Curvelet-domain matched filtering with frequency-domain regularization and an application to primary-multiple separation

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SUMMARY

In Herrmann et al. (2008a), it is shown that zero-order pseudodifferential operators, which model the migration-demigration operator and the operator mapping the predicted multiples to the true multiples, can be represented by a diagonal weighting in the curvelet domain. In that paper, a smoothness constraint was introduced in the phase space of the operator in order to regularize the solution to make it unique. In this paper, we use recent results in Demanet and Ying (2008) on the discrete symbol calculus to impose a further smoothness constraint, this time in the frequency domain. It is found that with this additional constraint, faster convergence is realized. Results on a synthetic pseudodifferential operator as well as on an example of primary-multiple separation in seismic data are included, comparing the model with and without the new smoothness constraint, from which it is found that results of improved quality are also obtained.

INTRODUCTION

In Herrmann et al. (2008a), it is shown that zero-order pseudodifferential operators, which model the migration-demigration operator and the operator mapping the predicted multiples to the true multiples in a seismic image, can be represented by a diagonal weighting in the curvelet domain. In that paper, because of non-uniqueness of coefficients in the curvelet domain, a smoothness constraint was introduced in the phase space of the operator. Here, recent results in Demanet and Ying (2008) on the discrete symbol calculus are used to impose a further smoothness constraint, this time in the frequency domain. It is found that including this additional constraint leads to faster convergence. Results on a synthetic pseudodifferential operator as well as on an example of primary-multiple separation in seismic data are included, showing that indeed both the rate of convergence and result quality are ameliorated.

PROBLEM STATEMENT

In Demanet and Ying (2008), the fact is derived and used that for zero-order pseudodifferential operators, a hierarchical spline representation may be used to represent the first-order Fourier transform of the symbol $a(\mathbf{x},\zeta)$ with respect to the spatial variable(s) x since this Fourier transform is smooth in the original frequency variable ζ. In this paper, we use this additional information by penalizing derivatives of this Fourier transform in ζ and obtain improved results.

MATCHED FILTERING IN THE CURVELET DOMAIN

As is well established in the literature, curvelets are an important tool in the modeling of seismic images due to their ability to represent such data in a sparse manner. The whole procedure of curvelet-domain matched filtering is based on the approximation of what are called pseudodifferential operators by a diagonal weighting in the curvelet domain.

Pseudodifferential Operators

The standard formula for a pseudodifferential operator Ψ defined in terms of its symbol $a(\mathbf{x},\zeta)$ is as follows:

$$\Psi(\mathbf{f})(\mathbf{x}) = \int_{\Omega} a(\mathbf{x},\zeta)e^{-i\mathbf{x}\cdot\zeta}\hat{\mathbf{f}}(\zeta)d\zeta, \quad (1)$$

where ζ is the wavenumber. In this paper, we restrict our attention to such operators in two spatial variables and two frequency variables, so that $\mathbf{x}$ and ζ are each 2D. A pseudodifferential operator corresponds to a spatially-dependent convolution, and may be used for many applications in seismic data processing that include amplitude-preserving estimation of a migration operator from a reference image and a remigrated image Herrmann et al. (2008a) and a curvelet-domain matched filter as part of primary-multiple separation Herrmann et al. (2008b), and ground-roll removal (Yan and Herrmann, private correspondence). It can be shown that for these applications, this operator possesses the property of smoothness in phase space, meaning that the symbol $a(\mathbf{x},\zeta)$ is smooth in both $\mathbf{x}$ and ζ, and that it is also positive-definite, which implies that the symbol $a(\mathbf{x},\zeta)$ is always positive.

In Herrmann et al. (2008a), it is proven that given a pseudodifferential operator, Ψ, can be represented by a curvelet-domain diagonal weighting. In other words, if a curvelet $\phi_{u,v}$ is input to Ψ, a scaled version of the same curvelet $\phi_{u,v}$ will be obtained (up to an exponentially small error in the scale of the curvelet). This scaling is in fact proven to be equal to the symbol of the operator Ψ evaluated at the center of the curvelet $\phi_{u,v}$ in phase space. Therefore the following relation is obtained:

$$\Psi(\mathbf{f})(\mathbf{x}) \approx \mathbf{C}^{\mathbf{T}}\mathbf{D}_{\Psi}\mathbf{C}\mathbf{f}(\mathbf{x}), \quad (2)$$

where the equality is approximate since this is a high frequency asymptotic result. In the above equation, $\mathbf{D}_{\Psi}$ is the diagonal curvelet-weighting matrix consisting of $a(\mathbf{x},\zeta)$ evaluated at the curvelet centres, and $\mathbf{C}$ the 2-dimensional curvelet-transform matrix ($\mathbf{C}^{\mathbf{T}}$ is its adjoint, which also happens to be its pseudo-inverse).

