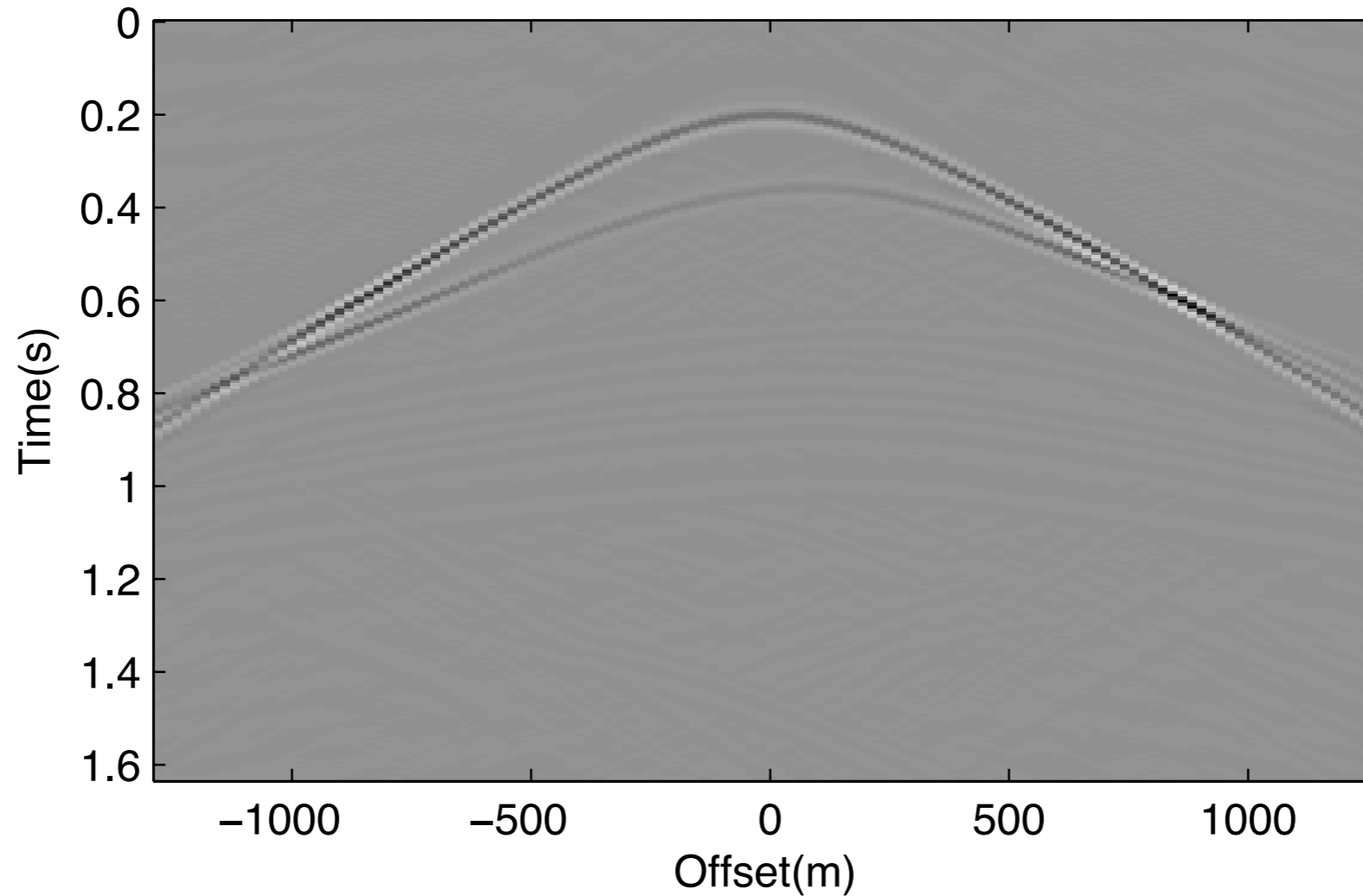
- multiple-free data

$$\mathbf{p}_1 = \mathbf{K}\delta\mathbf{m}$$

- data with multiples

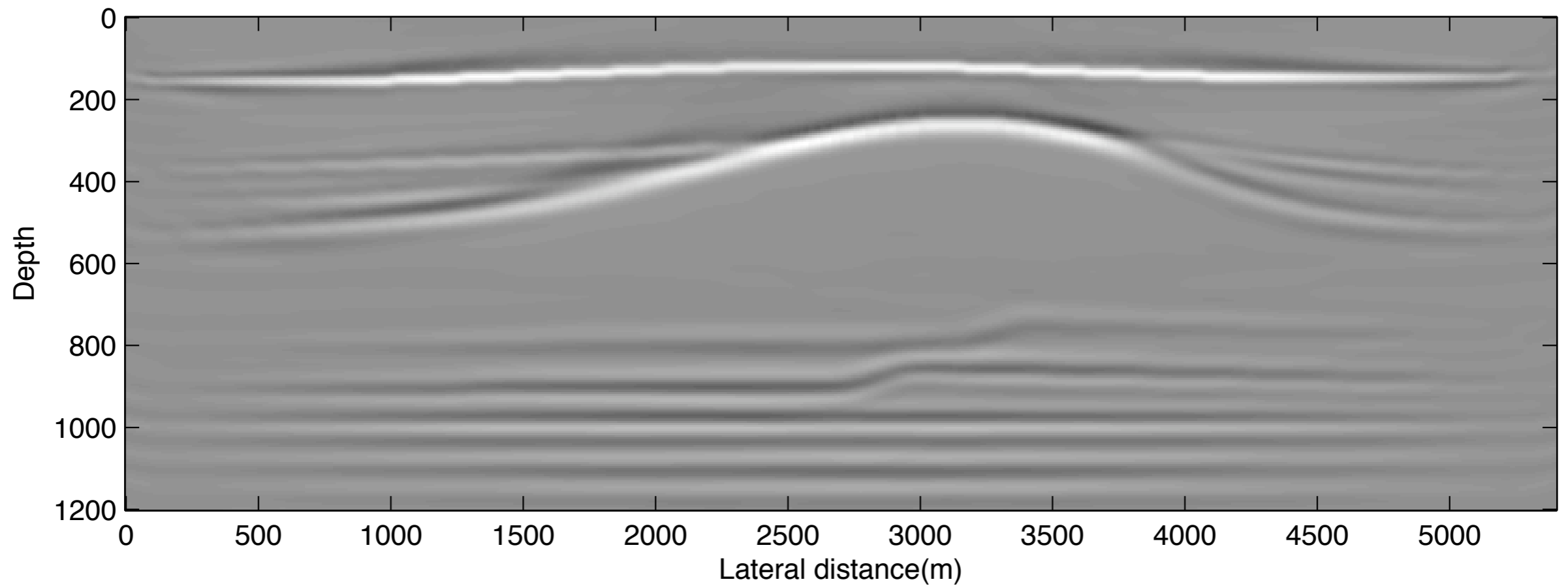
$$\mathbf{p}_2 = \mathbf{MK}\delta\mathbf{m}$$

