SEG HOUSTON 2009

Unified compressive sensing framework for **simultaneous acquisition** with **primary estimation**

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special thanks to G.J. van Groenestijn and Eric Verschuur

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key points

1) Establish seismic demultiplexing as a non-linear inversion process
   (Using techniques from aperture encoding, etc)

2) In that same process, also remove surface-related multiples via primary estimation

3) Joint inversion better than separate processing

outline

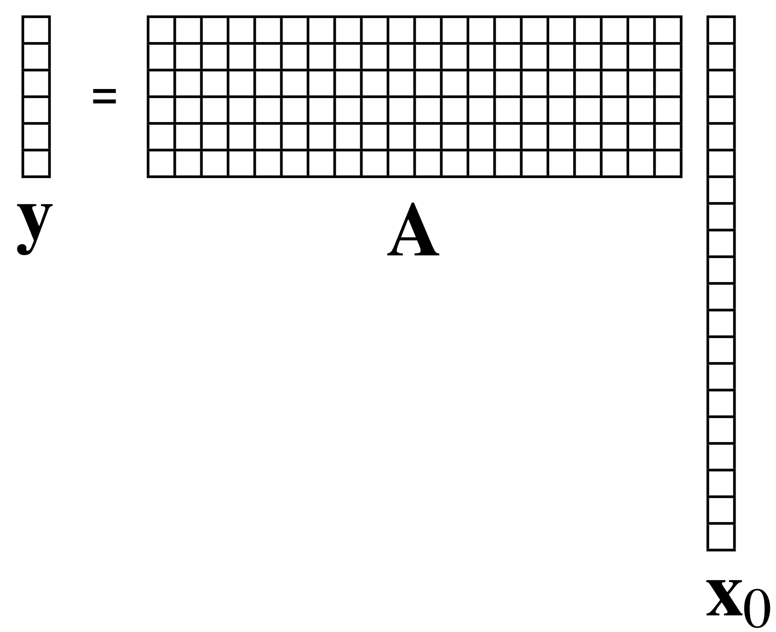
I. Simultaneous acquisition as compressive sensing

II. Inverting compressively sensed data

III. Primary estimation as inversion

IV. Joint CS and primary estimation inversion

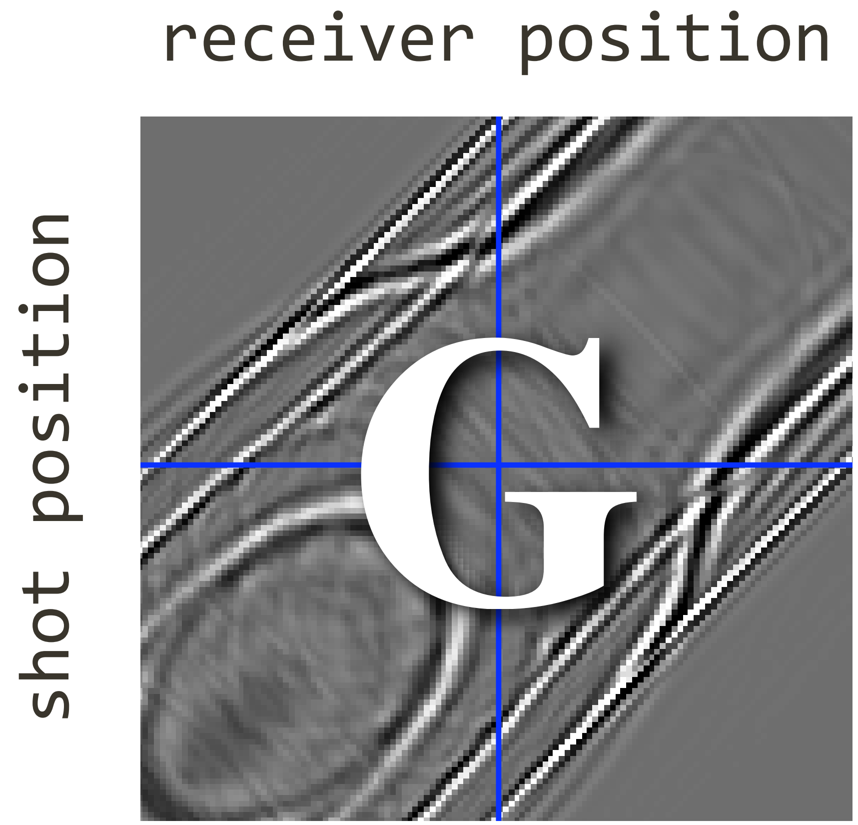
Compressed sensing



Is it possible to recover $\mathbf{x}_0$ accurately from $\mathbf{y}$?

(Candes, Romberg, Tao, 2006; Wakin, Baraniuk, Laska, 2006, Lustig, Donoho, Pauly, 2006)

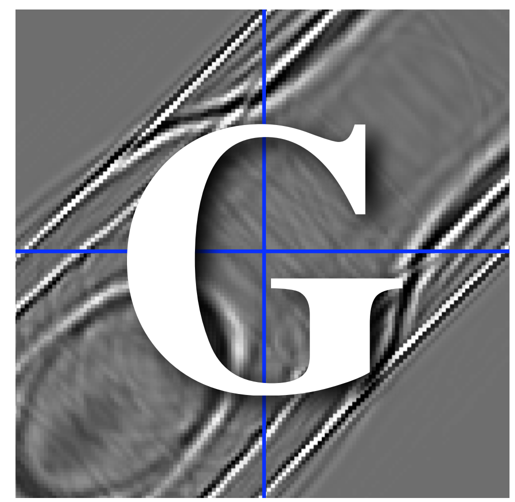
matrix view



Green's function

it’s linear algebra

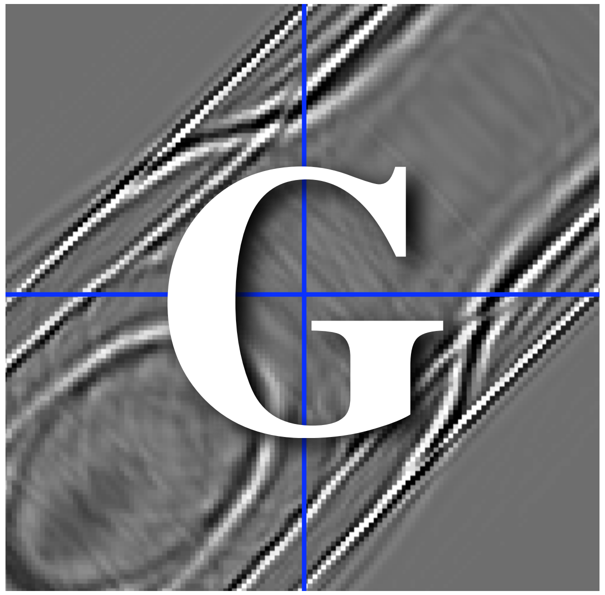
$$\mathbf{D} = \underbrace{\left[\;\mathbf{Q}\;\right]}_{\textbf{Shot}}\;\mathbf{G}\;\underbrace{\left[\qquad\right]}_{\textbf{Recv}}$$



represents acquisition of data

eg: ideal coverage

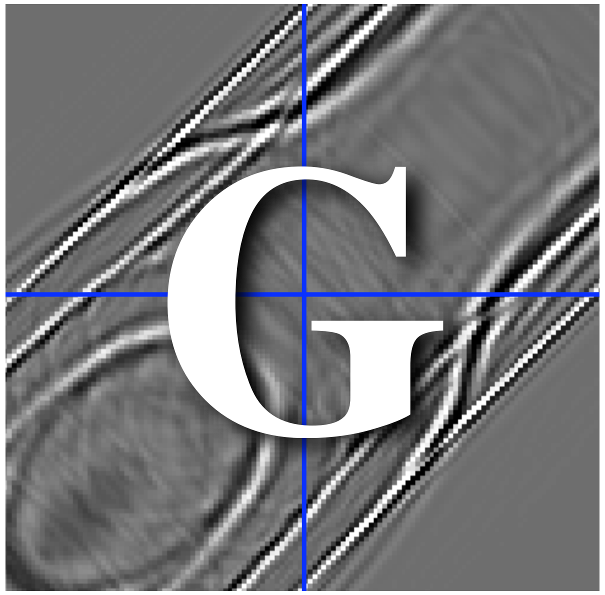
$$\mathbf{D} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \mathbf{G}$$



identity matrix

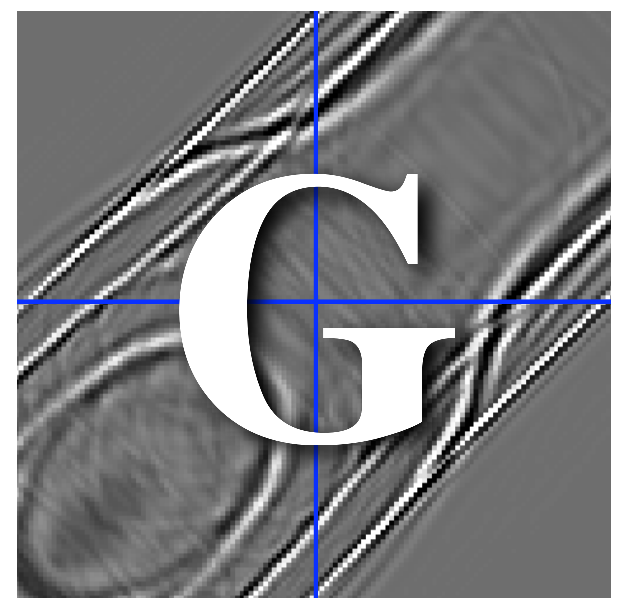
eg: 2x undersampled shots

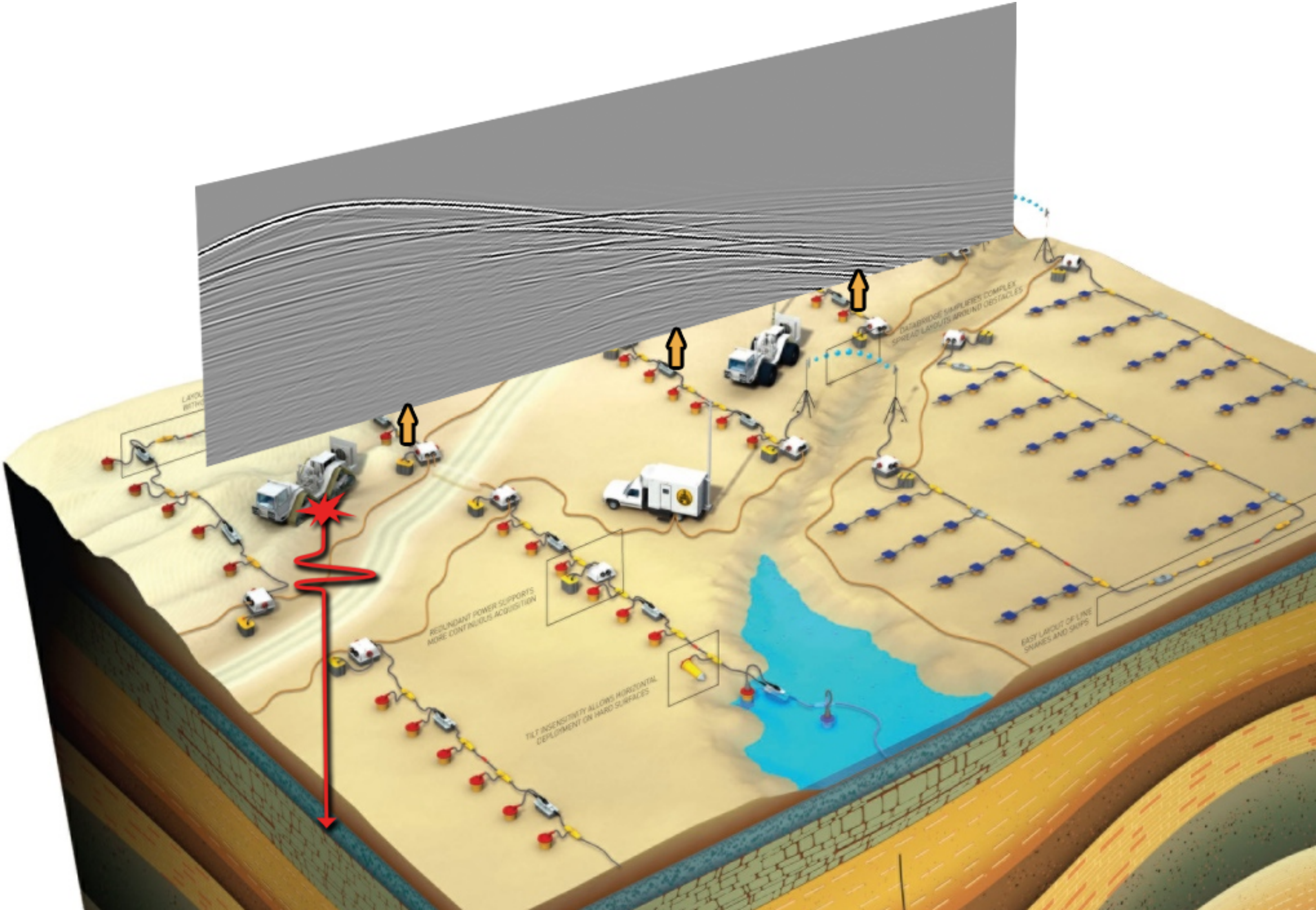
$$\mathbf{D} = \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix}$$



