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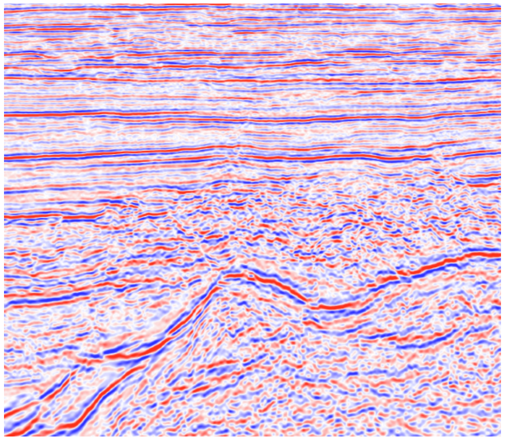
Incoherent noise suppression with curvelet-domain sparsity



Vishal Kumar & Felix Herrmann

SEG 2009, Houston

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Outline

- Motivation
- Background
  - Curvelets
  - Sparsity
- Curvelet-based Denoising
  - Algorithm
  - Results
- Wrap up
  - Conclusions
  - Challenges & Future work
  - Acknowledgments

Motivation

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- Conventional noise attenuation algorithm like F-X prediction (also called F-X deconvolution) cause harm to the coherent energy [Abma, R. and J. Claerbout, 1995; Neelamani et al., 2008].

- The signal-to-noise ratio (SNR) is low in deep crustal data.

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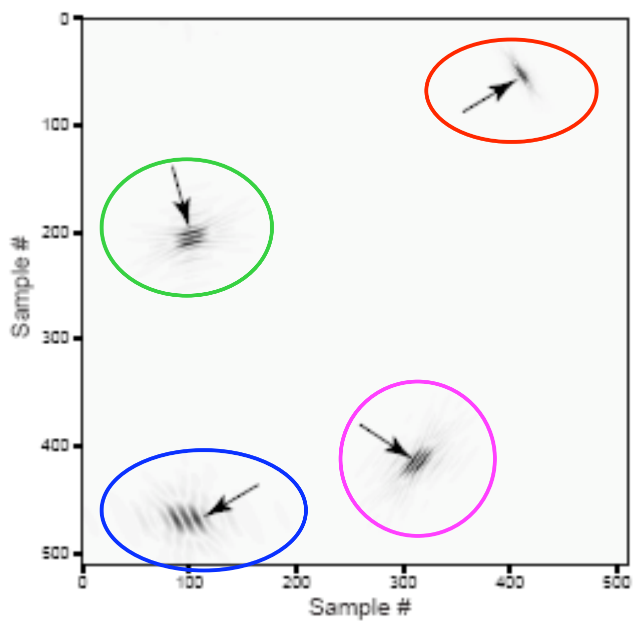
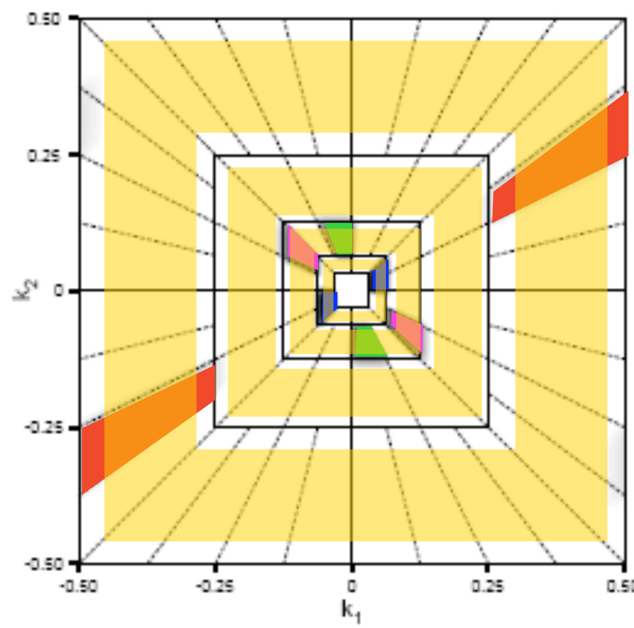
Why Curvelets

- Curvelets exploit the high-dimensional geometry of the seismic data [Herrmann et. al, 2008].
- Curvelets provide sparse representation for seismic data/model that facilitates use of sparsity-promoting solvers [Herrmann and Hennenfent, 2008].
- The signal and noise have minimum overlap in the curvelet-domain [Neelamani et al., 2008].

Curvelets



Spatial



Fourier

[Adapted from Herrmann et. al, 2008]

- **Multiscale:** tiling of the Fourier domain into dyadic coronae.
- **Multi-directional:** coronae sub-partition into angular wedges.
- **Pseudo-localized in spatial domain:** rapid decay in space.
- **Localized in frequency domain:** occupy certain wedges.

Curvelets vs. F-K

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- Curvelets:
  - decomposes into local and directional plane wave.
  - well suited to approximate a curved event by few elements.
- F-K:
  - decomposes into global monochromatic plane waves.
  - need lots of elements to approximate a curved event.

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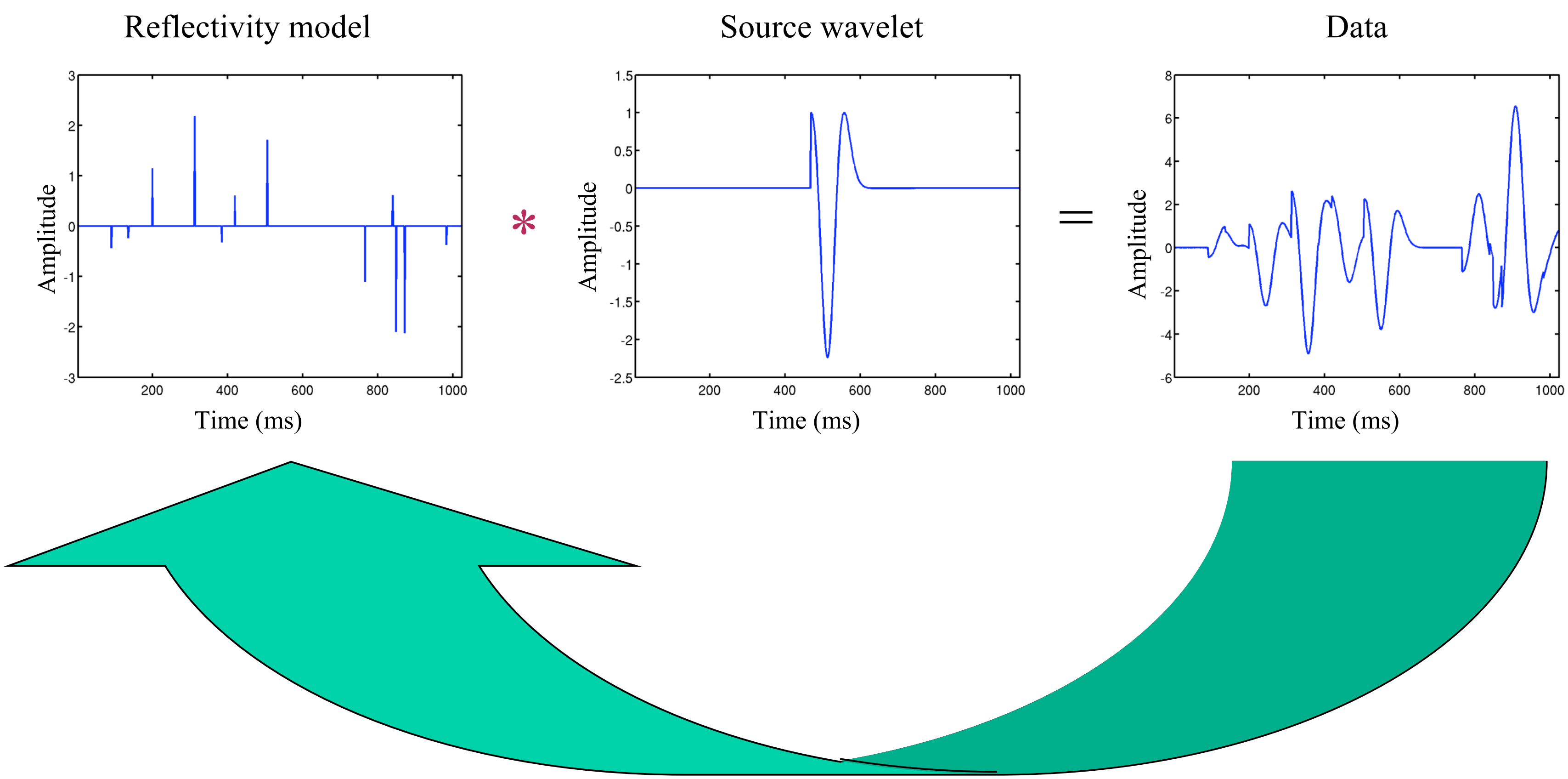
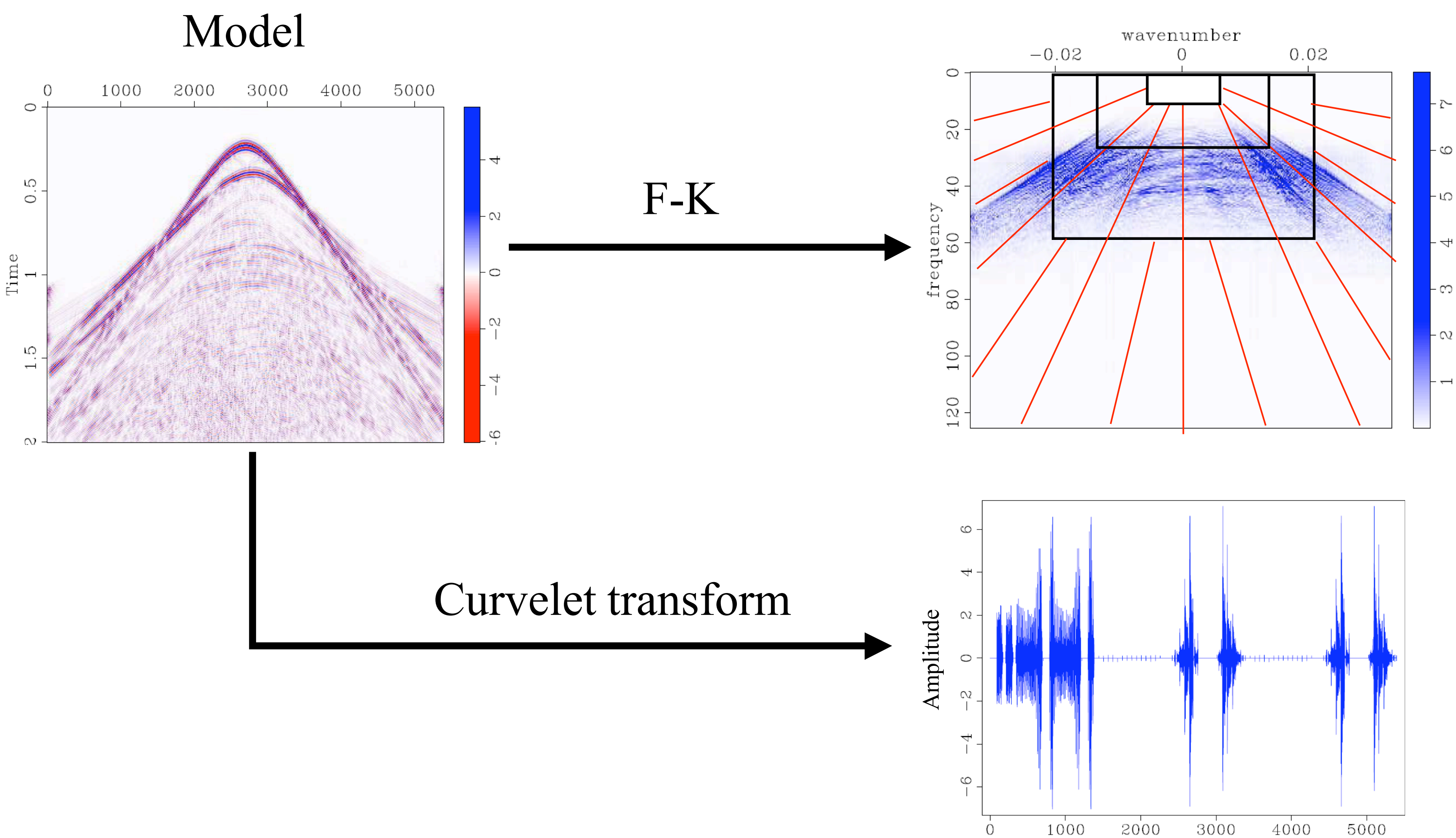
Sparsity

