Sub-Nyquist sampling and sparsity: getting more information from fewer samples

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SUMMARY

Seismic exploration relies on the collection of massive data volumes that are subsequently mined for information during seismic processing. While this approach has been extremely successful in the past, the current trend of incessantly pushing for higher quality images in increasingly complicated regions of the Earth continues to reveal fundamental shortcomings in our workflows to handle massive high-dimensional data volumes. Two causes can be identified as the main culprits responsible for this barrier. First, there is the so-called "curse of dimensionality" exemplified by Nyquist's sampling criterion, which puts disproportionate strain on current acquisition and processing systems as the size and desired resolution of our survey areas continues to increase. Secondly, there is the recent "departure from Moore's law" that forces us to lower our expectations to compute ourselves out of this curse of dimensionality. In this paper, we offer a way out of this situation by a deliberate *randomized* subsampling combined with structure-exploiting transform-domain sparsity promotion. Our approach is successful because it reduces the size of seismic data volumes without loss of information. Because of this size reduction both impediments are removed and we end up with a new technology where the costs of acquisition and processing are no longer dictated by the *size of the acquisition* but by the transform-domain *sparsity* of the end-product after processing.

BASIC METHODOLOGY IN A NUTSHELL

Coherent subsampling-related interferences are the main enemies of successful seismic acquisition and processing. These interferences can come from any number of places: traditionally they are caused by badly designed regular sub-Nyquist samplings of sources and receivers, and more recently by badly designed simultaneous sources. Conversely, well-designed *randomized* subsamplings—through jittered sampling (Hennenfent and Herrmann, 2008) of the source-receiver positions or through randomly phase encoded source signatures (Berkhout, 2008; Neelamani et al., 2008; Herrmann et al., 2009)—lead to manageable subsampling artifacts that manifest themselves as incoherent noise with a level that depends on the degree of subsampling; the more subsampled you are the higher your expected noise level becomes. Herein lies an unique opportunity: as long as we are able to separate subsampling noise from desired signal, we are in the position to remove the impediments of the "curse of dimensionality" and the apparent "departure from Moore's law". This is where transform-domain sparsity enters into the equation, because the sparser we can represent our desired signal—i.e., the more of the signal's energy we can store into the fewer largest transform-domain coefficients—the better we can separate this incoherent subsampling noise from signal. This task of recovering fully-sampled signals from deliberate subsampling is actually far less daunting as it may seem; we all know that seismic data contains structure that can be exploited with certain multiscale and multidirectional transforms such as curvelets (Candès et al., 2006). These transforms—possibly in conjunction with *focusing* procedures that map multiple energy onto primaries (see e.g. Herrmann and Wang (2008); van Groenestijn and Verschuur (2009) and our other contribution to the proceedings of this conference) or that collapse primary energy onto reflectors during imaging (see our other contribution to the proceedings of this conference)—translate this structure into transform domain *sparsity*. We exploit this sparsity with our sparsity-promoting recovery techniques. **Bottom line:** *As long as our object of interest permits a sparse enough representation, we are able to separate incoherent subsampling noise from our signal and therefore recover our signal from fewer samples than dictated by the Nyquist limit.*

Mathematically, our recovery from deliberate subsampling with sparsity promotion can be formulated as the inversion of a flat matrix $\mathbf{A}$ via

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 := \sum_i |x_i| \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{b}, \tag{1}$$

with

$$\mathbf{A} = \overbrace{\mathbf{R}}^{\text{randomized restriction}} \underbrace{\mathbf{M}}_{\text{randomized physics}} \overbrace{\mathbf{S}^*}^{\text{inv. sparse transform}}. \tag{2}$$

The above optimization procedure seeks amongst all possible transform-domain vectors $\mathbf{x}$ that which has the smallest one norm—i.e., it seeks the sparsest vector using a heuristically derived one-norm measure. The true miracle in this nonlinear recovery lies in the fact that the above optimization problem is able to recover the original coefficient vector—and hence the original data—with high accuracy from *randomized* subsampled data in $\mathbf{b}$. Here, the estimate for the recovered data is given by $\widetilde{\mathbf{d}} = \mathbf{S}^*\widetilde{\mathbf{x}}$ with $\widetilde{\mathbf{x}}$ the vector that solves the above optimization program. The accuracy of the recovery depends on the following design principles:

- the length of the measurement vector $\mathbf{b}$ = hight of the *randomized* restriction matrix $\mathbf{R}$, which randomly removes rows. The longer $\mathbf{b}$ the better the recovery.
- properties of the measurement matrix $\mathbf{M}$ that preferably represents *randomized* physics underlying the measurements. The more *randomized* this matrix—i.e., the more its action on $\mathbf{S}^*$ resembles a matrix with random Gaussian noise—the better the recovery.
- the *sparsity* attained by the transformed domain spanned by $\mathbf{S}$. The sparser this domain the better the recovery.