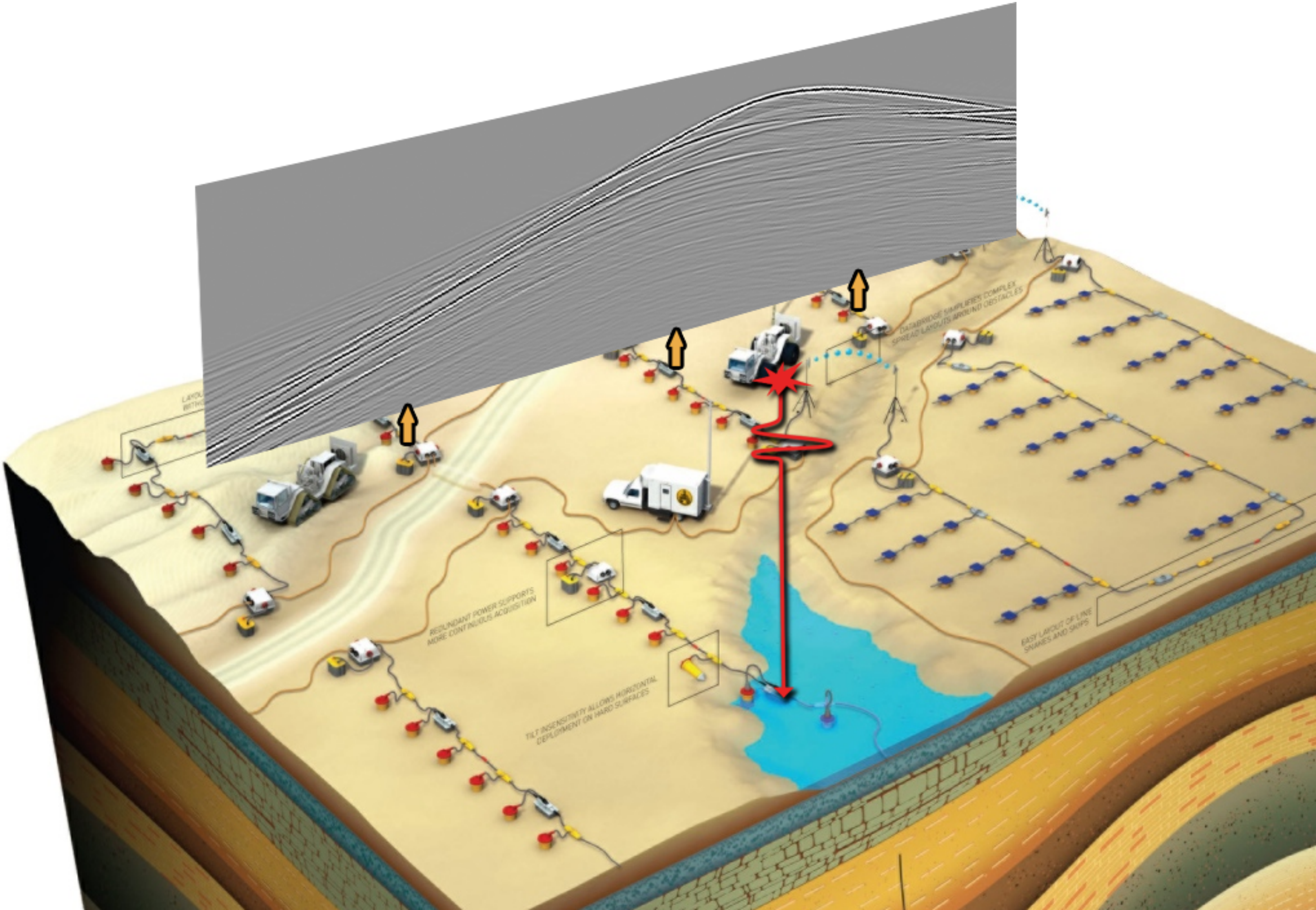
eg: Blend every other shot

$$\mathbf{D} = \begin{bmatrix} 1 & 1 & & & & \\ & & 1 & 1 & & \\ & & & & 1 & 1 \end{bmatrix} \mathbf{G}$$





REDUNDANT POWER SUPPORTS MORE CONTINUOUS ACQUISITION
TILT INSENSITIVITY ALLOWS HORIZONTAL DEPLOYMENT ON HARD SURFACES
DATABRIDGE SIMPLIFIES COMPLEX SPREAD LAYOUTS AROUND OBSTACLES
EASY LAYOUT OF LINE SNAKES AND SKIPS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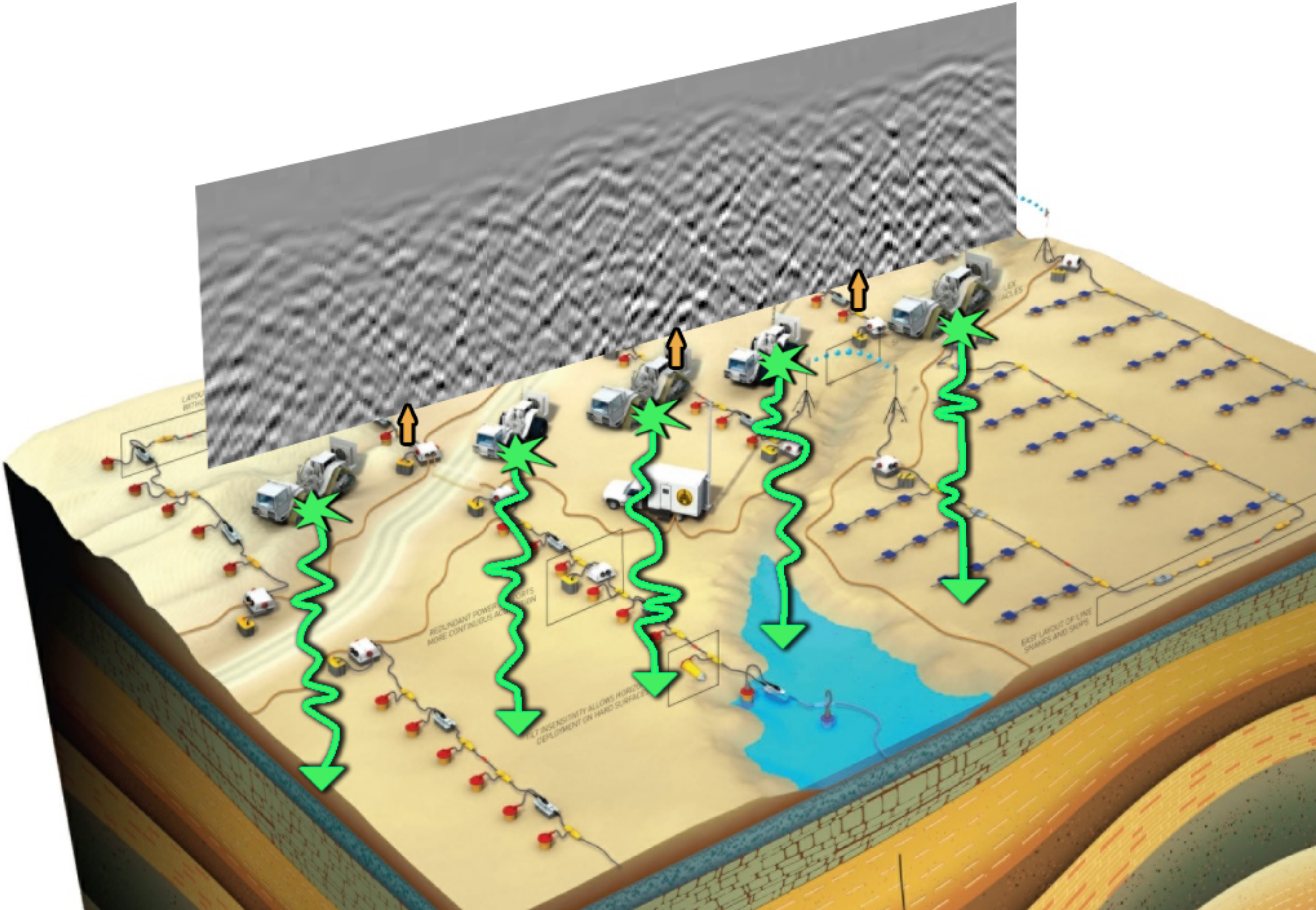


Compressed sensing

conditions:

- **A** obeys the *restricted isometry principle*
- $\mathbf{x}_0$ is *sufficiently sparse*

procedure:

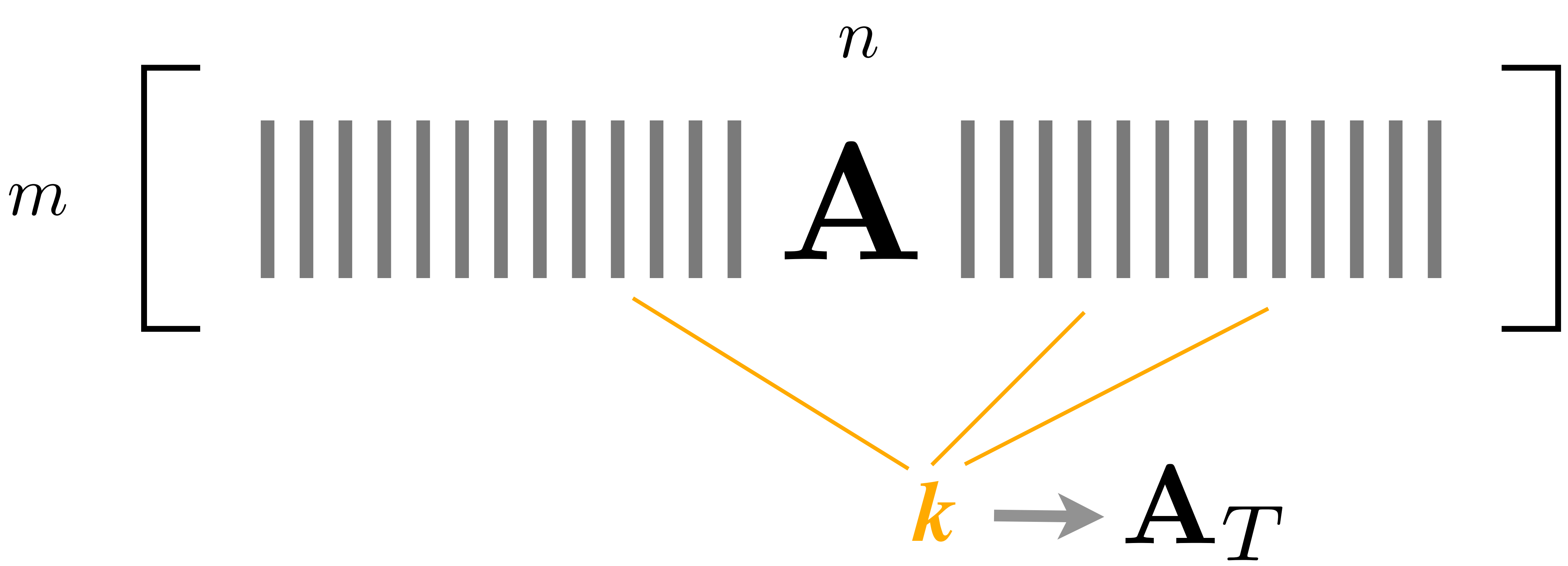
$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

performance:

- *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

RIP for $k \leq m \ll n$

$$(1-\delta_k)\|\mathbf{x}_T\|_{\ell 2} \leq \|\mathbf{A}_T\mathbf{x}\|_{\ell 2} \leq (1+\delta_k)\|\mathbf{x}_T\|_{\ell 2}$$



RIP for $k \leq m \ll n$

$\mathbf{A}_T$ how close is it to an orthonormal basis?

(if close enough, then if $\mathrm{NNZ}(\mathbf{x}) \leq k/2$,

$\mathbf{x_0} = \mathbf{x}$ with overwhelming probability)

bad, bad examples

$$\mathbf{A} = \begin{bmatrix} 1\ 0 & & \\ & 1\ 0 & \\ & & 1\ 0 \end{bmatrix}$$

(2x shot undersampling)

bad, bad examples

$$\mathbf{A} = \begin{bmatrix} 1\ 1 & & \\ & 1\ 1 & \\ & & 1\ 1 \end{bmatrix}$$

(Blend every-other shot)

good example

$$\mathbf{A} = \begin{bmatrix} \textbf{Gaussian} \\ \textbf{noise} \end{bmatrix}$$

(Completely blended shots)

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Compressed sensing

Some popular choices for A in literature

- Restricted random gaussian projections
- Restricted random phase encoding $\mathcal{O}(n \log n)$
- Restricted random signs projections
- Restricted Fourier transform

Call these kinds of matrices $\mathbf{RM}$ for literature consistency

Enforcing sparsity

$$\mathbf{A} = \mathbf{RMS^T}$$

- Using Curvelet transform for shot and receiver coordinates
  - **Frequency-domain** restrictions perform well under **Wavelet** transform for seismic data (Lin et. al. ’08)
  - **Spatial-domain** restrictions perform well under **Curvelet** transform for seismic data (Hennefent et. al. ’07)

- **Combine both transforms in the coordinate they are most suited for**
  - Wavelet sparsity on temporal-frequency coordinate
  - 2D Curvelet sparsity on shot and receiver plane

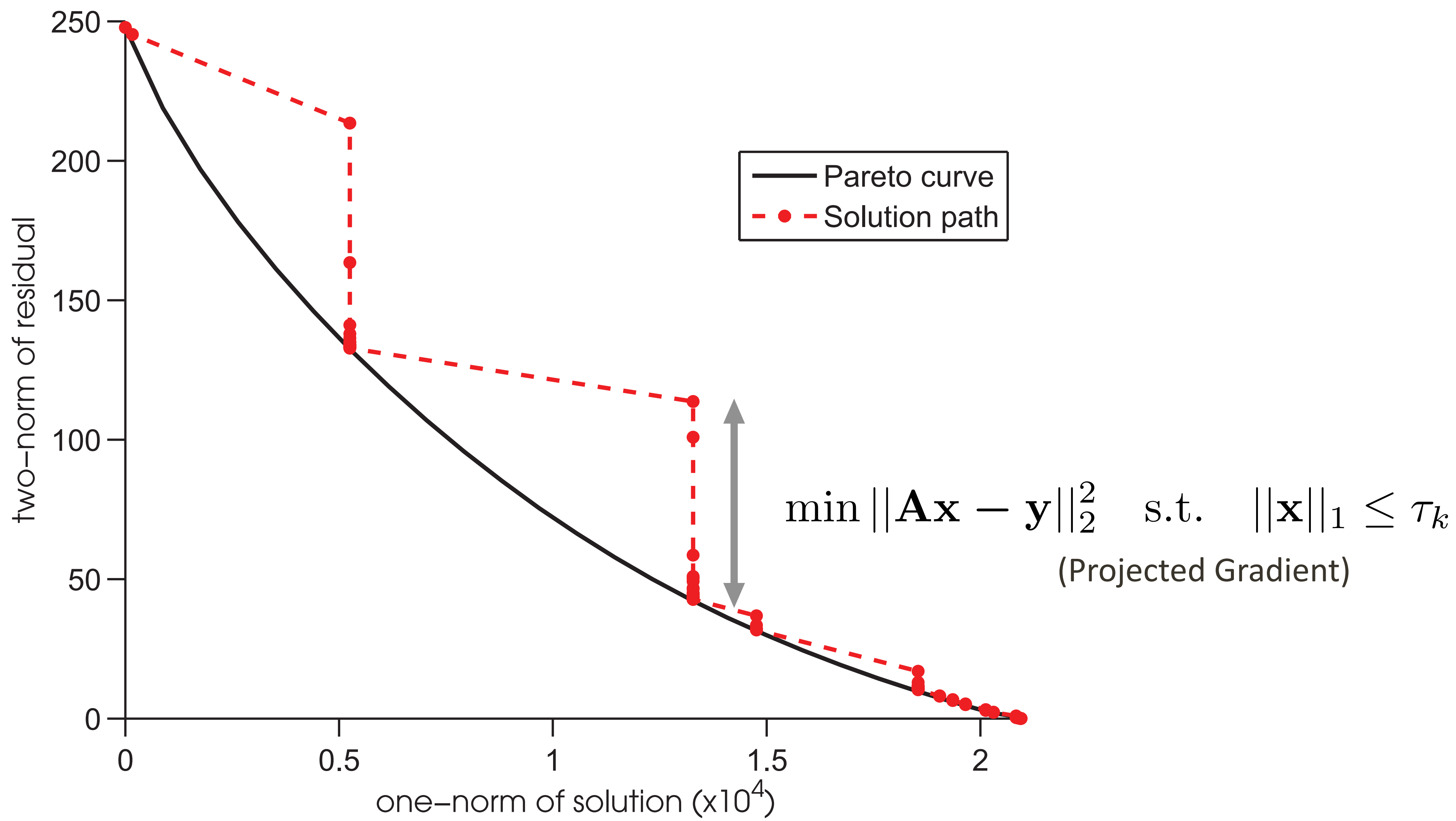
$$\mathbf{S} = \mathbf{C}_{2d} \otimes \mathbf{W}$$

L1 minimization

$$\min ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{Ax} = \mathbf{y}$$

Use SPGl1 (van den Berg, Friedlander, 2008)