The idea started with sparse spike deconvolution. [Oldenburg et. al, 1981]



Sparsity


Model
Time
F-K
wavenumber
frequency
Curvelet transform
Amplitude

Sparsifying transforms

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Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

[Courtesy: Gilles Hennenfent]

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Denoising- Forward Problem

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$$
\begin{aligned}
\mathbf{y} &= \mathbf{m} + \mathbf{n} \\
\mathbf{y} &= \mathbf{C}^T\mathbf{x} + \mathbf{n} \\
\mathbf{y} &= \text{Noisy data} \\
\mathbf{m} &= \text{Noise-free data (model)} \\
\mathbf{n} &= \text{Zero-centred Gaussian Noise} \\
\mathbf{C}^T &= \text{Curvelet synthesis operator} \\
\mathbf{x} &= \text{Curvelet coefficient}
\end{aligned}
$$

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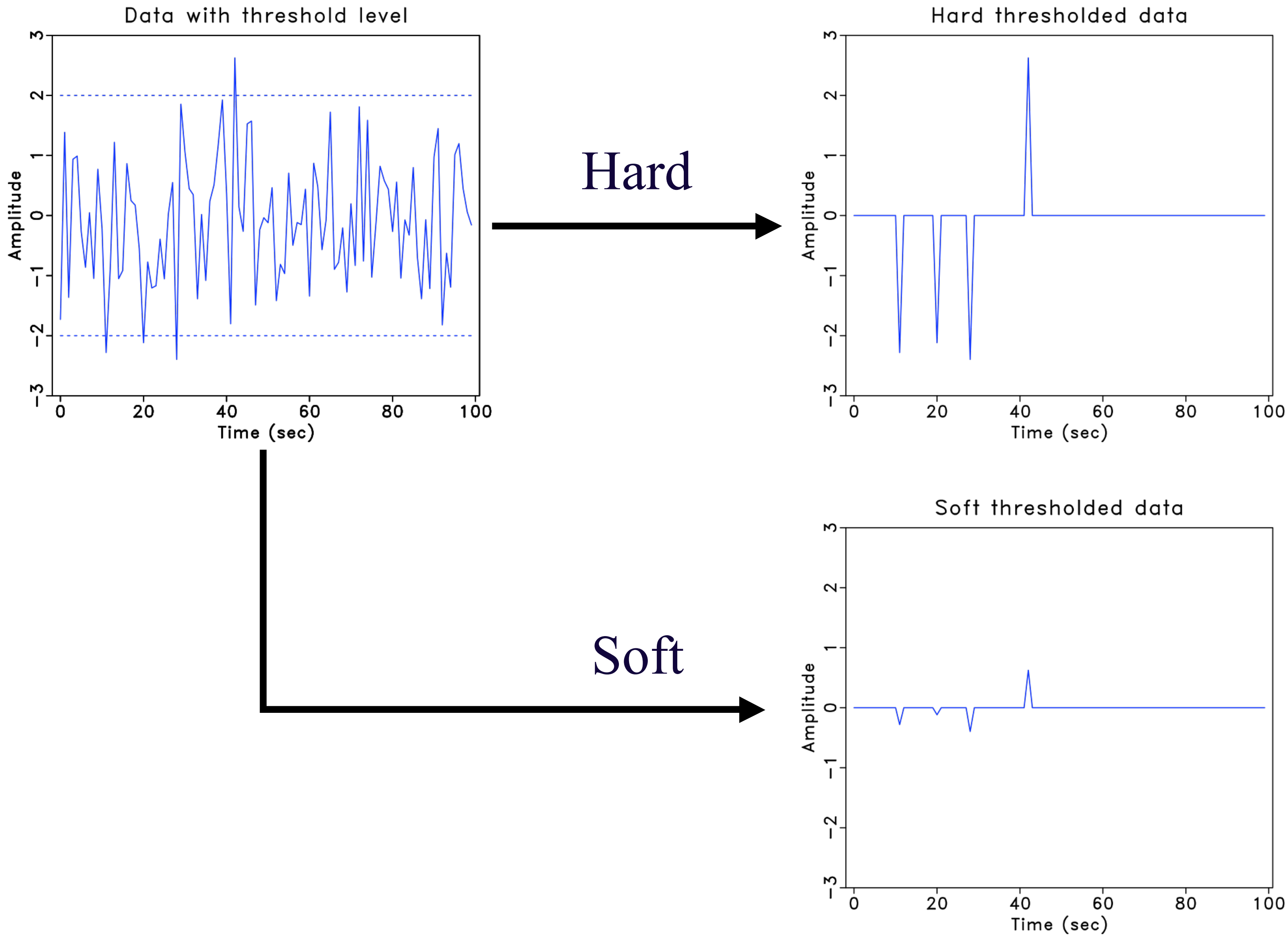
Denoising - Synthetic data

- Element-wise Hard thresholding of curvelet coefficients
  - discards any values that are less than the threshold level.
  - tested for a series of constant threshold values to choose the best result.

- Element-wise Soft thresholding of curvelet coefficients
  - discards any values that are less than the threshold level.
  - shrinks others by threshold value.
  - tested for a series of constant threshold values to choose the best result.

Hard & Soft thresholding



Denoising: One-norm

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- One-norm optimization-promote sparsity of curvelet coefficients
- solve the optimization problem:

$$\begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{s.t.} \quad ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2 \leq \varepsilon, \\ \tilde{\mathbf{m}} = \mathbf{C}^T \tilde{\mathbf{x}} \\ \mathbf{A} = \mathbf{C}^T \mathbf{R} \end{cases}$$

- **R** defines a restriction on the curvelet coefficients.
- **R** is created based on the best hard threshold result.
- Exact value of epsilon is chosen for synthetic data as we know the noise.
- Solution is found by iterative soft thresholding with cooling (ISTc). [Daubechies et al, 2005; Hennenfent et al., 2008]

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One-norm optimization

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Choose: L, K

Initialize: $k = 1$,

$\|\mathbf{A}^H\mathbf{y}\|_\infty > \lambda_1 > \cdots > \lambda_K$,

$\mathbf{x} = \mathbf{0}$

while $\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 > \epsilon$ **and** $k \leq K$ **do**

for $l = 1$ **to** L

$\mathbf{x} = S_{\lambda_k}\left(\mathbf{x} + \mathbf{A}^H(\mathbf{y} - \mathbf{A}\mathbf{x})\right)$

end for

$k = k + 1$

end while

[Adapted from Herrmann & Hennenfent, 2008]

Soft thresholding operator

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Synthetic data example

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- Our aim: NOT to harm the coherent energy.
- All three approaches are applied on synthetic data.
- Best threshold values for hard and soft thresholding are found by choosing the threshold level for maximum signal-to-noise ratio (SNR)

$$\text{SNR} = 20 * \log_{10} \frac{||\mathbf{m}||_2}{||\mathbf{m} - \widetilde{\mathbf{m}}||_2}$$

- All results are compared based on SNR and difference image.
- Both soft thresholding and One-norm introduce a bias.
- We limit our iterations for One-norm to avoid bias.