Ideally, given the input and output of Ψ, say $\mathbf{f}$ and $\mathbf{g}$ respectively, the problem of finding the underlying operator reduces to finding the elements of the diagonal matrix $\mathbf{D}_{\Psi}$, which can be achieved by solving the least-squares problem,

$$\underset{\mathbf{z}}{\operatorname{argmin}} \frac{1}{2}||\mathbf{C}^{\mathbf{T}}\operatorname{diag}(\mathbf{C}\mathbf{f})e^{\mathbf{z}} - \mathbf{g}||_2^2, \quad (3)$$

with the diagonal of $\mathbf{D}_\Psi$ equal to $e^{\mathbf{z}}$ (the exponential is introduced to ensure positivity of the symbol). However because of non-uniqueness of the curvelet coefficient space (many coefficient vectors may map to the same image under the adjoint operator $\mathbf{C^T}$), a smoothness constraint on $\mathbf{z}$ was also introduced into the least squares problem 3 in Herrmann et al. (2008a) to regularize the problem. It was shown that this indeed led to a positive solution for $\mathbf{z}$, however it will be seen later that even better results may be obtained with additional information on the symbol.

The regularized least-squares problem becomes:

$$\underset{\mathbf{z}}{\operatorname{argmin}}\frac{1}{2}||\mathbf{C^T}\operatorname{diag}(\mathbf{Cf})e^{\mathbf{z}}-\mathbf{g}||_2^2+\lambda^2||\mathbf{L}e^{\mathbf{z}}||_2^2, \qquad (4)$$

where now the matrix $\mathbf{L}$ is a matrix whose product with the scaling vector equals the partial derivatives of the scaling vector with respect to spatial position and dip in the phase space $(\mathbf{x},\zeta)$, and λ is a regularization parameter.

REGULARIZATION IN FREQUENCY

Though the additional derivative 2-norm term in the previous selection is necessary to regularize the underlying estimation of the pseudodifferential operator Ψ, it will be shown that further information about the symbol can be introduced into the model to further improve estimation of Ψ by a curvelet-domain diagonal weighting.

In Demanet and Ying (2008), the symbol $a(\mathbf{x},\zeta)$ is assumed to be highly separable (i.e. a product of a function of $\mathbf{x}$ and ζ), and is expanded into the following series:

$$a(\mathbf{x},\zeta)=\sum_{\alpha,\beta}a_{\alpha,\beta}e_\alpha(\mathbf{x})g_\beta(\zeta)(1+|\zeta|^2)^{\frac{d_a}{2}},$$

where $\mathbf{x}$ is the spatial variable, ζ is the frequency variable, and $e_\alpha(\mathbf{x})$ and $g_\beta(\zeta)$ are basis functions and $a_{\alpha,\beta}$ are unknown series coefficients. The variable d_a refers to the order of the pseudodifferential operator, and in the context of the matched filter, $d_a=0$. In the same paper, the functions $e_\alpha(\mathbf{x})$ and $g_\beta(\zeta)$ are chosen to be the Fourier basis and a smooth function respectively ($e_\alpha(\mathbf{x})=e^{2\pi i\mathbf{x}\cdot\alpha}$ and $g_\beta(\zeta)$ a hierarchical spline function with control points made scarcer as $|\zeta|\to\infty$).

Therefore,

$$a(\mathbf{x},\zeta)=\sum_{\alpha,\beta}a_{\alpha,\beta}e^{2\pi i\mathbf{x}\cdot\alpha}g_\beta(\zeta).$$

Taking the Fourier Transform once with respect to $\mathbf{x}$ of each side of the above equation yields:

$$\hat{a}^{(\mathbf{x},1)}(\mathbf{x},\zeta):=\hat{a}(\eta,\zeta)=\sum_{\alpha,\beta}a_{\alpha,\beta}g_\beta(\zeta),$$

wtih η the new frequency variable in $\mathbf{x}$. Because each $g_\beta(\zeta)$ is a smooth spline, this Fourier Transform is also smooth in ζ.