- a projected gradient based method (seismic data-volumes are huge)
- uses root-finding to find the final one-norm



dsp.ece.rice.edu/cs

l1-Magic
SparseLab
GPSR
ell-1 LS
sparsify

solvers, **Jun 2007**

MPTK
Bayesian
SPGL1
sparseMRI
FPC
Chaining Pursuit
Regularized OMP
TwIST
Fast CS using
SRM
FPC_AS
Fast Bayesian
Matching Pursuit
SL0
PPPA
CoSAMP
CS via belief prop
SpaRSA
KF-CS: Kalman
Filtered CS
Fast Bayesian CS

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L1 minimization

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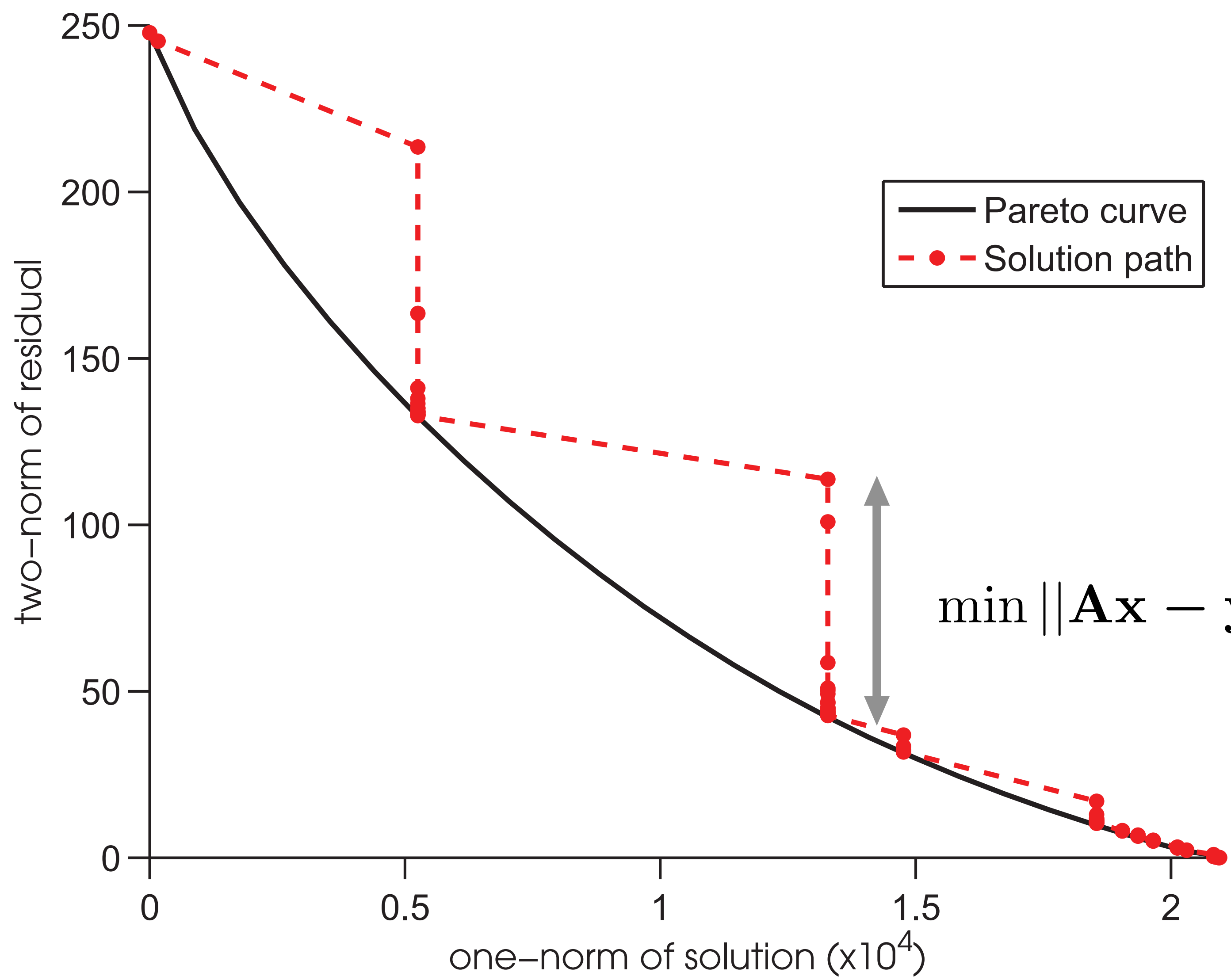

$$\min ||\mathbf{A}\mathbf{x} - \mathbf{y}||_2^2 \quad \text{s.t.} \quad ||\mathbf{x}||_1 \leq \tau_k$$

L1 minimization

$$\min ||\mathbf{x}||_1 \quad \text{s.t.} \quad \mathbf{Ax} = \mathbf{y}$$

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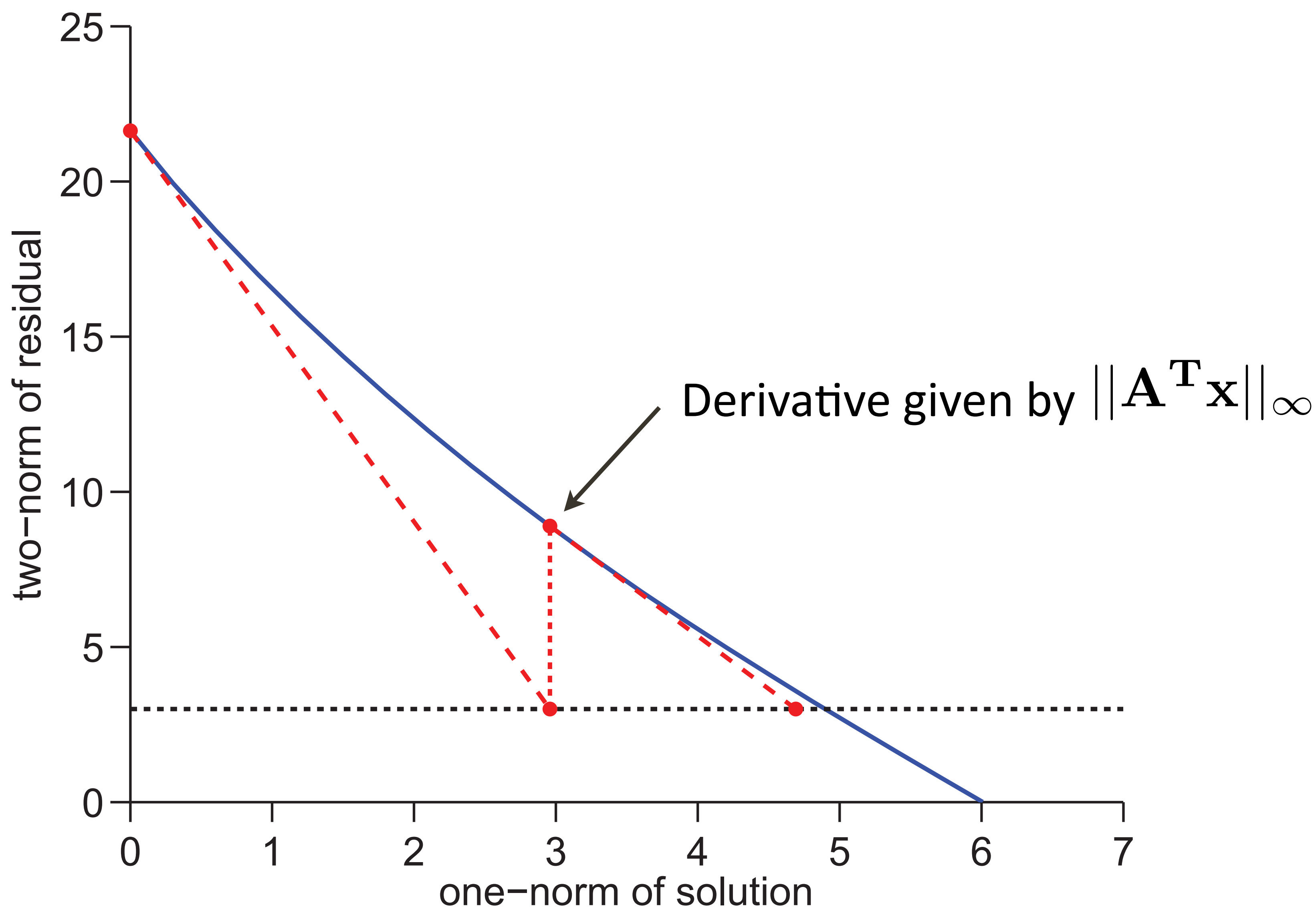
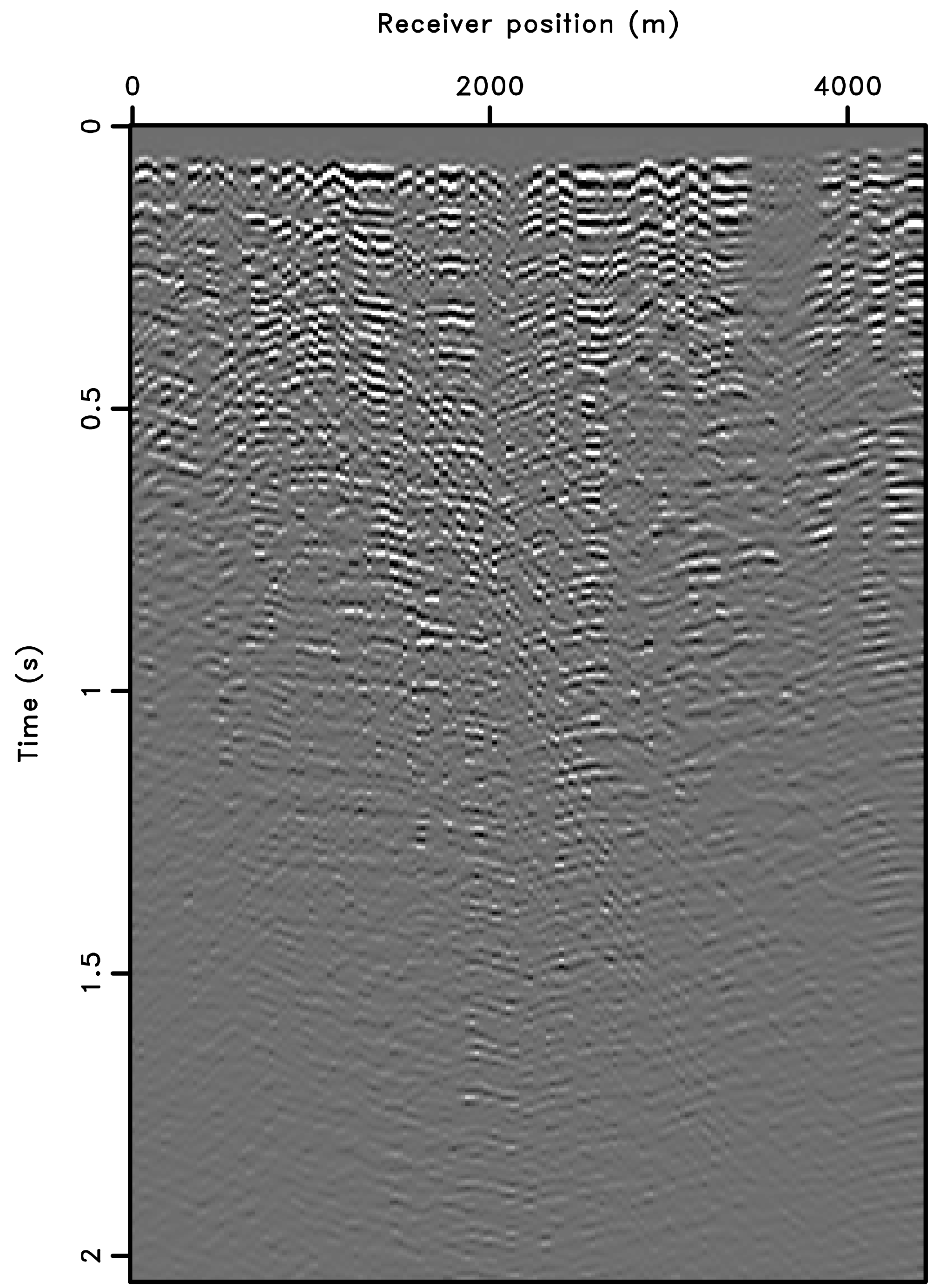
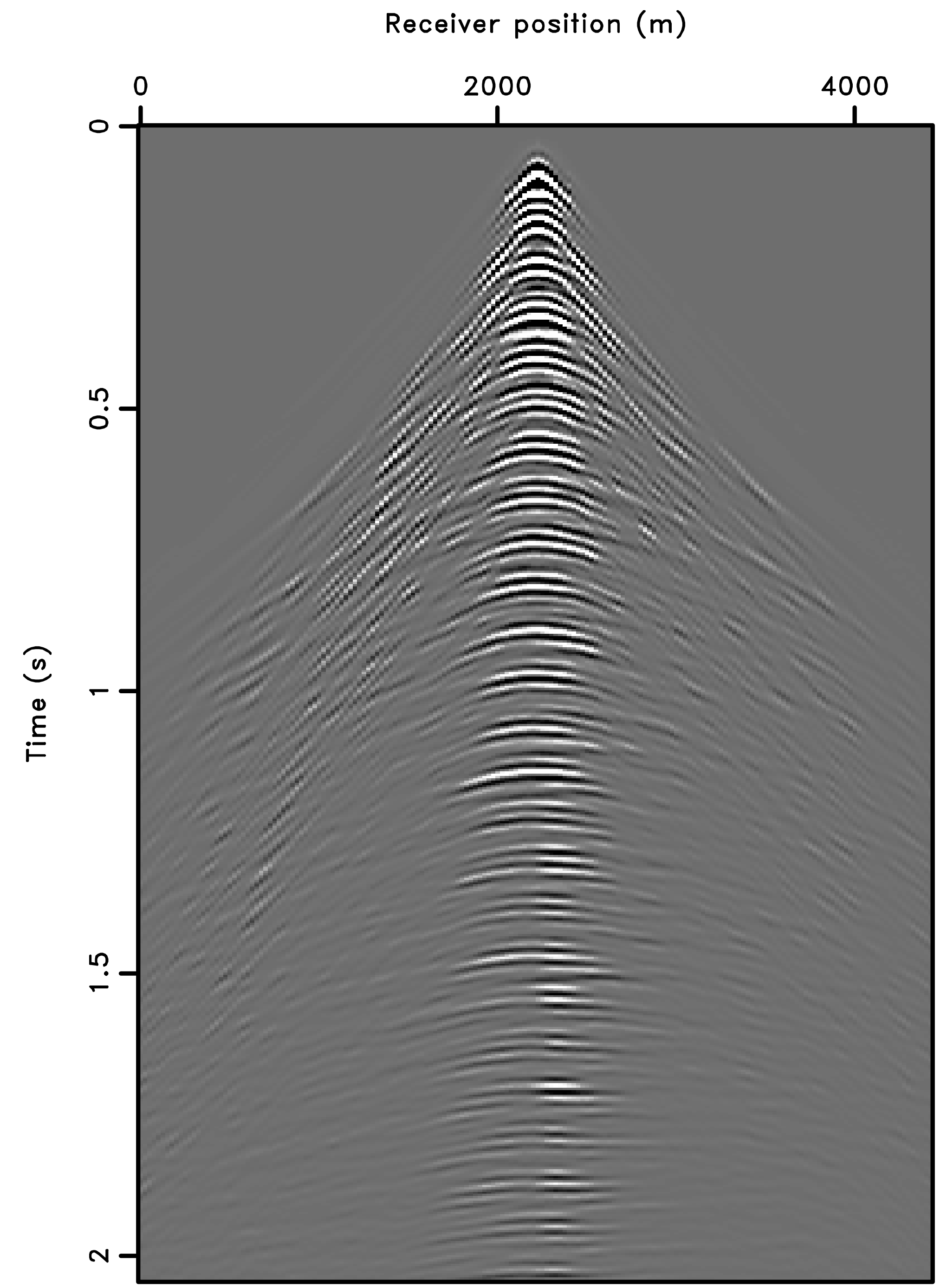