Data preview: multiple free



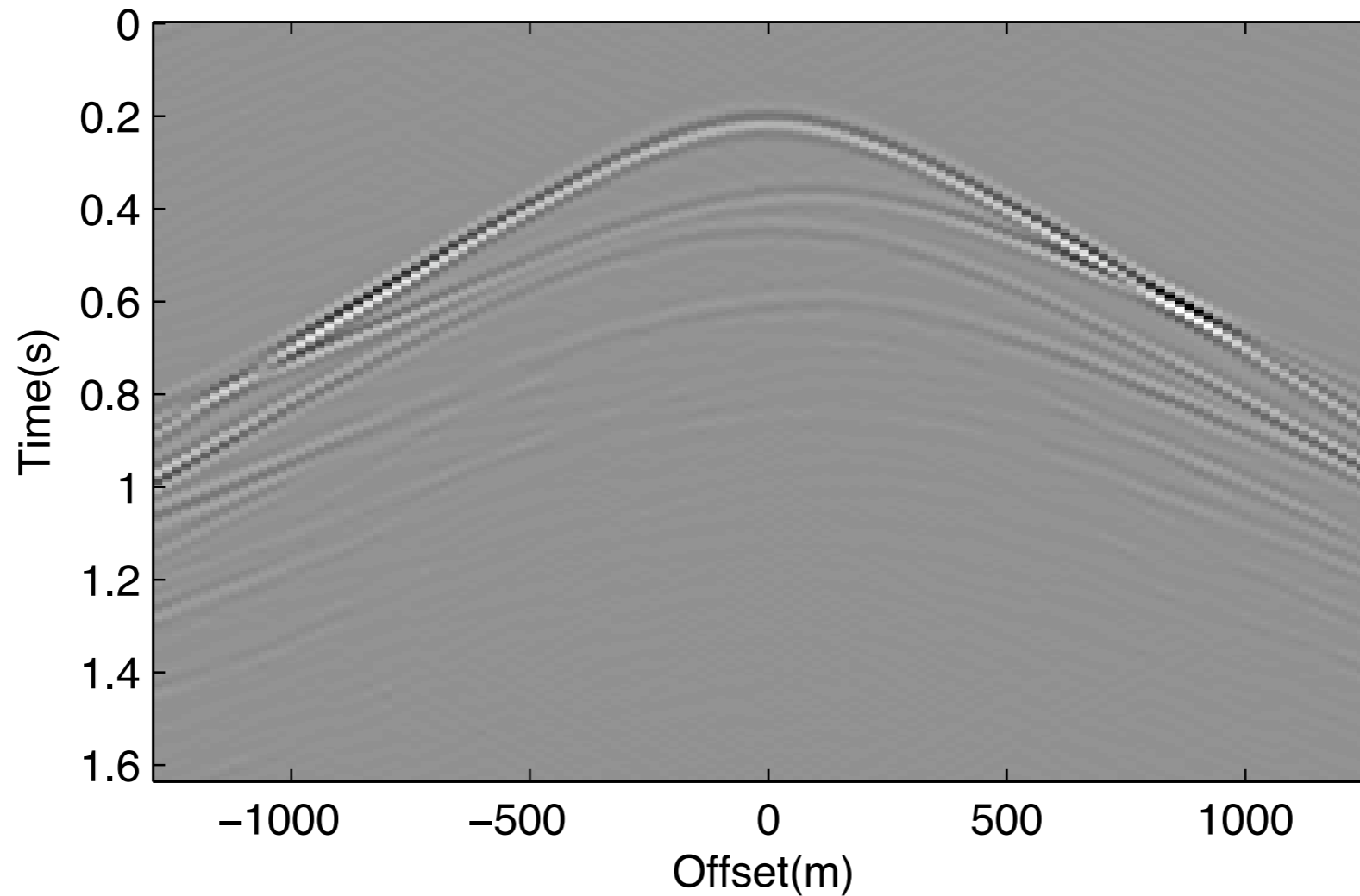
total shots: 128, shot number: 65

Image preview: multiple free



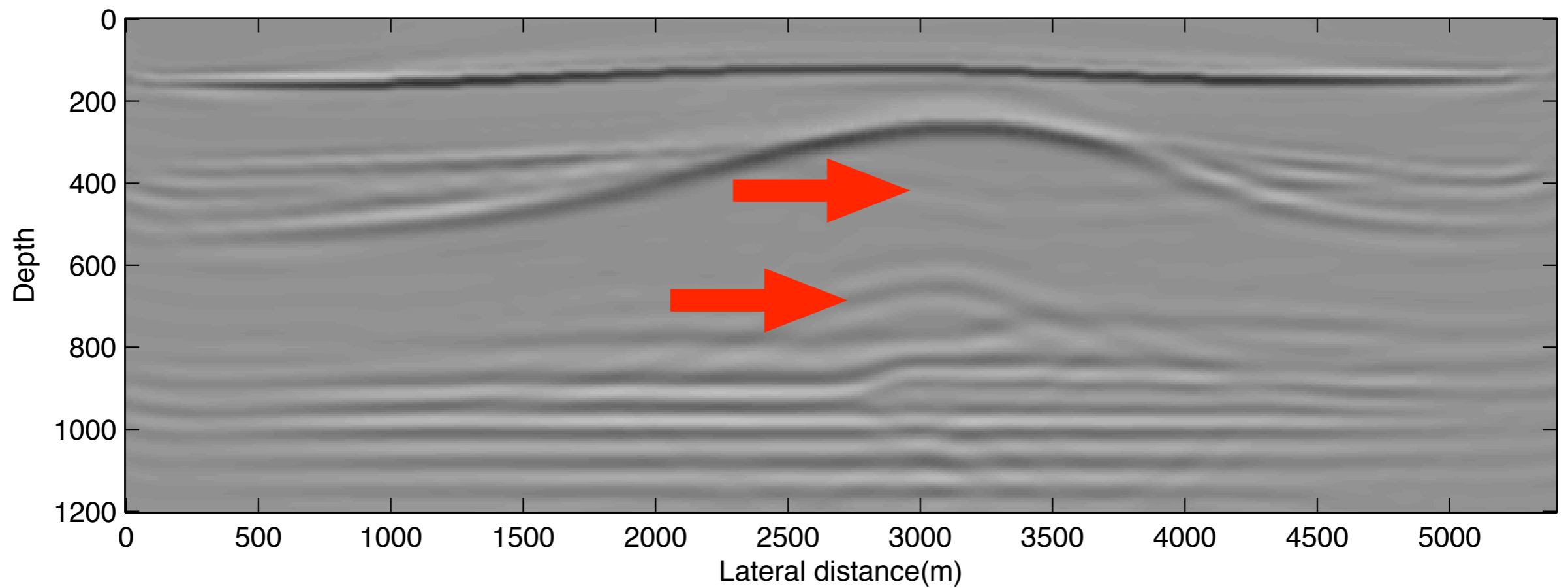
migrated section: time-weighted

Data preview: with multiples



total shots: 128, shot number: 65

Image preview: with multiples



migrated section: time weighted

Three scenarios: mig-multiple free

Migration from multiple free data:

$$\delta \tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta \mathbf{x}}{\operatorname{argmin}} \|\delta \mathbf{x}\|_1 \text{ s.t. } \|\mathbf{p}_1 - \mathbf{KS}^* \delta \mathbf{x}\|_2 \leq \sigma$$

Three scenarios: mig-with multiples

Migration from data with multiples:

$$\delta \tilde{\mathbf{m}} = \mathbf{S}^* \operatorname{argmin}_{\delta \mathbf{x}} \|\delta \mathbf{x}\|_1 \text{ s.t. } \|\mathbf{p}_2 - \mathbf{KS}^* \delta \mathbf{x}\|_2 \leq \sigma$$

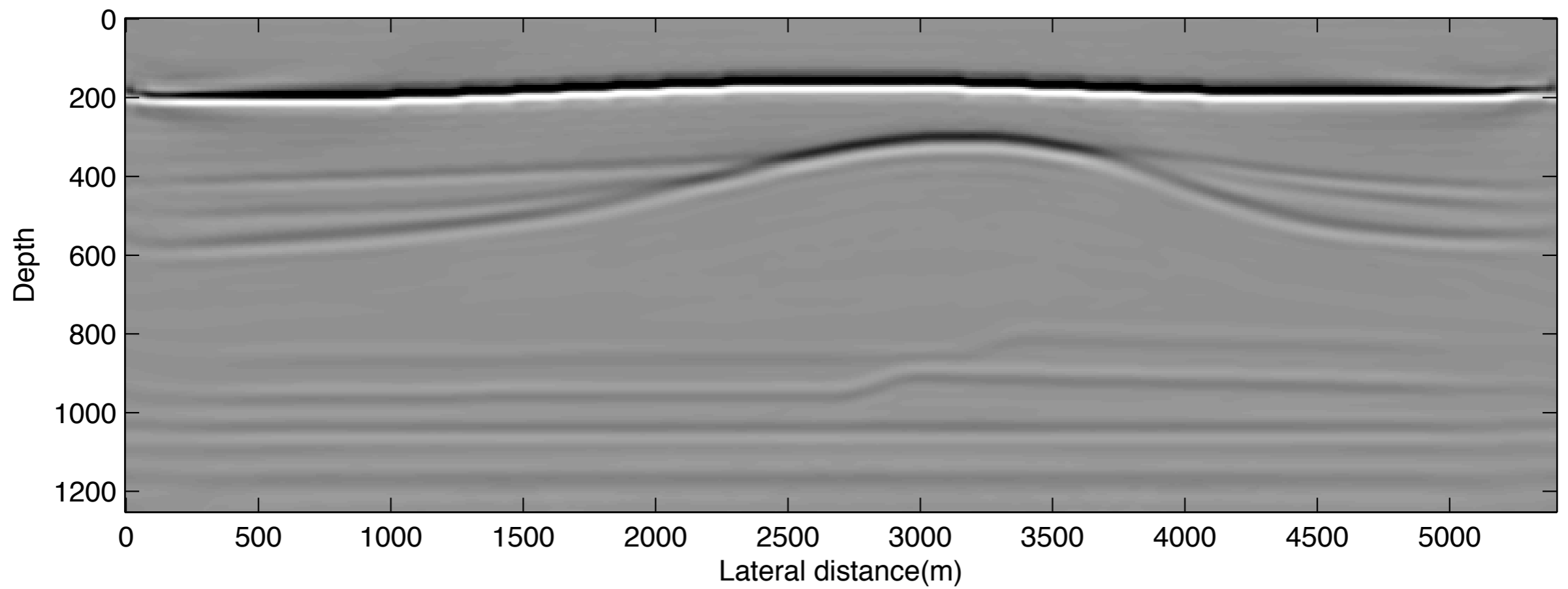
Three scenarios: mig/EPSt-with multiples

