Higher dimensional blue-noise sampling schemes for curvelet-based seismic data recovery

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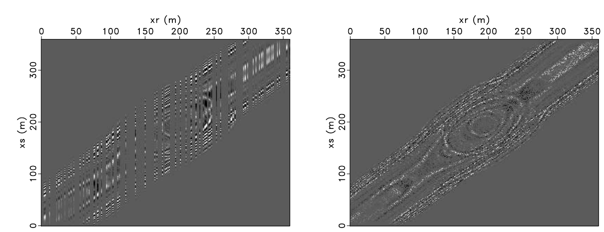
SUMMARY

In combination with compressive sensing, a successful reconstruction scheme called Curvelet-based Recovery by Sparsity-promoting Inversion (CRSI) has been developed, and has proven to be useful for seismic data processing. One of the most important issues for CRSI is the sampling scheme, which can greatly affect the quality of reconstruction. Unlike usual regular undersampling, stochastic sampling can convert aliases to easy-to-eliminate noise. Some stochastic sampling methods have been developed for CRSI, e.g. jittered sampling, however most have only been applied to 1D sampling along a line. Seismic datasets are usually higher dimensional and very large, thus it is desirable and often necessary to develop higher dimensional sampling methods to deal with these data. For dimensions higher than one, few results have been reported, except uniform random sampling, which does not perform well. In the present paper, we explore 2D sampling methodologies for curvelet-based reconstruction, possessing sampling spectra with blue noise characteristics, such as Poisson Disk sampling, Farthest Point Sampling, and the 2D extension of jittered sampling. These sampling methods are shown to lead to better recovery and results are compared to the other more traditional sampling protocols.

INTRODUCTION

Seismic data volumes are often high dimensional and extremely large, and incomplete with missing traces due to complex acquisition conditions. Sometimes because of computational or economical reasons, we also wish to reduce the number of sources or receivers, i.e. only sample some traces. Thus the recovery from incomplete data becomes a very important issue. Obviously, in general if samples are added to an already selected sample set, we obtain higher-resolution reconstructed data, which is desirable. However, on the other hand, in order to save measurement costs in the field as well as the computational cost for reconstruction, the fewer samples that are acquired the better. These two competing requirements lead us to design sampling methodologies that minimize the number of samples necessary while at the same time maintaining the quality of the reconstructed data volume from these samples. Traditionally, the Shannon/Nyquist sampling theorem states that it is necessary to sample with at least twice the rate of the signal's bandwidth. However this is for uniform regular sampling. A newly developed theory called "compressed sensing" (CS) (Candès et al., 2006b; Donoho, 2006) provides new insights, and opens the possibility of reconstructing compressible images or signals of scientific interest accurately from only a few samples far smaller than the Nyquist rate.

Based on CS theory, a successful recovery method for seismic data, named Curvelet-based Recovery by Sparsity-promoting Inversion (CRSI), was developed in Herrmann and Hennenfent (2008). It is derived from a sparsifying transform in conjunction with an undersampling scheme that favors recovery. For the sparsifying transform, it has been proven that curvelets are a very good choice, with their curve-like basis and well-documented sparsity for seismic data wavefronts (Candès et al., 2006a; Herrmann et al., 2007; Ma and Plonka, 2009). When regular undersampling is used as the underlying sampling scheme input to this reconstruction, performance is poor due to the presence of well-known periodic aliases. On the other hand, stochastic undersampling can render coherent aliases into easy-to-remove incoherent noise in the frequency domain, so that CRSI reconstrunction becomes a simple denoising problem. Unfortunately, the commonly used uniform random sampling cannot control gap lengths between missing traces, and this can greatly affect the quality of reconstruction. Thus, jittered sampling was introduced to mitigate this issue (Hennenfent and Herrmann, 2007, 2008), so that gap size is limited while at the same time aliases are converted to noise. Tang et al. (2009) proposed an optimized sampling strategy to improve the sampling scheme for CRSI. Compared to jittered sampling, it reduces the spectral leakage and also controls the maximum gap size directly. But most of these other methods only apply to 1D sampling, i.e. only sample some traces along one space axis of the data, as shown in Fig. 1(a). However, most seismic datasets are higher dimensional, for example, 3D volumes, so sampling along one axis with a 1D sampling method is not sufficient. In this paper, some higher dimensional sampling schemes are presented for use with seismic data interpolation by CRSI, for example, 2D sampling for 3D (i.e., seismic lines organized into time-source-receiver volumes) recovery, as shown in Fig. 1(b).