real marine data

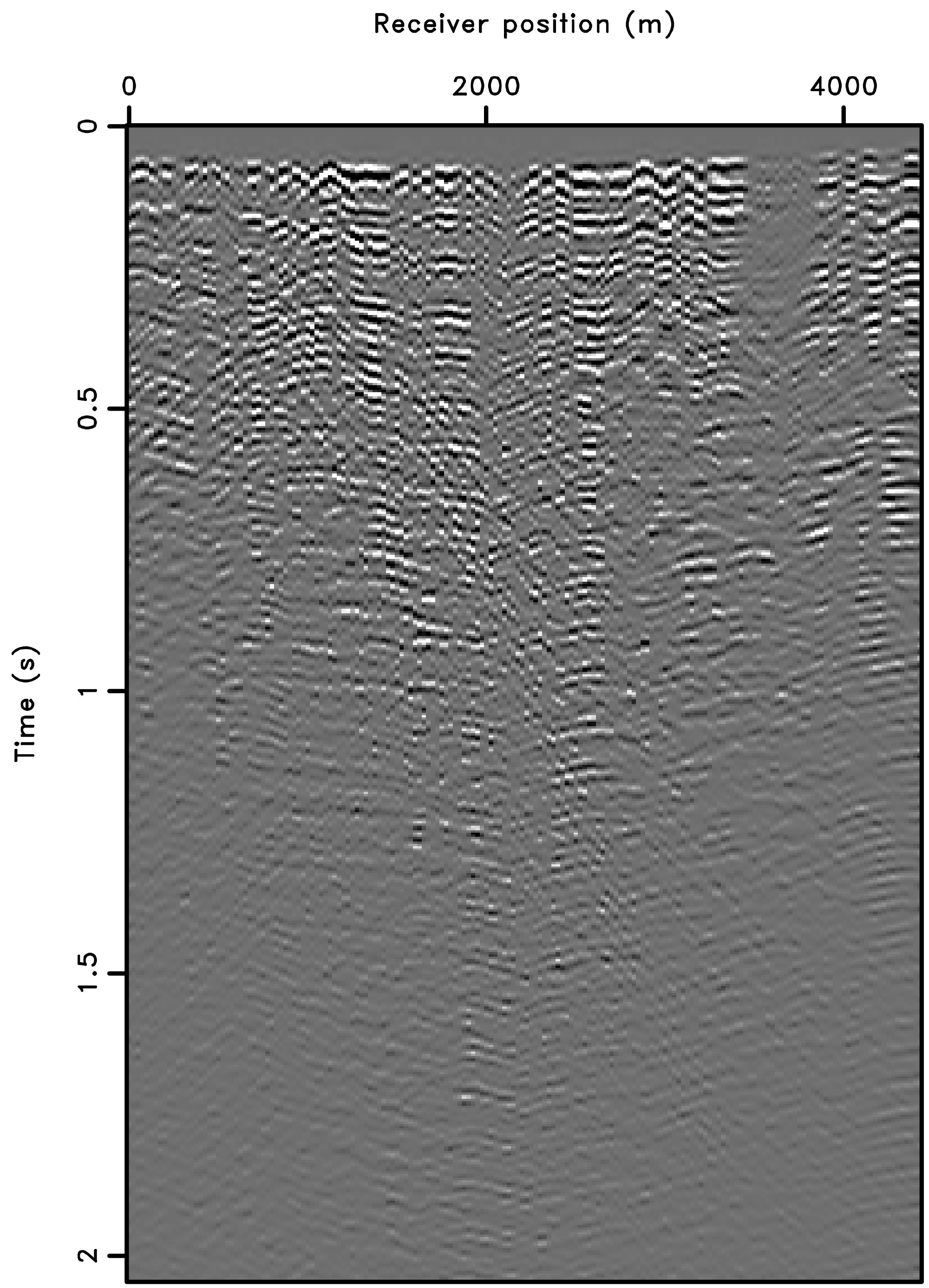


Single simultaneous single

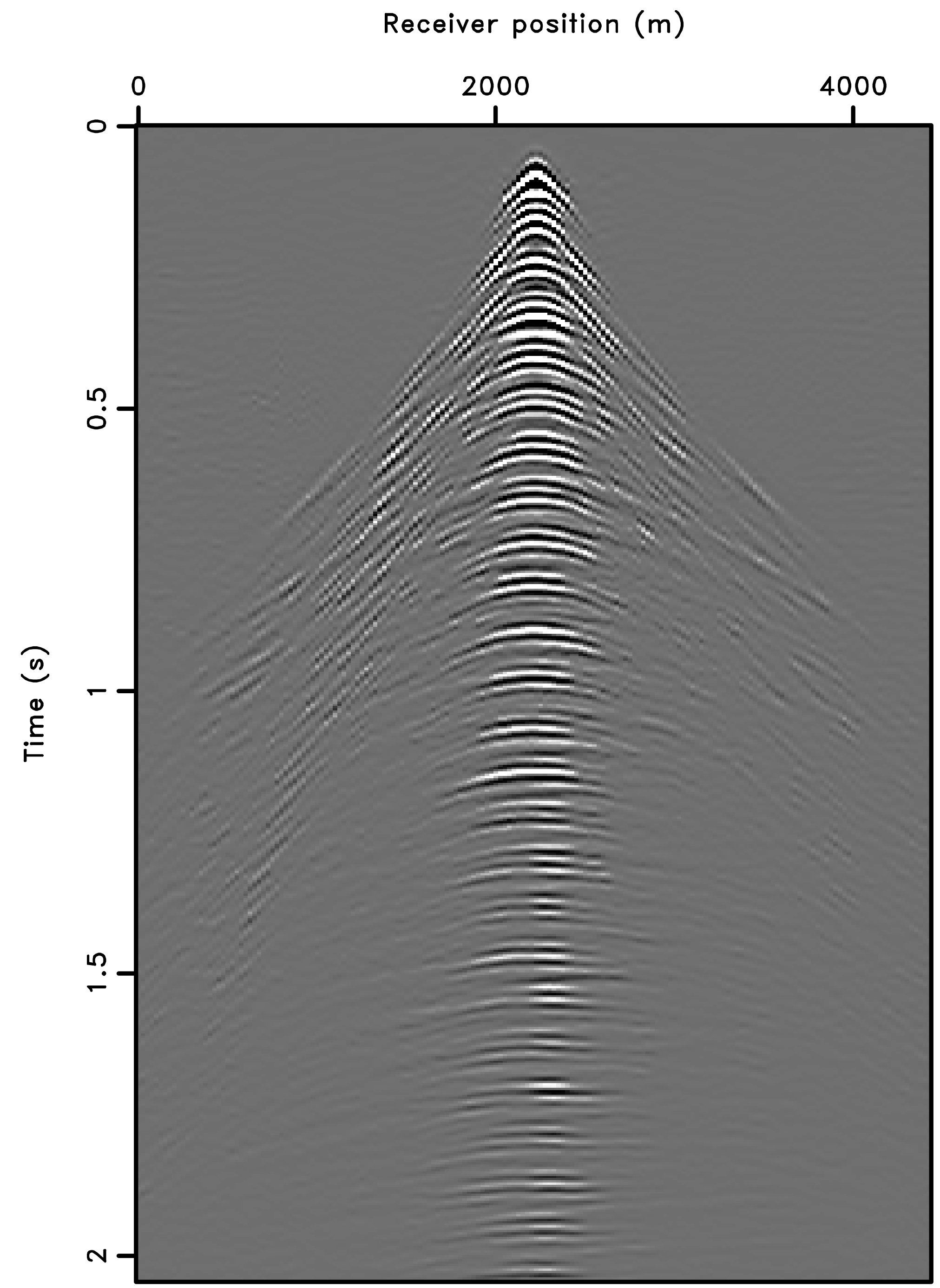


Single shot

real marine data



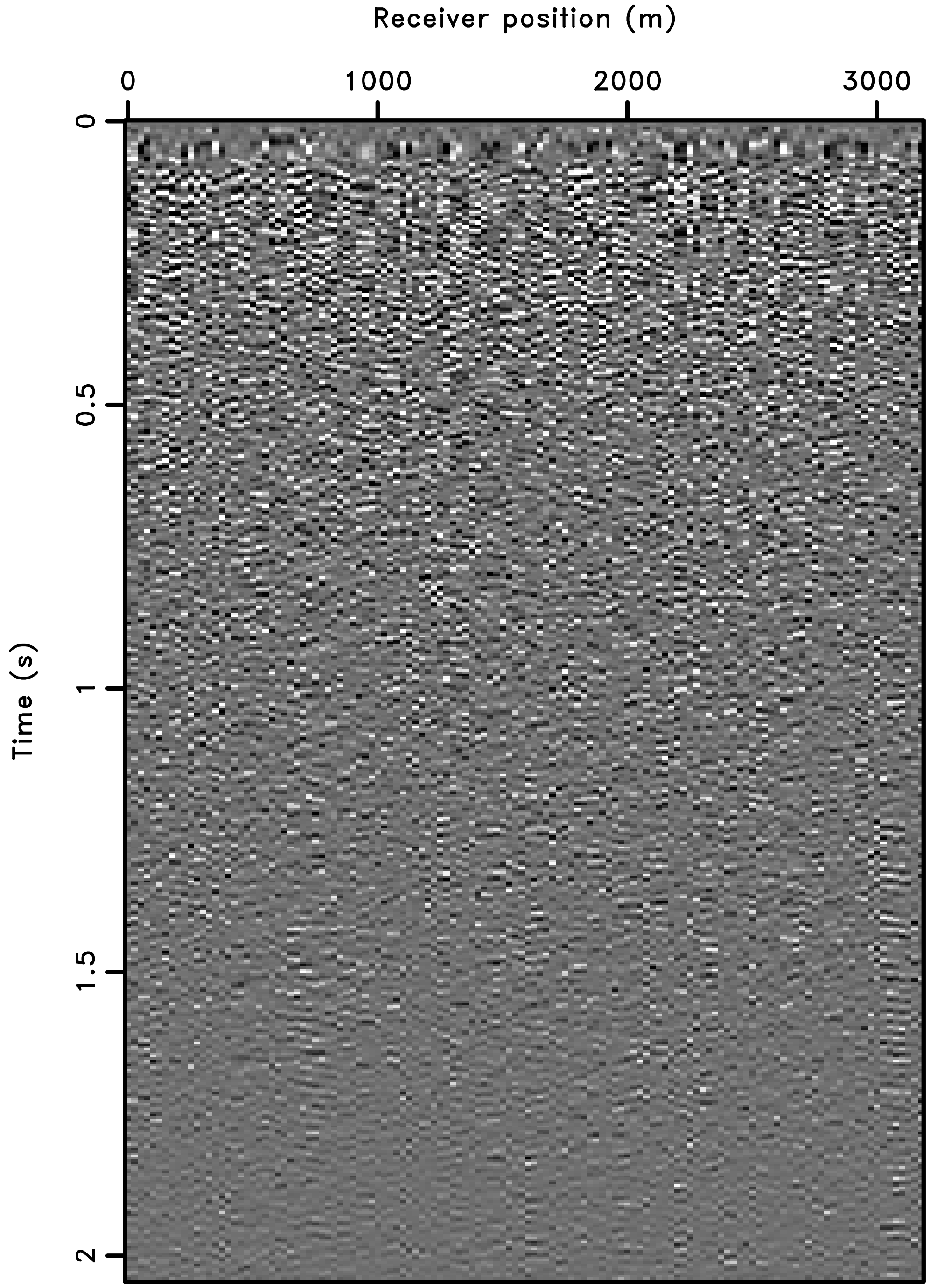
Single simultaneous shot



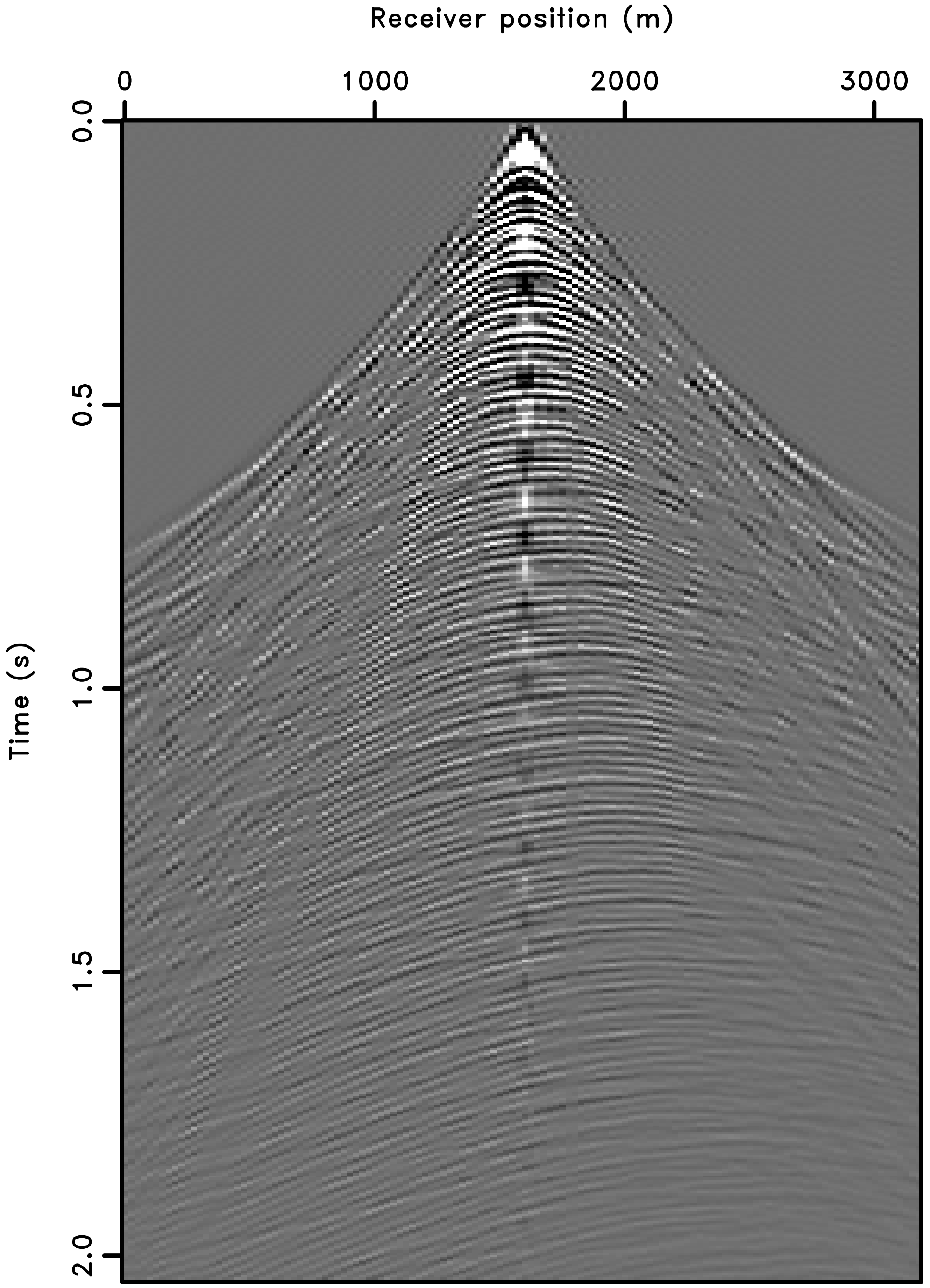
recovered from 25% number of realizations