Migration combined with EPSt from data
with multiples:

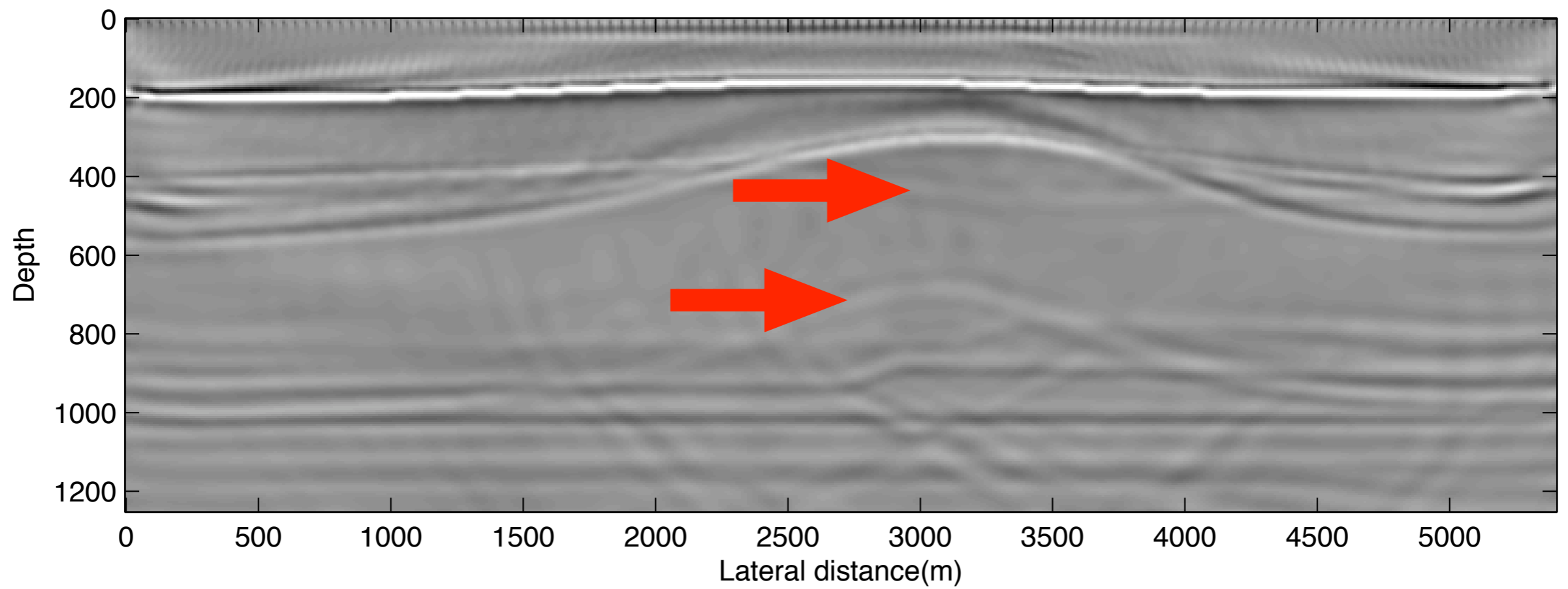
$$\delta \tilde{\mathbf{m}} = \mathbf{S}^* \underset{\delta \mathbf{x}}{\operatorname{argmin}} \|\delta \mathbf{x}\|_1 \text{ s.t. } \|\mathbf{p}_2 - \mathbf{MKS}^* \delta \mathbf{x}\|_2 \leq \sigma$$