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Synthetic Data & Model

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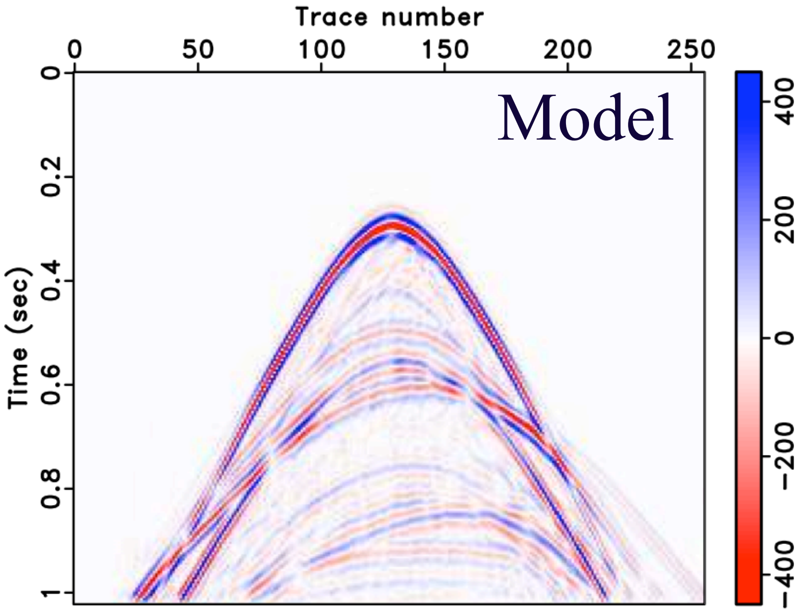
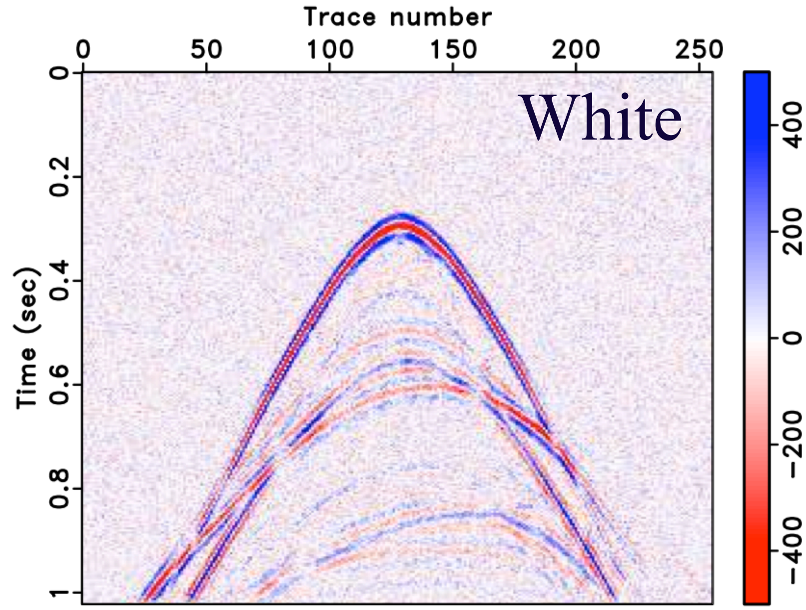
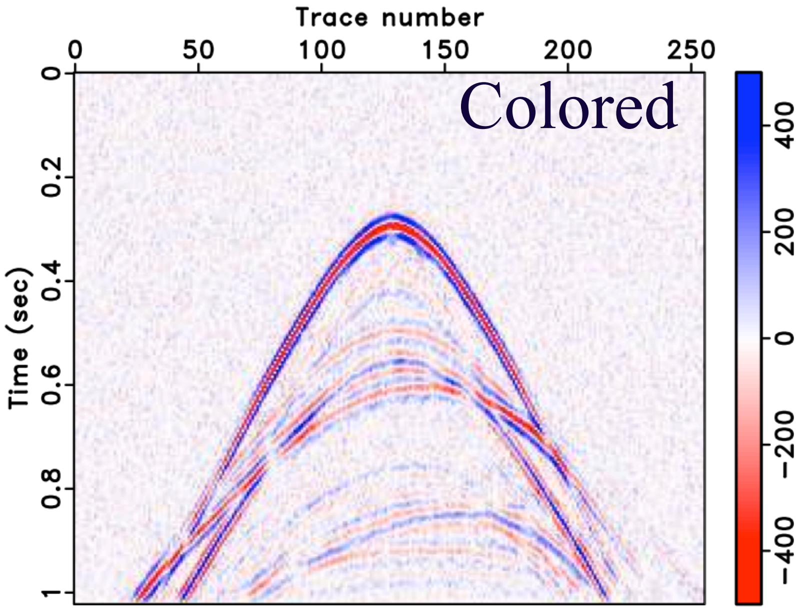




Colored noise is obtained by bandpass filter (5-60 Hz) on the white noise.



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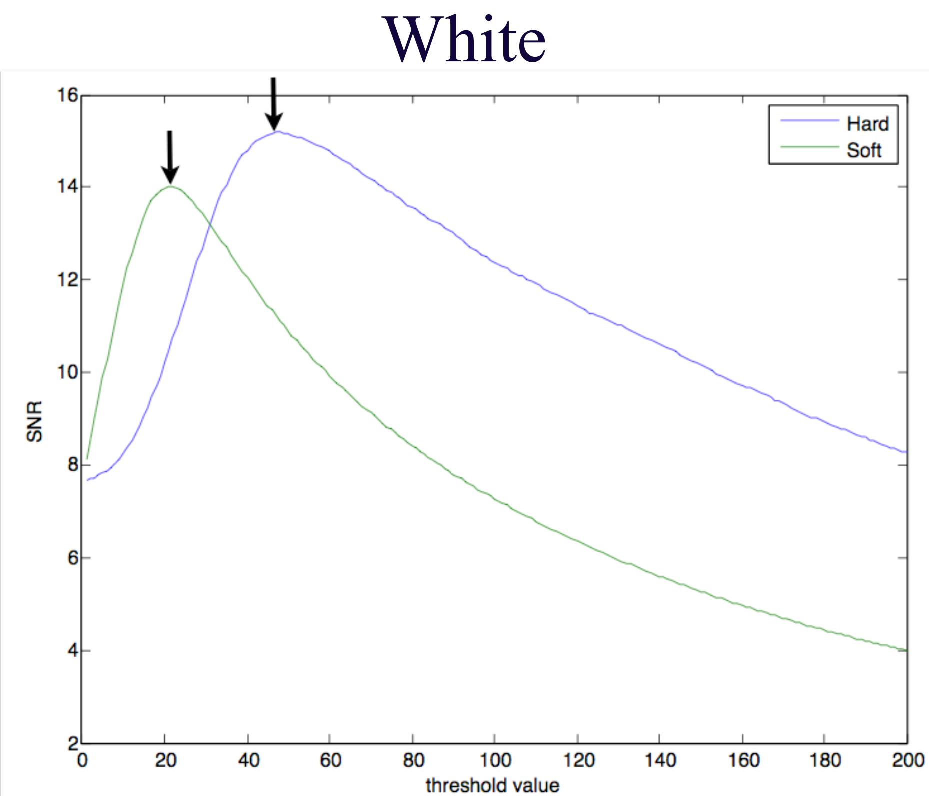
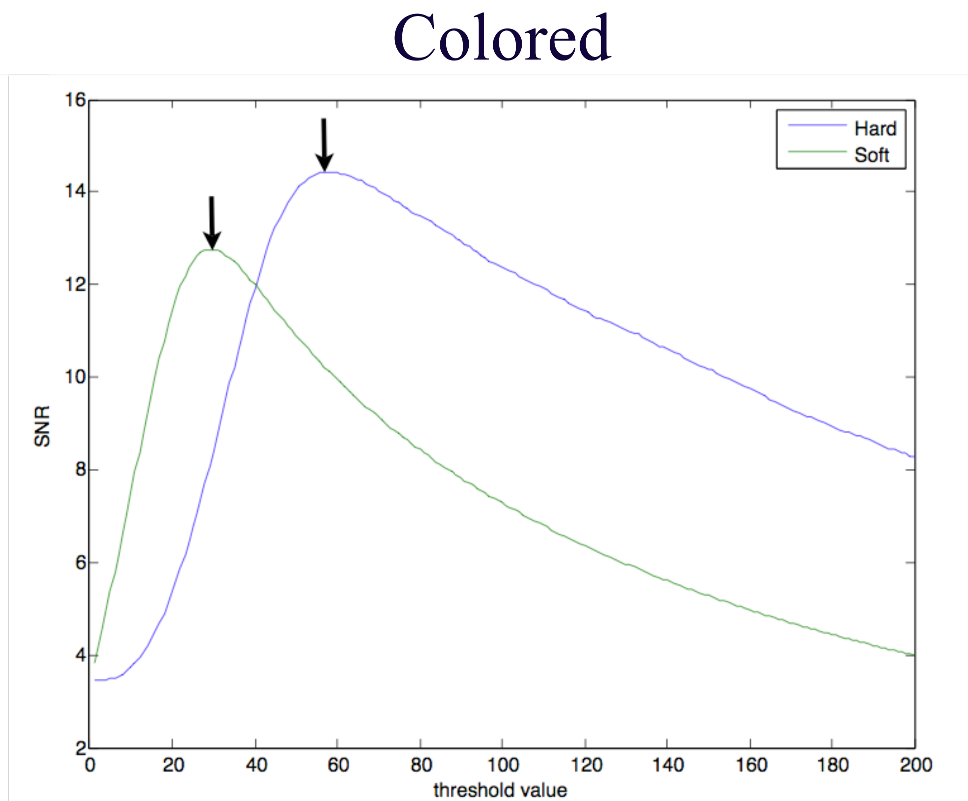
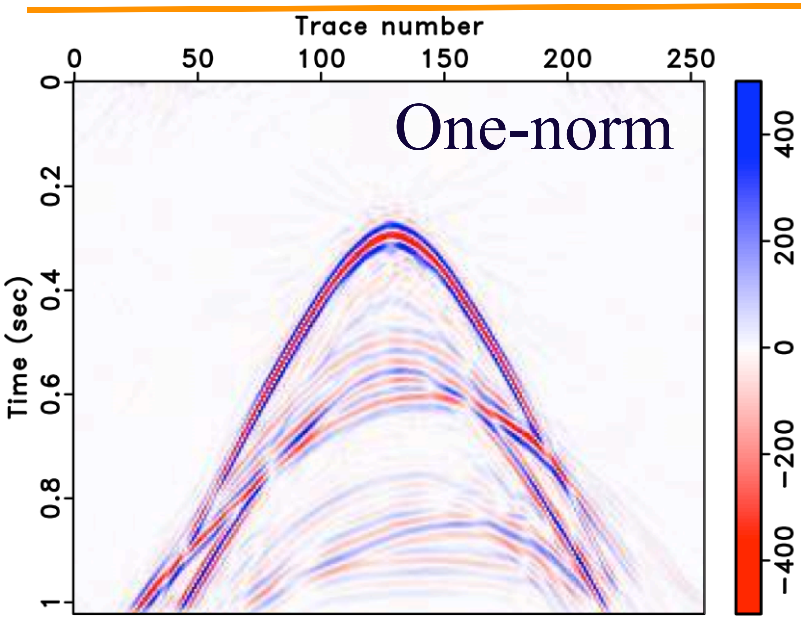
SNR vs threshold