~100 projected gradient

real marine data



Single simultaneous shot



Single shot

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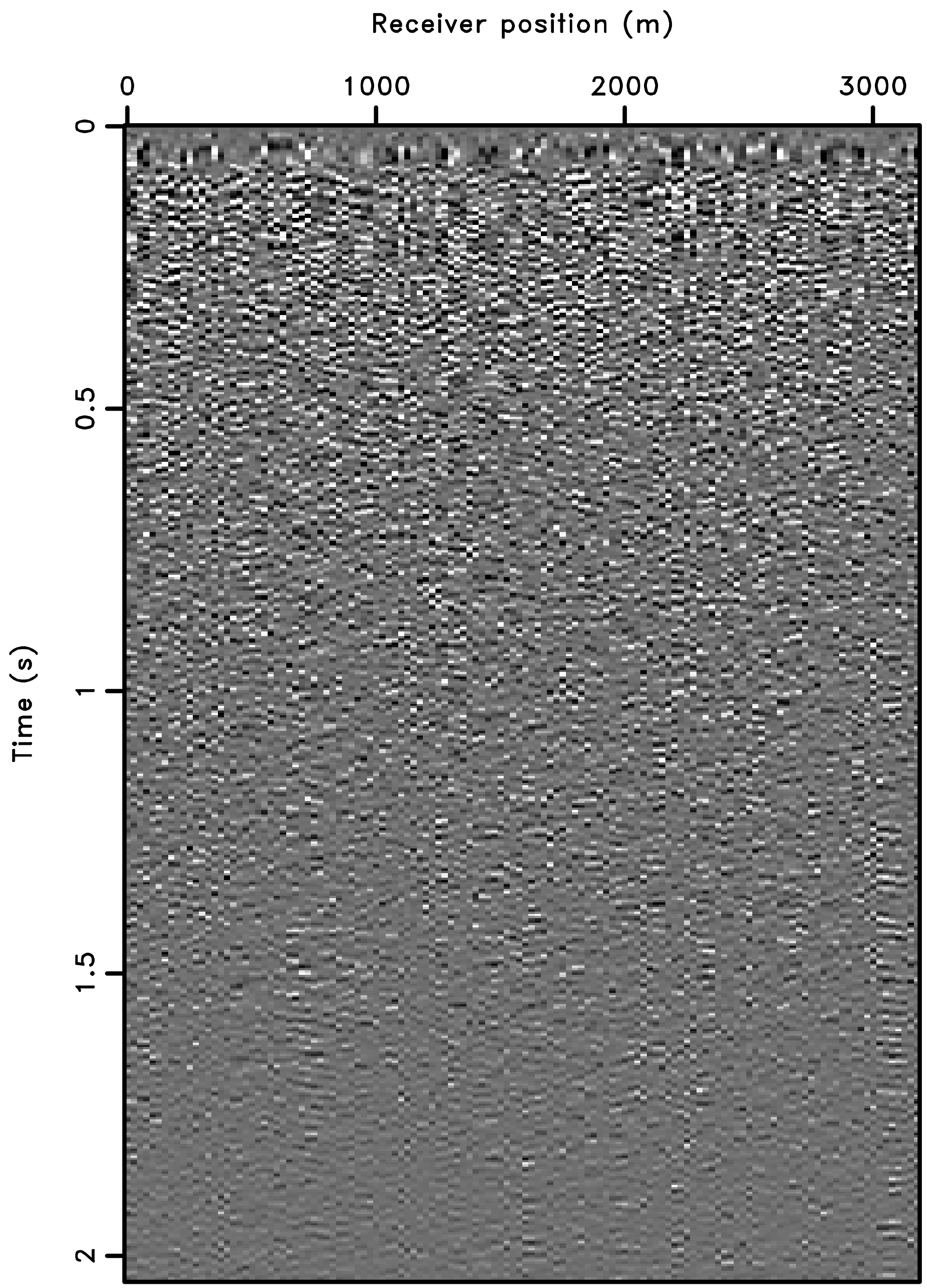
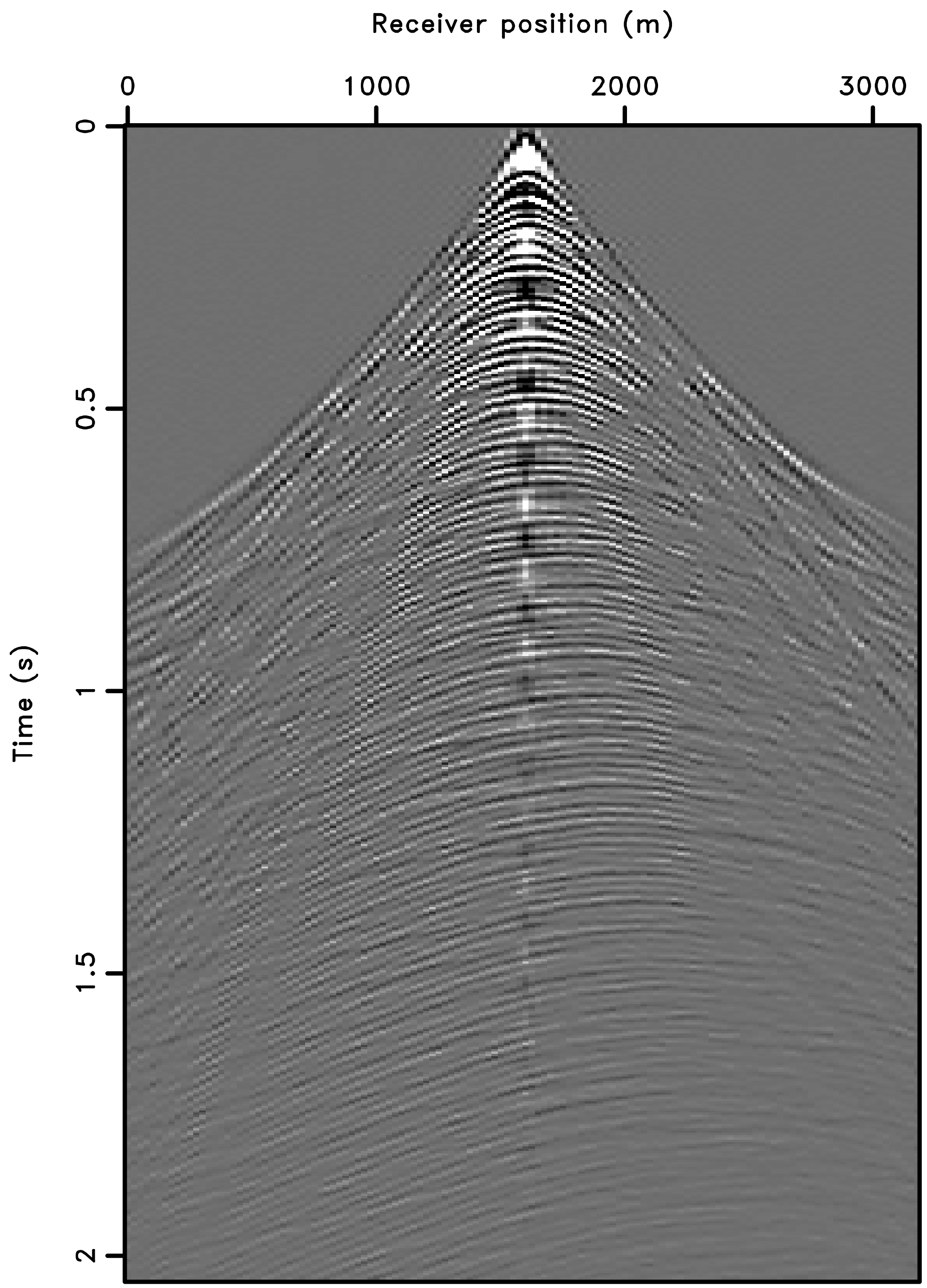
real marine data



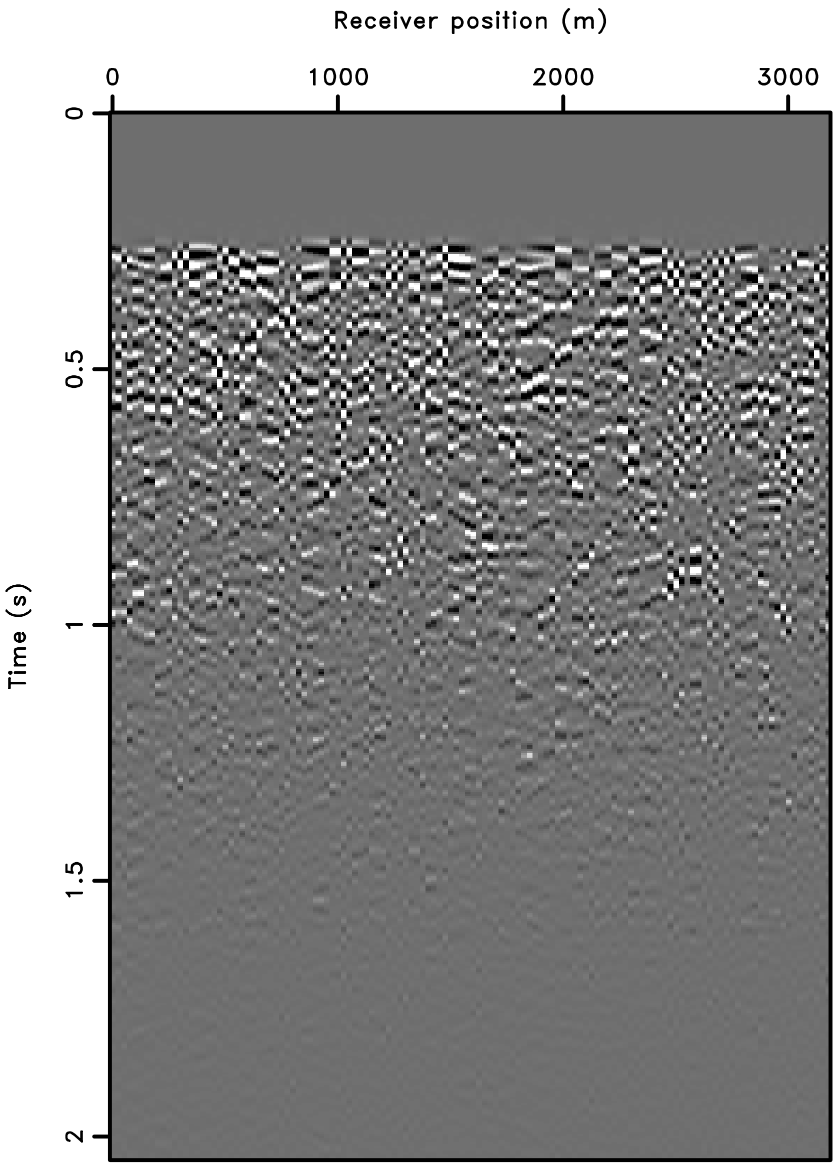
Single simultaneous shot



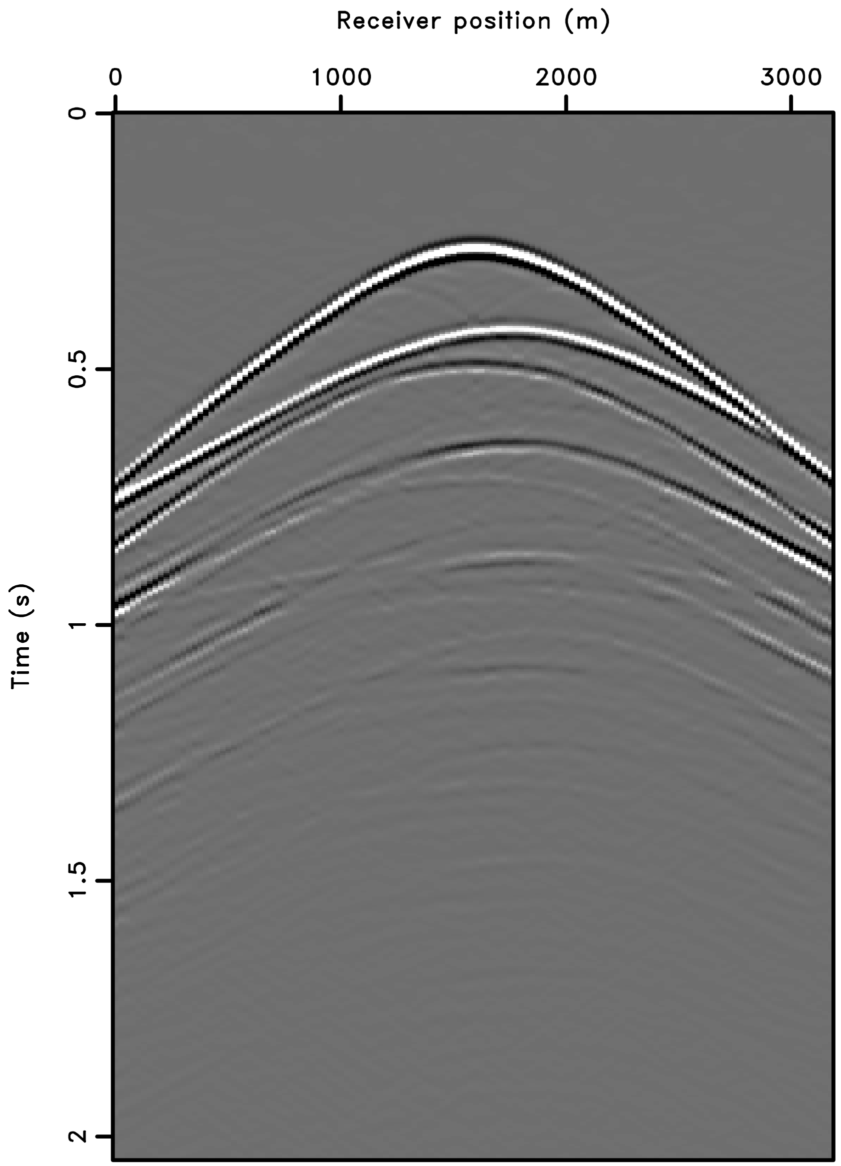
recovered from 25% number of realizations