Note that this smoothness result is contingent both on the smoothness in ζ of the original symbol as well as the separability of the symbol in $\mathbf{x}$ and ζ. That is, if the symbol were not known to be strongly separable, then the spatial Fourier transform would not necessarily be smooth in ζ. A simple example would be $a(x,\zeta)=\sin(x+\frac{|\zeta|}{100})$, for which the Fourier transform is

$$\hat{a}(x,\zeta)=\delta\left(\eta-\left(x+\frac{|\zeta|}{100}\right)\right)/2i-\delta\left(\eta+\left(x+\frac{|\zeta|}{100}\right)\right)/2i.$$

This Fourier transform is clearly not smooth for all ζ, since at the positions of the impulses, ζ will change from an infinite value to zero as ζ changes infinitesimally.

This smoothness in the frequency domain of this symbol can be leveraged in the solution of the curvelet domain diagonal estimation procedure in Herrmann et al. (2008a) to provide extra regularization.

Therefore, we may obtain the following new regularized least-squares problem:

$$\underset{\mathbf{z}}{\operatorname{argmin}}\frac{1}{2}||\mathbf{C^T}\operatorname{diag}(\mathbf{Cf})e^{\mathbf{z}}-\mathbf{g}||_2^2+\lambda^2||\mathbf{L}e^{\mathbf{z}}||_2^2+\mu^2||\mathbf{M}_\zeta\mathbf{R}\mathbf{F_X}e^{\mathbf{z}}||_2^2. \qquad (5)$$

In equation 5, the new matrix $\mathbf{M}_\zeta$ is a matrix producing partial derivatives of its input scaling vector in ζ with respect to the frequency angle θ and the curvelet scale. The matrix $\mathbf{F_X}$ is a spatial Fourier transform operator acting on each curvelet wedge separately. Since the Fourier transform of a real-signal is in general complex-valued, though with conjugate symmetry, the restriction matrix $\mathbf{R}$ is used to only select elements (both real and imaginary parts) of the spectrum corresponding to non-negative frequencies in one of the variables. Finally, λ and μ are two scalar-valued regularization parameters, though in general more complicated adaptive non-scalar values of these parameters may be chosen.

EXPERIMENTS

Synthetic Pseudodifferential Operator

First, we applied a known pseudodifferential operator

$$\Psi(\mathbf{x},\zeta)=\cos^2(\theta),$$

once again with θ the frequency angle (corresponding to an angle perpendicular to reflector dip) to a test image, the Laplacian of the well-known marmousi image (the Laplacian is taken to amplify higher-frequency information). This application of the operator was implemented with the method of Symes et al Bao and Symes (1996).

The first figure, is the test image that is input to the known pseudodifferential operator Ψ. The second figure corresponds to the exact value of the symbol plotted for all scales, and wedge-by-wedge, which scale increasing from the center to the perimeter of the image and angle increasing clockwise around the image. The third figure is the symbol obtained after 50 iterations of LSQR minimization of Equation 4 (the exponential $e^{\mathbf{z}}$ is not used here, so that the problem becomes linear, though the obtained solution is still all positive in this case). Finally, the fourth figure is the symbol obtained with 20 iterations of LSQR minimization of Equation 5. Clearly, the

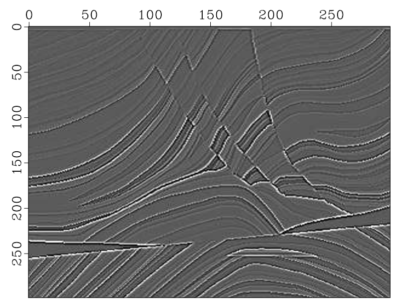

Figure 1: Test image (Laplacian of marmousi model) used as input to known pseudodifferential operator $\Psi = \cos(\theta)$