(a) (b)

Figure 1: 25% random samples on time slice: (a) 1D and (b) 2D sampling

In the field of image processing, sampling patterns with blue noise spectra have been proven to be able to scatter aliasing artifacts throughout the spectrum out of the signal band as broad-band noise, which is easily filtered out (Dippé and Wold, 1985; Ignjatovic and Bocko, 2005). Blue noise refers to a signal whose energy is concentrated at high frequencies with little energy concentrated at lower non-zero frequencies. Poisson

Disk sampling and Farthest Point Sampling are two such kinds of methods, which yield good blue noise spectra. This feature is very effective to obtain better reconstructions. Fig. 2 shows the blue-noise frequency features of these methods. We can see that Poisson Disk sampling, for example, gives a very good blue-noise spectrum, scattering the noise over a wide band and making it of low amplitude, so that it is easy to filter out.

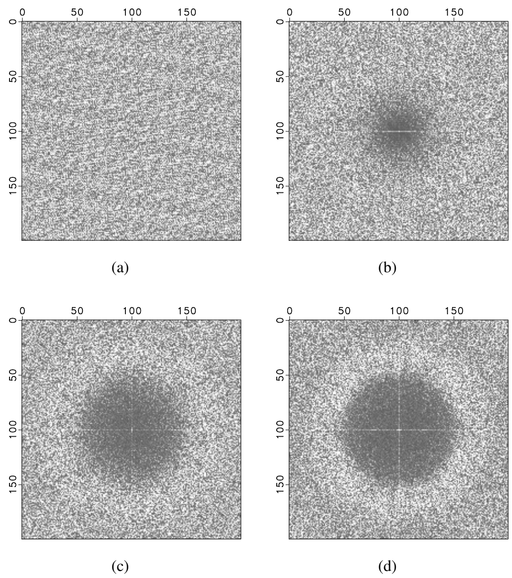


Figure 2: Spectra of 2D sampling with 10% points: (a) random sampling, without blue noise spectrum, (b) sub-optimal jittered sampling, (c) Poisson Disk sampling and (d) Farthest Point Sampling

With regard to higher dimensional sampling for curvelet-based seismic data recovery, there are a few papers in the literature. Herrmann and Hennenfent (2008) used 2D uniform random sampling while performing 3D interpolation, however their results are not as good as expected. Knowing that jittered sampling is better than random sampling in one dimension, it is natural to extend it to 2D (László, 1995). We found that by doing this, the results are good but not excellent, as shown in Fig. 2(b) and Fig. 3(c). Then we would like to introduce another two types of blue noise sampling schemes, Poisson Disk sampling (Cook, 1986) and Farthest Point Sampling (Eldar et al., 1997), which we apply to CRSI in this paper.

CRSI RECONSTRUCTION METHOD

The reconstruction from an incomplete seismic dataset follows the forward model

$$\mathbf{y} = \mathbf{R}\mathbf{m} \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^n$ represents the acquired incomplete data with missing traces, $\mathbf{m} \in \mathbb{R}^m$ is the model, to be recovered, i.e. the adequately sampled data, and $\mathbf{R} \in \mathbb{R}^{n\times m}$ is the restriction operator that collects the acquired samples from $\mathbf{m}$, $m \gg n$. Thus $\mathbf{R}$ is a sampling matrix, on which both the acquired data $\mathbf{y}$, and the recovery of model $\mathbf{m}$ depend. We will present more about sampling schemes that define this matrix in the next section.