~100 projected gradient

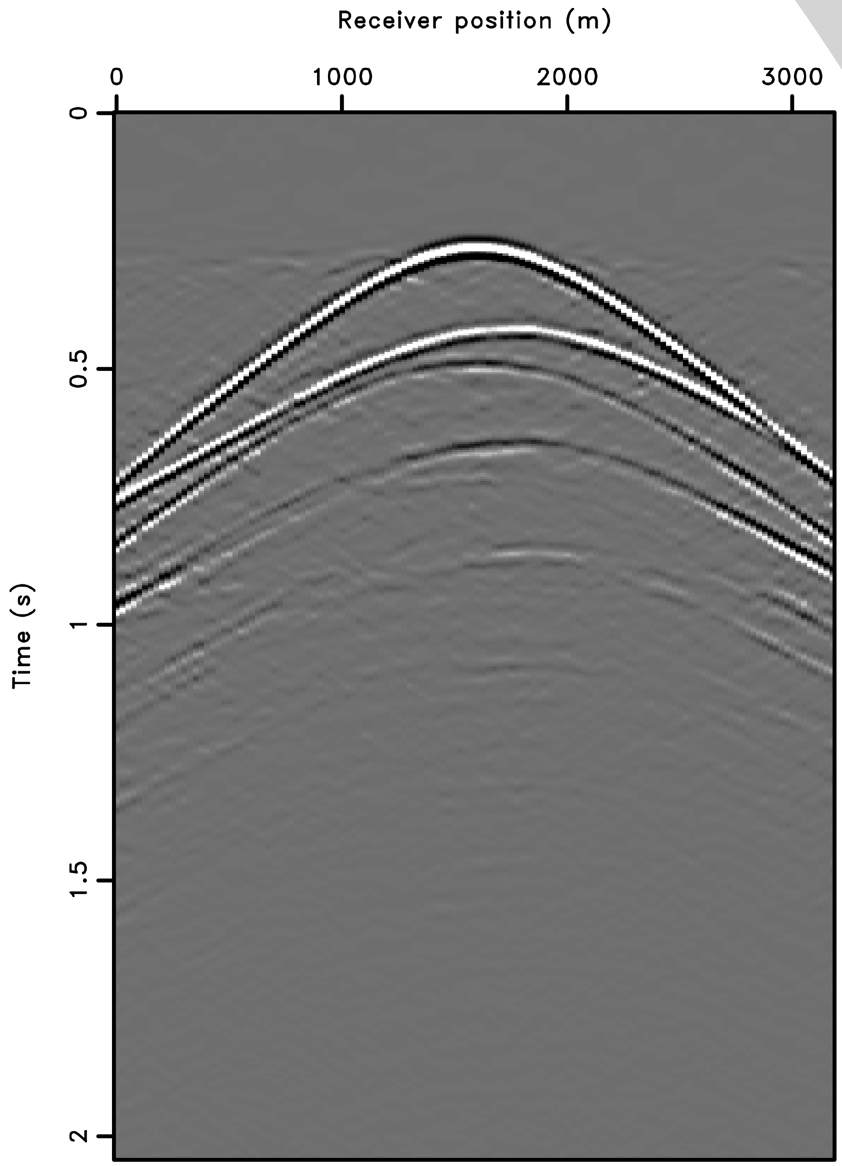
Reconstruction from different number of realizations of simultaneous simulation (measured in % of number of single-shots)



Single simultaneous shot



30% number of realizations



20% number of realizations

~100 projected gradient

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Primary estimation

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EPSI

Estimation of Primaries by Sparse Inversion (van Groenestijn and Verschuur, 2009)

-based on Amundsen inversion, division of up/down going wavefields

-additional sparsity regulation in the inversion process

$$\mathbf{P}^- = \mathbf{X_o}(\mathbf{S}^+ + \mathbf{R}\mathbf{P}^-)$$

$\mathbf{P}^-$ total up-going wavefield

$\mathbf{S}^+$ down-going source signature

$\mathbf{R}$ reflectivity of free surface (assume -1)

$\mathbf{X_o}$ primary impulse response

(all single-frequency slices, implicit ω)

EPSI

Uses sparsity assumption on $\mathbf{X_o}$

$$\underset{\mathbf{S^+},\mathbf{X_o}}{\textbf{minimize}} \quad \mathrm{nnz}(\mathbf{X_o}) \quad \textbf{s.t.} \quad ||\mathbf{P^-} - \mathbf{X_o}(\mathbf{S^+} + \mathbf{RP^-})||_2^2 \leq \sigma$$

But approximates the solution with k iterations of projected gradient

$$\underset{\mathbf{S^+},\mathbf{X_o}}{\textbf{minimize}} \quad ||\mathbf{P^-} - \mathbf{X_o}(\mathbf{S^+} + \mathbf{RP^-})||_2^2 \quad \textbf{s.t.} \quad \mathrm{nnz}(\mathbf{X_o}) \leq \frac{\tau}{k}$$

Nonetheless, a non-convex problem:

- existence of local minima
- no convergence guarantees

Compressed sensing

conditions:

- **A** obeys the *restricted isometry principle*
- $\mathbf{x}_0$ is *sufficiently sparse*

procedure:

$$\underbrace{\min_{\mathbf{x}} \|\mathbf{x}\|_1}_{\text{sparsity}} \quad \text{s.t.} \quad \underbrace{\mathbf{A}\mathbf{x} = \mathbf{y}}_{\text{perfect reconstruction}}$$

performance:

- *S*-sparse vectors recovered from roughly on the order of *S* measurements (to within constant and *log* factors)

Convex relaxation

Use L1-norm relaxation for the sparsity objective

$$\underset{\mathbf{S^+}, \mathbf{X_o}}{\textbf{minimize}} \quad ||\mathbf{X_o}||_1 \quad \textbf{s.t.} \quad ||\mathbf{P^-} - \mathbf{X_o}(\mathbf{S^+} + \mathbf{R}\mathbf{P^-})||_2^2 \leq \sigma$$

Bi-convex problem, but turns into two convex problems we know how to solve via alternating optimization

-Standard approach in blind image deconvolution

-no need for windowing primary events at each iteration

Convex relaxation

Use L1-norm relaxation for the sparsity objective

$$\underset{\mathbf{X_o}}{\textbf{minimize}} \quad ||\mathbf{X_o}||_1 \quad \textbf{s.t.} \quad ||\mathbf{P}^- - \mathbf{X_o}(\mathbf{S_k^+} + \mathbf{R}\mathbf{P}^-)||_2^2 \leq \sigma$$

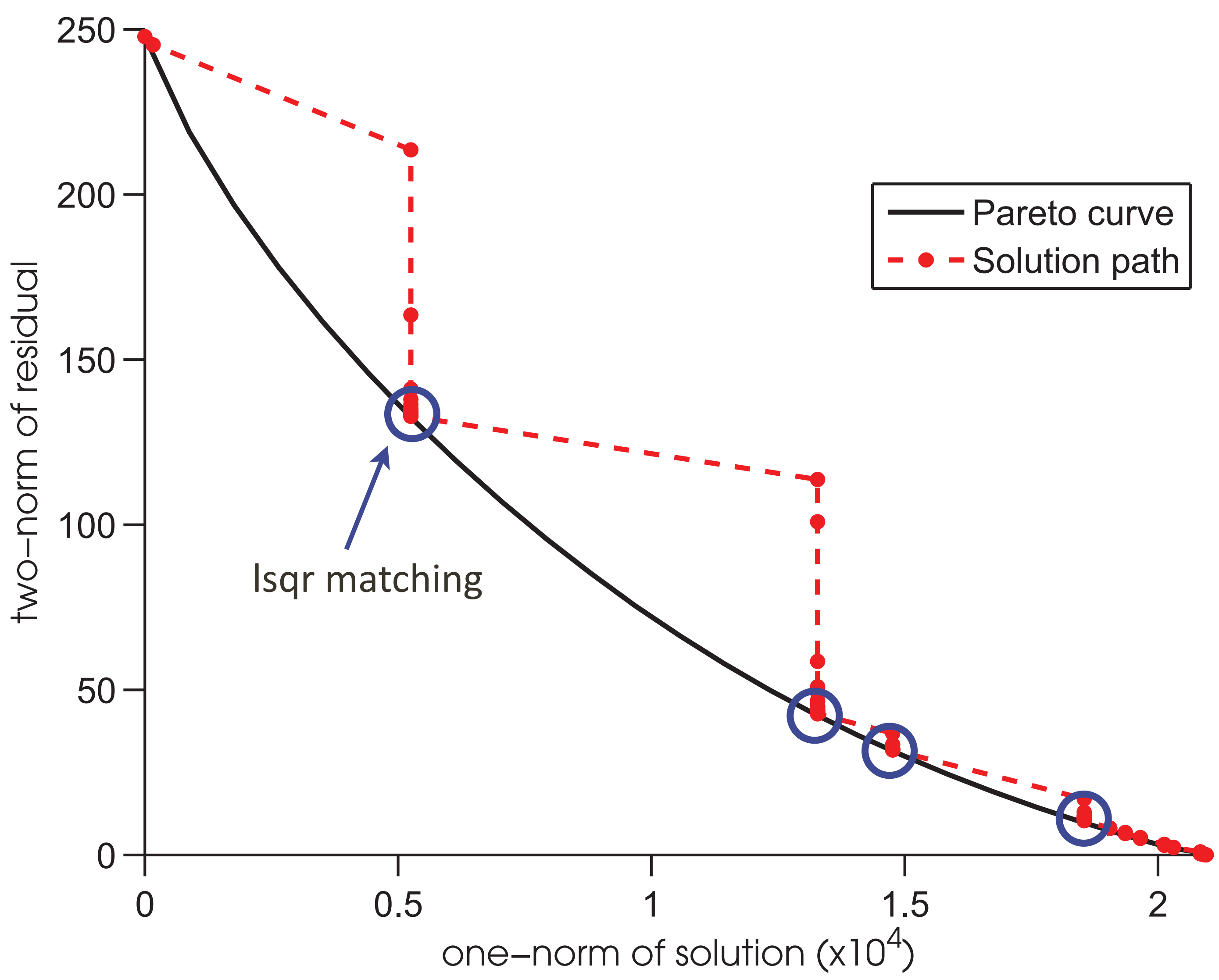
Fix source signature, turns into L1-minimization (SPGl1)

Convex relaxation

Use L1-norm relaxation for the sparsity objective

$$\underset{\mathbf{S}^+}{\textbf{minimize}} \quad ||\mathbf{X}_{\mathbf{O}k}||_1 \quad \textbf{s.t.} \quad ||\mathbf{P}^- - \mathbf{X}_{\mathbf{O}k}(\mathbf{S}^+ + \mathbf{R}\mathbf{P}^-)||_2^2 \leq \sigma$$

Fix primary impulse response, get least-squares matching for $\mathbf{S}^+$ upto tolerance σ



In SPGl1 solution path, do least-square matching of source everytime we reach an optimal solution along pareto

all together now

$$\mathbf{P}^{-} = \mathbf{X_o}(\mathbf{S}^{+} + \mathbf{R}\mathbf{P}^{-})$$

Define linear operator P:

$$\mathbf{P}\mathbf{x} := \mathcal{F}_{\omega}{}^{-1}\left[(\mathcal{F}_{\omega}\mathbf{x})(\mathbf{S}^{+} + \mathbf{R}\mathbf{P}^{-})\right]$$

all together now

And then composite together RM and P

$$\mathbf{A} = \mathbf{RMPS^T}$$

Solve CS problem

$$\min ||\mathbf{x}||_1 \quad \text{s.t.} \quad ||\mathbf{y} - \mathbf{Ax}||_2^2 \leq \sigma$$

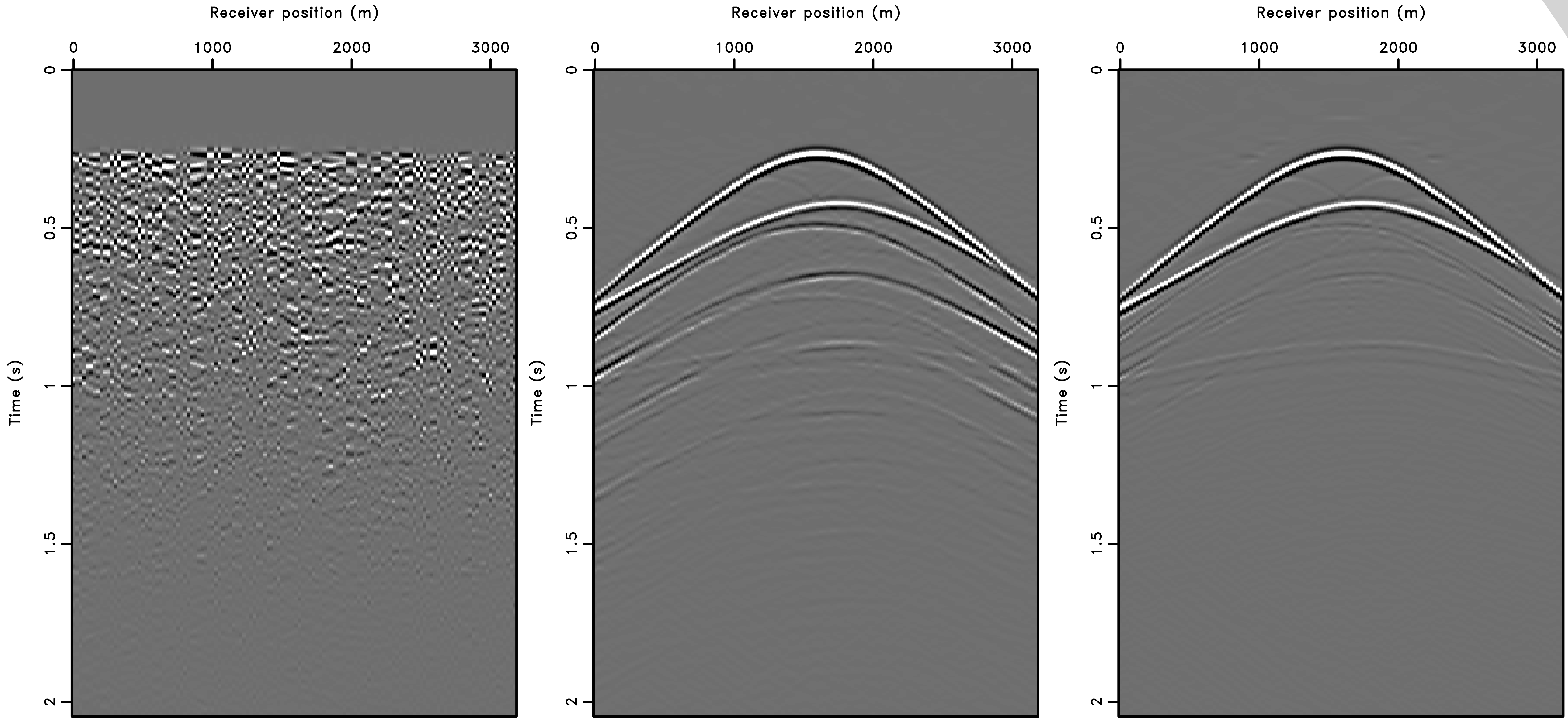
$\mathbf{y}$ is data measured according to $\mathbf{RM}$