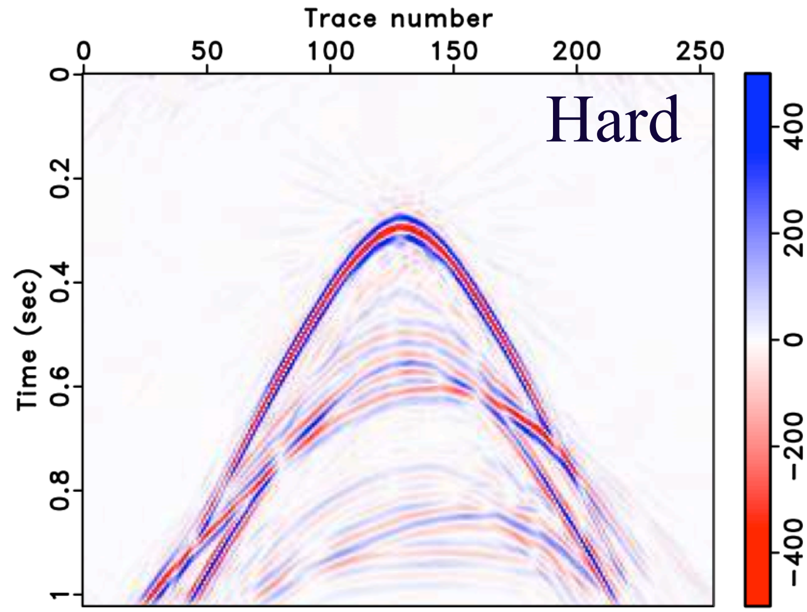


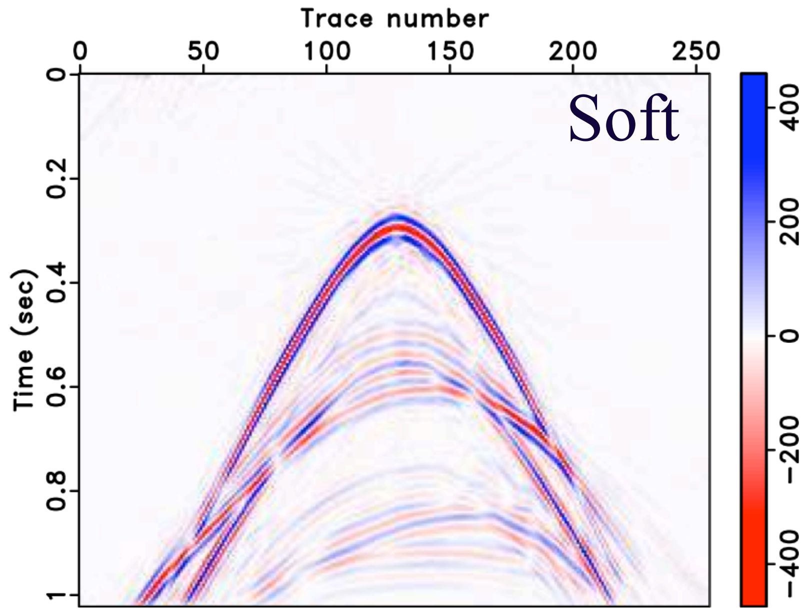


Arrows refer to the chosen threshold for maximum SNR.

Denoising -white noise







Difference - white noise

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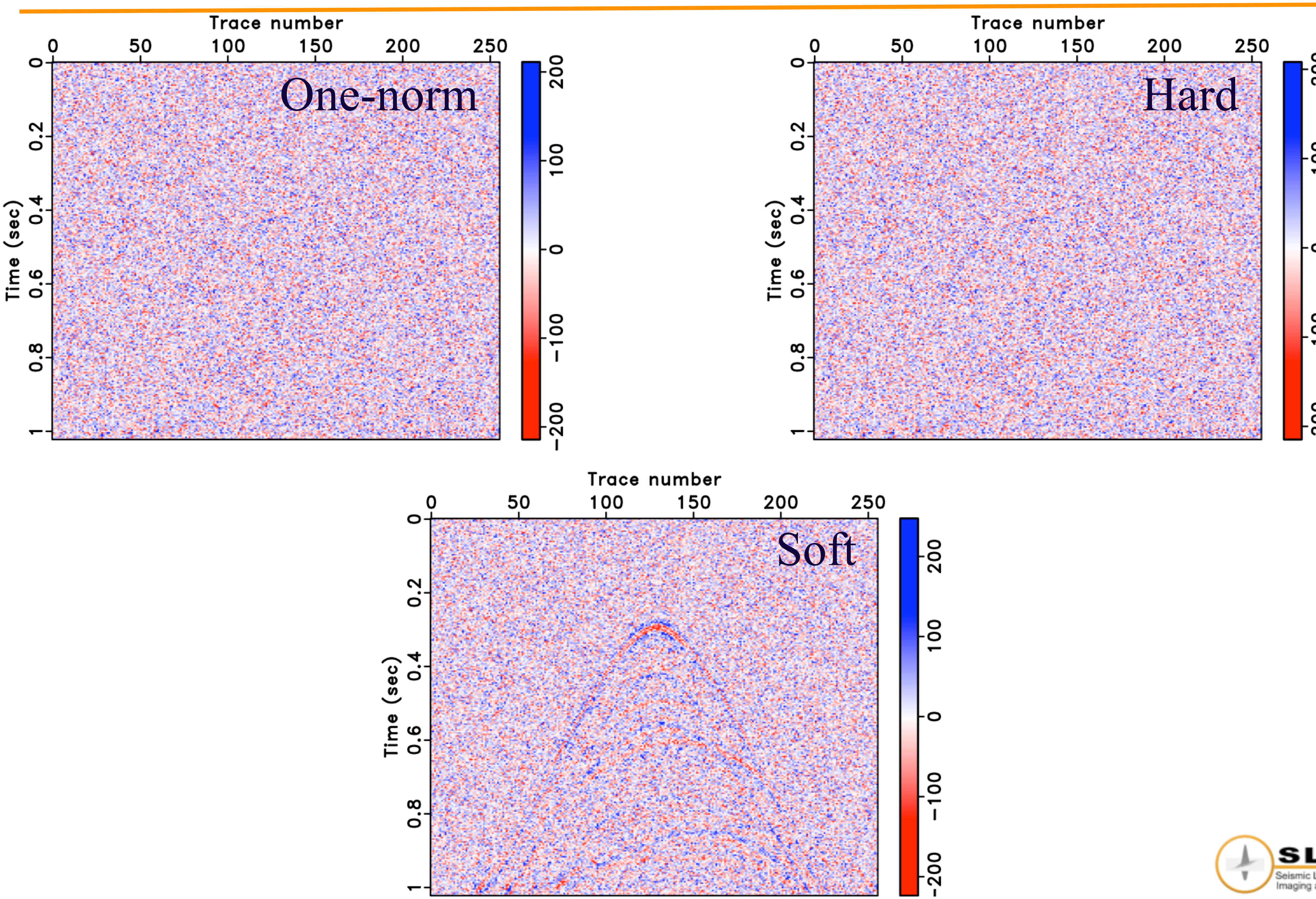