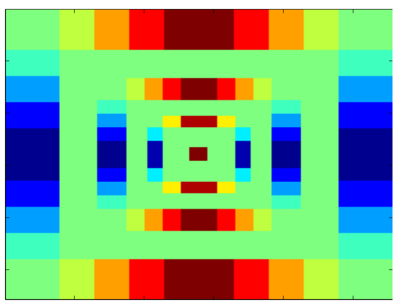

Figure 2: Plot of exact symbol for $\Psi = \cos(\theta)$

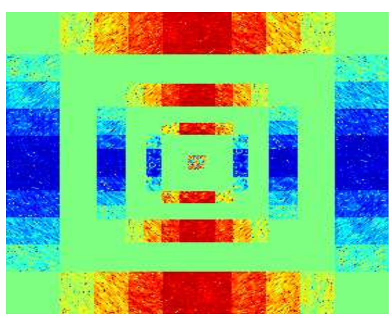

Figure 3: LSQR Solution of Optimization without Frequency Smoothness Constraint (50 iterations)

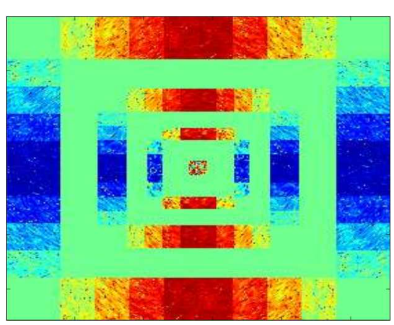

Figure 4: LSQR Solution of Optimization with Frequency Smoothness Constraint ($\mu = 200$, 20 iterations)

result in the final figure, corresponding to the new regularization of the model in frequency is closer to the exact symbol than the result in the second figure which does not include this additional regularization. What makes this more remarkable is that this better result is from only 20 iterations of LSQR, which is substantially faster than the 50 iterations required by the result without the new frequency smoothness term.

Primary-Multiple Separation

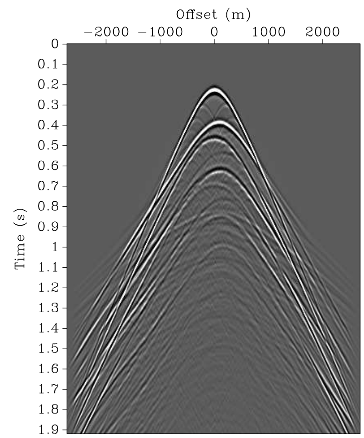

Figure 5: Total data consisting of both primaries and multiples

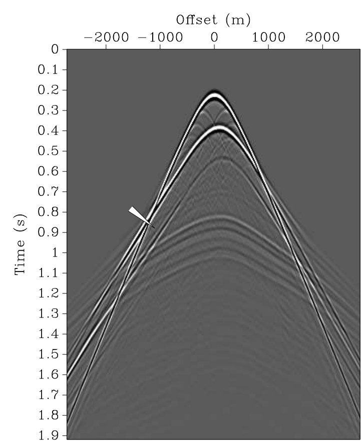

Figure 6: Ground truth multiple-free data

Previously, the curvelet-domain matched filter had been used with great success for the problem of primary-multiple separation Herrmann et al. (2008b), giving much better separation results than the traditional methods in the literature, e.g. Verschuur et al. (1992). In Herrmann et al. (2008b), the map between the predicted multiples and true multiples in seismic data was modeled as a linear operator $\mathbf{B}$ which applies a location, frequency and dip-dependent scaling. If the true multiples are $\mathbf{m_T}$, and the predicted multiples are $\mathbf{m_P}$, then this scaling can be represented by the equation:

$$\mathbf{B}\mathbf{m_P} = \mathbf{m_T}.$$

After removing source wavelet and directivity effects via a

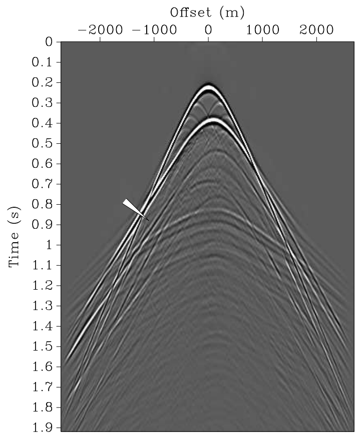

Figure 7: Primaries predicted from SRME

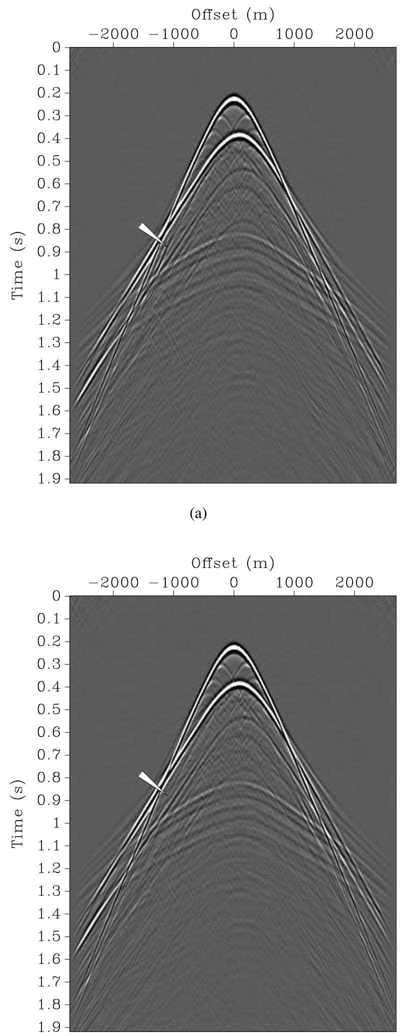

Figure 8: Estimated Primaries from Curvelet Matching, $\lambda = 0.5$: (a) without new Regularization (50 iter), SNR=8.996 and (b) with new Regularization (μ=1, 35 iter), SNR=9.148

conventional global matched-filtering procedure (possibly for each offset separately), it can be shown that the scaling can be represented by a zero-order pseudodifferential operator, which is approximately diagonalizable in the curvelet domain, so that

$$\mathbf{m_T} \approx \mathbf{C^T}\mathrm{diag}\left(\mathbf{w}\right)\mathbf{C}\mathbf{m_P}, \qquad (6)$$

with the elements of $\mathbf{w}$ all positive, and as before, $\mathbf{C}$ denoting the discrete curvelet transform. Since the true multiples are unknown, the total data $\mathbf{d}$ is used to approximate $\mathbf{m_T}$.

The initial guess used for the L-BFGS Liu and Nocedal (1989) optimization procedure to solve 5 was $\frac{|\mathbf{Cd}|}{|\mathbf{Cm_P}|}$. The curvelet-domain matched filter was implemented both with and without the new frequency smoothness regularization term. When compared to the ground truth primaries in Figure 6, the signal-to-noise ratio (computed using the formula $SNR = 20\log_{10}\frac{||f_0||_2}{||f-f_0||_2}$ for the primaries estimated without the new frequency-domain smoothness regularization was 8.996, while for the same parameters except for μ, the coefficient of the new regularization term, set to 1, we obtained an SNR of 9.148. This improved SNR is also obtained in fewer iterations of the L-BFGS optimization - 35 iterations were run for the new model and 50 iterations for the old one. Though the SNR difference between the two results may not seem to be substantial, it is clear from the figures that the new result is visually much closer to the ground truth primaries in Figure 6, especially in the part of the image pointed to by the arrow, where there is clearly some multiple activity in the obtained primaries without the new frequency-domain regularization but less so with the new regularization.

Currently, scale derivatives are not taken for the frequency term, however to make things more correct this would have to be done. This is expected to even further improve the obtained results and speed up convergence.

CONCLUSIONS

In this paper, an extension to the curvelet-domain matched filtering procedure is proposed that leverages additional smoothness in the spatial frequency variable. We use this smoothness to further regularize the estimation of coefficients of the curvelet-domain matched filter. Experimental results indicate that the new regularized model produces better results in far fewer iterations. We also showed that our matching procedure yields improved results in fewer iterations when applied to primary-multiple separation.

Future work involves analysis based on L-curves to more systematically select the regularization parameters in the model as well as application of the new model to 3D seismic data, and other problems in seismic data analysis, such as estimation of the migration operator and ground roll removal.

ACKNOWLEDGMENTS

This work was in part financially supported by the NSERC Discovery Grant 22R81254 and CRD Grant DNOISE 334810-05 of F.J. Herrmann.

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