-Demultiplex and EPSI now share the same solver and the same overhead

-Primary is sparser than full data

-CS predicts less measurement needed for same quality

all together now



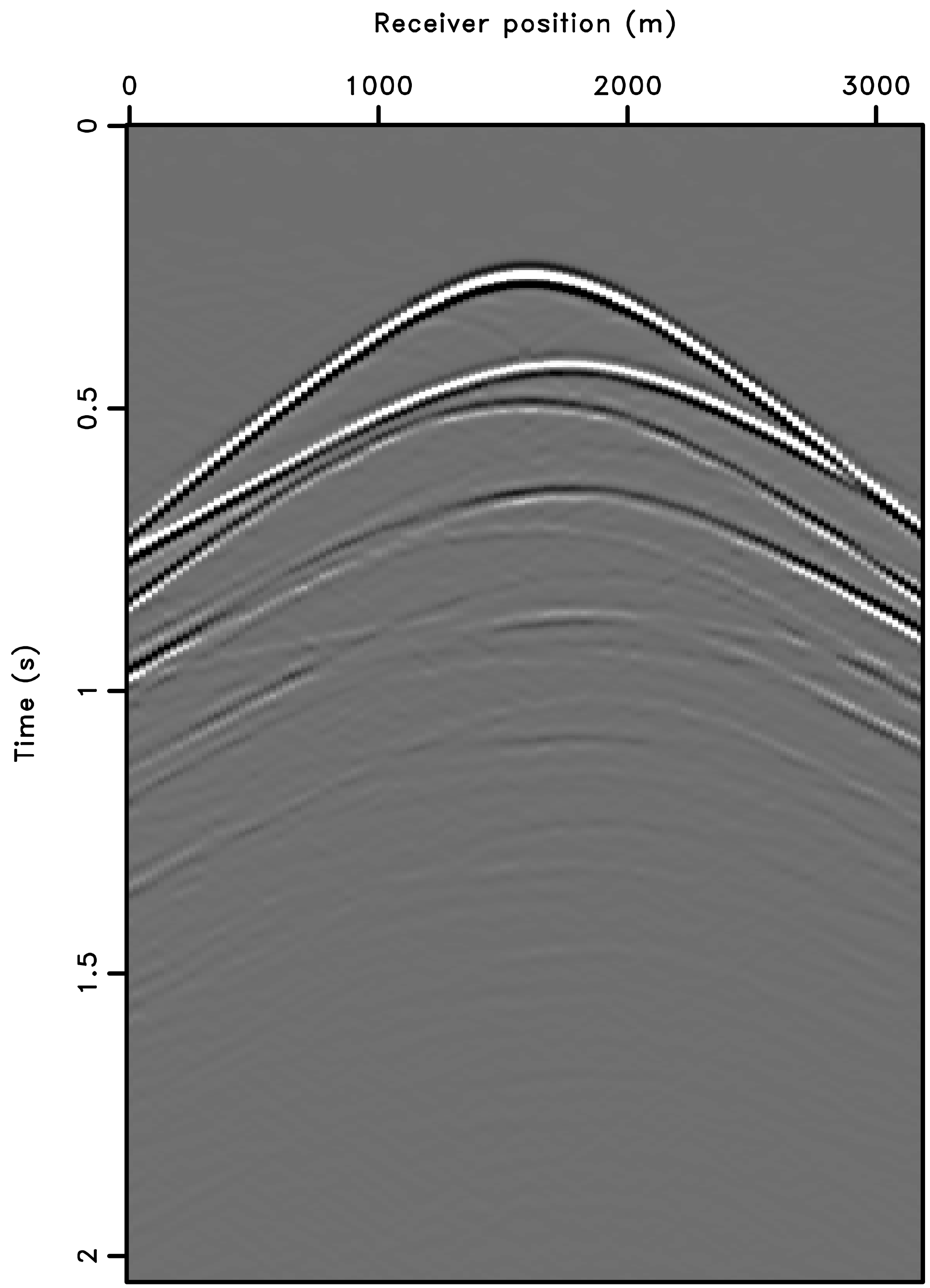
Single simultaneous shot

$\mathbf{A} = \mathbf{RMS}^\mathrm{T}$

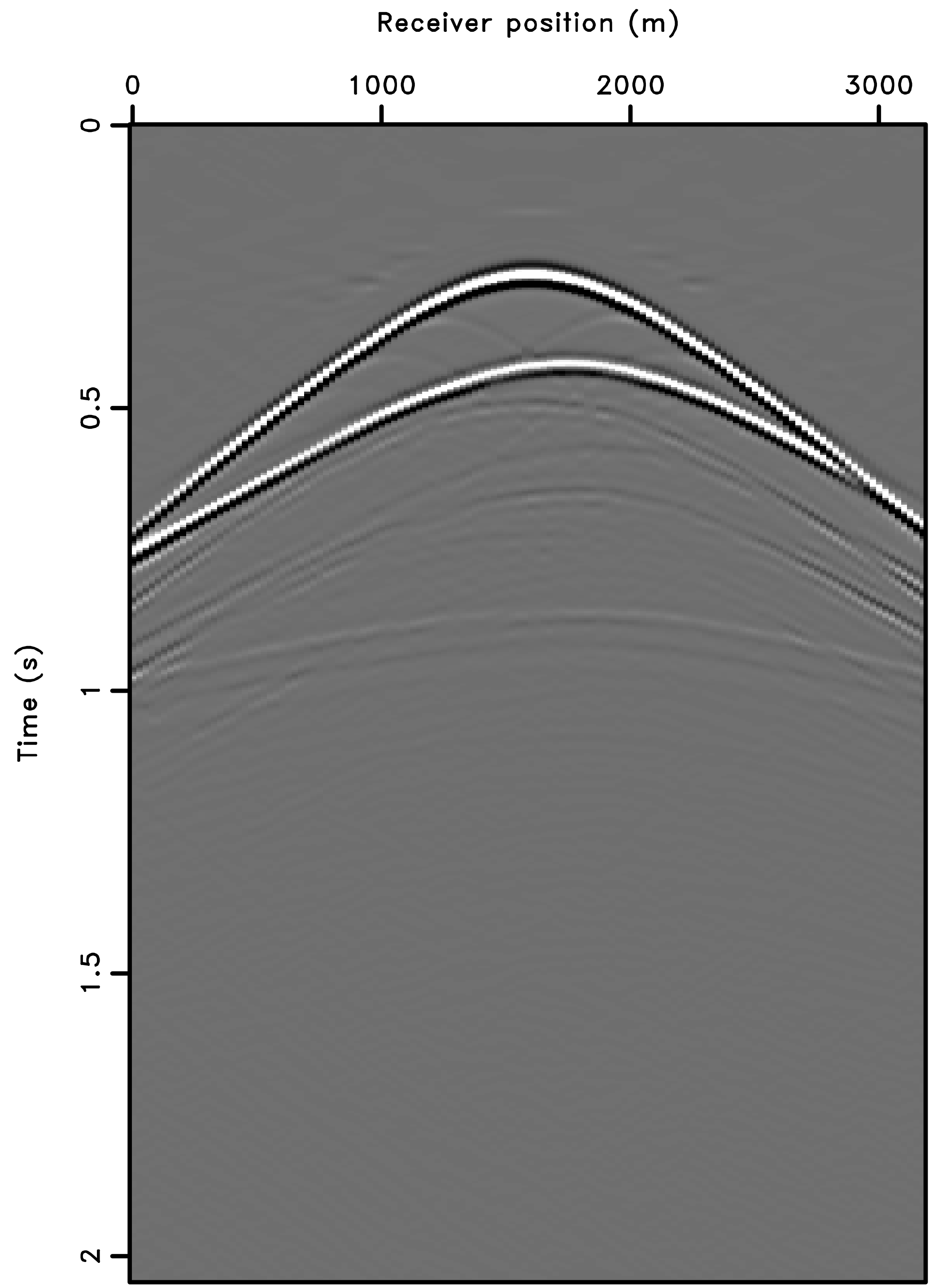
$\mathbf{A} = \mathbf{RMPS}^\mathrm{T}$

~100 projected gradient, 5 source matching

50% measurement

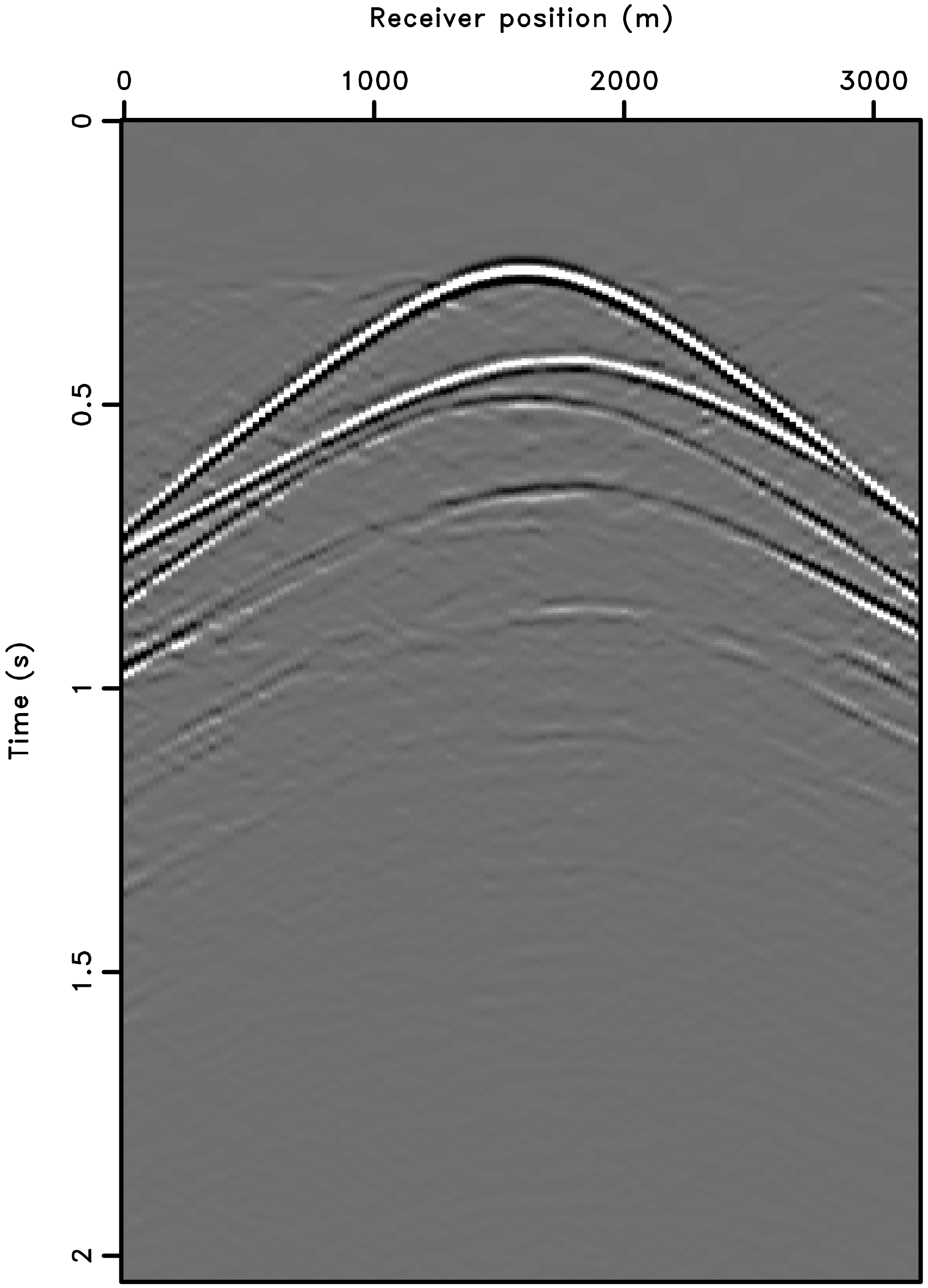


$$\mathbf{A} = \mathbf{RMS}^{\mathbf{T}}$$

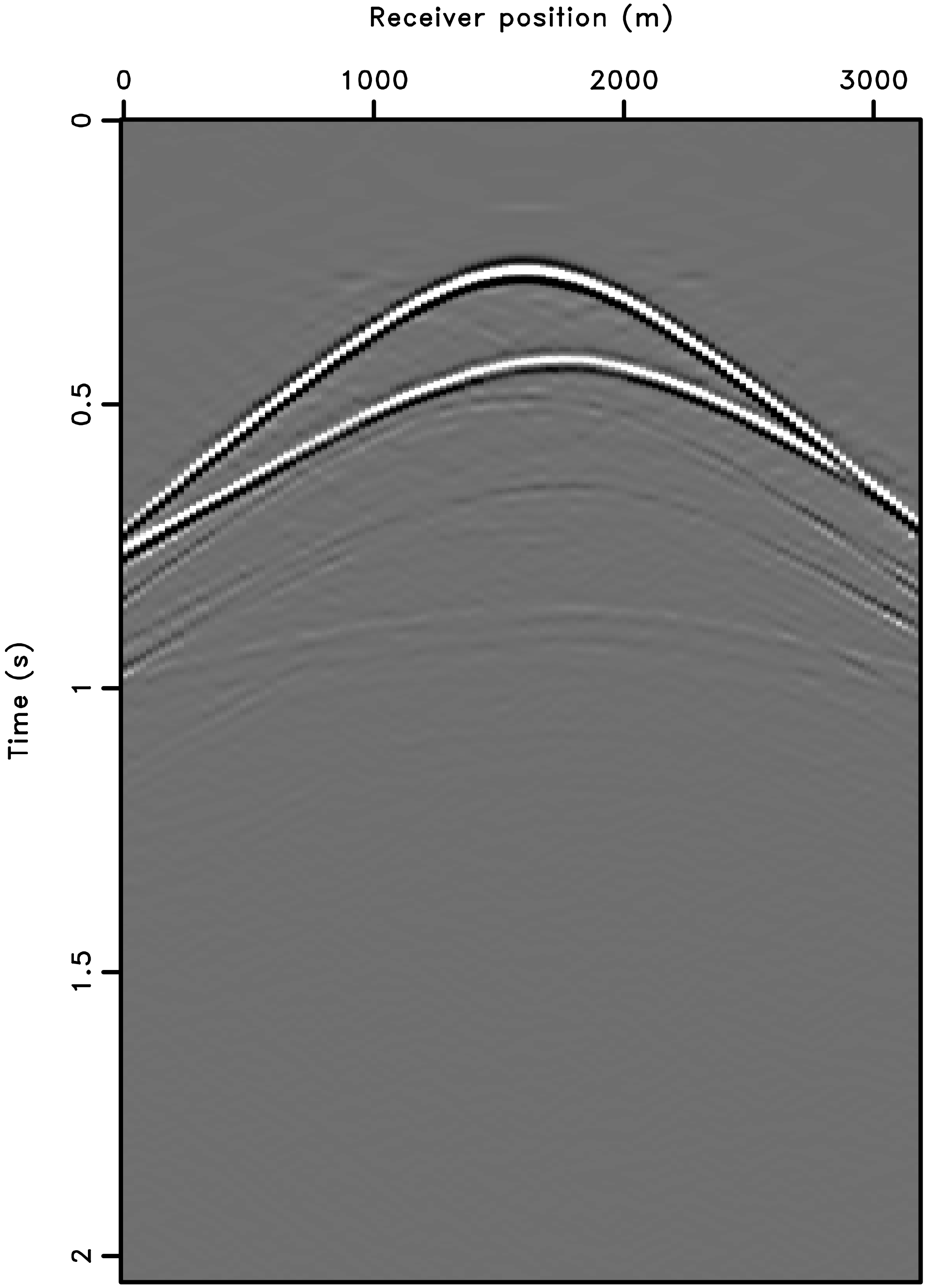


$$\mathbf{A} = \mathbf{RMPS}^{\mathbf{T}}$$

20% measurement



$\mathbf{A} = \mathbf{RMS}^{\mathbf{T}}$