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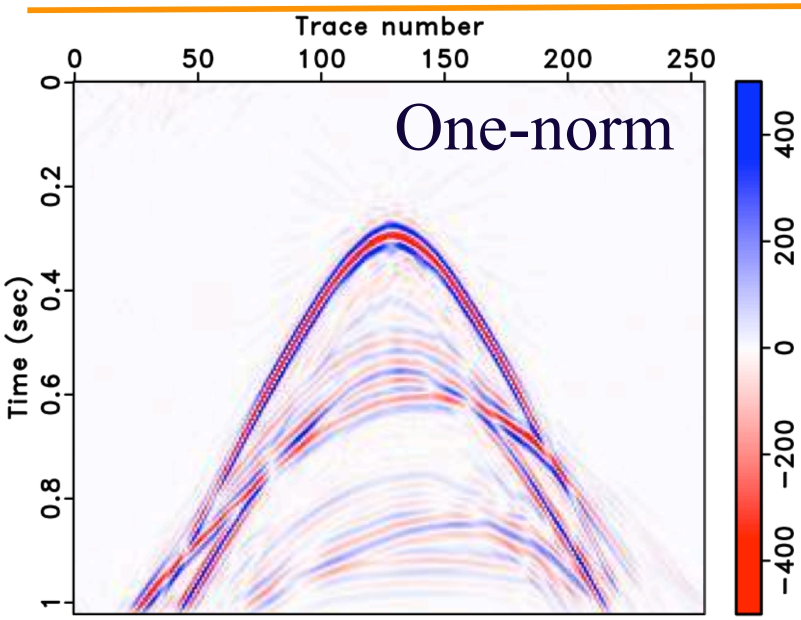
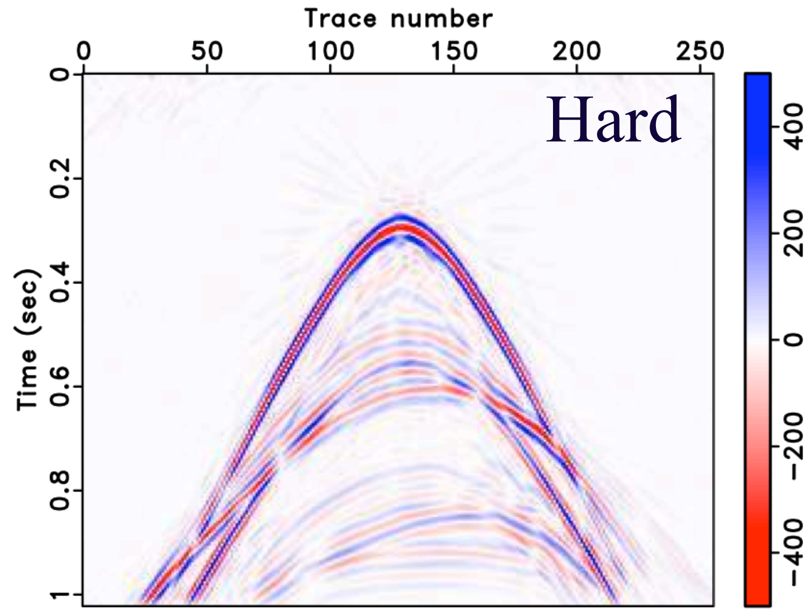
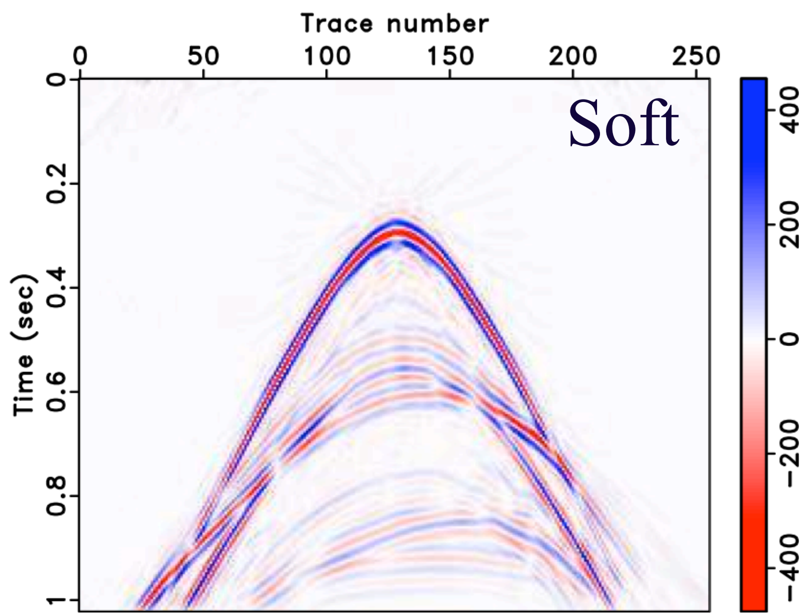
Denoising - colored noise







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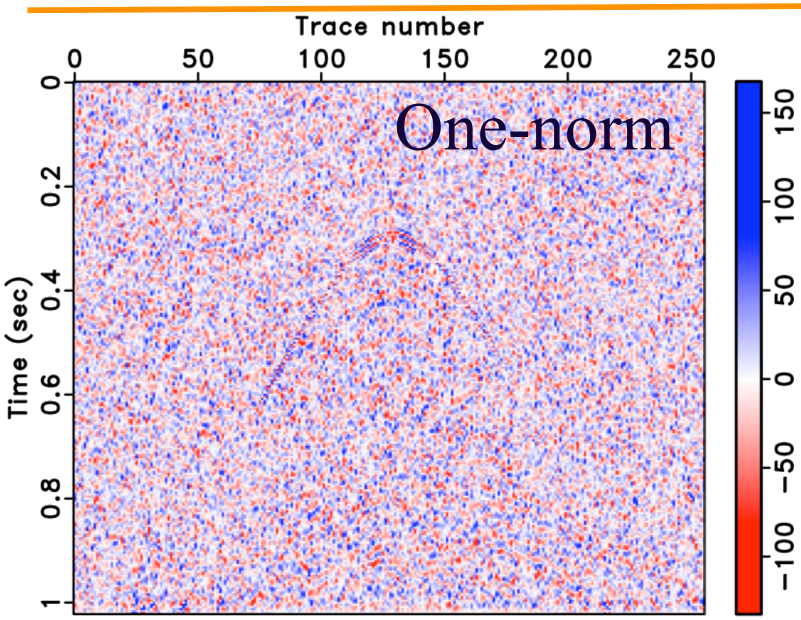
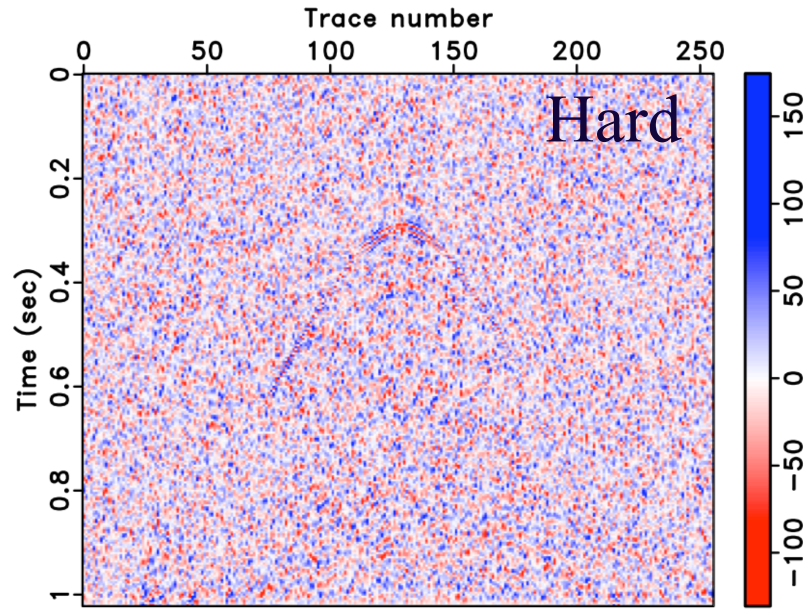
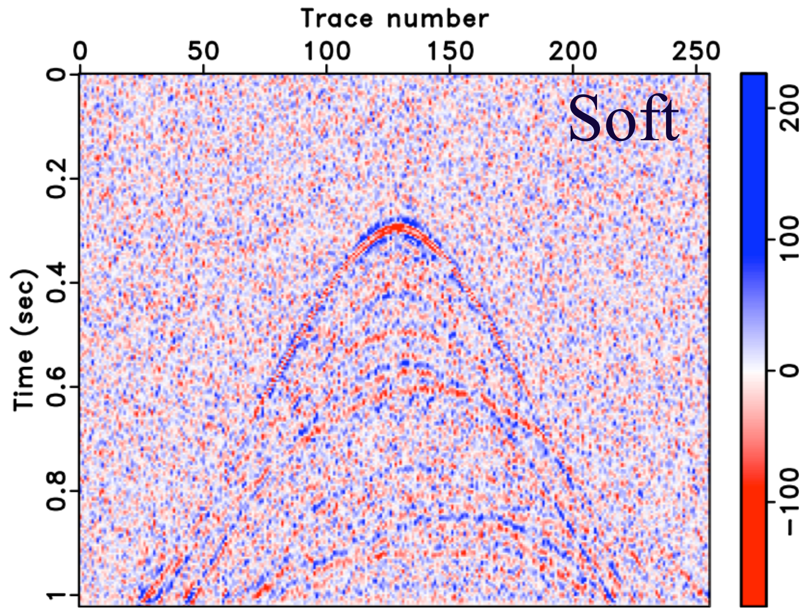
Difference - colored noise







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Summary- SNR

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Noise	Data	One-norm	Hard	Soft
White	3.44	14.69	14.44	12.77
Color	7.67	15.44	15.20	14.01

One-norm approach is chosen for real data application.