These design principles find their origin and theoretical justification in a new field of mathematics known as *compressive sampling*(Donoho, 2006; Candès et al., 2006), where sparse signals are recovered from *randomized* samples using sparsity-promoting programs. In the next two sections, we illustrate the above examples by two concrete examples that underline the importance of following the above design principles.

EXAMPLE I: RECOVERY OF FULLY SAMPLED DATA FROM JITTERED SAMPLING AND SIMULTANEOUS ACQUISITION

To illustrate the importance of selecting the appropriate *randomized* measurement matrix **M**, we compare recovery from three incomplete (50 % of shots missing) data sets that are equal in size but that differ in acquisition strategy, mainly according to two different scenarios. In the first scenario, we recover from *deterministic* impulsive shots chosen either at *regular* or at *random* (jittered) source positions. In the second scenario, we also fire 50 % of the time but now with incoherent *randomized* sweeps that go off simultaneously at *all* source positions. Results yielded by sparsity-promoting recovery from data collected according to both scenarios are summarized in Fig. 1. Comparison of the recovery from regular, jittered, and simultaneous shots shows a drastic improvement in the recovery quality as we move from regular subsampled, to *randomized* jittered source locations all the way to *randomized* simultaneous sources. These findings clearly underline the importance of *randomization* in the collection of seismic data. This example also nicely illustrates that *randomization* of the source locations by itself is not optimal and that a lot is to be gained by designing *randomized* incoherent simultaneous-source acquisitions such as acquisitions using phase-encoded sweeps.

EXAMPLE II: ESTIMATION OF PRIMARIES FROM SIMULTANEOUS DATA

The above scheme of recovery from *randomized* data can even be carried a step further by including more information on the physics, i.e., *focusing* in the matrix **M**. For instance, if we include in this matrix—aside from the randomization of the sources—an operator that generates surface related multiples, our inversion procedure will map surface-related multiples to "primaries" (that include internal multiples). This approach has two advantages. First, "primaries" are sparser then multiples. Second, multiples are mapped to primaries and thereby facilitate the decoding by sparsity promoting. To illustrate how this works we consider the following two scenarios. First, we recover the total data, including the surface-related multiples, from simultaneous acquired data followed by a prediction of the primaries. Second, we estimate the primaries directly from the simultaneously collected data. As we can see from Fig. 2, the recovery according to the second scenario is far superior because we incorporated more physics into the formulation of our problem (see our other contribution to the proceedings of this conference).

DISCUSSION

The above examples illustrate that we are at the cusp of very exciting developments where our design principles for acquisition and processing no longer need to be dominated by our fear of creating coherent subsampling related artifacts. Instead, we arrive at a formulation where we have control over these artifacts—by turning harmful coherent interferences into harmless incoherent noise. In this way, we facilitate the removal of subsampling related artifacts by our sparsity-promoting inversion procedure. This opens enticing new perspectives towards a new formulation of seismic data acquisition and processing. To summarize, the success of this new formulation depends on three key design principles, namely

1. *randomize*—break coherent aliases by introducing randomness, e.g. by designing randomly perturbed acquisition grids, or by designing randomized simultaneous sources and blended receivers.
2. *sparsify*—utilize sparsifying transforms in conjunction with sparsity promoting programs to remove incoherent subsampling artifacts, e.g. by exploiting curvelet-domain sparsity.
3. *focus*–leverage physical focusing principles that concentrate seismic energy in order to further promote sparsity in the final solution, e.g. by turning multiples into primaries or primaries into images.

EXTENSIONS

The implications of *randomized incoherent sampling* go far beyond the examples presented in this paper. For instance, our approach is applicable to land acquisition for physically realizable sources (Krohn and Neelamani, 2008; Romberg, 2008) and can be used to faster compute solutions to the wave equation (Herrmann et al., 2009) or to compute image volumes with smaller memory imprint. Because randomized sampling is *linear* (Bobin et al., 2008), our simulation method is incremental—i.e., adding more samples improves recovery. This linearity allows us to do compressive processing (e.g. estimation of primaries) or compressive computations on compressively-sampled data, an observation made independently by Berkhout (2008).

CONCLUSIONS

We have made the case that information can be obtained from *randomized* subsamplings. This allows us to formulate rigorous and cost-effective acquisition and processing schemes based on the principles of *compressive sensing*. According to these principles, data can be reconstructed from *randomized* subsamplings commensurate with their *complexity*. We verified this behavior experimentally and this, in conjunction with the intrinsic linearity of the *randomized* sampling, opens a number of enticing new perspectives because acquisition and processing costs are decoupled from the acquisition area and grid size. Instead, these costs depend on sparsity. Because of this linearity, we envision a seamless incorporation of this new *paradigm* into seismic exploration.