The solutions of 1 are not unique or are acutely sensitive to changes in the data – this is an underdetermined inverse problem. In Herrmann and Hennenfent (2008), it was suggested to reformulate the problem as follows:

$$\mathbf{y} = \mathbf{R}\mathbf{C}^H\mathbf{x}, \tag{2}$$

where $\mathbf{C}$ is the curvelet transform and $\mathbf{C}^H$ is its adjoint— i.e., its conjugate transpose, and $\mathbf{x} \in \mathbb{R}^N$ with $N \gg n$ is the representation of $\mathbf{m}$ in the curvelet domain. The curvelet transform gives a sparse representation of $\mathbf{m}$, which means that the vector $\mathbf{x}$ has few non-zero coefficients. These properties make it possible to successfully recover $\mathbf{m}$ according to the theory of *compressive sampling* (Candès et al., 2006b; Candès, 2006). However to solve this underdetermined problem, additional information must be provided to *regularize* the problem. The CRSI method promotes sparsity as a regularization term and gives a solution to problem 2 by

$$\mathbf{P}_\sigma : \qquad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{R}\mathbf{C}^H\mathbf{x} - \mathbf{y}\|_2 \le \sigma \\ \widetilde{\mathbf{m}} = \mathbf{C}^H\widetilde{\mathbf{x}}, \end{cases} \tag{3}$$

where $\|\mathbf{x}\|_1 \stackrel{\text{def}}{=} \sum_{i=1}^N |x_i|$ is the ℓ_1 norm. The recovered vector that solves $\mathbf{P}_\sigma$ is $\widetilde{\mathbf{x}}$ and $\widetilde{\mathbf{m}} \in \mathbb{R}^m$ is the estimate of the recovered data obtained by applying $\mathbf{C}^H$. The data misfit is conditioned by σ, which is linked to the noise variance. In our case there is no noise, so $\sigma = 0$.

SAMPLING METHODS

From traditional sampling theory, we know that, with regular sampled points, aliasing will occur at frequencies higher than the Nyquist limit, due to the regular and periodic nature of the sampling. If we sample in an irregular manner to make the sizes of unsampled regions incoherent with each other, these aliases can be converted into easy-to-remove noise. Stochastic sampling is a way of achieving this, by distributing samples randomly, so that every point has a finite probability of being sampled (Dippé and Wold, 1985). There are many kinds of stochastic samplings that have been developed. In this paper, we concentrate on three such schemes, Poisson Disk sampling, Farthest Point Sampling, as well as the extension of jittered sampling to 2D, all of which have blue noise sampling spectra.

Pure uniform random sampling, where each location on a grid has exactly the same probability of being chosen, is too random since it cannot control the size of gaps between samples. Such pure random sampling converts aliases into white noise in the frequency domain, as shown in Fig. 2(a), which in fact is very difficult to remove. On the other hand, uniform jittered sampling first subdivides the space into n regions, with n the predetermined number of samples we wish to take, and then randomly takes one sample in each region. Because each region is sampled, and the regions form a partition of the space (they are contiguous) , the size of gaps can be controlled to some extent and a blue noise spectrum is obtained, as in Fig. 2(b). However, often the samples cluster. Poisson Disk sampling can solve these problems. It selects n points at random iteratively, and only keeps a sample if it is a sufficient distance away from all previously selected samples. This also leads to a blue noise spectrum. Finally, Farthest Point Sampling is another irregular sampling scheme with excellent anti-aliasing properties, and is based on the computational geometry concept of the Voronoi diagram (Eldar et al., 1997). The main idea for that scheme, as suggested by its name, is to repeatedly place the next sample point to be the farthest point from all previously selected samples. This ensures that there are no regions of the image that are not adequately sampled, while at the same time, some randomness is maintained by initially selecting a small number of randomly selected seed samples.

RESULTS

First, we applied these three sampling methods to 1D reconstruction, recording the SNR with different percentages of samples. We define $SNR = 20log_{10}\frac{\|f_0\|_2}{\|f-f_0\|_2}$, where f_0 is the original data, and f is the interpolated data.