Solver: spgl1

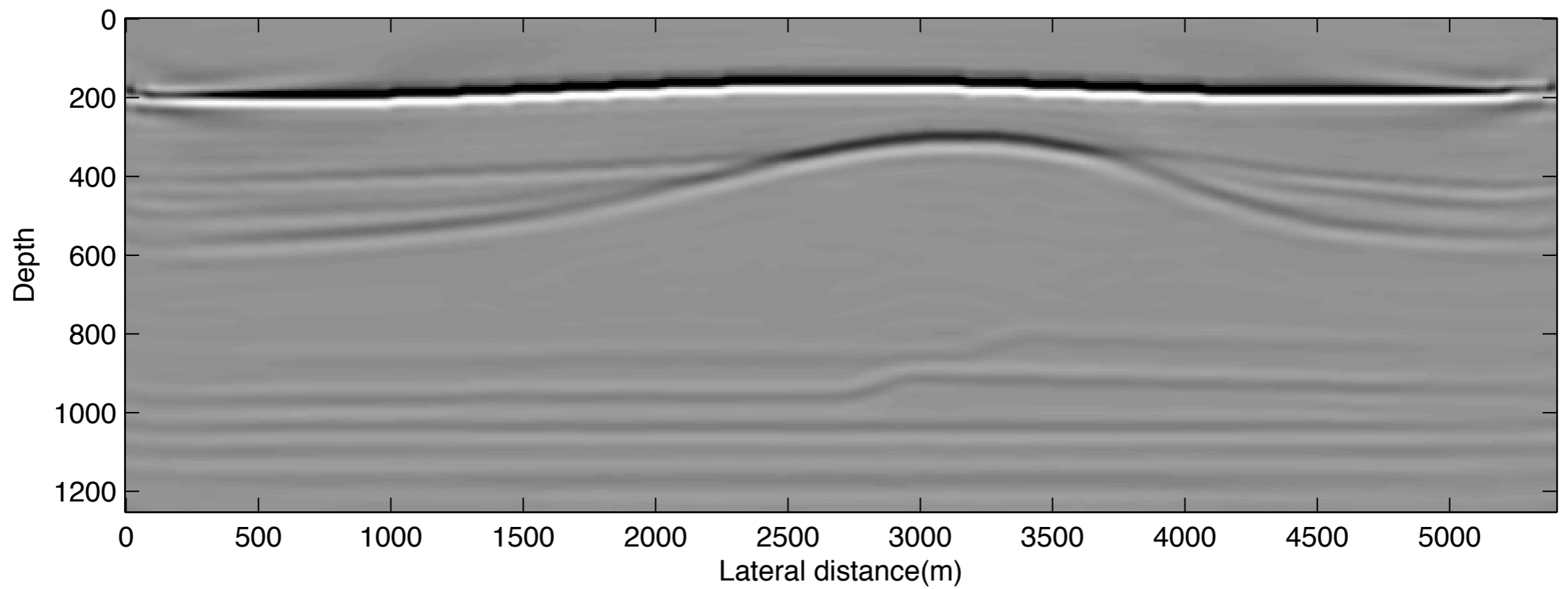
Mig-multiple free



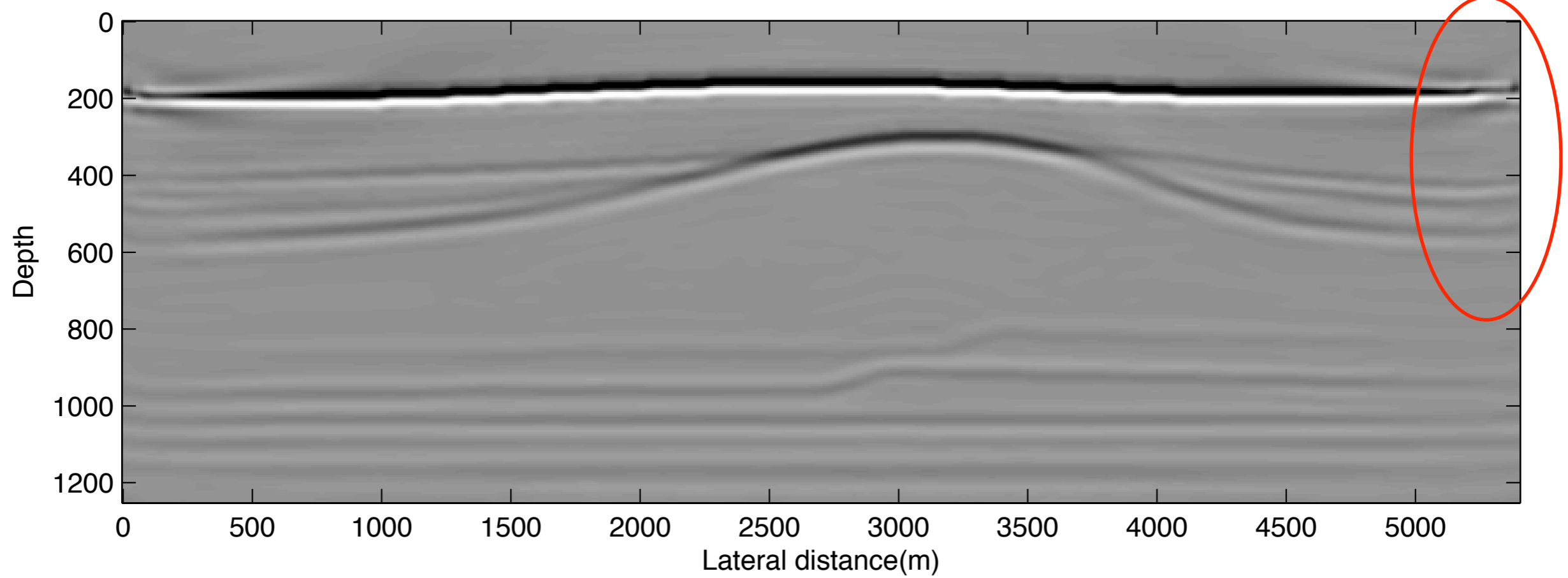
Mig-with multiples



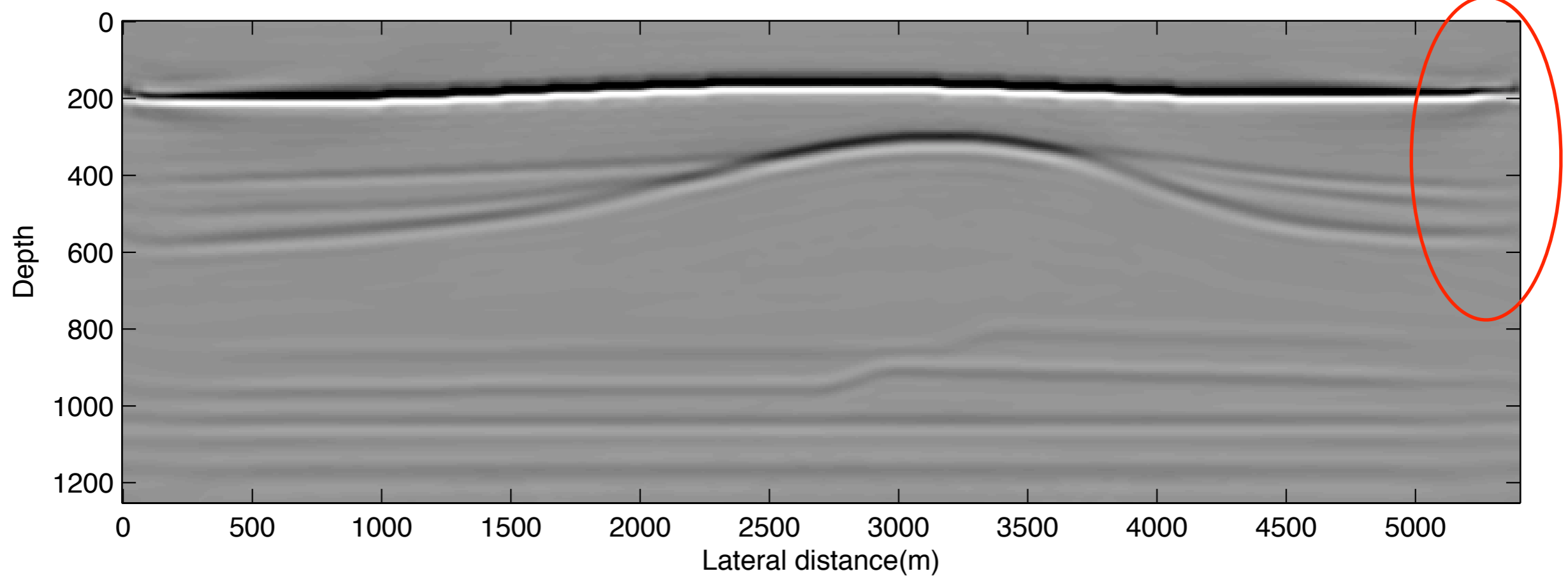
Mig/EPSt-with multiples



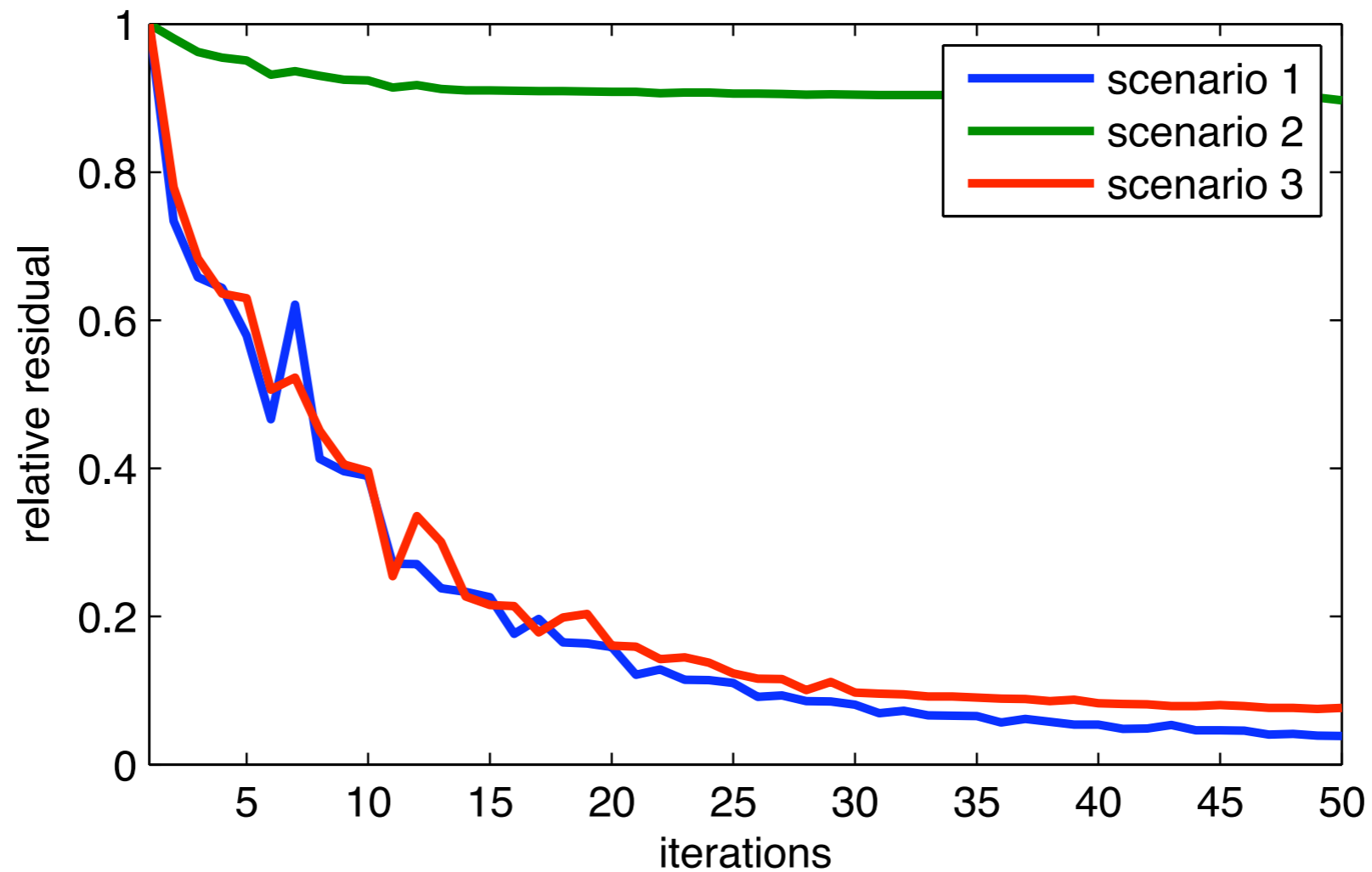
Mig/EPSt-with multiples



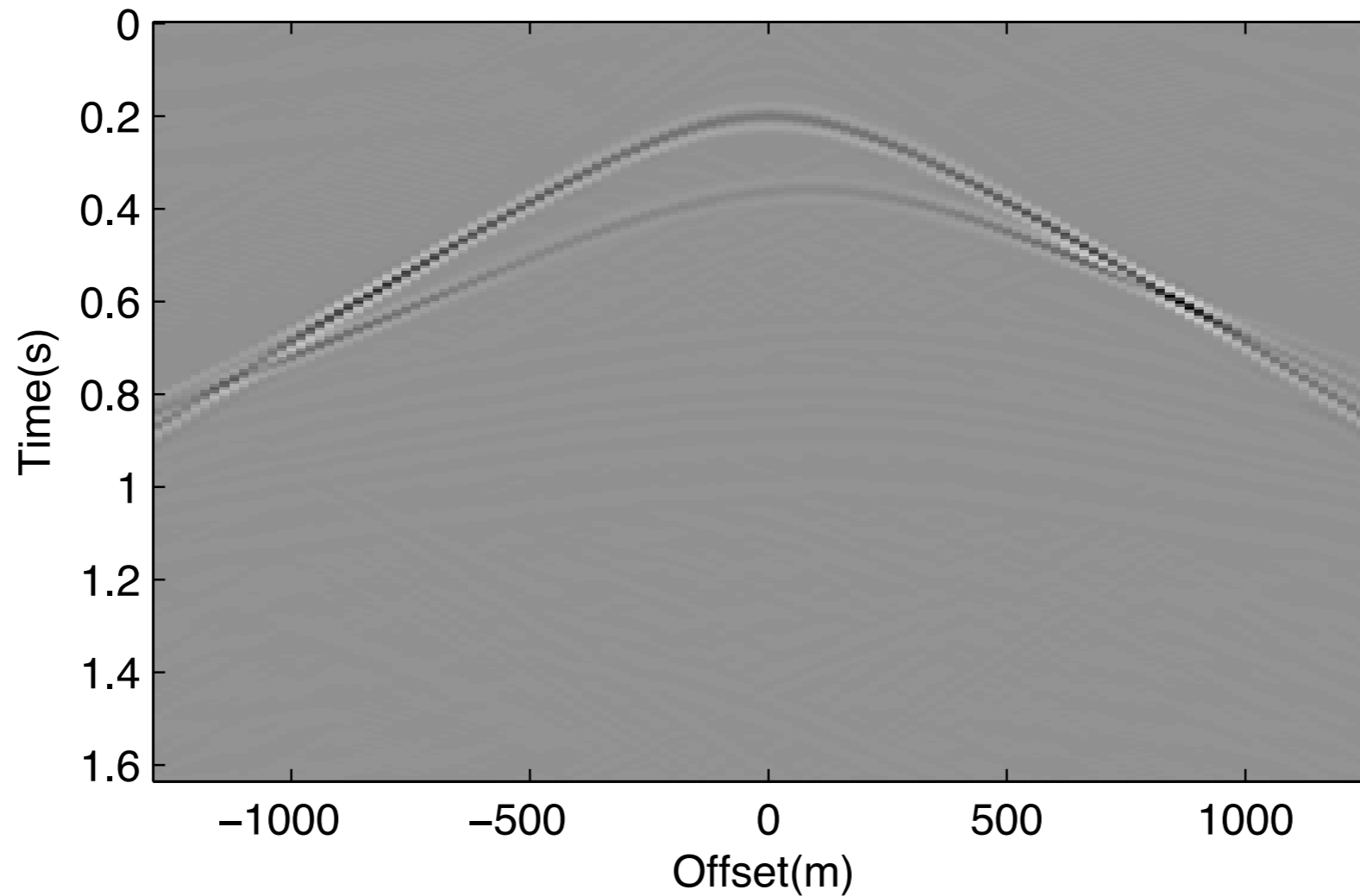
Mig-multiple free