ACKNOWLEDGMENTS

FJH would like to Eric Verschuur and Tim Lin for their input. FJH was in part financially supported by the NSERC Discovery Grant (22R81254) and CRD Grant DNOISE (334810-05). The industrial sponsors of the Seismic Laboratory for Imaging and Modelling (SLIM) are also gratefully acknowledged.

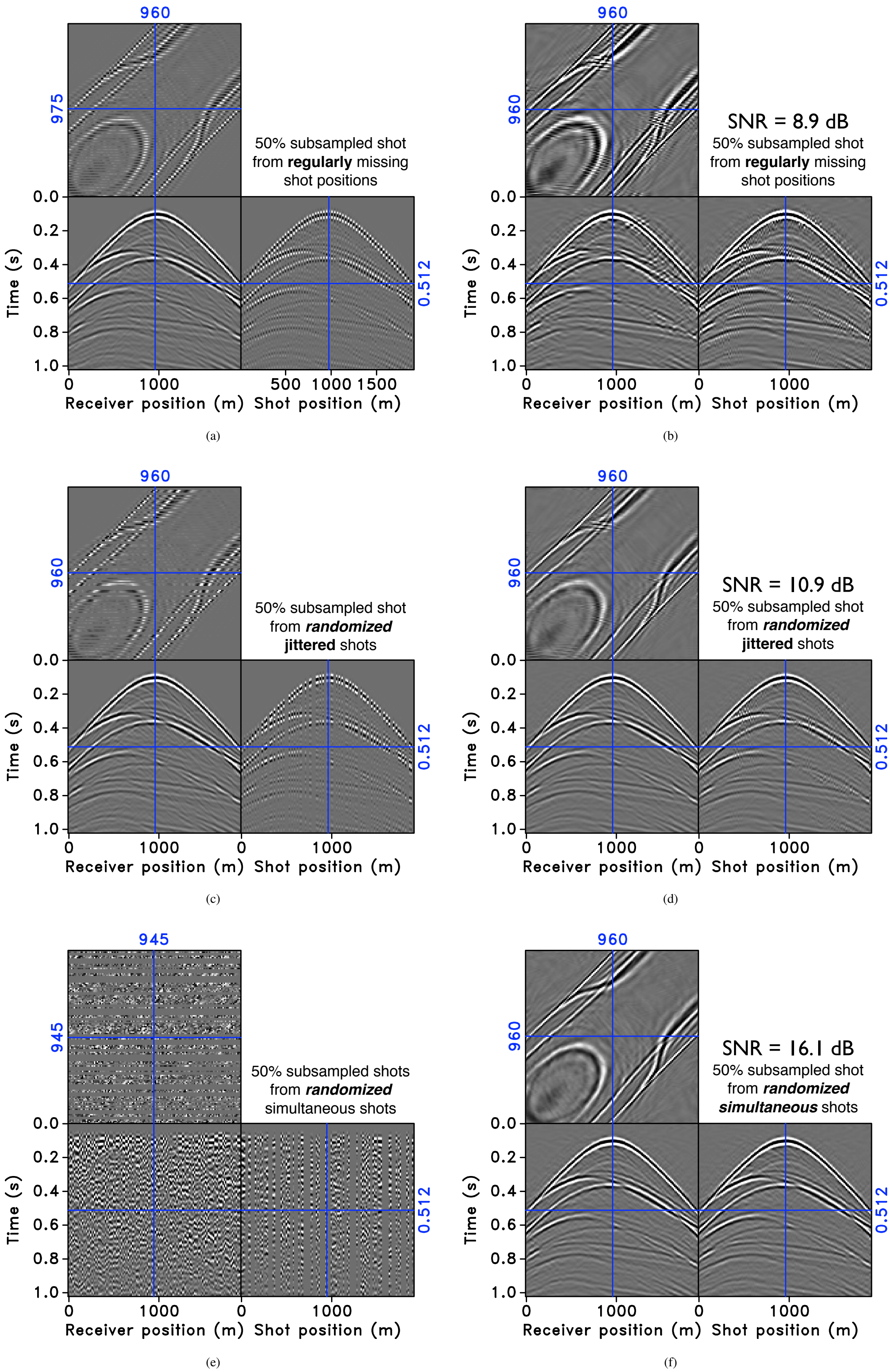

Figure 1: Sparsity-promoting recovery from 50 % of the shots missing. **(a)** Regularly subsampled shots. **(b)** Recovery from regularly subsampled shots. **(c)** Jittered subsampled shots **(d)** Recovery from jittered subsampled shots. **(e)** Subsampled *randomized* simultaneous shots. **(f)** Recovery from *randomized* simultaneous shots. Notice the remarkable improvement in recovery from the simultaneously acquired data.