$\mathbf{A} = \mathbf{RMPS}^{\mathbf{T}}$

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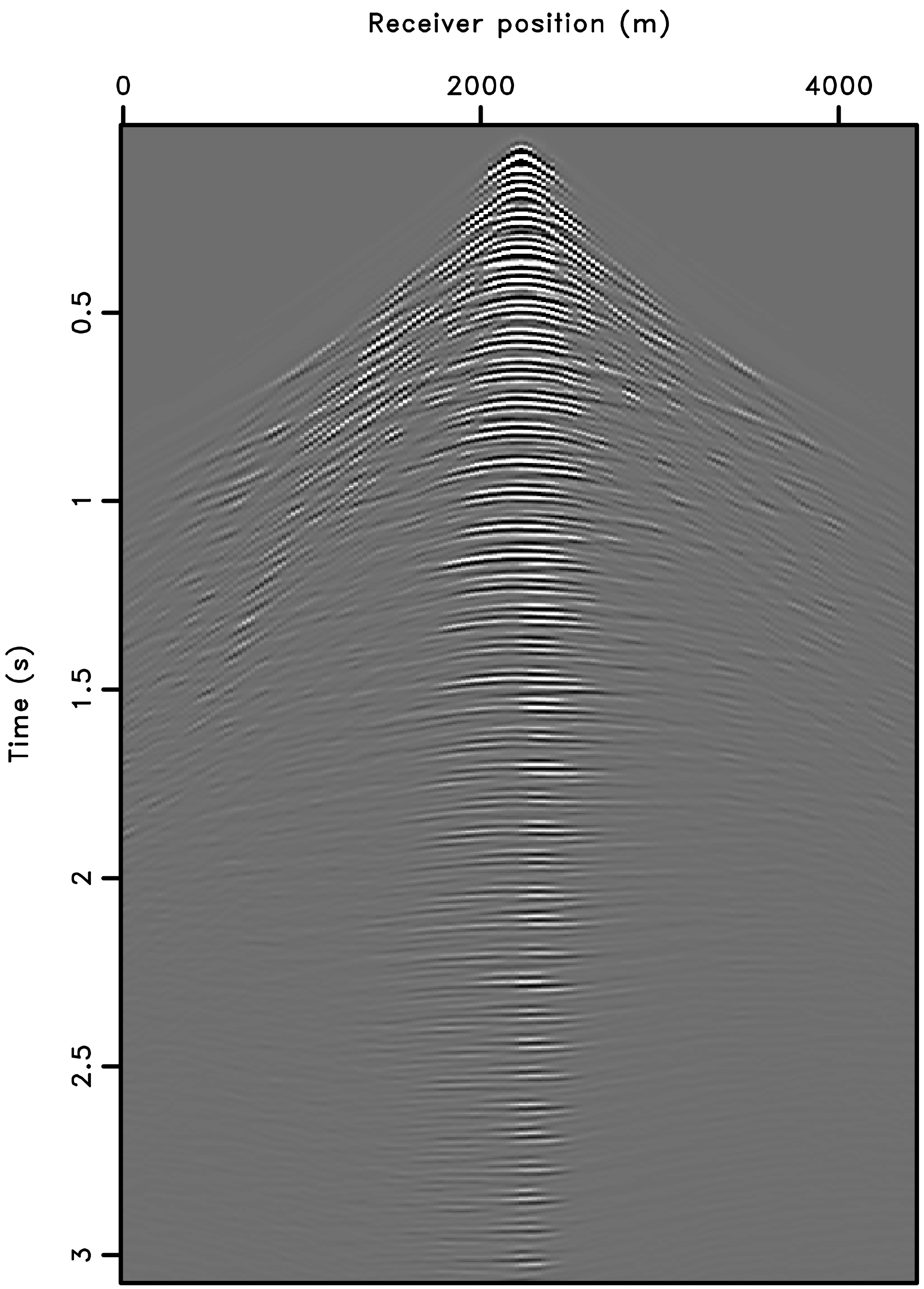
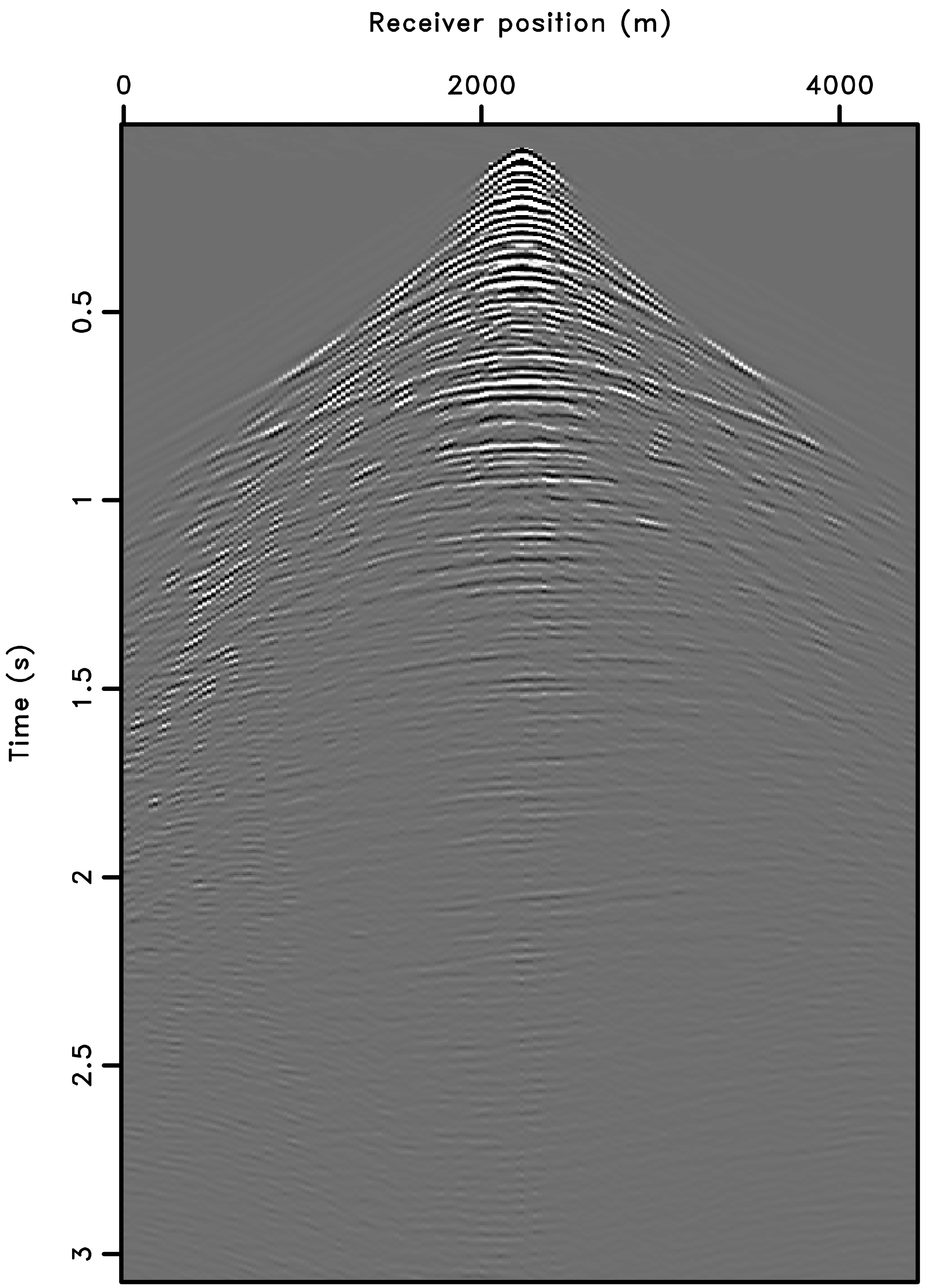
EPSI-L1

Gulf of Suez data
1024x178x178





~100 projected gradient, 5 source matching

EPSI-L1

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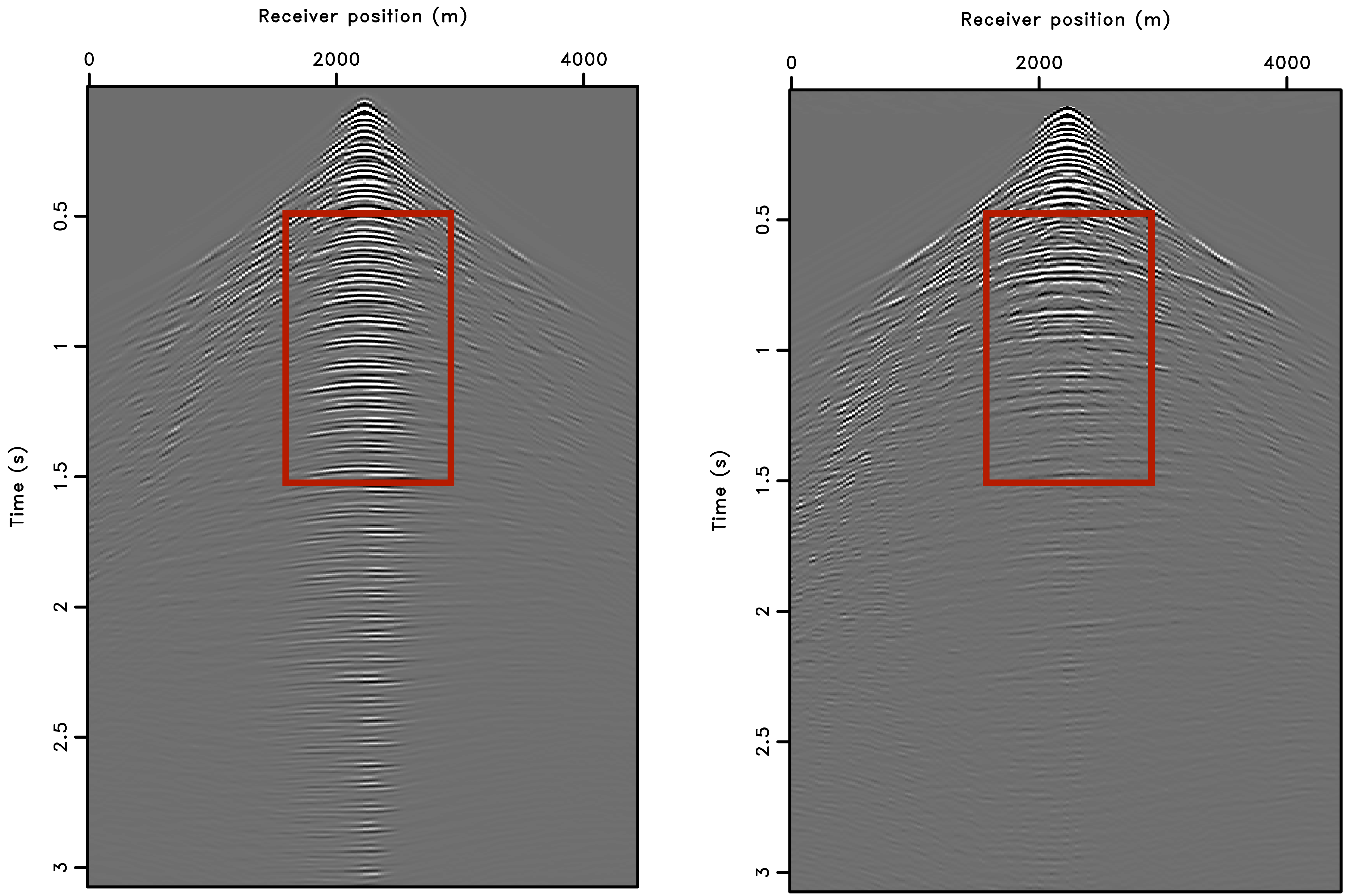
1024x178x178



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Gulf of Suez data
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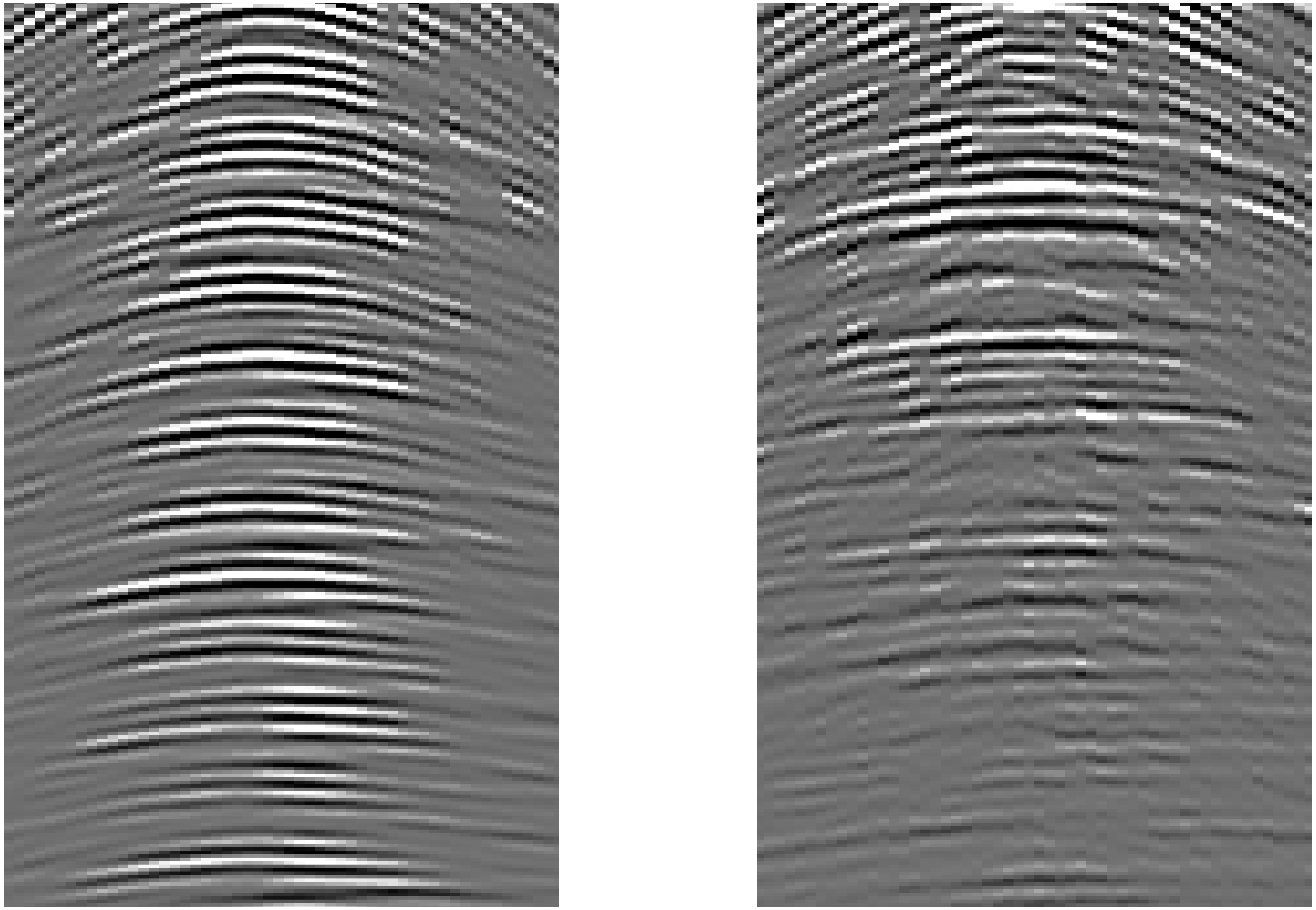

~100 projected gradient, 5 source matching

acknowledgements

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