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Denoising - Real data

- Our aim: NOT to harm the coherent energy.
- Parameter selection is always a challenge for real data.
- Restriction operator (R) is created based on the best hard threshold result (maximum threshold level such that there is NO harm to coherent energy) .
- The value of epsilon is kept as the L_2-norm of the noise removed by best hard threshold.
- The result is compared with those obtained from standard F-X prediction.
  - Sliding window prediction-error-filtering in the F-X domain.
  - Dimensions of window are fixed.
  - Replaces the sample with the predicted value.

Real Data: SNORCLE Line 1

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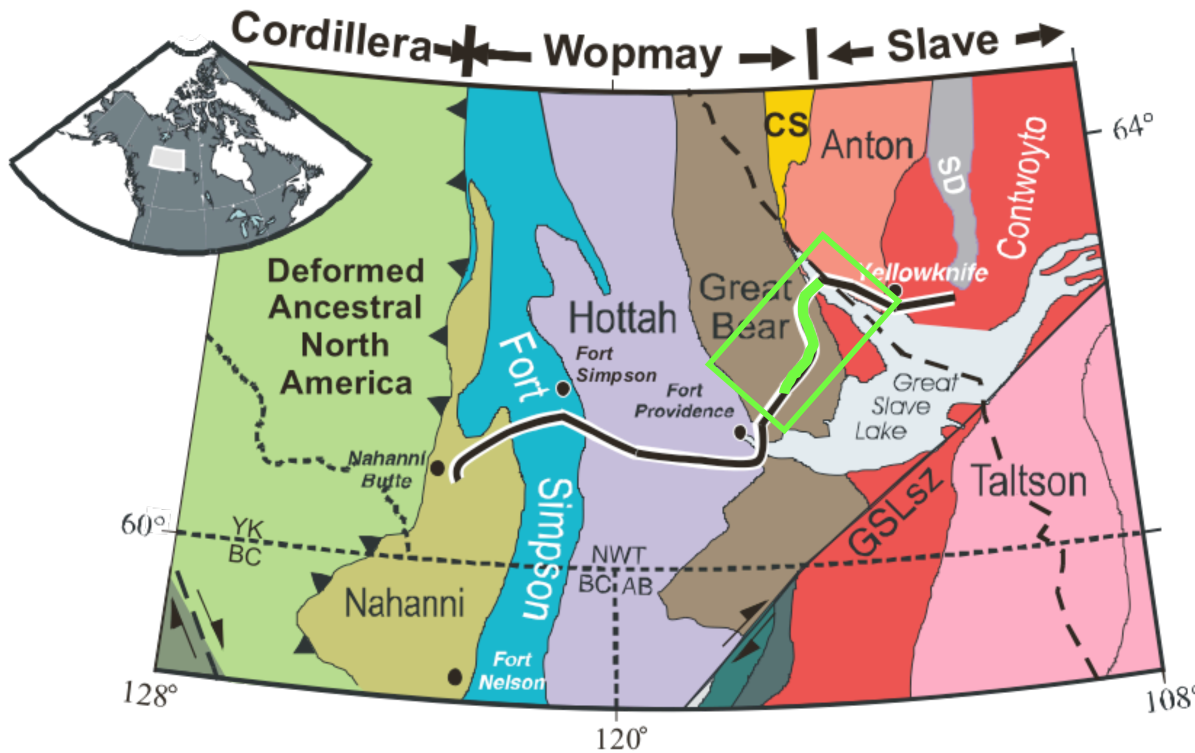
Parameter	Value
Number of vibrators	4 or 5
Number of sweeps	4 or 5
Sweep frequencies	10 - 80 Hz
Sample rate	4 ms
Sweep length	20 s
Record length	32 s
Number of channels	404
Receiver spacing	60 m
Number of geophones	9 per station
Vibrator point spacing	90 m
Geophone frequency	10 Hz
Nominal fold	134
Instruments	Sercel 388



[Cook et al., 1999]