Convergence rate with/ without EPSI

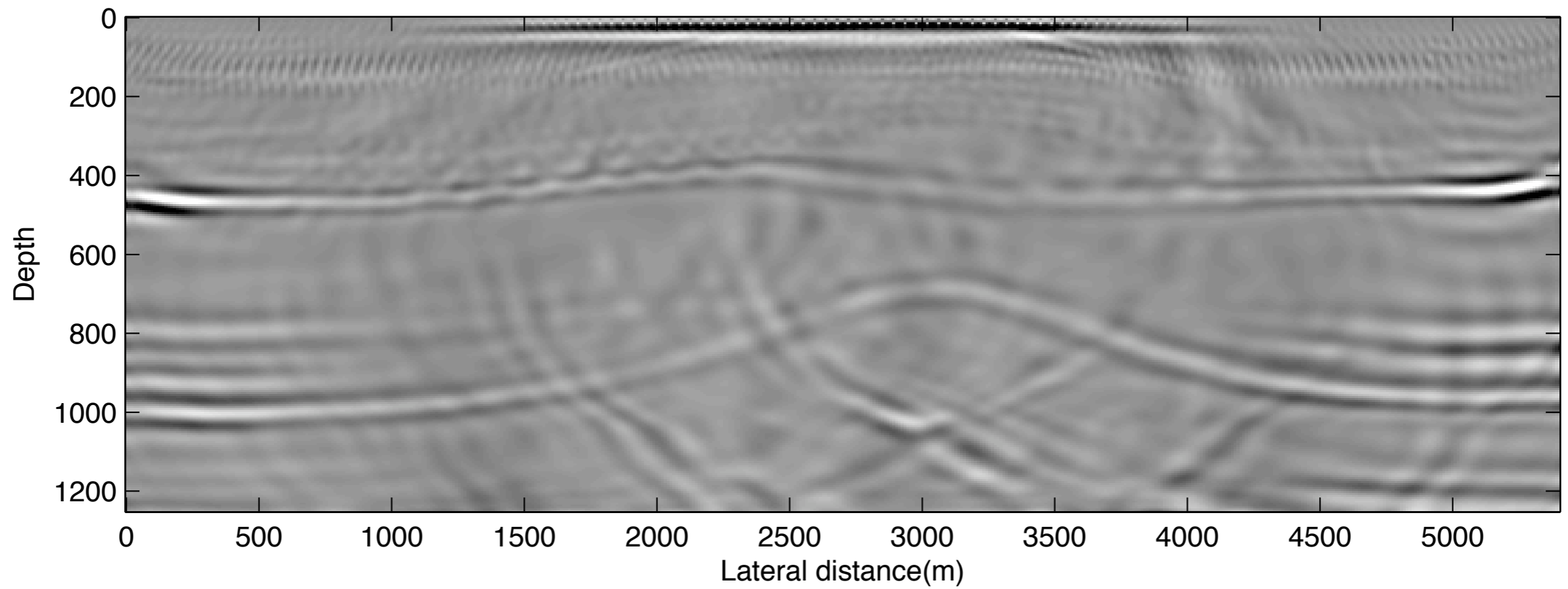


De-migrated section

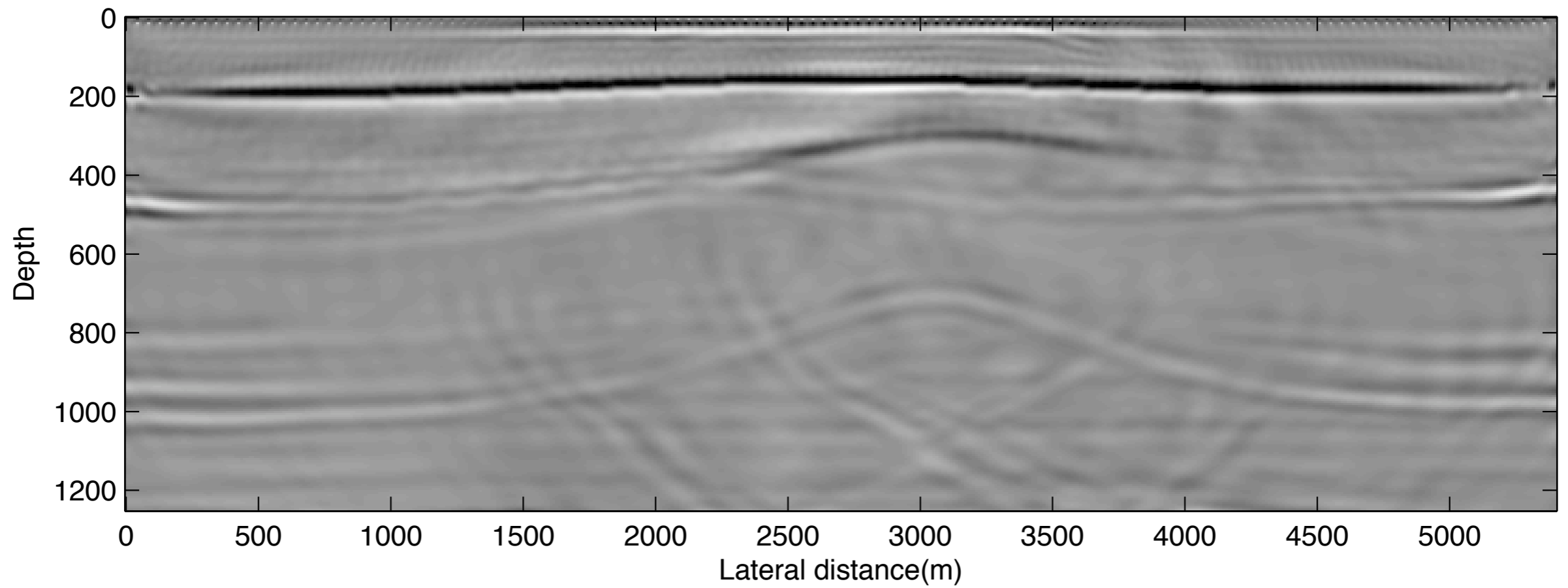


total shots: 128, shot number: 65, SNR: 23dB

Mig-multiples



Mig/EPSt-multiples



Conclusions

By combining EPSI with migration:

- multiples are well handled
- multiples actually help imaging

Future plans

Alternating optimization

- now EPSI operator is built using a pre-calculated wavelet
- wavelet will be estimated during the imaging process

Incorporate into full-waveform inversion

References

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Acknowledgement

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE II (375142-08).

This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BP, Chevron, ConocoPhillips, Petrobras, Total SA, and WesternGeco.

E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying for CurveLab (www.curvelet.org)

E. van der Berg and M. Friedlander for SPGL1 (www.cs.ubc.ca/labs/scl/spgl1/)

Thanks

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