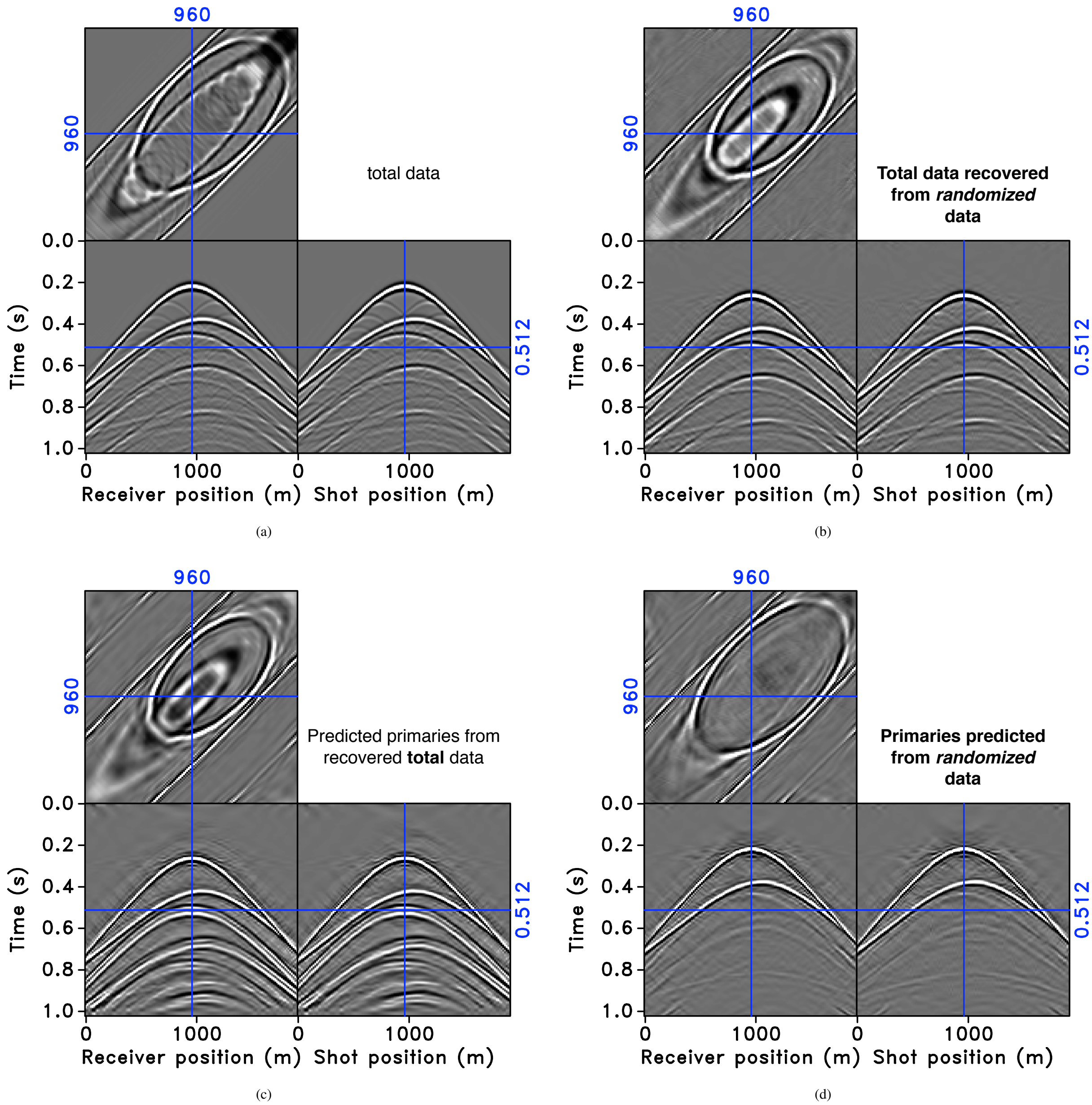

Figure 2: Sparsity-promoting recovery from simultaneously sampled data with 50 % of the shots missing. **(a)** Original data. **(b)** Estimation of the total data by sparsity promotion. **(c)** Estimation of primaries from recovered total data under **(b)**. **(d)** Estimation of primaries directly from the simultaneously acquired data. Notice the remarkable improvement in the estimation of the primaries *directly* from the simultaneously acquired data.

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