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Processing Flowchart

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Crooked line geometry

↓

Static correction

↓

Deconvolution

↓

NMO

↓

Stack

↙ ↘

F-X Prediction | **Curvelet Denoising**

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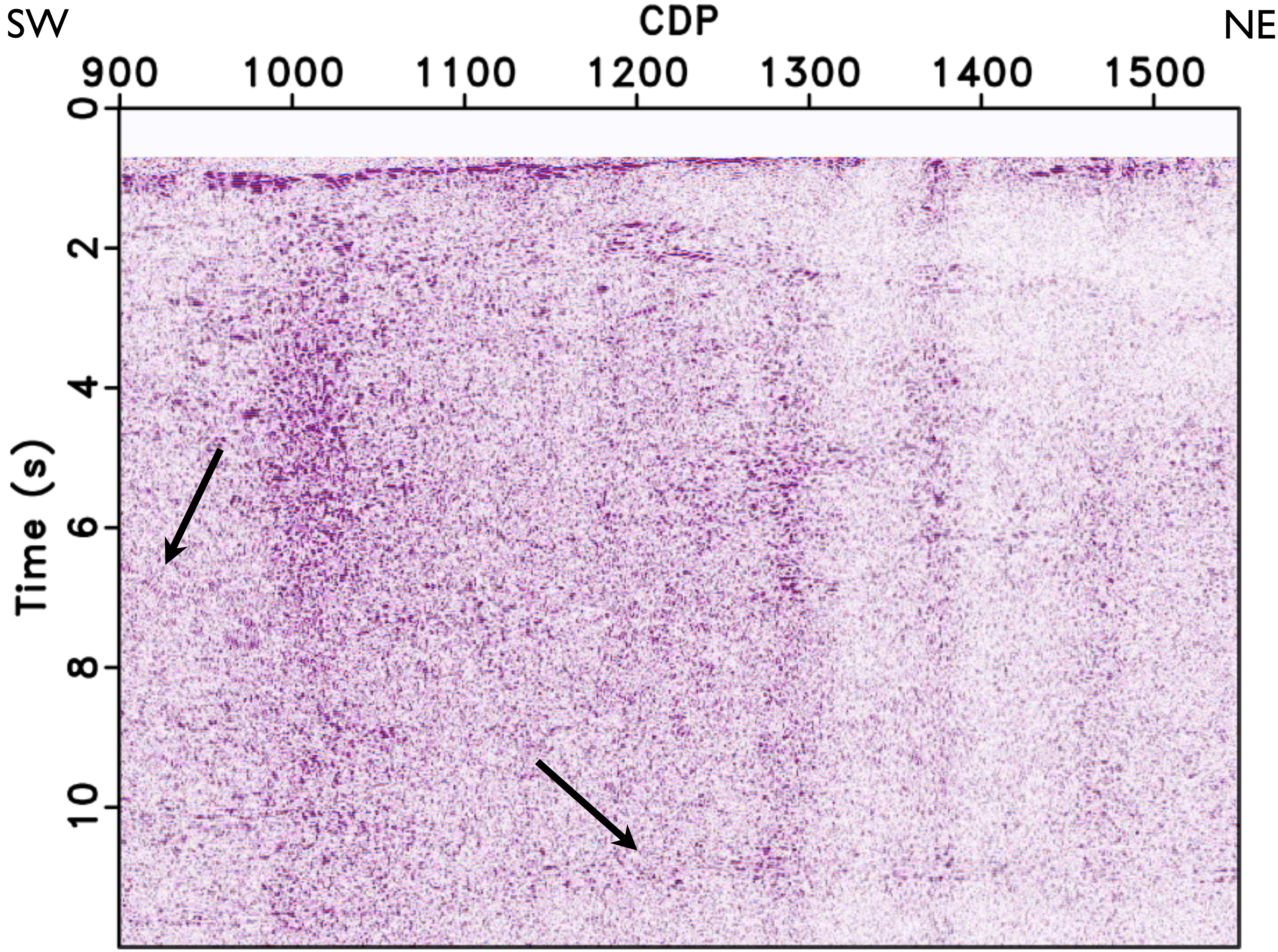
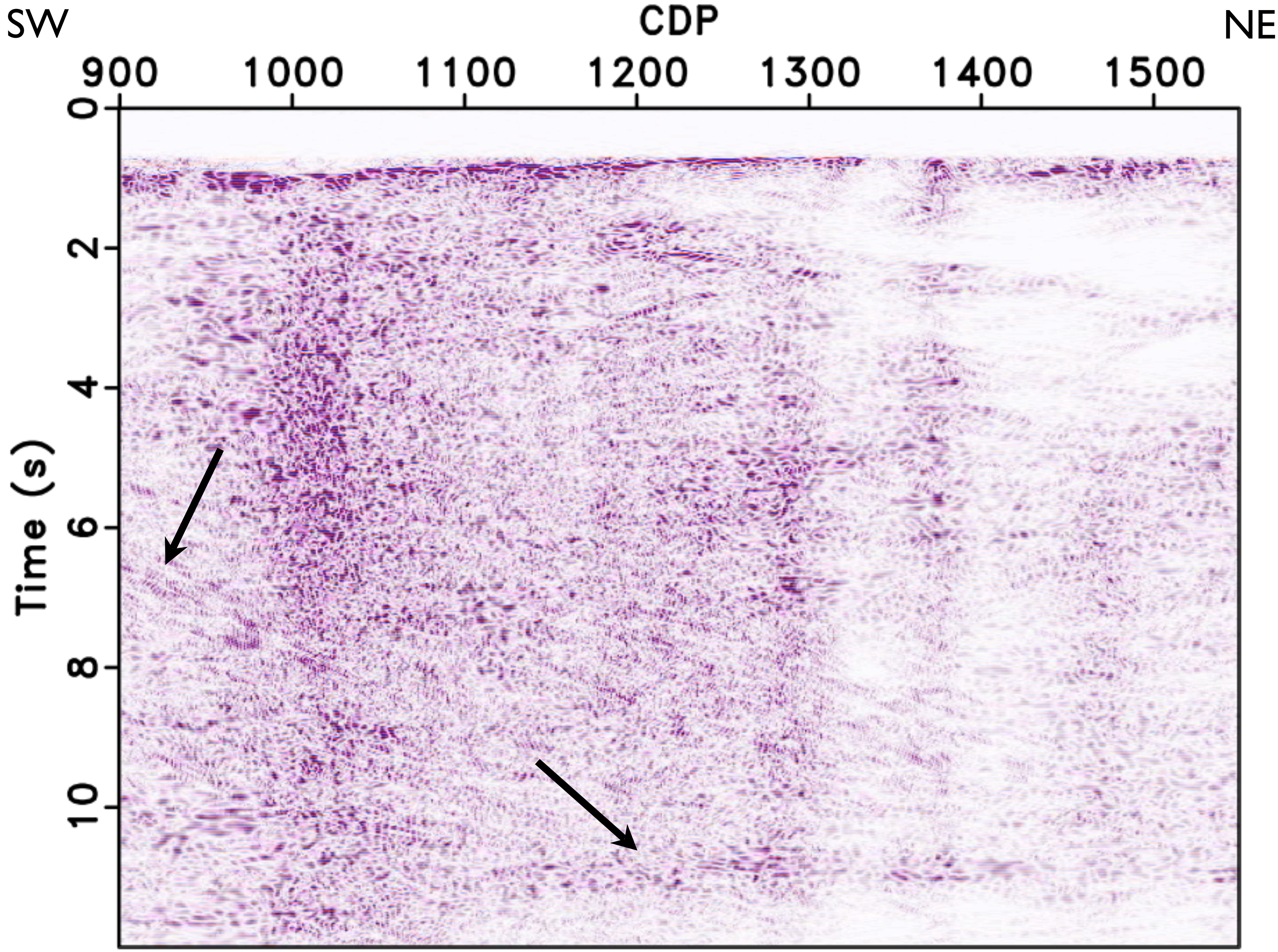
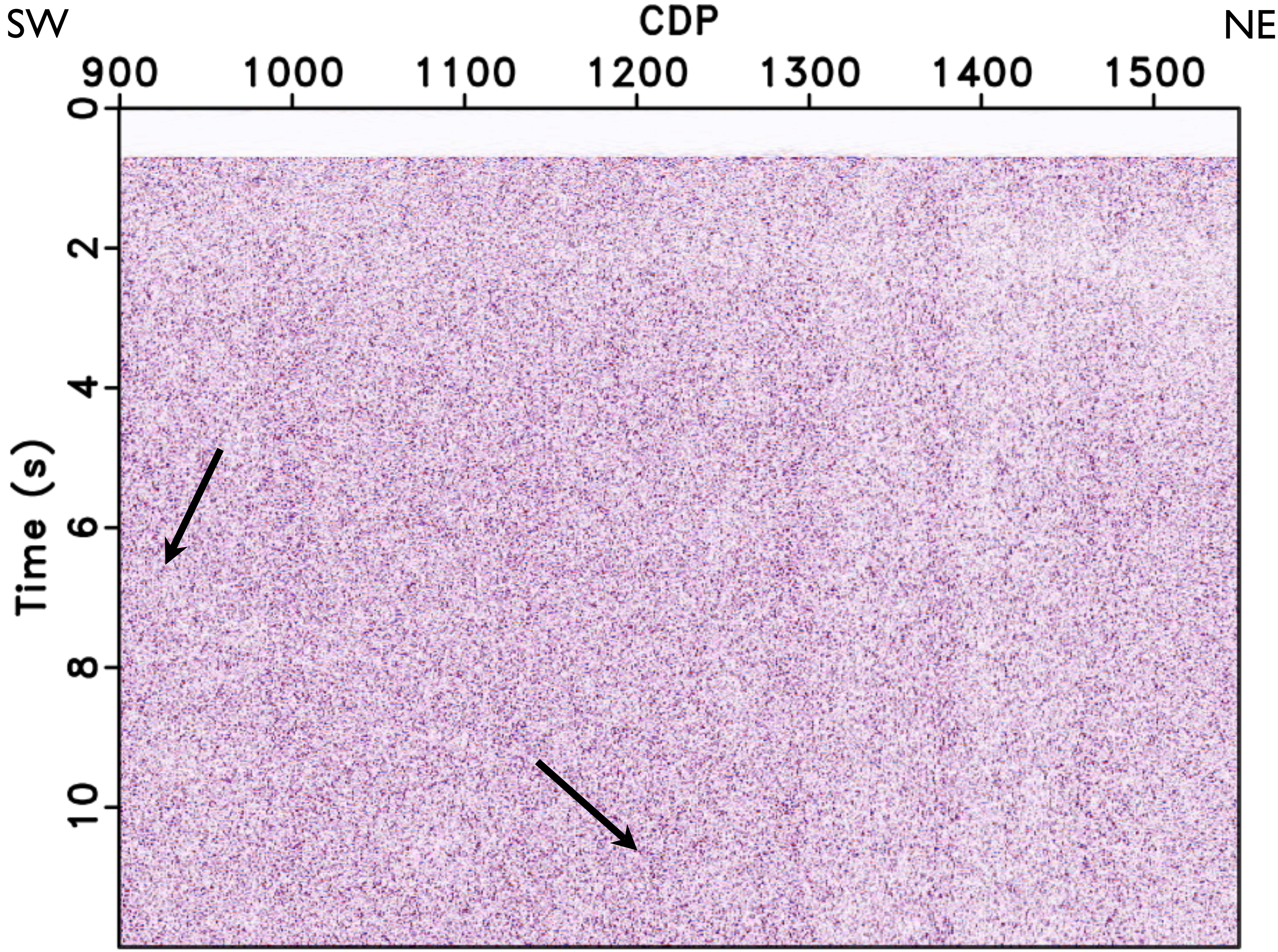


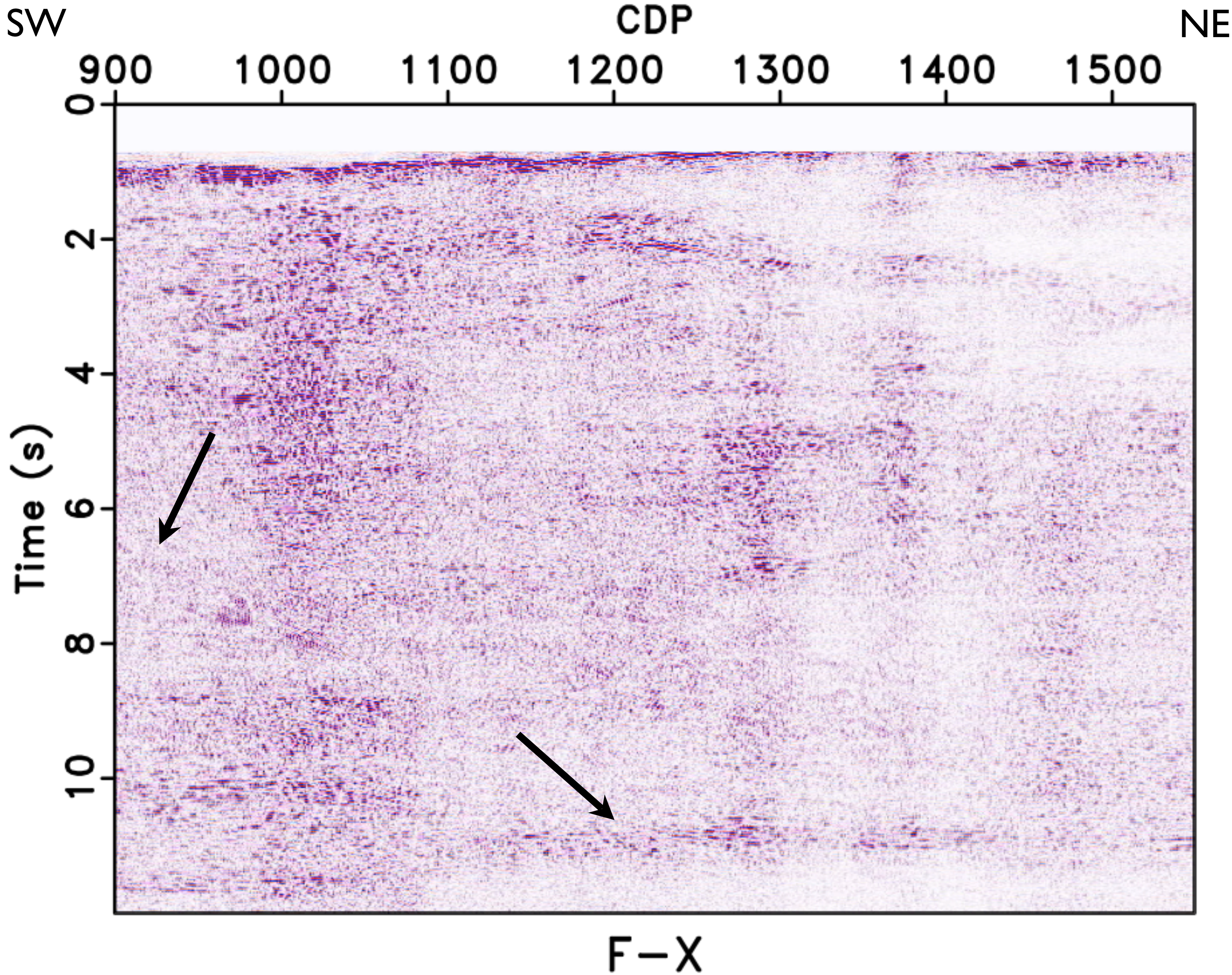
Stacked section with conventional processing