As shown in Fig. 4, both Poisson and Farthest Point Sampling give higher SNRs than uniform random sampling. Following that we show results for Poisson Disk sampling as an example of how a stochastic sampling scheme with blue noise characteristics leads to good results from the CRSI reconstruction process.

Then 2D Poisson Disk sampling is applied to a time slice, as shown in Fig. 3. Poisson Disk sampling gives better reconstruction than pure uniform random sampling, and also somewhat better than 2D jittered sampling.

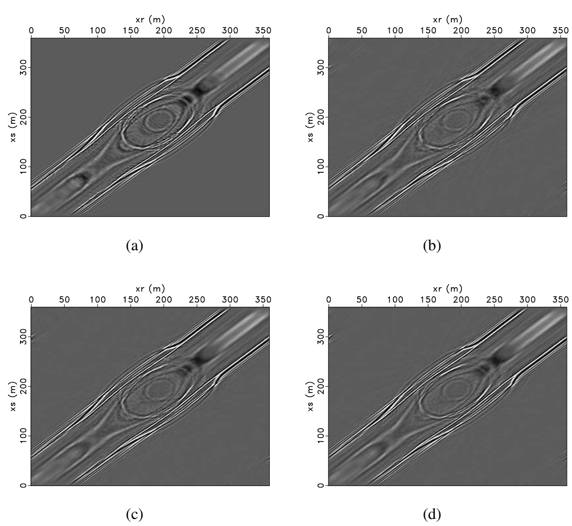


Figure 3: (a) time slice model, and recoveries from 25% samples by 2D sampling of (b) random, SNR = 9.979, (c) jittered, SNR = 10.594 and (d) Poisson Disk, SNR = 10.931

Finally we applied these methods to higher dimensional cubes, which are always very large, necessitating a strategy to reconstruct them from as few traces as possible, in order to design seismic data acquisition with less shots or receivers. Here we only sample 25% of the total traces with random and Poisson Disk sampling methods, then do the recovery by CRSI. As shown in Fig. 5, Poisson Disk sampling is much better than uniform random sampling.

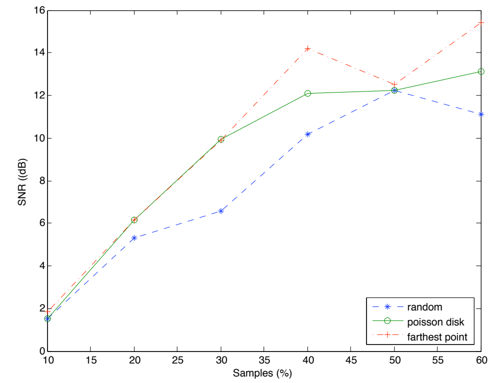


Figure 4: SNR, with different numbers of sampling points

CONCLUSIONS

In this paper, we explored some sampling schemes with blue-noise patterns for curvelet-based interpolation. We extended jittered sampling to 2D, and introduced Farthest Point and Poisson Disk sampling into the reconstruction. From the comparisons between these methods, including uniform random sampling, we know that 2D jittered sampling is still better than uniform random sampling, while Farthest Point and Poisson Disk sampling often give better results over jittered sampling. We applied these methods, choosing Poisson Disk sampling as an example, to 2D and 3D seismic data interpolation by CRSI, and obtained very good results, as compared to methods previously used in the literature.

The present work mainly used CRSI, where the curvelet transform is regular. However it is also possible use the non-equispaced fast discrete curvelet transform (NFDCT), which leads to a method called non-uniform CRSI (NCRSI). We are currently developing NCRSI for higher dimensions, in combination with the non-equispaced curvelet transform, and results from this higher-dimensional NCRSI will be presented in a future work along with extensions to adaptive sampling, where prior information on the image is used to more heavily sample image regions containing more information, e.g. edges.

ACKNOWLEDGMENTS

This work was in part financially supported by the NSERC Diskovery Grant 22R81254 and CRD Grant DNOISE 334810-05 of F.J. Herrmann. The first author would also like to thank Chinese Scholarship Council, National Basic Research Program of China (973 program, 2007CB209505).

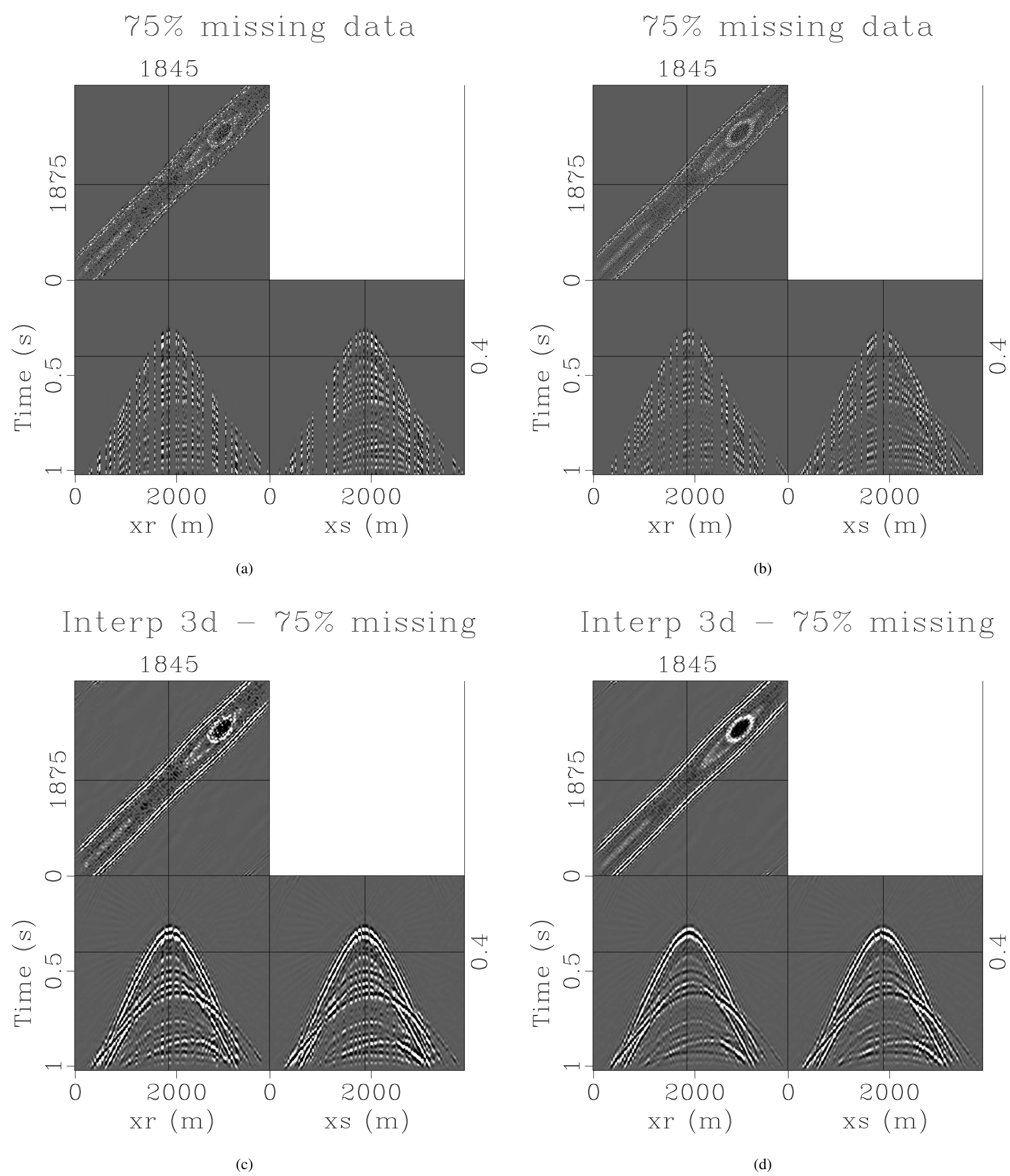


Figure 5: Sampled 3D data with 75% missing traces by: (a) random sampling (b) Poisson Disk sampling, (c), (d) are recovered by CRSI from (a) and (b) separately

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