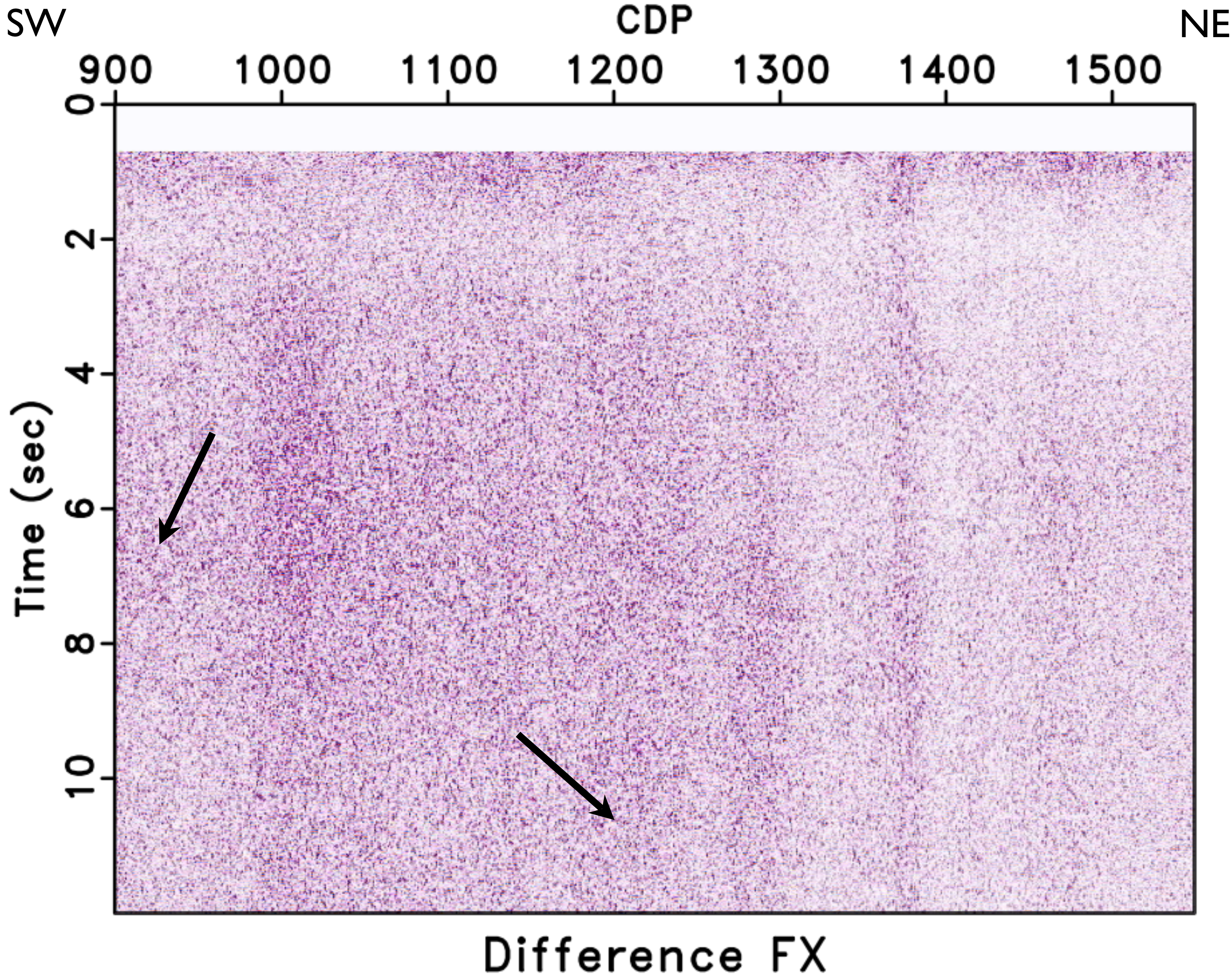


One-norm

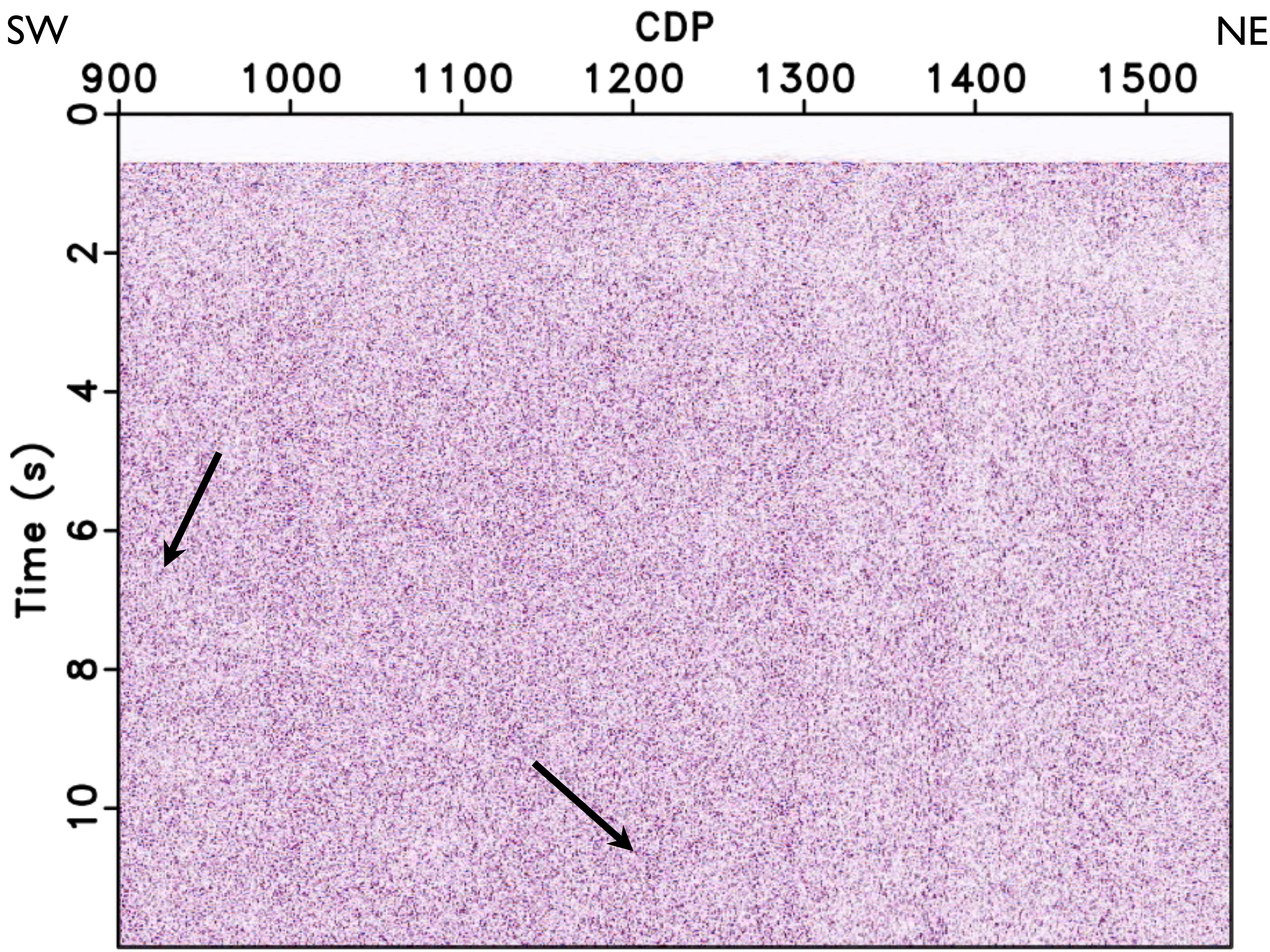


One-norm Difference


SW
CDP
NE
900
1000
1100
1200
1300
1400
1500
Time (s)
0
2
4
6
8
10
F-X



SW
CDP
NE
900
1000
1100
1200
1300
1400
1500
0
2
4
6
8
10
Time (sec)
Difference FX



One-norm Difference

Conclusions

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- The results produced by denoising shows improvement in the deeper area as pointed by arrows.

- Curvelet-based denoising is new addition to the tool-kit for seismic data processors, which could be advantageous for specific situations.

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Challenges & Future work

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- Curvelets are computationally expensive as they are *"redundant"* in nature (8 times in 2D and 24 times in 3D), hence require large memory storage and extra run time. Thus a parallel version is favorable.

- These methods can be extended to three dimensions to estimate *"3D model"* which has more spatial structure.

- Curvelets could be used to do velocity(dip) and frequency specific filtering.

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Acknowledgments

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- My supervisors: Felix Herrmann & Ron Clowes
- Jounada Oueity & SLIM team members for their help and support
- E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying for CurveLab (www.curvelet.org)
- S. Fomel, P. Sava, and the other developers of Madagascar (rsf.sourceforge.net)

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE (334810-05) of Felix J. Herrmann.

This research was carried out as part of the SINBAD II project with support from the following organizations: BG Group, BP, Petrobras, and Schlumberger.

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And finally

Thank you !!!

(Merci Beaucoup !!!)